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Article

Self-Consistent Physical Model of the Bubbles in a Gas-Solid Two-Phase Flow

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Abstract: In this work, we develop a self-consistent physical model of bubbles in a gas-solid two-phase flow. Based on PR state equation, and a detailed specific heat ratio equation of bubbles, we self-consistently evaluate kinetic equations of the bubbles on the basis of Ergun equation, thermodynamic equation and kinetic equations. It is found that the specific heat ratio of bubbles in such systems strongly depends on the bubble pressures and temperatures, which play an important role on the characteristics of the bubbles. The theoretical studies show that with a increasing of the height in the systems, the gas flow rate shows a downward trend. Moreover, the larger particles in the gas-solid flows are, the greater the gas velocity is. With a increasing the height, bubble sizes show a variation of first decreasing and then increasing (U type). The bubble velocity is affected by the gas velocity and the bubble size, which gradually declined and eventually stabilized. It shows that gas phases and solid phases in a gas-solid two-phase flow interact with each other and come into being a self-consistent system. The theoretical results have exhibited important guiding value for understanding the properties and effects of bubbles in the gas-solid two-phase flows.

Keywords: self-consistent physical model; bubbles; gas-solid two-phase flow

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1. Introductions

The gas-solid two-phase flow is a complex flow system composed of gas and solid particles, which is an important branch of fluid dynamics. The gas-solid two-phase flows widely exist in the nature and industrial productions. There is rich study value in this physical system, which have been extensively researched and developed, such as the gas-solid two-phase fluidization separation with a wide range of application and high selection efficiency. It is a efficient separation method that has been gradually applied to industrial productions[1,2]. With the gas-particle fluidization separation, the system is filled with uniform air flows in the fine particulate matter media bed, which makes the particle medium fluidization and forms the gas-solid suspended matter with certain density and fluid properties. The theoretical analysis of the two-phase flow is much more difficult than the single-phase flow because the general differential equations of describing the two-phase flow have not been established yet. Generally, the two theoretical simplified model can be applied in such systems. One of the models is that the system can be considered as a continuous medium model which is a mixture of two phases, where the concepts and methods of the single-phase flow are still suitable for the two-phase flow. The other is called the separated model, which considers that the concept and method of the single-phase flow can be respectively used in each phase of the two phase system, meanwhile the interaction between the two phases is included[3,4]. The second models have been widely associated with in physical application [5].

In the gas-solid two-phase flow, the mixture of the gas and the particles is not uniform and time-dependent, existing the two phases in the gas-solid two-phase flow. One of the phases is the continuous phase composed of the gas and the particles, called dense phase or emulsion phase; the other phase is the discontinuous phase appearing in the bubble state, called bubble phase. When the gas flowing in the gas-solid system is higher than the air flowing required for the critical fluidized state, the excess gas could be in the form of bubbles. Bubble dynamic behavior is the most basic characteristics and phenomena in the gas-solid two-phase flow system, which play an important significance for understanding and studying the properties of the gas-solid two-phase flow systems[4]. The study of bubbles in the gas-solid two-phase flow has been an important content in the field of fluidization state[6]. The pressure drop, height, swelling degree, uniform mixing of the gas-solid two-phase flow and stratified separation of particles in the gas-solid two-phase system strongly depend on the bubbles. The bubble size and rising velocity are the important factors which affect the fluidization stability, the dynamical of particles and the separation densities of fluidized beds[7]. Due to the existence of bubbles, the bubble phase and the dense phase exchange between heat and mass, the mixture of gas and solid is uniform. The growth of bubbles in the gas-solid two-phase flow can seriously affect the uniform stability of fluidization, affect the distribution of solid phase particles and affect the state of the whole two-phase flow. As a result, the research of bubble properties in the gas-solid two-phase flow system have been one of the most important and basic researches in this field[8]. Van Lare et al. have developed a statistical theory to study the properties of bubbles in the gas-solid systems using capacitance probe measurement experiment, and obtained the consistent results between the experiments and theories [9]. Ichiki etc. have studied the bubble phase and slugging phase in a fluidized bed by the numerical simulation. It is found that the convective motion more strongly effect on the bubble phase than on the slugging phase[10]. Farshi et al. have calculated the size of bubbles in the gas-solid fluidized bed, and found that the Mori-Wen-Rowe equation is a better choice for the studying of the bubble size comparing with the other theories[11]. Through the optical experimental study, Valverde et al. have found that the bubbles begin to be produced after fluidizations of the particles, and the interactions between the hydro-kinetic force and the particles can suppress the generation of bubbles for the sufficiently large particles in the two-phase flow[12]. Glasser et al. have studied the relationship between the bubbles and the clusters in the gas-solid two-phase flow and found that they are similar phenomenons in the systems[4]. Muller has measured the bubble phenomenon in two phase system using magnetic resonance imaging experiment in real time for the first time, and studied the bubble size, velocity and so on in detail[13]. The nonlinear characteristics of the gas-solid two-phase flow and the relationship with bubbles are studied by Homsy et al.[14]. Zahra researched the nonlinear dynamic characteristics of gas-solid system using the two phase theory[15]. Sasa adn Komatsu have investigated on the nonlinear dynamic characteristics of the fluidized states. Their results show that solitons, like waves exist in the system and play an important effect on the system. It indicates that the nonlinear characteristic has an important role and influence on the systems[16]. In the South Korean energy research institute, the bubble properties of the gas-solid fluidized bed system were studied by Choi, Son et al. using the electronic resistivity probe experiments, and the change of the bubble size with the height was obtained[17]. Al-Zahrani and Daous have established a simplified model to predict the average velocity of bubbles risings[18].

Bubble phenomenon is one of the most basic physical phenomena of the gas-solid two-phase flow system, and the relevant scholars at home and abroad have paid much attention to the research of bubbles in the gas-solid two-phase flow. It is noted that the present researches mainly focus on the improvement and research of productive processes in the actual systems, however, the basic theoretical research is relative lack[19]. The high-speed dynamic systems have been widely used to analysis the bubble behaviors of the gas-solid system in experiments. With the limited experimental approaches, we can observe and analyze the behavior of the system at boundary surface and can not go deep into the internal system for detailed analysis and research. Based on the experimental results, the relatively simplified models can be fitted out, which are usually based on fluid mechanics or continuum medium theory. The numerical simulation can also be used to analyze the motion characteristics of bubbles. Due to a certain gap between the numerical simulation models and the existing experimental conditions, the simulation results can only partly explain the experimental results. In this work, we hope to understand the the gas-solid two-phase flow system more from the point of view of physics.

By the theory of thermodynamics and fluid mechanics, we establish a new bubble physical model. The dynamic properties of bubbles in the gas-solid two-phase flow system are studied by the self-consistent theoretical method. Through establishing and researching the bubble thermodynamic equation, we examine the properties of bubbles in the gas-solid two-phase flow system on this basis of Ergun and bubble dynamics equations.

2. Theoretical model

The state equation of the general thermodynamic systems can be expressed as $\rho = \rho(P, T)$ or $f(\rho, P, T) = 0$, where ρ is the density of a system, P is the pressure and T is the system temperature. The state equation is very important for the general thermodynamic systems. With the help of it, the other thermodynamic properties which can not measured by experiments directly can be obtained. The ideal-gas state equation can be represented as $\rho = P/RT$, while for the real gas(thermodynamic systems), the state of equation can be written as $\rho = P/Z(P, T)RT$, here R is gas constant, $Z(P, T)$ is called compression coefficient, which indicates the deviation from the ideal gas state equation. The ideal gas model is a theoretical model which can be used as one of the criteria to measure the real gas state equations, which can be used for approximately estimating in a practical application. When the pressure is close to zero or the volume is close to infinity, any real gas state equations should be reduced to the ideal gas equation. A gas-solid two-phase flow is a complex thermodynamic systems, thus the theoretical model of the system simplified as an ideal system is no longer applicable.

2.1. PR equation and the heat capacity ratio of bubbles

None of the real thermodynamic systems is completely in conformity with the law of ideal gas. The deviation depends on the pressure, the temperature and the gas properties, especially on the degree of the difficulty of gas liquefactions. The gas-solid two-phase flow bubble systems contain not only gas interacting between air molecules, but also partial solid particles. With the change of ambient pressures and temperatures, the proportion of the particle phase and the gaseous phase is changed. Therefore, the system is a non-ideal complex system, which needs the non-ideal state equation for studying. The non-ideal gas state equation is widely used in engineering, such as the Van der waals equation, the Redlich-Kwong (RK) equation, the Virial equation and so on. With the RK equation used to calculate the strong polar compounds, it causes large deviation and rarely used to calculate liquid pressures, volumes and temperatures. The Virial equation is also not a good match for the polar compounds, and gas-solid two-phase flow can not be described by a set of the Virial coefficient[20]. There are big deviations for the RK equation and the RKS equations in calculating the critical compressibility factor Z_c and liquid density. However, the Peng-Robonson (PR) equation makes up for the obvious deficiencies, with a better accuracy in calculation of saturated vapor pressure, saturated liquid density and other aspects, especially in calculation of the multiphase fluid systems, which is one of the most commonly used equations in engineering design calculations. In this paper, the PR equation is developed to investigate on the gas-solid two-phase flow[21]. In this system, the PR equations which is corrected by the Stryjek and Vera are proposed to calculate the bubble properties of the gas-solid two-phase flow system, the specific form of the PR equation is

$$P = \frac{RT}{V - b} - \frac{a}{V^2 + 2bV - b^2} \tag{1}$$

$a = a_c \alpha$, $a_c = 0.457 R^2 T_c^2 / P_c$, $\alpha^{1/2} = 1 + (0.375 + 1.542\omega - 0.27\omega^2)[1 - (T/T_c)^{1/2}]$, $b = 0.078 RT_c/P_c$, P is the System pressure P_a, R is the gas constant $(J \cdot mol^{-1} \cdot K^{-1})$, V is the molar volume $(m^3 \cdot mol^{-1})$, T is the absolute temperature (K), ω is eccentric factor with the zero dimension, T_c is critical temperature, P_c is the critical pressure P_a[22]. The gas-solid two-phase flow system is a thermodynamic system consisting of a large number of the solid particles and the bubble phases, and the internal energy of the thermodynamic system is a state function of the system. According to internal energy differential formula, which is

$$dU = C_v dT + [T(\frac{\partial P}{\partial T})_v + P]dV, \tag{2}$$

where U is the internal energy of system, C_v is the constant volume molar specific heat of the system. The differential form of the internal energy of the non ideal thermodynamic system can be given by combining the thermodynamic state equations.

$$dU = C_v dT + \frac{a}{V^2 + 2bV - b^2} dV. \tag{3}$$

According to the first law of thermodynamics $dU = dW + dQ$, in the gas-solid two-phase flow system, the internal energy of gas changes after the quantitative non-ideal gas via adiabatic free expansion process, outside working for the gas in the adiabatic process with $dW = -PdV$, combining with the differential form of internal energy of non-ideal gas, we can obtain

$$C_v dT + (\frac{a}{V^2 + 2bV - b^2} + p)dV = 0 \tag{4}$$

In order to avoid tedious calculus, the PR equation can be simplified as $PV = ZRT$, where the deviation from the non-ideal gas to the ideal gas is attributed to the deviation factor Z. After solving the state equation for differential coefficient, $PdV + VdP = ZRdT$. According to the definition of the constant volume molar specific heat C_v, the expression $C_v = ZR/(\gamma - 1)$ can be got. Thus the differential equation of the state equation can be expressed as

$$PdV + VdP = C_v(\gamma - 1)dT, \tag{5}$$

Combining with the differential formula of the internal energy, we acquire

$$[\gamma P + \frac{a(\gamma - 1)}{V^2 + 2bV - b^2}]dV + VdP = 0. \tag{6}$$

Meanwhile, on the basis of the Newton velocity formula $v_s = \sqrt{dP/d\rho}$, v_s is the propagation velocity of sound wave in the system, ρ is the medium density, this process can be approximately regarded as a quasi-static adiabatic process during acoustic wave propagation, $v_s^2 = -V^2 \partial P/\partial V$. So the partial derivative of the equation is the partial derivative under the adiabatic condition, and the $V = 1/\rho$ is regarded as the volume of unit molar mass. Under equation (6), it can be written as

$$\frac{dP}{d\rho} = \frac{\gamma P}{d\rho} - \frac{a\rho(\gamma - 1)}{b^2\rho^2 - 2b\rho - 1}. \tag{7}$$

Here ρ is the density of bubble systems, which can be calculated by $\rho = 3.48 \times 10^{-3} P/T (kg/m^3)$, the speed of sound with $v_s = 331.3 + [0.606 \times (T - 273.5)]$ m/s. Considering the above calculations, the heat capacity ratio of the gas-solid two-phase flow bubble system is

$$\begin{gathered}\gamma = [(95.77P/T - 0.7 + 1.28PT) \times \\ (1.22 \times 10^5 b^2 P^2 - 6.97 \times 10^{-3} bPT - T^2) - 3.48 \times 10^{-3} aP^2 T] \\ /[1.22 \times 10^5 b^2 P^3 - 1.22 \times 10^5 aP^2 - 6.97 \times 10^{-3} bP^2 T - PT^2].\end{gathered} \tag{8}$$

It is clear that the heat capacity ratio γ is very different from the previous studies, which is not a constant, but a function of the system pressures and the temperatures.

2.2. *Gas velocity equation in the gas-solid two-phase flow system*

In the gas-solid two-phase flow system, the bubbles slowly rise from the bottom of the whole system, which are similar to the motions of bubbles in the water [23]. Consequently, our research focuses on the characteristics of bubbles in the direction of the motion (Z) with a steady state in x-y plane. Based on the results of the previous results and the experiments, Ergun obtains the comprehensive expression that the pressure of the gas-solid system decreases with the change of Z, which reads

$$\frac{\Delta P}{Z} = 150\frac{(1-\varepsilon)^2\mu_g u}{\varepsilon^3(\phi_s d_p)^2} + 1.75\frac{(1-\varepsilon)\rho_f u^2}{\varepsilon^3\phi_s d_p}. \tag{9}$$

ϕ_s is the spherical degree of solid particles. The first item of ϕ_s is the viscosity term, and it takes the leading role when the flow rate is low. The second one of ϕ_s is the inertia term, which play a major role when the flow rate is higher and the flow is turbulent. ε is the void content in the gas-solid two-phase flow which represents the proportion of space occupied by the gas and the space occupied by the system. u is the gas velocity of the gas-solid two-phase flow. d_p is the solid particle diameter. ρ_f is the density of the gas-solid flow. μ_g is the viscosity of this flow. The critical fluidization velocity u_f, is the fluidized velocity that the gas-solid flow pressure drop is equal to the weight of the solid particles, which can be derived from the Ergun equation. As a result and then,

$$\frac{(1-\varepsilon)\rho_f d_p u_f}{\varepsilon^3\phi_s^2\mu_g} + \frac{\rho_f^2 d_p^2 u_f^2}{100\phi_s\varepsilon^3\mu_g^2} = \frac{\rho_f d_p^3(\rho_p-\rho_f)g}{150\mu_g^2}, \tag{10}$$

The critical fluidization velocity can be obtained as

$$u_f = \frac{\mu_g}{\rho_f d_p}[\sqrt{1.84\times10^3\frac{(1-\varepsilon)^2}{\phi_s^2} + \frac{\phi_s\varepsilon^3}{1.75}\frac{g d_p^3\rho_f}{\mu_g^2}(\rho_p-\rho_f)} - 42.85\frac{1-\varepsilon}{\phi_s}], \tag{11}$$

g is the acceleration of gravity, ρ_p is the solid particle density. The relationship of the solid particle density ρ_p, the density of the entire system ρ_f and the gas/air density ρ_0 is $\rho_f = (1-\varepsilon)(\rho_p-\rho_0)$. Meanwhile, the average bubble diameter of the gas-solid two-phase flow in Mori and Wen models can be expressed as a function of the bubbles rise height Z, with the maximum bubble diameter is $d_{bm} = 1.64[(u-u_f)]^{0.4}$. In the whole two-phase flow system, bubbles rise slowly from low to high, and the size diameter rising along the height of Z is [24]

$$d_b(Z) = 0.54(u-u_f)^{0.4}(Z+4\sqrt{A_0})^{0.8}g^{-0.2}, \tag{12}$$

A_0 is the average area of each circular pinhole.

2.3. *Bubble size and velocity equation*

For such a thermodynamic system with the adiabatic model, the pressure inside the bubbles is related to the initial state of the bubbles and the volume of the bubbles, therefore the relationship between the bubble pressure P and the volume V is [19]

$$P = P_c + P_0(\frac{V_0}{V})^\gamma, \tag{13}$$

Where P_0 and V_0 are respectively the initial pressure and volume of bubbles, P_c is the saturated pressure. The simulation model of the bubbles is approximate considered as the sphere model. The

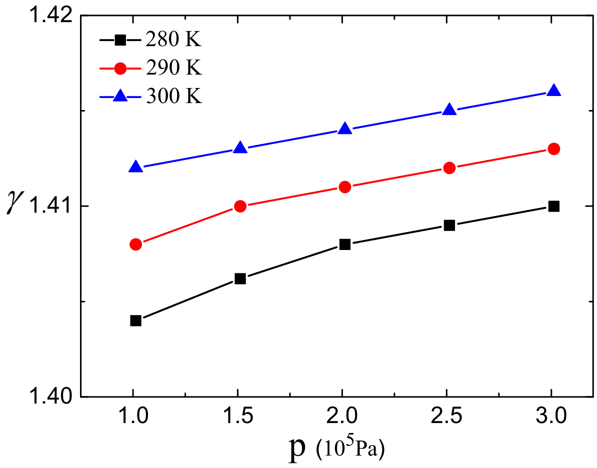

Figure 1. The heat capacity ratio versus the pressure for the different temperatures.

relationship of the bubble volume V with the rising velocity u and the height Z is obtained through the formulas of sphere volume and the diameter size, that is

$$V = \frac{\pi}{6}[0.54(u-u_f)^{0.4}(Z+4\sqrt{A_0})^{0.8}g^{0.2}]^3. \tag{14}$$

Based on the above deduction, u as the function of Z is followed,

$$\begin{aligned} P_0\left|V_0^\gamma - (\frac{\pi}{6})^\gamma[0.54(u-u_f)^{0.4}(Z+4\sqrt{A_0})^{0.8}g^{0.2}]^{3\gamma}\right| \\ = Z(\frac{\pi}{6})^\gamma\left[150\frac{(1-\varepsilon)^2\mu_g u}{\varepsilon^3(\phi_s d_p)^2} + 1.75\frac{(1-\varepsilon)\rho_f u^2}{\varepsilon^3\phi_s d_p}\right] \\ \times [0.54(u-u_f)^{0.4}(Z+4\sqrt{A_0})^{0.8}g^{0.2}]^{3\gamma} \end{aligned} \tag{15}$$

Self-consistent solving equations (2.11), (2.12), (2.14), (2.15) and (2.16), the change of the air velocity u with regards of the rising height Z in the gas-solid two-phase flow can be obtained, then the distribution of the bubble diameter $d_b(Z)$, as well as the distribution of the velocity of characteristic bubbles at the specific height Z in the gas-solid two-phase flow are as follows

$$u_b(Z) = 0.711\sqrt{gd_b(Z)} + u - u_f. \tag{16}$$

In the gas-solid two-phase flow system, the distributions and properties of the solid phase lead to the pressure and temperature variations of the bubble phase, then result in the changes of the bubble heat capacity ratio γ. Moreover, the changes of the heat capacity ratio γ affect the distributions and variations of the solid phases, thus strongly influence the property of the gas-solid two-phase flow system. The solid phases and gaseous phases in the system are the self-consistent system which mutually effect and restrict each other.

3. Results and discussion

The equations (8), (11), (12), (13), (14), (15) and (16), form a self consistent systems of equations. For a given Z and initial pressure P$_0$ and initial volume V$_0$, the gas velocity u, bubble size $d_b(Z)$ and the change law of the bubble rising velocity $u_b(Z)$ are simulated through the self-consistent calculations in the gas-solid two-phase flow systems. In this paper, three different parameters for solid particles are selected respectively in order to compare with each other, which are the black triangles ($\phi_s = 0.8$,

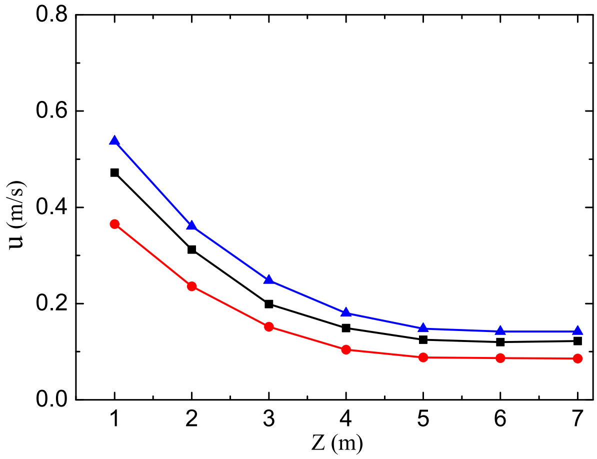

Figure 2. The speet of the flow versus the height Z for the different conditions.

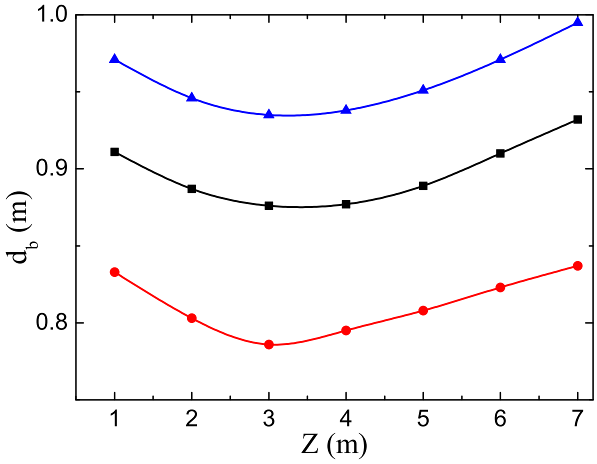

Figure 3. The size of the bubbles versus the height Z for the different conditions.

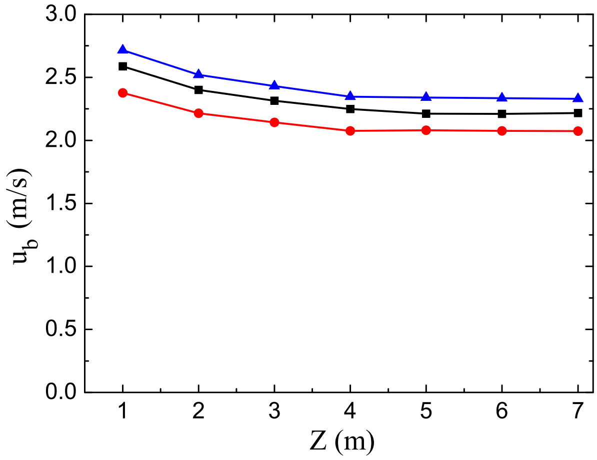

Figure 4. The speed of the bubbles versus the height Z for the different conditions.

$\rho_p = 1.3 \times 10^3\ kg/cm^3$, $d_p = 0.03\ m$), the black squares ($\phi_s = 0.8$, $\rho_p = 1.4 \times 10^3\ kg/cm^3$, $d_p = 0.02\ m$) and the black dots ($\phi_s = 0.8$, $\rho_p = 1.5 \times 10^3\ kg/cm^3$, $d_p = 0.15\ m$).

Figure 1 shows the heat capacity ratio γ of the bubble systems in the gas-solid two-phase flow as the function of the pressure P for the different temperatures. It can be seen from figure 1 that the heat capacity ratio of the bubbles increase with the increasing of pressure P at a fixed temperature T. It shows that our numerical result is obviously different from that in the ideal-gas systems, which is a function of the temperature and the pressure. The heat capacity ratio in ideal gas system is constant 1.4 and the value does not change with the state parameters of the system. In the past, when studying the characteristics of bubbles, the heat capacity ratio has usually been regarded as the ideal gas constant[23]. We obtain the expression of specific heat capacity of the gas-solid two-phase flow system through establishing a non ideal gas model. With the increasing of temperature, the heat capacity ratio also increases obviously. Meanwhile, it can be clearly seen from equation (15) that the equation is a complicatedly exponential root and the index variety can lead to the fundamental changes of the equation. Moreover, the small changes of the heat capacity ratio also lead to the extensive changes of u, d_z and u_b. As a result, the bubble system heat capacity γ indicates an important effect on the properties of the whole bubbles in the gas-solid two-phase flow.

In figure 2, the air velocity u as the function of Z in three sets of the parameters are obtained through the self-consistent calculations in the gas-solid two-phase flow. As can be seen in figure 2, with the increasing of the height Z, the air flow rate u in the gas-solid two-phase flow shows a downward trend, with the smaller and smaller decline rate for increasing Z, which gradually tends to stabilized and reaches a stable airflow state. It shows that the smaller the particle density, the greater the air flow rate in the gas-solid two-phase flow. The greater the diameter of solid particles, the greater the airflow velocity at the same height. The physical reason behind it is that the large particle diameter results in the large void ratio ε, which leas to large the flow velocity. Due to the differences of the particle size, the pressure and the temperature in the gas-solid two-phase, the whole system becomes very different and the environment around the bubbles also becomes very different. As shown in Figure 1, this effect is reflected in the specific heat capacity of the bubbles which changes significantly, resulting in the changes of gas speed and solid speed in the whole gas-solid two-phase flow. It is clear that the air velocity is gradually stable, which is beneficial to the stability of the gas-solid two-phase flow. The results indicate that such system can be used as a medium for particle separations.

Figure 3 shows the bubble diameter d_b as the function of the gas flow height Z by the self-consistent calculations in the gas-solid two-phase flow. It is shown that with increasing the height Z, bubble sizes show a variation of decreasing and then increasing, just like U type, in the gas-solid two-phase. It indicates that the gas velocity, the pressure, the particle density, the particle diameter combined together and collectively influence on the diameter of bubbles in the gas-solid two-phase flow. When the bubbles rise with the air flow, the external pressure decreases, since the external airflow rate is relatively large. At the same time, the solid particles suppress the increasing of bubble volume, resulting in the bubble sizes smaller in the initial stage. With the increasing of the height Z, the external air velocity, solid phase particle density, particle diameter, spherical degree and other factors can not suppress the increase of air bubbles because the pressure P is so weak. At this time the reducing of the external pressure plays the key and significant role. Consequently the bubble sizes increase with the increasing of the rising speed. It is found that the bubble phase depends strongly on the solid phase in the gas-solid two-phase flow. The bigger the density of solid phase particles, the smaller the bubble sizes in the two phase flow, which reveals that the high density of the particles can inhibit the increasing of the bubble sizes. Meanwhile, the particle size is smaller, the bubble size is smaller at the same height Z. This is because the particle size is smaller, the void fraction is smaller in the gas-solid flow, which can limit the growth of the bubble.

Figure 4 shows the change of the bubble rising velocity u_b with the height Z by the self consistent calculations in the gas-solid two-phase flow. In figure 4, the velocity of bubbles u_b is very different from the outside air velocity u in figure 2 in gas-solid two-phase flow. It is obvious that the bubble velocity is much faster than the air flow velocity. With the increasing of the height Z, the bubble velocity decreases and tends to be stable in gas-solid two-phase flow. In the gas-solid two-phase flow, the smaller the particle density, the bigger the bubble velocity is. The diameter of solid particles is larger, the bubble velocity is greater at the same height. This is because particle diameter becomes large by increasing the void fractions in the two-phase flow, which give rise to the large bubble phase velocity. The bubble velocity is influenced by the air velocity and the bubble size. Therefore, the properties of the bubble velocity are very similar to those of the gas flow velocity in gas-solid two-phase flow. Moreover, the bubble size firstly decreases and then increases. With the effect of the both aspects, the velocity of the bubble gradually declines and eventually stabilizes. We should also point out that the gradual stabilization of the bubble velocity is beneficial to the stability of the gas-solid two-phase flow systems and the separations of different particles in this system.

4. Conclusions

With the thermodynamic theory, the properties of the bubbles in the gas-solid systems are self-consistently investigated based on the fluid mechanics theory, the bubble thermodynamic equations and the kinetic equations. We find that the bubble heat ratio of the gas-solid two-phase flow significantly increases with the increasing of temperature and pressure in the bubble system. The bubble heat ratio plays an significant impact on the properties of bubbles. Theoretical study shows that the air velocity decreases with the increasing of the air flow height in the gas-solid two-phase flow, which finally tends to be stable with the height increasing and reaches the steady state. With increasing the height, bubble sizes show a variation of first decreasing and then increasing (U type). The rising velocity of bubbles depends strongly on the air velocity and the bubble sizes, with the speed gradually slowing down and eventually being stabilized. The physical reason for these phenomena is that the temperature and the pressure of gas phase determine the bubble heat capacity ratio in the gas-solid two-phase flow, and the heat capacity ratio is influenced by the density, the size and the granularity of the solid phase which affects the air speed and bubble size conversely. Since the bubble velocity depends strongly on the air velocity and the bubble size, the gas phase and the solid phase interact with each other and come into being a self-consistent system. Our theoretical results in this paper enrich and deepen the theory of the gas-solid two-phase flow. Our results presented and discussed in this paper can be used to understand the properties and effects of the bubbles in the gas-solid two-phase flow.

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