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A self-consistent physical model of the bubbles in a gas solid two-phase flow

H.M. Dong [1,†]**, J.F. He** [2]**, C.L. Duan** [2] **and Y.M. Zhao** [2,*]

[1] School of Physics, China University of Mining and Technology, Xuzhou 221116, P. R. China
[2] Key Laboratory of Coal Processing and Efficient Utilization, Ministry of Education, China University of Mining and Technology, Xuzhou 221116, P. R. China
[*] Correspondence: hmdong@cumt.edu.cn, ymzhao@cumt.edu.cn; Tel:+86-051683591580

Abstract: In this work, we develop a self-consistent physical model of bubbles in a gas solid two-phase flow. Using the Peng-Robonson state equation and a detailed specific heat ratio equation of bubbles, we obtain the kinetic equations of the bubbles on the basis of the Ergun equation, thermodynamic equations, and kinetic equations. It is found that the specific heat ratio of bubbles in such systems strongly depends on bubble pressures and temperatures, which play an important role in the characteristics of the bubbles. The theoretical studies show that with increasing height in the systems, the gas flow rate shows a downward trend. Moreover, the larger particles in the gas solid flows are, the greater the gas velocity is. The bubble sizes increase with the increasing heights of the gas solid systems, and then decrease. The bubble velocity is affected by the gas velocity and the bubble size, which gradually increase and eventually quasi-stabilize. This shows that gas and solid phases in a gas solid two-phase flow interact with each other and a self-consistent system comes into being. The theoretical results have exhibited important value as a guide for understanding the properties and effects of bubbles in gas solid two-phase flows.

Keywords: self-consistent physical model; bubbles; gas solid two-phase flow

1. Introduction

Gas solid two-phase flow is a complex flow system composed of gas and solid particles, and represents an important branch of fluid dynamics. Gas solid two-phase flows exist widely in nature and industrial productions. In this context, gas solid two-phase fluidization separation, for example, has been extensively researched and developed, with a wide range of applications and high selection efficiency. It is an efficient separation method that has been gradually applied to industrial production [1,2]. With gas article fluidization separation, the system is filled with uniform air flows in the fine particulate matter media bed, which creates particle medium fluidization and forms the gas solid suspended matter with certain density and fluid properties. The theoretical analysis of the two-phase flow is much more difficult than for single-phase flow because the general differential equations describing the two-phase flow have not yet been established. Generally, a simplified two-phase theoretical model can be applied in such systems. One of the models considers that the system is a continuous medium which is a mixture of two phases, where the concepts and methods of single-phase flow are still suitable for two-phase flow. The other is called the separated model, which considers that the concept and method of the single-phase flow can be respectively used in each phase of the two-phase system, while the interaction between the two phases is included [3,4]. The second type of model has been widely associated with physical applications [5].

In the gas solid two-phase flows, the mixture of the gas and the particles is not uniform and time-dependent, with the two phases existing in the gas solid two-phase flows (see Fig.1). One is a continuous phase, composed of gas and particles, and is called the dense phase or emulsion phase; the other is a discontinuous phase appearing in the bubble state, and is called the bubble phase. When the quantity of gas flowing in the gas solid system is greater than the air flow required for the critical

fluidized state, the excess gas can be in the form of bubbles. Bubble dynamic behavior is responsible for the most basic characteristics and phenomena in the gas solid two-phase flow system. This is significant for the understanding and study of the properties of gas solid two-phase flow systems [4]. The study of bubbles in gas solid two-phase flow forms an important part of the field of fluidization state [6]. The pressure drop, height, swelling degree, uniform mixing of the gas-solid two-phase flow, and stratified separation of particles in the gas-solid two-phase system strongly depend on the bubbles. The bubble size and rising velocity are important factors which affect the fluidization stability, the dynamics of particles, and the separation densities of fluidized beds [7]. The bubble phase and the dense phase transmit the heat and mass to each other, and then the gas-solid two-phase flows are not uniform. The growth of bubbles in the gas-solid two-phase flow can seriously affect the uniform stability of fluidization, affect the distribution of solid phase particles, and affect the state of the whole two-phase flow. As a result, research into bubble properties in the gas-solid two-phase flow system represent some of the most important and basic studies in the field [8]. Van Lare et al. developed a statistical theory to study the properties of bubbles in gas-solid systems using capacitance probe measurement experiments, and obtained consistent results between the experiments and theories [9]. Ichiki et al. studied the bubble phase and slugging phase in a fluidized bed by numerical simulations. It was found that the convective motion reveals a stronger effect on the bubble phase than on the slugging phase [10]. Farshi et al. calculated the size of bubbles in the gas-solid fluidized bed, and found that the Mori-Wen-Rowe equation is a better choice for the study of the bubble size, as compared with the other theories [11]. Through the optical experimental study, Valverde et al. found that bubbles begin to be produced after fluidization of the particles, and the interactions between the hydro-kinetic force and the particles can suppress the generation of bubbles for sufficiently large particles in the two-phase flow [12]. Glasser et al. studied the relationship between the bubbles and the clusters in the gas-solid two-phase flow and found that they are similar phenomenons in the s systems [4]. Muller measured the bubble phenomenon in a two-phase system using magnetic resonance imaging experiment in real time for the first time, and studied the bubble size, velocity, and so on in detail [13]. The nonlinear characteristics of the gas-solid two-phase flow and the relationship with bubbles were studied by Homsy et al. [14]. Zahra researched the nonlinear dynamic characteristics of gas-solid system using the two phase theory[15]. Komatsu and Hayakawa investigated the nonlinear dynamic characteristics of the fluidized states. Their results show that solitons, like waves, exist in the system and play an important role in the system. This indicates that the nonlinear characteristic has an important role and influence on the systems[16]. At the South Korean Energy Research Institute, the bubble properties of the gas-solid fluidized bed system were studied by Choi et al. using electronic resistivity probe experiments, and a change in bubble size with height was obtained [17]. Al-Zahrani and Daous established a simplified model to predict the average velocity of bubbles rising [18].

The bubble phenomenon represents one of the most basic physical phenomena of the gas-solid two-phase flow system, and relevant scholars worldwide have paid much attention to the research of bubbles in the gas-solid two-phase flow. It is noted that at present, studies mainly focus on the improvement and research of productive processes in the actual systems. However, basic theoretical research is relative lacking [19]. The high-speed dynamic systems have been widely used to analyze the bubble behaviors of the gas-solid system in experiments. With limited experimental approaches, we can observe and analyze the behavior of the system at the boundary surface but we cannot go deep into the internal system for detailed analysis and research. Based on the experimental results, the relatively simplified models can be fitted out, and are usually based on fluid mechanics or continuum medium theory. The numerical simulation can also be used to analyze the motion characteristics of bubbles. Due to a certain gap between the numerical simulation models and the existing experimental conditions, the simulation results can only partly explain the experimental results. In this work, we hope to understand the the gas-solid two-phase flow system more from the point of view of physics. Using thermodynamics and fluid mechanics theory, we establish a new bubble physical model. The dynamic properties of bubbles in the gas-solid two-phase flow system as shown in Fig. 1 are studied using the

self-consistent theoretical method. Through establishing and researching the bubble thermodynamic equation, we examine the properties of bubbles in the gas-solid two-phase flow system on the basis of Ergun and bubble dynamics equations.

2. Theoretical model

The state equation of the general thermodynamic systems can be expressed as $\rho = \rho(P,T)$ or $f(\rho, P, T) = 0$, where ρ is the density of a system, P is the pressure, and T is the system temperature. The state equation is very important for general thermodynamic systems. With this equation, the other thermodynamic properties which can not be measured by experiments directly can be obtained. The ideal gas state equation can be represented as $\rho = P/RT$, while for the real gas (thermodynamic systems), the state of equation can be written as $\rho = P/Z(P,T)RT$. Here, R is the gas constant, and $Z(P,T)$ is called the compression coefficient, which indicates the deviation from the ideal gas state equation. The ideal gas model is a theoretical model which can be used as one of the criteria to measure the real gas state equations, which can be used for approximate estimations in a practical application. When the pressure is close to zero or the volume is close to infinity, any real gas state equations should be reduced to the ideal gas equation. A gas-solid two-phase flow is a complex thermodynamic system, and thus the theoretical model of the system simplified as an ideal system is no longer applicable.

2.1. PR equation and the heat capacity ratio of bubbles

None of the real thermodynamic systems conform completely with the law of ideal gas. The deviation depends on the pressure, the temperature, and the gas properties, especially with respect to the degree of difficulty of gas liquefactions. The gas-solid two-phase flow bubble systems contain not only gas interacting between air molecules, but also partial solid particles. With the change of ambient pressures and temperatures, the proportion of the particle phase and the gaseous phase varies. Therefore, the system is a non-ideal complex system, which needs the non-ideal state equation for study. The non-ideal gas state equation is widely used in engineering, and includes the Van der waals equation, the Redlich Kwong (RK) equation, the virial equation and so on. When the RK equation is used to calculate strong polar compounds, it causes large deviations and is thus rarely used to calculate liquid pressures, volumes, and temperatures. The virial equation is not a good match for the polar compounds, and gas-solid two-phase flow cannot be described by a set using the virial coefficient [20]. There are large deviations for the RK and Redlich Kwong Soave (RKS) equations in calculating the critical compressibility factor Z_c and liquid density. However, the Peng-Robonson (PR) equation makes up for the obvious deficiencies, with a better accuracy in the calculation of saturated vapor pressure, saturated liquid density, and other aspects, especially in calculations of the multiphase fluid systems, which are some of the most commonly used equations in engineering design calculations. In this paper, the PR equation is developed to investigate the gas-solid two-phase flow [21]. In this system, the PR equations, which are corrected by Stryjek and Vera, are proposed to calculate the bubble properties of the gas-solid two-phase flow system [22]. The specific form of the PR equation is

$$P = \frac{RT}{V-b} - \frac{a}{V^2 + 2bV - b^2}, \tag{1}$$

where $a = a_c \alpha$, $a_c = 0.457 R^2 T_c^2 / P_c$, $\alpha^{1/2} = 1 + (0.375 + 1.542\omega - 0.27\omega^2)[1 - (T/T_c)^{1/2}]$, $b = 0.078 RT_c/P_c$, P is the system pressure P_a, R is the gas constant $(J \cdot mol^{-1} \cdot K^{-1})$, V is the molar volume $(m^3 \cdot mol^{-1})$, T is the absolute temperature (K), ω is eccentric factor with the zero dimension, T_c is critical temperature, and P_c is the critical pressure P_a [22]. The gas-solid two-phase flow system is a thermodynamic system consisting of a large number of solid particles and bubble phases, and

the internal energy of the thermodynamic system is a state function of the system. According to the internal energy differential formula, it is expressed as

$$dU = C_v dT + [T(\frac{\partial P}{\partial T})_v + P]dV, \tag{2}$$

where U is the internal energy of system, and C_v is the constant volume molar specific heat of the system. The differential form of the internal energy of the non-ideal thermodynamic system can be given by combining the thermodynamic state equations, which reads

$$dU = C_v dT + \frac{a}{V^2 + 2bV - b^2}dV. \tag{3}$$

According to the first law of thermodynamics ($dU = dW + dQ$), in the gas-solid two-phase flow system, the internal energy of gas changes through the quantitative non-ideal gas via the adiabatic free expansion process. This occurs outside for the gas in the adiabatic process with $dW = -PdV$.

Combined with the differential form of internal energy of non-ideal gas, we can obtain

$$C_v dT + (\frac{a}{V^2 + 2bV - b^2} + P)dV = 0. \tag{4}$$

In order to avoid tedious calculus, the PR equation can be simplified as $PV = \zeta RT$, where the deviation from the non-ideal gas to the ideal gas is attributed to the deviation factor ζ. After solving the state equation for the differential coefficient, $PdV + VdP = \zeta RdT$. According to the definition of the constant-volume molar-specific heat C_v, the expression $C_v = \zeta R/(\gamma - 1)$ can be obtained. Thus, the differential equation of the state equation can be expressed as

$$PdV + VdP = C_v(\gamma - 1)dT, \tag{5}$$

Combining with the differential formula of the internal energy, we acquire

$$[\gamma P + \frac{a(\gamma - 1)}{V^2 + 2bV - b^2}]dV + VdP = 0. \tag{6}$$

Furthermore, we consider the Newton velocity formula $v_s = \sqrt{dP/d\rho}$, where v_s is the propagation velocity of the sound wave in the system, and ρ is the medium density. This process can be approximately regarded as a quasi-static adiabatic process during acoustic wave propagation, $v_s^2 = -V^2 \partial P/\partial V$. Hence, the partial derivative of the equation is the partial derivative under the adiabatic condition, and $V = 1/\rho$ is regarded as the volume of unit molar mass. By Equation (6), it can be written as

$$\frac{dP}{d\rho} = \frac{\gamma P}{d\rho} - \frac{a\rho(\gamma - 1)}{b^2\rho^2 - 2b\rho - 1}. \tag{7}$$

Here, ρ is the density of bubble systems, which can be calculated by $\rho = 3.48 \times 10^{-3}P/T(kg/m^3)$, and the speed of sound with $v_s = 331.3 + [0.606 \times (T - 273.5)]$ m/s. Considering the above calculations, the heat capacity ratio of the gas-solid two-phase flow bubble system is obtained as

$$\gamma = \frac{(95.77P/T - 0.7 + 1.28PT) \times (1.22 \times 10^5 b^2 P^2 - 6.97 \times 10^{-3}bPT - T^2) - 3.48 \times 10^{-3}aP^2T}{1.22 \times 10^5 b^2 P^3 - 1.22 \times 10^5 aP^2 - 6.97 \times 10^{-3}bP^2T - PT^2}. \tag{8}$$

It is clear that the heat capacity ratio γ is very different from the previous studies, where it is not a constant, but a function of the system pressures and the temperatures.

2.2. Gas velocity equation in the gas-solid two-phase flow system

In the gas-solid two-phase flow system, the bubbles slowly rise from the bottom of the whole system as shown in Fig. 1. This is similar to the motions of bubbles in the water [23]. Consequently,

our research focuses on the characteristics of bubbles in the direction Z of the motion with a steady state in the X-Y plane. Based on the results of the previous results and the experiments, Ergun obtained the comprehensive expression that the pressure drop ΔP of the gas-solid system decreases with the change of Z, which reads

$$\frac{\Delta P}{Z} = 150\frac{(1-\varepsilon)^2\mu_g u}{\varepsilon^3(\phi_s d_p)^2} + 1.75\frac{(1-\varepsilon)\rho_f u^2}{\varepsilon^3\phi_s d_p}. \tag{9}$$

ϕ_s is the spherical degree of solid particles. The first item of the equation (9) at the right is the viscosity term, and it takes the leading role when the flow rate is low. The second item of the equation (9) at the right is the inertia term, which plays a major role when the flow rate is higher and the flow is turbulent. ε is the void content in the gas-solid two-phase flow which represents the proportion of space occupied by the gas and the space occupied by the system. u is the gas velocity of the gas-solid two-phase flow. d_p is the solid particle diameter. ρ_f is the density of the gas-solid flow. μ_g is the viscosity of this flow. The critical fluidization velocity u_f is the fluidized velocity where the gas-solid flow pressure drop ΔP is equal to the weight of the solid particles, which can be derived from the Ergun equation for the minimum fluidization conditions. The Ergun equation (9) can be written as

$$1.75\frac{Re_f^2}{\phi_s\varepsilon^3} + 150\frac{1-\varepsilon}{\phi_s^2\varepsilon^3}Re_f = A_r. \tag{10}$$

Here, $Re_f = d_p u_f \rho_f/\mu_g$, and $A_r = d_p^3\rho_f(\rho_p - \rho_f)g/\mu_g^2$. As a result, we can obtain the equation, which is

$$\frac{(1-\varepsilon)\rho_f d_p u_f}{\varepsilon^3\phi_s^2\mu_g} + \frac{1.75\rho_f^2 d_p^2 u_f^2}{150\phi_s\varepsilon^3\mu_g^2} = \frac{\rho_f d_p^3(\rho_p - \rho_f)g}{150\mu_g^2}, \tag{11}$$

The critical fluidization velocity is obtained as

$$u_f = \frac{\mu_g}{\rho_f d_p}\left[\sqrt{1.84\times10^3\frac{(1-\varepsilon)^2}{\phi_s^2} + \frac{\phi_s\varepsilon^3}{1.75}\frac{gd_p^3\rho_f}{\mu_g^2}(\rho_p - \rho_f)} - 42.85\frac{1-\varepsilon}{\phi_s}\right], \tag{12}$$

where g is the acceleration of gravity, and ρ_p is the solid particle density. The relationship of the solid particle density ρ_p, the density of the entire system ρ_f, and the gas/air density ρ_0 is $\rho_f = (1-\varepsilon)(\rho_p - \rho_0)$. Ergun's drag model is a standard and valid mode in two-phase flows, especially in fluidized beds, which has been used up to now [24]. Moreover, Syamlal O'Brien drag model has been developed to study the pressure drop in two-phase flows on the basis of Ergun model [25]. Sau et al. have suggested that the dimensionless empirical correlation approach can be used to calculate the pressure drop with the experimental results [26]. Recently, Zhang et al. have present a study to investigate on the pressure drop ratio in non-unform fluidizations by Prigogine's minimum of entropy production principle [27].

Moreover, the average bubble diameter of the gas-solid two-phase flow in the Mori and Wen models can be expressed as a function of the bubble rise height Z, with the maximum bubble diameter of $d_{bm} = 1.64[(u - u_f)]^{0.4}$. In the whole two-phase flow system, bubbles rise slowly from low to high, and the size diameter rising along the height of Z is [28]

$$d_b(Z) = 0.54(u - u_f)^{0.4}(Z + 4\sqrt{A_0})^{0.8}g^{-0.2}, \tag{13}$$

A_0 is the average area of each circular pinhole.

2.3. Bubble size and velocity equation

For such a thermodynamic system with the adiabatic model, the pressure inside the bubbles is related to the initial state of the bubbles and the volume of the bubbles. Therefore, the relationship between the bubble pressure P and the volume V is [19]

$$P = P_c + P_0 (\frac{V_0}{V})^\gamma, \tag{14}$$

Where P_0 and V_0 are respectively the initial pressure and volume of bubbles, and P_c is the saturated pressure. The simulation model of the bubbles is approximately considered as the sphere model. The relationship of the bubble volume V with the rising velocity u and the height Z is obtained through the formula of sphere volume and diameter size, which is

$$V = \frac{\pi}{6}[0.54(u - u_f)^{0.4}(Z + 4\sqrt{A_0})^{0.8}g^{0.2}]^3. \tag{15}$$

Based on the above deductions, u as the function of Z follows, we have

$$P_0\left|V_0^\gamma - (\frac{\pi}{6})^\gamma[0.54(u - u_f)^{0.4}(Z + 4\sqrt{A_0})^{0.8}g^{0.2}]^{3\gamma}\right|$$

$$= Z(\frac{\pi}{6})^\gamma\left[150\frac{(1-\varepsilon)^2\mu_g u}{\varepsilon^3(\phi_s d_p)^2} + 1.75\frac{(1-\varepsilon)\rho_f u^2}{\varepsilon^3\phi_s d_p}\right] \times [0.54(u - u_f)^{0.4}(Z + 4\sqrt{A_0})^{0.8}g^{0.2}]^{3\gamma}. \tag{16}$$

Through self-consistent solving equations (11), (12), (14), (15) and (16), the change of the air velocity u with regard to the rising height Z in the gas-solid two-phase flow can be obtained as shown in Fig. 1. Then, the distribution of the bubble diameter $d_b(Z)$, as well as the distribution of the velocity of characteristic bubbles at the specific height Z in the gas-solid two-phase flow are as follows:

$$u_b(Z) = 0.711\sqrt{gd_b(Z)} + u - u_f. \tag{17}$$

121 In the gas-solid two-phase flow system, the distributions and properties of the solid phase lead to the
122 pressure and temperature variations of the bubble phase, and then result in the changes of the bubble
123 heat capacity ratio γ. Moreover, the changes of the heat capacity ratio γ affect the distributions and
124 variations of the solid phases, and thus strongly influence the property of the gas-solid two-phase flow
125 system. The solid and gaseous phases in the system represent a self-consistent system, and mutually
126 affect and restrict each other.

3. Results and discussion

128 Equations (8), (11), (12), (13), (14), (15), and (16), form self-consistent systems of equations. For
129 a given Z, initial pressure P_0, and initial volume V_0, the gas velocity u, bubble size $d_b(Z)$, and the
130 change law of the bubble rising velocity $u_b(Z)$ are simulated through the self-consistent calculations in
131 the gas-solid two-phase flow systems as shown in Fig. 1. In this paper, three different parameters for
132 solid particles are selected respectively in order for comparison with each other. These parameters
133 are black triangles ($\phi_s = 0.8$, $\rho_p = 1.3 \times 10^3 \ kg/cm^3$, $d_p = 0.03 \ m$), black squares ($\phi_s = 0.8$, $\rho_p = $
134 $1.4 \times 10^3 \ kg/cm^3$, $d_p = 0.02 \ m$) and black dots ($\phi_s = 0.8$, $\rho_p = 1.5 \times 10^3 \ kg/cm^3$, $d_p = 0.15 \ m$) as shown
135 in Figs. 3, 4 and 5.
136 Figure 2 shows the heat capacity ratio γ of the bubble systems in the gas-solid two-phase flow as
137 the function of the pressure P for the different temperatures T. It can be seen from Figure 2 that the
138 heat capacity ratio of the bubbles increase with the increasing of pressure P at a fixed temperature T. It
139 shows that our numerical result is obviously different from that in the ideal gas systems, which is a
140 function of the temperature and the pressure for our mode. The heat capacity ratio in ideal gas system
141 is a constant (1.4) and the value does not change with the state parameters of the systems. In the
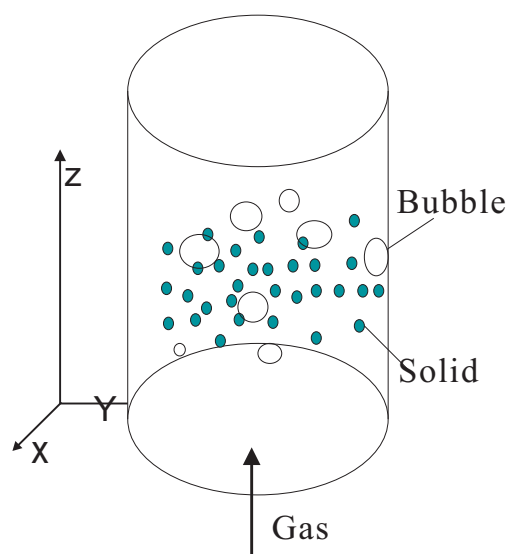
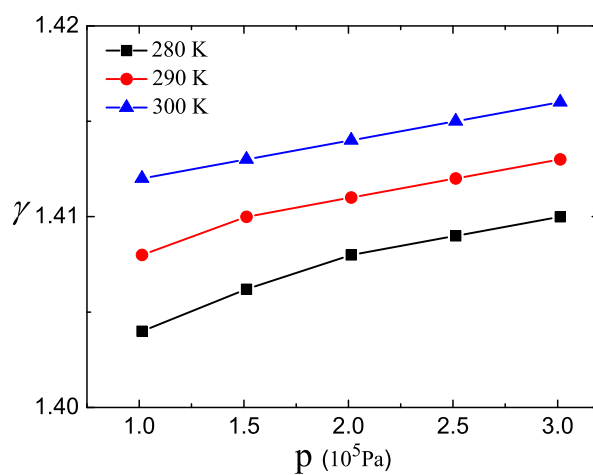**Figure 1.** The schematic for the gas-solid two-phase flows

Figure 2. The heat capacity ratio versus the pressure for the different temperatures.
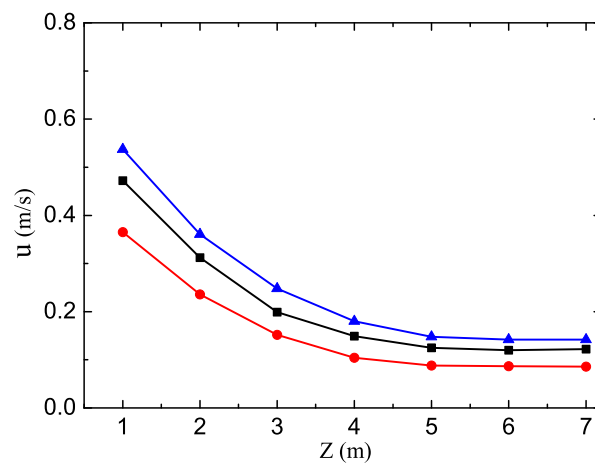
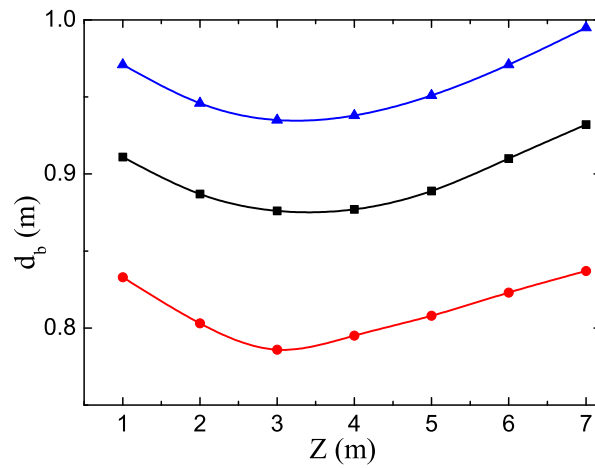**Figure 3.** The speed of the flow versus the height Z for the different conditions.

Figure 4. The size of the bubbles versus the height Z for the different conditions.
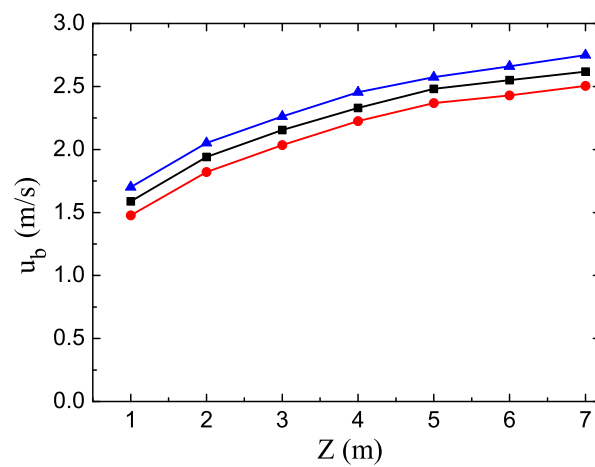

Figure 5. The speed of the bubbles versus the height Z for the different conditions.

142 past, when studying the characteristics of bubbles, the heat capacity ratio has usually been regarded
143 as the ideal gas constant [23]. We obtain the expression of specific heat capacity of the gas-solid
144 two-phase flow system through establishing a non-ideal gas model. With the increasing temperature,
145 the heat capacity ratio also increases clearly. Meanwhile, it can be clearly seen that Equation (16) is
146 a complicated exponential root and the index variety can lead to the fundamental changes of the
147 equation. Moreover, the small changes of the heat capacity ratio also lead to extensive changes of u, d_z,
148 and u_b. As a result, the bubble system heat capacity γ indicates an important effect on the properties
149 of the overall bubbles in the gas-solid two-phase flow.

150   In Figure 3, the air velocity u as the function of Z in three sets of parameters is obtained through
151 the self-consistent calculations in the gas-solid two-phase flow. As can be seen in Figure 3, with the
152 increase of the height Z, the air flow rate u in the gas-solid two-phase flow shows a downward trend,
153 with a smaller and smaller decline rate with increasing Z, gradually stabilizing and reaching a stable
154 airflow state. This shows that the smaller the particle density, the greater the air flow rate in the
155 gas-solid two-phase flow. The greater the diameter of solid particles, the greater the airflow velocity at
156 the same height. The physical reason behind this is that the large particle diameter results in a large
157 void ratio ε, which leads to a large flow velocity. Due to the differences in the particle size, the pressure,
158 and the temperature in the gas-solid two-phase flow, the whole system becomes very different and
159 the environment around the bubbles also becomes very different. As shown in Figure 2, this effect is
160 reflected in the specific heat capacity of the bubbles, which changes significantly, resulting in changes
161 of gas speed and solid speed in the whole gas-solid two-phase flow. It is clear that the air velocity
162 gradually stabilizes, which is beneficial to the stability of the gas-solid two-phase flow. The results
163 indicate that such system can be used as a medium for particle separations.

164   Figure 4 shows the bubble diameter d_b as the function of the gas flow height Z by the self-consistent
165 calculations in the gas-solid two-phase flow. It is shown that with increasing the height Z, bubble sizes
166 show a variation of decreasing and then increasing, just like the U type, in the gas-solid two-phase
167 flow. This indicates that the gas velocity, the pressure, the particle density, and the particle diameter
168 are combined together and collectively influence the diameter of bubbles in the gas-solid two-phase
169 flow. When the bubbles rise with the air flow, the external pressure decreases, since the external airflow
170 rate is relatively large. At the same time, the solid particles suppress the increase of bubble volume,
171 resulting in the bubble sizes being smaller in the initial stage. With increasing height Z, the external
172 air velocity, solid phase particle density, particle diameter, spherical degree, and other factors cannot
173 suppress the increase of air bubbles because the pressure P is so weak. At this time, the reduction of
174 external pressure plays a key role. Consequently, the bubble sizes increase with the increase of the
175 rising speed. It is found that the bubble phase depends strongly on the solid phase in the gas-solid
176 two-phase flow. The greater the density of solid phase particles, the smaller the bubble sizes in the
177 two-phase flow, which reveals that the high density of the particles can inhibit the increasing of the
178 bubble sizes. Meanwhile, the particle size is smaller, and the bubble size is smaller at the same height
179 Z. This is because the particle size and void fraction are smaller in the gas-solid flow, which can limit
180 the growth of the bubble. We find that our theoretical results can be compared with the recent reported
181 results. It is experimentally found that the size of gas bubble increases gradually from the bottom of
182 the systems (see e.g. Fig 7(a))[29]. Chen et al. have found that the bubble size enlarges by increasing
183 the height position base of the numerical simulations (see e.g. Fig. 10)[30]. However, we should notice
184 that a local minimum of the bubble sizes is observed for our self-consistent physical model, which are
185 not demonstrated experimentally at present. Our theoretical results are agreement with the reported
186 results after the local minimum sizes. Moreover, we hope our theoretical findings can be verified by
187 experiments.

188   Figure 5 shows the change of the bubble rising velocity u_b with the height Z by the self-consistent
189 calculations in the gas-solid two-phase flow. In Figure 4, the velocity of bubbles u_b is very different
190 from the outside air velocity u in Figure 3 in the gas-solid two-phase flow. It is obvious that the bubble
191 velocity is much faster than the air flow velocity. With the increasing of the height Z, the bubble

velocity increases and tends to be quasi-stable in gas-solid two-phase flows. In the gas-solid two-phase flow, the smaller the particle density, the larger the bubble velocity is. The diameter of solid particles is larger, and the bubble velocity is greater at the same height. This is because particle diameter becomes larger on increasing the void fractions in the two-phase flow, which gives rise to the large bubble phase velocity. The bubble velocity is influenced by the air velocity and the bubble size. Therefore, the properties of the bubble velocity are very similar to those of the gas flow velocity in gas-solid two-phase flow. Moreover, the bubble size firstly decreases and then increases. With the effect of the both aspects, the velocity of the bubble gradually increases and eventually stabilizes. We should also point out that the gradual stabilization of the bubble velocity is beneficial to the stability of gas-solid two-phase flow systems and the separations of different particles in this system. The tracer testing technology has been widely applied to study the properties of bubbles in multi-phase flows. The very recent experimental research by the technology shows that the bubble velocity become large by increasing the bed locations (see e.g. Fig 7(b))[29]. We find that our theoretical results are very consistent with the experimental results.

4. Conclusions

The properties of the bubbles in the gas-solid systems have been self-consistently investigated based on fluid mechanics theory, bubble thermodynamic equations, and kinetic equations. We found that the bubble heat ratio of the gas-solid two-phase flow significantly increases with the increase of temperature and pressure in the bubble system. The bubble heat ratio has a significant impact on the properties of bubbles. Theoretical study has shown that the air velocity decreases with the increase of the air flow height in the gas-solid two-phase flow, which finally stabilizes, with the height increasing and reaching a steady state. A local minimum in bubble sizes as a function of height can be observed. The rising velocity of bubbles depends strongly on the air velocity and the bubble sizes, with the speed gradually increase and eventually quasi-stabilizing. The physical reason for these phenomena is that the temperature and the pressure of gas phase determine the bubble heat capacity ratio in the gas-solid two-phase flow, and the heat capacity ratio is influenced by the density, the size, and the granularity of the solid phase, which affects the air speed and bubble size conversely. Since the bubble velocity depends strongly on the air velocity and the bubble size, the gas phase and the solid phase interact with each other and a self-consistent system comes into being. Our theoretical results enrich and deepen the theory of gas-solid two-phase flow. The results presented and discussed in this paper can be used to understand the properties and effects of the bubbles in gas-solid two-phase flows.

References

1. Kadanoff Leo P. *Reviews of Modern Physics* **1999**, 71, 435.
2. Wang S.; He Y.Q.; Wei H.; Xie W.N. *International Journal of Analytical Chemistry* **2017**, 2017, 4763937.
3. Yuan Z.L.; Zhu L.P.; Geng F.; Peng Z.B. *Gas-solid Two Phase Flow and Numerical Simulation (Southeast University Press)*, **2012**.
4. Glasser B.J.; Sundaresan S. and Kevrekidis I.G. *Physical Review Letters* **1998**, 81, 1849.
5. Martyushev L.M.; Celezneff V. *Entropy* **2017**, 19, 126.
6. Harrision D.; Leung L.S. *Nature* **1961**, 190, 433.
7. Pak H.K; Behringer R.P. *Nature* **1994**, 371, 231.
8. Xiong Q.G.; Lia B.; Zhou G.F. et al. *Chemical Engineering Science* **2012**, 71, 422.
9. Lare V; Piepers H.W.; Sehoonderbeek J.N. et al. *Chemical Engineering Science* **1997**, 52, 829.
10. Ichiki K.; Hayakawa H. *Physical Review E* **1995**, 52, 658.
11. Farshi A.; Javaherizaden H.; Hamzavi Abedi M.A. *Petroleum & Coal* **2008**, 50, 11.
12. Valverde J.M.; Castellanos A.; Mills P.; Quintanilla M.A.S. *Physical Review E* **2003**, 67, 051305.
13. Müller C.R.; Davidson J.F.; Dennis J.S. et al. *Physical Review Letters* **2006**, 96, 154504.

14.   Homsy G.M. *Applied Scientific Research* **1998**, 58, 251.

15.   Tafreshi Z.M.; Kingsley O.G.; Adesoji A.A. *The Canadian Journal of Chemical Engineering* **2000**, 78, 815.

16.   Komatsu T.S.; Hayakawa H. *Phys. Lett. A* **1993**, 183, 56.

17.   Choi J.H.; Son J.E.; Kim S.D. *Journal of Chemical Engineering of Japan* **1988**, 21, 171.

18.   Al-Zahrani A.A.; Daous M.A. *Powder Technology* **1996**, 87, 255.

19.   Best J.P. *Journal of Fluid Mechanics* **1993**, 251, 79.

20.   Tian J.X.; Jiang H.; Guic Y.X.; Mulerod A. *Phys. Chem. Chem. Phys.* **2009**, 11, 11213.

21.   Martín C; Jrgen M. *Fluid Phase Equilibria* **2005**, 232, 74.

22.   Gasem K.A.M.; Gao W.; Pan Z.; Robinson Jr.R.L. *Fluid Phase Equilibria* **2001**, 181, 113.

23.   Vandermeulen J.; Leliaert J.; Dupre L.; Van Waeyenberge B. *Journal of Physics D* **2017**, 50, 495007.

24.   Wei L.P.; Lu Y.J. *Chemical EngineeringScience* **2016**, 147, 21-29.

25.   Syamlal M.; O'Brien T.J. *AICHE Journal* **2003**, 49, 2793-2801.

26.   Sau D.C.; Mohanty S.; Biswal K.C. *Chemical Engineering and Processing* **2008**, 47, 2114-2120.

27.   Zhang C.X.; Qian W.Z.; Wei F. *Chemical Engineering Science* **2017**, 173, 187-195.

28.   Darton R.C.; Lanauze R.D.; Davidson J.F.; Harrison D. *Trans IChemE.* **1977**, 55, 274.

29.   Zhang C.X.; Li P.L.; Lei C.; Qian W.Z.; Wei F. *Chemical Engineering Science* **2018**, 176, 515-523.

30.   Chen L.M.; Yang X.G.; Li G. et al. *Powder Technology* **2017**, 322, 461-470.