**NS-Cross Entropy Based MAGDM under Single Valued Neutrosophic Set Environment**

Surapati Pramanik\(^1\), Shyam Dalapati\(^2\), Shariful Alam\(^2\), F. Smarandache\(^3\), Tapan Kumar Roy\(^2\)

\(^1\) Department of Mathematics, Nandalal Ghosh B.T. College, Panpur, P.O.-Narayanpur, District –North 24 Parganas, Pin code-743126, West Bengal, India. *E-mail: sura_pati@yahoo.co.in

\(^2\) Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, P.O.-Botanic Garden, Howrah-711103, West Bengal, India. E-mail: shyamal.rs2015@math.iiests.ac.in (S. D.), salam50in@yahoo.co.in (S. A.), roy_t_k@yahoo.co.in (T. K. R)

\(^3\) University of New Mexico, Mathematics & Science Department, 705 Gurley Ave., Gallup, NM 87301, U.S.A.; smarand@unm.edu

* Correspondence: e-mail: sura_pati@yahoo.co.in; Tel.: +91-9477035544

**Abstract:** Single valued neutrosophic set has king power to express uncertainty characterized by indeterminacy, inconsistency and incompleteness. Most of the existing single valued neutrosophic cross entropy bears an asymmetrical behavior and produce an undefined phenomenon in some situations. In order to deal with these disadvantages, we propose a new cross entropy measure under single valued neutrosophic set (SVNS) environment namely SN-cross entropy and prove its basic properties. Also we define weighted SN-cross entropy measure and investigate its basic properties. We develop a new multi attribute group decision making (MAGDM) strategy for ranking of the alternatives based on the proposed weighted SN-cross entropy measure between each alternative and the ideal alternative. Finally, a numerical example of MAGDM problem of investment potential is solved to show the validity and efficiency of proposed decision making strategy. We also present comparative analysis of the obtained result with the results obtained form the existing solution strategies in the solution.

**Keywords:** neutrosophic set; single valued neutrosophic set; SN-cross entropy function; multi-attribute group decision making

1. **Introduction**

Ye [20] defined a new cross entropy in IVIFS environment and presented an optimal decision-making strategy. Xia and Xu [21] put forward new entropy and cross entropy and employed them for multi-attribute criteria group decision making (MAGDM) strategy in IFS environment. Tong and Yu [22] defined cross entropy in IVIFS environment and applied it to MADM problems.

The study of uncertainty entered into a new direction after the publication of neutrosophic set (NS) [23] and single valued neutrosophic set (SVNS) [24]. SVNS draws more appeal to the researchers for its applicability in decision making [25-54], conflict resolution [55], educational problems [56, 57], image processing [58-60], cluster analysis [61, 62], social problems [63, 64], etc. The research on SVNS gets momentum after the inception of the international journal “Neutrosophic Sets and Systems”. Combining with neutrosophic set, a number of hybrid sets such as neutrosophic soft set [65-70], neutrosophic complex set [71], interval complex neutrosophic set [72], rough neutrosophic set [73-80], neutrosophic soft expert set [81, 82], rough neutrosophic bipolar set [83], rough neutrosophic tri complex set [84], neutrosophic rough hyper complex set [85], are reported in the literature. Wang et al. [86] defined interval neutrosophic set (INS). Majumdar and Samanta [87] defined an entropy measure and presented an MCDM strategy under SVNS environment. Ye [88] defined cross entropy for SVNS by extending the intuitionistic fuzzy cross entropy [7] and proposed MCDM strategy under SVNS environment. Sahin [89] proposed two cross entropy measures for INSs and proposed MCGDM strategy. Tian et al. [90] proposed a cross entropy for INSs and developed a MCDM strategy based on the cross entropy and TOPSIS. Ye [91] defined cross entropy measures for SVNSs and INSs to overcome the drawback of the existing cross entropy measures. Due to little research of cross entropy measures, we define a new cross entropy measure in SVNSs environment based on the distance function of two points and prove its basic properties. Also, we define single valued weighted cross entropy measure and investigate its properties. Getting motivation from the work of Ye [91] for MCDM, we propose a novel MAGDM strategy using the proposed weighted cross entropy.

The remaining of the paper is presented as follows:

Section 2 describes some concepts of SVNSs. In Section 3 we propose a new cross entropy measure between two SVNSs and investigate its properties.

In section 4, we develop a novel MAGDM strategy based on the proposed SN-cross entropy with SVNS information. In Section 5 we present comparative study and discussion. In section 6 an illustrative example is solved to demonstrate the applicability and efficiency of the developed MAGDM strategy under SVNS environment. Section 7 offers conclusions and perspectives of future work.

2. Preliminaries

This section presents a short list of mostly known definitions pertaining to this paper.

Definition 1. [23] NS

Let U be a space of points (objects) with a generic element in U denoted by u, i.e. \( u \in U \). A neutrosophic set A in U is characterized by truth-membership function \( T_A(u) \), indeterminacy-membership function \( I_A(u) \) and falsity-membership function \( F_A(u) \), where \( T_A(u), I_A(u), F_A(u) \) are the functions from U to \([0, 1] \) i.e. \( T_A(u), I_A(u), F_A(u) : U \rightarrow [0, 1] \). NS can be expressed as \( A = \{ u \mid T_A(u), I_A(u), F_A(u) \} \) \( \forall u \in U \). Since \( T_A(u) \), \( I_A(u) \), \( F_A(u) \) are the subsets of \([0, 1] \), the sum \( (T_A(u) + I_A(u) + F_A(u)) \) lies between \( 0 \) and \( 3^+ \).

Example 1. Suppose that \( U = \{ u_1, u_2, u_3, \ldots \} \) be the universal set. Let \( R_1 \) be any neutrosophic set in U. Then \( R_1 \), expressed as \( R_1 = \{ u_i \mid (0.6, 0.3, 0.4) \} : u_i \in U \).

Definition 2. [24] SVNS

Assume that U be a space of points (objects) with generic elements \( u \in U \). A SVNS H in U is characterized by a truth-membership function \( T_H(u) \), an indeterminacy-membership function \( I_H(u) \), and a falsity-membership function \( F_H(u) \), where \( T_H(u), I_H(u), F_H(u) \in [0, 1] \) for each point u in U. Therefore, a SVNS H can be expressed as \( H = \{ u, (T_H(u), I_H(u), F_H(u)) \mid \forall u \in U \} \), whereas, the sum
of $T_H(u)$, $I_H(u)$ and $F_H(u)$ satisfy the condition $0 \leq T_H(u) + I_H(u) + F_H(u) \leq 3$ and $H(u) = <(T_H(u), I_H(u), F_H(u))>$ call a single valued neutrosophic number (SVNN).

**Example 1.**
A SVNS $H$ in $U$ can be expressed as: $H = \{ u, (0.7, 0.3, 0.5) \mid u \in U \}$ and SVNN presented $H = <0.7, 0.3, 0.5>$.

**Definition 3.** [24] Inclusion of SVNSs

The inclusion of any two SVNS sets $H_1$ and $H_2$ in $U$ is denoted by $H_1 \subseteq H_2$ and defined as follows:

$H_1 \subseteq H_2$ iff $T_{H_1}(u) \leq T_{H_2}(u)$, $I_{H_1}(u) \geq I_{H_2}(u)$, $F_{H_1}(u) \geq F_{H_2}(u)$ for all $u \in U$.

**Example 2.**
Let $H_1$ and $H_2$ be any two SVNNs in $U$ presented as follows: $H_1 = <(7, .3, .5)>$ and $H_2 = <(8, .2, .4)>$ for all $u \in U$. Using the property of inclusion of two SVNNs, we conclude that $H_1 \subseteq H_2$.

**Definition 4.** [24] Equality of two SVNSs

The equality of any two SVNS $H_1$ and $H_2$ in $U$ denoted by $H_1 = H_2$ and defined as follows:

$H_1 = H_2$ iff $T_{H_1}(u) = T_{H_2}(u)$, $I_{H_1}(u) = I_{H_2}(u)$, and $F_{H_1}(u) = F_{H_2}(u)$ for all $u \in U$.

**Definition 5.** Complement of any SVNS

The complement of any SVNS $H$ in $U$ denoted by $H^c$ and defined as follows:

$H^c = \{ u, 1 - T_H, 1 - I_H, 1 - F_H \mid u \in U \}$.

**Example 3.**
Let $H$ be any SVNN in $U$ presented as follows: $H = <(7, .3, .5)>$. Then compliment of $H$ is obtained as $H^c = <(3, .7, .5)>$.

**Definition 6.** [24] Union

The union of two single valued neutrosophic sets $H_1$ and $H_2$ is a neutrosophic set $H_3$ (say) written as $H_3 = H_1 \cup H_2$.

Let $H_1$ and $H_2$ be two SVNSs in $U$ presented as follows:

$H_1 = <(0.6, 0.3, 0.4)>$ and $H_2 = <(0.7, 0.3, 0.6)>$. Then union of them is presented as:

$H_1 \cup H_2 = <(0.7, 0.3, 0.4)>$.

**Definition 7.** [24] Intersection

The intersection of two single valued neutrosophic sets $H_1$ and $H_2$ denoted by $H_4$ and defined as $H_4 = H_1 \cap H_2$.

Let $H_1$ and $H_2$ be two SVNSs in $U$ presented as follows:

$H_1 = <(0.6, 0.3, 0.4)>$ and $H_2 = <(0.7, 0.3, 0.6)>$. Then intersection of $H_1$ and $H_2$ is presented as follows:

$H_1 \cap H_2 = <(0.6, 0.3, 0.6)>$. 
Some operations of SVNSs [24]:

Let $H_1$ and $H_2$ be any two SVNSs. Then, addition and multiplication are defined as:

1. $H_1 \oplus H_2 = \langle T_{H_1}(u) + T_{H_2}(u), I_{H_1}(u) + I_{H_2}(u), F_{H_1}(u) + F_{H_2}(u) \rangle \quad \forall \ u \in U.$

2. $H_1 \otimes H_2 = \langle T_{H_1}(u) \cdot T_{H_2}(u), I_{H_1}(u) \cdot I_{H_2}(u), F_{H_1}(u) \cdot F_{H_2}(u) \rangle \quad \forall \ u \in U.$

Example 6. Let $H_1$ and $H_2$ be two SVNSs in $U$ presented as follows:

$H_1 = \langle 0.6, 0.3, 0.4 \rangle$ and $H_2 = \langle 0.7, 0.3, 0.6 \rangle$.

Then,

1. $H_1 \oplus H_2 = \langle 0.88, 0.09, 0.24 \rangle$

2. $H_1 \otimes H_2 = \langle 0.42, 0.51, 0.76 \rangle$.

3. SN-cross entropy function

In this section, we define a new single valued neutrosophic cross-entropy function for measuring the deviation of single valued neutrosophic variables from an a priori one.

Definition 6. 1. SN-cross entropy function

Let $H_1$ and $H_2$ be any two SVNSs in $U = \{ u_1, u_2, \ldots, u_n \}$. Then, the single valued cross-entropy of $H_1$ and $H_2$ is denoted by $CESN(H_1, H_2)$ and defined as follows:

$$
CESN(H_1, H_2) = \frac{1}{2} \left[ \sum_{i=1}^{n} \left( \frac{2|T_{H_1}(u_i) - T_{H_2}(u_i)|}{1 + |T_{H_1}(u_i)|^2 + |T_{H_2}(u_i)|^2} + \frac{2|(1-T_{H_1}(u_i)) - (1-T_{H_2}(u_i))|}{1 + |1-T_{H_1}(u_i)|^2 + |1-T_{H_2}(u_i)|^2} \right) + \frac{2|I_{H_1}(u_i) - I_{H_2}(u_i)|}{1 + |I_{H_1}(u_i)|^2 + |I_{H_2}(u_i)|^2} + \frac{2|(1-I_{H_1}(u_i)) - (1-I_{H_2}(u_i))|}{1 + |1-I_{H_1}(u_i)|^2 + |1-I_{H_2}(u_i)|^2} \right) + \frac{2|F_{H_1}(u_i) - F_{H_2}(u_i)|}{1 + |F_{H_1}(u_i)|^2 + |F_{H_2}(u_i)|^2} + \frac{2|(1-F_{H_1}(u_i)) - (1-F_{H_2}(u_i))|}{1 + |1-F_{H_1}(u_i)|^2 + |1-F_{H_2}(u_i)|^2} \right] 
$$

Example 4.

Let $H_1$ and $H_2$ be two SVNSs in $U$, which are given by $H_1 = \{ u, (0.7, 0.3, 0.4) | u \in U \}$ and $H_2 = \{ u, (0.6, 0.4, 0.2) | u \in U \}$. Using Equation (1), the cross entropy value of $H_1$ and $H_2$ is obtained as $CESN(H_1, H_2) = 0.707$.

Theorem

Single valued neutrosophic cross entropy $CESN(H_1, H_2)$ for any two SVNSs $H_1, H_2$, satisfies the following properties:

i) $CESN(H_1, H_2) \geq 0$.

ii) $CESN(H_1, H_2) = 0$ if and only if $T_{H_1}(u_i) = T_{H_2}(u_i), I_{H_1}(u_i) = I_{H_2}(u_i), F_{H_1}(u_i) = F_{H_2}(u_i), \ \forall u_i \in U$.

iii) $CESN(H_1, H_2) = CESN(H_1', H_2')$.

iv) $CESN(H_1, H_2) = CESN(H_2, H_1)$.

Proof: i)
For all values of $u_i \in U$, 
\[ |T_{H_1}(u)| \geq 0, \quad |T_{H_2}(u)| \geq 0, \quad |T_{H_1}(u) - T_{H_2}(u)| \geq 0, \quad |T_{H_1}(u) - F_{H_1}(u)| \geq 0, \quad |T_{H_2}(u) - F_{H_2}(u)| \geq 0, \quad |1 + T_{H_1}(u)| \geq 0, \quad |1 + T_{H_2}(u)| \geq 0. \]

Then,
\[ \left[ \frac{2|T_{H_1}(u) - T_{H_2}(u)|}{\sqrt{1 + |T_{H_1}(u)|^2} + \sqrt{1 + |T_{H_2}(u)|^2}} + \frac{2|1 - T_{H_1}(u) - (1 - T_{H_2}(u))|}{\sqrt{1 + |1 - T_{H_1}(u)|^2} + \sqrt{1 + |1 - T_{H_2}(u)|^2}} \right] \geq 0, \]

Similarly,
\[ \left[ \frac{2|I_{H_1}(u) - I_{H_2}(u)|}{\sqrt{1 + |I_{H_1}(u)|^2} + \sqrt{1 + |I_{H_2}(u)|^2}} + \frac{2|1 - I_{H_1}(u) - (1 - I_{H_2}(u))|}{\sqrt{1 + |1 - I_{H_1}(u)|^2} + \sqrt{1 + |1 - I_{H_2}(u)|^2}} \right] \geq 0, \]

\[ \left[ \frac{2|F_{H_1}(u) - F_{H_2}(u)|}{\sqrt{1 + |F_{H_1}(u)|^2} + \sqrt{1 + |F_{H_2}(u)|^2}} + \frac{2|1 - F_{H_1}(u) - (1 - F_{H_2}(u))|}{\sqrt{1 + |1 - F_{H_1}(u)|^2} + \sqrt{1 + |1 - F_{H_2}(u)|^2}} \right] \geq 0. \]

So, $\text{CE}_{_{\text{CS}}}(H_1, H_2) \geq 0$. Hence complete the proof.

\[ \text{iii) Using definition 5, we obtain the following expression} \]

\[ \text{CE}_{_{\text{CS}}}(H_1', H_2') = \frac{1}{2} \left[ \sum \left( \frac{2|1 - T_{H_1}(u) - (1 - T_{H_2}(u))|}{\sqrt{1 + |1 - T_{H_1}(u)|^2} + \sqrt{1 + |1 - T_{H_2}(u)|^2}} + \frac{2|T_{H_1}(u) - T_{H_2}(u)|}{\sqrt{1 + |T_{H_1}(u)|^2} + \sqrt{1 + |T_{H_2}(u)|^2}} \right) \right]. \]
\[
\begin{align*}
& \frac{2 | (1 - I_{H_1} (u)) - (1 - I_{H_2} (u)) |}{\sqrt{1 + | (1 - I_{H_1} (u)) |^2 + \sqrt{1 + | (1 - I_{H_2} (u)) |^2}} } + \frac{2 | I_{H_1} (u) - I_{H_2} (u) |}{\sqrt{1 + | I_{H_1} (u) |^2 + \sqrt{1 + | I_{H_2} (u) |^2}} } \\
& \frac{2 | (1 - F_{H_1} (u)) - (1 - F_{H_2} (u)) |}{\sqrt{1 + | (1 - F_{H_1} (u)) |^2 + \sqrt{1 + | (1 - F_{H_2} (u)) |^2}} } \\
& \frac{2 | F_{H_1} (u) - F_{H_2} (u) |}{\sqrt{1 + | F_{H_1} (u) |^2 + \sqrt{1 + | F_{H_2} (u) |^2}} } = CE_{SN} (H_1, H_2)
\end{align*}
\]

So, \( CE_{SN} (H_1, H_2) = CE_{SN} (H_1^*, H_2^*) \).

Hence complete the proof.

iv) Since,

\[
| I_{H_1} (u) - I_{H_2} (u) | = | I_{H_2} (u) - I_{H_1} (u) |, \quad | I_{H_1} (u) - I_{H_2} (u) | = | I_{H_2} (u) - I_{H_1} (u) |,
\]

\[
| I_{H_2} (u) - I_{H_1} (u) | = | I_{H_1} (u) - I_{H_2} (u) |,
\]

\[
| 1 - I_{H_1} (u) | = | 1 - I_{H_2} (u) |, \quad | 1 - F_{H_1} (u) | = | 1 - F_{H_2} (u) |,
\]

then

\[
\sqrt{1 + | I_{H_1} (u) |^2 + \sqrt{1 + | I_{H_2} (u) |^2}} = \sqrt{1 + | I_{H_2} (u) |^2 + \sqrt{1 + | I_{H_1} (u) |^2}} ,
\]

\[
\sqrt{1 + | F_{H_1} (u) |^2 + \sqrt{1 + | F_{H_2} (u) |^2}} = \sqrt{1 + | F_{H_2} (u) |^2 + \sqrt{1 + | F_{H_1} (u) |^2}} ,
\]

\[
\sqrt{1 + | I_{H_1} (u) |^2 + \sqrt{1 + | I_{H_2} (u) |^2}} = \sqrt{1 + | I_{H_2} (u) |^2 + \sqrt{1 + | I_{H_1} (u) |^2}} ,
\]

So, \( CE_{SN} (H_1, H_2) = CE_{SN} (H_2, H_1) \).
Hence complete the proof.

**Definition 7. Weighted SN-cross entropy function**

Considering the weight of the element \( u_i \), \( i = 1, 2, \ldots, n \) into account, we introduce a weighted SN-cross entropy.

We consider the weight \( w_i \) (\( i = 1, 2, \ldots, n \)) for the element \( u_i \) (\( i = 1, 2, \ldots, n \)) with the conditions

\[
0 \leq w_i \leq 1, \sum_{i=1}^{n} w_i = 1.
\]

Then the weighted cross entropy between SVNSs \( H_1 \) and \( H_2 \) can be defined as follows:

\[
CE_{SN}^w (H_1, H_2) = \frac{1}{2} \sum_{i=1}^{n} \left[ \frac{2|T_{n_i}(u_i) - T_{n_2}(u_i)|}{\sqrt{[1+|T_{n_1}(u_i)|]^2 + \sqrt{[1+|T_{n_2}(u_i)|]^2}}} + \frac{2|(1-T_{n_1}(u_i))-(1-T_{n_2}(u_i))|}{\sqrt{[1+|1-T_{n_1}(u_i)|]^2 + \sqrt{[1+(1-T_{n_2}(u_i))|^2}}} \right] + \frac{2|F_{n_i}(u_i) - F_{n_2}(u_i)|}{\sqrt{[1+|F_{n_1}(u_i)|]^2 + \sqrt{[1+|F_{n_2}(u_i)|]^2}}} + \frac{2|(1-F_{n_1}(u_i))-(1-F_{n_2}(u_i))|}{\sqrt{[1+|1-F_{n_1}(u_i)|]^2 + \sqrt{[1+(1-F_{n_2}(u_i)|]^2}}} \right]
\]

(2)

**Theorem 2.**

Single valued neutrosophic weighted SN-cross-entropy (defined in Equation (2)) satisfies the following properties:

i). \( CE_{SN}^w (H_1, H_2) \geq 0 \), if and only if \( T_{n_1}(u_i) = T_{n_2}(u_i), I_{n_1}(u_i) = I_{n_2}(u_i), F_{n_1}(u_i) = F_{n_2}(u_i) \), \( \forall u_i \in U \).

ii). \( CE_{SN}^w (H_1, H_2) = CE_{SN}^w (H_2, H_1) \).

iii). \( CE_{SN}^w (H_1, H_2) = CE_{SN}^w (H_1, H_3) + CE_{SN}^w (H_3, H_2) \).

iv). \( CE_{SN}^w (H_1, H_2) = CE_{SN}^w (H_2, H_1) \).

**Proof:** i).

For all values of \( u_i \in U \),

\[
\left| T_{n_1}(u_i) \right| \geq 0, \left| T_{n_2}(u_i) \right| \geq 0,
\]

\[
\left| T_{n_1}(u_i) - T_{n_2}(u_i) \right| \geq 0, \sqrt{1+\left| T_{n_1}(u_i) \right|^2} \geq 0, \sqrt{1+\left| T_{n_2}(u_i) \right|^2} \geq 0, \left| 1-T_{n_1}(u_i) \right| \geq 0, \left| 1-T_{n_2}(u_i) \right| \geq 0,
\]

\[
\left| 1-T_{n_1}(u_i) \right| - \left| 1-T_{n_2}(u_i) \right| \geq 0, \sqrt{1+\left| 1-T_{n_1}(u_i) \right|^2} \geq 0, \sqrt{1+\left| 1-T_{n_2}(u_i) \right|^2} \geq 0, \text{ then}
\]

\[
\frac{2\left| T_{n_1}(u_i) - T_{n_2}(u_i) \right|}{\sqrt{1+\left| T_{n_1}(u_i) \right|^2} + \sqrt{\left| T_{n_2}(u_i) \right|^2}} + \frac{2\left| 1-T_{n_1}(u_i) \right| - \left| 1-T_{n_2}(u_i) \right|}{\sqrt{1+\left| 1-T_{n_1}(u_i) \right|^2} + \sqrt{\left| 1-T_{n_2}(u_i) \right|^2}} \geq 0
\]

Similarly,

\[
\frac{2\left| I_{n_1}(u_i) - I_{n_2}(u_i) \right|}{\sqrt{1+\left| I_{n_1}(u_i) \right|^2} + \sqrt{\left| I_{n_2}(u_i) \right|^2}} + \frac{2\left| 1-I_{n_1}(u_i) \right| - \left| 1-I_{n_2}(u_i) \right|}{\sqrt{1+\left| 1-I_{n_1}(u_i) \right|^2} + \sqrt{\left| 1-I_{n_2}(u_i) \right|^2}} \geq 0
\]

and

\[
\frac{2\left| F_{n_1}(u_i) - F_{n_2}(u_i) \right|}{\sqrt{1+\left| F_{n_1}(u_i) \right|^2} + \sqrt{\left| F_{n_2}(u_i) \right|^2}} + \frac{2\left| 1-F_{n_1}(u_i) \right| - \left| 1-F_{n_2}(u_i) \right|}{\sqrt{1+\left| 1-F_{n_1}(u_i) \right|^2} + \sqrt{\left| 1-F_{n_2}(u_i) \right|^2}} \geq 0.
\]

Since \( w_i \in [0,1] \) and \( \sum_{i=1}^{n} w_i = 1 \), therefore, \( CE_{SN}^w (H_1, H_2) \geq 0 \).

Hence complete the proof.

ii).

Since,
\[
\left[ \frac{2\|T_{\text{H}1}(u) - T_{\text{H}2}(u)\|}{\sqrt{1 + \|T_{\text{H}1}(u)\|^2}} + \frac{2\|1 - T_{\text{H}1}(u)\| - (1 - T_{\text{H}2}(u))\|}{\sqrt{1 + \|1 - T_{\text{H}1}(u)\|^2}} \right] = 0,
\]

\[
\Longleftrightarrow T_{\text{H}1}(u) = T_{\text{H}2}(u),
\]

\[
\left[ \frac{2\|I_{\text{H}1}(u) - I_{\text{H}2}(u)\|}{\sqrt{1 + \|I_{\text{H}1}(u)\|^2}} + \frac{2\|1 - I_{\text{H}1}(u)\| - (1 - I_{\text{H}2}(u))\|}{\sqrt{1 + \|1 - I_{\text{H}1}(u)\|^2}} \right] = 0,
\]

\[
\Longleftrightarrow I_{\text{H}1}(u) = I_{\text{H}2}(u),
\]

\[
\left[ \frac{2\|F_{\text{H}1}(u) - F_{\text{H}2}(u)\|}{\sqrt{1 + \|F_{\text{H}1}(u)\|^2}} + \frac{2\|1 - F_{\text{H}1}(u)\| - (1 - F_{\text{H}2}(u))\|}{\sqrt{1 + \|1 - F_{\text{H}1}(u)\|^2}} \right] = 0,
\]

\[
\Longleftrightarrow F_{\text{H}1}(u) = F_{\text{H}2}(u) \quad \text{and} \quad w_i \in [0,1], \sum_{i=1}^{n} w_i = 1, w_i \geq 0. \text{ So, } CE_{\text{w}}^n (H_1, H_2) = 0 \iff T_{\text{H}1}(u) = T_{\text{H}2}(u),
\]

\[
I_{\text{H}1}(u) = I_{\text{H}2}(u), F_{\text{H}1}(u) = F_{\text{H}2}(u), \forall u_i \in U.
\]

Hence complete the proof.

\[
\text{iii). Using definition 5, we obtain the following expression}
\]

\[
CE_{\text{w}}^n (H_1', H_2') = \frac{1}{2} \sum_{i=1}^{n} w_i \left[ \frac{2\|T_{\text{H}1}(u) - T_{\text{H}2}(u)\|}{\sqrt{1 + \|T_{\text{H}1}(u)\|^2}} + \frac{2\|1 - T_{\text{H}1}(u)\| - (1 - T_{\text{H}2}(u))\|}{\sqrt{1 + \|1 - T_{\text{H}1}(u)\|^2}} \right] +
\]

\[
\left[ \frac{2\|I_{\text{H}1}(u) - I_{\text{H}2}(u)\|}{\sqrt{1 + \|I_{\text{H}1}(u)\|^2}} + \frac{2\|1 - I_{\text{H}1}(u)\| - (1 - I_{\text{H}2}(u))\|}{\sqrt{1 + \|1 - I_{\text{H}1}(u)\|^2}} \right] +
\]

\[
\left[ \frac{2\|F_{\text{H}1}(u) - F_{\text{H}2}(u)\|}{\sqrt{1 + \|F_{\text{H}1}(u)\|^2}} + \frac{2\|1 - F_{\text{H}1}(u)\| - (1 - F_{\text{H}2}(u))\|}{\sqrt{1 + \|1 - F_{\text{H}1}(u)\|^2}} \right] = CE_{\text{w}}^n (H_1, H_2)
\]
9 of 20

Hence complete the proof.

iv).

Since  \( |T_{i1}(u) - T_{i2}(u)| = |T_{i2}(u) - T_{i1}(u)| \), \( |I_{i1}(u) - I_{i2}(u)| = |I_{i2}(u) - I_{i1}(u)| \),

we obtain

\[
\sqrt{1 + |T_{i1}(u)|^2} + \sqrt{1 + |T_{i2}(u)|^2} = \sqrt{1 + |T_{i2}(u)|^2} + \sqrt{1 + |T_{i1}(u)|^2},
\]

\[
\sqrt{1 + |I_{i1}(u)|^2} + \sqrt{1 + |I_{i2}(u)|^2} = \sqrt{1 + |I_{i2}(u)|^2} + \sqrt{1 + |I_{i1}(u)|^2},
\]

\[
\sqrt{1 + |F_{i1}(u)|^2} + \sqrt{1 + |F_{i2}(u)|^2} = \sqrt{1 + |F_{i2}(u)|^2} + \sqrt{1 + |F_{i1}(u)|^2},
\]

\[
\sqrt{1 + |I_{i1}(u)|^2} + \sqrt{1 + |I_{i2}(u)|^2} = \sqrt{1 + |I_{i2}(u)|^2} + \sqrt{1 + |I_{i1}(u)|^2},
\]

\[
\sqrt{1 + |F_{i1}(u)|^2} + \sqrt{1 + |F_{i2}(u)|^2} = \sqrt{1 + |F_{i2}(u)|^2} + \sqrt{1 + |F_{i1}(u)|^2}, \quad \forall u_i \in U.
\]

and \( w_i \subseteq [0, 1], \sum w_i = 1 \).

So,  \( CE_{SN}^w (H_1, H_2^*) = CE_{SN}^w (H_1^*, H_2) \).

Hence complete the proof.

4. MAGDM strategy using proposed SN-cross entropy measure under SVNS environment

In this section, we develop a new MAGDM strategy using the proposed NS-cross entropy measure.

4.1 Description of the MAGDM problem

Assume that  \( A = \{A_1, A_2, A_3, \ldots, A_m\} \) and  \( G = \{G_1, G_2, G_3, \ldots, G_n\} \) be the discrete set of alternatives and attributes respectively and \( W = \{w_1, w_2, w_3, \ldots, w_n\} \) be the weight vector of attributes  \( G_j \) (\( j = 1, 2, 3, \ldots, n \)), where \( w_j \geq 0 \) and \( \sum w_j = 1 \). Assume that \( E = \{E_1, E_2, E_3, \ldots, E_p\} \) be the set of decision makers who are employed to evaluate the alternatives. The weight vector of the decision makers \( E_k (k = 1, 2, 3, \ldots, p) \) is \( \lambda = \{\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{p}\} \) (where, \( \lambda \geq 0 \) and \( \sum \lambda_k = 1 \)), which can be determined according to the decision makers expertise, judgment quality and domain knowledge.

Now, we describe the steps of the proposed MAGDM strategy using SN-cross entropy measure.

4.1.1. MAGDM strategy using SN-cross entropy function

Step 1. Formulate the decision matrices

For MAGDM with SVNSs information, the rating values of the alternatives \( A_i (i = 1, 2, 3, \ldots, m) \) based on the attribute \( G_j (j = 1, 2, 3, \ldots, n) \) provided by the \( k \)-th decision maker can be expressed in terms of SVNN as \( a_{ij}^k \leq T_j^k, I_j^k, F_j^k \) (\( i = 1, 2, 3, \ldots, m; j = 1, 2, 3, \ldots, n; k = 1, 2, 3, \ldots, p \)). We present these rating values of alternatives provided by the decision makers in matrix form as follows:
Step: 2. Formulate the weighted aggregated decision matrix
For obtaining one group decision, we aggregate all individual decision matrices to an aggregated decision matrix using the Equation (9) as follows:

\[
M^k = \begin{pmatrix}
G_1 & G_2 & \ldots & G_n \\
A_1 & a_{11}^k & a_{12}^k & \ldots & a_{1n}^k \\
A_2 & a_{21}^k & a_{22}^k & \ldots & a_{2n}^k \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
A_m & a_{m1}^k & a_{m2}^k & \ldots & a_{mn}^k \\
\end{pmatrix}
\] (7)

Step: 3. Formulate priori/ideal decision matrix
In the MAGDM, the priori decision matrix has been used to select the best alternatives among the set of collected feasible alternatives. In decision making situation, we use the following decision matrix as priori decision matrix.

\[
P = \begin{pmatrix}
G_1 & G_2 & \ldots & G_n \\
A_1 & a_{11}^* & a_{12}^* & \ldots & a_{1n}^* \\
A_2 & a_{21}^* & a_{22}^* & \ldots & a_{2n}^* \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
A_m & a_{m1}^* & a_{m2}^* & \ldots & a_{mn}^* \\
\end{pmatrix}
\] (10)

where, \( a_{ij}^* = \min (T_{ij}^k), \min (T_{ij}^k), \min (F_{ij}^k) \) and \( i = 1, 2, 3, \ldots, m; j = 1, 2, 3, \ldots, n \).

Step: 4. Calculate the weighted SN-cross entropy measure
Using equation (2), we calculate weighted cross entropy value between aggregate matrix and priori matrix. The cross entropy values can be presented in matrix form as follows:

\[
^{SN}M_{CE}^w = \begin{pmatrix}
CE_{SN}^w (A_1) \\
CE_{SN}^w (A_2) \\
\vdots \\
CE_{SN}^w (A_m) \\
\end{pmatrix}
\] (11)

Step: 5. Rank the priority
Smaller value of the cross entropy reflects that an alternative is closer to the ideal alternative. Therefore, the preference priority order of all the alternatives can be determined according to the increasing order of the cross entropy values \( CE_{SN}^w (A_i) \) \( (i = 1, 2, 3, \ldots, m) \). Smallest cross entropy value indicates the best alternative and greatest cross entropy value indicates the worst alternative.

Step: 6. Select the best alternative
From the preference rank order (from step 5), we select the best alternative.

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doi:10.20944/preprints201801.0006.v1
Preparatory Phase

Multi attribute group decision making problem

Decision making analysis phase

Formation of decision matrix provided by decision makers

Formation of weighted aggregated decision matrix

Formation of ideal decision matrix

Calculation weighted SN- cross entropy measure

Rank the priority

Selection the best alternative

Figure.1 Decision making procedure of proposed MAGDM method
In this section, we solve an illustrative example adapted from [12] of MAGDM problems to reflect the feasibility, applicability and efficiency of the proposed strategy under SVNS environment. Now, we use the example [12] for cultivation and analysis. A venture capital firm intends to make evaluation and selection to five enterprises with the investment potential:

1) Automobile company (A1)
2) Military manufacturing enterprise (A2)
3) TV media company (A3)
4) Food enterprises (A4)
5) Computer software company (A5)

On the basis of four attributes namely:
1) Social and political factor (G1)
2) The environmental factor (G2)
3) Investment risk factor (G3)
4) The enterprise growth factor (G4).

The investment firm makes a panel of three decision makers \( E = \{ E_1, E_2, E_3 \} \) having their weight vector \( \lambda = \{42, 28, 30\} \) and weight vector of attributes is \( W = \{24, 25, 28\} \).

The steps of decision making strategy (4.1.1.) to rank alternatives are presented as follows:

**Step: 1. Formulate the decision matrices**

We represent the rating values of alternatives \( A_i \) (i = 1, 2, 3, 4, 5) with respects to the attributes \( G_j \) (j = 1, 2, 3, 4) provided by the decision makers \( E_k \) (k = 1, 2, 3) in matrix form as follows:

**Decision matrix for \( E_1 \) decision maker**

\[
M^1 = \begin{pmatrix}
G_1 & G_2 & G_3 & G_4 \\
A_1 & (0.9, 0.5, 0.4, 0.4) & (0.7, 0.4, 0.4, 0.4) & (0.7, 0.3, 0.4, 0.9) \\
A_2 & (0.7, 0.2, 0.3) & (0.8, 0.4, 0.3) & (0.9, 0.6, 0.5) & (0.9, 0.1, 0.3) \\
A_3 & (0.8, 0.4, 0.4) & (0.7, 0.4, 0.2) & (0.9, 0.7, 0.6) & (0.7, 0.3, 0.3) \\
A_4 & (0.5, 0.8, 0.7) & (0.6, 0.3, 0.4) & (0.7, 0.2, 0.5) & (0.5, 0.4, 0.7) \\
A_5 & (0.8, 0.4, 0.3) & (0.5, 0.4, 0.5) & (0.6, 0.4, 0.4) & (0.9, 0.7, 0.5) \\
\end{pmatrix}
\]

**Decision matrix for \( E_2 \) decision maker**

\[
M^2 = \begin{pmatrix}
G_1 & G_2 & G_3 & G_4 \\
A_1 & (0.7, 0.2, 0.3) & (0.5, 0.4, 0.5) & (0.9, 0.4, 0.5) & (0.6, 0.5, 0.3) \\
A_2 & (0.7, 0.4, 0.4) & (0.7, 0.3, 0.4) & (0.7, 0.3, 0.4) & (0.6, 0.4, 0.3) \\
A_3 & (0.6, 0.4, 0.4) & (0.5, 0.3, 0.5) & (0.9, 0.5, 0.4) & (0.6, 0.5, 0.6) \\
A_4 & (0.7, 0.5, 0.3) & (0.6, 0.3, 0.6) & (0.7, 0.4, 0.4) & (0.8, 0.5, 0.4) \\
A_5 & (0.9, 0.4, 0.3) & (0.6, 0.4, 0.5) & (0.8, 0.5, 0.6) & (0.5, 0.4, 0.5) \\
\end{pmatrix}
\]

**Decision matrix for \( E_3 \) decision maker**

\[
M^3 = \begin{pmatrix}
G_1 & G_2 & G_3 & G_4 \\
A_1 & (0.7, 0.2, 0.5) & (0.6, 0.4, 0.4) & (0.7, 0.4, 0.5) & (0.9, 0.4, 0.3) \\
A_2 & (0.6, 0.5, 0.5) & (0.9, 0.3, 0.4) & (0.7, 0.4, 0.3) & (0.8, 0.4, 0.5) \\
A_3 & (0.8, 0.3, 0.5) & (0.9, 0.3, 0.4) & (0.8, 0.3, 0.4) & (0.7, 0.3, 0.4) \\
A_4 & (0.9, 0.3, 0.4) & (0.6, 0.3, 0.4) & (0.5, 0.2, 0.4) & (0.7, 0.3, 0.5) \\
A_5 & (0.8, 0.3, 0.3) & (0.6, 0.4, 0.3) & (0.6, 0.3, 0.4) & (0.7, 0.3, 0.5) \\
\end{pmatrix}
\]

**Step: 2. Formulate the weighted aggregated decision matrix**

Using the equation (9), the aggregated decision matrix is presented as follows:
Aggregated decision matrix

\[
\begin{bmatrix}
G_1 & G_2 & G_3 & G_4 \\
A_1 & (8.0,4.0,4.0) & (8.0,4.0,4.0) & (8.0,4.0,4.0) & (8.0,4.0,4.0) \\
A_2 & (8.0,4.0,4.0) & (8.0,4.0,4.0) & (8.0,4.0,4.0) & (8.0,4.0,4.0) \\
A_3 & (8.0,4.0,4.0) & (8.0,4.0,4.0) & (8.0,4.0,4.0) & (8.0,4.0,4.0) \\
A_4 & (8.0,4.0,4.0) & (8.0,4.0,4.0) & (8.0,4.0,4.0) & (8.0,4.0,4.0) \\
A_5 & (8.0,4.0,4.0) & (8.0,4.0,4.0) & (8.0,4.0,4.0) & (8.0,4.0,4.0)
\end{bmatrix}
\]

(25)

Step: 3. Formulate priori/ideal decision matrix

Priori/ideal decision matrix

\[
P = \begin{bmatrix}
G_1 & G_2 & G_3 & G_4 \\
A_1 & (1.0,0.0) & (1.0,0.0) & (1.0,0.0) & (1.0,0.0) \\
A_2 & (1.0,0.0) & (1.0,0.0) & (1.0,0.0) & (1.0,0.0) \\
A_3 & (1.0,0.0) & (1.0,0.0) & (1.0,0.0) & (1.0,0.0) \\
A_4 & (1.0,0.0) & (1.0,0.0) & (1.0,0.0) & (1.0,0.0) \\
A_5 & (1.0,0.0) & (1.0,0.0) & (1.0,0.0) & (1.0,0.0)
\end{bmatrix}
\]

(26)

Step: 4. Calculate the weighted SVNS cross entropy matrix

Using the equation (2), we calculate the single valued weighted cross entropy values between ideal matrix and weighted aggregated decision matrix.

\[
\begin{bmatrix}
0.935 \\
0.775 \\
0.840 \\
1.000 \\
0.980
\end{bmatrix}
\]

(27)

Step: 5. Rank the priority

The cross entropy values of alternatives are arranged in increasing order as follows:

0.775 < 0.840 < 0.935 < 0.980 < 1.000.

Alternatives are then preference ranked as follows:

A₂ > A₃ > A₁ > A₅ > A₄.

Step: 6. Select the best alternative

From step 5, we identify A₂ is the best alternative. Hence, military manufacturing enterprise (A₂) is the best alternative for investment.

Figure 2. Bar diagram of alternatives versus cross entropy values of alternatives
In Figure 3, we represent the relation between cross entropy values and acceptance values of alternatives. The range of acceptance level for five alternatives is taken five points. The high acceptance level of alternative indicates the best alternative for acceptance and low acceptance level of alternative indicates the poor acceptance alternative.

We see from Figure 3 that alternative A2 has the smallest cross entropy value and the highest acceptance level. Therefore A2 is the best alternative for acceptance. Figure 3 indicates that alternative A4 has highest cross entropy value and lowest acceptance value that means A4 is the worst alternative. Finally, we conclude that the relation between cross entropy values and acceptance value of alternatives is opposite in nature.

6. Comparative study and discussion

In literature only MADM strategy [88, 91] have been proposed. So the proposed MAGDM is non-comparable. However, for comparison purpose, the MADM strategies [88, 91] are transformed into MAGDM and for calculation purpose we assume the same set of weights for the decision makers. Then the obtained result derived from the proposed method is compared the results obtained from two existing strategies [88, 91] under SVNS environment. We present ranking order of the alternatives (see Table 1) using same illustrative example for the proposed strategy and two [88, 91].

Table 1. Ranking order of alternatives using different single valued neutrosophic cross entropy function

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>CE_{NS}^w (A_1) = .935</td>
<td>N_u(A_1) = .493</td>
<td>D(A_3) = .365</td>
</tr>
<tr>
<td>CE_{NS}^w (A_2) = .775</td>
<td>N_u(A_2) = .367</td>
<td>D(A_2) = .244</td>
</tr>
<tr>
<td>CE_{NS}^w (A_3) = .840</td>
<td>N_u(A_3) = .415</td>
<td>D(A_3) = .288</td>
</tr>
<tr>
<td>CE_{NS}^w (A_4) = 1.00</td>
<td>N_u(A_4) = .410</td>
<td>D(A_4) = .414</td>
</tr>
<tr>
<td>CE_{NS}^w (A_5) = .980</td>
<td>N_u(A_5) = .510</td>
<td>D(A_5) = .431</td>
</tr>
</tbody>
</table>

Preference ranking order \( A_2 > A_3 > A_1 > A_5 > A_4 \) Preference ranking order \( A_2 > A_4 > A_3 > A_1 > A_5 \) Preference ranking order \( A_2 > A_3 > A_1 > A_4 > A_5 \)

i). The MADM strategies [88] and [91] are not applicable for MAGDM problems. The proposed MAGDM strategy is free from such drawbacks.

ii). Ye [88] proposed cross entropy that does not satisfy the symmetrical property straightforward and is undefined for some situation [91] but the proposed strategy satisfies symmetry property and free from undefined phenomenon.
iii) The best alternative is the same for the three strategies. However, the preference ranking orders are not the same.

Figure 4. Graphical representation of ranking order of five alternatives based on three strategies.

7. Conclusion

In this paper we have defined a new cross entropy measure in SVNS environment which is free from all the drawback of existence cross entropy measures. We have proved the basic properties of the SN-cross entropy measure. We also defined weighted SN-cross entropy measure and proved its basic properties. Based on the weighted SN-cross entropy measure we have developed a novel MAGDM strategy to solve neutrosophic group decision making problems. We have at first proposed MAGDM strategy based on SN-cross entropy measure. Other existing cross entropy measures can deal only MADM problem with single decision maker. So in general, our proposed MAGDM strategy is not comparable with the existing MADM strategies. However, for comparison with the existing strategies, at first we have made them MAGDM strategies and consider the same set of weights of the decision makers and presented comparison analysis. Finally, we solve a MAGDM problem to show the feasibility, applicability and efficiency of the proposed MAGDM strategy. In future study, the proposed MAGDM strategy based on SN-cross entropy can be applied in teacher selection, pattern recognition, weaver selection, medical treatment selection option, and other practical problems.

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