A New Model Transfer Mechanism Framework for SLEUTH Model Performance Evaluation

Fang Liu 1,2, Jinling Quan 3,4, Ling Zhu 1,2, Qiang Chen 1,8, Yizhou Meng 1, Zhechen Na 1, Yanfang Li 1, Ang Gao 1, Sinan Li 4

1 School of Geomatics and Urban Spatial Information, Beijing University of Civil Engineering and Architecture, Beijing 100044, China; lf@bucea.edu.cn (F.L.); zhuling@bucea.edu.cn (L.Z.); chenqiang@bucea.edu.cn (Q.C.); 18630526196@163.com (Y.Z.M.); 870391757@qq.com (ZC.N.); 2287014558@qq.com (YF.L.); hlyuansh@163.com (A.G.); lsncageo@outlook.com (SN.L.)

2 Key Laboratory for Modern Urban Surveying and Mapping of National Administration of Surveying, Mapping and Geoinformation, Beijing 100044, China;

3 Institute of Geographic Sciences and Natural Resources Research, CAS, Beijing 100101, China; quanjl@lreis.ac.cn (JL.Q)

4 State Key Laboratory of Resources and Environmental Information System;

* Correspondence: chenqiang@bucea.edu.cn; Tel.: +86-152-1098-0906

Abstract: SLEUTH Model (slope, landuse, exclusion, urban extent, transportation and hillshade) is an important tool for landuse planning and land policy. To evaluate the performance of SLEUTH model, implementation of Sensitivity Analysis (SA) is essential. The main limitation of SA in SLEUTH application is a lack of insight into model input self-modification parameters (SMPs) variation, namely, uncertainty involved in the model transfer metrics and model presumptions, which often misled the decision makers and model users. To address this issue, this study divided the forward process into two stages. Firstly, during the transfer process ①, the contribution scores of five SMPs were drawn, and parameters highly sensitive to model output were given. Apart from that, the recommended initial value for SMPs of 0.11, 0.2, 0.87, 1.13, 15, 1.01, 0.49 were found to be subordinated to such a heterogeneous urban area simulation. Secondly, during the transfer process ②, SMP caused imagery metrics indicated the disparity between parameters with Fixed Reference and with Successive Reference. Reversely, it derives reasonable threshold for the best fit values of five prediction coefficients' initialization by comparing the real image with the predicted one. The framework of SLEUTH model transfer mechanism not only could distinguish highly sensitive SMPs with higher contribution scores, but also could give parametric analysis for simulation imagery based on metrics. The study was found to be a practical tool for quantization response of model input variables for modelling complex urban systems. So, this insight can help geographic information scientists decide how to find out the inner forward transfer mechanism of SLEUTH model for further make good use of it and improve the model.

Keywords: SLEUTH model; Sensitivity Analysis; uncertainty assessment; urban expansion

1. Introduction

SLEUTH models (slope, landuse, exclusion, urban extent, transportation and hillshade) of landuse change are popular and useful tools for simulating complex urban systems (e.g. Andreas Rienow & Roland Goetzke, 2015; Anil Akin et al., 2014; Gargi Chaudhuri & Keith C. Clarke, 2014; Javad Jafarnezhad et al., 2016; Mahesh Kumar Jat et al., 2017; Haiwei Yin, 2016). A review on the SLEUTH land use change model was published in paper (Gargi Chaudhuri & Keith Clarke, 2013). However, morphology and structure form of urban sprawl are full of variety, considering urbanization has hastened the process in region land use. Meanwhile the urbanization model research is always a hot issue for geographical scientists. Paper “Urban growth models: progress and perspective” (Xuecao Li & Peng Gong, 2016) researched hundreds of models and abstracted the evolution of urban growth models from two perspectives: from macro to micro, from static to dynamic (Xiping Cheng & Hu Sun, 2012; Na Li, 2011; Chi Xu, 2015). Besides, this paper outlined “An evolution tree of CA-based urban growth models” according to importance dimension and age
dimension. At present, these models developed in terms of modeling mechanisms, data-driven mechanisms (based on statistical empirical relationships) and process-driven approach mechanisms (based on feedback and interactions). A kind systematic retrospect model is needed for future new model development.

Sensitivity analysis (SA) represents an important step in improving the understanding and use of landuse change prediction models. The results inaccuracies and uncertainty of model might be attributed to the structure and nature of model, SA is the only critical tool for quantifying response of model input variables for modelling complex urban systems, and avoiding unusually high or low growth rate of urban expansion. There are many different SA approaches. Overall, they can be categorized into two groups: local SA and global SA. The local SA explores the changes of model response by varying one parameter while keeping other parameters constant. The simplest and most common approach is differential SA (DSA), which uses partial derivatives or finite differences of parameters at a fixed parameter location as the measure of parametric sensitivity. On the other hand, the global SA examines the changes of model response by varying all parameters at the same time, allowing them to provide robust measures in the presence of nonlinearity and interactions among the parameters (Haruko M. Wainwright, 2014), and thus are generally preferred due to their global properties (Andrea Saltelli, 2008). Generalized SA (GSA) method is one of the global SA methods that are designed to overcome the limitations of local SA methods. A version of GSA method, the Generalized Likelihood Uncertainty Estimation (GLUE) method was developed (Keith Beven & Andrew Binley, 1992). GSA is simple to implement and can work with different pseudo-likelihood (i.e., goodness of fit) measures (Keith J. Beven, 2011), but it is computationally inefficient. One-At-a-time (OAT) approach have gained popularity recently because they offer the most representative local sensitivity measures while maintaining computational efficiency. In sum, the complexity of these problems highlights the need to understand the sources and range of uncertainty associated with different aspects of the modeling process (Erqi Xu & Hongqi Zhang, 2013; Amin Tayyebi et al., 2016). Because it is easy to implement, computationally inexpensive, and useful to provide a glimpse at the model behavior. Recently, Jiri Nossent & Willy Bauwens (2012) adopted the OAT method for parameter SA of the Soil and Water Assessment Tool (SWAT) model, considering three values for each of the seven parameters. By exploring three values for each of the two factors, Zoras et al. (2007) conducted a parameter screening experiment based on the OAT method for an air pollution model. SA for Cellular Automata urban land use model have been done by someone (Verda Kocabas & Suzana Dragicevic, 2006; Hossein Shafizadeh-Moghadam et al., 2017; Richard Hewitt & Jaime Díaz-Pacheco, 2017). However, as a popular and important method derived from it (Xuecao Li & Peng Gong, 2016), SA for SLEUTH has not been recorded yet.

In this paper, two issues would be dealt with. Forwardly, by means of SA, modelers may probe the response relationship between independent variables and dependent variables using sample data, and discriminate important model factors from non-influential model factors. Reversely, it derives reasonable threshold for the best fit values of five prediction coefficients’ initialization for predicting the real images. These questions are addressed by OAT SA method using sample image data from SLEUTH3.0beta_p01 LINUX released 6/2005 sponsored by Project Gigalopolis, downloaded from website: http://www.ncgia.ucsb.edu/projects/gig/Dnload/download.htm.

2. Materials and methods

2.1. SLEUTH model & Data

SLEUTH is characterized by a series of rules in a nested loop to simulate urban growth, that derived by four growth rules as follows (Matteo Caglioni et al., 2006; Keith Clarke, 2008; Anl Akın, 2014): 1) the spontaneous growth defines the probability for any non-urbanized cell to be developed into an urban cell in the next step, which is determined by the diffusion and slope parameters; 2) the new spreading center growth defines the probability that the new, spontaneously urbanized cells will become new urban spreading centers, which is determined by the breed and slope parameters; 3) the edge growth defines the probability for existing urban spreading centers to expand outward or inward, which is determined by the spread and slope parameters; and 4) the road influenced growth
simulates the tendency of new urban cells to appear near existing transportation networks, which is determined by the breed, road gravity, diffusion, and slope coefficients.

SLEUTH uses a brute-force method, making attempts on sufficient combinations of model self-modification parameters (SMPs) to perform a calibration and derive a set of parameters that can best capture the historic growth trend of a study area. The SMPs includes seven parameters as: ROAD_GRAV_SENSITIVITY, SLOPE_SENSITIVITY, CRITICAL_LOW, CRITICAL_HIGH, CRITICAL_SLOPE, BOOM, BUST. These initial empiric values setting decide the processing and final results; at the same time, and govern the urban simulation imagery. If there is one real imagery, new initial value could simulate “Ideal imagery” with no significant discrepancy with real one.

Tested object is demo200 image package download along with “SLEUTH3.0.beta_p01_linux”, which is dependent of model input parameters SA test. Figure 1 illustrates the input maps in 1m resolution with size of 200×200.

Figure 1. The input maps of demo200 with size of 200×200
Figure 1 illustrates the construction of SLEUTH model input image. In figure 1, it is consisted of six types of data as: slope, landuse, exclude, urban, roads and hillshade, respectively.

2.2. Simulation workflow design

In the forward stage, three goals were accomplished.
1) The **Processing results** response to the **Initialization parameters** variation have been recorded and a new rule was established.
2) The **Final results** response to the **Initialization parameters** variation have been recorded and the above rule was supplemented.
3) An important **Feedback mechanism** from the rules for monitoring the control governance was extracted.

The forward simulation workflow govern by SMPs is combined by three stages (in Figure 2):
1. Firstly, little alteration to SMPs has transmitted corresponding information to PBFs, which is called stage ①: SMP+δ→PBF+Δ; secondly, little alteration to SMPs has acted on predicted images and avg files indirectly, which is called stage ②: SMP+δ→Final results+Θ. Thirdly, new alteration value δ' is drawn from the feedback mechanism, which is called stage ③.

**Figure 2. Workflow chart of SLEUTH Simulation framework**

**SMP** is the initial empiric value setting, which is expressed as \{N_i, i=1,2,3,4,5,6\}. The coefficients effect how the growth rules are applied to the data. They are, N_1: ROAD_GRAV_SENSITIVITY, N_2: SLOPE_SENSITIVITY, N_3: CRITICAL_LOW, CRITICAL_HIGH, N_4: CRITICAL_SLOPE, N_5: BOOM, N_6: BUST respectively. One suite of parameter set only with one δ alteration added upon one Ni parameter, expressed as \{N_1, N_2, ..., N_i+δ, ..., N_6\}. There are 65 suite of parameter sets totally, as one suitable alteration added to one each time.

**PBF** is the processing best fit record by altering the suite of parameter set calculated from the four steps calibration, which is expressed as \{G_k, k=1,2,3,4,5\}. They are, G_1: PREDICTION_DIFFUSION_BEST_FIT, G_2: PREDICTION_SPREAD_BEST_FIT, G_3: PREDICTION_BREED_BEST_FIT, G_4: PREDICTION_SLOPE_RESISTANCE_BEST_FIT, G_5: PREDICTION_ROAD_GRAVITY_BEST_FIT. PBF, which is purposed to predict the final images and record, is inseparably related with SMP. In order to detect the SA details, different relationship expression strategies are applied. It employed: 1) Weight; 2) absolute value; 3) difference unitization using the initial empiric value; 4) difference unitization using the sorting values.

The final results are including: **Predicted images** and **avg files**, which are the indirect result from PBF, which is also the record of model input parameters set SMPs varying. Prediction images are landuse charts from 1991 to 2010, totally twenty years. Considering too many images, 20*65, simplified metrics are borrowed for measuring charts. They are, M_1: Error Ellipse, M_2: Clusters Aggregation, M_3: Urban Area percentage, M_4: Roads’ correlationship with urban. The alteration of...
Final chart is expressed as $[M_1 + \Theta_1, M_2 + \Theta_2, \ldots, M_j + \Theta_j, \ldots, M_4 + \Theta_4]$. As for avg files, they are coefficients record which decides how the growth rules are applied year by year.

2.3. Experiment set up: Model parameters initialization and variation design

The initial states setup of model were parameters $\{N_i\}$ with experience values. However, this paper tends to explore the model with different initial states set different values, which could be applied to heterogeneous urban area simulation. The initial model parameters are experience values included along with SLEUTH code, supplied by original authors. They are, $N_1$: ROAD_GRAV_SENSITIVITY=0.01, $N_2$: SLOPE_SENSITIVITY=0.1, $N_3$: CRITICAL_LOW=0.97, CRITICAL_HIGH=1.03, $N_4$: CRITICAL_SLOPE=15, $N_5$: BOOM=1.01, $N_6$: BUST=0.09.

The aim of variation design is to make a clear of model dynamic transmission mechanism from beginning to the end. According to the idea of OAT sensitivity analysis methodology, this paper measures both system sensitivity and system result response to each independent parameter variation with different size and tendency. There are seven parameters divided into six groups, considering $N_3$ consisting of CRITICAL_LOW and CRITICAL_HIGH, which are widened or narrowed simultaneously. There are altogether six groups of initial input parameters are controlled, and 65 experiment suites are recorded. One parameter suite with only one $\delta$ alteration to one parameter, the minimum step size is set to 0.02 or 0.2 (CRITICAL_SLOPE only), while the study ranges as ±100 times. Considering their physical meaning, there are 65 experiment suites are produced induced from six groups of initial input parameters with range of variable from minus to positive (Figure 3, Table 1).

![Variation of model initialization parameters](image)

Figure 3. Variation of model initialization parameters (in chart form). X axis stands for increment change $\delta$ of SMPs, and Y axis are double axes, standing for dependent variable $\Delta$ for PBFs, where, left axis indicating all PBFs results range except CRITICAL_SLOPE, right axis indicating PBFs results range of CRITICAL_SLOPE.

2.4. Two Derivatives from Absolute Value

When SA information mining is dealt with, two derivatives from Absolute Value are developed, First Difference to fix reference and First Difference to relative reference(formula (1)-(2)). First Difference to fix reference is applied for analysing the differences ($\Delta$) between the output predicted maps/data and standard reference map, the latter with no change on control parameters set. Additionally, First Difference to successive Reference is used for evaluating the differences ($\Delta$)
between two consecutive output predicted maps/data. In order to unify the rate of output results variation, the above-mentioned differences (δ) have both been divided by the change of input parameter (Δ). The expression formulam are:

\[
PBF_1^k = \frac{\Delta_{ref+δ}^k - \Delta_{ref}^k}{|δ|},
\]

\[
PBF_2^k = \frac{\Delta_{ref+δ'}^k - \Delta_{ref}^k}{|δ-δ'|},
\]

where, SMP+δ is expressed as \([N_i, N_2, \ldots, N_i+δ, \ldots N_n]\); PBF+Δ is expressed as \([G_i+1, G_2+Δ, \ldots, G_k+Δ_k, \ldots G_5+Δ_5]\); i is \([N_i, i=1,2,3,4,5,6]\), which decides which SMPs to deal with. \(δ, δ'\) are change of input parameter which are controlled, which means \(δ\) or \(δ'\) times of 0.02 (or 0.2 for CRITICAL_SLOPE only) added to the reference values . Δ is change of PBFs caused by \(δ, δ'\).

**First Difference to fix reference** \((PBF^1)\): its role is to reflect where the initial empirical value is located properly on the X axis.

**First Difference to Successive reference** \((PBF^2)\): its role is to reflect the difference of response between adjacent points has any relationship with location of X.

### 2.5. Charts Metrics

In order to evaluate the geographical expansion of cities, from the predicted imagery, four evaluation indicators are employed, as Directional Distribution, Clusters Aggregation, Urban Area percent, Roads’ correlationship with urban, respectively (Table 1).

### Table 1. Metrics for imagery

<table>
<thead>
<tr>
<th>Indices</th>
<th>Description</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\theta) is the azimuthal orientation angle with due North. The rotation angle (\theta) is calculated as: (\theta = \arctan \left( \frac{A + B}{C} \right))</td>
<td></td>
</tr>
<tr>
<td>Urban expansion</td>
<td>The main direction, Center point xy-coordinates of Urban expansion from 1990 to 2010</td>
<td></td>
</tr>
<tr>
<td>Directional Distribution</td>
<td>(A = \left( \sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} y_i^2 \right))</td>
<td>(B = \sqrt{\left( \sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} y_i^2 \right)^2 + 4 \left( \sum_{i=1}^{n} \bar{x}_i \bar{y}_i \right)^2})</td>
</tr>
<tr>
<td></td>
<td>(C = 2 \sum_{i=1}^{n} \bar{x}_i \bar{y}_i)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\bar{x}_i) and (\bar{y}_i) are the derivations of the xy-coordinates from the Mean Center.</td>
<td></td>
</tr>
</tbody>
</table>
\[
\bar{y}_i = \frac{\sum_{i=1}^{n} (y_i - \bar{Y})^2}{n}
\]

where \(x_i\) and \(y_i\) are the coordinates for feature \(i\), \([\bar{X}, \bar{Y}]\) represents the Mean Centre for the features, and \(n\) is equal to the total number of features.

<table>
<thead>
<tr>
<th>Urban Clusters Aggregation</th>
<th>Urban spatial aggregation degree in 2010</th>
<th>(CAI = 2\sqrt{\pi A/P})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>where (A): Urban Clusters Area, (P): Urban Clusters Perimeter</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Urban Area percent /%</th>
<th>Urban proportion in 2010</th>
<th>(UAI = A/A_T)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>where (A): Urban Expansion Area; (A_T): Imagery total Area</td>
</tr>
</tbody>
</table>

\[
r_{xy} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}}
\]

where \(x_i\): Urban pixel value; \(y_i\): Road pixel value; \(\bar{x}, \bar{y}\): the average value of cluster values of \(x_i, y_i\)

### 3. Results

In order to detect the SA details, process record and final graphic and data results are carried out quantitative analysis with input model parameters. Two research tasks were done.

1) The biggest contributors are screened out and sorted in descending order by the Weight values. Meanwhile, the volatility features of trend are described by the Absolute Value curve.

2) Evaluate whether the initial empiric value setting is appropriate one for each index.
Figure 4. The output predicted landuse maps and the growth types of demo200

Figure 4 depicts the predicted maps derived from the above input image for simulations. The Legends clearly shows LANDUSE CLASS and GROWTH TYPE.

3.1 Model PBF Response to SMP variations

There are two types of data as the Model PBF Response results to SMP variations, Weight (0-100%) and Absolute Value (0.00-100.00). Table 2 illustrate the stage \( ① \) in figure 2, that is, six groups input model parameters variation and the corresponding Model PBF Responses, including: ROAD_GRAV_SENSITIVITY, SLOPE_SENSITIVITY, CRITICAL_HIGH/LOW, CRITICAL_SLOPE, BOOM and BUST. Interval of independent variable is set to be regular pattern, and the fine distinction caused by model initialization parameters A-F series variation are studied.

Table 2. ROAD_GRAV_SENSITIVITY index series variations and the corresponding Model PBF Response

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Dependent Variable Weight (0-100%)</th>
<th>Dependent Variable Absolute Value (0.00-100.00)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>N Times</td>
<td>Diff</td>
</tr>
<tr>
<td>0</td>
<td>-5</td>
<td>1</td>
</tr>
<tr>
<td>0.01</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0.11</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>0.21</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>0.31</td>
<td>15</td>
<td>1</td>
</tr>
<tr>
<td>0.41</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>0.61</td>
<td>30</td>
<td>1</td>
</tr>
<tr>
<td>0.81</td>
<td>40</td>
<td>1</td>
</tr>
<tr>
<td>1.01</td>
<td>50</td>
<td>1</td>
</tr>
<tr>
<td>1.21</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>1.41</td>
<td>70</td>
<td>1</td>
</tr>
<tr>
<td>1.61</td>
<td>80</td>
<td>1</td>
</tr>
<tr>
<td>1.81</td>
<td>90</td>
<td>1</td>
</tr>
<tr>
<td>2.01</td>
<td>100</td>
<td>1</td>
</tr>
<tr>
<td>RANGE</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>STDEV</td>
<td>0.00</td>
<td>1.20</td>
</tr>
</tbody>
</table>

Note: Diff, Brd, Sprd, Slp, Rg are abbreviation forms of DIFFUSION, BREED, SPREAD, SLOPE_RESISTANCE, ROAD_GRAVITY.

Table 2 shows that one of six groups input model parameters variation and the corresponding Model PBF responses.
Figure 5. SMP indices variations and the corresponding Model PBF Response in Weight form

From the Figure 5, the weight ranges of SMP indices variations are shown. The following table summarizes their behavior and lists them in sensitivity degree rank, which explains how much SMPs variation \( \delta \) contribute to the PBF alteration \( \Delta \).

<table>
<thead>
<tr>
<th>Dependent Variable Weight (0-100%)</th>
<th>Sensitivity degree (%)</th>
<th>Diff</th>
<th>Brd</th>
<th>Sprd</th>
<th>Slp</th>
<th>Rg</th>
</tr>
</thead>
<tbody>
<tr>
<td>80-100</td>
<td></td>
<td>\checkmark</td>
<td>\checkmark</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60-80</td>
<td></td>
<td>\checkmark</td>
<td>\checkmark</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40-60</td>
<td></td>
<td>\checkmark</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20-40</td>
<td></td>
<td>\checkmark</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0-20</td>
<td></td>
<td>\checkmark</td>
<td>\checkmark</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average STDEV</td>
<td></td>
<td>1.62</td>
<td>4.21</td>
<td>2.88</td>
<td>13.05</td>
<td>15.99</td>
</tr>
<tr>
<td>Average Range</td>
<td></td>
<td>4.42</td>
<td>13.66</td>
<td>8.78</td>
<td>44.91</td>
<td>38.54</td>
</tr>
</tbody>
</table>

From Table 3, the weight ranking of the SMPs for all criteria is as follows: (SLP,GR)>BRD>(DIFF,SPRD). SLP and GR which are assigned weight of 60%-80%, are high-sensitivity indices. Similarly, BRD are low sensitivity index, while DIFF and SPRD are extrem low sensitivity indices.

First Difference to a fixed Reference is used to reflect where the initial empirical value is located properly on the X axis, while First Difference to a Successive Reference method is employed to judge the difference of response between adjacent points has certain relationship with location of X.
Compare to a Fixed Reference in Line Plot (Fluctuation/Trend)

SMP variations to ROAD_GRAY_SENSITIVITY

Compare to a Successive Reference in Radar Plot (Dispersion)

SMP variations to ROAD_GRAY_SENSITIVITY

SMP variations to SLOPE_SENSITIVITY

SMP variations to CRITICAL_LOW/CRITICAL_HIGH

SMP variations to CRITICAL_LOW/CRITICAL_HIGH
Figure 6. SMP indices variations and the corresponding Model PBF Response in difference form

In Figure 6, when SA details of stage ① are explored, two chart forms as First Difference to a fixed Reference of SMP indices variations are employed, line plot and radar plot. Line plot is applied for exploring the rules of trend and fluctuation as each input parameter is ascending. As a supplement, radar plot is used to observe the dispersion of each series variation, as well as outliers’ position.

According to the First Difference to the Fixed Reference results, the trend line of RD, SLP, BREED and SPREAD have inflexion points in the known empirical value $x=0.11$, namely, ascending before the point, descending after that point. Generally speaking, considering parameters’ weight, the recommended initial value located on the X axis should be 0.11 for ROAD_GRAV_SENSITIVITY index.
Still the same method, the appropriate values of four SMPs is set as 0.2, 0.87, 1.13, 15, 1.01, 0.49 respectively, compared with the original empirical value at 0.1, 0.97, 1.03, 15, 1.01, 0.09, for SLOPE_SENSITIVITY, CRITICAL_LOW/HIGH, CRITICAL_SLOPE, BOOM, BUST, respectively (Table 4).

Table 4. Sensitivity Rank of Dependent Variables’ Weight

<table>
<thead>
<tr>
<th></th>
<th>ROAD_G</th>
<th>SLOPE_SENSITIVITY</th>
<th>CRITICAL_LOW/HIGH</th>
<th>CRITICAL_SLOPE</th>
<th>BOOM</th>
<th>BUST</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empirical Value Location</td>
<td>0.01</td>
<td>0.1</td>
<td>0.97, 1.03</td>
<td>15</td>
<td>1.01</td>
<td>0.09</td>
</tr>
<tr>
<td>Appropriate Value Location</td>
<td>0.11</td>
<td>0.2</td>
<td>0.87, 1.13</td>
<td>15</td>
<td>1.01</td>
<td>0.49</td>
</tr>
<tr>
<td>Change Value</td>
<td>+0.10</td>
<td>+0.10</td>
<td>-0.10, +0.10</td>
<td>0</td>
<td>0</td>
<td>+0.40</td>
</tr>
</tbody>
</table>

According to the First Difference to a Successive Reference results, conclusion from figure 6 is drawn: for different initialization parameters, the sensitive position of the same PBF parameter are different, which proved that specific position has certain relationship with sensitivity extent. According to the radar diagram, the position sensitive point of ROAD_GRAV_SENSITIVITY is 0.01. Still the same method, the position sensitive point to PBF parameter are set as 0.77, 1 and 0.39 in CRITICAL_LOW/HIGH, CRITICAL_SLOPE, BUST, while SLOPE_SENSITIVITY and BOOM indic keep stationary situations.

3.2 Model Imagery Response to SMP variations

In this part, output predicted imagery and its imagery description indices results are discussed. Considering too many output predicted images (in Figure 3), 20*65, simplified metrics are borrowed for measuring charts. They are, six groups input model parameters variations and the corresponding Model Imagery indices Responses results, including: Directional Distribution, Clusters Aggregation, Urban Area percent, Roads’ correlationship with urban, respectively (Table 1).

Figure 7 (a) – (d) illustrate the four groups input model parameters variation and the corresponding model imagery metrics’ responses.
Figure 7(a). Imagery metrics for Model PBF Response in difference form. Urban expansion Directional Distribution Index indicates the main direction of Urban expansion from 1990 to 2010. From the figures above, the Directional Distribution Index are 18.4° to 33.8° from the due North.
Figure 7(b). Imagery metrics for Model PBF Response in difference form. Urban Clusters Index indicates the urban patches spatial aggregation degree in 2010 predicted imagery. From the figures above, the Average Aggregation Index are 0.8148 to 0.8452, reflecting a certain degree of clustering tendency.

Figure 7(c). Imagery metrics for Model PBF Response in difference form. Urban Area percent Index indicates the urban proportion in 2010 predicted imagery. From the figures above, the Urban Area percent Index are 8.88% to 18.11%, reflecting a relative proportion of urban cover area.
Figure 7(d). Imagery metrics for Model PBF Response in difference form. Correlation between Existed Road & Simulated Urban Imagery Index indicates the Correlation between road buffer in 1990 with urban in 2010 predicted imagery. From the figures above, the Correlation Index are from 0.5360 to 0.5967, reflecting a significant correlation.

From stage ② the relation between SMPs and Final results (imagery indices) could be established as follows:

\[ N \times X = M, \quad (3) \]

where, \( N \) is SMP variation groups of \( \{ N_i, i=1...6 \} \), \( M \) is the Final imagery response groups of \( \{ M_j, j=1...4 \} \). \( X \) stands for the transition relationship from SMP to imagery variation, which is what we need to explain the quantitative transition mechanism of stage ②.

With it, the desired alteration added upon one group of \( N_i \) parameters could be calculated according to the difference between predicted imagery and the real one. The significance of SA is to establish the parametric knowledge database for predicting the fitting imagery close to the real one.

Using MATLAB, an example of the \( \{ X \} \) as below is calculated.

\[ X = N^{-1}M = \begin{bmatrix} N_1 & N_2 & N_3 & N_4 & N_5 & N_6 & N_7 \end{bmatrix}^{-1} \begin{bmatrix} M_1 & M_2 & M_3 & M_4 \end{bmatrix}, \quad (4) \]

The analytic hierarchy process (AHP) method has been used to find weight deviation of \( \{ X \} \) (Thomas L. Saaty, 2013). It is first to establish indicators framework, then use the Analytic Hierarchy Process, by constructing a judgment matrix to calculate the maximum eigenvalue of the matrix and vector features, the largest eigenvalue calculation, a one-time inspection steps to determine the weights. An establishment matrix is constructed with \( X \)-\( X \). Weights of seven factors are got by calculating eigenvectors of corresponding characteristic roots that are maximum. Finally, the consistency checks value (CK), the maximum eigenvalue (CI) and the consistency ratio (CR) are 4.01, 0.05 and 0.006, while consistency ratio (CR) less than 0.01 the rationality of consistency and weight can be accepted (Equation (5)).
4. DISCUSSION: Strengths and limitations of the proposed framework

Superiority of the proposed framework is shown in three points. Firstly, the aim of the forward and backward transition rule in this paper is clearly and valuable. For the prediction results to model parameters variation, as well as the uncertainty involved in the model metrics and presumptions, often misled the decision makers (Marleen Schouten, 2014). This study aims to propose a framework for sensitivity analysis (SA) of The SLEUTH model. Forwardly, it performs SA using sample data to probe the response relationship between independent variables and dependent variables. Reversely, it derives reasonable threshold for the best fit values of five prediction coefficients’ initialization by comparing the real image with the predicted one.

Secondly, among the controlling parameters of SLEUTH, SMP parameters variation is chosen as one of the controlling tools. The model’s behavior is affected by its controlling parameters as: Working Grids, Random Number Seed, Monte Carlo Iteration, Excluded Map, Calibration Parameters setup or Self-Modification Parameters setup (Feng Hui-Hui, 2012). When other parameters being equal, Calibration Parameters setup is procedures results of SMP variation. Here, we picked up SMP parameters as operable parameters, between the lower bound and the upper bound with fixed step, to explore structure and nature of SLEUTH model.

Thirdly, this paper adopted graph-level-based data mining, rather than pixel-level-based or feature-level-based one (Junping Zhang et al., 2016). This paper creatively proposes making use of four imagery indices as the metrics measuring the gap between the predicted imagery and the real one, and to reconstruct the real one according to the SMP experience dataset using the One-At-a-time SA method. The Urban Area percent index and the Directional Distribution index, are used to measure the expansion intensity and direction. The Roads’ correlation with urban index is used to evaluate the convergent relationship between road network and urban expansion (lines and planes). The Clusters Aggregation index, is used to evaluate the fragmentation and diversity of patches. Clearly, the experiment results show correlation of the SMP variations and imagery index responses.

Two questions raised in our job are testing samples and testing method adopted. Firstly, sample data adopted in this paper is a typical case, not universal adaptable one. Even including 0–7 classes, the test data could not be standardized and representative for different cases, and remote sensing data should be multi-scale and feature representation diversification (Toshio MichaelChin, 2017). Secondly, One-At-a-time (OAT) approach adopted to explore the SLEUTH model scheme is acceptable for its computation efficiency. Since it cost cumulative 11 hours for four-stage calibration and prediction, local SA approaches are more feasible than global SA approaches.

Prospective job should be done in the following aspects. Firstly, enrich the knowledge library in forward stage, which need hundreds times of experiment accomplishment instead of dozens of times with optimized two-stage SA experiment designing. Secondly, enrich the imagery metrics with more reasonable urban spatial morphology indices (Francois Racine, 2016; WANG Fei, 2016).

\[
\begin{bmatrix}
W_{x1} \\
W_{x2} \\
W_{x3} \\
W_{x4} \\
W_{x5} \\
W_{x6} \\
W_{x7}
\end{bmatrix} =
\begin{bmatrix}
0.11 \\
0.14 \\
0.31 \\
0.32 \\
0.11 \\
0 \\
0.01
\end{bmatrix}
\]
5. Conclusions

This study provided a paradigm of sensitivity analysis for SLEUTH model as well as other urban expansion prediction modeling through adjusting the settings of inherent operational parameters.

In summary, contribution achieved below are accomplished.

1) In the forward stage ①, two derivatives from Absolute Value are developed, First Difference to fix reference ($\theta^1_{\theta^1}$) and First Difference to Successive reference ($\theta^1_{\theta^2}$), to tick sensitive location and quantify parametric variation response. The Initialization parameters variation caused response have been recorded and a new rule was established. Applying above two derivatives, Three points could be drawn. a)The biggest contributors are screened out and sorted in descending order by the Weight values (Table 4). b) Evaluate whether the initial empiric value setting is appropriate one for each index (Table 5). c)Meanwhile, the volatility features of trend are described by the Absolute Value curve (Figure 6).

2) In the forward stage ②, four imagery evaluation indicators are employed, Directional Distribution, Clusters Aggregation, Urban Area percent, Roads’ correlationship with urban, as urban morphology quantify metrics. They could perform well parametric variation response (Figure 7).

3) In the reverse stage, based on the SA training sample database, the transition mechanism could be expressed as an matrix $X$. Using it, an important Feedback mechanism from the rules for monitoring the control governance was extracted, and weight of $X_1$-$X_7$ could be drawn (equation (5)).

This process could supply a routine when apply this framework for analyzing other model inherent operational parameters. However, the forward process with limited sample training times; meanwhile, the wide range land-use changes with dynamic change of driving forces are not considered. Moreover, global SA supporting wider parameters variation show efficiency, accuracy and more robustness that were propose for improvement. Results of show that better transition rules could be obtained with more sample screen out (e.g. two-level fractional factorial screening method or deriving-based global sensitivity method. Furthermore, multiple sampling method is a promising way for further development of SLEUTH urban growth models.

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