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Multivariable Static Output Feedback Control of a Binary Distillation Column using Linear Matrix Inequalities and Genetic Algorithm

Kalpana R.¹, Harikumar K.^{2*}, Senthilkumar J.¹, Balasubramanian G.¹, and Abhay S. Gour³

¹ School of Electrical and Electronics Engineering, SASTRA University, Thanjavur, Tamilnadu, India; e-mail: kalpana.elango@src.sastra.edu; jsenthilkumar18@src.sastra.edu; balu_eie@eie.sastra.edu.

² School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore; e-mail: harikumar100@gmail.com; kharikumar@ntu.edu.sg.

³ Centre for Cryogenic Engineering, Indian Institute of Technology, Kharagpur, India; e-mail: abhay.s.gour@gmail.com

* Correspondence: harikumar100@gmail.com; kharikumar@ntu.edu.sg; Tel.: +6590547402

Abstract: The current work addresses the control of two-input two-output (TITO) Wood and Berry model of a binary distillation column. The controller design problem is formulated in terms of multivariable H_∞ control synthesis. The controller structure takes the form of simplest static output feedback (SOF) control. The controller synthesis is performed using a hybrid approach of blending linear matrix inequalities (LMI) and genetic algorithm (GA). The performance of the static output feedback controller is compared with three other controllers designed for Wood and Berry model available in the literature. The first simulation study is performed for the case of tracking a unit step command in the presence of a step change in output disturbance. A second simulation study is performed for rejecting a change in sinusoidal output disturbance.

Keywords: Distillation column, Disturbance rejection, Genetic algorithm, H_∞ control, Linear matrix inequalities, Static output feedback.

1. Introduction

Multivariable chemical processes are arduous to control due to the coupling between multiple input channels and multiple output channels. Distillation control is a challenging problem due to inherent non-linear nature of the process, severe coupling between various inputs and outputs, nonstationary behavior of the process and effect of unmeasured disturbances. In this paper, we address the issue of multivariable nature of the distillation control having interaction among different control loops and the effect of unmeasured output disturbances. Wood and Berry proposed a popular two-input two-output (TITO) transfer function model of a pilot scale binary distillation column [1]. A plethora of multivariable control techniques are available in the literature, applied to TITO model of binary distillation column proposed by Wood and Berry, [2]-[9]. A method based on the static decoupler design using the steady state gain matrix is proposed in [2]. The decentralized proportional-integral-derivative (PID) controller designed for the decoupled process is then combined with the static decoupler to yield a multivariable controller. A similar methodology of decoupling followed by design of decentralized controllers are discussed in [3] - [9]. The performance of the decentralized controllers depends upon the degree of decoupling achieved. Under the ubiquity of strong coupling between the different control loops, the performance of decentralized controller degrades. Multivariable control design techniques inherently handle the coupling related issues found common

29 in distillation control.

30 In this paper, a multivariable static output feedback (SOF) controller is designed using a hybrid
 31 approach of blending linear matrix inequalities (LMI) and genetic algorithm (GA). Static output
 32 feedback control has found a lot of applications due to the ease in practical implementation [10] - [14].
 33 LMI is a powerful tool for synthesizing output feedback controllers satisfying multiple requirements.
 34 Multiple objectives like robust pole placement, H_2/H_∞ performance measures and frequency domain
 35 specifications can be easily realized through a system of LMI's [15] - [20]. The conventional solution of
 36 LMI's utilizes convex programming methods [21]. Multi-objective design requirements often leads to a
 37 system of bilinear matrix inequalities (BMI). In this paper, BMI is transformed to LMI by restructuring
 38 the additional slack variable of BMI as an input variable to GA. The solution provided by GA evolves
 39 over several iterations based on probabilistic rules and leads to an optimal/suboptimal solution [22].
 40 The paper is organized as follows. Section 2, discuss about the transfer function and state space model
 41 of Wood and Berry distillation column. A multivariable H_∞ control formulation is given in section 3.
 42 An algorithm for the synthesis of SOF controller is given in section 4. Section 5 discusses the simulation
 43 results, comparing the performance of SOF controller with other three controllers available in the
 44 literature for Wood and Berry distillation column. Section 6 concludes the paper.

45 2. Model of Binary Distillation Process

The TITO transfer function model for methanol-water separation in a distillation column is given in equation 1, [1]. The controlled variables (outputs) are the composition of top product (Y_1) and bottom product (Y_2), expressed in percentage of methanol in weight. The manipulated variables (control inputs) are the reflux steam flow rate (U_1) and the reboiler steam flow rate (U_2) in the units of lb/min. All the time constants are expressed in the unit of minutes (min).

$$\begin{pmatrix} Y_1(s) \\ Y_2(s) \end{pmatrix} = \begin{pmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{pmatrix} \begin{pmatrix} U_1(s) \\ U_2(s) \end{pmatrix} \quad (1)$$

Using a first order Pade-approximation for time delay in equation 1 yields a rational transfer function for the distillation column model as given in equation 2.

$$\begin{pmatrix} Y_1(s) \\ Y_2(s) \end{pmatrix} = \begin{pmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{pmatrix} \begin{pmatrix} U_1(s) \\ U_2(s) \end{pmatrix} \quad (2)$$

The expression for the terms $g_{11}(s)$ to $g_{22}(s)$ are given in equations from 3 to 6.

$$g_{11}(s) = \frac{12.8(-0.5s + 1)}{(16.7s + 1)(0.5s + 1)} \quad (3)$$

$$g_{12}(s) = \frac{-18.9(-1.5s + 1)}{(21s + 1)(1.5s + 1)} \quad (4)$$

$$g_{21}(s) = \frac{6.6(-3.5s + 1)}{(10.9s + 1)(3.5s + 1)} \quad (5)$$

$$g_{22}(s) = \frac{-19.4(-1.5s + 1)}{(14.4s + 1)(1.5s + 1)} \quad (6)$$

The state space model of the system given in equation 2 is given in equations 7 and 8. The A matrix for the state space model is given in equation 9. The B matrix for the state space model is given in equation 10 and C matrix is given in equations 11.

$$\dot{X} = AX + BU \quad (7)$$

$$Y = CX \quad (8)$$

$$A = \begin{pmatrix} -2.06 & -0.479 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.378 & -0.210 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.125 & 0 & -0.714 & -0.127 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.736 & -0.185 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.25 & 0 \end{pmatrix} \quad (9)$$

$$B = \begin{pmatrix} 2 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 2 \\ 0 & 0 \\ 0 & 2 \\ 0 & 0 \end{pmatrix} \quad (10)$$

$$C = \begin{pmatrix} -0.383 & 3.066 & 0 & 0 & 0.45 & -1.2 & 0 & 0 \\ 0 & 0 & -0.606 & 1.384 & 0 & 0 & 0.674 & -1.796 \end{pmatrix} \quad (11)$$

46 3. Multivariable H_∞ control formulation

47 Optimization in H_∞ space is suitable for systems having uncertainty in dynamics and disturbances. A
 48 diagram representing the H_∞ control formulation for the binary distillation column is given in figure 1.
 49 Along with the weighted sensitivity function (S), weighted complementary sensitivity function (T) is
 50 also minimized leading to multi-objective design problem [24]. Generic form of the weights $W_1(s)$ and
 51 $W_2(s)$ are given in the equations 12 and 13 respectively. The weighting function $W_2(s)$ is taken as high
 52 pass filter to attenuate the detrimental effects of unmodeled dynamics at higher frequencies. To retain
 53 good tracking and disturbance rejection properties at low frequencies, an ideal choice for the weighting
 54 function $W_1(s)$ is a low pass filter. The performance outputs to be minimized are $Z = [Z_1, Z_2]$ and are
 55 given in equations 14 and 15 respectively.

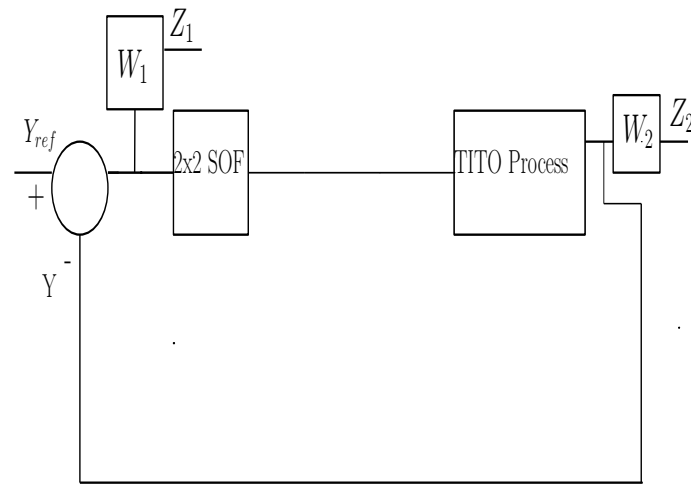


Figure 1. Diagram representing H_∞ control formulation.

$$W_1(s) = \begin{pmatrix} W_{e_1}(s) & 0 \\ 0 & W_{e_2}(s) \end{pmatrix} \quad (12)$$

$$W_2(s) = \begin{pmatrix} W_{y_1}(s) & 0 \\ 0 & W_{y_2}(s) \end{pmatrix} \quad (13)$$

$$Z_1 = [Z_{e_1}, Z_{e_2}]^T \quad (14)$$

$$Z_2 = [Z_{y_1}, Z_{y_2}]^T \quad (15)$$

56 The linear state space equation for the generalized plant are given in equations 16 to 19. Generalized
 57 plant is obtained by augmenting the state space model of binary distillation column given in equations
 58 7 to 11 with the weighting transfer functions $W_1(s)$ and $W_2(s)$. In equations 16 to 19, X_{tg} is the state
 59 vector of the generalized plant, Y is the vector of measurements (Y_1 and Y_2), U_{tc} is the vector of control
 60 inputs (U_1 and U_2) and W_{td} is the vector of disturbance inputs. The disturbance inputs W_{td} corresponds
 61 to the reference inputs Y_{1ref} and Y_{2ref} .

$$\dot{X}_{tg} = A_{tg}X_{tg} + B_{tu}U_{tc} + B_{tw}W_{td} \quad (16)$$

$$Z_1 = C_{t1}X_{tg} + D_{t11}U_{tc} + D_{t12}W_{td} \quad (17)$$

$$Z_2 = C_{t2}X_{tg} + D_{t21}U_{tc} + D_{t22}W_{td} \quad (18)$$

$$Y = C_tX_{tg} \quad (19)$$

65 Selection of weighting transfer functions $W_1(s)$ and $W_2(s)$ follow the guidelines presented in [23]. The
 66 low pass filter weights $W_{e1}(s)$ and $W_{e2}(s)$ are given in equations 20 and 21 respectively. The high pass
 67 filter weights $W_{y1}(s)$ and $W_{y2}(s)$ are given in equations 22 and 23 respectively.

$$W_{e1}(s) = \frac{0.1}{(s + 0.2)} \quad (20)$$

$$W_{e2}(s) = \frac{0.1}{(s + 0.3)} \quad (21)$$

$$W_{y1}(s) = \frac{0.01(s + 0.1)}{(s + 2)} \quad (22)$$

$$W_{y2}(s) = \frac{0.01(s + 0.1)}{(s + 3)} \quad (23)$$

The objective of the control design problem is to synthesis a SOF controller with the structure, $U_{tc} = KY$ that minimizes the norm given in equation 24.

$$\| T_{zwd} \|_{\infty} = \left\| \begin{pmatrix} W_1 S \\ W_2 T \end{pmatrix} \right\|_{\infty} \quad (24)$$

68 4. Static output feedback controller synthesis using LMI

69 Applying SOF control $U_{tc} = KY$ for the system given in equations 16 to 19, the resulting state space
 70 model for the closed loop generalized plant are given in equations 25 to 27.

$$\dot{X}_{tg} = (A_{tg} + B_{tu}KC_t)X_{tg} + B_{tw}W_{td} \quad (25)$$

$$Z_1 = (C_{t1} + D_{t11}KC_t)X_{tg} + D_{t12}W_{td} \quad (26)$$

$$Z_2 = (C_{t2} + D_{t21}KC_t)X_{tg} + D_{t22}W_{td} \quad (27)$$

73 Let $A_{clt} = A_{tg} + B_{tu}KC_t$ be the closed loop system matrix. The Lyapunov condition for stability of
 74 the closed loop system can be written as the following matrix inequality given in equation 28, [17].
 75 In equation 28, $Q = A_{clt}^T P + P A_{clt}$, $M = N B_{tu} B_{tu}^T P$ and $R = B_{tu}^T P + K C_t$, P is a positive definite
 76 matrix and N is any square matrix. The matrix inequality given in equation 28 is bilinear. The bilinear
 77 matrix inequality (BMI) in equation 28 can be converted into LMI by fixing one of the unknowns P or
 78 N . Since there is a constraint on P ($P > 0$), N is taken as a decision variable for genetic algorithm (GA).
 79 Choosing N as the decision variable for GA instead of P is less conservative and results in a enlarged
 80 search space. Multiple choices for N is selected by GA and the resultant LMI is solved by minimizing
 81 a performance index. The performance index is given in equation 29.

$$\begin{pmatrix} Q - M - M^T + N B_{tu} B_{tu}^T N & R^T \\ R & -I \end{pmatrix} < 0 \quad (28)$$

$$J_{PI} = q_{t1}(100) + q_{t2}(\| T_{zwd} \|_{\infty}) \quad (29)$$

82 The variables q_{t1} and q_{t2} in equation 29, takes the binary value 0 or 1 based on closed loop stability
 83 requirements. The binary value of variables q_{t1} and q_{t2} are obtained as follows. Let γ_{cli} denote the i^{th}
 84 eigenvalue of the closed loop system matrix A_{clt} , let n_{gclt} be the dimension of A_{clt} and $Re(\gamma)$ denotes
 85 the real part of the complex variable γ .

86 **CASE I** IF $Re(\gamma_{cli}) \geq 0$, for any $i=1,2,3,\dots,n_{gclt}$, THEN $q_{t1} = 1$ and $q_{t2} = 0$.

87 **CASE II** IF $Re(\gamma_{cli}) < 0, \forall i=1,2,3,\dots,n_{gclt}$, THEN $q_{t1} = 0$ and $q_{t2} = 1$.

88 Therefore, either $J_{PI}=100$ or $0 < J_{PI} < 1$ (since $\|T_{zw_{td}}\|_{\infty} \in (0,1)$). The SOF control gains are found
 89 iteratively using the algorithm given below.

90

- 91 1. Choose N as decision variable and J_{PI} as the performance index for GA.
- 92 2. Solve the LMI given in equation 28 to obtain P and K for the value of N given by GA.
- 93 3. Evaluate the performance index J_{PI} , given in equation 29.
- 94 4. Iterate the steps 1,2 and 3 until J_{PI} is minimized.

95 5. Numerical simulation results and discussion

96 For solving the LMI given by the equation 28, the matrix N is set to a diagonal matrix. Table 1 gives
 97 the initialization details of the genetic algorithm toolbox in MATLAB 7.5. The SOF gain obtained is
 98 given in equation 30 with $\|T_{zw_{td}}\|_{\infty}=0.498$. A comparison of open loop poles and closed loop poles
 99 of the system given in equations 7 to 11 is given in Table 2. Please note that there is no one to one
 100 correspondence between the open loop and closed loop poles given in Table 2. The SOF gain stabilizes
 101 the closed loop system and also minimizes $\|T_{zw_{td}}\|_{\infty}$.

Table 1. Initialization of GA toolbox in MATLAB 7.5.

Attribute	Value
Population size	500
Population range	[-20,20]
Number of generations	100
Selection function	Stochastic uniform
Crossover function	Scattered
Mutation function	Gaussian

$$K = \begin{pmatrix} -0.8145 & 0.6059 \\ -0.2200 & 0.3025 \end{pmatrix} \quad (30)$$

Table 2. Open loop and closed loop poles

Open loop poles	Closed loop poles
-0.0599	-0.9807 - 1.1092i
-0.2857	-0.2763 + 0.2829i
-0.0917	-0.2763 - 0.2829i
-0.6667	-0.6667
-0.6667	-0.0405
-0.0694	-0.0998 + 0.0395i
-0.0476	-0.0998 - 0.0395i

102 A comparative study is conducted to evaluate the performance of the SOF controller with the other
 103 controllers designed for TITO Wood and Berry distillation column given in [2] to [4]. The model

104 given in equation 1 is simulated in SIMULINK for a time duration of 100 minutes. As a first case, a
 105 step change in output disturbance is given in both Y_1 and Y_2 from a time instant of 50 minutes. The
 106 objective is to track a unit step command in both Y_1 and Y_2 channels simultaneously in the presence of
 107 a step output disturbance acting in both of the output channels. The performance is quantified in terms
 108 of integral square error (ISE) in tracking Y_1 and Y_2 and also the integral square value (ISV) of control
 109 inputs U_1 and U_2 . An expression for ISE and ISV are given in the equations 31 and 32 respectively
 110 (similar equations are used for Y_2 and U_2). Please note that the integration is performed numerically
 111 using SIMULINK with a fixed time step used for simulation.

$$ISE = \int_0^{100} (Y_1 - Y_{1ref})^2 dt \quad (31)$$

$$ISV = \int_0^{100} U_1^2 dt \quad (32)$$

112 Figure 2 gives the output Y_1 and figure 3 gives the output Y_2 respectively of various controllers in
 113 tracking a unit step command in both of the output channels (Y_1 and Y_2), in the presence of a unit step
 114 change in output disturbance in both of the output channels. The solid blue curve shows the response
 115 of the SOF controller given in equation 30. The response of the controllers presented in [2], [3] and [4]
 116 is given by black curve (dashed), brown curve (dotted) and green curve (dash-dot) respectively. After
 117 the introduction of output disturbance at a time instant of 50 minutes, the response of SOF controller is
 118 superior in terms of lowest undershoot and smallest settling time when compared to other controllers.
 119 The steady state error for tracking is poor for SOF controller for initial 50 minutes due to the lack of
 120 integrator. The performance metric ISE and ISV is given in Table 3 for the case of tracking a unit step
 121 command in both of the output channels (Y_1 and Y_2), in the presence of a unit step change in output
 122 disturbance in both of the output channels. The performance of the controllers designed in [3] and [4]
 123 are similar, but consumes very large control input when compared to SOF controller. Clearly, the SOF
 124 controller outperforms the one designed in [2]. Unmeasured output sinusoidal disturbances of very
 125 small frequencies are also prominent in process control applications. Figure 4 gives the output Y_1 and
 126 figure 5 gives the output Y_2 respectively of various controllers, in the presence of a sinusoidal change in
 127 output disturbance of frequency 0.1 rad/min in both of the output channels. The performance metric
 128 ISE and ISV is given in Table 4 for the case of rejecting a sinusoidal change in output disturbance of
 129 frequency 0.1 rad/min in both of the output channels. The sinusoidal disturbance rejection property
 130 of SOF controller is comparable to that of the controller designed in [3] and [4] and superior to the
 131 one designed in [2]. From Table 4, clearly the control effort (ISV of U_1) is on the lower side for SOF
 132 controller when compared to other controllers.

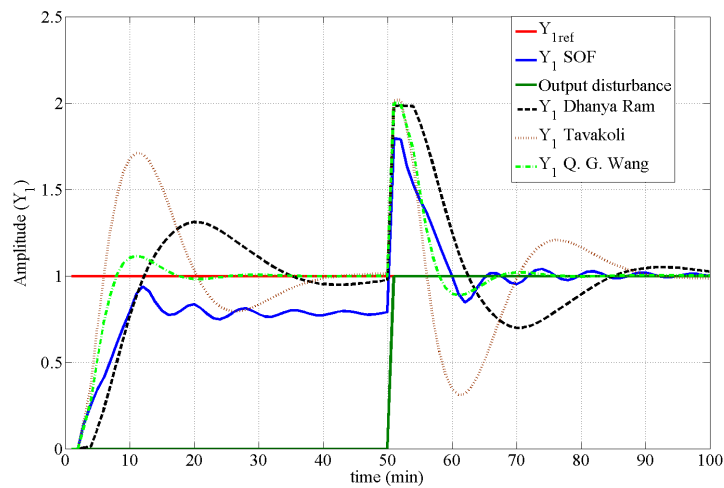


Figure 2. Comparison of the performance of various controllers under step change in output disturbance (Y_1).

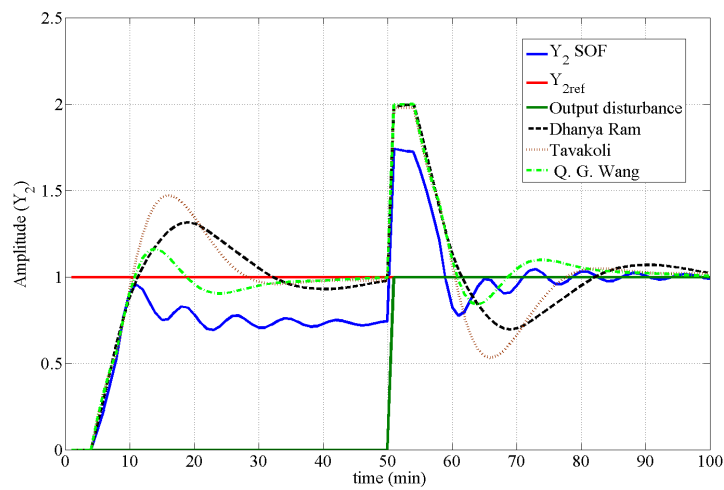


Figure 3. Comparison of the performance of various controllers under step change in output disturbance (Y_2).

Table 3. Results for a step change in output disturbance.

Controller	ISE (Y_1)	ISE (Y_2)	ISV (U_1)	ISV (U_2)
SOF Controller	7.91	10.91	0.206	0.161
Dhanya Ram	13.28	12.31	0.100	0.283
Tavakoli	6.20	10.61	0.348	0.183
Wang	6.23	10.64	0.344	0.187

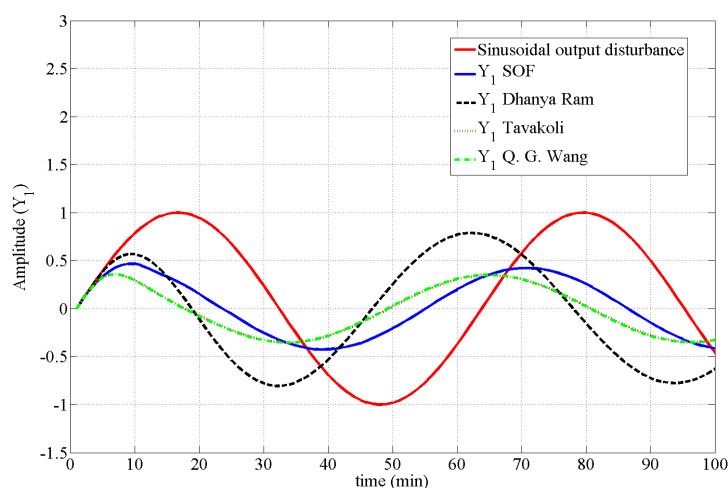


Figure 4. Comparison of the performance of various controllers under sinusoidal change in output disturbance (Y_1).

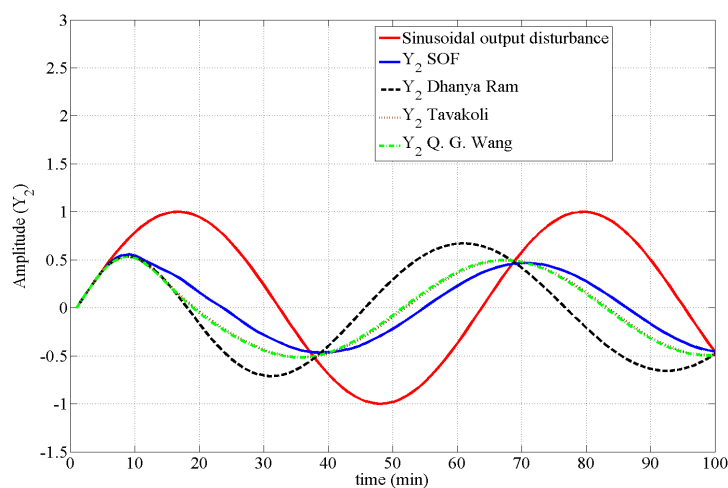


Figure 5. Comparison of the performance of various controllers under sinusoidal change in output disturbance (Y_2).

Table 4. Results for sinusoidal change in output disturbance of frequency 0.10 rad/min.

Controller	ISE (Y_1)	ISE (Y_2)	ISV (U_1)	ISV (U_2)
SOF Controller	9.10	11.36	0.177	0.127
Dhanya Ram	29.98	22.65	0.309	0.507
Tavakoli	6.26	12.92	0.486	0.119
Wang	6.35	13.03	0.481	0.128

133 6. Conclusions

134 Design of multivariable static output feedback controller for a binary distillation column is presented
 135 in this paper. The method adopted for controller synthesis is generic and can be applied to any
 136 multivariable systems with unstable poles or non-minimum phase zeros. The designed controller

137 has excellent disturbance rejection properties with low control effort. The comparative study with
138 other controllers shows the performance and effectiveness of the controller. The simple structure
139 of the controller yields accurate practical implementation when compared to other higher order
140 controllers. Linear matrix inequalities are powerful tool for designing multivariable static output
141 feedback controllers and further design constraints can be easily incorporated to yield better control
142 solutions.

143 **Author Contributions:** Kalpana and Harikumar formulated the problem and written the manuscript.
144 Senthilkumar, Balasubramanian and Abhay. S. Gour did the simulations and also edited the manuscript. All the
145 authors have read and approved the manuscript.

146 **Conflicts of Interest:** The authors declare no conflict of interest.

147 Abbreviations

148 The following abbreviations are used in this manuscript:

149	BMI	Bilinear matrix inequality
	GA	Genetic algorithm
	ISE	Integral square error
150	ISV	Integral square value
	LMI	Linear matrix inequality
	SOF	Static output feedback
	TITO	Two input two output

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