

1 Article

# 2 A Soft Curtailment of Wide-area Central Air 3 Conditioning Load

4 Leehter Yao <sup>1,\*</sup>, Lei Yao <sup>1</sup> and Wei Hong Lim <sup>2</sup>

5 <sup>1</sup> Department of Electrical Engineering, National Taipei University of Technology, Taipei, Taiwan;  
6 t105319401@ntut.edu.tw

7 <sup>2</sup> Faculty of Engineering, Technology and Built Environment, UCSI University, Kuala Lumpur, Malaysia;  
8 limweihong87@gmail.com

9 \* Correspondence: ltyao@ntut.edu.tw; Tel.: +886-2-77365827

10 **Abstract:** A real-time two-way direct load control (TWDLC) of central air-conditioning chillers in  
11 wide area is proposed to provide demand response. The proposed TWDLC scheme is designed to  
12 optimize the load shedding ratio of every customer under control to ensure the target load to be  
13 shed is met at every scheduling period. In order to overcome the load reduction uncertainties of  
14 TWDLC, an innovative solution is proposed by applying a certain degree of loosening on the  
15 constraint of the actual shed load. Fuzzy linear programming is utilized to solve the optimization  
16 problem with fuzzy constraints. The proposed fuzzy linear programming problem is solved by  
17 delicately transforming it into a regular liner programming problem. A selection scheme used to  
18 obtain the feasible candidates set for load shedding at every sampling interval of TWDLC is also  
19 designed along with the fuzzy linear programming.

20 **Keywords:** fuzzy linear programming, direct load control, scheduling optimization, chillers, air  
21 condition, demand response.  
22

## 23 1. Introduction

24 The rapid development of smart grid [1-3] integrated with advanced metering infrastructure  
25 (AMI) and two-way communication capability offers a new opportunity for utility company to  
26 revolutionize the existing electrical systems. The emergence of these cyber-infrastructures allows  
27 utility to exploit demand side capability of electricity users in order to achieve certain grid-level  
28 operation objectives such as the reduction of peak demand and forced outage [4]. Utilities tend to  
29 deploy different demand response (DR) programs to fully realize the benefits of smart grid. Existing  
30 DR programs are categorized into two types, namely the price-based and incentive-based schemes  
31 [5-6]. For latter schemes, electricity users are incentivized by utility or curtail service provider (CSP)  
32 for being able to reduce their energy consumption for a certain periods of time upon request.

33 Direct load control (DLC) is one common incentive-based DR programs used by utility or CSP  
34 to reshape load profile by scheduling the operation cycles of customer's high-power appliances.  
35 Central air conditioning chillers of industrial and commercial customers are the excellent candidates  
36 to achieve a cost effective DLC because the potential load reduction capacity delivered can reach up  
37 to several hundred kilowatts. Given the impressive strides made in metering and intelligent control  
38 technologies in facilitating a continuous bidirectional communication between the utility or CSP and  
39 its customers [7-8], a two-way direct load control (TWDLC) scheme for central chillers can further be  
40 envisioned as an emergency DR program to deliver the real-time load shedding effect. In particular,  
41 the utility or CSP can transmit the load shedding signals to its controlled customers while  
42 monitoring the load shedding results continuously via the Internet. The TWDLC of central chillers  
43 can even serve as an ancillary service if a huge amount of air-conditioning loads can be aggregated,  
44 monitored and managed in smart grid [9-10].

45 Computational intelligence approach has gained popularity in recent years to solve complex  
46 DLC scheduling problems. An iterative deepening search strategy was incorporated into genetic  
47 algorithm [11] and genetic programming [12] to produce a DLC schedule of air conditioning load  
48 capable of meeting the target load shed profile with minimum cost. A DLC scheduling problem with  
49 multiobjective framework was solved from different perspectives using an interactive evolutionary  
50 algorithm [13], [14]. An optimal DLC dispatch of air-conditioning loads was proposed in [15] using  
51 an imperialist competitive algorithm to minimize the total deviation between the actual and target  
52 load shed profiles. A new DLC model for air-conditioning loads was coordinated along with unit  
53 commitment in [16] using distributed imperialist competitive algorithm to minimize system  
54 operational cost. Differential evolution was used in [17] to solve a DLC model aiming to minimize  
55 the operational cost of a microgrid with high penetration of solar power and air conditioning loads.  
56 A hierarchical DLC framework for large-scale air conditioning load dispatch was proposed in [18]  
57 using differential evolution algorithm to minimize the operational cost. With a two-way  
58 communication platform, a real-time load shedding of central air conditioning chillers was  
59 optimized with linear programming [19]. By considering the uncertainties of electricity prices and  
60 ambient temperature change, a DLC of air-conditioning loads was solved in [20] by mixed-integer  
61 linear programming to enhance wind power utilization level and minimize system operational cost.  
62 By considering the transmission system reliability, an optimal DLC schedule of air conditioning  
63 loads was obtained in [21] by a fuzzy DR to attain a tradeoff between peak load and system  
64 operation cost reduction. In [22], a nonlinear programming approach was formulated by considering  
65 DLC as a part of integrated resource planning to minimize the investment cost of microgrid. A DLC  
66 union was formed for the retailer and residential users in [23] using cooperative game to minimize  
67 the regulation cost of retailer by providing users an indirect access into balance market to improve  
68 market efficiency. A model estimator controller was designed in [24] using Markov chain model to  
69 coordinate aggregation of air-conditioning loads in order to address energy imbalance issue in  
70 power systems. Both of the distributed load shedding and micro-generator dispatching was  
71 coordinated in [25] using a probabilistic method to provide an emergency DR. A novel DLC scheme  
72 was proposed using queuing system model to control the air-conditioning loads without comprising  
73 users' cooling comfort [26].

74 Most existing DLC scheduling strategies have not, to the authors' best knowledge, considered  
75 the uncertainties of demand reduction provided by air-conditioning loads. The previous works  
76 assumed the target load shed to be met are fixed and crisp optimization constraints were formulated  
77 to guarantee the actual load shed is not less than the predefined target. In practical scenarios, the  
78 load reduction of air-conditioners in wide area vary with time of day, ambient temperature, number  
79 of people in the cooling environment, communication network signal strength, etc. [27]. The  
80 optimality of DLC schedules produced without taking these uncertainties into account is hence  
81 questionable. The main contribution of this paper is to propose an innovative approach based on  
82 fuzzy linear programming [30] allowing more flexibility in solving the optimization of TWDLC for  
83 central air conditioning systems. Particularly, a fuzzy inference system is designed to model the  
84 uncertainties of load reduction by allowing a certain degree of constraint loosening to obtain an  
85 optimal TWDLC schedule of central air conditioning systems. With these optimization flexibilities, a  
86 soft curtailment for the TWDLC of central air conditioning systems is achieved. With the proposed  
87 approach, a tolerance range of load reduction uncertainty is provided to the aggregators as they  
88 negotiate DR capacity and purchase prices with CSPs [28-29].

89 This paper is structured as follows. The problem to be solved for TWDLC of central air  
90 conditioning chillers is mathematically defined in Section 2. Section 3 describes the optimization  
91 approach determining the best set of candidates for control in every sampling interval. The  
92 implementation of fuzzy linear programming for TWDLC is described in Section 4. Computer  
93 simulations verifying the performance of the proposed scheduling optimization approach are shown  
94 in Section 5. Conclusions are made in Section 6.

## 95 2. Two-way Direct Load Control of Central Chiller

96 Assume that  $N$  customers are recruited by utility company or CSP to participate the TWDLC  
 97 program and  $C_i$  chillers at every  $i$ -th customers are under control,  $i = 1 \dots N$ . An optimal scheduling  
 98 scheme for real time TWDLC is designed in this paper. Denote  $\lambda_{ij}(t)$  as the running status of the  $j$ -th  
 99 central chiller unit at the  $i$ -th customer's building at time  $t$ , where  $\lambda_{ij}(t) = 1$  if the  $j$ -th central chiller  
 100 belonging to the  $i$ -th customer is turned on at  $t$ , and  $\lambda_{ij}(t) = 0$ , otherwise,  $j = 1 \dots C_i$ ,  $i = 1 \dots N$ . The  
 101 load shedding for a centrifugal compressor in the chiller is technically achieved by partially reducing  
 102 the load instead of turning it off. Chiller energy efficiency is measured using the coefficient of  
 103 performance (COP), which varies with chiller's load ratio. The COP drops drastically if load ratio is  
 104 less than 50% for most chillers. As chiller's load is partially reduced for DLC, it needs to be  
 105 assigned a lower bound of load ratio preventing chiller from having low COP. Denote  $W_{ij}^f$  and  
 106  $W_{ij}^c(t)$  as chiller's capacity and the load measured at time  $t$ , respectively, and  $v_{ij}$  as the lower bound of  
 107 the load ratio for the  $j$ -th chiller of the  $i$ -th customer. The controllable load for the  $i$ -th customer,  
 108 denoted as  $W_a^i(\cdot)$ , is then defined as the total controllable load among all chillers belonging to that  
 109 customer, i.e.,

$$W_a^i(t) = \sum_{j=1}^{C_i} \lambda_{ij}(t) W_{ij}^c(t). \quad (1)$$

110 where the controllable load for the  $j$ -th chiller of the  $i$ -th customer  $W_{ij}^a(t)$  is defined as:

$$W_{ij}^a(t) = \begin{cases} W_{ij}^c(t) - v_{ij} W_{ij}^f, & \text{if } W_{ij}^c(t) \geq v_{ij} W_{ij}^f; \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

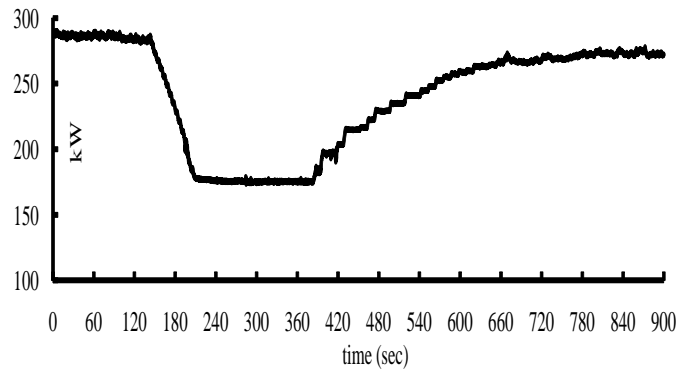
111 The utility company or CSP installs the gateway and chiller control network inside the building  
 112 of customers willing to participate in the curtailed service program or similar demand response  
 113 programs. Once the gateway receives the shedding command through the Internet over the  
 114 broadband network from the control center at time  $t$ , it calculates the required load reduction for  
 115 every central chiller and activates the load shedding through the chiller control network. Let the  
 116 control interval for the entire TWDLC be  $T_c$ . If the sampling interval for the control center to  
 117 conduct TWDLC is defined as  $T_s$ , the number of evaluations for TWDLC through an entire control  
 118 interval is defined as  $M = T_c / T_s$ .

119 The main computer in control center reviews available customers for load control at the  $k$ -th  
 120 sampling interval  $kT_s$ ,  $k = 1 \dots M$ . As long as the customer is available for control, the average  
 121 controllable load is measured at every sampling interval. Denote  $\bar{W}_a^i(kT_s)$  as the average  
 122 controllable load for the  $i$ -th available customer at  $kT_s$  and  $T_m$  as the averaging interval for calculating  
 123  $\bar{W}_a^i(kT_s)$ . Then,

$$\bar{W}_a^i(kT_s) = \frac{1}{T_m} \int_{kT_s - T_m}^{kT_s} W_a^i(t) dt. \quad (3)$$

124 As soon as reviewing all the available customers for control, the main computer in control center  
 125 selects certain number of available customers for load shedding by sending out load shedding  
 126 commands through internet to the gateway at customer site. Figure 1 shows the load variation of a  
 127 typical chiller starts shedding load to a certain ratio, maintains the load ratio for certain period of  
 128 time and restores the load back to the original load before shedding. It is shown in Figure 1 that a  
 129 period of time is required for a chiller to conduct load shedding and load restoration. If the  
 130 customer is selected for load shedding, the load that a chiller actually shed is measured by the  
 131 gateway and sent back to control center through internet. Denote  $\bar{W}_b^i(kT_s)$  as the average load after  
 132 load shedding for the  $i$ -th available customer and  $T_w$  as the time interval to wait until the chiller  
 133 finishes load shedding. Then,

$$\bar{W}_b^i(kT_s) = \frac{1}{T_m} \int_{kT_s+T_w}^{kT_s+T_w+T_m} W_a^i(t) dt. \quad (4)$$



**Figure 1.** Load variation as a typical chiller conducts load shedding and load restoration.

134

135

136

137 With the average loads defined in (3) and (4), the average shed load  $\bar{W}_d^i(kT_s)$  is defined as

$$\bar{W}_d^i(kT_s) = \max((\bar{W}_a^i(kT_s) - \bar{W}_b^i(kT_s)), 0). \quad (5)$$

138

139

140

141

142

143

144

145

146

147

148

149

At every time step, the average controllable load for every customer under control,  $\bar{W}_a^i(\cdot)$ ,  $i = 1 \dots N$ , is measured and sent to control center through the Internet by the gateway installed at customer's site. This paper proposes a real-time optimization approach for determining the combination of load shedding ratios for all customers at every time step through the entire control interval  $T_c$  based on the received average controllable load  $\bar{W}_a^i(\cdot)$ ,  $i = 1 \dots N$ . Denote  $\eta_i(\cdot)$  as the expected load shedding ratio calculated by the main computer in the control center for the  $i$ -th customer. The calculated load shedding ratio  $\eta_i(\cdot)$  is the ratio of the expected shed load with respect to the average controllable load. The proposed optimization approach aims to find the best combination of load shedding ratio of every central air-conditioning chiller in real time so that the overall target shed load is individually achieved at every time step. The overall target shed load is the total amount of load required to shed by the direct control of entire set of central air-conditioning chillers.

150

151

152

153

154

155

156

157

158

159

Denote the overall target shed load based on the load forecast as  $P(\cdot)$  and the set containing all available customers for TWDLC as  $J(\cdot)$ . The utility aims to minimize the shed load by TWDLC in order to minimize utility's electricity sale loss while satisfy overall and regional target shed load. Since the required shed load is based on the load forecast, it allows a certain degree of precision tolerance in response to weather, temperature, control timing, customer conditions, etc. In other words, the constraint for the calculated shed load being greater than the overall target shed load allows a certain degree of loosening. With this constraint loosening, more calculation flexibility is given to the optimization. Fuzzy linear programming is utilized to solve the optimization problem. The fuzzy linear programming categorized as linear programming with fuzzy resources is designed as follows.

$$\min_{\eta_i(kT_s) \in [0,1], i \in J(kT_s)} \left( \sum_{\eta_i(kT_s), i \in J(kT_s)} \eta_i(kT_s) \bar{W}_a^i(kT_s) \right). \quad (6)$$

160 subject to

$$0.1 \leq \eta_i(kT_s) \leq 1 \quad \text{if } \eta_i(kT_s) > 0, i \in J(kT_s); \quad (7)$$

161 and

$$\sum_{\eta_i(kT_s), i \in J(kT_s)} \eta_i(kT_s) \bar{W}_a^i(kT_s) \lesseqgtr P(kT_s). \quad (8)$$

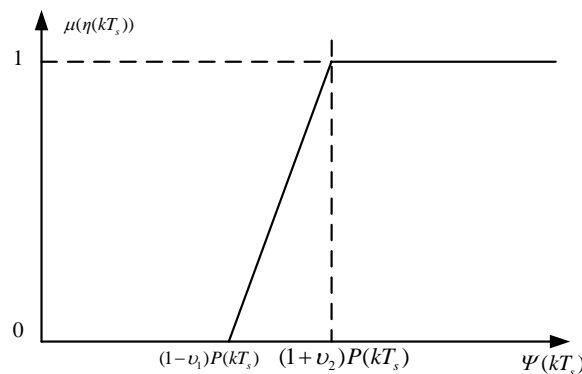
162

163 The expected shedding ratios  $\eta_i(\cdot)$  range between 0 and 1. However, if the calculated  $\eta_i(\cdot)$  is too  
 164 small, the required shed load at the corresponding customer might be barely greater than the  
 165 disturbance, leading to insignificant shedding contribution to the entire load reduction. Even  
 166 though some of the calculated shedding ratios are insignificantly small, the control center still needs  
 167 to send these ratios one by one to the corresponding gateways at customer sites, leading to  
 168 inefficient communication efforts. To avoid obtaining insignificantly small values, a lower bound is  
 169 assigned to the shedding ratio. Therefore, the calculated shedding ratio in the optimization is  
 170 constrained between 0.1 and 1 as in (7). The sign " $\lesseqgtr$ " in (8) symbolizes that the inequality is in  
 171 essence with fuzziness. The load curtailment due to TWDLC in (8) could be considered as a soft  
 172 curtailment because the load shed quantity is allowed to vary within a soft range. The fuzzy  
 173 constraint in (8) is characterized by the membership function  $\mu(\cdot)$  defined in Figure 2, where the  
 174 tolerance for the fuzzy constraint is characterized using two coefficients  $\nu_1, \nu_2 \in [0, 1]$ . Denote  
 175  $\Psi(kT_s)$  as the calculated shed load, i.e.,

$$\Psi(kT_s) = \sum_{\eta_i(kT_s), i \in J(kT_s)} \eta_i(kT_s) \bar{W}_a^i(kT_s). \quad (9)$$

176 The fuzzy constraint characterized by the membership function  $\mu(\cdot)$  in Figure 2 is defined as:

$$\mu(\eta(kT_s)) = \begin{cases} 1, & \text{if } \Psi(kT_s) > (1+\nu_2)P(kT_s) \\ \frac{\Psi(kT_s) - (1-\nu_1)P(kT_s)}{(\nu_1 + \nu_2)P(kT_s)}, & \text{if } (1-\nu_1)P(kT_s) \leq \Psi(kT_s) \leq (1+\nu_2)P(kT_s) \\ 0, & \text{if } \Psi(kT_s) < (1-\nu_1)P(kT_s) \end{cases} \quad (10)$$



177

178

Figure 2. Membership function characterizing the fuzzy constraint in (8).

179

### 3. Determination of Candidates for TWDLC

180 The monitoring and control of every chiller for the main computer is through the gateway  
 181 installed at every customer's location. To ease the computational and communication effort, the main  
 182 computer determines the load to be shed and sends the shedding control commands customer by  
 183 customer rather than chiller by chiller. The customer determination for shedding at every time step is  
 184 to fulfill the target shedding capacity  $P(\cdot)$  and level off the contribution to the entire load shedding for  
 185 every customer. To measure the shedding contribution of the  $i$ -th customer, a coefficient called  
 186 shedding contribution ratio, denoted as  $\hat{\eta}_i(\cdot)$ , is defined as a ratio of the average load actually shed  
 187 with respect to the average controllable load:

$$\hat{\eta}_i(kT_s) = \frac{\bar{W}_d^i(kT_s)}{\bar{W}_a^i(kT_s)}. \quad (11)$$

188 The main computer records the accumulated time under control for every customer and this  
 189 accumulated times is used as one of the reference indices when determining customers for shedding  
 190 at each time step. For the  $i$ -th customer, denote the effective accumulated time under control and off  
 191 control up to the  $k$ -th time step as  $\tau_i^{uc}(kT_s)$  and  $\tau_i^{oc}(kT_s)$ , respectively. The effective accumulated  
 192 times  $\tau_i^{uc}(kT_s)$  and  $\tau_i^{oc}(kT_s)$  are calculated by practically adding up the time intervals under control  
 193 and off control for the  $i$ -th customer. The shedding contribution ratios are also taken into account. For  
 194 instance, the customers with lower shedding contribution ratios lose cooling comfort less than those  
 195 with high shedding contribution ratios. The effective increments of the accumulated times under  
 196 control are considered to be shorter than the ones for the customer with high shedding contribution  
 197 ratios. An adjustment scheme for the accumulated time under control and off control is proposed  
 198 according to the load shedding control experience and customers' response. Customers with  $\hat{\eta}_i(\cdot)$   
 199  $> 0.75$  require no adjustment. But for customers with  $\hat{\eta}_i(\cdot) \leq 0.15$ , their equivalent shedding intervals  
 200 are discounted to 1/3 since less cooling comfort loss was brought by TWDLC during this shedding  
 201 control period. For customers with  $0.15 < \hat{\eta}_i(\cdot) \leq 0.75$ , their equivalent shedding intervals are  
 202 linearly adjusted between 1 and 1/3. Let the adjustment coefficient for the  $i$ -th customer be  $\xi_i(\cdot)$ ,  
 203  $\xi_i(\cdot)$  is defined as:

$$\xi_i(\cdot) = \begin{cases} \frac{1}{3}, & \text{if } 0 < \hat{\eta}(\cdot) \leq 0.15; \\ \frac{1}{3} + \frac{10}{9}(\hat{\eta}(\cdot) - 0.15), & \text{if } 0.15 < \hat{\eta}(\cdot) \leq 0.75; \\ 1, & \text{if } \hat{\eta}(\cdot) > 0.75 \text{ or } \hat{\eta}(\cdot) = 0. \end{cases} \quad (12)$$

204 Let  $s_i(kT_s) \in \{0,1\}$  be the control status of the  $i$ -th customer at the  $k$ -th sampling interval.  $s_i(kT_s) = 1$  if  
 205 the  $i$ -th customer is under control while  $s_i(kT_s) = 0$  if the  $i$ -th customer is uncontrolled or restored  
 206 from being controlled.  $\tau_i^{uc}(\cdot)$  and  $\tau_i^{oc}(\cdot)$  are effectively adjusted with reference to  $\xi_i(\cdot)$  as follows:

$$\tau_i^{uc}(kT_s) = (\tau_i^{uc}((k-1)T_s) + s_i(kT_s) \times \xi_i(kT_s - T_s) \times T_s); \quad (13)$$

$$\tau_i^{oc}(kT_s) = (\tau_i^{oc}((k-1)T_s) + \bar{s}_i(kT_s) \times (1 - \xi_i(kT_s - T_s) + 1/3) \times T_s) \times \bar{s}_i(kT_s); \quad (14)$$

207  $\forall i = 1 \dots N$  and  $k = 1 \dots M$ , where  $\tau_i^{uc}(0) = 0$  and  $\tau_i^{oc}(0) = 0$ .

208 Note that  $\bar{s}_i(\cdot)$  in both (13) and (14) denotes the complement of  $s_i(\cdot)$ .  $\tau_i^{uc}(\cdot)$  and  $\tau_i^{oc}(\cdot)$  are reset  
 209 to zero as the control status changes. As shown in (13) and (14), customers with larger shedding  
 210 contribution ratios in the previous time step have effectively more accumulated  $\tau_i^{uc}(\cdot)$  and vice  
 211 versa. The accumulated time under control needs an upper limit in order not to affect too much  
 212 cooling comfort due to load shedding. For the  $i$ -th customer, let  $T_i^{uc}$  be the maximum time allowed  
 213 for continuous shedding control, then

$$\tau_i^{uc}(kT_s) \leq T_i^{uc}, \forall i = 1 \dots N, k = 1 \dots M. \quad (15)$$

214 If the customer is off control, it means that the customer is restored from the previous shedding  
 215 control. It takes time for the building to regain cooling comfort before it is controlled again. Let  $T_i^{oc}$   
 216 be the least time the  $i$ -th customer needs to remain in off-control status, then

$$\tau_i^{oc}(kT_s) \geq T_i^{oc} \quad \forall i = 1 \dots N, k = 1 \dots M. \quad (16)$$

217 Every  $i$ -th customer becomes a candidate for load shedding if both constraints in (15) and (16)  
 218 are satisfied. On the contrary, if the constraint in either (15) or (16) is violated, the  $i$ -th customer is  
 219 removed from the candidate list for load shedding in the next time step. Therefore,

$$s_i((k+1)T_s) = 0, \text{ if } \tau_i^{uc}(kT_s) > T_i^{uc} \text{ or } \tau_i^{oc}(kT_s) < T_i^{oc}. \quad (17)$$

220 Every customer's shedding contribution ratio is accumulated and recorded in the main computer  
 221 at the control center. Denote  $\Omega_i(kT_s)$  and  $\bar{\Omega}_i(kT_s)$  as the accumulated shedding contribution ratio  
 222 and its average value, respectively, for the  $i$ -th customer building at the  $k$ -th time step, then

$$\Omega_i(kT_s) = \Omega_i(kT_s - T_s) + \hat{\eta}_i(kT_s); \quad (18)$$

$$\bar{\Omega}_i(kT_s) = \frac{1}{N} \sum_{i=1}^N \Omega_i(kT_s); \quad (19)$$

223 where the initial value  $\Omega_i(0) = 0$ . As the TWDLC is conducted day by day, the load shedding  
 224 control fairness needs to be watched because it is a long-term change in cooling comforts for  
 225 customers. To prevent some of the customers from being controlled too often and too long and thus  
 226 biasing the fairness, every customer's accumulated shedding contribution compared to the average  
 227 value among customers is monitored at the main computer. For the  $i$ -th customer, if  
 228  $\Omega_i(kT_s) \geq \bar{\Omega}_i(kT_s)$ , the customer can be removed from the candidate list for load shedding control in  
 229 the next time step. Therefore,

$$s_i((k+1)T_s) = 0, \text{ if } \Omega_i(kT_s) > \bar{\Omega}_i(kT_s). \quad (20)$$

230 Conversely, if  $\Omega_i(kT_s) < \bar{\Omega}_i(kT_s)$ , it is expected that more contribution to the TWDLC is required and  
 231 the customer is taken as a candidate for load shedding. Let  $J((k+1)T_s)$  be the set of candidates available  
 232 for load shedding at the  $(k+1)$ -th time step based on the records calculated up to the  $k$ -th time step.  
 233 Referring to (15), (16) and (20),  $J((k+1)T_s)$  is defined as:

$$J((k+1)T_s) = \{i | i \in \{1 \dots N\}, \tau_i^{uc}(kT_s) \leq T_i^{uc}, \tau_i^{oc}(kT_s) \geq T_i^{oc}, \Omega_i(kT_s) \leq \bar{\Omega}_i(kT_s), k = 1 \dots (M-1)\}. \quad (21)$$

234 Referring to (6), the set of decision variables for the optimization,  $J(\cdot)$ , is defined as in (21).

#### 235 4. Fuzzy Linear Programming

236 The optimization in (6) with the crisp constraint (7) and the fuzzy constraint (8) that is  
 237 characterized by the membership function in (10), can be solved by first solving the following two  
 238 standard linear programming problems:

$$\left\{ \begin{array}{l} \min_{\eta_i(kT_s) \in [0,1], i \in J(kT_s)} \sum_{\eta_i(kT_s) \in [0,1], i \in J(kT_s)} \eta_i(kT_s) \bar{W}_a^i(kT_s) \\ \text{subject to } 0.1 \leq \eta_i(kT_s) \leq 1, \text{ if } \eta_i(kT_s) > 0, i \in J(kT_s) \\ \sum_{\eta_i(kT_s) \in [0,1], i \in J(kT_s)} \eta_i(kT_s) \bar{W}_a^i(kT_s) \geq (1 + \nu_2) P(kT_s) \end{array} \right. \quad (22)$$

$$\left\{ \begin{array}{l} \min_{\eta_i(kT_s) \in [0,1], i \in J(kT_s)} \sum_{\eta_i(kT_s) \in [0,1], i \in J(kT_s)} \eta_i(kT_s) \bar{W}_a^i(kT_s) \\ \text{subject to } 0.1 \leq \eta_i(kT_s) \leq 1, \text{ if } \eta_i(kT_s) > 0, i \in J(kT_s) \\ \sum_{\eta_i(kT_s) \in [0,1], i \in J(kT_s)} \eta_i(kT_s) \bar{W}_a^i(kT_s) \geq (1 - \nu_1) P(kT_s) \end{array} \right. \quad (23)$$

239 where  $v_1$  and  $v_2$  are defined in the membership function in (8). Assume that the optimal solution of  
 240 (19) and (20) are  $\eta_i^0(kT_s)$  and  $\eta_i^1(kT_s)$ , respectively. Denote the shed load corresponding to  $\eta_i^0(kT_s)$   
 241 and  $\eta_i^1(kT_s)$  as  $\Psi^0(kT_s)$  and  $\Psi^1(kT_s)$ , respectively, i.e.,

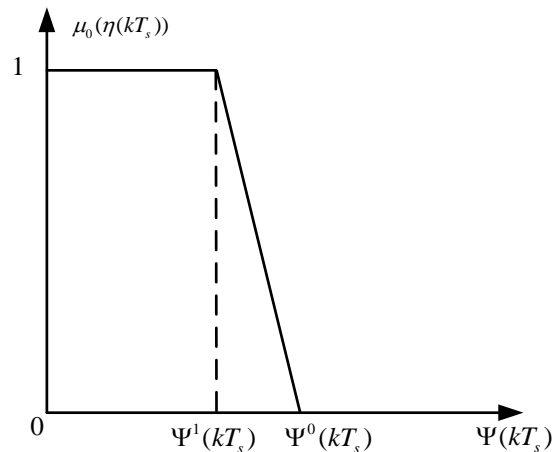
$$\Psi^0(kT_s) = \sum_{\eta_i^0(kT_s), i \in J(kT_s)} \eta_i^0(kT_s) \bar{W}_a^i(kT_s); \quad (24)$$

$$\Psi^1(kT_s) = \sum_{\eta_i^1(kT_s), i \in J(kT_s)} \eta_i^1(kT_s) \bar{W}_a^i(kT_s). \quad (25)$$

242 The degree of optimality is characterized by the following membership function  $\mu_0(\cdot)$  shown in  
 243 Figure 3 based on  $\Psi^0(kT_s)$  and  $\Psi^1(kT_s)$  as following :

$$\mu_0(\eta(kT_s)) = \begin{cases} 1, & \text{if } \Psi(kT_s) < \Psi^1(kT_s); \\ \frac{\Psi^0(kT_s) - \Psi(kT_s)}{\Psi^0(kT_s) - \Psi^1(kT_s)}, & \text{if } \Psi^1(kT_s) \leq \Psi(kT_s) \leq \Psi^0(kT_s); \\ 0, & \text{if } \Psi(kT_s) > \Psi^0(kT_s); \end{cases} \quad (26)$$

244 With the fuzzy constraint being transformed into the membership function  $\mu$  in (10) and the  
 245 objective function associated with the fuzzy constraint being transformed into the membership  
 246 function  $\mu_0$  in (26), the optimization in (6)-(8) is solved using a max-min approach as follows:



247  
 248 **Figure 3.** Membership function characterizing the degree of optimality.  
 249

$$\begin{cases} \max_{\eta_i(kT_s) \in [0,1], i \in J(kT_s)} \min(\mu_0(\eta(kT_s)), \mu(\eta(kT_s))) \\ \text{subject to } 0.1 \leq \eta_i(kT_s) \leq 1, \text{ if } \eta_i(kT_s) > 0, i \in J(kT_s). \end{cases} \quad (27)$$

250 The constrained max-min optimization problem in (27) can be implemented as a standard linear  
 251 programming problem:

$$\max_{\alpha \in [0,1], \eta_i(kT_s) \in [0,1], i \in J(kT_s)} \alpha \quad (28)$$

$$\text{subject to } 0.1 \leq \eta_i(kT_s) \leq 1, \text{ if } \eta_i(kT_s) > 0; \quad (29)$$

$$\mu_0(\eta(kT_s)) \geq \alpha; \quad (30)$$

$$\mu(\eta(kT_s)) \geq \alpha \quad (31)$$

252 where  $\eta_i(kT_s) \in [0,1], i \in J(kT_s)$ .

253 Substituting (26) into (30), the constraint in (30) is equivalent to:



$$\sum_{\eta_i(kT_s) \in [0,1], i \in J(kT_s)} \eta_i(kT_s) \bar{W}_a^i(kT_s) \leq (1-\alpha)\Psi^0(kT_s) + \alpha\Psi^1(kT_s) \quad (32)$$

254 Similarly, substituting (10) into (31), the constraint in (31) is equivalent to:

$$\sum_{\eta_i(kT_s) \in [0,1], i \in J(kT_s)} \eta_i(kT_s) \bar{W}_a^i(kT_s) \geq (1-\nu_1)P(kT_s) + \alpha(\nu_1 + \nu_2)P(kT_s). \quad (33)$$

255 Note that the constraint in (29) is not in a typical form of constraint for linear programming.  
 256 Define the surplus decision variables  $\gamma_i(kT_s) \in \{0, 1\}$   $i \in J(kT_s)$  and denote  $Q$  as a large constant, i.e.,  $Q$   
 257  $\gg 1$ . The constraint in (29) can be restated as the constraints as follows:

$$\eta_i(kT_s) \leq \gamma_i(kT_s) \times Q, \quad (34)$$

$$0.1 \times \gamma_i(kT_s) \leq \eta_i(kT_s), \quad \gamma_i(kT_s) \in \{0,1\}, \quad i \in J(kT_s). \quad (35)$$

258 Therefore, the fuzzy linear programming in (6)-(8) is solved based on the equivalent linear  
 259 programming problem in (28) with constraints in (32)-(35).

## 260 5. Computer Simulation

261 A set of 30 customers are selected to test the effectiveness and efficiency of the load aggregation  
 262 and the proposed TWDLC algorithm using fuzzy linear programming. The control interval was set  
 263 as a period of 5 consecutive days, 10:00 to 17:00 every day. The sampling interval  $T_s$  for the main  
 264 computer retrieving every customer's controllable load and conducting load shedding through the  
 265 gateway is set as 15 minutes. The sampling time for the gateway measuring the controllable load of  
 266 every chiller was is as 1 minute. The averaging interval  $T_m$  in (3) and the waiting interval  $T_w$  in (4) for  
 267 the gateway to calculate the average controllable load before and after every sampling time are both  
 268 set as 3 minutes. The capacity, time constraints and the maximum controllable load during the 5 day  
 269 control period are listed in Table 1.

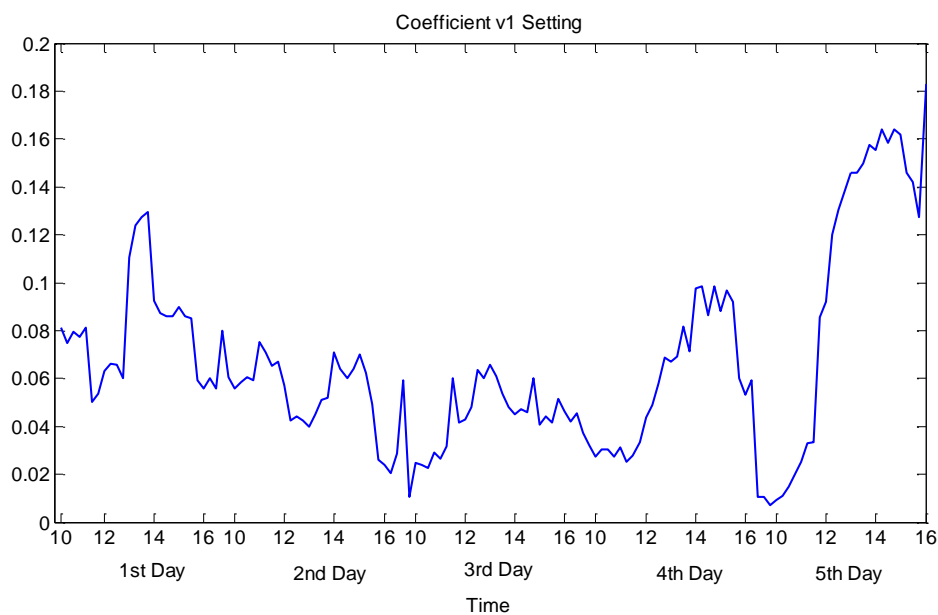
270  
271 **TABLE 1.**

272 List of capacity, maximum load within control interval and time constraints of customers under control

$i$	Capacity (kW)	Max( $w_a^i(kT_s)$ ) (kW)	$T_j^{uc}$ (min)	$T_j^{oc}$ (min)	$i$	Capacity (kW)	Max( $w_a^i(kT_s)$ ) (kW)	$T_j^{uc}$ (min)	$T_j^{oc}$ (min)
1	250	109	15	30	16	1300	727	15	16
2	350	219	30	15	17	1380	846	30	17
3	750	296	30	15	18	1550	1018	30	18
4	800	466	30	15	19	2000	1083	15	19
5	1000	585	30	15	20	2400	1198	30	20
6	1150	727	30	15	21	320	193	15	21
7	1350	772	30	15	22	530	253	30	22
8	1430	930	30	15	23	800	461	30	23
9	1800	1093	30	30	24	930	510	30	24
10	2100	1194	30	15	25	1100	669	30	25
11	250	140	30	15	26	1320	740	30	26
12	500	237	15	30	27	1400	905	30	27
13	800	347	15	30	28	1720	1045	30	28
14	800	494	30	30	29	2100	1131	30	29
15	1100	698	30	15	30	2400	1427	30	30

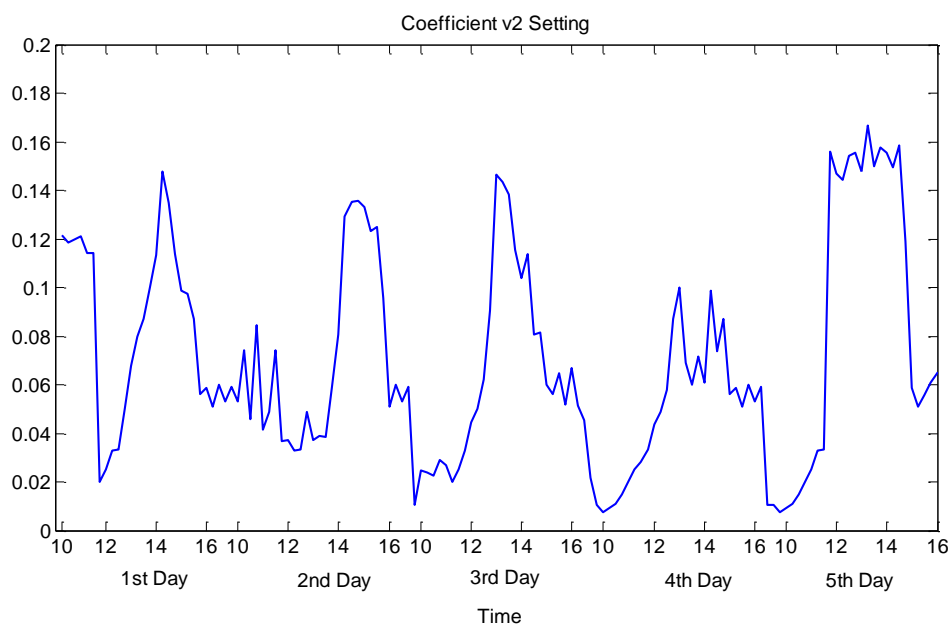
273

274 Recall that  $v_1$  and  $v_2$  in association with the membership function  $\mu(\cdot)$  in (10) characterize  
 275 the fuzzy constraint in (8) that the calculated load shed greater than or equal to the target load  $P(\cdot)$  to  
 276 a certain degree of precision tolerance. Both coefficients  $v_1$  and  $v_2$  could be either constants or  
 277 time-varying functions since the degree of precision tolerance for the fuzzy constraint in (8) can vary  
 278 with the temperature, load in regional power system, or time of a day, etc. In this paper, the  
 279 variations of  $v_1$  and  $v_2$  are both set as time-varying functions, as shown in Figures 4 and 5,  
 280 respectively. Using the proposed optimal real-time scheduling approach based on fuzzy linear  
 281 programming, the calculated load shed,  $\Psi(kT_s)$ , is shown in Figure 6. The upper and lower limits  
 282 for the fuzzy constraints in (8), i.e.,  $(1-v_1)P(kT_s)$  and  $(1+v_2)P(kT_s)$ , as well as the target load  
 283 expected to shed  $P(kT_s)$  are also compared with  $\Psi(kT_s)$  in Figure 6. It is shown in Figure 6 that  
 284 the calculated load shed  $\Psi(kT_s)$  matches the target load  $P(kT_s)$  well in response to the variations  
 285 in target load tolerance. The calculated shedding ratios and the corresponding controllable load of  
 286 the customer with the largest capacity are shown in Figures 7(a) and 7(b), respectively.



287  
288

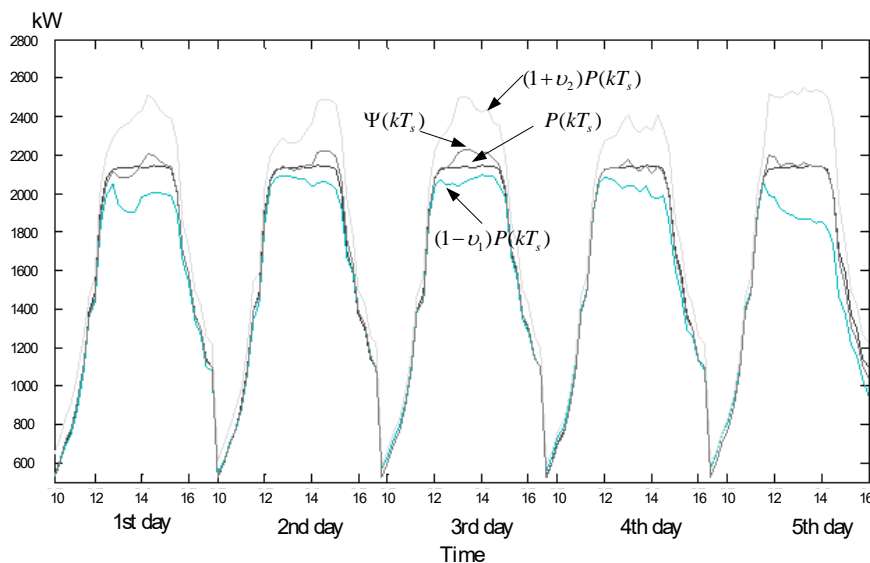
**Figure 4.** The variation of coefficient  $v_1$ .



289  
290

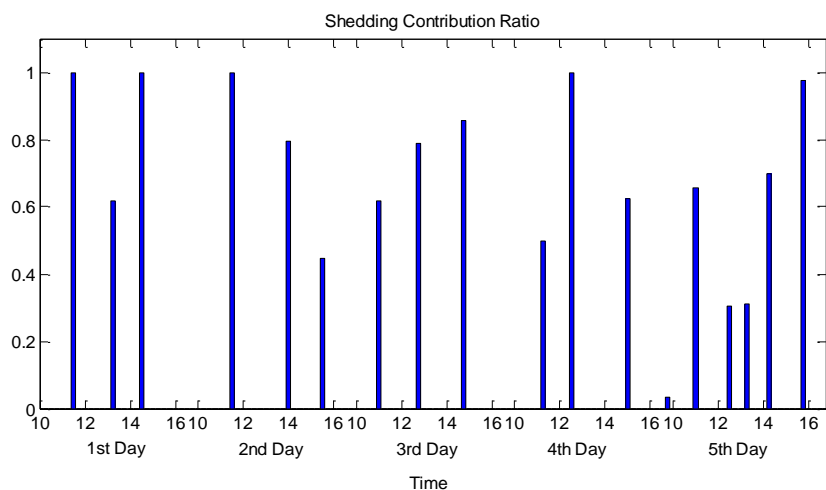
**Figure 5.** The variation of coefficient  $v_2$ .

291



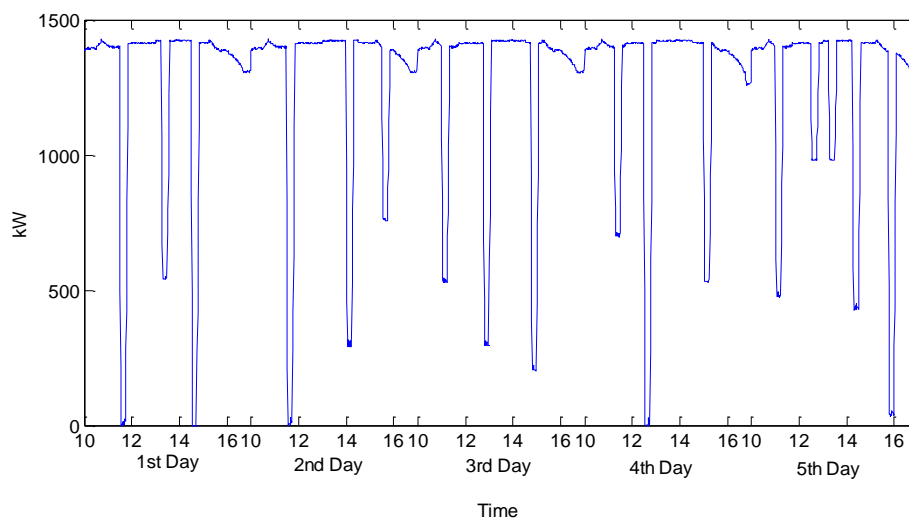
292  
293  
294  
295

**Figure 6.** Comparison of calculated load shed  $\Psi(kT_s)$ , the target load required to shed  $P(kT_s)$ , the upper and lower bound of target  $(1-\nu_1)P(kT_s)$  and  $(1+\nu_2)P(kT_s)$ .



296  
297

(a)



298  
299  
300  
301

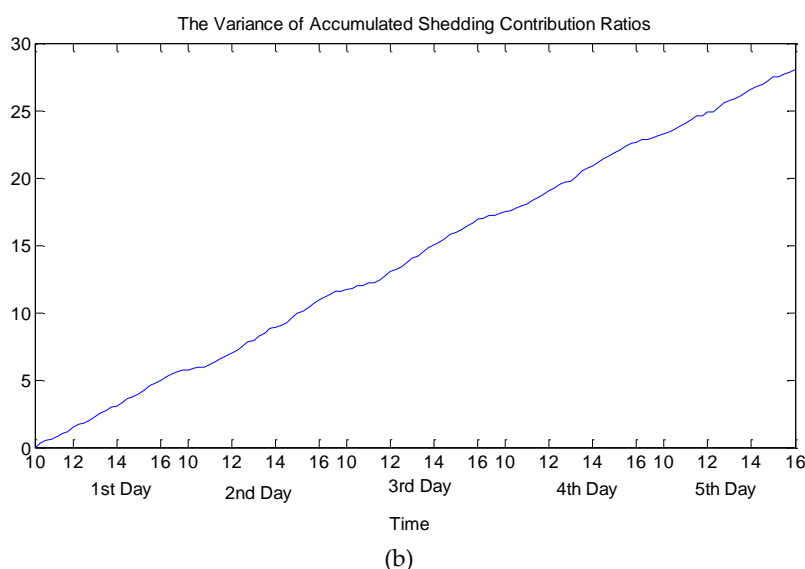
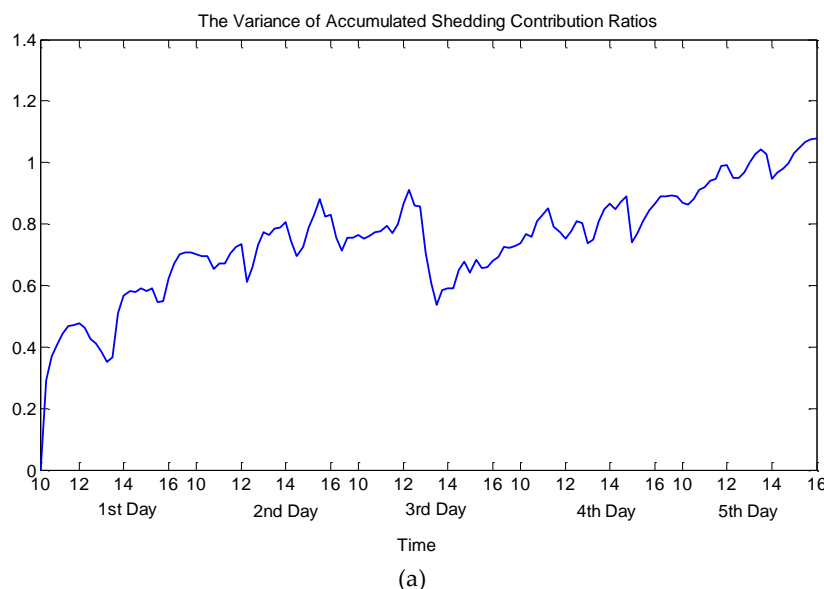
(b)

**Figure 7.** (a) Variation of expected shedding contribution ratios for the customer with the largest capacity. (b) Profile of controllable load for the customer with the largest capacity.

302 Both the number of shedding times and shedding ratio distributions vary among customers. In  
 303 order to show the effectiveness of the filtering scheme defined in (20), define the standard deviation  
 304 of the accumulated shedding contribution ratios at the  $k$ -th time step defined as:

$$\sigma(kT_s) = \sqrt{\frac{1}{N} \sum_{i=1}^N (\Omega_i(kT_s) - \bar{\Omega}(kT_s))^2} \quad (33)$$

305 where the accumulated shedding contribution ratio of the  $i$ -th customer  $\Omega_i(\cdot)$  and the average  
 306 accumulated shedding contribution ratio  $\bar{\Omega}(\cdot)$  are shown in (18) and (19), respectively. The variation  
 307 profiles of  $\sigma(\cdot)$  with and without the filtering scheme in (20) are both shown in Figures 8(a) and  
 308 8(b), respectively. Figure 8(a) shows that the standard deviation of accumulated shedding  
 309 contribution ratios varies within a limited range as time goes on if the filtering scheme in (20) is  
 310 applied. This shows that the filtering scheme levels off every customer's contribution to the load  
 311 shedding and achieves load shedding fairness. Conversely, if the filtering scheme in (20) is removed  
 312 from the customer selection process, the standard deviation of the accumulated shedding  
 313 contribution ratios increased drastically with time.



**Figure 8.** Variation of standard deviation of shedding contribution ratios (a) with (b) without filtering scheme in (20).

## 321 6. Conclusion

322 A real-time TWDLC optimization scheme is proposed as an effective demand response  
323 approach by scheduling the direct load control of the central air-conditioning chillers in wide area.  
324 The proposed TWDLC works well through the broadband network with gateway installed at the site  
325 of every customer under control. Fuzzy linear programming is utilized for optimization providing  
326 more optimization flexibility by allowing a precision tolerance for the shed load constraints. It is  
327 shown in simulation that the proposed TWDLC scheme is computationally efficient and effective,  
328 hence feasible for real-time optimization and time-varying precision tolerance in response to  
329 variable target load profile.

330 For future work, the degree of precision tolerance for the fuzzy constraints can be linked with  
331 weather condition, regional load, time of a day, etc., depending on the application scenario. Delicate  
332 modeling can be designed to automatically adjust the precision tolerance in response to environment  
333 changes. Type-2 membership functions can also be used to define precision tolerance of constraints  
334 for future work.

336 **Author Contributions:** Leehter Yao conceived and designed the main ideas proposed in the paper. He also  
337 wrote the paper. Lei Yao designed and performed the experiments. Wei Hong Lim designed the fuzzy linear  
338 programming model and analyzed the data.

339 **Conflicts of Interest:** The authors declare no conflict of interest.

## 340 References

- 341 1. F. Rahimi and A. Ipakchi, "Demand response as a market resource under the smart grid paradigm," *IEEE*  
342 *Trans. Smart Grid*, vol. 1, no. 1, pp. 82-88, Jun. 2010.
- 343 2. J. Medina, N. Muller, and I. Roytelman, "Demand response and distribution grid operations: opportunities  
344 and challenges," *IEEE Trans. Smart Grid*, vol. 1, no. 2, pp. 193-198, Sep. 2010.
- 345 3. P. P. Varaiya, F. F. Wu, and J. W. Bialek, "Smart operation of smart grid: risk-limiting dispatch," *IEEE Proc.*,  
346 vol. 99, no. 1, pp. 40-57, Jan. 2011.
- 347 4. S. Pullins and J. Westerman, San Diego Smart Grid Study Final Report, Science Applications Int. Corp.,  
348 Oct. 2006.
- 349 5. M. H. Albadi and E. F. El-Saadany, "Demand response in electricity markets: an overview," in Proc. IEEE  
350 PES General Meeting, June 2007, pp. 1-5.
- 351 6. U.S. Department of Energy, "Benefits of demand response in electricity markets and recommendations for  
352 achieving them," Report to the United State Congress, Feb. 2006. Available online: <http://eetd.lbl.gov>
- 353 7. P. Reusens, D. V. Bruyssel, J. Sevenhans, S. V. D. Bergh, B. V. Nimmen, and P. Spruyt, "A practical ADSL  
354 technology following a decade of effort," *IEEE Commun. Mag.*, pp. 145-151, Oct. 2001.
- 355 8. F. Ouyang, P. Duvaut, O. Moreno, and L. Pierrugues, "The first step of long-reach ADSL: smart DSL  
356 technology, READSL," *IEEE Commun. Mag.*, pp. 124-131, Sep. 2003.
- 357 9. M. Parvania and M. Fotuhi-Firuzabad, "Demand response scheduling by stochastic SCUC," *IEEE Trans.*  
358 *Smart Grid*, vol. 1, no. 1, pp. 89-98, Jun. 2010.
- 359 10. A. Brooks, E. Lu, D. Reicher, C. Spirakis, and B. Wehl, "Demand dispatch," *IEEE Power Energy Mag.*, vol.  
360 8, no. 3, pp. 20-29, May-Jun. 2010.
- 361 11. L. Yao, W.-C. Chang, and R.-L. Yen, "An iterative deepening genetic algorithm for scheduling of direct  
362 load control," *IEEE Trans. Power Syst.*, vol. 20, no. 3, pp. 1414-1421, Aug. 2005.
- 363 12. L. Yao, Y.-C. Chou, and C.-C. Lin, "Scheduling of direct load control using genetic programming," *Int. J.*  
364 *Innov. Comput. I.*, vol. 7, no. 5, pp. 2515-2528, May 2011.
- 365 13. A. Gomes, C. H. Antunes, and A. G. Martins, "A multiple objective approach to direct load control using  
366 an interactive evolutionary algorithm," *IEEE Trans. Power Syst.*, vol. 22, no. 3, pp. 1004-1011, Aug. 2007.
- 367 14. A. Gomes, C. Antunes, and E. Oliveira, "Direct load control in the perspective of an electricity retailer - a  
368 multi-objective evolutionary approach," in *Soft Comput. Ind. Appl.*, ed: Springer Berlin Heidelberg, Apr.  
369 2011, pp. 13-26.
- 370 15. F. Luo, J. Zhao, Z. Y. Dong, X. Tong, Y. Chen, H. Yang, et al., "Optimal dispatch of air conditioner loads in  
371 southern china region by direct load control," *IEEE Trans. Smart Grid*, vol. 7, no. 1, pp. 439-450, Jan. 2016.

- 372 16. F. Luo, J. Zhao, H. Wang, X. Tong, Y. Chen, and Z. Y. Dong, "Direct load control by distributed imperialist  
373 competitive algorithm," *J. Mod. Power Syst. Clean Energy*, vol. 2, no. 4, pp. 385-395, Dec. 2014.
- 374 17. F. Luo, Z. Xu, K. Meng, and Z. Y. Dong, "Optimal operation scheduling for microgrid with high  
375 penetrations of solar power and thermostatically controlled loads," *Sci. Technol. Build Environ.*, vol. 22,  
376 no. 6, pp. 666-673, Jul. 2016.
- 377 18. F. Luo, Z. Y. Dong, K. Meng, J. Wen, H. Wang, and J. Zhao, "An operational planning framework for  
378 large-scale thermostatically controlled load dispatch," *IEEE Trans. Ind. Informat.*, vol. 13, no. 1, pp.  
379 217-227, Feb. 2017.
- 380 19. L. Yao and H.-R. Lu, "A two-way direct control of central air-conditioning load via Internet," *IEEE Trans.*  
381 *Power Del.*, vol. 24, no. 1, pp. 240-248, Jan. 2009.
- 382 20. D. Wang, K. Meng, X. Gao, C. Coates, and Z. Dong, "Optimal air-conditioning load control in distribution  
383 network with intermittent renewables," *J. Mod. Power Syst. Clean Energy*, vol. 5, no. 1, pp. 55-65, Jan.  
384 2017.
- 385 21. L. Goel, Q. Wu, and P. Wang, "Fuzzy logic-based direct load control of air conditioning loads considering  
386 nodal reliability characteristics in restructured power system," *Electr. Pow. Syst. Res.*, vol. 80, no. 1, pp.  
387 98-107, Jan. 2010.
- 388 22. L. Zhu, Z. Yan, W.-J. Lee, X. Yang, Y. Fu, and W. Cao, "Direct load control in microgrids to enhance the  
389 performance of integrated resources planning," *IEEE Trans. Ind. Appl.*, vol. 51, no. 5, pp. 3553-3560,  
390 Sep.-Oct. 2015.
- 391 23. Q. Cui, X. Wang, X. Wang, and Y. Zhang, "Residential appliances direct load control in real-time using  
392 cooperative game," *IEEE Trans. Power Syst.*, vol. 31, no. 1, pp. 226-233, Jan. 2016.
- 393 24. J. L. Mathieu, S. Koch, and D. S. Callaway, "State estimation and control of electric loads to manage  
394 real-time energy imbalance," *IEEE Trans. Power Syst.*, vol. 28, no. 1, pp. 430-440, Feb. 2013.
- 395 25. R. Argiento, R. Faranda, A. Pievatolo, and E. Tironi, "Distributed interruptible load shedding and  
396 micro-generator dispatching to benefit system operations," *IEEE Trans. Power Syst.*, vol. 27, no. 2, pp.  
397 840-848, May 2012.
- 398 26. S. C. Lee, S. J. Kim, and S. H. Kim, "Demand side management with air conditioner loads based on the  
399 queuing system model," *IEEE Trans. Power Syst.*, vol. 26, no. 2, pp. 661-668, May 2011.
- 400 27. M. Sullivan, J. Bode, B. Kellow, S. Woehleke, and J. Eto, "Using residential AC load control in grid  
401 operations: PG&E's ancillary service pilot," *IEEE Trans. Smart Grid*, vol. 4, no. 2, pp. 1162-1170, Jun. 2013.
- 402 28. D. T. Nguyen, M. Negnevitsky, and M. d. Groot, "Walrasian market clearing for demand response  
403 exchange," *IEEE Trans. Power Syst.*, vol. 27, no. 1, pp. 535-544, Feb. 2012.
- 404 29. L. Gkatzikis, I. Koutsopoulos, and T. Salonidis, "The role of aggregators in smart grid demand response  
405 markets," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 7, pp. 1247-1257, Jul. 2013.
- 406 30. L.-X. Wang, *A Course in Fuzzy Systems and Control*, Pearson Education Taiwan, 2005.