Applying petroleum pressure buildup well test procedure on Thermal Response Test - A novel method to improve accuracy of thermal conductivity determination

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Abstract: Theory of the Thermal Response Testing (TRT) is a well-known part of sizing process of the geothermal exchange system. Multiple parameters influence accuracy of effective ground thermal conductivity measurement; like testing time, variable power, climate interferences, groundwater effect etc. To improve accuracy of the TRT we introduced procedure to additionally analyze falloff temperature decline after power test. Method is based on a premise of analogy between TRT and petroleum well testing, since origin of both procedures lies in diffusivity equation with solutions for heat conduction or pressure analysis during radial flow. Applying pressure buildup test interpretation technique to the borehole heat exchanger testing, greater accuracy could be achieved since ground conductivity could be obtained from this period. Analysis was conducted on coaxial exchanger with five different power steps, and with both direct and reverse flow regime. Each test was set with 96hr of a classical TRT, followed by 96hr of temperature decline, making it almost 2000 hours of cumulative borehole testing. Results showed that ground conductivity value could vary as much as 25% depending on test time, seasonal period and power fluctuations while thermal conductivity obtained from a falloff period gives more stable values with only 10% value variation.

Keywords: Shallow geothermal resource; Borehole heat exchanger; Thermal Response Test; TRT

1. Introduction

Number of installed systems for utilization of shallow geothermal energy is constantly increasing during the last decade. Beside common practice in modeling borehole heat exchanger fields with power less than 30kW with various software for estimate thermogeological parameters of the soil, Thermal Response Test (TRT) is the only justifiable method to get correct properties for certain location.

Catalogue properties of soil, which are source for modeling phase with software, often lead to underside system and functional problems. Soil sample analysis in laboratory are too expensive for everyday use. Considering that, TRT is also the most profitable method for modeling long-term lasting systems. Inaccuracies in estimation thermal properties from TRT measuring are mostly caused by outdoor effects. It is necessary to recognize and reduce atmospheric influences and changes in ambient temperature. Energy dissipation from the pipes above the ground results in higher value of estimated thermal conductivity and may lead to undersize of the future system installation.

Steep changes caused by demand of consumers in local electrical grid will cause fluctuation of the power provided for electric heaters in TRT equipment which leads to variation in heat rate transmitted to borehole fluid. That directly affects entering source temperature (EST) and orderliness of the results of the measurement, change of the temperature during the time, which is a source for...
calculating the thermal conductivity of the found and borehole thermal resistance. Therefore TRT should be performed during the whole daily cycles because in that way it is easier to connect discrepancies in measurement with pattern of daily oscillation of the power in electrical grid. Prerequisites for credible calculations from TRT measurement are described in numerous researches and handbooks.

Novel method of TRT presented in this paper could achieve more precise determination of thermal properties of the ground. It is based on theoretical and experiential methods from petroleum engineering utilized for almost century in practice, where it is possible to find similarities due to the same theoretical origin of describing pressure behavior and heat conduction with diffusivity equation solution. Therefore, our recommendation is to always conduct analysis of thermal recovery period after performing classical TRT. Such prolonged TRT procedure could decrease chance errors in interpretation due to fluctuation of voltage in public electrical grid and effect of test duration on final calculations of ground thermal properties. Furthermore, presented method it suitable to diminish effects of ambient temperature interference, which can notably affect values of obtained ground thermal conductivity. This is especially true during winter season where there is high difference in temperature of the air and fluid inside of the pipes during TRT, yet significantly smaller during recovery period due to smaller heat flux between surface equipment and air.

2. Literature overview

Duration of Thermal Response Test is one of the most discussed parameters. ASHRAE Handbook [1] recommends that Thermal Response Test should be performed for 36 to 48 hours. Bujok et all [2] presented measurement done on experimental underground heat storage where TRT was performed on eight boreholes with possibility to compare software simulations and various durations of measuring on identical ground environment. Software simulation with data for first 24 hours of TRT for thermal conductivity deviates up to 7,8% comparing with results of TRT provided for 70h. Borehole thermal resistance calculated with software differs up to 17.9% from values calculated form 70h TRT.

The least discussed influence on credibility of TRT is variation of ambient temperature which can strongly affect results of measured temperature of circulating fluid. Bandos et al [3] introduced first method of subtracting effect of atmospheric conditions by using data of the air temperature at the time of providing the test. Application of the method reduces the oscillation of the value of thermal conductivity from 30% to 10% from the mean one. It is shown in research that delay of the ambient to mean temperature is about 3 hours. Borinaga-Trevino et al [4] developed a new method to analyze and reduce the influence of atmospheric conditions on the TRT with main advantage that it is not necessary to know its physical origin but is based on analyzing influence of chosen time interval to fit the data of the ILS theory and formulas to predict the ground thermal conductivity. Two TRT were analyzed, each with the different equipment and level of isolation from the environment. For the less isolated TRT performance thermal conductivity for a 5% of confidence interval and a log-normal probability function was determined to be 2,7-2,76 W/m K and 2,49-2,53 W/m K for the uncorrected and corrected solutions, while in better isolated TRT values were 2,75-2,76 W/m K and 2,73-2,74 W/m K. Therefore, poorly isolated TRT equipment exposed to variable ambient condition may lead to ±33% of mistake in determine thermal conductivity of the ground, depending on the season of measuring. The reduction of the thermal conductivity variation is more evident on the standard deviation obtained. For the first TRT with poor isolation, the ground thermal conductivity for a 5% of confidence interval and a log-normal probability function was determined to be 2,70-2,76 W/m K and 2,49-2,53 W/m K for the uncorrected and corrected solutions, respectively. For the other one these intervals were 2,75-2,76 W/m K and 2,73-2,74 W/m K.

Signorelli et al [5] tested duration of the TRT for evaluation of thermal properties, comparing the results from 3-D FE numerical model with results from synthetic line source solution. Research concentrated on estimation of time t0, as a time after start of the test, after which data will not be affected by lower thermal conductivity of the tubes and grouting material and effect of unsteady heat
transfer. In the case, the simulated 200 h test from the numerical model was evaluated for values of 
$t_0$ of 10 hr, 20 hr, 40 hr, and 60 hr, and variable values of end of the test, $t_E$, in extent $t_0 < t_E < 200$ h.

According to results, accurate defining of $t_0$ has more significant impact on values of calculated 
thermal conductivity than the full duration of test.

Novel method for determination of duration of unsteady state is presented by Kurevija et al [6] where 
derivation curve, method from well testing in petroleum engineering, is applied on TRT data. It is 
based on relation between change of temperature in time vs. time ($\Delta T/\Delta t$ vs. $t$) and following chart 
precisely shows time after which change of temperature below 0.25°C/10min which is arbitrarily 
chosen value. Such determination of the start of steady state heat transfer and segregation of data 
which will be used for calculating thermal properties is much more accurate that well known from 
Gehlin [7], in which is necessary to assume value of thermal diffusivity, $\alpha$, and in that way increasing 
the possibility of mistake.

Badenes et al [8] investigated influence of operational parameters during TRT distinguishing the 
importance of heat injection control. Research presents how by using PID controller decreases the 
impact of environmental temperature fluctuations, fluctuations of inlet and outlet temperature are 
negligible and values of thermal conductivity and borehole thermal resistance are closer to values 
standard deviation of input power, directly affecting injected heat from the electric heaters, should 
be less than $\pm$1.5% of the average value with peaks less than $\pm$10% of the average. Electric heaters 
can be provided with electrical power either from local electrical grid or from the generator. Demand 
for electrical energy changes during the day according to habits and obligations in each of the 
household but mostly regular pattern can be found. Peak of the demand happens during the morning 
where huge amount of energy has to be brought to users mostly for the need of water-heaters, around 
7-9 AM. Other peak usually happens between 4 and 9 PM [9].

Henk J.L. Witte [10] in his research about possible errors of TRT analysis asserts that average 
temperature of the fluid, mostly calculated as arithmetic mean of the entering source temperature 
and leaving source temperature, is correct only when the heat flux is constant along the entire 
borehole. That confirms the importance of reaching the semi-steady heat transfer. From various 
possibility of errors and influences of parameters it important to remark that according to assumption 
of ILS, conduction of the heat around the borehole heat exchanger is the only way of heat transport. 
But classical TRT use heat injection at high rates. ASHRAE Handbook [1] recommends rates from 50- 
80 W/m as those are expected peak loads of the actual heat pump system, thermally induced 
convection can occur which doesn’t fit with ILS assumptions.

Unexpected events, such as electrical power outage, can interrupt TRT before sufficient duration is 
reached to estimate thermal properties of the ground from measured data. ASHRAE Handbook [1] 
recommends that in that case loop temperature should be allowed to return within 0.3 °C of pretest 
initial ground temperature. It is assumed it will require 10 to 12 days in mid-t-high-conductivity 
formations and 14 days in low-conductivity formations in case if 48 hr test has been performed.

3. Theoretical background

3.1. Solutions of the diffusivity equations for the case of infinite medium and line source well

The TRT analysis in most cases is based on assumption that borehole heat exchanger of sufficient 
length and negligible radius can be regarded as a line source. In that case Kelvin’s line source theory 
analytical solution can be used to describe radial heat transfer and behavior of temperature in 
function of time and radius during the test [11][12]. Theory emanates from Fourier’s law which 
describes heat conduction in solids, which was later applied for solutions of pressure distribution 
during radial fluid flow in porous media (1), since both are derived from diffusivity partial equation. 
For a case of radial flow into well in homogenous, isotropic and porous medium, for a fluid of small
and constant compressibility, constant viscosity and small pressure gradients diffusivity equation can be written as:
\[ \frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} = \frac{\phi \mu c}{k} \frac{\partial p}{\partial t} \]
(1)
where \( \frac{k}{\phi \mu c} = \eta \) is hydraulic diffusivity factor in petroleum engineering.

For a case of a heat conduction:
\[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} = \frac{c_r \rho}{\lambda} \frac{\partial T}{\partial t} \]
(2)
where \( \frac{\lambda}{c_r \rho} = \alpha \) describes thermal diffusivity factor in thermogeology.

Due to same theoretical origin, it is possible to correlate some practical methods from petroleum engineering with thermogeology engineering. From that assumption arise aim of this research: to implement novel improved TRT method based on the petroleum engineering well testing methods. Since these methods are based on the same model of diffusivity equation for describing the behavior of pressure in function of time and radius during pressure build-up test, analogously behavior of temperature in borehole heat exchanger during TRT temperature recovery phase could also be described.

Solution to the diffusivity equation for the line source theory is well known in petroleum engineering and utilized almost a century for pressure transient analysis in the drawdown well testing:
\[ p(r,t) = p_i - \frac{Q \mu}{2\pi kh} \left[ \frac{1}{2} Ei\left(-\frac{\phi \mu c r^2}{4kt}\right) \right] \]
(3)

In geothermal applications line source theory is used for describing temperature change in function of time and borehole radius during heat exchange between borehole and surrounding ground.

For a case of heat extraction from the ground during heat pump heating phase:
\[ T(r_0,t)_{\text{ext}} = T_i + \frac{q'}{2\pi \lambda} \left[ -\frac{1}{2} Ei\left(\frac{r_0^2}{4\alpha t}\right) \right] = T_i - q' \frac{1}{4\pi \lambda} Ei\left(\frac{r_0^2}{4\alpha t}\right) \]
(4)

For a case of heat rejection to the ground during heat pump cooling phase or TRT:
\[ T(r_0,t)_{\text{req}} = T_i + \frac{q'}{2\pi \lambda} \left[ \frac{1}{2} Ei\left(\frac{r_0^2}{4\alpha t}\right) \right] = T_i + q' \frac{1}{4\pi \lambda} Ei\left(\frac{r_0^2}{4\alpha t}\right) \]
(5)

Term \( Ei \) represents exponential integral function with solution:
\[ -Ei(-x) = \frac{\text{e}^x}{x} \]
(6)

In cases where \( x < 0.02 \), or term \( \frac{4\alpha t}{r_w^2} > 50 \), exponential integral can be approximated with function of natural logarithm, with error less than 0.6% [13] as follows:
\[ -Ei(-x) \equiv -\ln(\gamma x) \equiv -\ln(e^x) \equiv \ln\left(\frac{1}{x}\right) - 0.5772 \]
(7)
where \( \gamma \) represents Euler’s constant equal to 0.5772 and \( e \) Euler’s number equal to 2,7183.

Considering this, equation (4) can be derived for heat extraction from the ground as follows:
$$T(r_w,t)_{aw} = T_i + \frac{q'}{4\pi\lambda} \ln \left(e^{r_w^2/4\alpha t} + \frac{r_w^2}{4\alpha t} + 0.5772\right) = T_i + \frac{q'}{4\pi\lambda} \ln \left(\frac{r_w^2}{\alpha t} - 0.8097\right)$$  \hspace{1cm} (8)

Analogously, for heat rejection to the ground equation (5) becomes:

$$T(r_w,t)_{aw} = T_i - \frac{q'}{4\pi\lambda} \ln \left(e^{r_w^2/4\alpha t} + \frac{r_w^2}{4\alpha t} + 0.5772\right) = T_i - \frac{q'}{4\pi\lambda} \ln \left(\frac{r_w^2}{\alpha t} - 0.8097\right)$$  \hspace{1cm} (9)

Where \( \kappa = \frac{q'}{4\pi\lambda} \) is the slope of the line when plotting \( T \) vs. \( \ln(t) \) and it is standardized principle to obtain effective ground thermal conductivity.

3.2. Application of Horner’s pressure build-up method in applied thermogeology and TRT

Analogy of the classic TRT is equal to the flow test in petroleum engineering. However, flow tests are rarely used for the determination of formation properties because of the long periods acquired to reach fully developed transient flow region. Pressure buildup test is the most frequently used test to determine formation permeability (equivalent to thermal conductivity) and near-well damaged formation or skin factor (equivalent to borehole thermal resistance). Considering similarities in origin of equations for describing behavior of pressure and temperature in function of time and radius arises the hypothesis for suggested novel method of TRT: “thermal properties of the ground and borehole thermal resistance (skin) could be also estimated from observing temperature recovery period after classical TRT, which is analog to observing pressure build-up (recovery) after constant flow rate in petroleum well testing”.

Method of petroleum pressure build-up test consists of constant rate of production and achieved stabilized bottom-hole pressure for certain time after which well is closed from the surface allowing pressure to build up – recover. During recovery phase bottom-hole pressure is monitored in a function of the time. Acquired data is used to estimate physical properties of the reservoir and skin, indicator of damage and disturbance of initial near-wellbore formation permeability [13].
Interpretation of collected pressure data is most often performed with Horner’s method [14], where measured pressure and \((\Delta p + \Delta \Delta t) / \Delta t\) are plotted on semi-log paper (Figure 1). Term \(t_p\) is duration of constant production rate before well shut-in, while \(\Delta t\) is time following well shut-in. Typical Horner method procedure is to analyze data and identify where curve becomes linear on semi-log graph, as this is period of special interest from which slope of the curve, \(m\), could then be determined. This factor describes pressure change (or temperature change during TRT) for a one logarithmic cycle of time. Curve from semilog paper can be arbitrary and logically divided in three regions, Early Time Region (ETR), Middle Time Region (MRT) and Late Time Region (LTR). In petroleum well testing, ETR is affected with altered (damaged) permeability of near–well zone and afterflow effect from reservoir into the wellbore after shut-in. In thermogeology and particularly TRT analysis, ETR describes time necessary to accomplish semi-steady heat flow state from borehole heat exchanger to surrounding ground. Like in petroleum well testing, initial data distortion in form of unsteady state heat flow occurs under influence of borehole thermal resistances; primarily because of low conductivity of pipe material and grout properties, as described in subsection 3.4.

MTR period starts when radius of investigation overcomes the influence of altered zone near test wells or with beginning of semi-steady state heat flow in TRT. Accurate determination of this period is crucial for valid interpretation of measured data. In MTR, slope of the straight line, \(m\), is directly dependent on effective permeability of formation in petroleum engineering, or in case of thermogeology slope of the line is dependent on effective thermal conductivity of the ground. Absolute change in pressure or temperature for a one log cycle, following Figure 1., can be presented as:

\[
m = 2,303 \frac{Q_B \mu}{4 \pi k h}
\]

in petroleum well testing (10) and,

\[
m = 2,303 \frac{q^*}{4 \pi \lambda} \Rightarrow \lambda = 2,303 \frac{q^*}{4 \pi m}
\]

in thermogeology and TRT interpretation (11).
Constant factor 2,303 is the result of transferring linear source solution with natural logarithm into decimal logarithm required for semilog paper in Horner's method:

\[
\log_e x = \frac{\log_{10} x}{\log_{10} e} = \frac{\log_{10} x}{0.4343} \quad \text{therefore} \quad \ln(x) = 2.303 \log(x) \quad (12)
\]

Straight line from MRT will continue until radius of investigation reaches natural reservoir boundaries or interference effects of pressure with other wells from the oil or geothermal reservoir appear. After this moment deviation of pressure from straight line can be again observed. In applied thermogeology and thermal response test, MRT period will also cease after interferences due to intersection of heat flow lines from other surrounding borehole heat exchangers appear. As seen from Figure 1., by extrapolating the linear part of the curve to the value of \((t_p + \Delta t)/\Delta t = 1\), related value of pressure will be equal to initial, undisturbed static pressure in the reservoir. By following same principal of investigation, initial effective temperature alongside the borehole heat exchanger can be determined.

Describing pressure behavior curve during multi-flow rates of production in petroleum engineering is based on mathematical technique called principle of superposition. Considering the solution of line source theory for pressure behavior during pressure drawdown or flow test (3) and constant flow rate, \(q\), reduced to time of interest with approximation of value of exponential integral (7), bottom-hole pressure drop is:

\[
p_i - p_{sof} = -\frac{Q \mu}{4 \pi k h} \left[ \ln \left( \frac{e^{r^2}}{4 \eta t_r} \right) \right] \quad (13), \text{after the end of production period } t_p.
\]

By shut-in the well for a time \(\Delta t\) after production period \(t_p\), required for performing pressure build up test by Horner's method (F(14),g.2.), pressure drop at time \(\Delta t\) could be obtained by the principle of superposition as:

\[
p_i - p_{sof} = (\text{pressure drop caused by rate } Q \text{ for the time } t_p + \Delta t) + (\text{pressure drop caused by rate change } -Q \text{ for the time } \Delta t)
\]

(-Q) for the time \(\Delta t\)

Pressure \(p_{sof}\) can be replaced with static pressure \(p_{stat}\) at certain time \(\Delta t\) after closing the well, therefore according to Horner method corresponding equation for pressure build-up becomes:

\[
p_i - p_{stat} = -\frac{Q \mu}{4 \pi k h} \left[ \ln \left( \frac{e^{r^2}}{4 \eta (t_r + \Delta t)} \right) \right] + \frac{Q \mu}{4 \pi k h} \left[ \ln \left( \frac{e^{r^2}}{4 \eta \Delta t} \right) \right], \quad (14) \text{ therefore}
\]

\[
p_{stat} = p_i - \frac{q \mu}{4 \pi k h} \ln \left( \frac{t_i + \Delta t}{\Delta t} \right) \quad (15), \text{which is the basic pressure buildup equation.}
\]

Considering the required decimal logarithm for plotting in Horner's semi-log paper, as shown in subsection 3.2., final form of equation (15) becomes:

\[
p_{stat} = p_i - 2,303 \frac{q \mu}{4 \pi k h} \log \left( \frac{t_i + \Delta t}{\Delta t} \right) \quad (16)
\]

This equation confirms that if pressure \(p_{stat}\) observed during a shut in period is plotted versus logarithm of \((t_i + \Delta t)/\Delta t\) straight line should be obtained (Figure 1.). Correspondent to that, TRT period is equal to pressure drawdown test and injector test (Figure 2.), quadrant a1,a2 and c1/c2). After TRT is performed with constant heat rate \(q\) during time \(t_r\), turning off the heaters and changing the heat rate for \((-q)\) for the time \(t_i + \Delta t\), temperature recovery (equal to pressure build-up test and fall-off test) (Figure 2. quadrant b1,b2 and d1,d2) can be derived as:
\[ \Delta T_{EST} = -2.303 \frac{q'}{4 \pi \lambda} \log \left( \frac{t_p + \Delta t}{t_p} \right) = -m \log \left( \frac{t_p + \Delta t}{t_p} \right) \quad (17) \]

Figure 2. Display of corresponding behavior of pressure/temperature during different types of well tests/thermal response tests

3.3. Application of petroleum engineering derivation curves on TRT interpretation

As mentioned, precise determination of MTR period threshold is crucial for valid interpretation of measured data, either to determine formation permeability from pressure data or ground conductivity from temperature data. In thermogeology, currently most used method to determine beginning of semi-steady heat flow is by using formula [15][7]:

\[ t > \frac{5r^2}{\alpha} \text{ with 10% error, or } t > \frac{20r^2}{\alpha} \text{ with 2% error} \]

In the nominator this equation contains a value of thermal diffusivity, \( \alpha \), which is assumed from the drilling data or catalogue values for different types of soil. Since this is not the exact value, this method of determining the duration of the transition period can cause further error in interpretation, especially for highly heterogeneous ground. We suggest much more accurate grapho-analytical method, so called derivation curve principle, which is widely used in petroleum engineering. In TRT interpretation it could be used to precisely estimate transition from ETR into MTR period, or transition from unsteady state heat flow to relevant semi-steady state heat flow regime. Typical shape of derivation curve for a TRT is shown on Figure 3., where each data point is derivate as:

\[ \left( \frac{dy}{dx} \right)_i = \frac{\left( \frac{y_2 - y_1}{x_2 - x_1} \right)}{(x_i + x_j)} \quad (18) \]
As seen from Figure 3., it can be argued that MTR period obviously appears for a value of \(\Delta t\) of approximately 0.25°C/\(\Delta t\). This threshold between ETR and MTR could be used for any type of TRT procedure, independently of power rate or test duration. Small distortions on Figure 3., are usual in TRT when electric grid is used as a source of power, due to daily fluctuations of voltage.

![Derivation curve](image)

**Figure 3.** Determination of semi-steady heat flow period via derivation curve technique

Derivation curve could also be replaced with simple \(\Delta T/\Delta t\) principle. This method monitors segment of change in entering borehole fluid temperature versus some small segment of time. Duration of this segment during TRT should not be longer than 5 - 10 minutes, as this is roughly time of one flow cycle through the coaxial or 2U heat exchanger of 100m in depth. This method could be interpreted like real derivation curve because derivative of a dot on certain curve is tangent on that curve. If we are looking at change of \(\Delta T\) vs. \(\Delta t\) on curve, line that connects these two dots is secant. If \(\Delta T\) versus \(\Delta t\) is small enough, then secant and tangent fall almost in the same line, therefore making change of temperature in time so small it can be neglected.

### 3.4. Interpretation of borehole thermal resistance analogue to well testing skin effect

Inclined coaxial heat exchanger, as a pipe in pipe type of system, is used as the geothermal source for this research. There are two principal cases of fluid circulation; CXA where inlet of fluid is through annular space and outlet through central pipe and CXC with inlet of fluid through central pipe and outlet through annular space. Coaxial borehole heat exchanger consists of three components (Figure 4.): one pipe-in (marked as \(i1\)), one pipe-out (marked as \(o1\)) and grout material considered as only one zone (marked as \(g1\)) [16]. In the case of CXA arrangement, heat exchange to the grout material \(g1\), which is in contact with soil \(s\), is only performed through the pipe-in \(i1\), while the pipe-in \(i1\) exchanges heat only with the pipe-out \(o1\) component. For a case of CXC arrangement, heat exchange to the grout material \(g1\), is only performed through the pipe-out \(o1\), while the pipe-out \(o1\) only exchanges heat with the pipe-in \(i1\).

According to Diersch et al [17], it is possible to divide thermal resistance for case of coaxial borehole heat exchangers in three components:

1. Thermal resistance due the advective heat transport in the pipes and between pipes (\(R_0\))
2. Thermal resistance due to pipes wall material and grout transition (\(R_{0g}, R_{gs}\))
3. Thermal resistance due to grout – soil exchange (\(R_{gs}\))
Thermal resistances distribution is presented in Figure 4. for both cases of coaxial borehole heat exchanger flow setting.

![Figure 4. Components of borehole thermal resistance for a) CXA arrangement b) CXC arrangement](image)

In petroleum engineering, permeability of the formation near the wellbore is reduced as a result of invasion of drilling fluids into the permeable zone, dispersion of clays and presence of mudcake on wellbore wall [18]. The effect of reduction in near-well permeability can be taken into account as an additional pressure drop \( \Delta p \) proportional to the rate of production \( Q \). The zone of reduced permeability has been called “skin”, and the resulting effect a “skin effect”.

With knowing the value of original formation permeability (from pressure build-up tests) it is possible to calculate skin factor, which is a dimensionless factor for evaluating near-borehole damage and the cause of decreased permeability. Skin factor can be presented as an additional pressure drop [19]:

\[
\Delta p_{\text{skin}} = s \left( \frac{Q \mu}{2 \pi k h} \right) \tag{19}
\]

If the same principle is linked to applied thermogeology, skin could represent cumulative thermal resistances to heat flow inside the borehole, as described above for a case of coaxial heat exchanger, and can be expressed as initial temperature rise during TRT:

\[
\Delta T_{\text{skin}} = s \left( \frac{q'}{2 \pi \lambda} \right) = R_s \times q' \tag{20}
\]

After introducing equations (19) (20) into the equations (13) (8), well pressure or borehole heat exchanger temperature could be expressed after a producing time \( t_p \) for a pressure drawdown test or extraction of heat from the ground:

\[
p_{\text{ex}} = p_i + \frac{Q \mu}{4 \pi k h} \ln \left( \frac{e^{r_u^2}}{4 \pi \eta} \right) - \Delta p_{\text{skin}} = p_i + \frac{Q \mu}{4 \pi k h} \left\{ \ln \left( \frac{e^{r_u^2}}{4 \pi \eta} \right) - 2s \right\} \tag{21}
\]

\[
T_{\text{ex}} = T_i + \frac{q'}{4 \pi \lambda} \ln \left( \frac{e^{r_u^2}}{4 \alpha \eta} + s \right) - \Delta T_{\text{skin}} = T_i + \frac{q'}{4 \pi \lambda} \ln \left( \frac{e^{r_u^2}}{4 \alpha \eta} \ln \frac{r_u^2}{0.8097 - 2s} \right) \tag{22}
\]

For a case of injector well test in petroleum engineering or heat rejection to the ground during TRT:

\[
p_{\text{ex}} = p_i - \frac{Q \mu}{4 \pi k h} \ln \left( \frac{e^{r_u^2}}{4 \pi \eta} \right) + \Delta p_{\text{skin}} = p_i - \frac{Q \mu}{4 \pi k h} \left\{ \ln \left( \frac{e^{r_u^2}}{4 \pi \eta} \right) - 2s \right\} \tag{23}
\]

\[
T_{\text{ex}} = T_i - \frac{q'}{4 \pi \lambda} \ln \left( \frac{e^{r_u^2}}{4 \alpha \eta} \right) + \Delta T_{\text{skin}} = T_i - \frac{q'}{4 \pi \lambda} \ln \left( \frac{e^{r_u^2}}{4 \alpha \eta} + s \right) \tag{24}
\]
In the case of heat rejection to the ground (classic TRT), skin effect could be calculated from equation (24) for a case of initial conditions and conditions just before heaters shut-in.

\[
s = \frac{1}{2} \ln \left( e^{r_w^2} \frac{(T_i - T_{p})}{4\alpha} \right) - \frac{2q_{r}^{'}\Delta t}{q'} \quad (25)
\]

The order of magnitude of the skin effect (or effective borehole thermal resistance) for a pressure build-up test or TRT recovery period, can be estimated from the difference between the pressure or fluid temperature before shut-in and the one shortly after. By combining equation (21)/(22), which gives the pressure/temperature before shut-in, with equation (16)/(17) which gives the pressure/temperature after shut-in, it can be found that:

\[
T_i - T_p = -\frac{q_{r}^{'}\Delta t}{4\alpha} \ln \left( e^{r_w^2} \frac{(t_{p} + \Delta t)}{4\alpha} \right) - 2s \quad (26)
\]

For \( \Delta t \) small compared to \( t_p \) it could be approximated that \((t_{p} + \Delta t)/t_p = 1\). Rearranging equation (25) and arbitrary choosing that \( \Delta t = 1 \) hour so the \( T_i = T_{hr} \) it could be written [18]:

\[
s = 1,151 \left[ \frac{T_{hr} - T_p}{m} + \log \left( e^{r_w^2} \frac{r^2}{4\alpha} \right) \right] = 1,151 \left[ \frac{T_{hr} - T_p}{m} + \log \left( \frac{r^2}{\alpha} \right) - 0,351 \right] \quad (27)
\]

This equation is one of the most important expressions in well testing. In this shape it could be also used to define skin effect during TRT recovery period and to compare it with skin derived from classic TRT (equation 25). Value of \( T_{hr} \) is not strictly set, changing value of \( \Delta t \) in equation to different value than 1 hour would merely change constant 0.351 in equation (27).

In ideal case this two values should be the same but due to fluctuations in voltage during classic TRT and different ambient thermal interferences for two periods they are slightly different (different heat flux depending on temperature difference between air temperature and borehole temperature during measurement).

To describe entire curve for a two steps (TRT period + TRT recovery) including borehole thermal resistances, principle of superposition applies. For a first TRT period general equation (5) could be used with addition of temperature rise term, due to borehole skin:

\[
T(r_w, t)_{R_1} = T_i + q_{r}^{'} \left[ Ei \left( \frac{r_w^2}{4\alpha t} \right) \right] + \Delta T_{dis} \quad (28)
\]

To fit second part of the curve (TRT recovery period) superposition technique is applied:

\[
T(r_w, t)_{R_2} = T_i + q_{r}^{'} \left[ Ei \left( \frac{r_w^2}{4\alpha t} \right) \right] - q_{r}^{'} \left[ Ei \left( \frac{r_w^2}{4\alpha (\Delta t)} \right) \right] \quad (29)
\]

Where \( \Delta t \) is difference between cumulative time in certain point and \( t_f \) as a time of first period.

4. Experimental site setting

The ground TR tests were conducted on the coaxial borehole heat exchanger system in the city of Zagreb/Croatia, at the Faculty of Geology, Mining and Petroleum Engineering. Installation serves as the testing heat exchanger for students.

System comprise of two boreholes, each with length of 50m, hydraulically connected in series to provide effectively one borehole of 100m in length. Boreholes were drilled with standard diameter of 110 mm, while drilling was performed with specialized equipment that allows setting of the drilling angle from 35° to 65° and in all directions (Figure 5b.). Each borehole has angle of 45°, being placed oppositely to one another, and inside the polyethylene shaft with diameter of 1 m and depth of 1 m. (Figure 5c.). Coaxial heat exchangers is set out of an outer polyethylene pipe of 63 mm with standard
dimension ratio SDR11, where the inner polyethylene pipe is 32 with SDR11. Thermal response
testing was conducted on both possible flow arrangements, CXC and CXA setup (see Section 3.4.).
Cementing with thermally enhanced grout was not possible due to total losses of material into the
high-permeable gravel layer. Therefore, borehole was cemented with mixture of water, bentonite and
cement, with somewhat lower thermal conductivity of 1.2 W/m K, measured with needle probe
method.

Detailed geological setting of location, and the city of Zagreb in general, consists of Middle and
Upper Pleistocene sediments, where lateral changes of gravel, silt, sand and clay are frequent, and
Holocene sediments which consist of yellow-brown gravel, sand and limestone pebbles. Faculty
location is set in the northern part of Zagreb aquifer just near the outer boundary, with thin aquifer
layer present at the depth between 6.5 and 12.0 m beneath the surface. Lithological profile from
drilling data is shown on Figure 5f. Undisturbed ground temperature and geothermal gradient at the
Zagreb location were investigated in our previous research [6], [20] with geothermal gradient
corresponding to 5.5 °C per 100 m of depth while undisturbed ground temperature is approximately
14.5°C at the 10m of depth.

As seen from Figure 5e., geometry of the inclined coaxial system implies that final depth of
boreholes is 35 m beneath the surface. Since aquifer is present in shallow thin layer with thickness of
5.5 m, cumulative length of the pipes being affected by additional convective heat transfer from
ground water is 16 m out of 100 m in total. From regional hydrogeological research data, hydraulic
conductivity of the aquifer near the outer boundary is only 0.3 cm/s [21], therefore convective
component of heat transfer was neglected in thermal response test.

Measurements on coaxial heat exchanger were conducted from September-2016 until June-2017
with TRT apparatus Geocube GC500. Equipment has maximum available power of electric heaters
of 9.0 kW @ 240V. Internal logger collects 5-min interval data about inlet and outlet fluid temperature,
air temperature, flow, voltage and electric current. Sealed temperature sensors (resistance
temperature detectors - RTD) on inlet and outlet connection have accuracy of ±0.2 °C from 0°C to 50
°C.

Testing procedure was organized as a classic TRT heat rejection step with duration of 96 hr,
followed each time by a recovery period of the additional 96 hr (only circulation). Five different heat
steps were used: 35, 42, 54, 61 and 71 W/m for each of the two possible flow setups, CXC and CXA.
Ultimately, total testing time on the coaxial heat exchanger was 192 hours for each of the ten different
conditions, making it cumulatively 1920 hours of data with 5min logging interval. After each testing
condition pause was made for a duration of seven days, allowing recovery of ground and borehole
fluid temperature to static initial conditions. Entire collected TRT raw data can be

Unlike classic 2U-loop vertical heat exchangers of 100 m in depth, coaxial heat exchangers exploit
much more shallow heat source, with depths only up to 35 m. Hence, initial temperature conditions
due to different climate seasons are much more affected from the surface air temperature for this kind
of installation. Also, in practical installations coaxial heat exchangers are densely radially drilled from
a single shaft, causing much more thermal interference between adjacent boreholes in a first few
meters of depth.
Table 1. represents results of MS Excel descriptive statistics related to TRT heating power, for each of the ten different heat steps; five for CXC flow arrangement and five for CXA arrangement. The acceptable maximum standard deviation for input power is +/-1.5% from the average power level, according to ASHRAE standard 1118-TRP. Furthermore, peak variations must be kept at less than 10% of the average power level. It could be seen from test data that entire power data fits inside the provided industry guidelines, with largest deviations seen for highest power steps, as expected. Table 2. gives descriptive statistics results related to air temperature measurements for an 1-hour step. Entire test procedure prolonged over six-month period due to ~2000hr of testing, in addition plus one-week waiting time for ground to return to initial conditions. Therefore, it could be seen that mean air temperatures data are scattered for each of the heat steps, especially minimum and maximum values. This also affects initial temperature conditions for such shallow geothermal installation in some degree, since solar perturbation affects ground up to 10m in depth for test location and boreholes final depth are 35m (Figure 5e.).
there was a fully developed turbulent regime in both the annular space (Re = 7100) and column pipe (D63SDR11/D32SDR11), volume flow and velocity, viscosity and density of water and pipe roughness during 30 minutes of solely fluid circulation. Flow for each of the ten steps was set to 0.42 l/s and for each step classic TRT analysis was performed during heat power condition. Before turning on the electric heaters initial borehole temperature was recorded during 30 minutes of solely fluid circulation. Flow for each of the ten steps was set to 0.42 l/s and circulating fluid was pure water. Considering coaxial pipe arrangement and dimensions (D63SDR11/D32SDR11), volume flow and velocity, viscosity and density of water and pipe roughness there was a fully developed turbulent regime in both the annular space (Re = 7100) and column pipe (Re=23000).

5. Results and Discussion

As explained in section 4. TRT + recovery period was performed for ten heat steps and two different flow arrangement. For each of the steps classic TRT analysis was performed during heat power condition. Before turning on the electric heaters initial borehole temperature was recorded during 30 minutes of solely fluid circulation. Flow for each of the ten steps was set to 0.42 l/s and circulating fluid was pure water. Considering coaxial pipe arrangement and dimensions (D63SDR11/D32SDR11), volume flow and velocity, viscosity and density of water and pipe roughness there was a fully developed turbulent regime in both the annular space (Re = 7100) and column pipe (Re=23000).
Figure 6a. presents standard analysis for each step, where entering source temperature (EST), leaving source temperature (LST) and unit heat power is plotted in a function of test time (96 hr + 96 hr). Since TRT is usually performed trough the principle of heat rejection into the ground, an inversed mirror flow/return curves have been charted to represent the cycle when the heat pump would be in heating mode (subcooling the ground). This approach can give better presentation of working conditions during the extraction of heat from the ground (heat pump heating cycle for example).

To determine effective ground thermal conductivity for each of the ten analyzed cases, emersion time of the semi-steady state heat flow had to be identify from the initial unsteady state. Based on theory presented in section 3.3, derivation curves were created for each of ten TRT periods. Results are provided by Figure 7. It is evident that semi-steady state appears after ~10th hour of investigation where change of the temperature per unit time reaches 0,25 °C per 5min time step. Since most of the lithology column is made out of damp clay (except thin saturated gravel near surface) effective thermal diffusivity, according to catalogue soil data, could be anywhere between 0,030 – 0,060 m²/d, depending on the moist content. As a reasonable estimation we considered value of 0,050 m²/d in further analysis. If appearance of SS state is deduced from standardized equation by Mogensen presented in section 3.3., then corresponding time would be 7,5 hr for a borehole diameter of 110 mm. To achieve maximum accuracy, ground thermal conductivities were derived from interval 15-96 hr for all test conditions in TRT period and recovery period.
From Figure 6.a and 6.b procedure of deriving ground thermal conductivities could be seen. For a case of TRT period semi-log axes were used and borehole $T_{avg}$ vs. $t_p$ was plotted for SSS interval. Since semi-log plot is used, as stated by equation (10) there is a need to determine change of temperature for a one log cycle of time ($m$) and then calculate effective ground conductivity. Same result can be obtained by using standardized principle of plotting $T_{avg}$ vs. $\ln(t_p)$ on normal graph and then use equation (9) and slope of the line $\kappa$.

When analyzing recovery period, slightly different approach has to be used, as explained in section 3.2. Reversed semi-log graph has to be used and $T_{avg}$ is plotted vs. $(t_p+\Delta t)/\Delta t$, as shown by Figure 6.c. Effective ground thermal conductivity is then derived from equation (17) by knowing log slope $m$, just like in the case for a standard TRT period. When temperature recovery line is extended until $(t_p+\Delta t)/\Delta t = 1$ are initial conditions are reached, or undisturbed ground temperature. Figure 8. shows entire Horner procedure for all ten heat step conditions. When extending data trendline until $(t_p+\Delta t)/\Delta t = 1$ it could be seen that initial temperature value spreads between 14,0°C and 15,2°C. As mentioned before, reason for this is shallow installation of coaxial system with final depth of 35m, while solar perturbation progresses to the 10m of depth. Therefore, initial conditions values are somewhat dependent on climate season when measurements are taking place.

**Figure 7.** EST derivation curves for CXA and CXC flow arrangement and each TRT heat step

**Figure 8.** Horner semi-log method to determine ground thermal conductivity applied on TRT recovery period for each heat power step and flow regime
Since total duration of TRT is not strictly defined, only minimum time is required as explain in section 2., in practical field testing it always leads to certain degree of analysis error. Since final duration time is arbitrary chosen by TRT operator, different test times could lead to quite different thermal conductivities. Figure 9. shows analysis of defining thermal conductivity by variable time between 36–96hr. When looking at thick lines on both graphs, for CXC and CXA, it could be seen that choosing different TRT periods could lead to as much as 20% difference in final result. From a standpoint of modelling geothermal heat pump with multiple boreholes, such discrepancies could have significant impact on oversizing or undersizing geoexchange system and on technoeconomical benefit.

![Figure 9.](image)

**Figure 9.** Results of obtained ground thermal conductivity value for classic TRT and recovery period in a function of variable test time 36-96hr

Such difference in results is influenced primarily by two factors: fluctuations of voltage in public electrical grid, which is usual method of powering TRT, and interferences between surface equipment and air temperature, especially during winter months. As explained in section 2. fluctuations in voltage from public grid are regularly seen following recognizable pattern in 24-hr cycle, depending on specific demand during the day and night. Therefore, to at least try to minimize this effect on development of line slope, as shown in Figure 6b., TRT duration should follow 24 hr multiplication factor (i.e. 48, 72, 96 hr or 36, 60, 84 hr). The other major concern is interferences from ambient temperature to the TRT equipment and header pipes. As seen from Figure 5e. we used 3 m header pipes length from borehole shaft to the TRT equipment on purpose to magnify this effect, although entire setup was properly insulated with 12mm of caoutchouc rubber insulation. This effect could be noticed in higher conductivity deviation at CXC, oppose to CXA flow arrangement. Theoretically, flow direction arrangement does not have impact on conductivity measurement procedure, rather on borehole resistance, but CXC tests were carried out during autumn/winter months and CXA during spring months. Data shown in Table 2. conclusively point out to this claim and explaining reason for higher measured thermal conductivity for CXC compare to CXA tests.

Prolonging TRT procedure by conducting additional recovery period, and using Horner method to interpret data, could lead to higher accuracy in approaching actual ground thermal conductivity. This is clearly seen from Figure 9. where thermal conductivity obtained by Horner method from recovery period gives more symmetrical results, for both CXC and CXA setup. This is explained by the fact that recorded data is smooth since there is no heat power applied (coefficient of determination...
from 0.996 to 1.0 as seen from Figure 8.) and fact that during winter months interference heat flux between ambient and TRT equipment is lower due to lower temperature difference between air temperature and fluid temperature. Entire analysis presented with two-step TRT principle could give geothermal engineer more precise method to qualitatively interpret field data.

Furthermore, as stated in Section 3.4. both TRT period and the recovery period could be used to determine equivalent borehole resistance or skin effect, an equal important parameter for the efficient design of geothermal exchange system. By applying equations (24) and (25) on TRT data for each heat step, and equation (27) for Horner method of recovery period following values are obtained, as shown in Table 3. It can be noticed that CXA flow arrangement generally shows higher skin effect or equivalent borehole heat resistance (approximately +5%), which is in line with previous research in this field, as discussed in section 2.

<table>
<thead>
<tr>
<th>TRT Heat step, Mean</th>
<th>CXC</th>
<th>CXA</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRT period</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thermal conductivity @96hr, W/m K</td>
<td>70.9</td>
<td>70.8</td>
</tr>
<tr>
<td>Recovery period</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thermal conductivity @96hr, W/m K</td>
<td>2.62</td>
<td>2.50</td>
</tr>
<tr>
<td>Initial temperature, TRT circulation, °C</td>
<td>14.6</td>
<td>14.7</td>
</tr>
<tr>
<td>EST after 96 hr of TRT, °C</td>
<td>36.0</td>
<td>37.8</td>
</tr>
<tr>
<td>EST after 96 hr of Recovery period, °C</td>
<td>15.7</td>
<td>16.7</td>
</tr>
<tr>
<td>Initial temperature, Horner, °C</td>
<td>13.9</td>
<td>14.8</td>
</tr>
<tr>
<td>Borehole resistance, Horner, m K/W</td>
<td>0.138</td>
<td>0.135</td>
</tr>
<tr>
<td>m slope, Horner method °C/log cycle</td>
<td>5.6</td>
<td>6.2</td>
</tr>
<tr>
<td>skin factor, Horner method, °C</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>T1hr, Horner method, °C</td>
<td>25.1</td>
<td>27.0</td>
</tr>
<tr>
<td>Tp, Horner method, °C</td>
<td>36.0</td>
<td>37.7</td>
</tr>
</tbody>
</table>

Discrepancies in obtained thermal conductivity between TRT period and recovery period are rather high (TRT analysis is showing 10-20% higher values than the Horner method), as seen from Table 3 and Figure 9. We already explained causes for this phenomena but using simple statistical analysis, such as sum of squares of difference or SSD, could provide exactly which thermal conductivity coefficients are of statistical significance. Sum of Squares or Variation (SUMXMY2 function in MS Excel) is a statistical technique used in regression analysis to determine the dispersion of data points. In a regression analysis, the goal is to determine how well a data series (in this case measured EST) can be fitted to a function which might help to explain how the data series was generated (in this case ILS with $E_i$ function with different thermal conductivities). The sum of squares is used as a mathematical way to find the function which best fits (varies least) from the measured data.

Procedure was carried out for every heat step separately with its obtained two values of thermal conductivity factor; one from TRT period and one from recovery period. Then entire curve was fitted with equation (28) for first step period, and equation (29) for second step period. Example is shown on Figure 10. for a case of CXC and 71 W/m. In this way total of 20 points were obtained, each data set fitted with two $E_i$ function curves with two different thermal conductivities. Results are presented in Figure 11a.
Same kind of variation analysis was performed with EST hourly measured data and hourly air temperatures for each period (TRT or recovery) and heat step. Results are shown in Figure 11b. were associated thermal conductivities were plotted against sum of squares between EST and $T_{air}$.

![Graph showing temperature vs. hours with trendlines and data points.](image)

**Figure 10.** Example of fitting measured EST data with infinite line source equation and two different values of thermal conductivity (one obtained from TRT period and second from recovery period)

![Graph showing sum of squares differences vs. effective thermal conductivity.](image)

**Figure 11.** Results of sum of squares of differences analysis: a) SUMXMY2 between ILS fitted curve and EST measured data; b) SUMXMY2 between air temperature and EST measured data

Both statistical analysis from Figure 11 shows trendline were for value of SUMXMY2 = 0 ground thermal conductivity coefficient is in range between 2.0 – 2.2 W/m K. This is in fact ground thermal conductivity which was obtained by most of the tests conducted in recovery period, as seen from Figure 9. Such results confirm hypothesis of this paper that prolonged TRT should always be applied to give more accurate results in thermal conductivity measurement. Test should consist of classic TRT test and corresponding recovery test of same duration, interpreted with Horner technique derived from petroleum well testing experiences. In this way errors caused by voltage fluctuations if public electric grid is used, and interferences from ambient climate, could be reduced.
References


**Nomenclature**

- \( p(r,t) \) – pressure in function of time and radius (Pa)
- \( p_i \) – initial pressure (Pa)
- \( p_{wf} \) – bottom hole flow pressure (Pa)
- \( p_{ws} \) – bottom hole static pressure (Pa)
- \( r \) – radius (m)
- \( r_c \) – wellbore radius (m)
- \( \phi \) – porosity, fraction
- \( \mu \) – viscosity (Pa * s)
- \( c \) – compressibility (Pa\(^{-1}\))
- \( k \) – permeability (m\(^2\))
- \( t \) – time (h)
- \( t_p \) – duration of constant production rate (h)
- \( \Delta t \) – shut-in time (h)
- \( \eta \) – hydraulic diffusivity factor
- \( \rho \) – density of formation (kg / m\(^3\))
- \( T(r,t) \) – temperature in function of radius and time (°C)
- \( T_i \) – initial borehole temperature (°C)
- \( T_{ent} \) – entering source temperature (°C)
- \( T_{rej} \) – rejected temperature into the ground (°C)
- \( \alpha \) – soil thermal diffusivity (m\(^2\) / h)
- \( \lambda \) – soil thermal conductivity (W / mK)
- \( Q \) – rate of production (m\(^3\))
- \( q^t \) – heat power per meter of borehole (W / m)
- \( Ei \) – exponential integral
- \( \gamma \) – Euler’s constant (0.5572)
- \( u \) – integral parameter
- \( B \) – formation volume factor (m\(^3\) / m\(^3\))
- \( s \) – skin factor, dimensionless
- \( Rb \) – equivalent borehole resistance (m K / W)