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Article

Recursive Abduction and Universality of Physical Laws: A Logical Analysis Based on Case Studies

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Abstract: The paper studies some cases in physics such as Galilean inertia motion and etc., and hereby, presents a logical schema of recursive abduction, from which we can derive the universality of physical law in an effective logical path without infinite induction asked. Recursive abduction provides an effective logical path to connect a universal physical law with finite empirical observations basing on the both quasi-law tautology and suitable recursive dimension, the two new concepts introduced in this paper. Under the viewpoint of recursive abduction, the historically lasting difficulty from Hume's problem naturally vanishes. In Hume's problem one always misunderstood the universality of natural law as a product of empirical induction and the time-recursive issue as an infinitely inductive problem and, thus, sank into the inescapable quagmire. The paper gives a concluding discussion to Hume's problem in the new effective logical schema.

Key words: Abduction; Recursion; Physical law; Hume's problem

MSC classifications: 00A79; 03A05; 03B60; 03C57

1. Introduction

The paper clarifies the properties of physical law from three aspects: abduction, the foundation of universality of physical laws, and Hume's problem.

Abduction has revealed an intuitively vivid logical process in empirical judgments, but left the logical uncertainty to physical laws just as to commonsense. It significantly conflicts with our general impression that the physical law is successful much more than the commonsense. From the initial illustration of abduction at beginning, by Pierce [1], to the more rigorous formal formulations of abduction in the present, e.g., by Meheus and Batens [2], Lycke [3], Soler-Toscano, et al. [4], Beirlaen and Aliseda [5], and so on, during one and half centuries, physical law and commonsense were always indiscriminately viewed as similar objects in abductive analyses. Below, we shall call such kind of abduction as ***normal abduction***, which always cannot discriminate the strict natural law from the commonsense. In the framework of normal abduction, the abductive studies are not enough to explain the difference between the discovery processes of physical law and other objects, and usually focus on their similarity, e.g., see Pombo and Gerner [7], McNally, et al. [8], Chattopadhyay and Lipson [9], Haig [10], Khemlani, et al. [11], Pedemonte and Reid [12], Bajc [13], Singer [14], and so on, in which the logical inferential processes in physics, biology, psychology, pedagogy, ethnography, jurisprudence, and etc. are almost indifferent.

Developing from Pierce's initial proposal [1], abduction as the non-standard logic follows the reasoning schema

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$$B(x),\ (A(x) \to B(x))/A(x), \tag{1.1}$$

where A and B must be different predicates. Distinguishing from the empirical induction, different predicates in the antecedent and consequent in $A(x) \to B(x)$ are the necessary characteristic of abduction. The empirical induction is always carried on only in different quantifiers but with the same predicate. In such a characteristic framework for identifying the abductive reasoning, an abduced physical law and an abduced commonsense are usually indifferent. It is a long-standing fatal mistake misleading the abductive discussion upon physical laws.

Emphasizing on the distinction from deduction and induction, normal abduction deals with all abduced objects in postulations as the following, e.g., see Nubiola [6]:

i) it cannot be logically derived out;

ii) it is purely an empirical guess.

Namely, in the mathematical logical terminologies, any abduced object in normal abduction is logically inderivable and empirically indecidable in the rigorous logical sense. The major concern in this paper is not the distinction from deduction and induction but the difference of physical law from not so rigorous abduced objects. The case studies will present a significantly different view upon the abductive reasoning for physical laws in Sections 2 and 3.

Tracing back to earlier times, Hume's problem also mixed natural laws with normal empirical inductions and denied any certainty of natural laws, of course, including the most successful physical laws. Intensive question to natural law was incisively claimed by Hume since more than two and half centuries ago, Hume [15][16], and left unsolved up to resent time, both in the classical reasoning and in the non-standard abductive reasoning. The fatal problem is how to know the conformability of past and future for the natural law in empirical test, which asks infinite empirical induction. From Hume's time, philosophers and logicians had made great effort to rescue the empirical origin of natural laws' certainty and finally laid out two leading roundabout but unsuccessful tactics in modern time to fight Hume's skepticism. One was to justify the feasibility or reliability of empirical inductive procedure for yielding a natural law, and some of them eventually drew support from the probability argument, e.g., Fisher [17], Carnap [18-21], Reichenbach [22,23], and others showed the empirical inductive procedure always more reliable than Hume's reasoning path, e.g., Worrall [24], Kelly [25]. Another tried to dissolve the skeptical threat by cancelling the core sense of Hume's problem to defense the natural law system, such as using falsification to replace verification, Popper [26-29], using unsovability of Hume's problem in a three-valued logical schema to evade from Hume's critique, Shier [30], and etc.; but they only used empirical inductive falsification or empirical inductive unsovability to replace empirical inductive verification, and still concentrated on empirical induction to find resolution for Hume's problem. The two tactics all sank into the circulation dilemma of using induction to justify induction to which Hume had already denounced far earlier than those arguments were proposed, see Howson [31]. All those arguments are unsuccessful to thoroughly free the natural law system from Hume's critique, see Skyrms [32], Howson [31], and, especially, are incapable to surpass the cordon delimited by Hume and to bring enterprising studies to the natural law system on a level higher than what Hume had reached. Up to resent time, as a methodological problem in natural science, the core issue unsolved in Hume's radical skepticism is always the problem of infinite induction, e.g., Boulter [33], Okasha [34], Hetherington [35], Schurz [36]. Different from the above two tactics, this paper will open up a new route to deal with Hume's problem in a recursive pattern to overcome the problem of infinite induction.

Another development from the discussions of Hume's problem is Goodman's discovery [37,38]. Goodman proposed a triple propositional category of accidentalness, lawlikeness, and natural law to deal with the analysis involving Hume's problem, in which only the lawlike proposition, but not all empirical propositions, is possible to serve as a candidate for becoming a natural law. For a long time lawlikeness had been discussed repeatedly, e.g., Carnap [19], Barker and Achinstein [39], Small [40], Ullion [41], Quine [42], Hesse [43], Skyrms [33], Israsel [44], Chart [45], Godfrey-Smith [46], Schwartz [47]. Nevertheless, how to characterize the lawlikeness had little progress and, in fact, fell into difficulties up to now. All efforts were seemingly destined to come back to the endless old argument for inductive procedure similar to those in Hume's problem. The resent paper will involve nothing with Goodman and his successors' approach of and explanation to the lawlikeness. But the idea of lawlikeness indicating the candidate of a natural law is adopted to make up an important logical judgment leading to the natural law.

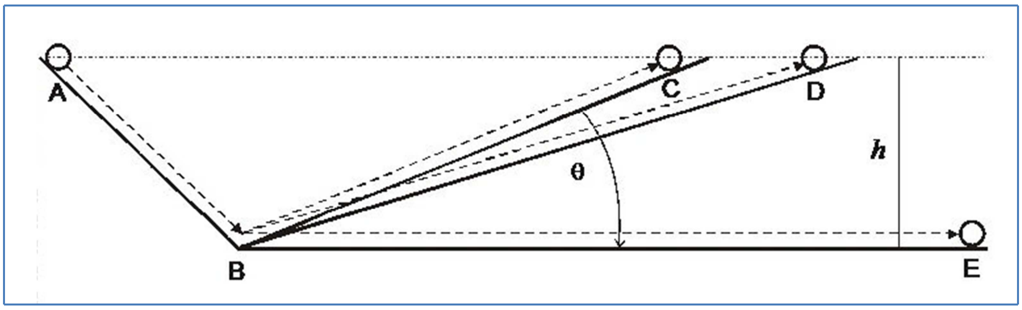
The paper is organized as follows: Section 2, first clarifies the realistic logical path for yielding a physical law by case studies in the mathematical logic framework, discusses the relation among characteristics of normal abduction, lawlikeness, and physical law, gets rid of some long-term misunderstandings, and finally introduces the new abductive framework, called law-deriving abduction. Section 3 defines the recursive abduction in the logical and operational senses and discusses the universality of physical law. Section 4 gives a conclusive solution to Hume's problem basing on the framework of recursive abduction. Section 5 summarizes the main conclusions.

2. Analysis on Physical Laws Derived from Experiments

In order to precisely re-clarify the objective ground on which the physical law is yielded, we first focus on the case study of some physical laws in the mathematical logical form.

2.1. Case 1: Galilean Derivation for the Inertia Motion

Fig. 1 Illustration of Galilean inference in his slant experiment



Galilean discovery, Galilei [48], can be illustrated in Fig. 1. First, a metal ball freely moves down from "A" to "B" along slant "A-B", proceeds to move up to "C" along slant "B-C", and during moving on "B-C", it slows down to zero at the highest point "C"; if neglect the resistance, the heights of points "A" and "C" are equivalent h. Second, keeping the point "B" fixed and turning slant "B-C" to "B-D", repeating the process in the first time, the metal ball reaches the final point "D" along fold line "A-B-D"; if neglect the resistance, the height of point "D" is yet h. Turning the slant between ∠CBE has no impact on height h. So far, we induce from the slant experiment that the metal ball will arrives at h and slows down to zero at its terminal point on the slant B-C for any θ between ∠CBE. Galilei further thought that if slant "B-C" turns to close the horizontal plane more and more, to reach height h the velocity of the metal ball will slow down slowly more and more, and finally if let slant "B-C" infinitely tends to coincide with horizontal plane "B-E", to reach the final height h the metal ball will have to always move on and therefore to keep its velocity constant. So Galilei concluded that if there is no any external force imposed on an object, the object will keep its velocity constant to move on horizontally. It is the preliminary description of inertia motion which bred Newton's inertia law in a more general form, eventually.

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Restate Galilean discovery more rigorously in the form of propositional logic as follows:

1) Neglect the resistance, denoted e.

2) For all observable θ between ∠CBE, if a metal ball freely moves down from height h and along slant "A-B" from "A" to "B", denoted $h_\downarrow$, it will get maximal velocity v_0 at "B", denoted v_0, and proceeds to move up along slant "B-C" , denoted $h_\uparrow$, and finally reaches height h too, during which v_0 slows down to zero, denoted $v_{0\downarrow}$: $(h_\downarrow \to v_0) \to (e \to h_\uparrow \wedge v_{0\downarrow})$. Inductive conclusion p_I from the experiment is $p_I = e \to h_\uparrow \wedge v_{0\downarrow}$.

3) As a reasonable supposition, change angle θ such that slant "B-C" infinitely tends to coinciding with horizontal "B-E", denoted θ_{BE}, $h_\downarrow$ will also result $h_\uparrow$, and to do so unless the metal ball keeps v_0 constant infinitely to move on along "B-E", denoted $\neg v_0 \to \neg h_\uparrow$, thus, the metal ball will keep v_0 constant always to move on along "B-E", denoted $h_\uparrow \to (e \to v_0)$: $\theta_{BE} \to ((\neg(e \to v_0) \to \neg h_\uparrow) \to (h_\uparrow \to (e \to v_0)))$.

4) Summing up from "1)" ~ "3)", obtain

$$((h_\downarrow \to v_0) \to (e \to h_\uparrow \wedge v_{0\downarrow})) \to (\theta_{BE} \to ((\neg(e \to v_0) \to \neg h_\uparrow) \to (h_\uparrow \to (e \to v_0)))). \tag{2.1}$$

Concluding from (2.1),

$$\theta_{BE} \to ((\neg(e \to v_0) \to \neg h_\uparrow) \to (h_\uparrow \to (e \to v_0))). \tag{2.2}$$

(2.1) completely describes Galilean analysis and (2.2) is the right-hand side in (2.1). (2.2) is a tautology, denoted $\mathcal{L}_G$, called Galilean "quasi-law tautology" (in the case without specific assignment, denoted $\mathcal{L}$). If merely concern with the conclusion, they are equivalent. However, (2.2) cannot be directly tested in the slant experiment, and it is the result of tautological inference from the experimentally observable process under Galilean premise $h_\uparrow$. Only the left-hand side in (2.1), denoted I, is an inductive examination, and can be tested by the experiment. The inductive conclusion in I is $p_I = e \to (h_\uparrow \wedge v_{0\downarrow})$. (2.1) includes all contents covering the experimentally observable process and the tautological inference in Galilean analysis. It is neither a pure empirical induction nor a pure tautological inference but some kind of their combination to enable Galilei to derive his conclusion.

(2.1) is a tautology too, denoted by $\mathcal{T}_G$ called Galilean "lawlike tautology" (in the case without specific assignment, denoted $\mathcal{T}$). The last formula $e \to v_0$ in $\mathcal{L}_G$, called inertia motion, is Galilean conclusion. However, we cannot accept it as a physical law only basing on the analysis of slant experiment, for it is not the unique reasonable conclusion from the empirical inductive conclusion p_I. Galilei only used $h_\uparrow$ in inductive conclusion $e \to h_\uparrow \wedge v_{0\downarrow}$ in I, and disused $v_{0\downarrow}$. Galilean analysis does not include all conclusions objectively implied in the slant experiment.

We might convert our theoretical stand to agree with Aristotle's claim. From Aristotle, the basic empirical evidence contained in this experimental induction should be that the metal ball always naturally stops in the midway and never always freely moves on, Aristotle [49], that is, in inductive conclusion $e \to h_\uparrow \wedge v_{0\downarrow}$, $v_{0\downarrow}$ but not Galilean $h_\uparrow$ is Aristotle's premise for the further analysis. Hence, from Aristotle's ideas $h_\uparrow$ is impossible to happen in θ_{BE} for it must request $\neg v_{0\downarrow}$. Following Aristotle, (2.1) should become

$$((h_\downarrow \to v_0) \to (e \to h_\uparrow \wedge v_{0\downarrow})) \to (\theta_{BE} \to ((h_\uparrow \to \neg v_{0\downarrow}) \to (v_{0\downarrow} \to \neg h_\uparrow))), \tag{2.3}$$

and correspondingly (2.2) becomes

$$\theta_{BE} \to ((h_\uparrow \to \neg v_{0\downarrow}) \to (v_{0\downarrow} \to \neg h_\uparrow). \tag{2.4}$$

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Similar to (2.1) and (2.2), (2.3) and (2.4) are also tautologies. But (2.3) and (2.4) conclude that when slant "B-C" infinitely tends to coinciding with horizontal plane "B-E", the metal ball will stop in the midway and will be impossible to reach height h. The conclusion from (2.3) or (2.4) is opposite to that from (2.1) or (2.2). Denoting $\mathcal{L}_A$ the right-hand side in (2.3), a quasi-law tautology, and commonly denoting I the left-hand sides in (2.1) and (2.3) (notice: they are the same), similar to (2.1), (2.3) is a lawlike tautology, denoted $\mathcal{T}_A$, and I in $\mathcal{T}_A$ is the initial empirical induction. The last formula $\neg h_\uparrow$ in $\mathcal{L}_A$ is Aristotle's conclusion.

In evident, the empirical induction in slant experiment cannot determine which of $e \to v_0$ or $\neg h_\uparrow$ to be a physical law. They are merely the candidate for a physical law so far. Galilean $e \to v_0$ and Aristotle's $\neg h_\uparrow$ are two typical lawlike propositions, denoted p_L. In $\mathcal{L}_G$, $h_\uparrow$ combining with the particular logical consistent explanation turns out $e \to v_0$ and excludes $v_{0\downarrow}$, while, in $\mathcal{L}_A$, $v_{0\downarrow}$ combining with the other particular logical consistent explanation turns out $\neg h_\uparrow$ and excludes $h_\uparrow$, although the experimental inductive conclusion $p_I = e \to h_\uparrow \wedge v_{0\downarrow}$ is the same for both. The difference in logical consistent explanations, respectively indicated by $\mathcal{L}_G$ and $\mathcal{L}_A$, plays an important role to determine different candidates of physical law. The consistent explanation presented in a quasi-law tautology is an indispensable logical ground to construct a lawlike proposition, and the initial inductive observation in the slant experiment only offers the reasoning clue for inspiring scientists to suggest their own lawlike propositions.

The initial experimental induction cannot uniquely exclude one and support another between $\mathcal{L}_G$ and $\mathcal{L}_A$. Only the further empirical test is possible to determine which of them to be a physical law. In the history, the further corollaries from Galilean inertia motion, such as parabolic motion, Galilean transformation in the inertial system, static phenomena in the rotating earth, and etc., contributed the further empirical support to make Galilean lawlike proposition eventually to become a natural law. Whereas Aristotle's motion law was rejected for its corollaries, beside of static phenomena in the rotating earth, were not supported by further empirical observations. They involve nothing of Kuhn's incomparable paradigms, Kuhn [50]. Abduction offers litmus test for their qualities.

Taking parabolic motion as an example, and denoting g the acceleration of gravity and m_p the parabolic motion, we have

$$(e \to v_0) \to ((e \to v_0) \wedge (g \to m_p)), \qquad (2.5)$$

in which $(e \to v_0) \wedge (g \to m_p)$ gives a new empirical inductive observation, denoted p_I'. The judgment to (2.5) supporting Galilean inertia motion has the reasoning schema of

$$p_I',\ (p_L \to p_I')/p_L,$$

a typical abduction, where, $p_I' = ((e \to v_0) \wedge g \to m_p)$ and $p_L = (e \to v_0)$.

As well-known in abduction, all natural laws have this feature: in Einstein's relativity, it is principle of constancy of light velocity to interpret Michelson-Morley experiment but not vice versa, meanwhile Michelson-Morley experiment to give the empirical truth to principle of constancy of light velocity; in Darwin's evolution theory, it is principle of natural selection to interpret the phenomena of species' evolution but not vice versa, meanwhile the phenomena of species' evolution to give the empirical truth to principle of natural selection; and so on. They finally become natural laws all through p_I', $(p_L \to p_I')/p_L$, see, e.g., Pombo and Gerner [7]. Here the further issue is that why physical laws are always most excellent in their universality comparing with other empirical sciences and commonsense.

2.2. Further Discussions on the Abduction for Deriving Physical Laws

Denoting $\mathcal{L}p_L$ as tautology $\mathcal{L}$ containing conclusion p_L, we define quasi-law tautology, lawlike proposition, and natural law as follows:

Definition 1. $\mathcal{L}p_L$ is a quasi-law tautology if the corollary of p_L is an empirical induction p_I', and p_L is called lawlike proposition.

Definition 2. p_L is a natural law, denoted p_L^t, if and only if p_I', the corollary of p_L, is empirically true.

In Definition 2, p_L and p_L^t are the same in terms of their sentence contents, but they are distinguished by the different empirical truth value statuses. p_L is merely the conclusion in $\mathcal{L}p_L$ without the empirical truth value but p_L^t is the empirically valued p_L by abduction.

In Case 1, Galilean $p_L = e \to v_0$ in $\mathcal{L}_G$ obtains empirical truth by abduction in $p_L \to p_I'$ to become a natural law p_L^t, whereas Aristotle's $p_L = \neg h_\uparrow$ in $\mathcal{L}_A$ fails in obtaining empirical truth by abduction in $p_L \to p_I'$ and is rejected as a natural law.

Using the symbols in Definitions 1 and 2, we always denote the abduction for deriving a physical law as follows

$$p_I',\ (p_L \to p_I')/(p_L = p_L^t). \tag{2.6}$$

It is still an abduction of course, in which we discriminate p_L and p_L^t, a lawlike proposition and a physical law. A lawlike proposition is the conclusion in quasi-law tautology $\mathcal{L}p_L$, and a natural law is a lawlike proposition obtaining the empirical truth in abduction. The form of (2.6) slightly differs from the current abductive reasoning schema and emphasizes on the different empirical truth statuses of p_L and p_L^t.

2.3. A New Kind of Abduction

(2.6) indicates a new kind of abduction, called ***law-deriving abduction***, briefly ***L-abduction***. It means that normal abduction in $p \to p_I'$ is merely the necessity but not the sufficiency for deriving a physical law. No any strict natural law such as physical law is sufficiently derivable from the normal abduction. On the other hand, it reveals yet that the combination of quasi-law tautology, indicating the lawlikeness following Definition 1, with empirical abduction, indicating the empiricalness following Definition 2, to give the sufficient derivability of a physical law, though the empirical abduction alone is always insufficient to do it. This new kind of abduction introduces the new sufficiency for deriving a strict natural law, and is grounded on the combination of lawlikeness with empirical abduction. In order to establish the sufficient abduction in empirical induction for deriving a physical law we should focus on the combination of lawlikeness with empirical abduction but not the normal abduction alone.

In terms of the simplest case in which there is only p_I', i.e., there is only one corollary of p_L considered in abduction, the above analysis on Case 1 is generally summarized in (2.7) and (2.8),

$$\begin{cases} (p_I \to \mathcal{L}p_L) \to (p_L \to p_I'); & (2.7) \\ p_I',\ (p_L \to p_I')/(p_L = p_L^t). & (2.8) \end{cases}$$

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By the similar analysis conducted in Case 1, it is easily to examine that (2.7) and (2.8) describes the general reasoning process followed by strict natural laws such as Newtonian three laws of motion, Darwin's law of evolution, mass conservation law in chemistry, Mendeleev periodic law of elements, Mendel's law of inheritance, principle of constancy of light velocity, and so on. Every strict natural law first has its own lawlikeness indicated by a quasi-law tautology, in which particular consistent explanation is presented, and then, is supported by empirical observations in abduction.

So far we have clarified at least that a lawlike proposition and a strict natural law are never a normal abductive result. In (2.7), only p_I is a purely descriptive empirical result and it merely offers preliminary raw materials for inspiring scientists to find the lawlikeness for the candidate of natural law. It contributes the inspiration psychologically but neither the necessity nor the sufficiency for deriving a natural law. Moreover, (2.2) derived from (2.1) and (2.4) from (2.3) all obey $(p_I \to C) \vdash p_L$, where C indicates an interpreting choice from p_I. We derive different lawlike results p_L=$e \to v_0$ or p_L=$\neg h_\uparrow$ by introducing the interpretation $h_\uparrow$ or $v_{0\downarrow}$ as C. Initial empirical observation in I at most delivers p_I. Logically irrelevant to p_I, the logical consistent explanation in $\mathcal{L}$, rather than the great number of cumulative evidences in the normal empirical abduction, to yield a lawlike proposition. Lawlike tautology $\mathcal{T}$ asks both a consistent logical relationship and a set of observables. Based on $\mathcal{L}$ in $\mathcal{T}$ but not I in $\mathcal{T}$, the candidate of a natural law is eventually proposed.

Theoretically, initial empirical induction I is unnecessary, and it is only a psychological factor to inspiring scientists. For example, more than two-thousand and three-hundred years ago Aristotle proposed his motion law without the empirical inductive result p_I=$e \to (h_\uparrow \wedge v_{0\downarrow})$ in a slant experiment. $\mathcal{L}p_L$ and p_I' together consist of sufficiency to abduct p_L^t. So (2.7) can be simplified into (2.9) as the following

$$\begin{cases} \mathcal{L}p_L \to (p_L \to p_I'); & (2.9) \\ p_I',\ (p_L \to p_I')/(p_L = p_L^t). & (2.10) \end{cases}$$

Henceforth, we always refer L-abduction to (2.9) and (2.10).

The case study clearly reveals that a strict natural law is logically derivable in (2.9). It is an important feature differing from the normal abduction and formally distinguishes a strict natural law from a commonsense. The next issue left is how the empirical truth of a strict natural law is decidable in observational test from the further analysis on (2.10).

It is concluded now that

1. *Beginning with an initial empirical inductive conclusion* p_I*, one cannot determine* p_L *unless it associates with a consistent relationship presented in quasi-law tautology* $\mathcal{L}$*; and* p_L *is the conclusion in* $\mathcal{L}$*, namely* $\mathcal{L}p_L$.

2. p_L *cannot become* p_L^t *unless its corollaries* p_I', p_I''*,... are empirically true.*

3. *Not the initial inspiring* p_I *but the combination of lawlikeness with sequent empirical inductive* p_I', p_I''*,... gives the sufficiency to infer a natural law. It is an L-abduction.*

4. *(2.9) and (2.10) completely define the L-abduction.*

Normal abduction always well prepares for the non-monotonicity and dynamical reasoning but overlooks the sufficiency of derivation for the strict natural law. The normal abduction as a general concept is certainly over-pessimistic. It is necessary to distinguish L-abduction from normal abduction. Their difference can be summarized as: the normal abduction deals with loose phenomena of A and B in speculative $A \to B$, just like a physician facing with a patient to conjecture what is the explanation to the patient's symptom, in which usually no sufficiency exists for deriving a conclusion, Meheus and Batens [2]; L-abduction deals with the logically compact relation between p_L and p_I' consistently based on $\mathcal{L}p_L$ and the empirical truth of p_I', just like one using originally consistent geometry in the measurement of realistic space to determine whether the conclusions from the geometrical theorems are empirically true, in which there is the sufficiency to derive the conclusion.

2.4. Case 2: Difference between Empirical Inductive Conclusion and Physical Law

Sentence "the sun rises from the east everyday" is an empirical inductive conclusion (purely as a repeated result from numerous empirical observations) or a physical law (as an empirically true corollary from inertia rotation of the earth following inertia law). The difference between a physical law and a normal empirical inductive result is not their empirical contents but their intensions concerning the logical consistent explanation presented in a quasi-law tautology. Empirical observation at most produces the inductive conclusion, only L-abduction possibly produces the strict natural law. Here, three things should be distinguished: empirical induction, normal abduction, and L-abduction. If "Tom always dresses red color" is concluded from the observable result of "Tom dressed red color every day up to now", this is an empirical inductive conclusion. If one from "Tom likes to dress red color" inferring "Tom dresses red color" empirically examines "Tom likes to dress red color", this is a normal abductive reasoning, for ***like*** and ***dress*** involve different predicates. However, it will be impossible to derive a lawlike proposition and, further, a strict natural law, unless a quasi-law tautology makes sense to explain Tom's dressing in a consistent relation. Only L-abduction is possible to finally yield a strict natural law.

2.5. Case 3: Galilean Law of Free Fall

Aristotle regarded from the intuitive impression that the heavier a body ($w^{\uparrow}$) is, the faster its free fall ($g^{\uparrow}$) is, and, naturally, the lighter a body ($w^{\downarrow}$) is, the slower its free fall ($g^{\downarrow}$) is: $(w^{\uparrow} > w^{\downarrow}) \rightarrow (g^{\uparrow} > g^{\downarrow})$. By using reduction to absurdity, Galilei refuted upon Aristotle's viewpoint, Galilei, [48]: Supposing Aristotle's assertion right, if a heavier body is bundled with a lighter one, the total weight $w^{\uparrow\downarrow}$ will increase to their plus $w^{\uparrow\downarrow} = (w^{\uparrow} + w^{\downarrow}) > w^{\uparrow}$, and if they together freely fall, then their free fall ($g^{\uparrow\downarrow}$) will be faster than $g^{\uparrow}$:

$$(w^{\uparrow} + w^{\downarrow}) \rightarrow (g^{\uparrow\downarrow} > g^{\uparrow}); \tag{2.11}$$

on the other hand, if a faster falling heavier body is bundled with a slower falling lighter one, then the slower one will slow down the faster one to result in their common free fall ($g^{\uparrow\downarrow}$) slower than $g^{\uparrow}$: $g^{\uparrow\downarrow} < g^{\uparrow}$, that is,

$$(w^{\uparrow} + w^{\downarrow}) \rightarrow (g^{\uparrow\downarrow} < g^{\uparrow}); \tag{2.12}$$

combining (2.11) with (2.12), obtain

$$(w^{\uparrow} + w^{\downarrow}) \wedge (g^{\uparrow} \wedge g^{\downarrow}) \rightarrow (g^{\uparrow\downarrow} > g^{\uparrow}) \wedge (g^{\uparrow\downarrow} < g^{\uparrow}); \tag{2.13}$$

it is contradictory. And the contradiction is avoided unless $g^{\uparrow} = g^{\downarrow}$, that is, free falls of heavier and lighter bodies are the same. Galilei therefore proposed $g^{\uparrow} = g^{\downarrow}$ by the following reasoning

$$(\neg(g^{\uparrow} = g^{\downarrow}) \rightarrow ((g^{\uparrow\downarrow} > g^{\uparrow}) \wedge (g^{\uparrow\downarrow} < g^{\uparrow}))) \rightarrow (\neg((g^{\uparrow\downarrow} > g^{\uparrow}) \wedge (g^{\uparrow\downarrow} < g^{\uparrow})) \rightarrow g^{\uparrow} = g^{\downarrow}). \tag{2.14}$$

(2.14) is a quasi-law tautology, in which $g^{\uparrow} = g^{\downarrow}$ is the conclusion, a lawlike proposition p_L. In this case, the empirical inductive result is $p_I = (g^{\uparrow} > g^{\downarrow}) \rightarrow ((w^{\uparrow\downarrow} > w^{\uparrow}) \rightarrow (g^{\uparrow\downarrow} > g^{\uparrow})) \wedge ((g^{\uparrow} \wedge g^{\downarrow}) \rightarrow (g^{\uparrow\downarrow} < g^{\uparrow}))$, which is omitted in (2.14) for simplifying the expression. Galilean reasoning in (2.14) led to the discovery of gravity acceleration constant g on the earth surface. Here g is p_I' derived from Galilean lawlike proposition and supported the law of free fall in abduction, namely p_I', $(p_L \to p_I')/p_I' = p_L^t$. It is an example without interpreting choice by dividing p_I. It still follows (2.9) and (2.10). If Galilei proposed $g^{\uparrow} = g^{\downarrow}$ merely from the said experimental observation to the free falling bodies in Leaning Tower of Pisa without the consistent explanation in (2.14), it would had been called equal free falling phenomenon, a pure empirical inductive conclusion, but not the law of free fall. The consistent explanation in $\mathcal{L}$ is the indispensible character of a lawlike proposition preparing for a natural law.

2.6. Relationship between the Empirical Truth Values of $\mathcal{L}p_L$ and p_L

A tautology is always true for its semantics of connectives, namely connective-semantically true, but its contents may be empirically true or false. In Case 1, the contents of $\mathcal{L}_G$ is empirically true revealing the realistic empirical process, whereas $\mathcal{L}_A$ empirically false involving nothing with the realistic empirical process. It enlightens us that a quasi-law tautology $\mathcal{L}p_L$ is empirically true or false because p_L is empirically true or false, and vice versa. Below, we proof this result as Theorem 1 in the general sense.

Theorem 1. $(p_L \to p_I') \leftrightarrow (\mathcal{L}p_L \to p_I')$

Proof of Theorem 1:

There are three steps in the proof:

Step 1, proof for $(\mathcal{L}p_L \to (p_L \to p_I')) \to (\mathcal{L}p_L \to p_I')$:

① $(\mathcal{L}p_L \to (p_L \to p_I')) \to ((\mathcal{L}p_L \to p_L) \to (\mathcal{L}p_L \to p_I'))$ Axiom

② $\mathcal{L}p_L \to (p_L \to p_I')$ (2.9)

③ $(\mathcal{L}p_L \to p_L) \to (\mathcal{L}p_L \to p_I')$ ①②MP

④ $\mathcal{L}p_L \to p_L$ $\mathcal{L}p_L \coloneqq \mathcal{L}$ containing conclusion p_L

⑤ $\mathcal{L}p_L \to p_I'$ ③④MP

Step 2, proof for $(\mathcal{L}p_L \to (p_L \to p_I')) \to (p_L \to p_I')$:

①′ $(\mathcal{L}p_L \to (p_L \to p_I')) \to (p_L \to p_I')$ Theorem

②′ $\mathcal{L}p_L \to (p_L \to p_I')$ (2.9)

③′ $p_L \to p_I'$ ①②′MP

Step 3, proof for $(\mathcal{L}p_L \to p_I') \leftrightarrow (p_L \to p_I')$:

①′′ $((\mathcal{L}p_L \to (p_L \to p_I')) \to (\mathcal{L}p_L \to p_I')) \wedge ((\mathcal{L}p_L \to (p_L \to p_I')) \to (p_L \to p_I')) \to (\mathcal{L}p_L \to p_I') \leftrightarrow (p_L \to p_I')$ Theorem

②′′ $((\mathcal{L}p_L \to (p_L \to p_I')) \to (\mathcal{L}p_L \to p_I')) \wedge ((\mathcal{L}p_L \to (p_L \to p_I')) \to (p_L \to p_I'))$ Step 1 and Step 2

③′′ $(\mathcal{L}p_L \to p_I') \leftrightarrow (p_L \to p_I')$ ①′②′′MP

Denoting empirically true $\mathcal{L}p_L$ as $\mathcal{L}p_L{}^t$, we have

Corollary 1. $\mathcal{L}p_L{}^t \equiv p_L^t$.

In $\mathcal{L}p_L{}^t$ and p_L^t in Corollary 1, superscript t indicates the empirical truth of $\mathcal{L}p_L$ and p_L obtained from L-abduction. The meanings of Corollary 1 are only that the empirical truth value of $\mathcal{L}p_L$ is equivalent to the empirical truth value of p_L obtained from L-abduction, but not $\mathcal{L}p_L \equiv p_L$.

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Proof of Corollary 1: From Theorem 1, if p_L obtains empirical truth by L-abductive reasoning in $p_L \to p_I'$ to become p_L^t, then $\mathcal{L}p_L$ will obtain empirical truth by L-abductive reasoning in $\mathcal{L}p_L \to p_I'$ to become $\mathcal{L}p_L{}^t$, and vice versa.

Corollary 1 reveals a physical law p_L^t must correspond to an empirically true $\mathcal{L}p_L$. It explicitly appeals us, beside of the connective-semantically truth of a tautology, to concentrate on the empirical truth value of sentence contents in quasi-law tautology $\mathcal{L}p_L$ though it is never the subject in classical formal logic. Empirically true $\mathcal{L}p_L$ is always meaningful in physics. In Case 1, $\mathcal{L}_G$ implies the conservation of mechanical energy, in which the summation of potential energy, indicated by $h_\uparrow$ of the metal ball, and kinetic energy, indicated by v_0 of the metal ball, keeps constant. It was clearly proposed almost one-hundred and fifty years later from Galilean time. Also, in Case 3, the empirically true quasi-law tautology (2.14) implies the equivalence of gravitational mass and inertial mass, which was unambiguously proposed almost three-hundred years later. They have revealed the importance of empirical truth value of a quasi-law tautology. Classical formal logic involves nothing such as whether or not the contents of a tautology are empirically true and needs not discriminate the connective-semantic truth and empirical truth. However, derivation for a physical law is not such a reasoning, and we must answer whether or not the contents of a quasi-law tautology are empirically true, just as we had seen in $\mathcal{L}_G$ and $\mathcal{L}_A$. Their empirical truth values are not inconsequential and always make sense in physics. It is another non-ignorable characteristic of L-abduction distinguished from classical logic. Usually we always paid our attention to the natural law rather than to the quasi-law tautology. The analysis to the quasi-law tautology might be neglected too much in physical studies.

3. Recursive Abduction Based on Quasi-Law Tautology

3.1. A Comparison between Physics and Geometry

Physics as empirical science and geometry as formal system are always viewed as essentially different knowledge fields. However, under the viewpoint of L-abduction presented by (2.9) and (2.10), they are the same.

All theorems of geometry are just lawlike propositions and have the same logical status to the candidate of physical law p_L without any particularity until they are tested in the empirical observation by L-abduction. The logical procedure for yielding the theorems of geometry is also $\mathcal{L}p_L$, in which the relation between axiom and theorem is presented. When they pass the empirical test by L-abduction, they become the natural law, and the complete logical procedure is the combination of (2.9) and (2.10) as well. This logical procedure is indifferent between physical law and geometrical theorem. Geometry as the feasible descriptive system to the nature differs from physics merely by nothing but being yielded usually without referring to specific I in $\mathcal{T}$, a non-essential feature for physical laws. No mater of the axioms of geometry or the fundamental explanations of physical laws, they all are presented in $\mathcal{L}p_L$; geometrical theorems and physical laws such as Newtonian motion laws, Einstein's principle of constancy of light velocity, and so on, all correspond to the conclusions in $\mathcal{L}p_L$, unlike the current misunderstanding that corresponds the physical law to the axiom in geometry. Obviously in L-abduction, the physical law corresponds to the theorem of geometry, and the relationship between geometrical theorem and corresponding axiom is presented in $\mathcal{L}p_L$.

In $\mathcal{T}$, antecedent I is an initial empirical induction inspiring the scientist to propose a lawlike proposition, it is merely to describe the most feasible discovery manner in practice for the scientist who always makes effort to eliminate the psychological illusion so that straightly to stimulates the insight into the objective nature.

Theoretically, the scientist can directly uses observables to construct quasi-law tautology $\mathcal{L}p_L$ without the initial specific empirical induction I in $\mathcal{T}$, and in this case, the lawlike proposition directly derived from $\mathcal{L}p_L$ will likewise become a natural law as long as its logical corollary p_I' is empirically true in L-abduction. Geometry as the description of realistic physical space is just such a natural law system. Without drawing support from specific induction I in $\mathcal{T}$, geometrists directly construct a set of consistent inferences (corresponding to a set of quasi-law tautologies) to set up a lawlike system, in which idealized spatial descriptive elements, such as point, line, plane, and so on, are organized in consistent logical contexts to compose of possible consistent spatial descriptions, such as Euclidean geometry and non-Euclidean geometries; if anyone of them infers the true empirical results in L-abduction, it will become a natural law system. This process is completely similar to determining which of the conclusions in $\mathcal{L}_G$ and $\mathcal{L}_A$ into a natural law. In Newton's time, Euclidean geometry was a natural law system, and in Einstein's time, it became non-Euclidean geometry. Natural law system is always verifiable and falsifiable in L-abduction.

Physics and geometry as applicable systems to the realistic world similarly contain two parts: consistent formality part presented by the quasi-law tautology and empirically tested part presented by the L-abduction. Traditionally, one was customary to concentrating more on empirically tested part in physics and on consistent formal part in geometry. L-abduction reveals to us that two parts are equally important for physics and geometry. We shall intensively discuss what the quasi-law tautology in L-abduction has brought to physics.

3.2. Recursive Abduction

First, let us come back to (2.2). For time T, we can always define measurable recursive partitions t_k, $k = 0,1,2,\cdots$, such that

$$T = \Sigma t_k, \qquad k = 0,1,2,\cdots.$$

The most familiar t_k includes second, minute, hour, day, month, year, century, and so on. When we choose time as a recursive dimension, the recursive feature of time is certainly treated as the ultimate attribute of time itself in the explanation given in a quasi-law tautology. What we want to clarify here is the recursion how to function in L-abduction differently from that in a normal abduction.

Denoting p_L in t_k as $p_L^{t_k}$, $k = 0,1,2,\cdots$, in (2.2), $p_L^{t_k} = (e \to v_0)^{t_k}$. From the tautological character of (2.2), naturally,

$$((\neg(e \to v_0) \to \neg h_\uparrow) \to (h_\uparrow \to (e \to v_0))) \to ((\neg(e \to v_0) \to \neg h_\uparrow) \to (h_\uparrow \to (e \to v_0)^{t_k})). \tag{3.1}$$

(3.1) means that as the tautological result the conclusion in (2.2) will always keep original in any t_k. It only presents the request from logical consistent explanation given in (2.2), and is purely formality so far. It is not a positive result in empirical world, for example, empirically false Aristotle's $(\neg h_\uparrow)^{t_k}$ also holds in $\mathcal{L}_A$ similarly to (3.1).

However, following (3.1), the empirical test to $(e \to v_0)^{t_k}$ in (2.5) becomes (3.2) correspondingly

$$(e \to v_0)^{t_k} \to ((e \to v_0) \wedge g \to m_p)^{t_k}, \qquad k = 0,1,2,\cdots. \tag{3.2}$$

In (3.2), $((e \to v_0) \wedge g \to m_p)^{t_k}$ as p_I' is an experimental observation happening in realistic t_k, and it is not formality but practice in the empirical world. (3.1) requires the empirical test to $e \to v_0$ in (2.2) to conduct in the temporal series t_k, $k = 0,1,2,\cdots$. In other words, the empirical test to $e \to v_0$ of (2.2) is requested recursive in time realistically. That is,

if

a) $(e \to v_0)^{t_0}$ is empirically true in (3.2) by L-abduction,
b) $(e \to v_0)^{t_k}$ is empirically true in (3.2) by L-abduction,
c) $(e \to v_0)^{t_{k+1}}$ is empirically true in (3.2) by L-abduction,

then

d) $(e \to v_0)$ in (2.5) will be empirically true in t_k, $k = 0,1,2,\cdots$, namely, passes the empirical test in L-abduction;

and

e) otherwise, $(e \to v_0)$ in (2.5) will be false in t_k, $k = 0,1,2,\cdots$, namely, fails in the empirical test in L-abduction.

For example, in Case 1, Galilean $e \to v_0$ obtains the empirical truth in t_k, $k = 0,1,2,\cdots$, while Aristotle's $\neg h_\uparrow$ not. This time-recursive requirement from a quasi-law tautology is generally the same to both physical laws and geometrical theorems used in realistic space description.

The above discussion is made in terms of time-recursive abduction. In fact, for any recursively measurable dimension we can introduce the recursive test in an L-abduction as long as $\mathcal{L}p_L$ is uncontradictory to this dimension. And in this case, we call such a dimension ***the suitable recursive dimension***. Not all dimensions must be the suitable recursive dimension, e.g., all quantum dimensions are contradictory to $\mathcal{L}_G$ for $\mathcal{L}_G$ disagrees with Heisenberg's uncertainty relation.

Letting r_k, $k = 0,1,2,\cdots$, be a suitable recursive dimension, we have the general definition of recursive abduction as follows:

Definition 3. A recursive abduction refers to:

if

a) $p_L^{r_0}$ is empirically true in $p_L^{r_0} \to p_I'^{r_0}$ by L-abduction,
b) $p_L^{r_k}$ is empirically true in $p_L^{r_k} \to p_I'^{r_k}$ by L-abduction,
c) $p_L^{r_{k+1}}$ is empirically true in $p_L^{r_{k+1}} \to p_I'^{r_{k+1}}$ by L-abduction,

then

d) p_L in $p_L \to p_I'$ will be empirically true in r_k, $k = 0,1,2,\cdots$, and becomes p_L^t, namely, passes the empirical test in L-abduction;

and

e) otherwise, p_L in $p_L \to p_I'$ will be empirically false in r_k, $k = 0,1,2,\cdots$, namely, fails in the empirical test in L-abduction.

Different from the mathematical recursive function, if Definition 3 is used in a physical measurement, the practical operational sense of r_k and r_{k+1} is that k is randomly selected from $k = 0,1,2,\cdots$. Namely, in physical measurement, we use ***randomness*** of k in place of ***arbitrariness*** of k in mathematics to make up the recursion. The randomness interpretation of k contains two meanings: first, it allows a new probability argument of natural law's universality; and second, it is just the recursive expression for the well familiar repeatability of experimental or empirical observations in physical studies.

Recursive abduction relies on the combination of (2.9) and (2.10), and impossibly comes true in classical logic and normal abduction. Recursive abduction reveals the essence of sufficiency for deriving a physical law in L-abduction. In other words, the universality of a physical law is nothing else but the physical law holding in one or more suitable recursive dimensions.

Recursive abduction can be used in all suitable recursive dimensions, such as space-recursion, mass-recursion, speed-recursion, and etc. For space-recursion, it refers to different locations in space; for mass-recursion, it refers to different quantities of mass; for speed-recursion, it refers to different speeds of motion; and so on. They not only consist of the verifiable universality of physical laws but also give the possibility to recursively falsify a physical theory in its inapplicable range, e.g., Newtonian mechanics was falsified in the space-recursion of micro field, in the mass-recursion of quantum field, and in the speed-recursion in high-speed field. Recursive abduction is the general character of physics system.

By the similar proof to Corollary 1, we have

Corollary 2. From Theorem 1, if p_L^t from $p_L \to p_I'$ holds in r_k, $r = 0,1,2,\cdots$, $\mathcal{L}p_L$ will be empirically true in r_k, $r = 0,1,2,\cdots$.

Corollary 2 means that corresponding to p_L^t, a set of empirically true sentences in $\mathcal{L}p_L$ is consistent in r_k, $r = 0,1,2,\cdots$.

Such a set of sentences in $\mathcal{L}{p_L}^t$ makes up the foundational explanation to p_L^t, and is meaningful in corresponding discipline such as physics, geometry, or etc., as mentioned above. Those consistent sentences involve p_L^t but usually contain more than p_L^t. They are usually distinct from p_L^t, and have their own analytical value.

From Definition 3, L-abduction is recursive as long as there is a suitable recursive dimension. The recursive L-abductive schema can be given as follows

$$\begin{cases} \mathcal{L}p_L \to (p_L \to p_I'); & (3.3) \\ {p_I'}^{r_{0,k,k+1}},\ \forall_{r_k}(p_L^{r_k} \to {p_I'}^{r_k})/(p_L^{r_{0,k,k+1}} = p_L^{tr_k}), \qquad k = 0,1,2,\cdots & (3.4) \end{cases}$$

where ${p_I'}^{r_{0,k,k+1}} = \{{p_I'}^{r_0}, {p_I'}^{r_k}, {p_I'}^{r_{k+1}}\}$, and $p_L^{r_{0,k,k+1}} = \{p_I^{r_0}, p_I^{r_k}, p_I^{r_{k+1}}\}$.

(3.3) and (3.4) present an effective logical foundation for physical laws: the physical law is derivable in (3.3), and its empirical truth value is decidable in (3.4). Specifically, its universality is derivable in the quasi-law tautological sense and its empirical truth is decidable in the recursive abduction sense. Here the recursion is used as a feasible thinking manner grounded on natural number system.

For the current probability argument, the insuperable obstacle in Hume's problem is that the universality of natural law is produced from and, meanwhile, tested by the same empirical induction; if the induction confirms the universality it must be infinitely carried on, thus, impossibly finished; all empirical inductions are doomed to be limited; in this way, any empirical test to the universal natural law always offers a limited inductive sample which always corresponds to a zero probability in the required infinite induction.

In the schema presented by (3.3) and (3.4), the things all have been changed. The universality of strict natural law only refers to a requirement of logical consistence from the quasi-law tautology rather than it necessarily contains something equivalent to the unlimited universal applicability in the empirical world so that it needs the infinite empirical induction to support. The logical foundation of such universality is formed before, by no means after, the empirical inductive test p_I'. It is not the product of empirical induction but a result of effective logical extension from a natural law to its corresponding quasi-law tautology. The logical extension is ensured by Theorem 1. Namely, the induction happening in empirical test does nothing to produce the universality but merely

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to carry the ready-made universality to a suitable recursive dimension. This explanation is quite different from the traditional understanding in the famous inductive problem. In the recursive abduction, theoretically, the quasi-law tautologically based universality can be tested empirically in limited steps in any suitable recursive dimension without the problem of infinite induction; and, practically, the ready-made universality can be tested by empirical induction in a limited inductive sample, in which the probability argument is always feasible. So the universality of natural law is **probabilistically testable** by the recursive abduction. The dilemma of inductive problem has been overcome for it is just resulted from the natural law's universality **producable** in a probabilistic examination.

The above recursive feature in L-abduction interprets the acceptance or rejection of a physical law not from numerously piling up enumerated empirical inductive evidences. And the repeatability of observation in testing a physical law (for acceptance or rejection) can be interpreted in the above recursive sense as well.

Quasi-law tautology discriminates the strict and non-strict empirical sciences. Physics is the strictest empirical science because almost of all its laws have their own quasi-law tautologies, and thus, its empirical truth is determined by recursive abduction, just like geometry. Psychology, biology, and clinical medicine, and etc., all are sub-strict empirical sciences for some of their empirical truths come from recursive abduction and some from normal abduction. It is the recursive abduction to contribute the universality to physics and endow the particular connotation to the universality of physics. In recursive abduction, the universality of physics is verifiable and falsifiable recursively, just like a geometry used in physical space measurement. Recursive abduction gives a clear, smooth, and coherent non-monotonic and dynamic reasoning path.

4. Discussion on Hume's Problem

In Hume's problem, the continuously besetting difficulty in the history was that all universal natural laws were sweepingly regarded as the result of empirical induction mixed with common senses; however, any empirical induction was predestined to be limited and, thus, failed to offer the universal result. There is no effective logical path for creating a universal natural law purely from empirical induction. Now, from the L-abductive point of view, we have known that in Hume's problem one always misunderstood a formal universality derivable from quasi-law tautology as an empirical conformability about past to future and a time-recursive issue as an infinitely inductive problem and, hence, sank into the inescapable quagmire.

(3.3) and (3.4) presents an effective logical path for connecting a universal natural law with the finite empirical observation, in which the universality of a natural law is derivable in (3.3) as a formal result similar to the geometrical theorem before it is examined empirically, and then, the empirical truth value is decidable in recursive (3.4) without infinite induction asked. It provides a new foundation for the resolution of Hume's problem. This new logical framework clearly distinguishes the universality and the time recursion for a natural law and excludes the confusion commonly made by Hume and his opponents. The time recursion is a more ultimate feature independent from the contents of a natural law. They are not equivalently ultimate as thinking elements just like the natural number in mathematics.

In a quasi-law tautology such as (3.1), letting $t_k,\ k = 0,1,2,\cdots$, cover the past and future, we go into typical Hume's problem. Time recursion as the most ultimate attribute bases natural laws, Hume's critique, and counter-proposal to Hume's critique, and is thus the common ground for they can engaging in a battle. In other words, Hume's problem also bases itself on the time recursion. When we discuss Hume's problem, it is enough to go on under the concept of time recursion but need not discuss time recursion itself. Or in short, if recursive abduction is the first-order logic, the natural law, Hume's critique, and the counter-proposal to Hume's critique all are the second-order system with respect to time.

Under the viewpoint of L-abduction, as well-discussed above, the quasi-law tautology and its conclusion will

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keep original irrelevant to the past or future. It is requested by the logical consistent explanation in the quasi-law tautology, and is a formal requirement just like that in geometry. Hume misunderstood the future truth only possibly coming from the past inductive one, and was innocence to both the consistent request from a quasi-law tautology and the empirical test in time-recursive abduction. In fact, the logical consistent explanation in a quasi-law tautology not only requests that the empirical truth of future must be coherent to the past but also requests that the past one must be coherent to the future. It is just the feature of any empirical science based on L-abduction. Hume's problem to the strict natural law system is a pseudo problem.

In Case 1, introduce the temporal characteristics to $\mathcal{L}_G$ as follows

$$(\neg v_0 \to \neg h_\uparrow) \to (e \to (h_\uparrow \to v_0)^P), \tag{4.1}$$
$$(\neg v_0 \to \neg h_\uparrow) \to (e \to (h_\uparrow \to v_0)^F), \tag{4.2}$$

where, $(\quad)^P$ and $(\quad)^F$ indicate the sentences within ( ) belonging to the past and future, respectively.

Hume's critique is equivalent to say that one can only accept (4.1) and must reject (4.2). As we had discussed, (4.1) and (4.2) are equivalent in the time-recursive sense and the consistent explanations and conclusions of (4.1) and (4.2) keep the same to original $\mathcal{L}_G$. It is emphasized that (4.1) and (4.2) equivalently and simultaneously hold just like (2.2), by no means (4.2) must be derived from (4.1). It is a request from the logical consistent explanation in (2.2) but not an additional empirical hypothesis regarding conformability in the past and future. Its empirical truth value is determined by L-abduction in time-recursive empirical test. Here, the key point is that $\mathcal{L}_G$ requests the same consistent explanation in the past and future among observables $h_\downarrow$, $h_\uparrow$, v_0, and $v_{0\downarrow}$. This feature is well familiar by us in geometry when we use it to describe the realistic world. And this feature does not exist in normal empirical inductive conclusion for which there is no any logical consistent explanation given in a quasi-law tautology. Hume and his opponents all overlooked the logical intercommunity represented by $\mathcal{L}p_L$ in the natural law and geometry, and completely mistook the highlight on natural law.

We can further explain (4.1) and (4.2) by examples to eliminate the possible divergence as follows: Newtonian mechanics as a theoretical system in its applicable field always keeps its own consistent explanation regardless of it as a precise empirical theory before the twentieth century or as an approximation after the twentieth century, just like Euclidean geometry always keeps its own consistence regardless of the more precise empirical observation accepting or rejecting it. The consistence asks all observables to be consistently related with each other whatever they are past or future. Hume's skepticism only suits for questioning the normal empirical inductive process but not the quasi-law-tautologically based natural law. The twentieth-century negative evidence to Newtonian mechanics must change the meanings of its ever positive evidence in the seventeenth century, which in fact can be explained as the good approximation to the twentieth-century one in low-speed and macroscopic phenomena; and vice versa, the seventeenth-century positive evidence had ever asked the twentieth-century evidence in the future to consistently agree with it. This is not the attribute of a normal inductive result. For a normal inductive conclusion such as "Tom always dresses red color", if we saw Tom dressed green color today, we would regard the inductive conclusion wrong today but unnecessarily have to change the meanings of past evidence "Tom dressed red color yesterday"; and vice versa, "Tom dressed red color yesterday" does nothing to consistently ask "Tom would dress red color in the future". To the normal empirical inductive result, Hume's question is proper. The difference between quasi-law-tautologically based natural laws and normal empirical inductive conclusions can be illustrated as follows: Newtonian mechanics has consistent explanation excluding contradictory evidences regardless of past and future, but a normal empirical inductive conclusion only contains the simple accumulation of observable evidences un-exclusive to the contradiction, without any consistent ask to its contents in any dimension including in the past-future dimension and, therefore, is suitable to be questioned by Hume's skepticism. It is all too obvious that any empirical support to a consistent theory is logically indifference to the past and future for its logical consistent explanation is a logical attribute and regardless of time; empirical observables as the contents in the physical law are always organized together consistently to make the whole so that the future (or past) negative evidence will be invalid unless it meanwhile consistently changes the past (or future) one and includes the past (or future) one. The future suitability of a physical law merely refers to nothing but the consistent restriction for all observables no mater of past or future if time is a suitable recursive dimension. Facing with the empirical test, a normal empirical inductive conclusion is only a weak and fragile defendant, but a strict natural law has the strong consistence power to persist in its own "ask" for accepting or rejecting it. By the way, not involving rigorous logical contents and mainly focusing on the factual outcome, Lakatos [51] from a historical and philosophical sight, to some extent, had similarly discussed the same result called "hard core" and "protection zone" of scientific theory. In this paper, the conclusion is more clear and explicit logically. In this way, a natural law is always universally applicable to past and future as well as it is universally falsifiable in any time. It is the logical consistent explanation in a quasi-law tautology to request but not to examine the physical law applicable from past to future in recursive time dimension. And this attribute of a physical law completely resembles to the consistence between theorems and their axioms in geometries which need not have to additionally suppose that in a natural law the contents of future must be conformable to the past.

In recursive L-abduction, any strict natural law is requested consistent in past and future, and simultaneously to change its meanings in past and future if any. Namely, past requests future, meanwhile, future requests past, they are originally equivalent in recursive L-abduction; and the future never requests a one-way support from the past. This is the sense of universality of natural law. Hume and his opponents all mistook the universality as the inductive result of one-way support from past to future.

This analysis reveals the key role of quasi-law tautology in a physical law system. It is the quasi-law tautology to distinguish a physical law from the normal empirical commonsense and to have the physical law immunized to the infection of Hume's skepticism.

Under the interpretation from L-abduction, the function of empirical induction p_I' in a scientific theory is to anchor the logical consistent explanation in empirical phenomena by recursive abduction, but not to provide an enormous factual aggregation to pile up a natural law. The minimal observational set of p_I' is $\{p_I'^{r_0}, p_I'^{r_k}, p_I'^{r_{k+1}}\}$. In the recursive abduction, the effective empirical anchoring asks the clear logical contexts but not the huge number of evidence accumulation.

Of course, observational repeatability is one of the important derived features of recursive abduction. Researchers always unconsciously use the repeatability as a practically equivalent substitute for the recursion to promote the efficiency of scientific discovery and, meanwhile, it absorbs researchers' sight to deviate from the essential character of recursive abduction. Numerous empirical evidences are in favor of the discovery of natural law for two reasons that, first, people easily mistakes in merely one set of observations and the repeatability is helpful to rule out one's carelessness; and second, a result derived from recursive L-abduction must contain the repeatability in all recursive cases, hence, it will be more possible to find a recursive natural law in repeatable observations than in repeatability-unknown cases. In the first reason, the repeated inductive observation is a *management measure* to overcome the human being's behavioral fault; and in the second reason, the repeated inductive observation will reduce the discovery cost in scientific activities, is *a cost-saving principle*. They all are not the primitive feature of a natural law. In current discussions about Hume's problem, all participants were often at sea for they misunderstood *a behavioral management measure* and *a cost-saving principle* as the primitive attribute of natural law.

The discovery of scientific theory is a process to look for consistent explanation among empirical phenomena, just like geometry to be used in the measurement of physical space. The logical consistent explanation given in

$\mathcal{L}p_L$ requests all elements uncontradictory from each other in all suitable recursive dimensions including, but not limiting to, the past and future. Hume's analysis only suits for the critique to the inductive commonsense, in which however originally no rigorous logical consistent explanation is sought, and to the immature sub-science, such as penal data model in economics, traditional folk medical diagnosis, and etc., which are however originally liquid without durable conformability. Hume's problem is hence not only pseudo to strict natural laws but also redundant, though is not false, as rational critique to the empirical induction or commonsense.

5. Conclusions

It is concluded from this paper:

1. Case studies reveal the quasi-law tautology functioning in the derivation for any physical law. Quasi-law tautology combines with abduction naturally lead to recursive abduction.

2. The empirical universality of physical law bases itself on recursive abduction. It refers to nothing else but a set of consistent physical relations holding in one or more suitable recursive dimensions, just like a consistent geometry used in the measurement of realistic physical world. In the schema of recursive abduction, a physical law is always derivable logically and decidable empirically.

3. Hume's problem and its typical opponent proposals all misunderstood a formal universality derivable from quasi-law tautology as an empirical conformability about past to future and a time-recursive issue as an infinitely inductive problem and, thus, sank into the inescapable quagmire. Under the viewpoint of recursive abduction Hume's problem naturally vanishes.

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