A cosmological view on Milgrom's acceleration constant

Engel Roza

Stripperwei 1, 5551 ST Valkenswaard, The Netherlands Email: <u>engel.roza@onsbrabantnet.nl</u>

Summary

In this article a two-parameter model is developed for the universe. The two parameters are the age of the universe and the value of Einstein's Cosmological Constant. It is shown that these are sufficient to calculate the amounts of matter and dark energy in the universe, as well as the contributions of dark matter and baryonic matter in the matter part. All this, not only for present time, but also as a function of cosmological time. Moreover, the model allows establishing the numerical value for Milgrom's acceleration parameter for present time. The developed theory gives an adequate explanation for the phenomena of the accelerated scaling of the universe and the anomaly of the stellar rotation curves in galaxies. The numerical results are in agreement with those of the Lamda-CDM model.

Keywords: Cosmological Constant; MOND; dark matter; dark energy; Lamda-CDM model

1. Introduction

This article is meant as an extension to a previous study [1]. It is my aim to give some view on the cosmological space, in which I will apply the concepts that I have developed to give an explanation for the negative pressure executed by the spatial fluid that must be present in vacuum to justify a positive value of the Cosmological Constant Λ in Einstein's Field Equation. Such a value is required to remove the anomaly of particular cosmological phenomena, like the rotation curves of stars in galaxies and the accelerated expansion of the universe. In my previous article, it has straightforwardly been derived that in a gravitational system with a central mass M in vacuum, the Cosmological Constant, while independent of space-time coordinates, amounts to

$$\Lambda = a_0 / 5MG , \tag{1}$$

where $a_0 (\approx 10^{-10} \text{ m/s}^2)$ is Milgrom's acceleration constant [2,3,4] and *G* the gravitational constant. It is my aim to show that this relationship between Milgrom's acceleration constant and the Cosmological Constant not only applies to galaxies, but holds for the universe as a whole as well, in spite of the obvious difficulty to identify a central mass. The study is prompted by the wish to find a means to assess the numerical value of Milgrom's acceleration constant by theory, for which no clue could be found within the scope of galaxies.

Satisfying Einstein's Field equation in vacuum under absence of a massive source under the condition of a positive Cosmological Constant Λ requires the presence of a background energy [5]. Under inclusion of the massive source, the background energy will of course still be there and shows up as polarized dipoles [1,6], which explains the 1/M dependency of the Cosmological Constant. It may seem that a gravitational dipole concept, in the sense of a

1



bond between a positive mass and a negative mass, violates physics, because a negative mass is not a viable concept. It showed up, however, in a model that explains the weak limit solution of Einstein's equation with a positive Cosmological Constant. As is well known, such a solution forces viewing the vacuum as a fluidal space with a negative pressure (corresponding with a positive background mass density). A close inspection will reveal (see appendix in [1]) that this solution is nothing else but the result of a modulation of the background energy density, due to a disturbance caused by the insertion of a central mass M. This allows to conceive the disturbance as gravitational dipoles on the pedestal of the background energy density. What may seem as a negative mass in the gravitational dipole is a dip in the background energy. The dipoles constitute grains in the fluidal vacuum with a negative pressure that neutralizes the gravitational force between the poles. This makes the gravitational dipole a valid concept.

The vacuum fluid is an ideal one. This implies that its stress-energy tensor contains diagonal elements only and that in space-time ((ict, x, y, z) with (+,+,+,+) metric they all have the same value. The pressure p_v is due to the background energy of the fluidal space, expressed by,

$$p_v = -\rho c^2 = -\frac{c^4}{8\pi G}\Lambda, \qquad (2)$$

where c is the light velocity in vacuum and where ρ is the fluidal mass density. It must be present for giving a solution of Einstein's Field Equation with $\Lambda \neq 0$ for the vacuum without any baryonic sources. Assuming the correctness of (1), the fluidal pressure would rise to infinity under the absence of baryonic source. Baryonic sources "eat" from the fluid, thereby gaining mass and reducing the energy level of the fluid. This model is applicable to cosmological objects with a massive kernel, like galaxies. Interestingly, though, it can be evolved to a model that fits to the cosmos where mass is distributed. For reasons to be explained later, a semantic difference is made between something denoted as cosmos and something else denoted as universe.

2. Cosmological model

Let us model the cosmos as a sphere with distributed matter and some radius L. This distributed matter is a mixture of fluidal matter M_D as meant by (2) and baryonic matter M_B as meant by (1). Effectively, the latter is the total of encapsulated mass. Hence, from (1) and (2),

$$M_{D}c^{2} = \int_{0}^{L} \frac{a_{0}}{5M_{B}G} \frac{c^{4}}{8\pi G} 4\pi r^{2} dr .$$
(3)

Let ΔM_D the difference between the fluidal matter in a sphere $L + \Delta L$ and the fluidal matter in a sphere L. It follows readily that

$$c^{2} \Delta M_{D} = \frac{a_{0}}{10M_{B}G^{2}} c^{4} L^{2} \Delta L .$$
(4)

Because of the influence of the baryonic mass on the energy level of the fluidal mass, it is fair to state that,

$$M_{B} = \beta M_{D}, \qquad (5)$$

where β is a dimensionless quantity. Hence, by integrating (4),

$$\frac{1}{2}M_D^2 c^2 = \frac{a_0}{10\beta G^2}c^4 \frac{L^3}{3}.$$
(6)

The total gravitational massive energy $M_G c^2$ (fluidal plus baryonic), therefore, is

$$M_G c^2 = M_D c^2 + M_B c^2 = (1+\beta) \sqrt{\frac{a_0}{5\beta G^2} c^6 \frac{L^3}{3}}.$$
(7)

Hence, the massive energy density $ho_{_G}c^2$ in the sphere with radius L is given by

$$\rho_G c^2 = (1+\beta) \sqrt{\frac{a_0}{5\beta G^2} c^6 \frac{L^3}{3}} \cdot \frac{4}{3\pi L^3} = \frac{4(1+\beta)}{3\pi} \sqrt{\frac{a_0}{15\beta G^2} \frac{c^6}{L^3}}.$$
(8)

This unification of baryonic mass and fluidal mass allows us to compare this massive energy with the one as obtained in the present cosmological model, known as the Lamda-CDM model (CDM \equiv Cold Dark Mass, Lamda \equiv Einstein's Cosmological Constant), [7]

The CDM-Lamda model has been evolved from the Einstein-de Sitter model that has been the preferred one for the universe up to the 1980s. It has been refined to the present one to cope with certain cosmological phenomena, like for instance the discovery of the accelerating universe in 1998, [8,9].

3. The Lamda-CDM model

The model evolves from the solution of Einstein's Field Equation under the constraint of a particular metric.

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad \text{with} \quad G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \,, \tag{9}$$

where $T_{\mu\nu}$ is the stress-energy tensor, which describes the energy and the momenta of the source(s) and where $R_{\mu\nu}$ and R are respectively the so-called Ricci tensor and the Ricci

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scalar, which can be calculated if the metric tensor components $g_{\mu\nu}$ are known [10,11,12]. The adopted metric, known as the Friedmann-Lemairtre-Robertson-Walker (FLRW) metric [13], is

$$ds^{2} = dq_{0}^{2} + a^{2}(t)(\frac{dr^{2}}{1 - kr^{2}} + r^{2}\sin^{2}\vartheta d\varphi^{2} + r^{2}d\vartheta^{2}), \qquad (10)$$

where $q_0 = ict$ is the normalized time coordinate ($i = \sqrt{-1}$), and where k is a measure for the curving of space-time. The scale factor a(t) expresses the time-dependence of the size of the universe. The ratio

$$\frac{\dot{a}}{a} = H(t), \tag{11}$$

is known as the Hubble factor. It is the main observable of the universe, because its numerical value can be established from red shift observations on cosmological objects ($\dot{a} \equiv da / dt$).

By moving the term $\Lambda g_{\mu\nu}$ to the right side of (9), it can be conceived as an additional contribution to the energy density,

$$\rho(t) \to \rho(t) + \rho_{\Lambda}; \quad \rho_{\Lambda} = \frac{\Lambda}{\kappa}; \quad \kappa = \frac{8\pi G}{c^2}.$$
(12a)

$$p(t) \rightarrow p(t) - \rho_{\Lambda} c^2$$
 (12b)

The solution of (9) under constraint of the metric (10) are the two Friedmann equations [14],

$$(\frac{\dot{a}}{a})^2 + \frac{kc^2}{a^2} = \frac{8\pi G}{3}\rho$$
 (13a)

$$2\frac{\ddot{a}}{a} + (\frac{\dot{a}}{a})^2 + \frac{kc^2}{a^2} = -\frac{8\pi G}{c^2} p.$$
 (13b)

Under the constraint k = 0 (flat universe), and taking into consideration (12a,b), the first Friedmann equation evolves as,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho + \rho_\Lambda) \to \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_c; \quad \rho_t = \rho + \rho_\Lambda.$$
(14a)

The second Friedmann equation reads as,

$$\frac{\ddot{a}}{a} = \frac{4}{3}\pi G(2\rho_{\Lambda} - \rho) \rightarrow \frac{\ddot{a}}{a} = \frac{4}{3}\pi G(2\rho_{\Lambda} + 2\rho - 3\rho) = \frac{8}{3}\pi G\rho_t(1 - \frac{3}{2}\alpha); \alpha = \frac{\rho}{\rho_t}.$$
(14b)

Differentiating the mass density ho_t in (14a) gives,

$$\dot{\rho}_{t} = \frac{3}{8\pi G} \frac{\mathrm{d}}{\mathrm{d}t} (\frac{\dot{a}}{a})^{2} = \frac{3}{8\pi G} [2\frac{\dot{a}}{a} \{\frac{\ddot{a}}{a} - (\frac{\dot{a}}{a})^{2}\}] =$$

$$2\frac{3}{8\pi G} \frac{\dot{a}}{a} \{\frac{8}{3}\pi G(1 - \frac{3}{2}\alpha) - \frac{8\pi G}{3}\}\rho_{t} = 2\frac{\dot{a}}{a} \{(1 - \frac{3}{2}\alpha) - 1\}\rho_{t} = -3\alpha\frac{\dot{a}}{a}\rho_{t} \rightarrow$$

$$\dot{\rho}_{t} = -3\alpha\frac{\dot{a}}{a}\rho_{t} \rightarrow \frac{1}{\rho_{t}}\frac{\mathrm{d}\rho_{t}}{\mathrm{d}t} = -3\frac{\rho}{\rho_{t}}\frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}t} \rightarrow \frac{\mathrm{d}\rho_{t}}{\mathrm{d}t} = -3\rho\frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}t} \rightarrow$$

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + \frac{\mathrm{d}\rho_{\Lambda}}{\mathrm{d}t} = -3\rho\frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}t}.$$
(15)

Because the background massive density ρ_{Λ} is time-independent (Λ is independent of space-time coordinates), (15) is satisfied if,

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -3\rho \frac{1}{a} \frac{\mathrm{d}a}{\mathrm{d}t} \to \rho = \Omega_m H_0 \alpha^{-3} \quad \text{and} \quad \rho_\Lambda = H_0 \Omega_\Lambda, \tag{16a,b}$$

where Ω_m , Ω_Λ and H_0 are constants. The quantity H_0 is the Hubble parameter \dot{a}/a at a(t) = 1. It is tempting to believe that Ω_m and Ω_Λ are, respectively, the relative amount of baryonic mass ρ / ρ_t and the relative amount of background mass ρ_Λ / ρ_t at a(t) = 1. This, however, is not necessarily be true, because (without further constraints) the differential equation (14) is satisfied for any distribution between Ω_m and Ω_Λ as long as $\Omega_m + \Omega_\Lambda = 1$.

Applying (16a,b) on the first Friedmann equation (14a), results into,

$$H(a) = \frac{\dot{a}}{a} = H_0 \sqrt{\Omega_m a^{-3} + \Omega_\Lambda} .$$
(17)

This equation represents the Lamda-CDM model in its most simple format (actually, more terms are heuristically added under the square root operator to model empirical evidence from certain cosmological phenomena). Eq. (17) can be analytically solved as [15],

$$a(t) = \left(\frac{\Omega_m}{\Omega_\Lambda}\right)^{1/3} \sinh^{2/3} \left(3\sqrt{\Omega_\Lambda}t/2t_H\right); t_H = H_0^{-1}.$$
(18)

At present time $t = t_p$, the scale factor equals unity (a = 1) and the Hubble parameter is the observable H_0 . Equating present time t_p with Hubble time t_H is justified if a(t) would have shown a linear increase over time up to now, under a constant rate of say c_0 , because in that case $a(t) = c_0 t$ and $\dot{a}(t) = c_0$. This is Hubble's empirical law. Equating $t_p = t_H$ in (18) as an axiomatic assumption, indeed results in a behavior of the scale curve that, up to present time $t \le t_H$, is pretty close to Hubble's empirical law. Hence, from (17),

$$a(t_H) = 1; \ H(a)|_{t_H} = H_0 \to \Omega_m + \Omega_\Lambda = 1.$$
 (19)

Hence, from (19) and (18),

$$\left(\frac{1-\Omega_{\Lambda}}{\Omega_{\Lambda}}\right)^{1/3}\sinh^{2/3}\left(3\sqrt{\Omega_{\Lambda}}/2\right) = 1 \to \Omega_{\Lambda} = 0.737; \ \Omega_{m} = 0.263.$$
(20)

These values are only slightly different from those in the six-parameter Lamda-CDM model. The difference is due to the simple format (17). Figure 1 demonstrates the viability of the axiomatic assumption to equate present time with Hubble time.



Fig.1: The scaling factor a(t) as a function of cosmological time. The lower curve represents Hubble's law. The upper curve shows the curve of accelerated scaling due to Einstein's Cosmological Constant.

4. Harmonizing the view on the universe with the view on the cosmos

Let us proceed by trying to harmonize the view on the cosmos as discussed in the second paragraph with the view on the universe as discussed in the third paragraph. This will be done by comparing the amount of massive energy in the universe that is presently observable with the massive energy that would be observable on the basis of the cosmological model. Note that we wish only taking care of observability, thereby ignoring possible massive energy in non-observable ranges.

The present massive density $\rho_t(a=1)$, known as critical mass ρ_c in the Lamda-CDM model observed within the Hubble horizon $L_H = ct_H$, can be established from (13a) as,

$$\rho_c = \frac{3H_0^2}{8\pi G}.$$
(21)

Its massive energy is equivalent with the massive energy as defined by (7). Hence,

$$\frac{3H_0^2}{8\pi G}c^2 = \frac{4(1+\beta)}{3\pi}\sqrt{\frac{a_0}{15\beta G^2}\frac{c^6}{L_H^3}}.$$
(22)

Let us eliminate H_0 and L_H from (22) by introducing a quantity a_L with the dimensionality of acceleration (m/s²),

$$a_L = \frac{c^2}{L_H},\tag{23}$$

Hence, from $L_H = ct_H$ and (19),

$$H_0 = \frac{a_L}{c} \,. \tag{24}$$

It has to be emphasized that a_L is not a free variable, but a known quantity that can be calculated from (23) as $a_L \approx 7 \times 10^{-10} \text{ m/s}^2$. Expression (22) evolves as,

$$\frac{3H_0^2}{8\pi G}c^2 = \frac{4(1+\beta)}{3\pi}\sqrt{\frac{a_0}{15\beta G^2}\frac{c^6}{L_H^3}} \to \frac{3a_L^2}{8G} = \frac{4(1+\beta)}{3}\sqrt{\frac{a_0}{15\beta G^2}a_L^3} \to (\frac{3a_L^2}{8})^2 = \{\frac{4(1+\beta)}{3}\}^2\frac{a_0}{15\beta}a_L^3 \to a_0 = \frac{81x15}{64x16}\frac{\beta}{(1+\beta)^2}a_L.$$
(25)

This expression establishes a relationship between the present ratio of baryonic matter over fluidal matter and Milgrom's acceleration constant. Taking Milgrom's acceleration constant for present time and the age of the universe as independent parameters, allows to establish the ratio of the values for baryonic matter and dark matter, which in the Lamda-CDM model are considered as primary parameters. From $t_H = 13.8$ Gyear, we have $a_L = c^2 / (ct_H) = 6.9$ m/s². For $a_0 = 1.07$ m/s² we get from (25) the result $\beta = 0.185$, which is the ratio baryonic matter/dark matter in the Lamda-CDM model. The correspondence of the calculated value for Milgrom's acceleration parameter in present time with evidence of observations gives a strong support for the viability of the theory developed in this article.

5. Dark matter and dark energy

The issue to be resolved still is the problem how to relate the theoretically established quantities Ω_m and Ω_Λ for matter and dark energy, which have their origin in Einstein's Cosmological Constant as well, with baryonic matter and dark matter. While under expansion of the universe the matter density decreases proportionally with the size of the universe, the baryonic part of still continues "eating" from the dark matter part. The density of the dark energy, however, remains invariant under expansion, possibly after an initial loss of matter. That means that the matter part is made up by baryonic matter and dark matter in a ratio that scales under expansion of the universe. The scaling behaviour can be established from the following considerations. For the massive energy of the universe, we have from (21),

$$Mc^{2} = c^{2} \int_{0}^{L_{H}} \frac{3H_{0}^{2}}{8\pi G} 4\pi r^{2} dr = \frac{H_{0}^{2}}{2G} L_{H}^{3} c^{2}.$$
 (26)

From (26) and (1), the Cosmological Constant Λ for the universe (which is different from the Λ of galaxies) can be established as

$$\Lambda = \frac{2}{5} \frac{a_0}{MG} = \frac{2}{5} \frac{a_0}{G} \frac{2G}{H_0^2 L_H^3} = \frac{4}{5} a_0 \frac{L_H^2}{c^2 L_H^3} = \frac{4}{5} a_0 \frac{1}{c^2 L_H} \frac{c^2}{c^2} = \frac{4}{5} \frac{a_0 a_L}{c^4} .$$
(27)

(Note that, because of the time-independency of Λ and the scaling of $\alpha_L = c^2 / L_H$, Milgroms acceleration constant scales with L_H)

Hence, from (27) and (25),

$$\frac{4}{5}\frac{a_0a_L}{c^4} = \frac{4}{5}\frac{81x15}{64x16}\frac{\beta}{(1+\beta)^2}\frac{a_L^2}{c^4} = \text{constant}$$
(28)

$$\rightarrow \frac{\beta}{\left(1+\beta\right)^2} \frac{1}{L^2} = \text{constant.}$$

Hence,

$$\frac{\beta}{(1+\beta)^2} = k_0 \left(\frac{t}{t_H}\right)^2 \to \beta = \frac{1-2k_0 \left(t/t_H\right)^2 \pm \sqrt{1-4k_0 \left(t/t_H\right)^2}}{2k_0 \left(t/t_H\right)^2},$$
(29)

where the constant k_0 can be established from the known value of $\beta = 0.185$ at present time $t = t_H$. Figure 2 shows the ratios β and $1 - \beta$ as a function of cosmological time. Because the baryonic content "eats" from the fluidal mass content, the baryonic part cannot exceed the dark matter part. Therefore $\beta \le 1$, such as shown in the graph.



Fig.2. The upper curve shows the relative amount of dark matter as a function of cosmological time. The lower curve shows the relative amount of baryonic matter.

6. Conclusion

Putting everything together, we arrive at a very simple model for the universe, which nevertheless explains phenomena as dark matter, dark energy, the anomaly of the stellar rotation curves in the galaxies and the accelerated expansion of the universe. It gives an explanation as well for the empirical laws of Hubble and Milgrom. Next to Newton's gravitational constant G, only two other parameters are needed. These are the age of the universe t_H and Einstein's Cosmological Constant Λ for the universe. Everything else follows from Einstein's Field Equation in conjunction with the FRLW-metric. Let me summarize the basic equations and their implications.

The time behaviour of the scaling factor of the universe is a solution of Einstein's Field Equation under the FRLW-metric,

$$a(t) = \left(\frac{\Omega_m}{\Omega_\Lambda}\right)^{1/3} \sinh^{2/3} \left(3\sqrt{\Omega_\Lambda}t/2t_H\right); t_H = H_0^{-1}.$$
(30)

As a consequence of $a(t_H) = 1$, the relative values for matter density and dark energy are established as,

$$\Omega_m = 0.263 \text{ and } \Omega_\Lambda = 0.737.$$
 (31)

The relationship between the Cosmological Constant of the universe and Milgrom's acceleration parameter is established as,

$$\Lambda = \frac{4}{5} \frac{a_0 a_L}{c^4}; \ a_L = \frac{c^2}{L}; \ L = c t_H.$$
(32)

Accepting the life time of the universe $t_H = 13.8$ Gyear and $\Lambda = 7.29 \times 10^{-54} \text{ m}^{-2}$ as primary independent quantities, we get $a_0 = 1.07 \text{ m/s}^2$.

The relationship between Milgrom's acceleration parameter and the ratio β of baryonic matter over dark matter is established as,

$$a_0 = \frac{81 \times 15}{64 \times 16} \frac{\beta}{(1+\beta)^2} a_L.$$
(33)

The resulting value for the present value of this ratio amounts to $\beta = 0.185$. The evolution over time is shown in figure 2. Because the baryonic matter cannot exceed the initial amount of dark matter, the ratio β has the upper limit $\beta = 1$.

All this has been derived straightforwardly by this two-parameter theory. The calculated quantities correspond nicely with those of the six-parameter Lamda-CDM model, which is largely empirical. The benefit of including more parameters is the modelling of cosmological effects beyond the scope of the simple model. The benefit of the simple model is its strength to show the relationship between dark energy and dark matter as well as the relationship

between Milgrom's empirical MOND theory [2] and the Lamda-CDM model. Moreover, the two parameter theory strengthens the prediction made before that at large cosmological distance gravity turns on and-off into antigravity with some spatial periodicity [1].

7. Discussion

The previous paragraph contains the conclusions of this work. Harder than formulae is not possible. And a better proof than the match with known empirical evidence cannot be given. This leaves the problem of interpretation. This will be open for discussion and opinions might diverge. In the picture of the author, which he wants to give free for a better one, the universe seems appearing as a bubble in the cosmos. The cosmos is a sea of fluidal energy. Otherwise Einstein's Cosmological Constant would be zero (an empty universe does not allow a viable solution of Einstein's Field equation for $\Lambda \neq 0$). The universe is created from the subtraction of a matter bubble from this sea. The ratio of the fluidal energy and the matter energy is found from Einstein's Field Equation and the axiom that the universe is a flat one. The matter subtraction is the event that marks the birth of the universe. The matter bubble might be a "quid pro quo" for spontaneously created ubiquitus ones.

The matter bubble consists of dark matter, which is gradually converted into baryonic matter, in a rate that is determined by the value of Einstein's Cosmological Constant for the universe, which is natural constant next to the Gravitational Constant. The conversion rate is related with the expansion rate of the universe, which is accelerated because of the non zero-value of the Cosmological Constant, hence because of the energetic fluid in the cosmos.

As shown in the previous study [1], in which the relationship between Milgrom's acceleration constant and the Cosmological Constant has been derived, the sea of fluidal energy contains grains with a gravitational dipole. The negative pole is a dip in the sea of energy. That means that the grains are ripples in the sea and that the gravitational dipole has a pedestal. In a galaxy, the grains influence the gravitational force, because the field from the central baryonic mass polarizes their gravitational dipole moment, thereby creating the gravitational equivalent of a displacement charge that adds to the baryonic one. Effectively, it means that matter from outside is pushed in within the cosmological horizon. This "displaced" matter is the dark matter.

This dark matter model is less clear for the universe, where baryonic matter has a uniform distribution, thereby causing an overall random distribution of the gravitational dipole moments of the grains. Nevertheless, the universe is an assembly of galaxies as well, which all show the mechanism of attracting mass from beyond their cosmological horizons. It might well be that all their contributions sum up to the dark matter of our universe. The developed theory predicts the conversion of dark matter into baryonic matter. However, in spite of an adequate description of the process, as a consequence of Einstein's Cosmological Constant, it does not reveal the very physical structure of the conversion. This conversion seems to be a an irreversible process, akin to osmosis, in which, for some reason, stable baryonic structures are created from the entropic behaviour of the dark matter kernels. Further research is required to gain more insight in this.

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