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NS-Cross Entropy Based MAGDM under Single Valued Neutrosophic Set Environment

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Abstract: Single valued neutrosophic set has king power to express uncertainty characterized by indeterminacy, inconsistency and incompleteness. Most of the existing single valued neutrosophic cross entropy bears an asymmetrical behavior and produce an undefined phenomenon in some situations. In order to deal with these disadvantages, we propose a new cross entropy measure under single valued neutrosophic set (SVNS) environment namely SN- cross entropy and prove its basic properties. Also we define weighted SN-cross entropy measure and investigate its basic properties. We develop a new multi attribute group decision making (MAGDM) strategy for ranking of the alternatives based on the proposed weighted SN-cross entropy measure between each alternative and the ideal alternative. Finally, a numerical example of MAGDM problem of investment potential is solved to show the validity and efficiency of proposed decision making strategy. We also present comparative anslysis of the obtained result with the results obtained form the existing solution strategies in the solution.

Keywords: neutrosophic set; single valued neutrosophic set; SN-cross entropy function; multi-attribute group decision making

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1. Introduction

To tackle uncertainty and modeling real and scientific problems, Zadeh [1] first introduced the fuzzy set by definig membership function in 1965. Bellman and Zadeh [2] contributed an imporatnt research on fuzzy decision making using max and min operators. Atanassov [3] established intuitionistic fuzzy set (IFS) in 1986 by adding non-membership function as an indepent component to the fuzzy set. Theoretical and practical applications of IFSs in multi-criteria decision making (MCDM) have been reported in the literature [4-12]. Zadeh [13] introduced entropy measure in fuzzy environment. Burillo and Bustince [14] proposed distance measure between IFSs and offered an axiomatic definition of entropy measure. In IFS environment, Szmidt and Kacprzyk [15] proposed a new entropy measure based on geometric interpretation of IFS. Wei et al. [16] developed an entropy measure for interval-valued intuitionistic fuzzy set (IVIFS)and presented applications in pattern recognition and MCDM. Li [17] presented a new MADM strategy combining entropy and TOPSIS in IVIFS environment. Shang and Jiang [18] introduced the cross entropy in fuzzy environment. Vlachos and Sergiadis [19] presented intuitionistic fuzzy cross entropy by extending fuzzy cross entropy [18].

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Ye [20] defined a new cross entropy in in IVIFS environment and presented an optimal decision-making strategy. Xia and Xu [21] put forward new entropy and cross entropy and employed them for multi- attribute criteria group decision making (MAGDM) strategy in IFS environment. Tong and Yu [22] defined cross entropy in IVIFs environment and applied it to MADM problems.

The study of uncertainty entered into a new direction after the publication of neutrosophic set (NS) [23] and single valued neutrosophic set (SVNS) [24]. SVNS draws more appeal to the rersearchers for its applicability in decision making [25-54], conflict resolution [55], educational problems [56, 57], image processing [58-60], cluster analysis [61, 62], social problems [63, 64], etc. The research on SVNS gets momentum after the inception of the international journal "Neutrosophic Sets and Systems". Combining with neutrosophic set, a number of hybrid sets such as neutrosophic soft set [65-70], neutrosophic complex set [71], interval complex neutrosophic set [72], rough neutrosophic set [73-80], neutrosophic soft expert set [81, 82], rough neutrosophic bipolar set [83], rough neutrosophic tri complex set [84], neutrosophic rough hyper complex set [85], are reported in the literature. Wang et al. [86] defined interval neutrosophic set (INS). Majumdar and Samanta [87] defined an entropy measure and presented an MCDM strategy under SVNS environment. Ye [88] defined cross entropy for SVNS by extending the intuitionistic fuzzy cross entropy [7] and proposed MCDM strategy under SVNS environment. Sahin [89] proposed two cross entropy measures for INSs and proposed MCGDM strategy. Tian et al. [90] proposed a cross entropy for INSs and developed a MCDM strategy based on the cross entropy and TOPSIS. Ye [91] defined cross entropy measures for SVNSs and INSs to overcome the drawback of the existing cross entropy measures. Due to little research of cross entropy measures, we define a new cross entropy measure in SVNSs environment based on the distance function of two points and prove its basic properties. Also, we define single valued weighted cross entropy measure and investigate its properties. Getting motivation from the work of Ye [91] for MCDM, We propose a novel MAGDM strategy using the proposed weighted cross entropy.

The remaining of the paper is presented as follows:

Section 2 describes some concepts of SVNSs. In Section 3 we propose a new cross entropy measure between two SVNSs and investigate its properties.

In section 4, we develop a novel MAGDM strategy based on the proposed SN-cross entropy with SVNS information. In Section 5 we present comparative study and discussion. In section 6 an illustrative example is solved to demonstrate the applicability and efficiency of the developed MAGDM strategy under SVNS environment. Section 7 offers conclusions and perspectives of future work.

2. Preliminaries

This section presents a short list of mostly known definitions pertaining to this paper.

Definition 1. [23] NS

Let U be a space of points (objects) with a generic element in U denoted by u, i.e. $u \in U$. A neutrosophic set A in U is characterized by truth-membership function $T_A(u)$, indeterminacy-membership function $I_A(u)$ and falsity-membership function $F_A(u)$, where $T_A(u)$, $I_A(u)$, $F_A(u)$ are the functions from U to $]^-0, 1^+[$ i.e. $T_A(u), I_A(u), F_A(u): U \to\ ]^-0, 1^+[$. NS can be expressed as $A = \{<u; (T_A(u), I_A(u), F_A(u))>: \forall u \in U\}$. Since $T_A(u), I_A(u), F_A(u)$ are the subsets of $]^-0, 1^+[$, there the sum $(T_A(u) + I_A(u) + F_A(u))$ lies between $^-0$ and 3^+.

Example 1. Suppose that $U = \{u_1, u_2, u_3, ...\}$ be the universal set. Let R_1 be any neutrosophic set in U. Then R_1 expressed as $R_1 = \{<u_1; (0.6, 0.3, 0.4)>: u_1 \in U\}$.

Definition 2. [24] SVNS

Assume that U be a space of points (objects) with generic elements $u \in U$. A SVNS H in U is characterized by a truth-membership function $T_H(u)$, an indeterminacy-membership function $I_H(u)$, and a falsity-membership function $F_H(u)$, where $T_H(u)$, $I_H(u)$, $F_H(u) \in [0, 1]$ for each point u in U. Therefore, a SVNS A can be expressed as $H = \{u, (T_H(u), I_H(u), F_H(u)) \mid \forall u \in U\}$, whereas, the sum

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of $T_H(u)$, $I_H(u)$ and $F_H(u)$ satisfy the condition $0 \le T_H(u) + I_H(u) + F_H(u) \le 3$ and $H(u) = <(T_H(u), I_H(u), F_H(u)>$ call a single valued neutrosophic number (SVNN).

Example 1.

A SVNS H in U can be expressed as: $H = \{u, (0.7, 0.3, 0.5) \mid u \in U\}$ and SVNN presented $H = < 0.7, 0.3, 0.5>$.

Definition 3. [24] Inclusion of SVNSs

The inclusion of any two SVNS sets H_1 and H_2 in U is denoted by $H_1 \subseteq H_2$ and defined as follows:

$H_1 \subseteq H_2$, iff $T_{H_1}(u) \le T_{H_2}(u), I_{H_1}(u) \ge I_{H_2}(u), F_{H_1}(u) \ge F_{H_2}(u)$ for all $u \in U$.

Example 2.

Let H_1 and H_2 be any two SVNNs in U presented as follows: $H_1 = <(.7, .3, .5)>$ and $H_2 = <(.8, .2, .4)>$ for all $u \in U$. Using the property of inclusion of two SVNNs, we conclude that $H_1 \subseteq H_2$.

Definition 4. [24] Equality of two SVNSs

The equality of any two SVNS H_1 and H_2 in U denoted by $H_1 = H_2$ and defined as follows:

$T_{H_1}(u) = T_{H_2}(u), I_{H_1}(u) = I_{H_2}(u)$ and $F_{H_1}(u) = F_{H_2}(u)$ for all $u \in U$.

Definition 5. Complement of any SVNSs

The complement of any SVNS H in U denoted by H^c and defined as follows:

$H^c = \{u, 1 - T_H, 1 - I_H, 1 - F_H \mid u \in U\}$.

Example 3.

Let H be any SVNN in U presented as follows:

$H = <(.7, .3, .5)>$. Then compliment of H is obtained as $H^c = <(.3, .7, .5)>$.

Definition 6. [24] Union

The union of two single valued neutrosophic sets H_1 and H_2 is a neutrosophic set H_3 (say) written as

$H_3 = H_1 \cup H_2$.

$$T_{H_3}(u) = \max\{T_{H_1}(u),\ T_{H_2}(u)\},\ I_{HJ_3}(u) = \min\{I_{H_1}(u),\ I_{H_2}(u)\},\ F_{H_3}(u) = \min\{F_{H_1}(u),\ F_{H_2}(u)\},\ \forall\ u \in U.$$

Example 4. Let H_1 and H_2 be two SVNSs in U presented as follows:

$H_1 = <(0.6, 0.3, 0.4)>$ and $H_2 = <(0.7, 0.3, 0.6)>$. Then union of them is presented as:

$H_1 \cup H_2 = <(0.7, 0.3, 0.4)>$.

Definition 7. [24] Intersection

The intersection of two single valued neutrosophic sets H_1 and H_2 denoted by H_4 and defined as

$H_4 = H_1 \cap H_2$

$$T_{H_4}(u) = \min\{T_{H_1}(u),\ T_{H_2}(u)\},\ I_{H_4}(u) = \max\{I_{H_1}(u),\ I_{H_2}(u)\}$$

$$F_{H_4}(u) = \max\{F_{H_1}(u),\ F_{H_2}(u)\},\ \forall\ u \in U.$$

Example 5. Let H_1 and H_2 be two SVNSs in U presented as follows:

$H_1 = <(0.6, 0.3, 0.4)>$ and $H_2 = <(0.7, 0.3, 0.6)>$. Then intersection of H_1 and H_2 is presented as follows:

$H_1 \cap H_2 = <(0.6, 0.3, 0.6)>$

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Some operations of SVNSs [24]:

Let H_1 and H_2 be any two SVNSs. Then, addition and multiplication are defined as:

1. $H_1 \oplus H_2 = < T_{H_1}(u) + T_{H_2}(u) - T_{H_1}(u) \,.\, T_{H_2}(u),\ I_{H_1}(u) \,.\, I_{H_2}(u),\ F_{H_1}(u) \,.\, F_{H_2}(u) > \ \forall\ u \in U.$
2. $H_1 \otimes H_2 = < T_{H_1}(u) \,.\, T_{H_2}(u),\ I_{H_1}(u) + I_{H_2}(u) - I_{H_1}(u) \,.\, I_{H_2}(u),\ F_{H_1}(u) + F_{H_2}(u) - F_{H_1}(u) \,.$
$F_{H_2}(u) >$
$\forall\ u \in U.$

Example 6. Let H_1 and H_2 be two SVNSs in U presented as follows:

H_1 = <0.6, 0.3, 0.4> and H_2 = <0.7, 0. 3, 0. 6>.

Then, 1. $H_1 \oplus H_2$ = <0.88, 0.09, 0.24>

2. $H_1 \otimes H_2$ = <0.42, 0.51, 0.76>.

3. SN-cross entropy function

In this section, we define a new single valued neutrosophic cross-entropy function for measuring the deviation of single valued neutrosophic variables from an a priori one.

Definition 6. 1. SN-cross entropy function

Let H_1 and H_2 be any two SVNSs in U = $\{u_1, u_2, u_3, ..., u_n\}$. Then, the single valued cross-entropy of H_1 and H_2 is denoted by CE_{SN} (H_1, H_2) and defined as follows:

$$CE_{SN}(H_1, H_2) = \frac{1}{2}\left\{\sum_{i=1}^{n}\left\langle\left[\frac{2\left|T_{H_1}(u_i) - T_{H_2}(u_i)\right|}{\sqrt{1+\left|T_{H_1}(u_i)\right|^2} + \sqrt{1+\left|T_{H_2}(u_i)\right|^2}} + \frac{2\left|(1-T_{H_1}(u_i)) - (1-T_{H_2}(u_i))\right|}{\sqrt{1+\left|(1-T_{H_1}(u_i))\right|^2} + \sqrt{1+\left|(1-T_{H_2}(u_i))\right|^2}}\right] + \right.\right.$$
$$\left[\frac{2\left|I_{H_1}(u_i) - I_{H_2}(u_i)\right|}{\sqrt{1+\left|I_{H_1}(u_i)\right|^2} + \sqrt{1+\left|I_{H_2}(u_i)\right|^2}} + \frac{2\left|(1-I_{H_1}(u_i)) - (1-I_{H_2}(u_i))\right|}{\sqrt{1+\left|(1-I_{H_1}(u_i))\right|^2} + \sqrt{1+\left|(1-I_{H_2}(u_i))\right|^2}}\right] +$$
$$\left.\left.\left[\frac{2\left|F_{H_1}(u_i) - F_{H_2}(u_i)\right|}{\sqrt{1+\left|F_{H_1}(u_i)\right|^2} + \sqrt{1+\left|F_{H_2}(u_i)\right|^2}} + \frac{2\left|(1-F_{H_1}(u_i)) - (1-F_{H_2}(u_i))\right|}{\sqrt{1+\left|(1-F_{H_1}(u_i))\right|^2} + \sqrt{1+\left|(1-F_{H_2}(u_i))\right|^2}}\right]\right\rangle\right\} \quad (1)$$

Example 4.
Let H_1 and H_2 be two SVNSs in U, which are given by H_1 = {u, (.7, .3, .4)| u ∈U} and H_2 = {u, (.6, .4, .2)| u ∈U}. Using Equation (1), the cross entropy value of H_1 and H_2 is obtained as $CE_{SN}(H_1, H_2) = 0.707$.
Theorem
Single valued neutrosophic cross entropy $CE_{SN}(H_1, H_2)$ for any two SVNSs H_1, H_2, satisfies the following properties:
i) $CE_{SN}(H_1, H_2) \geq 0$.
ii) $CE_{SN}(H_1, H_2) = 0$ if and only if $T_{H_1}(u_i) = T_{H_2}(u_i)$, $I_{H_1}(u_i) = I_{H_2}(u_i)$, $F_{H_1}(u_i) = F_{H_2}(u_i)$, $\forall u_i \in U$.
iii) $CE_{SN}(H_1, H_2) = CE_{SN}(H_1^c, H_2^c)$
iv) $CE_{SN}(H_1, H_2) = CE_{SN}(H_2, H_1)$
Proof: i)

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For all values of $u_i \in U$, $|T_{H_1}(u_i)| \geq 0$, $|T_{H_2}(u_i)| \geq 0$, $|T_{H_1}(u_i) - T_{H_2}(u_i)| \geq 0$, $\sqrt{1+|T_{H_1}(u_i)|^2} \geq 0$, $\sqrt{1+|T_{H_2}(u_i)|^2} \geq 0$, $|(1-T_{H_1}(u_i))| \geq 0$, $|(1-T_{H_2}(u_i))| \geq 0$, $|(1-T_{H_1}(u_i)) - (1-T_{H_2}(u_i))| \geq 0$, $\sqrt{1+|(1-T_{H_1}(u_i))|^2} \geq 0$, $\sqrt{1+|(1-T_{H_2}(u_i))|^2} \geq 0$

Then,

$$\left[\frac{2|T_{H_1}(u_i) - T_{H_2}(u_i)|}{\sqrt{1+|T_{H_1}(u_i)|^2} + \sqrt{1+|T_{H_2}(u_i)|^2}} + \frac{2|(1-T_{H_1}(u_i)) - (1-T_{H_2}(u_i))|}{\sqrt{1+|(1-T_{H_1}(u_i))|^2} + \sqrt{1+|(1-T_{H_2}(u_i))|^2}}\right] \geq 0$$

Similarly, $\left[\frac{2|I_{H_1}(u_i) - I_{H_2}(u_i)|}{\sqrt{1+|I_{H_1}(u_i)|^2} + \sqrt{1+|I_{H_2}(u)|^2}} + \frac{2|(1-I_{H_1}(u_i)) - (1-I_{H_2}(u_i))|}{\sqrt{1+|(1-I_{H_1}(u_i))|^2} + \sqrt{1+|(1-I_{H_2}(u_i))|^2}}\right] \geq 0$, and

$$\left[\frac{2|F_{H_1}(u_i) - F_{H_2}(u_i)|}{\sqrt{1+|F_{H_1}(u_i)|^2} + \sqrt{1+|F_{H_2}(u_i)|^2}} + \frac{2|(1-F_{H_1}(u_i)) - (1-F_{H_2}(u_i))|}{\sqrt{1+|(1-F_{H_1}(u_i))|^2} + \sqrt{1+|(1-F_{H_2}(u_i))|^2}}\right] \geq 0$$

So, $CE_{SN}(H_1, H_2) \geq 0$.

Hence complete the proof.

ii)

$$\left[\frac{2|T_{H_1}(u_i) - T_{H_2}(u_i)|}{\sqrt{1+|T_{H_1}(u_i)|^2} + \sqrt{1+|T_{H_2}(u_i)|^2}} + \frac{2|(1-T_{H_1}(u_i)) - (1-T_{H_2}(u_i))|}{\sqrt{1+|(1-T_{H_1}(u_i))|^2} + \sqrt{1+|(1-T_{H_2}(u_i))|^2}}\right] = 0,$$

$$\Leftrightarrow T_{H_1}(u_i) = T_{H_2}(u_i)$$

$$\left[\frac{2|I_{H_1}(u_i) - I_{H_2}(u_i)|}{\sqrt{1+|I_{H_1}(u_i)|^2} + \sqrt{1+|I_{H_2}(u)|^2}} + \frac{2|(1-I_{H_1}(u_i)) - (1-I_{H_2}(u_i))|}{\sqrt{1+|(1-I_{H_1}(u_i))|^2} + \sqrt{1+|(1-I_{H_2}(u_i))|^2}}\right] = 0$$

$\Leftrightarrow I_{H_1}(u_i) = I_{H_2}(u_i)$, and

$$\left[\frac{2|F_{H_1}(u_i) - F_{H_2}(u_i)|}{\sqrt{1+|F_{H_1}(u_i)|^2} + \sqrt{1+|F_{H_2}(u_i)|^2}} + \frac{2|(1-F_{H_1}(u_i)) - (1-F_{H_2}(u_i))|}{\sqrt{1+|(1-F_{H_1}(u_i))|^2} + \sqrt{1+|(1-F_{H_2}(u_i))|^2}}\right] = 0,$$

$$\Leftrightarrow F_{H_1}(u_i) = F_{H_2}(u_i)$$

So, $CE_{SN}(H_1, H_2) = 0$ iff $T_{H_1}(u_i) = T_{H_2}(u_i)$ $I_{H_1}(u_i) = I_{H_2}(u_i)$, $F_{H_1}(u_i) = F_{H_2}(u_i)$, $\forall u_i \in U$.

Hence complete the proof.

iii) Using definition 5, we obtain the following expression

$$CE_{SN}(H_1^c, H_2^c) = \frac{1}{2}\left\{\sum_{i=1}^{n}\left\langle\left[\frac{2|(1-T_{H_1}(u_i)) - (1-T_{H_2}(u_i))|}{\sqrt{1+|(1-T_{H_1}(u_i))|^2} + \sqrt{1+|(1-T_{H_2}(u_i))|^2}} + \frac{2|T_{H_1}(u_i) - T_{H_2}(u_i)|}{\sqrt{1+|T_{H_1}(u_i)|^2} + \sqrt{1+|T_{H_2}(u_i)|^2}}\right] + \right.\right.$$

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$$\left[\frac{2\left|(1-I_{H_1}(u_i))-(1-I_{H_2}(u_i))\right|}{\sqrt{1+\left|(1-I_{H_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{H_2}(u_i))\right|^2}}+\frac{2\left|I_{H_1}(u_i)-I_{H_2}(u_i)\right|}{\sqrt{1+\left|I_{H_1}(u_i)\right|^2}+\sqrt{1+\left|I_{H_2}(u)\right|^2}}\right]+$$
$$\left.\left.\left[\frac{2\left|(1-F_{H_1}(u_i))-(1-F_{H_2}(u_i))\right|}{\sqrt{1+\left|(1-F_{H_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{H_2}(u_i))\right|^2}}+\frac{2\left|F_{H_1}(u_i)-F_{H_2}(u_i)\right|}{\sqrt{1+\left|F_{H_1}(u_i)\right|^2}+\sqrt{1+\left|F_{H_2}(u_i)\right|^2}}\right]\right\rangle\right\}$$

$$=\frac{1}{2}\left\{\sum_{i=1}^{n}\left\langle\left[\frac{2\left|T_{H_1}(u_i)-T_{H_2}(u_i)\right|}{\sqrt{1+\left|T_{H_1}(u_i)\right|^2}+\sqrt{1+\left|T_{H_2}(u_i)\right|^2}}+\frac{2\left|(1-T_{H_1}(u_i))-(1-T_{H_2}(u_i))\right|}{\sqrt{1+\left|(1-T_{H_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{H_2}(u_i))\right|^2}}\right]+\right.\right.$$
$$\left[\frac{2\left|I_{H_1}(u_i)-I_{H_2}(u_i)\right|}{\sqrt{1+\left|I_{H_1}(u_i)\right|^2}+\sqrt{1+\left|I_{H_2}(u)\right|^2}}+\frac{2\left|(1-I_{H_1}(u_i))-(1-I_{H_2}(u_i))\right|}{\sqrt{1+\left|(1-I_{H_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{H_2}(u_i))\right|^2}}\right]+$$

$$\left.\left.\left[\frac{2\left|F_{H_1}(u_i)-F_{H_2}(u_i)\right|}{\sqrt{1+\left|F_{H_1}(u_i)\right|^2}+\sqrt{1+\left|F_{H_2}(u_i)\right|^2}}+\frac{2\left|(1-F_{H_1}(u_i))-(1-F_{H_2}(u_i))\right|}{\sqrt{1+\left|(1-F_{H_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{H_2}(u_i))\right|^2}}\right]\right\rangle\right\}=CE_{SN}(H_1,H_2)$$

So, $CE_{SN}(H_1,H_2)=CE_{SN}(H_1^c,H_2^c)$.

Hence complete the proof.

iv) Since,

$\left|T_{H_1}(u_i)-T_{H_2}(u_i)\right|=\left|T_{H_2}(u_i)-T_{H_1}(u_i)\right|$, $\left|I_{H_1}(u_i)-I_{H_2}(u_i)\right|=\left|I_{H_2}(u_i)-I_{H_1}(u_i)\right|$,

$\left|F_{H_1}(u_i)-F_{H_2}(u_i)\right|=\left|F_{H_2}(u_i)-F_{H_1}(u_i)\right|$, $\left|(1-T_{H_1}(u_i))-(1-T_{H_2}(u_i))\right|=\left|(1-T_{H_2}(u_i))-(1-T_{H_1}(u_i))\right|$,

$\left|(1-I_{H_1}(u_i))-(1-I_{H_2}(u_i))\right|=\left|(1-I_{H_2}(u_i))-(1-I_{H_1}(u_i))\right|$,

$\left|(1-F_{H_1}(u_i))-(1-F_{H_2}(u_i))\right|=\left|(1-F_{H_2}(u_i))-(1-F_{H_1}(u_i))\right|$, then

$$\sqrt{1+\left|T_{H_1}(u_i)\right|^2}+\sqrt{1+\left|T_{H_2}(u_i)\right|^2}=\sqrt{1+\left|T_{H_2}(u_i)\right|^2}+\sqrt{1+\left|T_{H_1}(u_i)\right|^2},$$

$$\sqrt{1+\left|I_{H_1}(u_i)\right|^2}+\sqrt{1+\left|I_{H_2}(u_i)\right|^2}=\sqrt{1+\left|I_{H_2}(u_i)\right|^2}+\sqrt{1+\left|I_{H_1}(u_i)\right|^2},$$

$$\sqrt{1+\left|F_{H_1}(u_i)\right|^2}+\sqrt{1+\left|F_{H_2}(u_i)\right|^2}=\sqrt{1+\left|F_{H_2}(u_i)\right|^2}+\sqrt{1+\left|F_{H_1}(u_i)\right|^2},$$

$$\sqrt{1+\left|(1-T_{H_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{H_2}(u_i))\right|^2}=\sqrt{1+\left|(-T_{H_2}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{H_1}(u_i))\right|^2},$$

$$\sqrt{1+\left|(1-I_{H_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{H_2}(u_i))\right|^2}=\sqrt{1+\left|(1-I_{H_2}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{H_1}(u_i))\right|^2},$$

$$\sqrt{1+\left|(1-F_{H_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{H_2}(u_i))\right|^2}=\sqrt{1+\left|(1-F_{H_2}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{H_1}(u_i))\right|^2},\ \forall u_i\in U.$$

So, $CE_{SN}(H_1,H_2)=CE_{SN}(H_2,H_1)$.

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Hence complete the proof.

Definition 7. Weighted SN-cross entropy function

Considering the weight of the element u_i, i = 1, 2, .., n into account, we introduce a weighted SN-cross entropy.

We consider the weight w_i (i= 1, 2, ..., n) for the element u_i (i = 1, 2, .., n) with the conditions $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$.

Then the weighted cross entropy between SVNSs H_1 and H_2 can be defined as follows:

$$CE_{SN}^{w}(H_1,H_2)=\frac{1}{2}\left\langle \sum_{i=1}^{n} w_i \left\{ \left[\frac{2\left|T_{H_1}(u_i)-T_{H_2}(u_i)\right|}{\sqrt{1+\left|T_{H_1}(u_i)\right|^2}+\sqrt{1+\left|T_{H_2}(u_i)\right|^2}} + \frac{2\left|(1-T_{H_1}(u_i))-(1-T_{H_2}(u_i))\right|}{\sqrt{1+\left|(1-T_{H_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{H_2}(u_i))\right|^2}} \right] + \left[\frac{2\left|I_{H_1}(u_i)-I_{H_2}(u_i)\right|}{\sqrt{1+\left|I_{H_1}(u_i)\right|^2}+\sqrt{1+\left|I_{H_2}(u)\right|^2}} + \frac{2\left|(1-I_{H_1}(u_i))-(1-I_{H_2}(u_i))\right|}{\sqrt{1+\left|(1-I_{H_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{H_2}(u_i))\right|^2}} \right] + \left[\frac{2\left|F_{H_1}(u_i)-F_{H_2}(u_i)\right|}{\sqrt{1+\left|F_{H_1}(u_i)\right|^2}+\sqrt{1+\left|F_{H_2}(u_i)\right|^2}} + \frac{2\left|(1-F_{H_1}(u_i))-(1-F_{H_2}(u_i))\right|}{\sqrt{1+\left|(1-F_{H_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{H_2}(u_i))\right|^2}} \right] \right\} \right\rangle \quad (2)$$

Theorem 2.

Single valued neutrosophic weighted SN- cross-entropy (defined in Equation (2)) satisfies the following properties:

i). $CE_{SN}^{w}(H_1,H_2) \geq 0$.

ii). $CE_{SN}^{w}(H_1,H_2) = 0$, if and only if $T_{H_1}(u_i) = T_{H_2}(u_i)$ $I_{H_1}(u_i) = I_{H_2}(u_i)$, $F_{H_1}(u_i) = F_{H_2}(u_i)$, $\forall u_i \in U$.

iii). $CE_{SN}^{w}(H_1,H_2) = CE_{SN}^{w}(H_1^c,H_2^c)$

iv). $CE_{SN}^{w}(H_1,H_2) = CE_{SN}^{w}(H_2,H_1)$

Proof: i).

For all values of $u_i \in U$,

$\left|T_{H_1}(u_i)\right| \geq 0$ $\left|T_{H_2}(u_i)\right| \geq 0$,

$\left|T_{H_1}(u_i)-T_{H_2}(u_i)\right| \geq 0$, $\sqrt{1+\left|T_{H_1}(u_i)\right|^2} \geq 0$, $\sqrt{1+\left|T_{H_2}(u_i)\right|^2} \geq 0$, $\left|(1-T_{H_1}(u_i))\right| \geq 0$, $\left|(1-T_{H_2}(u_i))\right| \geq 0$,

$\left|(1-T_{H_1}(u_i))-(1-T_{H_2}(u_i))\right| \geq 0$, $\sqrt{1+\left|(1-T_{H_1}(u_i))\right|^2} \geq 0$, $\sqrt{1+\left|(1-T_{H_2}(u_i))\right|^2} \geq 0$, then

$$\left[\frac{2\left|T_{H_1}(u_i)-T_{H_2}(u_i)\right|}{\sqrt{1+\left|T_{H_1}(u_i)\right|^2}+\sqrt{1+\left|T_{H_2}(u_i)\right|^2}} + \frac{2\left|(1-T_{H_1}(u_i))-(1-T_{H_2}(u_i))\right|}{\sqrt{1+\left|(1-T_{H_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{H_2}(u_i))\right|^2}} \right] \geq 0$$

Similarly, $\left[\frac{2\left|I_{H_1}(u_i)-I_{H_2}(u_i)\right|}{\sqrt{1+\left|I_{H_1}(u_i)\right|^2}+\sqrt{1+\left|I_{H_2}(u)\right|^2}} + \frac{2\left|(1-I_{H_1}(u_i))-(1-I_{H_2}(u_i))\right|}{\sqrt{1+\left|(1-I_{H_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{H_2}(u_i))\right|^2}} \right] \geq 0$ and

$$\left[\frac{2\left|F_{H_1}(u_i)-F_{H_2}(u_i)\right|}{\sqrt{1+\left|F_{H_1}(u_i)\right|^2}+\sqrt{1+\left|F_{H_2}(u_i)\right|^2}} + \frac{2\left|(1-F_{H_1}(u_i))-(1-F_{H_2}(u_i))\right|}{\sqrt{1+\left|(1-F_{H_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{H_2}(u_i))\right|^2}} \right] \geq 0.$$

Since $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$, therefore, $CE_{SN}^{w}(H_1,H_2) \geq 0$.

Hence complete the proof.

ii).

Since,

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$$\left[\frac{2\left|T_{H_1}(u_i)-T_{H_2}(u_i)\right|}{\sqrt{1+\left|T_{H_1}(u_i)\right|^2}+\sqrt{1+\left|T_{H_2}(u_i)\right|^2}}+\frac{2\left|(1-T_{H_1}(u_i))-(1-T_{H_2}(u_i))\right|}{\sqrt{1+\left|(1-T_{H_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{H_2}(u_i))\right|^2}}\right]=0,$$

$\Leftrightarrow T_{H_1}(u_i)=T_{H_2}(u_i)$,

$$\left[\frac{2\left|I_{H_1}(u_i)-I_{H_2}(u_i)\right|}{\sqrt{1+\left|I_{H_1}(u_i)\right|^2}+\sqrt{1+\left|I_{H_2}(u)\right|^2}}+\frac{2\left|(1-I_{H_1}(u_i))-(1-I_{H_2}(u_i))\right|}{\sqrt{1+\left|(1-I_{H_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{H_2}(u_i))\right|^2}}\right]=0,$$

$\Leftrightarrow I_{H_1}(u_i)=I_{H_2}(u_i)$,

$$\left[\frac{2\left|F_{H_1}(u_i)-F_{H_2}(u_i)\right|}{\sqrt{1+\left|F_{H_1}(u_i)\right|^2}+\sqrt{1+\left|F_{H_2}(u_i)\right|^2}}+\frac{2\left|(1-F_{H_1}(u_i))-(1-F_{H_2}(u_i))\right|}{\sqrt{1+\left|(1-F_{H_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{H_2}(u_i))\right|^2}}\right]=0,$$

$\Leftrightarrow F_{H_1}(u_i)=F_{H_2}(u_i)$ and $w_i\in[0,1], \sum_{i=1}^{n} w_i=1$, $w_i\geq 0$. So, $CE_{SN}^{w}(H_1,H_2)=0$ iff $T_{H_1}(u_i)=T_{H_2}(u_i)$, $I_{H_1}(u_i)=I_{H_2}(u_i)$, $F_{H_1}(u_i)=F_{H_2}(u_i)$, $\forall u_i\in U$.

Hence complete the proof.

iii). Using definition 5, we obtain the following expression

$$CE_{SN}^{w}(H_1^c,H_2^c)=\frac{1}{2}\left\{\sum_{i=1}^{n} w_i\left\langle\left[\frac{2\left|(1-T_{H_1}(u_i))-(1-T_{H_2}(u_i))\right|}{\sqrt{1+\left|(1-T_{H_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{H_2}(u_i))\right|^2}}+\frac{2\left|T_{H_1}(u_i)-T_{H_2}(u_i)\right|}{\sqrt{1+\left|T_{H_1}(u_i)\right|^2}+\sqrt{1+\left|T_{H_2}(u_i)\right|^2}}\right]+\right.\right.$$

$$\left[\frac{2\left|(1-I_{H_1}(u_i))-(1-I_{H_2}(u_i))\right|}{\sqrt{1+\left|(1-I_{H_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{H_2}(u_i))\right|^2}}+\frac{2\left|I_{H_1}(u_i)-I_{H_2}(u_i)\right|}{\sqrt{1+\left|I_{H_1}(u_i)\right|^2}+\sqrt{1+\left|I_{H_2}(u)\right|^2}}\right]+$$

$$\left.\left.\left[\frac{2\left|(1-F_{H_1}(u_i))-(1-F_{H_2}(u_i))\right|}{\sqrt{1+\left|(1-F_{H_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{H_2}(u_i))\right|^2}}+\frac{2\left|F_{H_1}(u_i)-F_{H_2}(u_i)\right|}{\sqrt{1+\left|F_{H_1}(u_i)\right|^2}+\sqrt{1+\left|F_{H_2}(u_i)\right|^2}}\right]\right\rangle\right\}$$

$$=\frac{1}{2}\left\{\sum_{i=1}^{n} w_i\left\langle\left[\frac{2\left|T_{H_1}(u_i)-T_{H_2}(u_i)\right|}{\sqrt{1+\left|T_{H_1}(u_i)\right|^2}+\sqrt{1+\left|T_{H_2}(u_i)\right|^2}}+\frac{2\left|(1-T_{H_1}(u_i))-(1-T_{H_2}(u_i))\right|}{\sqrt{1+\left|(1-T_{H_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{H_2}(u_i))\right|^2}}\right]+\right.\right.$$

$$\left[\frac{2\left|I_{H_1}(u_i)-I_{H_2}(u_i)\right|}{\sqrt{1+\left|I_{H_1}(u_i)\right|^2}+\sqrt{1+\left|I_{H_2}(u)\right|^2}}+\frac{2\left|(1-I_{H_1}(u_i))-(1-I_{H_2}(u_i))\right|}{\sqrt{1+\left|(1-I_{H_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{H_2}(u_i))\right|^2}}\right]+$$

$$\left.\left.\left[\frac{2\left|F_{H_1}(u_i)-F_{H_2}(u_i)\right|}{\sqrt{1+\left|F_{H_1}(u_i)\right|^2}+\sqrt{1+\left|F_{H_2}(u_i)\right|^2}}+\frac{2\left|(1-F_{H_1}(u_i))-(1-F_{H_2}(u_i))\right|}{\sqrt{1+\left|(1-F_{H_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{H_2}(u_i))\right|^2}}\right]\right\rangle\right\}=CE_{SN}^{w}(H_1,H_2)$$

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So, $CE_{SN}^{w}(H_1, H_2) = CE_{SN}^{w}(H_1^c, H_2^c)$.
Hence complete the proof.

iv).

Since $|T_{H_1}(u_i) - T_{H_2}(u_i)| = |T_{H_2}(u_i) - T_{H_1}(u_i)|$, $|I_{H_1}(u_i) - I_{H_2}(u_i)| = |I_{H_2}(u_i) - I_{H_1}(u_i)|$,

$|F_{H_1}(u_i) - F_{H_2}(u_i)| = |F_{H_2}(u_i) - F_{H_1}(u_i)|$, $|(1 - T_{H_1}(u_i)) - (1 - T_{H_2}(u_i))| = |(1 - T_{H_2}(u_i)) - (1 - T_{H_1}(u_i))|$,

$|(1 - I_{H_1}(u_i)) - (1 - I_{H_2}(u_i))| = |(1 - I_{H_2}(u_i)) - (1 - I_{H_1}(u_i))|$, $|(1 - F_{H_1}(u_i)) - (1 - F_{H_2}(u_i))| = |(1 - F_{H_2}(u_i)) - (1 - F_{H_1}(u_i))|$,

we obtain

$$\sqrt{1+|T_{H_1}(u_i)|^2} + \sqrt{1+|T_{H_2}(u_i)|^2} = \sqrt{1+|T_{H_2}(u_i)|^2} + \sqrt{1+|T_{H_1}(u_i)|^2},$$

$$\sqrt{1+|I_{H_1}(u_i)|^2} + \sqrt{1+|I_{H_2}(u_i)|^2} = \sqrt{1+|I_{H_2}(u_i)|^2} + \sqrt{1+|I_{H_1}(u_i)|^2},$$

$$\sqrt{1+|F_{H_1}(u_i)|^2} + \sqrt{1+|F_{H_2}(u_i)|^2} = \sqrt{1+|F_{H_2}(u_i)|^2} + \sqrt{1+|F_{H_1}(u_i)|^2},$$

$$\sqrt{1+|(1-T_{H_1}(u_i))|^2} + \sqrt{1+|(1-T_{H_2}(u_i))|^2} = \sqrt{1+|(-T_{H_2}(u_i))|^2} + \sqrt{1+|(1-T_{H_1}(u_i))|^2},$$

$$\sqrt{1+|(1-I_{H_1}(u_i))|^2} + \sqrt{1+|(1-I_{H_2}(u_i))|^2} = \sqrt{1+|(1-I_{H_2}(u_i))|^2} + \sqrt{1+|(1-I_{H_1}(u_i))|^2},$$

$$\sqrt{1+|(1-F_{H_1}(u_i))|^2} + \sqrt{1+|(1-F_{H_2}(u_i))|^2} = \sqrt{1+|(1-F_{H_2}(u_i))|^2} + \sqrt{1+|(1-F_{H_1}(u_i))|^2}, \forall u_i \in U.$$

and $w_i \in [0,1], \sum_{i=1}^{n} w_i = 1$.
So, $CE_{SN}^{w}(H_1, H_2) = CE_{SN}^{w}(H_2, H_1)$.
Hence complete the proof.

4. MAGDM strategy using proposed SN-cross entropy meaure under SVNS environment

In this section, we develop a new MAGDM strategy using the proposed NS-cross entropy measure.

4.1 Description of the MAGDM problem

Assume that $A = \{A_1, A_2, A_3, ..., A_m\}$ and $G = \{G_1, G_2, G_3, ..., G_n\}$ be the discrete set of alternatives and attributes respectively and $W = \{w_1, w_2, w_3, ..., w_n\}$ be the weight vector of attributes G_j (j = 1, 2, 3, …, n), where $w_j \geq 0$ and $\sum_{j=1}^{n} w_j = 1$. Assume that $E = \{E_1, E_2, E_3, ..., E_\rho\}$ be the set of decision makers who are employed to evaluate the alternatives. The weight vector of the decision makers E_k $(k = 1,2,3,...,\rho)$ is $\lambda = \{\lambda_1, \lambda_2, \lambda_3, ..., \lambda_\rho\}$ (where, $\lambda \geq 0$ and $\sum_{k=1}^{\rho} \lambda_k = 1$), which can be determined according to the decision makers expertise, judgment quality and domain knowledge.
Now, we describe the steps of the propsed MAGDM strategy using SN- cross entropy measure.

4.1.1. MAGDM strategy using SN- cross entropy function

Step: 1. Formulate the decision matrices

For MAGDM with SVNSs information, the rating values of the alternatives A_i $(i=1,2,3,...,m)$ based on the attribute G_j $(j=1,2,3,...,n)$ provided by the k-th decision maker can be expressed in terms of SVNN as $a_{ij}^k = \langle T_{ij}^k, I_{ij}^k, F_{ij}^k \rangle$ (i = 1, 2, 3, …, m; j = 1, 2, 3, …, n; k = 1, 2, 3, …, ρ). We present these rating values of alternatives provided by the decision makers in matrix form as follows:

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$$M^{k}=\begin{pmatrix} & G_1 & G_2 & \dots & G_n \\ A_1 & a_{11}^k & a_{12}^k & \dots & a_{1n}^k \\ A_2 & a_{21}^k & a_{22}^k & & a_{2n}^k \\ . & . & & \dots & . \\ A_m & a_{m1}^k & a_{m2}^k & \dots & a_{mn}^k \end{pmatrix} \quad (7)$$

Step: 2. Formulate the weighted aggregated decision matrix

For obtaining one group decision, we aggregate all individual decision matrices to an aggregated decision matrix using the Equation (9) as follows:

$$M=\begin{pmatrix} & G_1 & G_2 & \dots & G_n \\ A_1 & a_{11} & a_{12} & \dots & a_{1n} \\ A_2 & a_{21} & a_{22} & & a_{2n} \\ . & . & & \dots & . \\ A_m & a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \quad (8)$$

Here, $a_{ij}=<1-\prod_{k=1}^{\rho}(1-T_{ij}^{k})^{w_j}, \prod_{k=1}^{\rho}(I_{ij}^{k})^{w_j}, \prod_{k=1}^{\rho}(F_{ij}^{k})^{w_j}>$ ……(9) and $(i = 1, 2, 3, \dots, m; j = 1, 2, 3, \dots, n; k = 1, 2, 3, \dots, \rho)$.

Step: 3. Formulate priori/ ideal decision matrix

In the MAGDM, the priori decision matrix has been used to select the best alternatives among the set of collected feasible alternatives. In decision making situation, we use the following decision matrix as priori decision matrix.

$$P=\begin{pmatrix} & G_1 & G_2 & \dots & G_n \\ A_1 & a_{11}^* & a_{12}^* & \dots & a_{1n}^* \\ A_2 & a_{21}^* & a_{22}^* & & a_{2n}^* \\ . & . & & \dots & . \\ A_m & a_{m1}^* & a_{m2}^* & \dots & a_{mn}^* \end{pmatrix} \quad (10)$$

where, $a_{ij}^{*}=<\max(T_{ij}^{k}), \min(I_{ij}^{k}), \min(F_{ij}^{k})>$ and $(i = 1, 2, 3, \dots, m; j = 1, 2, 3, \dots, n)$.

Step: 4. Calculate the weighted SN- cross entropy measure

Using equation (2), we calculate weighted cross entropy value between aggregate matrix and priori matrix. The cross entropy values can be presented in matrix form as follows:

$$^{SN}M_{CE}^{w}=\begin{pmatrix} CE_{SN}^{w}(A_1) \\ CE_{SN}^{w}(A_2) \\ \dots\dots\dots\dots \\ \dots\dots\dots\dots \\ CE_{SN}^{w}(A_m) \end{pmatrix} \quad (11)$$

Step: 5. Rank the priority

Smaller value of the cross entropy reflects that an alternative is closer to the ideal alternative. Therefore, the preference priority order of all the alternatives can be determined according to the increasing order of the cross entropy values $CE_{SN}^{W}(A_i)$ $(i = 1, 2, 3, \dots, m)$. Smallest cross entropy value indicates the best alternative and greatest cross entropy value indicates the worst alternative.

Step: 6. Select the best alternative

From the preference rank order (from step 5), we select the best alternative.

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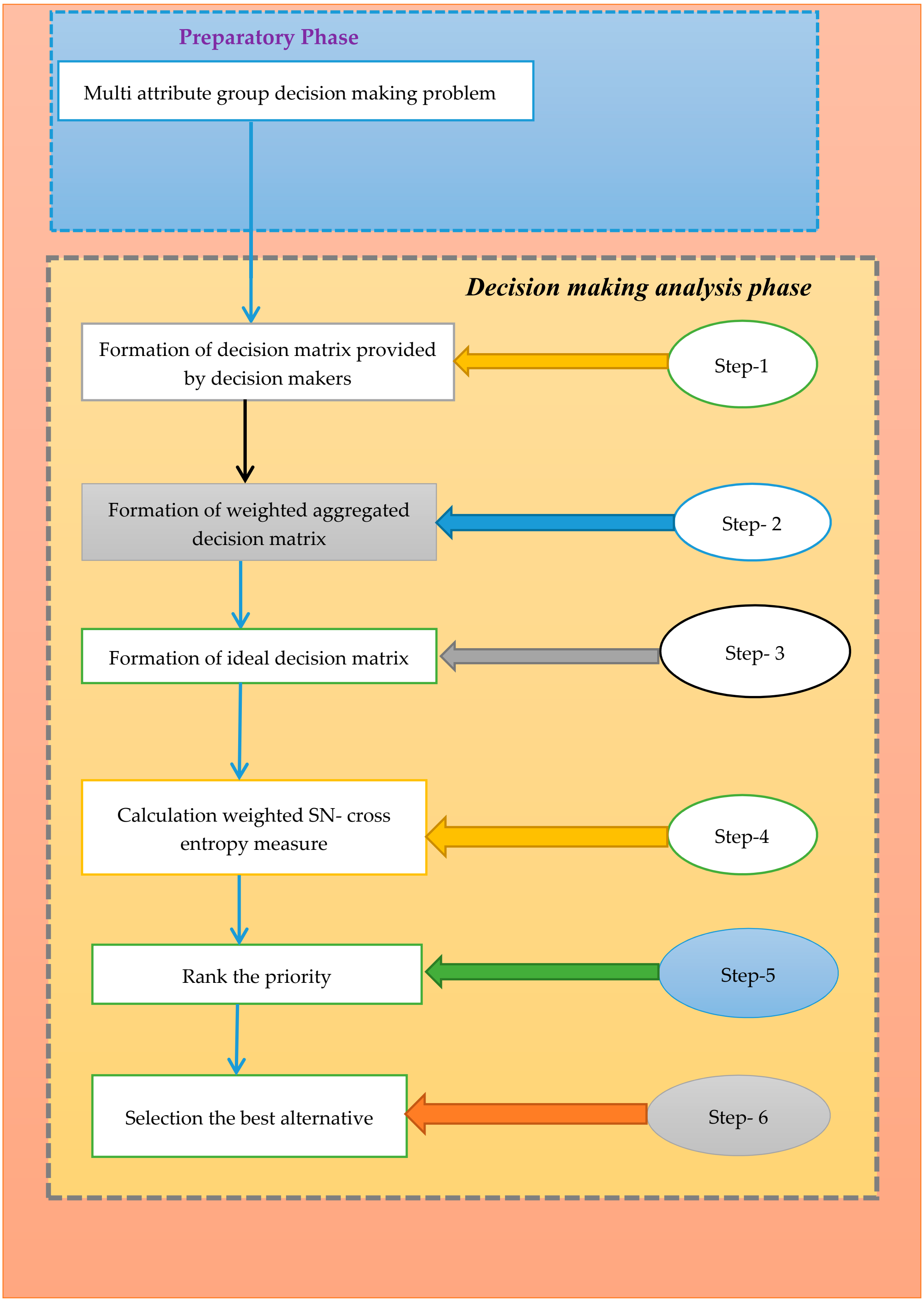

Figure.1 Decision making procedure of proposed MAGDM method

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5. Illustrative example

In this section, we solve an illustrative example adapted from [12] of MAGDM problems to reflect the feasibility, applicability and efficiency of the proposed strategy under SVNS environment. Now, we use the example [12] for cultivation and analysis. A venture capital firm intends to make evaluation and selection to five enterprises with the investment potential:

1) Automobile company (A_1)
2) Military manufacturing enterprise (A_2)
3) TV media company (A_3)
4) Food enterprises (A_4)
5) Computer software company (A_5)

On the basis of four attributes namely:

1) Social and political factor (G_1)
2) The environmental factor (G_2)
3) Investment risk factor (G_3)
4) The enterprise growth factor (G_4).

The investment firm makes a panel of three decision makers $E = \{E_1, E_2, E_3\}$ having their weight vector $\lambda = \{.42, .28, .30\}$ and weight vector of attributes is $W = \{.24, .25, .23, .28\}$.
The steps of decision making strategy (4.1.1.) to rank alternatives are presented as follows:

Step: 1. Formulate the decision matrices

We represent the rating values of alternatives A_i (i = 1, 2, 3, 4, 5) with respects to the attributes G_j (j = 1, 2, 3, 4) provided by the decision makers E_k (k = 1, 2, 3) in matrix form as follows:

Decision matrix for E_1 decision maker

$$M^1 = \begin{pmatrix} & G_1 & G_2 & G_3 & G_4 \\ A_1 & (0.9,0.5,0.4) & (0.7,0.4,0.4) & (0.7,0.3,0.4) & (0.5,0.4,0.9) \\ A_2 & (0.7,0.2,0.3) & (0.8,0.4,0.3) & (0.9,0.6,0.5) & (0.9,0.1,0.3) \\ A_3 & (0.8,0.4,0.4) & (0.7,0.4,0.2) & (0.9,0.7,0.6) & (0.7,0.3,0.3) \\ A_4 & (0.5,0.8,0.7) & (0.6,0.3,0.4) & (0.7,0.2,0.5) & (0.5,0.4,0.7) \\ A_5 & (0.8,0.4,0.3) & (0.5,0.4,0.5) & (0.6,0.4,0.4) & (0.9,0.7,0.5) \end{pmatrix} \quad (22)$$

Decision matrix for E_2 decision maker

$$M^2 = \begin{pmatrix} & G_1 & G_2 & G_3 & G_4 \\ A_1 & (0.7,0.2,0.3) & (0.5,0.4,0.5) & (0.9,0.4,0.5) & (0.6,0.5,0.3) \\ A_2 & (0.7,0.4,0.4) & (0.7,0.3,0.4) & (0.7,0.3,0.4) & (0.6,0.4,0.3) \\ A_3 & (0.6,0.4,0.4) & (0.5,0.3,0.5) & (0.9,0.5,0.4) & (0.6,0.5,0.6) \\ A_4 & (0.7,0.5,0.3) & (0.6,0.3,0.6) & (0.7,0.4,0.4) & (0.8,0.5,0.4) \\ A_5 & (0.9,0.4,0.3) & (0.6,0.4,0.5) & (0.8,0.5,0.6) & (0.5,0.4,0.5) \end{pmatrix} \quad (23)$$

Decision matrix for E_3 decision maker

$$M^3 = \begin{pmatrix} & G_1 & G_2 & G_3 & G_4 \\ A_1 & (0.7,0.2,0.5) & (0.6,0.4,0.4) & (0.7,0.4,0.5) & (0.9,0.4,0.3) \\ A_2 & (0.6,0.5,0.5) & (0.9,0.3,0.4) & (0.7,0.4,0.3) & (0.8,0.4,0.5) \\ A_3 & (0.8,0.3,0.5) & (0.9,0.3,0.4) & (0.8,0.3,0.4) & (0.7,0.3,0.4) \\ A_4 & (0.9,0.3,0.4) & (0.6,0.3,0.4) & (0.5,0.2,0.4) & (0.7,0.3,0.5) \\ A_5 & (0.8,0.3,0.3) & (0.6,0.4,0.3) & (0.6,0.3,0.4) & (0.7,0.3,0.5) \end{pmatrix} \quad (24)$$

Step: 2. Formulate the weighted aggregated decision matrix

Using the equation (9), the aggregated decision matrix is presented as follows:

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Aggregated decision matrix

$$M = \begin{pmatrix} & G_1 & G_2 & G_3 & G_4 \\ A_1 & (0.8,0.3,0.4) & (0.6,0.4,0.4) & (0.8,0.4,0.4) & (0.7,0.4,0.5) \\ A_2 & (0.7,0.3,0.4) & (0.8,0.3,0.4) & (0.8,0.4,0.4) & (0.8,0.2,0.3) \\ A_3 & (0.8,0.4,0.4) & (0.8,0.3,0.3) & (0.9,0.5,0.5) & (0.7,0.3,0.4) \\ A_4 & (0.7,0.5,0.5) & (0.6,0.3,0.4) & (0.6,0.2,0.4) & (0.7,0.4,0.5) \\ A_5 & (0.8,0.4,0.4) & (0.6,0.4,0.4) & (0.7,0.4,0.4) & (0.8,0.5,0.5) \end{pmatrix} \quad (25)$$

Step: 3. Formulate priori/ ideal decision matrix

Priori/ ideal decision matrix

$$P = \begin{pmatrix} & G_1 & G_2 & G_3 & G_4 \\ A_1 & (1,0,0) & (1,0,0) & (1,0,0) & (1,0,0) \\ A_2 & (1,0,0) & (1,0,0) & (1,0,0) & (1,0,0) \\ A_3 & (1,0,0) & (1,0,0) & (1,0,0) & (1,0,0) \\ A_4 & (1,0,0) & (1,0,0) & (1,0,0) & (1,0,0) \\ A_5 & (1,0,0) & (1,0,0) & (1,0,0) & (1,0,0) \end{pmatrix} \quad (26)$$

Step: 4. Calculate the weighted SVNS cross entropy matrix

Using the equation (2), we calculate the single valued weighted cross entropy values between ideal matrix and weighted aggregated decision matrix.

$${}^{SN}M^{w}_{CE} = \begin{pmatrix} 0.935 \\ 0.775 \\ 0.840 \\ 1.000 \\ 0.980 \end{pmatrix} \quad (27)$$

Step: 5. Rank the priority

The cross entropy values of alternatives are arranged in increasing order as follows:

$0.775 < 0.840 < 0.935 < 0.980 < 1.000$.

Alternatives are then preference ranked as follows:

$A_2 > A_3 > A_1 > A_5 > A_4$.

Step: 6. Select the best alternative

From step 5, we identify A_2 is the best alternative. Hence, military manufacturing enterprise (A_2) is the best alternative for investment.

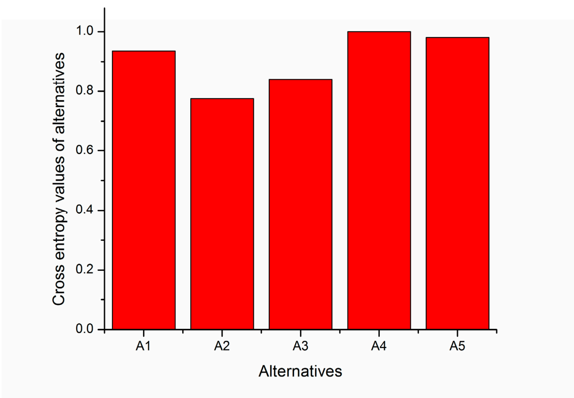

Figure.2. Bar diagram of alternatives versus cross entropy values of alternatives

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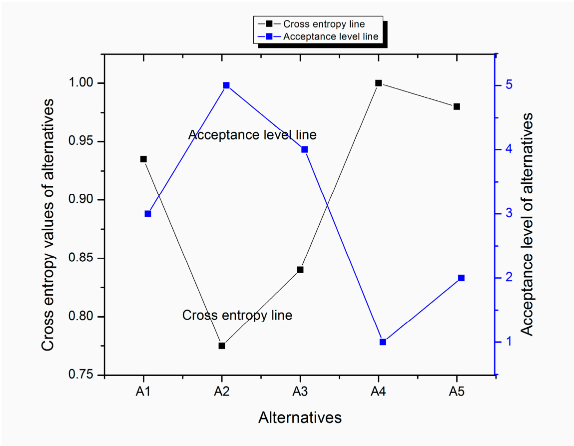

Figure.3. Relation between cross entropy values and acceptance level line of alternatives.

In Figure 3, we represent the relation between cross entropy values and acceptance values of alternatives. The range of acceptance level for five alternatives is taken five points. The high acceptance level of alternative indicates the best alternative for acceptance and low acceptance level of alternative indicates the poor acceptance alternative.

We see from Figure 3 that alternative A_2 has the smallest cross entropy value and the highest acceptance level. Therefore A_2 is the best alternative for acceptance. Figure 3 indicates that alternative A_4 has highest cross entropy value and lowest acceptance value that means A_4 is the worst alternative. Finally, we conclude that the relation between cross entropy values and acceptance value of alternatives is opposite in nature.

6. Comparative study and discussion

In literature only MADM strategy [88, 91] have been proposed. So the proposed MAGDM is non-comparable. However, for comparison purpose, the MADM strategies [88, 91] are transformed into MAGDM and for calculation purpose we assume the same set of weigts for the decision makers. Then the obtained result derived from the proposed method is compared the results obtained from two existing strategies [88, 91]under SVNS environment. We present ranking order of the alternatives (see Table 1) using same illustrative example for the proposed strategy and two [88, 91].

Table 1. Ranking order of alternatives using different single valued neutrosophic cross entropy function

Proposed Strategy	Ye [91] Strategy	Ye [88] Strategy
$CE_{NS}^{w}(A_1)$=.935 $CE_{NS}^{w}(A_2)$=.775 $CE_{NS}^{w}(A_3)$=.840 $CE_{NS}^{w}(A_4)$=1.00 $CE_{NS}^{w}(A_5)$=.980	$N_w(A_1)$=.493 $N_w(A_2)$=.367 $N_w(A_3)$=.415 $N_w(A_4)$=.410 $N_w(A_5)$=.510	$D(A_1)$=.365 $D(A_2)$=.244 $D(A_3)$=.288 $D(A_4)$=.414 $D(A_5)$=.431
Preference ranking order $A_2 \succ A_3 \succ A_1 \succ A_5 \succ A_4$	Preference ranking order $A_2 \succ A_4 \succ A_3 \succ A_1 \succ A_5$	Preference ranking order $A_2 \succ A_3 \succ A_1 \succ A_4 \succ A_5$

i). The MADM strategies [88] and [91] are not applicable for MAGDM problems.The proposed MAGDM strategy is free from such drawbacks.
ii). Ye [88] proposed cross entropy that does not satisfy the symmetrical property straightforward and is undefined for some situation [91] but the proposed strategy satisfies symmetry property and free from undefined phenomenon.

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iii) The best alternative is the same for the three strategies. However, the preference ranking orders are not the same.

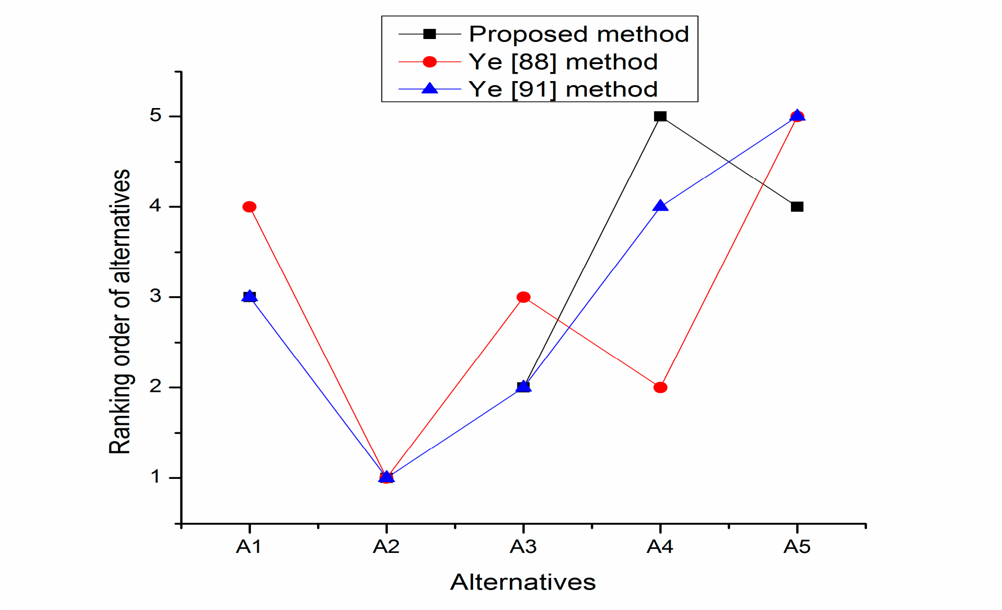

Figure.4. Graphical representation of ranking order of five alternatives based on three strategies.

7. Conclusion

In this paper we have defined a new cross entropy measure in SVNS environment which is free from all the drawback of existence cross entropy measures. We have proved the basic properties of the SN-cross entropy measure. We also defined weighted SN-cross entropy measure and proved its basic properties. Based on the weighted SN- cross entropy measure we have developed a novel MAGDM strategy to solve neutrosophic group decision making problems. We have at first proposed MAGDM strategy based on SN- cross entropy measure. Other existing cross entropy measures can deal only MADM problem with single decision maker. So in general, our proposed MAGDM strategy is not comparable with the existing MADM strategies. However, for comparision with the existing strategies, at first we have made them MAGDM strategies and considerd the same set of weights of the decision makers and presented comparisonanalysis. Finally, we solve a MAGDM problem to show the feasibility, applicability and efficiency of the proposed MAGDM strategy. In future study, the proposed MAGDM stragey based on SN- cross entropy can be applied in teacher selection, pattern recognition, weaver selection, medical treatment selection option, and other practical problems.

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Conflicts of Interest: The authors declare that there is no conflict of interest for publication of the article.

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