A Soft Curtailment of Wide-area Central Air Conditioning Load

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Abstract: A real-time two-way direct load control (TWDLC) of central air-conditioning chillers in wide area is proposed to provide demand response. The proposed TWDLC scheme is designed to optimize the load shedding ratio of every customer under control to ensure the target load to be shed is met at every scheduling period. In order to overcome the load reduction uncertainties of TWDLC, an innovative solution is proposed by applying a certain degree of loosening on the constraint of the actual shed load. Fuzzy linear programming is utilized to solve the optimization problem with fuzzy constraints. The proposed fuzzy linear programming problem is solved by delicately transforming it into a regular linear programming problem. A selection scheme used to obtain the feasible candidates set for load shedding at every sampling interval of TWDLC is also designed along with the fuzzy linear programming.

Keywords: fuzzy linear programming, direct load control, scheduling optimization, chillers, air condition, demand response.

1. Introduction

The rapid development of smart grid [1-3] integrated with advanced metering infrastructure (AMI) and two-way communication capability offers a new opportunity for utility company to revolutionize the existing electrical systems. The emergence of these cyber-infrastructures allows utility to exploit demand side capability of electricity users in order to achieve certain grid-level operation objectives such as the reduction of peak demand and forced outage [4]. Utilities tend to deploy different demand response (DR) programs to fully realize the benefits of smart grid. Existing DR programs are categorized into two types, namely the price-based and incentive-based schemes [5-6]. For latter schemes, electricity users are incentivized by utility or curtail service provider (CSP) for being able to reduce their energy consumption for a certain periods of time upon request.

Direct load control (DLC) is one common incentive-based DR programs used by utility or CSP to reshape load profile by scheduling the operation cycles of customer’s high-power appliances. Central air conditioning chillers of industrial and commercial customers are the excellent candidates to achieve a cost effective DLC because the potential load reduction capacity delivered can reach up to several hundred kilowatts. Given the impressive strides made in metering and intelligent control technologies in facilitating a continuous bidirectional communication between the utility or CSP and its customers [7-8], a two-way direct load control (TWDLC) scheme for central chillers can further be envisioned as an emergency DR program to deliver the real-time load shedding effect. In particular, the utility or CSP can transmit the load shedding signals to its controlled customers while monitoring the load shedding results continuously via the Internet. The TWDLC of central chillers can even serve as an ancillary service if a huge amount of air-conditioning loads can be aggregated, monitored and managed in smart grid [9-10].
Computational intelligence approach has gained popularity in recent years to solve complex DLC scheduling problems. An iterative deepening search strategy was incorporated into genetic algorithm [11] and genetic programming [12] to produce a DLC schedule of air conditioning load capable of meeting the target load shed profile with minimum cost. A DLC scheduling problem with a multiobjective framework was solved from different perspectives using an interactive evolutionary algorithm [13], [14]. An optimal DLC dispatch of air-conditioning loads was proposed in [15] using an imperialist competitive algorithm to minimize the total deviation between the actual and target load shed profiles. A new DLC model for air-conditioning loads was coordinated along with unit commitment in [16] using distributed imperialist competitive algorithm to minimize system operational cost. Differential evolution was used in [17] to solve a DLC model aiming to minimize the operational cost of a microgrid with high penetration of solar power and air conditioning loads. A hierarchical DLC framework for large-scale air conditioning load dispatch was proposed in [18] using differential evolution algorithm to minimize the operational cost. With a two-way communication platform, a real-time load shedding of central air conditioning chillers was optimized with linear programming [19]. By considering the uncertainties of electricity prices and ambient temperature change, a DLC of air-conditioning loads was solved in [20] by mixed-integer linear programming to enhance wind power utilization level and minimize system operational cost. By considering the transmission system reliability, an optimal DLC schedule of air conditioning loads was obtained in [21] by a fuzzy DR to attain a tradeoff between peak load and system operation cost reduction. In [22], a nonlinear programming approach was formulated by considering DLC as a part of integrated resource planning to minimize the investment cost of microgrid. A DLC union was formed for the retailer and residential users in [23] using cooperative game to minimize the regulation cost of retailer by providing users an indirect access into balance market to improve market efficiency. A model estimator controller was designed in [24] using Markov chain model to coordinate aggregation of air-conditioning loads in order to address energy imbalance issue in power systems. Both of the distributed load shedding and micro-generator dispatching was coordinated in [25] using a probabilistic method to provide an emergency DR. A novel DLC scheme was proposed using queuing system model to control the air-conditioning loads without comprising users’ cooling comfort [26].

Most existing DLC scheduling strategies have not, to the authors’ best knowledge, considered the uncertainties of demand reduction provided by air-conditioning loads. The previous works assumed the target load shed to be met are fixed and crisp optimization constraints were formulated to guarantee the actual load shed is not less than the predefined target. In practical scenarios, the load reduction of air-conditioners in wide area vary with time of day, ambient temperature, number of people in the cooling environment, communication network signal strength, etc. [27]. The optimality of DLC schedules produced without taking these uncertainties into account is hence questionable. The main contribution of this paper is to propose an innovative approach based on fuzzy linear programming [30] allowing more flexibility in solving the optimization of TWDLC for central air conditioning systems. Particularly, a fuzzy inference system is designed to model the uncertainties of load reduction by allowing a certain degree of constraint loosening to obtain an optimal TWDLC schedule of central air conditioning systems. With these optimization flexibilities, a soft curtailment for the TWDLC of central air conditioning systems is achieved. With the proposed approach, a tolerance range of load reduction uncertainty is provided to the aggregators as they negotiate DR capacity and purchase prices with CSPs [28-29].

This paper is structured as follows. The problem to be solved for TWDLC of central air conditioning chillers is mathematically defined in Section 2. Section 3 describes the optimization approach determining the best set of candidates for control in every sampling interval. The implementation of fuzzy linear programming for TWDLC is described in Section 4. Computer simulations verifying the performance of the proposed scheduling optimization approach are shown in Section 5. Conclusions are made in Section 6.

2. Two-way Direct Load Control of Central Chiller
Assume that $N$ customers are recruited by utility company or CSP to participate in the TWDLC program and $C_i$ chillers at every $i$-th customers are under control, $i = 1...N$. An optimal scheduling scheme for real time TWDLC is designed in this paper. Denote $\lambda_{ij}(t)$ as the running status of the $j$-th central chiller unit at the $i$-th customer’s building at time $t$, where $\lambda_{ij}(t) = 1$ if the $j$-th central chiller belonging to the $i$-th customer is turned on at $t$, and $\lambda_{ij}(t) = 0$, otherwise, $j = 1...C_i$, $i = 1...N$. The load shedding for a centrifugal compressor in the chiller is technically achieved by partially reducing the load instead of turning it off. Chiller energy efficiency is measured using the coefficient of performance (COP), which varies with chiller’s load ratio. The COP drops drastically if load ratio is less than 50% for most chillers. As chiller’s load is partially reduced for DLC, it needs to be assigned a lower bound of load ratio preventing chiller from having low COP. Denote $W_{ij}$ and $W_{ij}^f(t)$ as chiller’s capacity and the load measured at time $t$, respectively, and $v_{ij}$ as the lower bound of the load ratio for the $j$-th chiller of the $i$-th customer. The controllable load for the $i$-th customer, denoted as $W_{ij}(\cdot)$, is then defined as the total controllable load among all chillers belonging to that customer, i.e.,

$$W_{ij}(t) = \sum_{j=1}^{C_i} \lambda_{ij}(t)W_{ij}^f(t).$$

(1)

where the controllable load for the $j$-th chiller of the $i$-th customer $W_{ij}^c(t)$ is defined as:

$$W_{ij}^c(t) = \begin{cases} W_{ij}^f(t) - v_{ij}W_{ij}^f, & \text{if } W_{ij}^f(t) \geq v_{ij}W_{ij}^f; \\ 0, & \text{otherwise}. \end{cases}$$

(2)

The utility company or CSP installs the gateway and chiller control network inside the building of customers willing to participate in the curtailed service program or similar demand response programs. Once the gateway receives the shedding command through the Internet over the broadband network from the control center at time $t$, it calculates the required load reduction for every central chiller and activates the load shedding through the chiller control network. Let the control interval for the entire TWDLC be $T$. If the sampling interval for the control center to conduct TWDLC is defined as $T_m$, the number of evaluations for TWDLC through an entire control interval is defined as $M = T / T_m$.

The main computer in control center reviews available customers for load control at the $k$-th sampling interval $kT_m$, $k = 1...M$. As long as the customer is available for control, the average controllable load is measured at every sampling interval. Denote $\bar{W}_i^c(kT_m)$ as the average controllable load for the $i$-th available customer at $kT_m$ and $T_m$ as the averaging interval for calculating $\bar{W}_i^c(kT_m)$. Then,

$$\bar{W}_i^c(kT_m) = \frac{1}{T_m} \int_{t_{kT_m}}^{t_{kT_m + T_m}} W_i^c(t)dt.$$  

(3)

As soon as reviewing all the available customers for control, the main computer in control center selects certain number of available customers for load shedding by sending out load shedding commands through internet to the gateway at customer site. Figure 1 shows the load variation of a typical chiller starts shedding load to a certain ratio, maintains the load ratio for certain period of time and restores the load back to the original load before shedding. It is shown in Figure 1 that a period of time is required for a chiller to conduct load shedding and load restoration. If the customer is selected for load shedding, the load that a chiller actually shed is measured by the gateway and sent back to control center through internet. Denote $\bar{W}_i^c(kT_m)$ as the average load after load shedding for the $i$-th available customer and $T_m$ as the time interval to wait until the chiller finishes load shedding. Then,
With the average loads defined in (3) and (4), the average shed load $\tilde{W}_i(kT_i)$ is defined as
\[
\tilde{W}_i(kT_i) = \max((\tilde{W}_i(kT_i) - \tilde{W}_i(kT_i)), 0).
\] (5)

At every time step, the average controllable load for every customer under control, $\tilde{W}_i()$, $i = 1...N$, is measured and sent to control center through the Internet by the gateway installed at customer's site. This paper proposes a real-time optimization approach for determining the combination of load shedding ratios for all customers at every time step through the entire control interval $T_c$ based on the received average controllable load $\tilde{W}_i()$, $i = 1...N$. Denote $\eta_i()$ as the expected load shedding ratio calculated by the main computer in the control center for the $i$-th customer. The calculated load shedding ratio $\eta_i()$ is the ratio of the expected shed load with respect to the average controllable load. The proposed optimization approach aims to find the best combination of load shedding ratio of every central air-conditioning chiller in real time so that the overall target shed load is individually achieved at every time step. The overall target shed load is the total amount of load required to shed by the direct control of entire set of central air-conditioning chillers.

Denote the overall target shed load based on the load forecast as $P()$ and the set containing all available customers for TWDLC as $J()$. The utility aims to minimize the shed load by TWDLC in order to minimize utility's electricity sale loss while satisfy overall and regional target shed load. Since the required shed load is based on the load forecast, it allows a certain degree of precision tolerance in response to weather, temperature, control timing, customer conditions, etc. In other words, the constraint for the calculated shed load being greater than the overall target shed load allows a certain degree of loosening. With this constraint loosening, more calculation flexibility is given to the optimization. Fuzzy linear programming is utilized to solve the optimization problem. The fuzzy linear programming categorized as linear programming with fuzzy resources is designed as follows.
\[
\min_{\eta_i(kT_i) \in [0,1], \forall i \in J(kT_i)} \left( \sum_{\eta_i(kT_i) \in [0,1], \forall i \in J(kT_i)} \eta_i(kT_i)\tilde{W}_i(kT_i) \right).
\] (6)

subject to
\[
0.1 \leq \eta_i(kT_i) \leq 1 \quad \text{if} \quad \eta_i(kT_i) > 0, \ i \in J(kT_i);
\] (7)

and
The expected shedding ratios $\eta_i(\cdot)$ range between 0 and 1. However, if the calculated $\eta_i(\cdot)$ is too small, the required shed load at the corresponding customer might be barely greater than the disturbance, leading to insignificant shedding contribution to the entire load reduction. Even though some of the calculated shedding ratios are insignificantly small, the control center still needs to send these ratios one by one to the corresponding gateways at customer sites, leading to inefficient communication efforts. To avoid obtaining insignificantly small values, a lower bound is assigned to the shedding ratio. Therefore, the calculated shedding ratio in the optimization is constrained between 0.1 and 1 as in (7). The sign "\(\geq\)" in (8) symbolizes that the inequality is in essence with fuzziness. The load curtailment due to TWDLC in (8) could be considered as a soft curtailment because the load shed quantity is allowed to vary within a soft range. The fuzzy constraint in (8) is characterized by the membership function $\mu(\cdot)$ defined in Figure 2, where the tolerance for the fuzzy constraint is characterized using two coefficients $\nu_1, \nu_2 \in [0, 1]$. Denote $\psi(kT_i)$ as the calculated shed load, i.e.,

$$\psi(kT_i) = \sum_{\eta_i(kT_j) \neq \eta_i(kT_i)} \eta_i(kT_j) \bar{W}_i(kT_j).$$

(9)

The fuzzy constraint characterized by the membership function $\mu(\cdot)$ in Figure 2 is defined as:

$$\mu(\eta(kT_i)) = \begin{cases} 1, & \text{if } \psi(kT_i) > (1+\nu_2)P(kT_i) \\ \frac{\psi(kT_i)-(1-\nu_1)P(kT_i)}{(1+\nu_2)(1-\nu_1)P(kT_i)}, & \text{if } (1-\nu_1)P(kT_i) \leq \psi(kT_i) \leq (1+\nu_2)P(kT_i) \\ 0, & \text{if } \psi(kT_i) < (1-\nu_1)P(kT_i) \end{cases}$$

(10)

Figure 2. Membership function characterizing the fuzzy constraint in (8).

3. Determination of Candidates for TWDLC

The monitoring and control of every chiller for the main computer is through the gateway installed at every customer’s location. To ease the computational and communication effort, the main computer determines the load to be shed and sends the shedding control commands customer by customer rather than chiller by chiller. The customer determination for shedding at every time step is to fulfill the target shedding capacity $P(\cdot)$ and level off the contribution to the entire load shedding for every customer. To measure the shedding contribution of the $i$-th customer, a coefficient called shedding contribution ratio, denoted as $\hat{\eta}_i(\cdot)$, is defined as a ratio of the average load actually shed with respect to the average controllable load:
The main computer records the accumulated time under control for every customer and this accumulated times is used as one of the reference indices when determining customers for shedding at each time step. For the i-th customer, denote the effective accumulated time under control and off control up to the k-th time step as \( \hat{\eta}_i^{uc}(kT_s) \) and \( \hat{\eta}_i^{oc}(kT_s) \), respectively. The effective accumulated times \( \hat{\eta}_i^{uc}(kT_s) \) and \( \hat{\eta}_i^{oc}(kT_s) \) are calculated by practically adding up the time intervals under control and off control for the i-th customer. The shedding contribution ratios are also taken into account. For instance, the customers with lower shedding contribution ratios lose cooling comfort less than those with high shedding contribution ratios. The effective increments of the accumulated times under control are considered to be shorter than the ones for the customer with high shedding contribution ratios. An adjustment scheme for the accumulated time under control and off control is proposed according to the load shedding control experience and customers’ response. Customers with \( \hat{\eta}_i^{oc} \) > 0.75 require no adjustment. But for customers with \( \hat{\eta}_i^{oc} \leq 0.15 \), their equivalent shedding intervals are discounted to 1/3 since less cooling comfort loss was brought by TWDLC during this shedding control period. For customers with 0.15 < \( \hat{\eta}_i^{oc} \) ≤ 0.75, their equivalent shedding intervals are linearly adjusted between 1 and 1/3. Let the adjustment coefficient for the i-th customer be \( \hat{\xi}_i(\cdot) \), \( \hat{\xi}_i(\cdot) \) is defined as:

\[
\hat{\xi}_i(\cdot) = \begin{cases} 
\frac{1}{3}, & \text{if } 0 < \hat{\eta}_i(\cdot) \leq 0.15; \\
\frac{1}{3} + \frac{10}{9} (\hat{\eta}_i(\cdot) - 0.15), & \text{if } 0.15 < \hat{\eta}_i(\cdot) \leq 0.75; \\
1, & \text{if } \hat{\eta}_i(\cdot) > 0.75 \text{ or } \hat{\eta}_i(\cdot) = 0.
\end{cases}
\]

(12)

Let \( s_i(kT_s) \in \{0,1\} \) be the control status of the i-th customer at the k-th sampling interval. \( s_i(kT_s) = 1 \) if the i-th customer is under control while \( s_i(kT_s) = 0 \) if the i-th customer is uncontrolled or restored from being controlled. \( \tau_i^{uc}(\cdot) \) and \( \tau_i^{oc}(\cdot) \) are effectively adjusted with reference to \( \hat{\xi}_i(\cdot) \) as follows:

\[
\tau_i^{uc}(kT_s) = (\tau_i^{oc}(k-1)T_s) + s_i(kT_s) \times \hat{\xi}_i(kT_s - T_s) \times T_s;
\]

(13)

\[
\tau_i^{oc}(kT_s) = (\tau_i^{oc}(k-1)T_s) + \hat{\xi}_i(kT_s - T_s) + (1 - \hat{\xi}_i(kT_s - T_s) + 1/3) \times T_s \times \tau_i^{uc}(kT_s);
\]

(14)

\( \forall i = 1...N \) and \( k = 1...M \), where \( \tau_i^{uc}(0) = 0 \) and \( \tau_i^{oc}(0) = 0 \).

Note that \( \tau_i(\cdot) \) in both (13) and (14) denotes the complement of \( s_i(\cdot) \). \( \tau_i^{uc}(\cdot) \) and \( \tau_i^{oc}(\cdot) \) are reset to zero as the control status changes. As shown in (13) and (14), customers with larger shedding contribution ratios in the previous time step have effectively more accumulated \( \tau_i^{uc}(\cdot) \) and vice versa. The accumulated time under control needs an upper limit in order not to affect too much cooling comfort due to load shedding. For the i-th customer, let \( T_i^{uc} \) be the maximum time allowed for continuous shedding control, then

\[
\tau_i^{uc}(kT_s) \leq T_i^{uc}, \forall i = 1...N, k = 1...M.
\]

(15)

If the customer is off control, it means that the customer is restored from the previous shedding control. It takes time for the building to regain cooling comfort before it is controlled again. Let \( T_i^{oc} \) be the least time the i-th customer needs to remain in off-control status, then

\[
\tau_i^{oc}(kT_s) \geq T_i^{oc}, \forall i = 1...N, k = 1...M.
\]

(16)
Every $i$-th customer becomes a candidate for load shedding if both constraints in (15) and (16) are satisfied. On the contrary, if the constraint in either (15) or (16) is violated, the $i$-th customer is removed from the candidate list for load shedding in the next time step. Therefore,

$$s_i((k+i)T_\tau) = 0, \quad \text{if } \tau^\infty_1(kT_s) > T^\infty_s \quad \text{or} \quad \tau^\infty_2(kT_s) < T^\infty_s. \quad (17)$$

Every customer’s shedding contribution ratio is accumulated and recorded in the main computer at the control center. Denote $\Omega_i(kT_\tau)$ and $\bar{\Omega}_i(kT_\tau)$ as the accumulated shedding contribution ratio and its average value, respectively, for the $i$-th customer building at the $k$-th time step, then

$$\Omega_i(kT_\tau) = \Omega_i(kT_\tau - T_s) + \hat{\eta}_i(kT_\tau); \quad (18)$$

$$\bar{\Omega}_i(kT_\tau) = \frac{1}{N} \sum_{i=1}^{N} \Omega_i(kT_\tau); \quad (19)$$

where the initial value $\Omega_i(0) = 0$. As the TWDLC is conducted day by day, the load shedding control fairness needs to be watched because it is a long-term change in cooling comforts for customers. To prevent some of the customers from being controlled too often and too long and thus biasing the fairness, every customer’s accumulated shedding contribution compared to the average value among customers is monitored at the main computer. For the $i$-th customer, if $\Omega_i(kT_\tau) > \bar{\Omega}_i(kT_\tau)$, the customer can be removed from the candidate list for load shedding control in the next time step. Therefore,

$$s_i((k+i)T_\tau) = 0, \quad \text{if } \Omega_i(kT_\tau) > \bar{\Omega}_i(kT_\tau). \quad (20)$$

Conversely, if $\Omega_i(kT_\tau) < \bar{\Omega}_i(kT_\tau)$, it is expected that more contribution to the TWDLC is required and the customer is taken as a candidate for load shedding. Let $J((k+1)T_s)$ be the set of candidates available for load shedding at the $(k+1)$-th time step based on the records calculated up to the $k$-th time step. Referring to (15), (16) and (20), $J((k+1)T_s)$ is defined as:

$$J((k+1)T_s) = \{i | i \in \{1,...,N\}, \tau^\infty_1(kT_s) \leq T^\infty_s, \tau^\infty_2(kT_s) \geq T^\infty_s, \Omega_i(kT_s) \leq \bar{\Omega}_i(kT_s), k = 1,...(M-1)\}. \quad (21)$$

Referring to (6), the set of decision variables for the optimization, $f(\cdot)$, is defined as in (21).

4. Fuzzy Linear Programming

The optimization in (6) with the crisp constraint (7) and the fuzzy constraint (8) that is characterized by the membership function in (10), can be solved by first solving the following two standard linear programming problems:

$$\begin{aligned}
\min_{\eta_i(kT_s) \in [0,1]} & \sum_{\eta_i(kT_s) \in [0,1]} \eta_i(kT_s) \tilde{W}_s^i(kT_s) \\
\text{subject to} & \quad 0.1 \leq \eta_i(kT_s) \leq 1, \quad \text{if } \eta_i(kT_s) > 0, \quad i \in J(kT_s) \\
& \quad \sum_{\eta_i(kT_s) \in [0,1]} \eta_i(kT_s) \tilde{W}_s^i(kT_s) \geq (1+t_c)P(kT_s) 
\end{aligned} \quad (22)$$

$$\begin{aligned}
\min_{\eta_i(kT_s) \in [0,1]} & \sum_{\eta_i(kT_s) \in [0,1]} \eta_i(kT_s) \tilde{W}_s^i(kT_s) \\
\text{subject to} & \quad 0.1 \leq \eta_i(kT_s) \leq 1, \quad \text{if } \eta_i(kT_s) > 0, \quad i \in J(kT_s) \\
& \quad \sum_{\eta_i(kT_s) \in [0,1]} \eta_i(kT_s) \tilde{W}_s^i(kT_s) \geq (1-t_c)P(kT_s) 
\end{aligned} \quad (23)$$
where \( \upsilon_i \) and \( \upsilon_j \) are defined in the membership function in (8). Assume that the optimal solution of (19) and (20) are \( \eta_i^0(kT_i) \) and \( \eta_i^0(kT_i) \), respectively. Denote the shed load corresponding to \( \eta_i^0(kT_i) \) and \( \eta_i^0(kT_i) \) as \( \Psi_i^0(kT_i) \) and \( \Psi_i^0(kT_i) \), respectively, i.e.,

\[
\Psi_i^0(kT_i) = \sum_{\eta_i(kT_i) \in J(kT_i)} \eta_i^0(kT_i) \bar{W}_i(kT_i);
\]

(24)

\[
\Psi_i^0(kT_i) = \sum_{\eta_i(kT_i) \in J(kT_i)} \eta_i^1(kT_i) \bar{W}_i(kT_i).
\]

(25)

The degree of optimality is characterized by the following membership function \( \mu_h(\cdot) \) shown in Figure 3 based on \( \Psi_i^0(kT_i) \) and \( \Psi_i^0(kT_i) \) as following:

\[
\mu_h(\eta(kT_i)) = \begin{cases} 
0, & \text{if } \Psi_i^0(kT_i) \leq \Psi_i^0(kT_i) \\
1, & \text{if } \Psi_i^0(kT_i) > \Psi_i^0(kT_i)
\end{cases}
\]

(26)

With the fuzzy constraint being transformed into the membership function \( \mu \) in (10) and the objective function associated with the fuzzy constraint being transformed into the membership function \( \mu_h \) in (26), the optimization in (6)-(8) is solved using a max-min approach as follows:

\[
\max_{\alpha \in [0,1], \eta_i(kT_i) \in [0,1], i \in J(kT_i)} \alpha \\
\text{subject to } 0.1 \leq \eta_i(kT_i) \leq 1, \text{ if } \eta_i(kT_i) > 0, i \in J(kT_i).
\]

(27)

The constrained max-min optimization problem in (27) can be implemented as a standard linear programming problem:

\[
\max_{\alpha \in [0,1], \eta_i(kT_i) \in [0,1], i \in J(kT_i)} \alpha \\
\text{subject to } 0.1 \leq \eta_i(kT_i) \leq 1, \text{ if } \eta_i(kT_i) > 0; \eta_i(kT_i) \geq 0; \eta_i(kT_i) \geq 0
\]

(28)

(29)

(30)

(31)

where \( \eta_i(kT_i) \in [0,1], i \in J(kT_i) \).

Substituting (26) into (30), the constraint in (30) is equivalent to:
Similarly, substituting (10) into (31), the constraint in (31) is equivalent to:

$$\sum_{i \in J(k_T), j \in J(k_T)} \eta_i(k_T) \bar{W}_i(k_T) = (1 - \alpha) \bar{P}(k_T) + \alpha \bar{P}(k_T).$$ (33)

Note that the constraint in (29) is not in a typical form of constraint for linear programming.

Define the surplus decision variables \( \gamma(k_T) \in [0, 1] \) \( i \in J(k_T) \) and denote \( Q \) as a large constant, i.e., \( Q >> 1 \). The constraint in (29) can be restated as the constraints as follows:

$$\eta_i(k_T) \leq \gamma_i(k_T) \times Q,$$ (34)

$$0.1 \times \gamma_i(k_T) \leq \eta_i(k_T), \quad \gamma_i(k_T) \in [0, 1], \quad i \in J(k_T).$$ (35)

Therefore, the fuzzy linear programming in (6)-(8) is solved based on the equivalent linear programming problem in (28) with constraints in (32)-(35).

5. Computer Simulation

A set of 30 customers are selected to test the effectiveness and efficiency of the load aggregation and the proposed TWDLC algorithm using fuzzy linear programming. The control interval was set as a period of 5 consecutive days, 10:00 to 17:00 every day. The sampling interval \( T_s \) for the main computer retrieving every customer’s controllable load and conducting load shedding through the gateway is set as 15 minutes. The sampling time for the gateway measuring the controllable load of every chiller was as 1 minute. The averaging interval \( T_a \) in (3) and the waiting interval \( T_w \) in (4) for the gateway to calculate the average controllable load before and after every sampling time are both set as 3 minutes. The capacity, time constraints and the maximum controllable load during the 5 day control period are listed in Table 1.

<table>
<thead>
<tr>
<th>Capacity (kW)</th>
<th>Max(( \bar{W}_i(k_T) )) (kW)</th>
<th>( T_s^w ) (min)</th>
<th>( T_s^m ) (min)</th>
<th>Capacity (kW)</th>
<th>Max(( \bar{W}_i(k_T) )) (kW)</th>
<th>( T_s^w ) (min)</th>
<th>( T_s^m ) (min)</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>250</td>
<td>109</td>
<td>15</td>
<td>30</td>
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<td>1300</td>
<td>727</td>
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<td>15</td>
<td>17</td>
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Recall that $\nu_1$ and $\nu_2$ in association with the membership function $\mu(\cdot)$ in (10) characterize the fuzzy constraint in (8) that the calculated load shed greater than or equal to the target load $P(\cdot)$ to a certain degree of precision tolerance. Both coefficients $\nu_1$ and $\nu_2$ could be either constants or time-varying functions since the degree of precision tolerance for the fuzzy constraint in (8) can vary with the temperature, load in regional power system, or time of a day, etc. In this paper, the variations of $\nu_1$ and $\nu_2$ are both set as time-varying functions, as shown in Figures 4 and 5, respectively. Using the proposed optimal real-time scheduling approach based on fuzzy linear programming, the calculated load shed, $\Psi(kT_r)$, is shown in Figure 6. The upper and lower limits for the fuzzy constraints in (8), i.e., $(1-\nu_1)P(kT_r)$ and $(1+\nu_2)P(kT_r)$, as well as the target load expected to shed $P(kT_r)$ are also compared with $\Psi(kT_r)$ in Figure 6. It is shown in Figure 6 that the calculated load shed $\Psi(kT_r)$ matches the target load $P(kT_r)$ well in response to the variations in target load tolerance. The calculated shedding ratios and the corresponding controllable load of the customer with the largest capacity are shown in Figures 7(a) and 7(b), respectively.

![Figure 4. The variation of coefficient $\nu_1$.](image)

![Figure 5. The variation of coefficient $\nu_2$.](image)
Figure 6. Comparison of calculated load shed $\Psi(kT_s)$, the target load required to shed $P(kT_s)$, the upper and lower bound of target $(1-\nu_1)P(kT_s)$ and $(1+\nu_2)P(kT_s)$.

Figure 7. (a) Variation of expected shedding contribution ratios for the customer with the largest capacity. (b) Profile of controllable load for the customer with the largest capacity.
Both the number of shedding times and shedding ratio distributions vary among customers. In order to show the effectiveness of the filtering scheme defined in (20), define the standard deviation of the accumulated shedding contribution ratios at the $k$-th time step defined as:

$$\sigma(kT_n) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (\Omega_i(kT_n) - \bar{\Omega}(kT_n))^2}$$

(33)

where the accumulated shedding contribution ratio of the $i$-th customer $\Omega_i(\cdot)$ and the average accumulated shedding contribution ratio $\bar{\Omega}(\cdot)$ are shown in (18) and (19), respectively. The variation profiles of $\sigma(\cdot)$ with and without the filtering scheme in (20) are both shown in Figures 8(a) and 8(b), respectively. Figure 8(a) shows that the standard deviation of accumulated shedding contribution ratios varies within a limited range as time goes on if the filtering scheme in (20) is applied. This shows that the filtering scheme levels off every customer’s contribution to the load shedding and achieves load shedding fairness. Conversely, if the filtering scheme in (20) is removed from the customer selection process, the standard deviation of the accumulated shedding contribution ratios increased drastically with time.

**Figure 8.** Variation of standard deviation of shedding contribution ratios (a) with (b) without filtering scheme in (20).
6. Conclusion

A real-time TWDLC optimization scheme is proposed as an effective demand response approach by scheduling the direct load control of the central air-conditioning chillers in wide area. The proposed TWDLC works well through the broadband network with gateway installed at the site of every customer under control. Fuzzy linear programming is utilized for optimization providing more optimization flexibility by allowing a precision tolerance for the shed load constraints. It is shown in simulation that the proposed TWDLC scheme is computationally efficient and effective, hence feasible for real-time optimization and time-varying precision tolerance in response to variable target load profile.

For future work, the degree of precision tolerance for the fuzzy constraints can be linked with weather condition, regional load, time of a day, etc., depending on the application scenario. Delicate modeling can be designed to automatically adjust the precision tolerance in response to environment changes. Type-2 membership functions can also be used to define precision tolerance of constraints for future work.

Author Contributions: Leether Yao conceived and designed the main ideas proposed in the paper. He also wrote the paper. Lei Yao designed and performed the experiments. Wei Hong Lim designed the fuzzy linear programming model and analyzed the data.

Conflicts of Interest: The authors declare no conflict of interest.

References