A hybrid MCDM technique for risk management in construction projects

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Abstract: Multi-stakeholder based construction projects are subject to various risk factors due to dynamic business environments. These risks affect project activities which indirectly impact construction costs, resulting in delays and poor building quality. So, managing these project risks requires suitable risk mitigation strategies to evaluate and analyse their severity. Hence, risk evaluation and assessment of construction projects is a multi-criteria decision making (MCDM) problem. In present real-life problems, evaluation of project risks is often uncertain and even incomplete, and the prevailing methodologies fail to handle such situations. To address the problem, this paper extends the analytical network process (ANP) methodology in the D number domain to handle three types of ambiguous evaluations, viz. complete, uncertain, and incomplete, and assesses the weight of risk criteria. The D number based approach overcomes the deficiencies of the exclusiveness hypothesis and completeness constraint of Dempster-Shafer (D-S) theory. Here, preference ratings of the decision matrix for each decision-maker are determined using a D numbers extended consistent fuzzy preference relation (D-CFPR). An extended multi-attributive border approximation area comparison (MABAC) method in D numbers is then developed to rank and select the best alternative risk response strategy. Finally, an illustrative example from construction sector is presented to check the feasibility of the proposed approach. For checking the reliability of alternative ranking, a comparative analysis is performed with different MCDM approaches (D-COPRAS, D-ARAS, D-MABAC, and D-TOPSIS). Based on different criteria weights, a sensitivity analysis of obtained ranking of the hybrid D-ANP-MABAC model is performed for verify the robustness of the proposed method.

Keywords: D number, Analytical Network Process (ANP), MABAC, Multi-criteria decision making (MCDM), Consistent fuzzy preference relation (CFPR), Construction project risk, Risk Management.

1. Introduction

In recent decades, projects in the construction sector have become more complex and risky due to the diverse nature of activities among global companies [1–8]. In comparison to other sectors, construction projects encounter more risks due to uncertainties occurring because of various construction practices, working conditions, mixed cultures and political conditions between host and home countries [9–12]. Thus, in this scenario, risk management can be considered a vital part of the decision making process in construction projects. These projects may involve many stakeholders, in
addition to uncertain socio-economic conditions at the project site, bringing big challenges to practitioners of the industry in recent decades [13–14]. Construction project failure may cause higher costs and time over-runs, requiring a systematic risk assessment and evaluation procedure to classify and respond to changes [15–16].

Thus, prioritisation among construction based risk portfolios, and finding suitable risk mitigation strategies for construction projects, can be introduced as multi-criteria decision making (MCDM) problems. Researchers have recently proposed new methods for prioritising risks in construction based projects [10, 17–19]. Also, the increasing dynamism of construction projects have resulted in extensive impreciseness and subjectivities in this risk investigation procedure. With respect to the identification of risk criteria, a methodology is needed to sort and prioritise criteria weights, based on specific environments and domain experts’ judgment. In the real world, since various uncertainties occur in the decision making process due to subjective and qualitative judgment of DMs, so it is essential to develop a more optimised technique that can handle various types of uncertainties [15, 20–24].

In this scenario, a proper decision making methodology is required to solve multiple conflicting interdependent criteria when evaluating risks in construction projects. In recent years, the number of papers related to analytic hierarchy process (AHP) and analytic network process (ANP) methods, considering supplier selection procedure, has increased substantially [25–29]. But increasing the number of criteria or comparison levels causes confusion in decision makers’ (DMs) judgments, resulting in incomplete decision making and inconsistency in the evaluator’s judgments, thus reducing strategy selection ability. Under these circumstances, Herrera-Viedma et al. proposed consistent fuzzy preference relations (CFPR) to evade inconsistency in the decision making process [30], and thus models have been developed with CFPR structuring of the problem of multi-criteria knowledge based strategy selection both with AHP [31] and ANP [32, 33]. The CFPR methodology requires less time, has computational simplicity, and also guarantees consistency of decision matrices. But as CFPR is based on complete and certain information, there always exists a possible inconsistency risk due to the inability of DMs to deal with overcomplicated objects [34].

It is obvious that the approaches mentioned above can play a vital role under some special circumstances, but it also reveals more uncertainties due to the subjective judgment of experts’ assessment. For fuzzy set theory it is difficult to determine in advance membership function values before making any decision in an uncertain and vague environment [8, 23]. Interval theory has the same deficiency. Also the frame of discernment and basic probability assignment (BPS) present in Dempster-Shafer (D-S) evidence theory limits its ability to represent incomplete information in uncertain situations [34, 35]. In order to overcome the above shortcomings and effectively handle various uncertain and incomplete information, D numbers [36], a special kind of random set, is applied in the construction of CFPR. The D-CFPR method expresses the expert’s linguistic preference values using D numbers, and can also be converted to traditional CFPR [34]. In recent years, papers related to D numbers based MCDM methods have begun to appear, viz. D-VIKOR [37], D-GRP (grey relational projection) [38], D-AHP [39, 40], D-CFPR [34], D-TOPSIS [41], and D-DEMATEL (D number based decision making trial and evaluation laboratory) [42].

Besides managing the many risks associated with construction projects, this study also attempts to categorise and assess their risk mitigation strategies, thus setting up a proper framework which is accountable to investors in construction sectors. Thus, a survey on risk mitigation strategies is performed, based on the work [9, 23, 43], to alleviate construction projects risks. Based on the above considerations, this paper also develops an extended version of the MABAC methodology in an uncertain and incomplete decision environment, to evaluate risk mitigation strategies in construction based projects. Some papers related to MABAC have been published in recent years, viz. traditional DEMATEL-MABAC [44], Pythagorean fuzzy MABAC [45], interval type 2 fuzzy MABAC [46], interval valued intuitionistic fuzzy MABAC [47], and the fuzzy AHP-MABAC model [48].

Thus, the key motivation of this paper is to develop a D numbers based ANP-MABAC decision making methodology for prioritising construction project risks, and to find suitable mitigating strategies in uncertain and incomplete decision environments. In this paper, we choose the ANP
methodology due to its ability to represent the potential interactions, interdependences, and feedback among the risk based criteria and sub-criteria. Hesamamiri et al. developed a systematic framework combining ANP and CFPR to properly assess and select a knowledge management strategy [33]. In this paper, CFPR is taken into the decision matrix under D-ANP to find the priority vectors of the criteria for uncertain and incomplete environments. Thus the main objectives of this research are as follows:

- Categorise and describe proper risk issues concerning various socio-economic, technical, and geo-political based sectors correlated to construction projects.
- Develop a logical structure incorporating a D-CFPR based ANP model for risk prioritisation during construction.
- Identify and prioritise risk response factors in the construction industry based on the D-MABAC methodology.

This paper is organised as follows: Section 2 offers an outline of construction projects along with a review of risk assessment in the sector. Section 3 briefly discusses the preliminaries of D-S evidence theory and D numbers theory, along with their properties. In Section 4, algorithmic methodologies of D-CFPR, D-ANP, and D-MABAC are discussed. Section 5 demonstrates the effectiveness of the above methodology by presenting a numerical example on risk prioritisation and responses in the construction sector. A discussion based on sensitivity analysis is given in Section 6. Finally, the conclusions and future direction of the present research are given in Section 7.

2. Risks in construction projects

2.1. An overview

The construction industry in general, as well as individual construction projects, deals with various threats which are called risks. Risk assessment in construction projects has been applied differently from project to project (using various models of risk assessment) to evaluate the risk in certain activities of the project [49]. However, the socio-economic complexity involved in construction events makes it more risk prone, so that there may be negative effects on project sustainability [23]. Due to various complex factors, the construction industry is highly diverse and heterogeneous, experiencing a great deal of dynamic change with global sourcing and increasing price competition [50].

To overcome these risks contractors have generally used high mark-ups, but this approach is no longer effective as their margins have become smaller [51]. In recent decades, the stages involved in construction projects have become far more complex in nature, due to technological upgrading and stakeholder pressure, and are characterised by a number of uncertainties which have a negative influence on the projects [52]. As risk free construction projects are impossible in real life, a controlled risk assessment procedure is required to manage various risks in the projects. The aim of this study is to highlight the main risks that construction projects face and the risk mitigation strategies used to manage them.

2.2 Previous studies on risk assessment in construction projects

Materials and Methods should be described with sufficient details to allow others to replicate and build on published results. Please note that publication of your manuscript implicates that you must make all materials, data, computer code, and protocols associated with the publication available to readers. Please disclose at the submission stage any restrictions on the availability of materials or information. New methods and protocols should be described in detail while well-established methods can be briefly described and appropriately cited.

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Interventionary studies involving animals or humans, and other studies require ethical approval.

There have been substantial developments over the last four decades in research related to construction project management. Projects have been considered that are either exposed to risks, or have apparently inherited risk due to the participation of several stakeholders (as owners, contractors and designers); see [53] among others. For classifying and managing risks effectively, many methodologies have been suggested in the literature. In this section, a list of the existing approaches to construction project risk management are now briefly summarised.

Wang et al. developed an Alien Eyes’ Risk Model, categorising the project risks and their mutual relationships, and proposing a qualitative risk mitigation framework [4]. Schieg adopted a risk management process in construction project management, concentrating more on personal area risks [17]. Zavadskas et al. presented risk assessment based on the MCDM method and applying TOPSIS grey and COPRAS-G [23]. Wen integrated rough sets and artificial neural networks for risk evaluation of construction projects [54]. Fouladgar et al. proposed a risk evaluation outline faced during tunnelling operations based on fuzzy TOPSIS methodology [55]. Taroun and Yang hybridised the Dempster-Shafer (D-S) theory of evidence and evidential reasoning algorithm for structuring personal experience and professional judgment with a spreadsheet-based decision support system [56]. Mohammadi and Tavakolan combined fuzzy logic and AHP in traditional Failure Mode Effect Analysis (FMEA) for construction project risk assessment [25]. Serpella et al. applied a knowledge-based approach, based on a three-fold arrangement and risk management function, to address project risks in the construction management sector [18]. Ebrat and Ghodsi identified the risks in construction projects based on the adaptive neuro fuzzy inference system and stepwise regression model for the evaluation of project risks [14]. Taylan et al. proposed hybrid methodologies combining fuzzy AHP and fuzzy TOPSIS and applied the relative importance index method to prioritise the project risks based on the data obtained [15]. Iqbal et al. considered two types of risk management technique during project execution, viz. preventive techniques (to manage risk before the start of the project) and remedial techniques (after occurrence of the risk) [9]. Vafadarnikjoo et al. developed an intuitive fuzzy decision making trial and evaluation laboratory (DEMATEL) to prioritise risks associated with construction projects by using the risk breakdown structure [24]. Ahmadi et al. analysed the criteria for prioritising potential risk events and quantified it using fuzzy AHP [27]. The best response action for a risk event is then identified with respect to the same criteria using a scope expected deviation index. Santos and Jungles evaluates the completion of construction project risk by considering the correlation of delay and the schedule performance index along with any time overrun [19]. Shin et al. made a comparative analysis of AHP and fuzzy AHP to evaluate the potential risk factors at the construction site of a nuclear power plant [28]. Kao et al. proposed an integrated fuzzy ANP based balanced scorecard system for evaluation of relevant bilateral factors in Taiwanese construction projects [57]. Burcar Dunovic et al. assessed large infrastructure projects by integrating the risk impact cumulative distribution curve based MCDM approach [58]. Yousefi et al. proposed a neural network model for predicting emerging time and cost claims applied to Iranian construction projects [59]. Valipour et al. presented a fuzzy cybernetic ANP model for proper identification of public-private partnership project based risks [60]. Ulubeyli and Kazaz developed a fuzzy based subcontractor selection model (CoSMo) for global based construction projects [61]. Rajakallio et al. analysed the solution delivery from a network perspective in integrated business model renewal. For assessing risk in deep foundation excavation [62], Valipour et al. developed a SWARA-COPRAS framework to analyse Iranian construction project uncertainty [63]. Khanzadi et al. solved the dispute resolution problem in the construction sector by considering a grey number based discrete zero-sum two-person matrix game model [64]. Keshavarz Ghorabae et al. presented the hybrid fuzzy SWARA-EDAS model for assessment of the construction equipment by taking sustainability into account [65].

3. Preliminaries
3.1. Dempster-Shafer (D-S) evidence theory

To deal with real world information, which may be ambiguous, various imprecise decision making models based on probability theory, fuzzy set theory, and (D-S) evidence theory have been developed [66]. D-S evidence theory directly expresses uncertain information by allocating the probability to the multiple object based subsets in lieu of individual items [67]. Due to its ability to compare pairs of evidence or belief functions when deriving new ones, and its superiority in an uncertain environment, evidence theory has been widely applied in various domains [22, 34, 39, 68-70).

Certain basic definitions are now presented. Let $\mathcal{U}$ denote the frame of discernment representing a collective set of exhaustive and mutually exclusive events, and each element of $2^\mathcal{U}$ (a power set in $\mathcal{U}$) represents a proposition [22]. Based on these, basic probability assignment (BPA) is defined as follows:

**Definition: Basic probability assignment (BPA)**

A finite non-empty set of mutually exclusive and exhaustive hypotheses for any problem domain is called its frame of discernment. In lieu of a D-S evidence framework, the BPA is defined on a frame of discernment to express the imprecise judgements of experts [39]. Thus, if $2^\mathcal{U}$ denotes the power set of a finite non-empty set $\mathcal{U}$ then a BPA is defined as a mapping $m$ satisfying (1) and (2):

\[ m : 2^\mathcal{U} \rightarrow [0, 1], \]

\[ m(\emptyset) = 0 \text{ and } \sum_{A \in 2^\mathcal{U}} m(A) = 1 \]  

where $\emptyset$ is an empty set and $A$ is any element of $2^\mathcal{U}$. If $m(A) > 0$, $A$ is called its focal element, and the union of all focal elements is the core of the mass function.

**Example:** Suppose there exists a task to assess a project. In the frame of D-S theory, the frame of discernment is the proposed set $\{b_1, b_2, b_3\}$.

As per the expert view, a BPA can be created to express their assessment result:

$m(\{b_1\}) = 0.2; m(\{b_2\}) = 0.7; m(\{b_1, b_3\}) = 0.1$ where $b_1 = [0, 40], b_2 = [41, 70], b_3 = [71, 100]$. Also, the set $B = \{b_1, b_2, b_3\}$ is a frame of discernment in D-S theory with $m(\{b_1\}) + m(\{b_2\}) + m(\{b_2, b_3\}) = 1$.

3.2. D number theory

Although D-S evidence theory is operative in the data fusion problem, it is restricted to handling semantic information where linguistic variables are not mutually exclusive [39, 70]. Intuitively, if some hypotheses of D-S theory is removed reasonably, the ability to represent and handle uncertain information may greatly improve. Overall, in many practical scenarios, the experts fail to have complete information, and the assessment is done solely on the basis of partial information, resulting in an incomplete BPA. Based on this idea, Deng proposed D numbers theory, a generalisation of evidence theory, in order to characterise ambiguous data, as defined in (3) and (4) below [36]:

**Definition: D number [36]**

Let $\Omega$ be a finite nonempty set, D number is defined by a mapping:

\[ D : \Omega \rightarrow [0, 1], \]

\[ \sum_{B \subseteq \Omega} D(B) \leq 1 \text{ and } D(\emptyset) = 0 \]
where $\emptyset$ is an empty set and $B$ is a subset of $\Omega$.

Compared with D-S evidence theory, the $D$ numbers concept has the advantages that [39, 34, 42, 71]:

- **Firstly**, $D$ numbers with nonexclusive hypothesis in each element of the frame of discernment is more applicable for linguistic assessment.
- **Secondly**, for an evidence theory, a normal BPA must be complete, implying that the sum of all focal length elements in BPA is 1. $D$ numbers allows the experts to input incomplete and uncertain information to the framework resulting in an incomplete BPA, thus releasing the completeness constraint. Thus, if $\sum_{B \subseteq \Omega} D(B) = 1$, the information is said to be complete, and for $\sum_{B \subseteq \Omega} D(B) < 1$, the information is said to be incomplete.

Example: Let an expert give their assessment on the condition of a bridge, using $D$ numbers, in a scale interval of [0,100]. The expert gives a BPA to express their assessment as follows:

$D(\{a_1\}) = 0.5; \quad D(\{a_2\}) = 0.3; \quad D(\{a_1, a_2, a_3\}) = 0.1$

where, $a_1 = [0, 60]$, $a_2 = [45, 75]$, and $a_3 = [65, 100]$.

Hence the set of $\{a_1, a_2, a_3\}$ is not a frame of discernment, because the elements in the above set are not mutually exclusive. Also, since $D(\{a_1\}) + D(\{a_2\}) + D(\{a_1, a_2, a_3\}) = 0.9 < 1$, the above information is incomplete. This example shows that the definition of $D$ numbers is similar to the definition of the mass function.

**Definition: D numbers for a discrete set** [36]

For a discrete set $\Omega = \{b_1, b_2, ..., b_i, ..., b_n\}$, where $b_i \in R$ and $b_i \neq b_j \ (i \neq j)$, a special form of $D$ numbers can be expressed as:

$D(\{b_i\}) = v_i \ (i = 1, 2, ..., n)$

or denoted as,

$D = \{(b_1, v_1), (b_2, v_2), ..., (b_i, v_i), ..., (b_n, v_n)\}$

where $v_i > 0$ and $\sum_{i=1}^{n} v_i \leq 1$.

**Definition: Permutation invariability** [22, 71].

Two $D$ numbers $D_1 = \{(b_1, v_1), ..., (b_i, v_i), ..., (b_n, v_n)\}$ & $D_2 = \{(b_1, v_1), ..., (b_i, v_i), ..., (b_n, v_i)\}$ are said to be invariable if:

$D_1 \Leftrightarrow D_2 \ , \quad (6)$

**Example:** If there are two $D$ numbers:

$D_1 = \{(0.0, 0.7), (1.0, 0.3)\}$ and $D_2 = \{(1.0, 0.3), (0.0, 0.7)\}$, then, $D_1 \Leftrightarrow D_2$

**Definition: D numbers integration** [6, 36].

Let $D = \{(b_1, v_1), (b_2, v_2), ..., (b_i, v_i), ..., (b_n, v_n)\}$ be a $D$ number, then the integration representation $I(D)$ of the $D$ number is defined as:

$I(D) = \sum_{i=1}^{n} b_i v_i \ , \quad (7)$
Example: Let \( D = \{(1, 0.3), (2, 0.4), (3, 0.1), (4, 0.2), (5, 0.2)\} \), then
\[
I(D) = 1 \times 0.3 + 2 \times 0.4 + 3 \times 0.1 + 4 \times 0.2 + 5 \times 0.2 = 3.2
\]

4. Methodology

4.1. D-CFPR: D numbers extended CFPR

Based on the additive transitive property, put forward the consistent fuzzy preference relation (CFPR) for structuring a \( n \times n \) decision matrix [30], which requires only \((n-1)\) pair wise comparisons. Preserving both the reciprocity and transitivity properties, CFPR is constructed only on complete and certain information, but fails to deal with cases containing insufficient information [34, 36]. Citing the same deficiency as in the case of fuzzy preference relations, Deng et al. proposed the concept of a \( D \) numbers preference relation encompassing the fuzzy preference relations of the \( D \) numbers domain [39]. Also, considering the insufficiency in traditional CFPRs, Zuo et al. proposed a \( D \) numbers consistent fuzzy preference relation matrix, abbreviated as D-CFPR matrix. In the following steps, we will discuss the algorithmic methodology of D-CFPR, where distribution assessments are assessed under incomplete or missing information [42].

Step 1: Construct a D-CFPR matrix \( R_\beta \). The \( D \) numbers extended CFPR is advocated to reinforce the capability of CFPR to express uncertain and incomplete information. A \( D \) numbers based preference relation on a set of criteria \( C = \{C_1, C_2, ..., C_n\} \) is characterised by a \( n \times n \) matrix \( R_D = [D_{ij}]_{n \times n} \) on the product set \( C \times C \), whose elements are formulated as per Eqn. (8), and represented in matrix form (9).

\[
R_D : C \times C \rightarrow D,
\]
\[
C_1 \quad C_2 \quad \ldots \quad C_n
\]
\[
\begin{bmatrix}
D_{11} & D_{12} & \ldots & D_{1n} \\
D_{21} & D_{22} & \ldots & D_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
D_{n1} & D_{n2} & \ldots & D_{nn}
\end{bmatrix}
\]

where, \( D_\beta = \{(b_\beta^1, v_\beta^1), (b_\beta^2, v_\beta^2), ..., (b_\beta^n, v_\beta^n)\} \), \( D_\beta = \{(1 - b_\beta^1, v_\beta^1), (1 - b_\beta^2, v_\beta^2), ..., (1 - b_\beta^n, v_\beta^n)\} \), \( \forall i, j \in \{1, 2, ..., n\} \), \( b_\beta^i \in [0, 1], v_\beta^i > 0, \sum v_\beta^i = 1 \).

Obviously, \( D_\beta = \{(0.5, 1.0)\} \) \( \forall \beta \in \{1, 2, ..., n\} \) in \( R_\beta \).

In Eqn. (9), \( R_\beta \) called a D-CFPR as it is constructed based on \((n-1)\) pair wise comparisons, denoted as \( \{D_{12}, D_{23}, ..., D_{(n-1)n}\} \) representing a set of \( D \) numbers. Considering the set \( B = \{D_\beta, 1 \leq i < j \leq n\} \), the elements of which are denoted \( D_\beta = \{(b_\beta^1, v_\beta^1), (b_\beta^2, v_\beta^2), ..., (b_\beta^n, v_\beta^n)\} \), can also be represented as follows:

\[
D_\beta = \frac{j - i + 1}{2} - D_{i(j+1)} - D_{i(j+2)} - \cdots - D_{(j-1)j}
\]

where every component \((b_\beta^i, v_\beta^i)\) is obtained by:
\[
\begin{align*}
    b^j_{ji} &= \frac{j-i+1}{2} - b_{(i+1)(i+2)}^i - b_{(i+1)(i+2)}^i - \cdots - b_{(j-1)j}^i, \quad \forall (x, y, \ldots, z) \\
    v^j_i &= \sum_{(x, y, \ldots, z) \in \Omega} v^i_{(i+1)(i+2)} \times \cdots \times v^j_{(j-1)j}, \quad (b_{(i+1)i}^i, v_{(i+1)i}^i) \text{ is the } x^{th} \\
    \text{component of } D_{(i+1)i}^i, \quad (b_{(i+1)i}^i, v_{(i+1)i}^i) \text{ is the } y^{th} \\
    \text{component of } D_{(j-1)j}^j,
\end{align*}
\]
\[(10)\]

where \(\Omega = \{x, y, \ldots, z\}\) and \(\forall (x, y, \ldots, z) \in \Omega\).

Thus, based on the reciprocal property, the rest of the entries in D-CFPR \((9)\) are calculated as per \eqref{eq:11} as follows:

\[
D_j = -D_j, \quad \forall i, j \in \{1, 2, \ldots, n\}. \quad (11)
\]

As per Eqns. \((9)-(11)\), a D-CFPR can be constructed based on \((n-1)\) pair wise comparisons \(\{D_1, D_2, \ldots, D_{(n-1)n}\}\). In the D-CFPR generated above, some values of the \(b^i_j\)'s in \(D_j, \quad i, j \in \{1, 2, \ldots, n\}\) that fall in the interval \([-a, 1+a]\), \(\alpha > 0\), need to be transformed using Eqn. \ref{eq:12}:

\[
f : [-a, 1+a] \rightarrow [0, 1], \quad f(r) = \frac{r+a}{1+2a}, \quad (12)
\]

The transformation function \(f\) works as a normalisation, which transforms the values of \(b^i_j\)'s from \([-a, 1+a]\) to the interval \([0, 1]\).

**Step 2: Determine the crisp matrix \(R_c\).** Convert the matrix \(R_p = \left[D_j\right]_{n \times n}\) in \((9)\) to a crisp matrix \(R_c = \left[c^i_j\right]_{n \times n}\) \((13)\) using the integration representation of D numbers [as per Eqn. \ref{eq:7}], where each element \(c^i_j = I(D_j)\) for \(i, j = 1, 2, \ldots, n\).

\[
R_c = \left[c^i_j\right]_{n \times n} = \begin{bmatrix}
    c_{11} & c_{12} & \cdots & c_{1n} \\
    c_{21} & c_{22} & \cdots & c_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    c_{n1} & c_{n2} & \cdots & c_{nn}
\end{bmatrix}, \quad (13)
\]

**Step 3: Construct a probability matrix \(R_p\).** Based on the crisp matrix \(R_c\) \((13)\), construct a probability matrix \(R_p = \left[p^i_j\right]_{n \times n}\) \((14)\), representing the preference probability between a pair wise set of \(n\) criteria \(C = \{C_1, C_2, \ldots, C_n\}\):  

\[
R_p = \left[p^i_j\right]_{n \times n} = \begin{bmatrix}
    p_{11} & p_{12} & \cdots & p_{1n} \\
    p_{21} & p_{22} & \cdots & p_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    p_{n1} & p_{n2} & \cdots & p_{nn}
\end{bmatrix}, \quad (14)
\]

In \(R_p\), the element \(p^i_j = \Pr(C_i > C_j), \forall i, j \in \{1, 2, \ldots, n\}\), where "\(\succ\)" represents "prefer to". Based on \((13)\), a set of rules is suggested to generate \(R_p\):  

- For the elements satisfying \(c^i_j + c^j_i = 1.0\),
  - If \(c^i_j > 0.5\), then \(\Pr(C_i > C_j) = 1\) and \(\Pr(C_i \prec C_j) = 0\),
  - If \(c^i_j \leq 0.5\), then \(\Pr(C_i > C_j) = 1\) and \(\Pr(C_i > C_j) = 0\).
• When $c_{ij} + c_{ji} > 1.0$: $c_{ij} \geq 0.5$ and $c_{ji} \geq 0.5$,
  - If $c_{ij} \geq 0.5$, then $Pr(C_i > C_j) = 1$ and $Pr(C_j > C_i) = 0$,
  - If $c_{ji} \geq 0.5$, then $Pr(C_j > C_i) = 1$ and $Pr(C_i > C_j) = 0$.
• When $c_{ij} + c_{ji} < 1.0$: $c_{ij} < 0.5$ and $c_{ji} < 0.5$,

In this condition, the unallocated preference ($up$) is $c_{ap} = 1 - (c_{ij} + c_{ji})$. The probability of one criteria outperforming another criteria is as follows:
  - If $c_{ij} < 0.5$, then $Pr(C_i > C_j) = \frac{1}{c_{ap}} (0.5 - c_{ij})$,
  - If $c_{ji} < 0.5$, then $Pr(C_j > C_i) = \frac{1}{c_{ap}} (0.5 - c_{ji})$.

**Step 4: Construct a triangular probability matrix** $R_p^T$. Rank the criteria using the triangularisation procedure, viz. by maximising the sum of the values above, the main diagonal in the $n \times n$ square matrix in the final order is given as:
  - First, sum up of each row of the $n \times n$ matrix and determine the row number with maximum value.
  - Then, assuming the obtained row number is $k$, delete the $k$-th row and $k$-th column in the matrix.
  - Replicate the above two procedures until the matrix is empty.

Thus, by applying this defined row deletion order operation on the matrix $R_p = \left[p_{ij}\right]_{n \times n}$ (14), construct the triangular probability matrix $R_p^T$, shown in (15):

$$R_p^T = \left[p_{ij}^T\right]_{n \times n} = \begin{bmatrix} p_{11}^T & p_{12}^T & \cdots & p_{1n}^T \\ p_{21}^T & p_{22}^T & \cdots & p_{2n}^T \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1}^T & p_{n2}^T & \cdots & p_{nn}^T \end{bmatrix},$$ (15)

**Step 5: Construct triangulated crisp matrix** $R_c^T$. Firstly, the crisp matrix $R_c$ (13) is triangulated based on the triangular matrix $R_p^T$ (15). From that step, the triangulated crisp matrix $R_c^T$ (16) is derived:

$$R_c^T = \left[c_{ij}^T\right]_{n \times n} = \begin{bmatrix} c_{11}^T & c_{12}^T & \cdots & c_{1n}^T \\ c_{21}^T & c_{22}^T & \cdots & c_{2n}^T \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1}^T & c_{n2}^T & \cdots & c_{nn}^T \end{bmatrix},$$ (16)

Secondly, for elements satisfying $R_c^T(i,j) + R_c^T(j,i) < 1$, a new “polishing operation” (17) is implemented to obtain a triangulated crisp matrix $R_k^T$ (18):

$$R_k^T(i,j) = R_c^T(i,j) + \frac{1}{2} \left[R_c^T(i,j) + R_c^T(j,i) - 1\right],$$ (17)

$$R_k^T = \left[k_{ij}^T\right]_{n \times n} = \begin{bmatrix} k_{11}^T & k_{12}^T & \cdots & k_{1n}^T \\ k_{21}^T & k_{22}^T & \cdots & k_{2n}^T \\ \vdots & \vdots & \ddots & \vdots \\ k_{n1}^T & k_{n2}^T & \cdots & k_{nn}^T \end{bmatrix}. $$ (18)
Step 6: Calculate the relative priority weights of the criteria. Assume that the weight vector is \( W = (w_i)^T \), where \( w_i (i = 1, 2, \ldots, n) \) are the weights of criteria \( C_i (i = 1, 2, \ldots, n) \). The elements above and alongside the main diagonal of the matrix \( R_i^T = \begin{bmatrix} k_{ij}^T \end{bmatrix}_{n \times n} \) (18), indicate the weight relationship of the criteria. Thus, by adding some necessary constraints, a set of equations (19) is formed:

\[
\lambda (w_2 - w_1) = k_{22}^T - k_{12}^T \\
\lambda (w_3 - w_2) = k_{33}^T - k_{23}^T \\
\vdots \\
\lambda (w_n - w_{n-1}) = k_{nn}^T - k_{(n-1)n}^T 
\]

where \( \lambda \) indicates the granular information about the pairwise comparison, which reveals the expert’s perceptive competency. The value of each weight \( w_i (i = 1, 2, \ldots, n) \), subjected to the parameter \( \lambda \), relates to the expert’s cognitive aptitude. The values of \( \lambda \) highly depend on the influence of the experts’ judgment and belief function. Thus, a feasible outline (20) is obtained as follows:

\[
\lambda = \begin{cases} 
\left[ \frac{\lambda}{n} \right], & \text{The information has high credibility} \\
\frac{\lambda}{n}, & \text{The information has medium credibility} \\
\frac{\lambda}{2}, & \text{The information has low credibility}
\end{cases}
\]

where \( \lambda \) signifies the lower bound of \( \lambda \), \( \left[ \frac{\lambda}{n} \right] = \min \{ k \in \mathbb{Z} | k \geq \lambda \} \), and \( n \) is the number of alternatives. The concrete priority weight value of each criteria is calculated here when \( \lambda = n \).

Step 7: Inconsistency for the D-CFPR. The traditional CFPR is fully consistent when constructed under the transitivity property for \( n-1 \) preference values. The \( D \) numbers based CFPR is consistent, but when reduced to classical CFPR it shows inconsistency under uncertain or incomplete information. Thus, to express this inconsistency of D-CFPRs, an inconsistency degree (ID) (21) is defined, based on the triangular probability matrix \( R_p^T \) (18), as follows:

\[
I.D = \frac{\sum_{i,j}^{n} R_p^T(i,j)}{n(n-1)/2},
\]

where \( R_p^T(i,j) \) is an element of \( R_p^T \), and \( n \) is the number of comparison objects. This emphasises that the inconsistency of the D matrix, and its acceptable level, is decided by the DMs, whose subjective requirement regulate the tolerance level of the aforementioned inconsistency.

4.2. Evaluating the risk criteria weight using D-ANP

The evaluation of D-ANP methodology is composed of two phases.

The first phase emphasises the formation of pairwise judgments for every dependent relationship among given criteria \( C_i (i = 1, 2, \ldots, n) \), and determination of their priority weight. The priority weights thus obtained are input to the system-with-feedback supermatrix for computing the network influences amongst the different relationships.

The second phase, namely supermatrix evaluation, incorporates five steps: formation of the unweighted supermatrix, formation of the weighted supermatrix, normalisation of the weighted supermatrix (a column stochastic matrix), and finally convergence to a solution using the limited supermatrix.
The converged supermatrix will provide us with the relative priorities for each of the criteria (or sub-criteria) considered within clusters of the decision framework. Thus the following algorithmic steps of D-ANP are as follows:

Step 1: Model construction and problem structuring. First, we properly outline the decision problem with detailed criteria and sub-criteria (if taken) and then delimit the cluster’s (dimension’s) network and elements (criteria set) within the given clusters. Next, we decide which inter- and inner-dependencies will prevail in the decision problem and clusters of the over-all feedback system.

Step 2: Pairwise comparison matrices and priority vectors. The expert inputs prerequisite for the ANP method are the pairwise judgments of the elements within each cluster, from which inter-and-inner-dependence matrices are formed. Primarily three categories of pairwise comparisons are formed. First, the inner dependence comparison matrix, based on the criteria of different clusters (dimensions), is used to get the priority weight vector. Then, the pairwise comparison matrices obtained from criteria with respect to other factors (criteria) of the same cluster (dimension). Finally, the pairwise comparison matrix is developed by considering the factor (criteria) of a cluster (dimension) among factors (criteria) of the other cluster (dimension), which affect the criteria.

These pairwise comparison matrices, along with their valuation elicitation, constitute a modified version of D-AHP [22] and D-CFPR [34]. Based on Section 4.1, a brief summary for getting the priority weight of any pairwise comparison matrix is given below, the graphical description of which is shown in Fig.1.

- In the first step, the D-CFPR matrix \( R_D = \left[ D_{ij} \right]_{n \times n} \), is constructed for \( n \) criteria, by considering the system as an input using Eqns. (9)–(12).
- The D-CFPR matrix formed \( R_p \), is converted to a crisp matrix \( R_c = \left[ c_{ij} \right]_{n \times n} \), using the integration representation of D numbers, shown in Eqn. (13).
- The probability matrix \( R_p = \left[ p_{ij} \right]_{n \times n} \) is then constructed based on the derived crisp matrix \( R_c \) using Eqn. (14), and it satisfies a set of rules in Step 3 of Section 4.1.
- In the next step, using Eqn. (15), triangularisation \( R_p^T = \left[ p_{ij}^T \right]_{n \times n} \) is applied to the probability matrix using local information that contains the preference relations of pairwise criteria.
- Lastly, applying Eqns. (16)–(19), the crisp based triangular matrix \( R_c^T = \left[ c_{ij}^T \right]_{n \times n} \), is obtained, and relative priority weights \( w_i \) \( (i = 1, 2, ..., n) \) of each criteria \( C_i \) \( (i = 1, 2, ..., n) \), based on clusters (dimensions) are calculated, thereby checking its inconsistency as per Eqn. (21).

Step 3: Formation of the unweighted supermatrix. The local priority weights resulting from the pairwise comparison are used as input in suitable columns of the unweighted constructed supermatrix, to obtain the global priorities in a system. As a result, a supermatrix takes the form of a partitioned matrix, each segment of which represents an association between two clusters in the given system. Thus, the supermatrix \( W \) [Eqn. (22)] formed is represented by sub-matrices \( W_D^\theta \) as

\[
W = \left[ W_D^\theta \right]_{n \times n} (i, j = 1, 2, ..., n).
\]

The size of the sub-matrix \( W_D^\theta \) [Eqn. (23)] depends on the compared factors (criteria). Interdependency between clusters is illustrated in Eqn. (22) by analysing both inter- and intra-relations among clusters.

Arrange all priority vectors, representing the impact of a given set of elements in a cluster on another element in the network, as sub-columns of the corresponding column of an unweighted supermatrix \( W \) as in Eqns. (22)–(23). This is composed of \( k \) clusters \( \{ D_1, D_2, ..., D_k \} \), and linkages of these clusters \( \{ e_{11}, e_{12}, ..., e_{in} \} \), are elements of the cluster \( D_p \) \( (p = 1, 2, ..., k) \). Each column of the sub-matrix \( W_D^\theta \) [Eqn. (23)] is the priority vector acquired from the identical pairwise judgment, indicating the significance of the elements in the \( i^{th} \) cluster with respect to an element in the \( j^{th} \) cluster.
Step 4: Determine the normalised weighted supermatrix. If there is no linkage between clusters \( D_k \) and \( D_l \), then the sub-matrix \( W^{il} \) equals zero. We compute the weighted supermatrix \( W = \left[ \tilde{W}^q \right]_{n \times n} \), by multiplying the unweighted matrix \( W = \left[ W^{ij} \right]_{n \times n} \), [(Eqn. (23)] by priority of dimensions \( V_j (j = 1, 2, \ldots, n) \), in this stage. As the weighted supermatrix \( W^q \) needs to be stochastic, we normalise each column of it, and develop meaningful limiting priorities for determining overall cluster influences. This normalisation procedure ensures that the weighted supermatrix is column stochastic; it is finally represented by matrix \( \tilde{W} = \left[ \tilde{W}^q \right]_{n \times n} \).

Step 5: Compute the limiting priorities for criteria weights. Finally, the column stochastic weighted supermatrix is raised to an appropriately large power until it converges. Thus, the weighted supermatrix \( \tilde{W} \), is raised to limiting powers \( \left( \tilde{W}^q \right)^{m+1} \), \( m \) being an arbitrarily large number, to attain a steady-state limiting matrix \( W^q \). Details are shown in Eqn. (24):

\[
W^q = \lim_{k \to \infty} \left( \tilde{W}^q \right)^{2m+1},
\]

The priority weight of criteria for the corresponding clusters (dimensions) can now be found in the rows of the limiting supermatrix \( W^q \). The limiting supermatrix provides the priority information for the elements of each individual cluster. The strategy outcome with the highest value should be selected from the cluster of criteria. Other priority rankings in different clusters are also provided.

4.3. D-MABAC for ranking alternatives

The MABAC methodology was developed by Pamucar and Cirovic to handle problems in MCDM [44]. \( D \) numbers is a new representation of uncertain information that can denote the more imprecise based conditions. So, the combination of MABAC and \( D \) numbers is a new experiment to make decisions in an uncertain environment. The basic setting of the MABAC technique is revealed in the definition of the distance of the criterion function of each of the observed alternatives from the approximate border area [72]. The algorithmic steps displays the execution process for the aforesaid D-MABAC methodology in six steps as follows:

Step 1: Constructing the initial decision matrix \( M \). Initially, the assessment of \( m \) alternatives in respect of \( n \) criteria is carried out. The alternatives are represented in vector form \( A_i = (d_{i1}, d_{i2}, \ldots, d_{in}) \), where \( d_{ij} (j = 1, 2, \ldots, n) \) denotes the value (in \( D \) numbers format) for the \( i \)th alternative.
alternative \((i=1,2,...,m)\) according to \(j\) criteria for matrix \(M = [d_{ij}]\). Details shown in decision-
matrix (25).
\[
M = \begin{bmatrix}
    d_1(\{1\}) = d_{11} & d_1(\{2\}) = d_{12} & \cdots & d_1(\{n\}) = d_{1n} \\
    d_2(\{1\}) = d_{21} & d_2(\{2\}) = d_{22} & \cdots & d_2(\{n\}) = d_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    d_m(\{1\}) = d_{m1} & d_m(\{2\}) = d_{m2} & \cdots & d_m(\{n\}) = d_{mn}
\end{bmatrix}, \quad (25)
\]

Next applying Eqn. (8), (i.e. integration representation of \(D\) numbers) on the elements \(d_{ij}\) \((i=1,2,...,m; j=1,2,...,n)\) of decision matrix \(M = [d_{ij}]\), crisp decision matrix \(X = [I(d_{ij})]\) is formed as per Eqn. (26) as follows:
\[
X = \begin{bmatrix}
    I(d_{11}) & I(d_{12}) & \cdots & I(d_{1n}) \\
    I(d_{21}) & I(d_{22}) & \cdots & I(d_{2n}) \\
    \vdots & \vdots & \ddots & \vdots \\
    I(d_{m1}) & I(d_{m2}) & \cdots & I(d_{mn})
\end{bmatrix}_{\text{max}} = \begin{bmatrix}
    x_{11} & x_{12} & \cdots & x_{1n} \\
    x_{21} & x_{22} & \cdots & x_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    x_{m1} & x_{m2} & \cdots & x_{mn}
\end{bmatrix}_{\text{max}}, \quad (26)
\]

**Step 2. Normalisation of the elements of the initial matrix \(X\).** The elements of the normalised matrix \(N = [t_{ij}]\), are obtained using the following expressions:
\[
N = \begin{bmatrix}
    t_{11} & t_{12} & \cdots & t_{1n} \\
    t_{21} & t_{22} & \cdots & t_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    t_{m1} & t_{m2} & \cdots & t_{mn}
\end{bmatrix}, \quad (27)
\]
\[
t_{ij} = \begin{cases} 
    \frac{x_{ij}^+ - x_{ij}^-}{x_{ij}^+ - x_{ij}^-}, & \text{for "benefit type" criteria} \\
    \frac{x_{ij}^+ - x_{ij}^-}{x_{ij}^+ - x_{ij}^-}, & \text{for "cost type" criteria}
\end{cases}, \quad (28)
\]
The components \(x_{ij}^+ = \max(x_1, x_2, ..., x_n)\), and \(x_{ij}^- = \min(x_1, x_2, ..., x_n)\), of the decision matrix \((X)\) represent the maximum and minimum values of the criteria \(C_j (j=1,2,...,n)\), by alternatives \(A_j (i=1,2,...,m)\).

**Step 3. Calculation of the elements of weighted matrix \((V)\).** The elements of the weighted matrix \(V = [v_{ij}]\), are calculated using the following equations:
\[
V = \begin{bmatrix}
    v_{11} & v_{12} & \cdots & v_{1n} \\
    v_{21} & v_{22} & \cdots & v_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    v_{m1} & v_{m2} & \cdots & v_{mn}
\end{bmatrix}, \quad (29)
\]
\[
v_{ij} = w_j \cdot (t_{ij} + 1), \quad (30)
\]
where \(t_{ij}\) are the elements of the normalised matrix \(N = [t_{ij}]\) and \(w_j (j=1,2,...,n)\) the weight coefficients of criteria, respectively.
Step 4. Determine the approximate border area matrix (G). The elements of the border approximation area (BAA) for each criterion $C_j$ ($j = 1, 2, ..., n$), are determined as follows:

$$g_j = \left( \prod_{i=1}^{m} v_{ij} \right)^{1/m},$$  \hspace{1cm} (31)

where $v_{ij}$ are the elements of the weighted matrix $V = [v_{ij}]_{m \times n}$, and $m$ is the total number of alternatives. After calculating the value $g_j (j = 1, 2, ..., n)$, for each criterion, the BAA matrix $(G)$ is formed with format $n \times 1$ ($n$ = the total number of criteria, according to which the selection is made from the alternatives).

$$G = [g_1, g_2, \ldots, g_n]_{1 \times n},$$  \hspace{1cm} (32)

Step 5. Calculation of the distance of the alternative from the BAA for the matrix elements $(Q)$. The distance $(q_{ij})$ of the alternatives $A_i (i = 1, 2, ..., m)$ from the BAA matrix $G = [g_1, g_2, \ldots, g_n]$, is determined as the difference between the elements $v_{ij} (i = 1, 2, ..., m; j = 1, 2, ..., n)$ in the weighted matrix $V = [v_{ij}]_{m \times n}$, and the value $g_j (j = 1, 2, ..., n)$ of the BAA $(G)$. The distance $q_{ij} (i = 1, 2, ..., m; j = 1, 2, ..., n)$ are elements of a matrix $Q = [q_{ij}]_{m \times n}$ and are shown in Eqn. (33):

$$Q = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1n} \\ q_{21} & q_{22} & \cdots & q_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ q_{m1} & q_{m2} & \cdots & q_{mn} \end{bmatrix} \quad \text{where each } q_{ij} = v_{ij} - g_j,$$

Alternative $A_i (i = 1, 2, ..., m)$ can belong to BAA $(G)$, upper approximation area $(G^+)$, or lower approximation area $(G^-)$, i.e. $A_i \in \{ G \lor G^+ \lor G^- \}$ and are determined as per Eqn. (34):

$$A_i \in \begin{cases} G^+ & \text{if } q_{ij} > 0 \\ G & \text{if } q_{ij} = 0 \\ G^- & \text{if } q_{ij} < 0 \end{cases},$$  \hspace{1cm} (34)

Step 6. Ranking of the alternatives. A calculation of the values of the criterion functions for the alternatives is obtained as the sum of the distance of the alternatives from the BAA$(q_{ij})$. Using Eqn. (35), we calculate the sum of the elements $q_{ij}$ of matrix $(Q)$ by rows, and obtain the final values of the criterion functions $\hat{S}_i (i = 1, 2, ..., m)$, for the alternatives $A_i (i = 1, 2, ..., m)$:

$$\hat{S}_i = \sum_{j=1}^{n} q_{ij}, \hspace{0.5cm} i = 1, 2, ..., m; \hspace{0.5cm} j = 1, 2, ..., n,$$  \hspace{1cm} (35)

5. Numerical example: risk assessment in a construction project

5.1. Identification of construction projects risk indicators and their mitigation strategies

For categorising and managing construction project risks effectually, several methodologies are recommended in the literature [43, 73]. Keeping this in mind, we proposed the hybrid D-ANP-MABAC approach for the risk assessment of projects in the construction sector involving uncertain and incomplete information data. The authors employed a combination of questionnaire surveys.
involving literature reviews and subjective judgments of highly proficient experts to detect different
risk response strategies that optimise the performance of construction projects. The proposed
methodology incorporates the knowledge and experience of ten experts (five technical and five
management based experts) for risk identification and structuring, along with proper risk mitigation
strategies. The demographic profile of the respondents is given in Table 1.

Based on the literature review of papers discussed in Table 2, the DMs consider nine construction
project risk criteria identified under three dimensions: political instability (C1), economic risk (C2), and
social risk (C3). These are defined as environmental based external risks (D1). Technological risk (C4),
work quality risk (C5), and time and cost risk (C6) are defined as construction process based project
risks (D2). Resource risk (C7), documents and information risk (C8), and stakeholder’s risk (C9) are
defined under intrinsic criteria based internal risks (D3). The details shown in Table 3.

In past decades, construction businesses were constrained to use only a limited number of risk
management procedures, even though they were not suitable for all situations. For example, Lyons
and Skitmore found that brainstorming is the most common risk identification technique used in the
Queensland engineering construction industry [50]. Forbes et al. developed a matrix for selecting
appropriate risk management techniques such as artificial intelligence, probabilistic analysis,
sensitivity analysis, and decision trees in the built environment for each stage of risk management
[74]. In this paper, based on the brainstorming method, the experts sought five alternative
construction project risk response mitigation strategies, as detailed in Table 4.

5.2. Calculating risk based criteria weight using D-ANP framework

In this section, the weights of the risk based criteria in construction projects are calculated by D-
ANP. The ANP has substantial influence in MCDM problems involving a wide range of factors and
sub-factors. In the ANP, a decision problem is transformed into a network structure that allows both
inter-intra dependency and feedback among the decision clusters, and even amongst elements within
the same clusters. In this phase, the decision group is asked to make pairwise comparison matrices
for priority weights of three dimensions and nine criteria (as detailed in Table 3). Using Steps 1–6 (of
Section 4.1) of D-CFPR, the priority weights are calculated in the decision matrices. The algorithmic
steps of D-ANP are shown below:

Step 1: Construction of the hierarchy of criteria and alternative risks strategies. The clearly defined risk
based construction project model is decomposed into a logical system like a network. Based on the
hierarchy of Fig. 2, we have five risk mitigation strategies $A_i (i = 1, 2, ..., 5)$ in Level 4 hierarchical
position, three dimensions $D_j (j = 1, 2, 3)$ (external, project, and internal risk) in Level 2, and
corresponding to each dimension, a total of nine criteria $C_{ij} (j = 1, 2, ..., 9)$ in Level 3.

Step 2: Determination of the pairwise comparison matrices and priority vectors within clusters. The D
numbers based preference matrix is first constructed before calculating the weight of the indicators
using the ANP supermatrix. Here, we present the process of determining the priority weight of
criteria (C1, C2, and C3) for risk dimension external risk with respect to the criterion C6 under dimension
Project risk.

First, we calculate the relative significance of sub-criteria (C1, C2, and C3) relative to sub-criterion
C6, to construct the inner dependence matrix built on the D numbers based preference relation
$R_p = [D_{ij}]_{3x3}$. In this case, the preference modelled by a set of D numbers (based on DM’s choice)
involving both uncertain entry (i.e., $D_{ij} = \{(0.55,0.7),(0.65,0.3)\}$ and incomplete entry
$D_{ij} = \{(0.75,0.8)\}$, respectively.

$$R_p = \begin{bmatrix}
D_{11} & D_{12} & D_{13} \\
D_{21} & D_{22} & D_{23} \\
D_{31} & D_{32} & D_{33}
\end{bmatrix} = \begin{bmatrix}
\{(0.5,1.0)\} & \{(0.55,0.7),(0.65,0.3)\} & - \\
- & \{(0.5,1.0)\} & \{(0.75,0.8)\} \\
- & - & \{(0.5,1.0)\}
\end{bmatrix}$$
The standard CFPR cannot handle this case, but D-CFPR (Eqns. (9)–(12) in Section 3.1) is effective to fill up the rest of the matrix elements in $R_D = [D_{ij}]_{3x3}$ as follows:

$$R_D = \begin{bmatrix}
{(0.5,1.0)} & {(0.55,0.7),(0.65,0.3)} & {(0.80,0.56),(0.9,0.24)} \\
{(0.45,0.7),(0.35,0.3)} & {(0.5,1.0)} & {(0.75,0.8)} \\
{(0.2,0.56),(0.1,0.24)} & {(0.25,0.8)} & {(0.5,1.0)} \\
\end{bmatrix}.$$  

Then, the D numbers based CFPR matrix $R_D = [D_{ij}]_{3x3}$ is converted to a crisp mode, $R_c = [C_{ij}]_{3x3}$, using Eqn. (13), and the integration representation of D numbers [Eqn. (7)], shown below:

$$R_c = I(R_D) = \begin{bmatrix} 0.5 & 0.58 & 0.66 \\
0.42 & 0.5 & 0.60 \\
0.13 & 0.20 & 0.5 \end{bmatrix}.$$

Applying the preference rules proposed for D-CFPR (in Step 3 of Section 4.1) and using Eqn. (14), the probability matrix $R_p = [P_{ij}]_{3x3}$ is constructed:

$$R_p = \begin{bmatrix} 0 & 1 & 1 \\
0 & 0 & 1 \\
0 & 0 & 0 \end{bmatrix}.$$

Following the process (as mentioned in section 4.1 of Step 4) of the D-CFPR methodology, we obtain the triangular matrix $R_p^T = [P_{ij}^T]_{3x3}$. Using the triangularisation method, the ranking of the indicators is calculated and shown as: $I_1 > I_2 > I_3$, where the symbol "\( > \)" indicates preference, $R_p^T = \begin{bmatrix} 0 & 1 & 1 \\
0 & 0 & 1 \\
0 & 0 & 0 \end{bmatrix}$.

We next evaluate the relative weights of the risk criteria. First, based on the ranking of the risk criteria in the triangulated matrix $R_p^T$, the crisp matrix $R_c$, is converted to a triangular crisp matrix $R_c^T$, as per Eqn. (16):

$$R_c^T = \begin{bmatrix} 0.5 & 0.58 & 0.66 \\
0.42 & 0.5 & 0.60 \\
0.13 & 0.20 & 0.5 \end{bmatrix}.$$

Next, for elements satisfying $R_c^T(i,j) + R_c^T(j,i) < 1$, a new polishing operation [Eqn. (17)] is executed and, by also applying Eqn. (18), a novel triangulated crisp matrix $R_k^T = [k_{ij}]_{3x3}$ is obtained:

$$R_k^T = \begin{bmatrix} 0.5 & 0.58 & 0.76 \\
0.42 & 0.5 & 0.70 \\
0.21 & 0.24 & 0.5 \end{bmatrix}.$$

Finally, applying Eqn. (19), a group of equations is built to calculate the priority weight $w_i (i = 1, 2, \ldots, n)$ of each risk based criterion. Applying the weight relation of the indicators in matrix
mode, and incorporating necessary constraints, the weight equations are constructed and shown
below:

\[
\begin{align*}
\lambda (w_i - w_j) &= 0.58 - 0.5 \\
\lambda (w_j - w_k) &= 0.76 - 0.5 \\
w_i + w_j + w_k &= 1 \\
\lambda &> 0 \\
w_i &\geq 0, \; \forall i \in \{1, 2, 3\}
\end{align*}
\]

where \( w_i \) denotes the weight of the \( i^{th} \) indicator and \( \lambda \) indicates the granular information about the pairwise evaluation, which is connected to the cognitive aptitude of the experts. Setting \( \lambda = 3 \) and using Eqn. (20), the weight of risk criteria \( C_1, C_2, \) and \( C_3 \) for dimension \( D_1 \) relative to \( C_6 \) of dimension \( D_2 \) are calculated as \( (0.373, 0.347, 0.280) \), respectively.

For quantifying the consistency of the D-CFPR based matrix \( R^T \), an ID defined for the \( D \) numbers preference relation [as defined in Eqn. (21)] is used to express such inconsistency, and for the case study taken, it is found to be consistent.

Similarly, using the same process, the priority weights of remaining criteria, shown in Fig. 3 with respect to the same dimensions and criteria of other clusters (dimensions), are calculated.

**Step 3: Formation of unweighted supermatrix.** Arrange all priority vectors, indicating the influence of pre-set elements, in different cluster elements in the network, as sub-columns of the resultant column in an unweighted supermatrix \( W \), which is composed of \( k \) clusters \( D_k \) \((k = 1, 2, 3)\), with corresponding linkages (criteria) \( \{e_{11}, e_{12}, e_{13}\} \) for \( k = 1, 2, 3 \). Putting \( k = 1 \), we get the first cluster (dimension) \( D_1 \) along with three elements \( \{e_{11}, e_{12}, e_{13}\} \) representing three risk criteria \( \{C_1, C_2, C_3\} \), respectively. For \( k = 2 \), we get a second cluster (dimension) \( D_2 \) along with three elements \( \{e_{21}, e_{22}, e_{23}\} \) representing three risk criteria \( \{C_4, C_5, C_6\} \). Similarly, the remaining cluster (dimensions) \( D_3 \), along with its risk criteria, can be found. Thus, we get three clusters (dimensions) \( D_k = \{D_1, D_2, D_3\} \) and nine corresponding risk criteria \( C_j \) \((j = 1, 2, ..., 9)\). Based on the above, the unweighted supermatrix \( W \) is formed by placing priority vector elements in the particular column, where each criterion influences the other risk criteria. Details shown in Table 5.

**Step 4: Calculating the weighted supermatrix.** The weighted supermatrix \( (W^w) \) is calculated by multiplying unweighted supermatrix \( W \) (Table 5) by the inner dependence matrix of risk dimension \( D_j \) \((j = 1, 2, 3)\) (Table 6). Details are shown in Table 7.

**Step 5: Selecting the weight of criteria based on the limit matrix.** To make the matrix column stochastic in Table 7, we normalise the weighted supermatrix \( (W^w) \) column wise, and the result is shown in Table 8. The normalised weighted supermatrix \( \hat{W} \) (Table 8) is raised to its limiting power using Eqn. (22), to get the limiting supermatrix \( W^q \) (Table 9). The final ranking of risk criteria weight for the construction project is shown in Table 10.

From Table 10 it is concluded that the third cluster (dimension) internal risk \( (D_3) \) has a severe risk effect on the construction project sector. Document and information risk \( (C_9) \) is the most risky, followed by resource risk \( (C_7) \) and stakeholder’s risk \( (C_8) \). First cluster External risk \( (D_1) \) has less of a risk effect on construction business. Economic risk \( (C_2) \) and social risk \( (C_3) \) are in 8th and 9th position, respectively. But political instability attains 4th position in respect of risk category. Managers and stakeholders should keep this in view when choosing projects in large construction sectors.

5.3. Determination of final alternative ranking by D-MABAC
In this phase, the evaluation and ranking of risk response alternatives is performed by the application of a D numbers based MABAC (D-MABAC) methodology in construction project risk management. The step-by-step computational procedure is shown below.

Step 1: First, the five risk response alternative vectors, with respect to nine risk criteria $(j=1,2,...,9)$, are represented as $A_i = (d_{1i}, d_{2i}, ..., d_{9i})$ $(i=1,2,...,5)$ using incomplete and uncertain numbers expressed in D numbers. Using Eqns. (23)–(24), we develop an initial decision matrix $M = \left[ d_{ij} \right]_{5 \times 9}$ (Table 11) along with its crisp form $X = \left[ x_{ij} \right]_{5 \times 9}$.

Step 2: The elements of the crisp decision matrix $X = \left[ x_{ij} \right]_{5 \times 9}$ are normalised using Eqns. (24)–(25) to form a normalised decision matrix $N = \left[ n_{ij} \right]_{5 \times 9}$, shown in Table 12.

Step 3: Using Eqns. (26)–(27), the elements of the weighted normalised decision matrix $V = \left[ v_{ij} \right]_{5 \times 9}$ are calculated, and are shown in Table 13.

Step 4: Next, using Eqns. (28)–(29) we determine the BAA, $G = \left[ g_1, g_2, ..., g_9 \right]$ for each criterion $C_j$ $(j=1,2,...,9)$. This is followed by calculation of the distance $(q_i)$ for matrix elements $Q = \left[ q_{ij} \right]_{5 \times 9}$ for risk response alternatives $A_i$ $(i=1,2,...,5)$ from BAA $G = \left[ g_1, g_2, ..., g_9 \right]$ using Eqns. (30)–(31).

Step 5: Finally, using Eqn. (32) we calculate the sum function $\hat{S}_i (i=1,2,...,5)$ to obtain the ranking of alternatives $A_i$ $(i=1,2,...,5)$. A graphical representation of the process is shown in Fig. 4.

6. Results and discussion

In this section, a detailed comparative analysis of all alternative initiatives (with respect to criteria and dimensions) is conducted.

6.1. Comparison of alternative ranking using different MCDM methods

The hybrid MCDM methods viz. D-MABAC, D-TOPSIS, D-COPRAS, and D-ARAS were applied to the construction project based case study data to obtain the weighted normalised decision making matrix $V = \left[ v_{ij} \right]_{5 \times 9}$ (Table 13). The priority order of the risk response alternatives $A_i$ $(i=1,2,...,5)$ is compared and presented in Table 15. Ranking of the risk response alternatives according to the presented MCDM methods concluded that the optimal alternative risk response is $A_1$: Proper scheduling for getting updated project information, followed by $A_3$ (Information about local partner's credibility), $A_4$ (Transfer risks with other parties), $A_2$ (Adjust plans for scope of work). The worst performing risk response alternative is $A_5$ (Merger and diversification of projects).

Spearman’s rank correlation coefficient $r_s$ between ranks is applied for determining correlation of ranks obtained by various approaches. Here, this coefficient is applied to demonstrate the statistical importance of difference among the ranking obtained through pairwise correlation analysis of different MCDM methods. Based on the recommendation of Keshavarz Ghorabaee et al. [65], all $r_s$ values higher than 0.80 show considerably high correlation. As per Table 16, a strong correlation (1.000) among the MCDM approaches is shown, confirming the credibility of the proposed approach.
6.2. Sensitivity analysis

Ranking of results in MCDM problems are subject to the distribution of weight coefficients of the criteria. Sometimes, modifying these criteria weight coefficients may change the ranking order of alternatives, generally analysed by sensitivity analysis during the decision making process. The above weight coefficients are usually based on expert subjective perception, and thus the outcome of probable deviation of these weight values need to be properly assessed.

A sensitivity analysis was executed to measure the level of crosstalk amongst the criteria, revealing the variation in alternative rankings as per variation in criteria weight. Outcomes of the sensitivity analysis for prioritising specific project based criteria \(C_j\) \((j = 1, 2, \ldots, 9)\) weights are shown in Table 16, and its corresponding effect on ranking of risk response alternatives \(A_i\) \((i = 1, 2, \ldots, 5)\) in Table 17.

- The results (Table 16 and Fig. 5) shows that assigning various weights to project based criteria \(C_j\) \((j = 1, 2, \ldots, 9)\) through different scenarios \(\{S_1, \ldots, S_9\}\) results in changes to ranking of individual alternatives, thus proving that the model is sensitive to variations in weight coefficients.

- Analysis of the alternative ranking through eight scenarios (Table 17) showed that alternative \(A_1\) retained its rank in five scenarios \(\{S_2, S_3, S_4, S_5, S_6\}\) (best-ranked alternative), while in the remaining two scenarios \(\{S_1, S_7\}\) it was ranked second, and third in scenario \(S_8\).

- The worst-ranked alternative \(A_5\) retained its rank in six scenarios \(\{S_2, S_3, S_4, S_5, S_7, S_8\}\), while in two scenarios \(\{S_1, S_6\}\) it was ranked second worst. Therefore, changing the criteria weights through different scenarios resulted in changes to the ranks of the remaining alternatives.

- In addition, from Tables 16–17, it is clear that prioritising criteria \(C_9\) has less of an effect on ranking position of alternatives. However, prioritising criteria set \(\{C_4, C_6, \text{and } C_8\}\) in scenario \(S_7\), \(\{C_2, C_6, \text{and } C_7\}\) in scenario \(S_6\), along with \(\{C_4, C_6, \text{and } C_8\}\) in scenario \(S_8\) all altered the positions of risk response alternatives \(A_i\) \((i = 1, 2, \ldots, 5)\).

- The prioritising of criteria weight in scenarios \(\{S_2, S_3, S_4, S_5, S_6\}\) has no effect on ranking of best or worst risk response alternative \(A_1\) and \(A_5\), respectively, but it does have an effect on the ranking of the second best risk response alternative \(A_2\).

7. Conclusions and future research

In this paper, we rank uncertain risk strategies in construction projects using a D-ANP-MABAC multi-criteria decision making (MCDM) approach. In the new proposed method, the decision matrix determination from the MCDM problem is transformed to D numbers, which effectively represent the inevitable uncertainties such as incompleteness and imprecision, due to the subjective assessment of decision makers. By using D-CFPR decision matrices it allows all stakeholders (members) of a construction business to address multiple criteria involving various types of uncertainties, such as imprecision and fuzziness in the decision process. The weights of the risk criteria for any cluster are determined by the D-ANP method and alternative risk response strategy ranking is achieved using the D-MABAC method. In view of several categories of uncertainties, including incompleteness and imprecision, the proposed technique can effectively represent and address uncertain information weighting risk criteria and alternatives in a logical way. An example of selecting risk strategies in construction risk projects is shown here to demonstrate the efficiency of the proposed approach. The assessment of a real world application of D-ANP-MABAC methodology, along with its output result of sensitivity, clearly identifies its potential to provide stable solutions to the problematic choice of laying-up positions. Based on that, the proposed risk strategy alternatives are successfully ranked. Thus, it can be concluded that the above procedure has provided an alternate approach for sustainable risk analysis and decision making in the construction sector. In future research, the theoretical framework needs to be further perfected and applied to other real-life application areas such as supplier selection problems, project portfolio management, renewable energy selection etc., to further validate its effectiveness.
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**Author Contributions:**

The individual contribution and responsibilities of the authors were as follows:

- Kajal Chatterjee - designed the research, methodology, performed the development of the paper,
- Krishnendu Adhikary- collected and analyzed the data and the obtained results,
- Edmundas Kazimieras Zavadskas- provided extensive advice throughout the study,
- Jolanta Tamošaitienė- regarding the research design, revised the manuscript.
- Samarjit Kar- methodology, findings.

All the authors have read and approved the final manuscript.

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