Article

Chaotic Synchronizing Systems with Zero Time Delay and Free Couple via Iterative Learning Control

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Abstract: This research not only dedicated a less restrictive method of iteration-varying function for a learning control law to design a controller but also synchronize two nonlinear systems with free time-delay. In addition, the mathematical theory of system synchronization has proved rigorously and the theory verified through an example to demonstrate the behavior of each parameter in the theory. The design of a controller using the iterative learning control law is significant for robotic tracking. The controller in this research generates a feed-forward control input using the error dynamics among the drive-response systems. The error dynamics satisfies the Lyapunov function and the combination of output errors, which respectively represented relative estimated differences of the drive-response systems. The iterative learning control rule serves the function of a filter adding previous control error after the end of each iteration. The numerical example of a synchronous system is given a Lorenz system for driving and another with the iterative learning control law for response under different initial condition. The results verify and demonstrate the proposed mathematical theory. The simulation exhibits consistency in the behavior of each parameter to match mathematical theory.

Keywords: synchronization; chaos; chaotic system; Iterative Learning Control (ILC); Lyapunov function; error convergent

1. Introduction

The concept of Iterative Learning Control (ILC) theory [1] takes the errors of a system repeatedly executing similar tasks into consideration to improve overall performance by learning previous information of the original system. The system's learning control regards the same multiple operations under various operating conditions [1-2]. To conduct a betterment process for a mechanical robot is the original principle of ILC in 1984 [3]. The ILC differs from other learning control systems, such as repetitive control, adaptive control, and neural network [2] that it will adjust the input signal according to previous output of the same system whereas to modify the controller is an example of an adaptive control stages [2-3]. Instead, many studies described the iterative control process by mphasizing its periodic control and not considering its "Learning" process [2].

ILC is applicable to many fields in system modeling, PID-control, nonlinear dynamical system, and system synchronization as well as in the academic field [4-8]. Maria [9] also proposed the iterative method by using many theories of classical linear system to approximate nonlinear systems. Recently, the ILC theory has also employed chaotic secure communication into encrypt and decrypt a message [4]. ILC also becomes the focus of many industrial systems for manufacturing, robotics, and assembly line entailing repetition of mass production [1-2].

The goal of designing ILC controller is to generate a feed-forward control signal as an appropriate tracking reference to proceed or deny a repeating distance, which can improve the performance of systems and achieve low tracking error, while the tracking error exists on every transient-time repeated work [1]. Arimoto in the [3] examined the PID-type learning algorithm of betterment process during the operation of robots and ensured the error of system was convergent.

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Kuc [5] proposed the learning rule of nonlinear dynamic systems was through each iteration of linear feedback by uniformly bounded state error to track reference input. The iterative method of classical linear system theory was introduced by Mara [10] to approximate the nonlinear systems.

The betterment effect of iterative process [3] shows convergence in the vector-norm of errors, but the iterative learning control is not unique to non-linear system. The Lyapunov function provides the sufficient condition for convergence [2-3]. The tracking error is the asymptotically stable system is a bounded function constructed by Lyapunov function [1]. The Lyapunov function is the most commonly used strategies to study the stability of a control system. The Lyapunov function in [11] serves to guarantee the designed salve system in asymptotical synchronization with the master system.

In this research, the principle of learning operator is in Hauser [12], and the Lyapunov function of synchronization system follows the method by Zhou [13] for stability. The restrictive synchronization criteria of the Lyapunov function exhibited in [14]. The existence and stability conditions of two different continuous chaotic systems found in [16] and the synchronization manifold of two unidirectional systems showed equivalent state vectors indicating the possible perfect system synchronization [17]. The error norm in the dynamics error system would be monotonically convergent when the Markov parameters were used to find the time-varying learning gain [20], that such error convergence in dynamics systems was monotonic and independent of the iteration time duration as shown in [21]. Therefore, the system with couple and time-delay to synchronize was verified the stability by the Lyapunov function [13 -21].

This research dedicated a less restrictive method of iterative control learning law for the design of controller to synchronize two distinct nonlinear systems, which are free time-delay and non-couple. The rigorously proof of each parameter is in relative mathematical theory, such as the iterative learning control law is bounded and non-increasing, the dynamics error system is stable, and the tracking error between the drive and response systems is convergent. The results of example indicated the behavior of parameters in the synchronous process. The numerical example for synchronization of drive-response systems in which the drive is the Lorenz system [23] and response is the similar system with the iterative learning law, respectively. Their initial conditions are different. The results of example verify the theory in this paper and demonstrate the effectiveness of proposed concept.

The relevant studies herein include the followings: (1) the description of synchronization system; (2) the mathematically proof of related theory and proposed the scheme of iterative learning control; and (3) the simulation results of example as verification of the proposed mathematical theory in exhibiting the performance of ILC algorithm for system synchronization. Finally, this paper derives a conclusion and recommendation for future works.

2. The ILC Problem Formulation

2.1. System Description

The drive and response systems are chaotic systems to synchronize with zero time delay and free couple and the general formula of systems are described by following equations equation (1a) and equation (1b), respectively.

$$\begin{aligned} \dot{\boldsymbol{x}}_{\boldsymbol{m}}(t) &= f(\boldsymbol{x}_{\boldsymbol{m}}, t) = \boldsymbol{A}(x_m^i)\boldsymbol{x}_{\boldsymbol{m}}(t), \\ \boldsymbol{y}_{\boldsymbol{m}}(t) &= \boldsymbol{C}_{\boldsymbol{m}}\boldsymbol{x}_{\boldsymbol{m}}(t), \end{aligned} \tag{1a}$$

The response system adjusts the error between the drive and tracking systems approaching synchronization using the control input $\boldsymbol{Bu}^{(\boldsymbol{k})}$.

$$\begin{aligned} \dot{\boldsymbol{x}}_{\boldsymbol{s}}^{(k)}(t) &= f\left(\boldsymbol{x}_{\boldsymbol{s}}^{(k)}, t\right) + \boldsymbol{Bu}^{(k)}(t) = \boldsymbol{A}(x_m^i)\boldsymbol{x}_{\boldsymbol{s}}^{(k)}(t) + \boldsymbol{Bu}^{(k)}(t), \\ \boldsymbol{y}_{\boldsymbol{s}}^{(k)}(t) &= \boldsymbol{C}_{\boldsymbol{s}}\boldsymbol{x}_{\boldsymbol{s}}^{(k)}(t), \end{aligned} \tag{1b}$$

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The state vectors $\boldsymbol{x_m(t)}$, $\boldsymbol{x_s^{(k)}(t)}$ and the outputs $\boldsymbol{y_m}$ and $\boldsymbol{y_s^{(k)}}$ are of the drive-response systems in state space $\boldsymbol{R^n}$, respectively. Originally, $\boldsymbol{f(x_m, t)}$ and $\boldsymbol{f\left(x_s^{(k)}, t\right)}$ are similar systems. The response system with the iterative control input signal of $\boldsymbol{Bu^{(k)}(t)}$ and the output $\boldsymbol{y_s^{(k)}}$, after the *k*-th learning of iteration. The C_m , C_s, and $B = B^T$ are nonsingular constant matrices with appropriated dimensions and some entries of the nonsingular polynomial matrix $\boldsymbol{A(x_m^i)}$ are replaced by the *i*-th component of $\boldsymbol{x_m(t)}$ to be the factor of system synchronization, where i = 1, 2, 3. The input signal sequence $\{\boldsymbol{u^{(k)}(t)}\}_{\boldsymbol{k=1,2,\ldots}}$ in $\boldsymbol{R^m}$ is the control learning law after the *k*-th iterative learning for the response system synchronizing the drive system.

$\boldsymbol{\Delta}^{(k)} = (x_m - x_s^{(k)})^T$ is the synchronization error and the output error of drive-response system is given $\boldsymbol{y_m} - \boldsymbol{y_s^{(k)}} = \boldsymbol{C_m x_m} - \boldsymbol{C_s x_s^{(k)}}$. The output error is equal to the synchronization error when $\boldsymbol{C_m} = \boldsymbol{C_s} = \boldsymbol{1}$ is set. The dynamical system synchronization error between drive-response systems can be described in (1a) and (1b).

$$\dot{\boldsymbol{\Delta}}^{(k)} = \dot{\boldsymbol{x}}_{\boldsymbol{m}}(t) - \dot{\boldsymbol{x}}_{\boldsymbol{s}}^{(k)}(t) = \boldsymbol{A}(x_m^i)\left(\boldsymbol{x_m} - \boldsymbol{x_s^{(k)}}\right)^{\boldsymbol{T}} + \boldsymbol{Bu}^{(k)}(t) = \boldsymbol{A}(x_m^i)\boldsymbol{\Delta}^{(k)} + \boldsymbol{Bu}^{(k)}(t), \quad (2)$$

The the appropriate $\boldsymbol{Bu}^{(k)}(t)$ is given and the minus in (2) is absorbed by $\boldsymbol{B}$. The limitation of synchronization error must approach to zero that is $\lim_{n\to\infty} \boldsymbol{\Delta}^{(k)} = \left(x_s^{(k)} - x_m\right)^T = 0$, and the error dynamics should be less than or equal to zero, that is $\dot{\Delta} \leqq 0$, when iteration learning procedure applies to response system to track drive system in the time interval [0, T] after sufficiently large iterative number *k*.

The characters between drive and response systems are the drive system to be reference system and the tracking system for a response system in synchronization procedure, respectively. The response system traces the trajectory of the drive system by employing the output information of drive system. In order to achieve the goal of synchronization and search a system whose trajectory is closed to drive system, it is necessary to find an estimated system similar to drive system. The estimated system in [21] can be defined the measuring system of drive system as (3).

$$\dot{\boldsymbol{x}}_{\boldsymbol{m}}(t) = \boldsymbol{H}(\boldsymbol{x}, t) + \boldsymbol{\varepsilon} = \boldsymbol{A}(x_m^i)\hat{\boldsymbol{x}}_{\boldsymbol{m}}(t) + \boldsymbol{\varepsilon}, \quad (3)$$

The nonlinear problem is no general solution. The perturbation and linearization techniques will be applied to the equation (3). Least square linear estimation is familiar to minimize the errors in measure processes. The error criterion of equation (3) defined as

$$\boldsymbol{E}(\boldsymbol{\varepsilon}) = \boldsymbol{\varepsilon}^{\boldsymbol{T}}\boldsymbol{\varepsilon} = \left(\dot{\boldsymbol{x}}_{\boldsymbol{m}}(t) - \boldsymbol{A}(x_m^i)\hat{\boldsymbol{x}}_{\boldsymbol{m}}(t)\right)^{\boldsymbol{T}}\left(\dot{\boldsymbol{x}}_{\boldsymbol{m}}(t) - \boldsymbol{A}(x_m^i)\hat{\boldsymbol{x}}_{\boldsymbol{m}}(t)\right), \quad (4)$$

The minimization of ***E*** is to differentiate ***E*** with respect to the state vector $\hat{\boldsymbol{x}}_{\boldsymbol{m}}$ and equate to the result as:

$$\begin{gathered}\frac{\partial \boldsymbol{E}}{\partial \hat{\boldsymbol{x}}_{\boldsymbol{m}}} = -\boldsymbol{2}\left(\dot{\boldsymbol{x}}_{\boldsymbol{m}}{}^{\boldsymbol{T}}(t)\boldsymbol{A}(x_m^i)\right) + \boldsymbol{2}\left(\hat{\boldsymbol{x}}_{\boldsymbol{m}}(t)^{\boldsymbol{T}}\boldsymbol{A}(x_m^i)^{\boldsymbol{T}}\boldsymbol{A}(x_m^i)\right) = \boldsymbol{0}, \\ \hat{\boldsymbol{x}}_{\boldsymbol{m}}(t) = \left(\boldsymbol{A}(x_m^i)^{\boldsymbol{T}}\boldsymbol{A}(x_m^i)\right)^{-\boldsymbol{1}}\boldsymbol{A}(x_m^i)^{\boldsymbol{T}}\,\dot{\boldsymbol{x}}_{\boldsymbol{m}}{}^{\boldsymbol{T}},\end{gathered} \quad (5)$$

The parameter $\hat{\boldsymbol{x}}_{\boldsymbol{m}}(\boldsymbol{t})$ is the minimum value of the scale of error ***E***.

The design of the Lyapunov function reaches the synchronization of the chaotic system whose manifold $\boldsymbol{x_m} = \boldsymbol{x_s^{(k)}}$ must be stable [13-17]. This fact indicates systems synchronization of (1a) and (1b) through ILC procedure so that the error dynamics as (2) is stable by Lyapunov criterion and the local Lipschitz condition is satisfied during the period of the system.

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Lemma 1. The equation (2) has a trivial solution by the ILC procedure. $f(\boldsymbol{x_m}, t)$ (1a) and $f(\boldsymbol{x}_s^{(k)}, t)$ in (1b) are satisfied local Lipschitz condition in the interval [0, T].

Proof:

By the ILC procedure, there is a trivial solution $\boldsymbol{\Delta}^{(k)} = (\boldsymbol{x_m} - \boldsymbol{x}_s^{(k)})^T = 0$ which implies δ > 0 for all ε > 0 and k = 0, 1, 2,… such that $\|f(\boldsymbol{x_m}, t) - f(\boldsymbol{x}_s^{(k)}, t)\| < \boldsymbol{\varepsilon}$ as $\|\boldsymbol{x_m} - \boldsymbol{x}_s^{(k)}\| < \boldsymbol{\delta}$ which means the condition $\|f(\boldsymbol{x_m}, t) - f(\boldsymbol{x}_s^{(k)}, t)\| < \boldsymbol{\varepsilon}\|\boldsymbol{x_m} - \boldsymbol{x}_s^{(k)}\|$ is held. The $f(\boldsymbol{x_m}, t)$ is convergent to $f(\boldsymbol{x}_s^{(k)}, t)$ when $\boldsymbol{\Delta}^{(k)} = (\boldsymbol{x_m} - \boldsymbol{x}_s^{(k)})^T$ approaches to zero. Basically, the consequence of this lemma implies that $\|f(\boldsymbol{x_m}, t) - f(\boldsymbol{x}_s^{(k)}, t)\| < \boldsymbol{\varepsilon}\delta \to 0$.

□

The convergence of synchronization error, $\boldsymbol{\Delta}^{(k)}$, indicates the error dynamics $\dot{\boldsymbol{\Delta}}^{(k)}$ is non-increasing that is $\dot{\boldsymbol{\Delta}}^{(k)} \leq 0$ and dependent on the iterative learning control law $\boldsymbol{u}^{(k)}(t)$ in (1a) is chosen as

$$\boldsymbol{u}^{(k)}(t) = \boldsymbol{B_1}\boldsymbol{u}^{(k-1)}(t) + \boldsymbol{B_2}(\widehat{\boldsymbol{\Delta}}_{\boldsymbol{m}} + \widehat{\boldsymbol{\Delta}}_s^{(k)}) = \boldsymbol{B_1}\boldsymbol{u}^{(k-1)}(t) + \boldsymbol{B_2}\boldsymbol{\Delta}^{(k)}, \tag{6}$$

where the $\boldsymbol{B_1}$ and $\boldsymbol{B_2}$ are appropriate constant matrices and symmetry. The learning law $\boldsymbol{u}^{(k-1)}$ is previous of $\boldsymbol{u}^{(k)}$. $\widehat{\boldsymbol{\Delta}}_{\mathbf{m}} = (\boldsymbol{x_m} - \widehat{\boldsymbol{x}}_{\boldsymbol{m}})^T$ and $\widehat{\boldsymbol{\Delta}}_s^{(k)} = (\widehat{\boldsymbol{x}}_{\boldsymbol{m}} - \boldsymbol{x}_s^{(k)})^T$ are the errors between the estimated state vectors and the state vectors of $\boldsymbol{x_m}$ and $\boldsymbol{x}_s^{(k)}$, respectively.

□

The sum of errors of equation (6) is $\widehat{\boldsymbol{\Delta}}_{\mathbf{m}} + \widehat{\boldsymbol{\Delta}}_s^{(k)} = \boldsymbol{\Delta}^{(k)}$. If the error dynamics in the equation (2) is convergent then the iterative learning control law $\boldsymbol{u}^{(k)}(t)$ is decremented. The completed proof is going to exhibit in lemma 2.

Lemma 2. If the error dynamics in the equation (2) is convergent then the iterative learning control law in the equation (6) is non-increasing function.

Proof:

From equation (5) and lemma 1, the error in (2) can be rewrite as following

$$\boldsymbol{\Delta}^{(k)} = (\boldsymbol{x_m} - \widehat{\boldsymbol{x}}_{\boldsymbol{m}})^T + (\widehat{\boldsymbol{x}}_{\boldsymbol{m}} - \boldsymbol{x}_s^{(k)})^T = \widehat{\boldsymbol{\Delta}}_{\boldsymbol{m}} + \widehat{\boldsymbol{\Delta}}_s^{(k)}, \tag{7}$$

Suppose that $(\boldsymbol{x_m} - \widehat{\boldsymbol{x}}_{\boldsymbol{m}})^T \leq \boldsymbol{\varepsilon}_1^T$ and $(\widehat{\boldsymbol{x}}_{\boldsymbol{m}} - \boldsymbol{x}_s^{(k)})^T \leq \boldsymbol{\varepsilon}_2^T$, take the $\boldsymbol{\varepsilon} = \boldsymbol{Max}\{\boldsymbol{\varepsilon}_1^T, \boldsymbol{\varepsilon}_2^T\}$, the equation (7) can be rewritten the detail as $\boldsymbol{\Delta}^{(k)} = (\boldsymbol{x_m} - \widehat{\boldsymbol{x}}_{\boldsymbol{m}})^T + (\widehat{\boldsymbol{x}}_{\boldsymbol{m}} - \boldsymbol{x}_s^{(k)})^T = \widehat{\boldsymbol{\Delta}}_{\boldsymbol{m}} + \widehat{\boldsymbol{\Delta}}_s^{(k)} \leq \boldsymbol{2\varepsilon}$. The difference of iterative learning control law is $\boldsymbol{u}^{(k)}(t) - \boldsymbol{B_1}\boldsymbol{u}^{(k-1)}(t) = \boldsymbol{B_2}\boldsymbol{\Delta}^{(k)} \leq \|\boldsymbol{B_2}\boldsymbol{\Delta}^{(k)}\| \leq \|\boldsymbol{B_2}\|\|\boldsymbol{\Delta}^{(k)}\| = 0$ which means the sequence $\{\boldsymbol{u}^{(k)}(t)\}_{\boldsymbol{k=1,2,,\cdots}}$ is non-increasing because the error $\boldsymbol{\Delta}^{(k)}$ is convergent to zero.

□

The analytical approximation of the chaotic systems synchronizing the trajectory in equations (1a) and (1b) is the Lyapunov stable investigating [13-17]. The Lyapunov criterion introduced in the theorem 1 is a positive-definite function with non-time delay and free couple of the system (2).

Theorem 1. The iterative learning control law is chosen as the equation (6). The Lyapunov function can be defined as

$$\boldsymbol{V}^{(k)}(t) = \frac{\mathbf{1}}{\mathbf{2}}(\boldsymbol{\Delta}^{(k)})^{\boldsymbol{T}}(\boldsymbol{\Delta}^{(k)}) + \boldsymbol{\mu}\int_{t_{k-1}}^{t_k}(\boldsymbol{u}^{(k)})^{\boldsymbol{T}}(\boldsymbol{u}^{(k)})d\tau, \tag{8}$$

(a). When $\boldsymbol{\mu} = \mathbf{0}$, $\boldsymbol{V}^{(k)}(t) = \frac{1}{2}(\boldsymbol{\Delta}^{(k)})^{\boldsymbol{T}}(\boldsymbol{\Delta}^{(k)})$ is the Lyapunov function of an estimation system in the system (3).

(b). If $\boldsymbol{V}^{(k)}(t)$ is Lyapunov function of the system (2) then the system should be stable.

Proof:

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First part of proof is to prove the part (a) in this theorem. The derivative of the function $\boldsymbol{V}^{(k)}(t)$ along the track of the system (2) was in [24] discussion and written as

$$\dot{\boldsymbol{V}}^{(k)}(t) = \left(\boldsymbol{\Delta}^{(k)}\right)^{\boldsymbol{T}}\left(\dot{\boldsymbol{\Delta}}^{(k)}\right), \tag{9}$$

By using the lemma 1, the equation (9) has a trivial solution. The derivative of Lyapunov function is equal to zero or negative, which implies that the system (2) is stable.

Next, the proof of part (b) is a general case of a chaotic system with no time-delay and free couple. The derivative of the function $\boldsymbol{V}^{(k)}(t)$ along the track of the system (2) is introduced in [13 -17] and following as:

$$\dot{\boldsymbol{V}}^{(k)}(t) = \left(\boldsymbol{\Delta}^{(k)}\right)^{\boldsymbol{T}}\left(\dot{\boldsymbol{\Delta}}^{(k)}\right) + \boldsymbol{\mu}\left[\left(\boldsymbol{u}^{(k)}(t_k)\right)^{\boldsymbol{T}}\left(\boldsymbol{u}^{(k)}(t_k)\right) - \left(\boldsymbol{u}^{(k)}(t_{k-1})\right)^{\boldsymbol{T}}\left(\boldsymbol{u}^{(k)}(t_{k-1})\right)\right]$$

$$= \left(\boldsymbol{\Delta}^{(k)}\right)^{\boldsymbol{T}}\left(\dot{\boldsymbol{\Delta}}^{(k)}\right) + \boldsymbol{\mu}\left[\left(\boldsymbol{u}^{(k)}(t_k)\right)^{2} - \left(\boldsymbol{u}^{(k)}(t_{k-1})\right)^{2}\right]. \tag{10}$$

The first term in equation (10) proved in the equation (8) and the second term should be equal to zero or negative when the iterative learning control law is a non-increasing function. When the iterative control learning is divergent, the Lyapunov function of the dynamical system (1a) and (1b) would be divergent and the system (2) cannot be stable.

□

In the proof of the theorem, it is important to determine the learning control law, $\boldsymbol{u}^{(k)}(t)$, in Lyapunov function applied in the more complex system. The decision as to what suitable for iterative learning control law and parameters $\boldsymbol{B_1}$ and $\boldsymbol{B_2}$ to reduce the divergence of non-linear systems should be discussed and studied the synchronization of non-linear systems.

2.2. Proposed algorithm for Iterative Learning Control Law

The iterative learning control algorithm exhibits in figure 1. The diagram contains three systems, namely the drive system, the response system, and the estimated system, with three outputs, namely the output of drive system, the output of response system and the output of error, respectively. The initial conditions of the drive system and the response system are different. The iterative learning control law of the first stage exhibits the error of initial conditions between the drive system and the response system. The estimation system in the equation (3) provides for the estimated state vectors as expressed equation (4), to drive and response systems, respectively. The drive system and the response system are in closed-loop that the feedback in the former is the own output of drive system and the feedback in the latter is the result of iterative learning control law as the own output of response system.

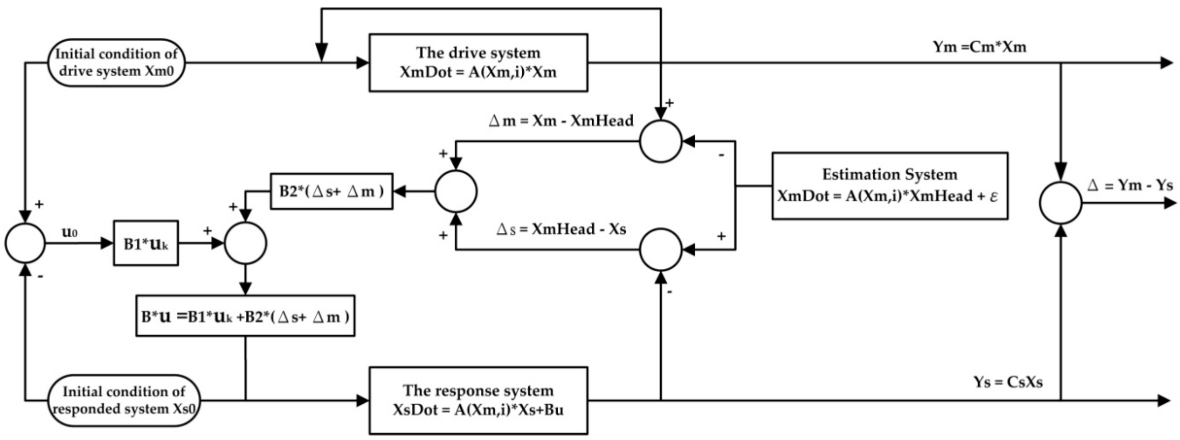

Figure 1. The iterative learning control algorithm

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The algorithm in the figure 1 of examines learning control input $\boldsymbol{u}^{(k)}$ is bounded convergence and satisfies the criteria of monotonically convergent conditions. The learning control input $\boldsymbol{u}^{(k)}$ in the equation (6) is concerned with the ability to adjust the feedback error of response system and track the trajectory of the drive system. Therefore, iterative learning control law, $\boldsymbol{u}^{(k)}$, must be bounded.

Corollary 1. The learning control input $\boldsymbol{u}^{(k)}$ in the equation (6) is monotonic decreasing and bounded.

Proof:

The learning control law in equation (5) is a updated law to refresh the input of the system (1b) proposed. The sequence $\{\boldsymbol{u}^{(k)}(t)\}_{\boldsymbol{k}=\mathbf{1},\mathbf{2},,\cdots}$ is non-increasing sequence such that the condition $\boldsymbol{Max}\{\|\boldsymbol{u}^{(k)}(t)\|\}_{k=1,2,,\cdots} = \|\boldsymbol{u}^{(0)}(t)\| \leq M$ is held with an upper bounded M of real number having in lemma 2.

The appropriate matrices $\boldsymbol{B}$, $\boldsymbol{B}_1$, and $\boldsymbol{B}_2$ making the sequence $\{\boldsymbol{u}^{(k)}(t)\}_{\boldsymbol{k}=\mathbf{1},\mathbf{2},,\cdots}$ is strictly decreasing are important. The ILC law $\{\boldsymbol{u}^{(k)}(t)\}_{\boldsymbol{k}=\mathbf{1},\mathbf{2},,\cdots}$ can be expanded it in initial learning law $\boldsymbol{u}^{(0)}(t)$ by induction as follows:

$$\boldsymbol{u}^{(k)}(t) = (\boldsymbol{B}_1)^k\boldsymbol{u}^{(0)}(t) + (\boldsymbol{B}_1)^{k-1}\boldsymbol{B}_2\boldsymbol{\Delta}^{(1)} + (\boldsymbol{B}_1)^{k-2}\boldsymbol{B}_2\boldsymbol{\Delta}^{(2)} + \cdots + \boldsymbol{B}_2\boldsymbol{\Delta}^{(k)}, \tag{11}$$

The learning operator $\boldsymbol{L}$ in this research follows the method of Hauser [12] as:

$$\boldsymbol{L} \equiv (\boldsymbol{B} * \boldsymbol{B}_1^k) \tag{12}$$

$$\|\boldsymbol{I} - \boldsymbol{L}\boldsymbol{B}_2\| \leq \delta < 1. \tag{13}$$

The consequence is from the monotonically decreasing sequence $\{\boldsymbol{u}^{(k)}(t)\}_{\boldsymbol{k}=\mathbf{1},\mathbf{2},,\cdots}$ of the ILC rule. The $\boldsymbol{u}^{(0)}$ is the maximum in the monotonically decreasing sequence $\{\boldsymbol{u}^{(k)}(t)\}_{\boldsymbol{k}=\mathbf{1},\mathbf{2},,\cdots}$ and $\|\boldsymbol{L}\boldsymbol{B}_2\| \leq 1$ in equation (13) . Theses matrices can be found by Linear Matrix Inequality (LMI) method, but not main object in this research.

□

3. Example Illustration and Demonstrated Results

The example in this section is going to demonstrate the results of synchronization approach, investigate the non-linear drive-response systems with free time-delay and non-couple, and synchronize two non-linear systems. The drive system is expressed by a Lorenz system as follows [23] and the response system is another with the ILC input.

3.1. The Example of Iterative Learning Algorithm to Decide Learning Law

In order to exhibit the synchronization of two non-linear systems and verify the algorithm of iterative learning control law in Figure 1, the drive-response systems with non-identical initial conditions are given by follows:

$$\begin{aligned}\dot{\boldsymbol{x}}_{\boldsymbol{m}}(t) &= f(\boldsymbol{x}_{\boldsymbol{m}}, t) = \boldsymbol{A}(x_m^1)\boldsymbol{x}_{\boldsymbol{m}}(t), \\ &= \begin{bmatrix} -10 & 10 & 0 \\ 30 & -1 & x_m^1 \\ 0 & x_m^1 & -{}^8/_3 \end{bmatrix}\boldsymbol{x}_{\boldsymbol{m}}(t), \\ \boldsymbol{y}_{\boldsymbol{m}}(t) &= \boldsymbol{x}_{\boldsymbol{m}}(t), \qquad \boldsymbol{x}_{\boldsymbol{m0}}(t=0) = (0.02, 0.01, 0.03),\end{aligned} \tag{15}$$

and

$$\dot{\boldsymbol{x}}_{\boldsymbol{s}}^{(k)}(t) = f\left(\boldsymbol{x}_{\boldsymbol{s}}^{(k)}, t\right) + \boldsymbol{B}\boldsymbol{u}^{(k)}(t) = \boldsymbol{A}(x_m^1)\boldsymbol{x}_{\boldsymbol{s}}^{(k)}(t) + \boldsymbol{B}\boldsymbol{u}^{(k)}(t) \tag{16}$$

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$$= \begin{bmatrix} -10 & 10 & 0 \\ 30 & -1 & x_m^1 \\ 0 & x_m^1 & -{}^{8}/_{3} \end{bmatrix} \boldsymbol{x_s}(t) + \boldsymbol{Bu}^{(k)}(t),$$

$$\boldsymbol{y}_s^{(k)}(t) = \boldsymbol{x}_s^{(k)}(t), \qquad \boldsymbol{x}_{s0}^{(0)} = (2, 1, 3),$$

From the drive-response system, the error dynamical system is expressed as

$$\dot{\boldsymbol{\Delta}}^{(k)} = \boldsymbol{A}(x_m^i)\left(\boldsymbol{x_m} - \boldsymbol{x}_s^{(k)}\right)^{\boldsymbol{T}} = \boldsymbol{A}(x_m^i)\boldsymbol{\Delta}^{(k)} + \boldsymbol{Bu}^{(k)}(t),$$

$$= \begin{bmatrix} -10 & 10 & 0 \\ 30 & -1 & x_m^1 \\ 0 & x_m^1 & -{}^{8}/_{3} \end{bmatrix} \boldsymbol{\Delta}^{(k)} + (-\boldsymbol{1}) * \boldsymbol{Bu}^{(k)}(t), \tag{17}$$

In this example, setting the $\boldsymbol{C_m} = \boldsymbol{C_s} = \boldsymbol{1}$, the output error is $\boldsymbol{y_m} - \boldsymbol{y}_s^{(k)} = \boldsymbol{\Delta}^{(k)} = \left(\boldsymbol{x_m} - \boldsymbol{x}_s^{(k)}\right)^T$ in $\boldsymbol{R}^3$. The paramaters are explained as $\boldsymbol{x_m} = (x_{m1}, x_{m2}, x_{m3})^T$, $\boldsymbol{x}_s^{(k)} = \left(x_{s1}^{(k)}, x_{s2}^{(k)}, x_{s3}^{(k)}\right)^T$, $and\ \dot{\boldsymbol{\Delta}}^{(k)} = \left(\dot{\boldsymbol{x}}_{\boldsymbol{m}}(t) - \dot{\boldsymbol{x}}_s^{(k)}(t)\right)^T$. The x_m^1 in polynomial matrix $\boldsymbol{A}(x_m^1)$ is the first component of the state vector in the drive system. The state vector in estimation system is $\hat{\boldsymbol{x}}_{\boldsymbol{m}}(t) = \left(\boldsymbol{A}(x_m^1)^{\boldsymbol{T}}\boldsymbol{A}(x_m^1)\right)^{-1}\boldsymbol{A}(x_m^1)^{\boldsymbol{T}}\,\dot{\boldsymbol{x}}_{\boldsymbol{m}}{}^{\boldsymbol{T}}$ and the iterative learning law with the initial condition $\boldsymbol{u}^{(0)}(t=0) = \boldsymbol{\Delta}^{(0)}(t=0) = \left(\boldsymbol{x_{m0}} - \boldsymbol{x}_{s0}^{(0)}\right)^T$. The $\boldsymbol{u}^{(k)}(t)$ in equation (6) and Lyapunov equation in equation (8), respectively. The matrices $\boldsymbol{B1}$ and $\boldsymbol{B2}$ in the ILC rule of equation (6) are as:

$$\boldsymbol{B1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-\lambda t_k} \end{bmatrix} \text{ and } \boldsymbol{B2} = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & -{}^{3}/_{8} \end{bmatrix}, \tag{18}$$

The metrix **B1** has an entry $e^{-\lambda t_k}$ with the minimal eigenvalue λ of polynomial matrix $\boldsymbol{A}(x_m^i)$. The matrix **B** in the equation (1b) is easy to choose the identity matrix. The results were conducted the simulations with MATLAB to verify the performance of ILC rule. The drive system is used an ode45 funtion in simulink and the response system is found using the the Euller method with the estimated state vectors of equation (5). The relevant simulation results show in the next section.

3.2. Simulation Results and Discussion

The trajectories in two-dimensional x1, x3-space of the drive system in black dash line, the response system by estimating in red, and the estimation of drive system in blue show in the figure 2, respectively. The initial condition of drive system differs from others. The trajectory of estimated systems quickly approach to the drive system after their initial condition but the approximation is not excellent, just as the iterative learning controlled law is not perfect.

According to the equation (2) and lemma 2, the error dynamics $\dot{\boldsymbol{\Delta}}^{(k)}$should be less than or equal to zeor and the demonstration in the figure 3. The two previous components in $\dot{\boldsymbol{\Delta}}^{(k)}$ are always negative satisfied the error convergence criterion. Nevertheless, the third component is vibration in the bounded interval around between 100 and 200 as well as to verify the bounded error of each iteration and design appropriate controller by ILC rule in equation (2) and lemma 2.

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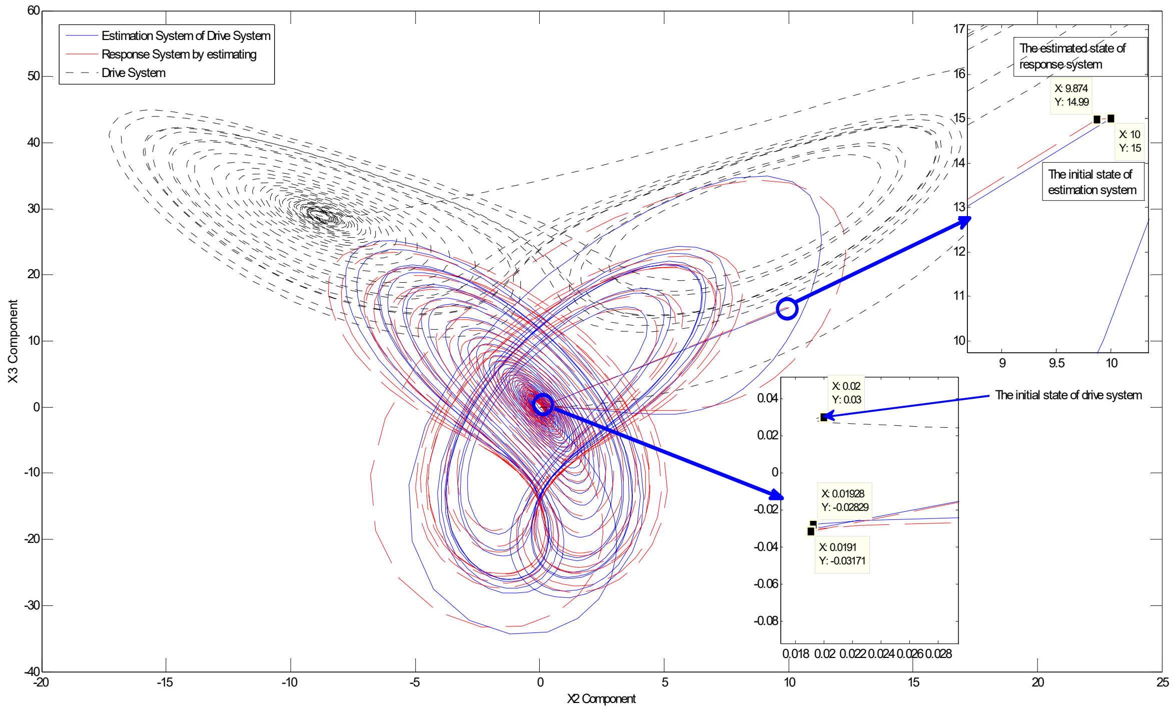

Figure 2. The trajectories in two-dimensional x1, x3-space of systems.

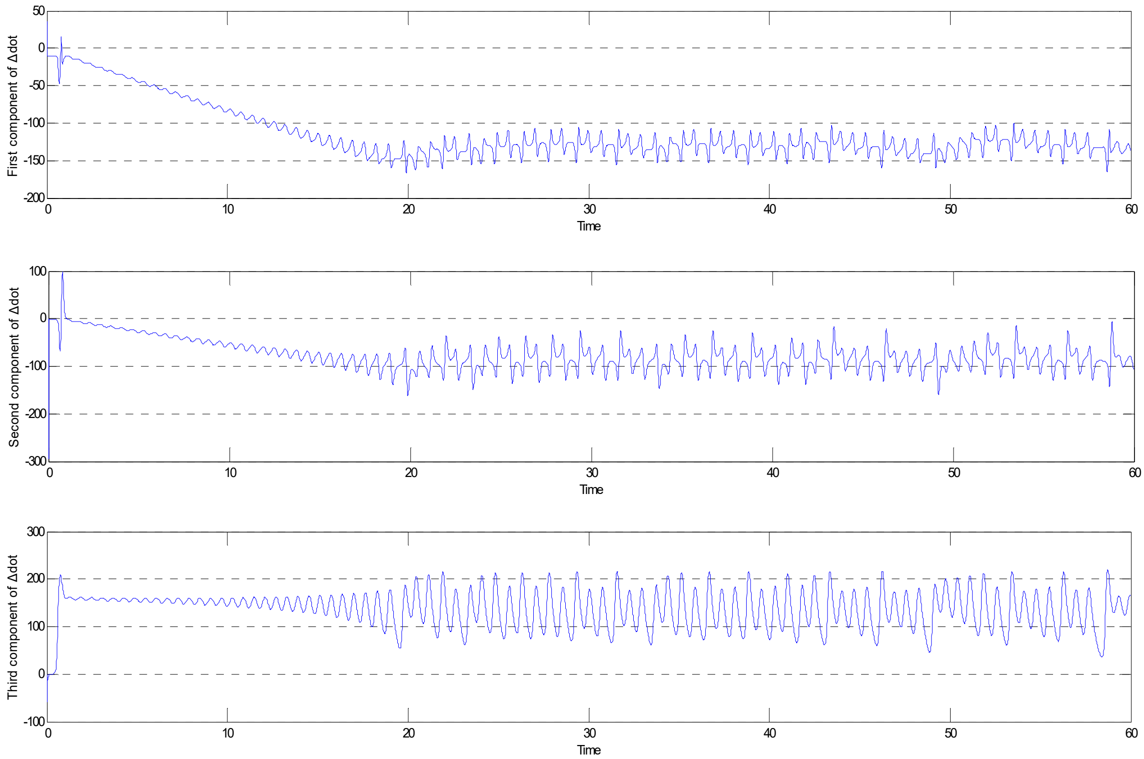

Figure 3. The simulation of each component of the error dynamics system $\dot{\Delta}^{(k)}$

The behaviors of ILC rule in the Figure 4 show two components are always decreasing and X3-component is non-increasing. The simulation results of equation (6) verify the lemma 2 and corollary 1. The behaviors are not identical for different ILC rule showed in the figure 5. The Figure 5a is chosen the ***B*1** is identity matrix and ***B*1** = [0.1, 0.1, 0.92] in Figure 5b, respectively. One of components is rose sharply and then the vibrating in a bounded interval and others are smoothly decreased in the

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previous ILC rule. In latter ILC rule exhibits one of components is more vibration in a bounded interval than previous and the other decreasing components are speed bumps.

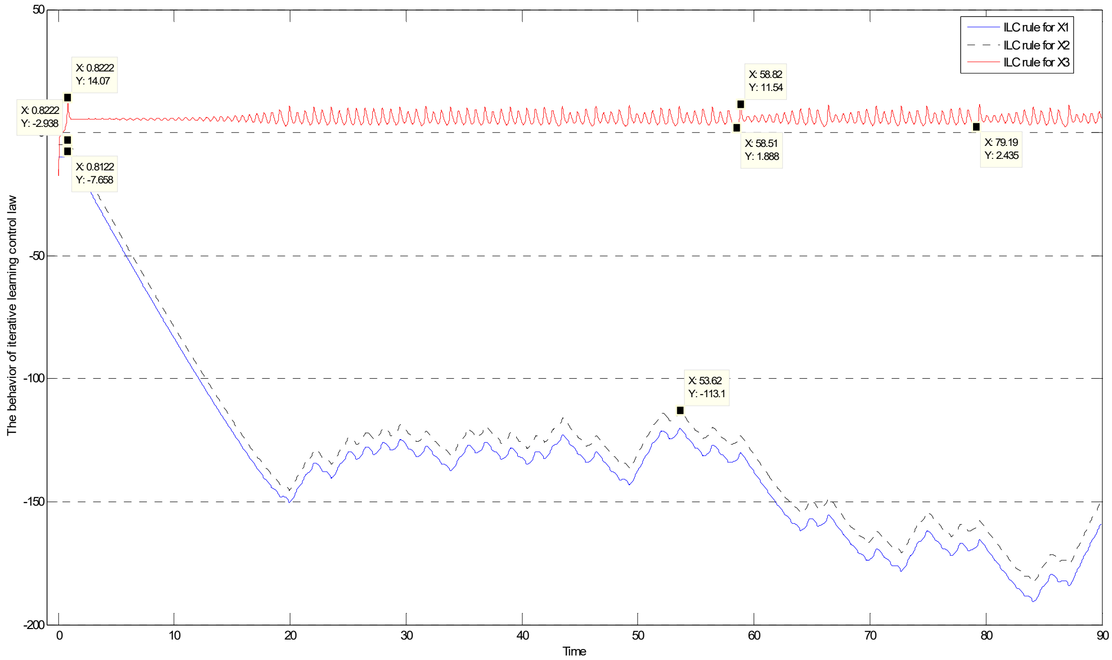
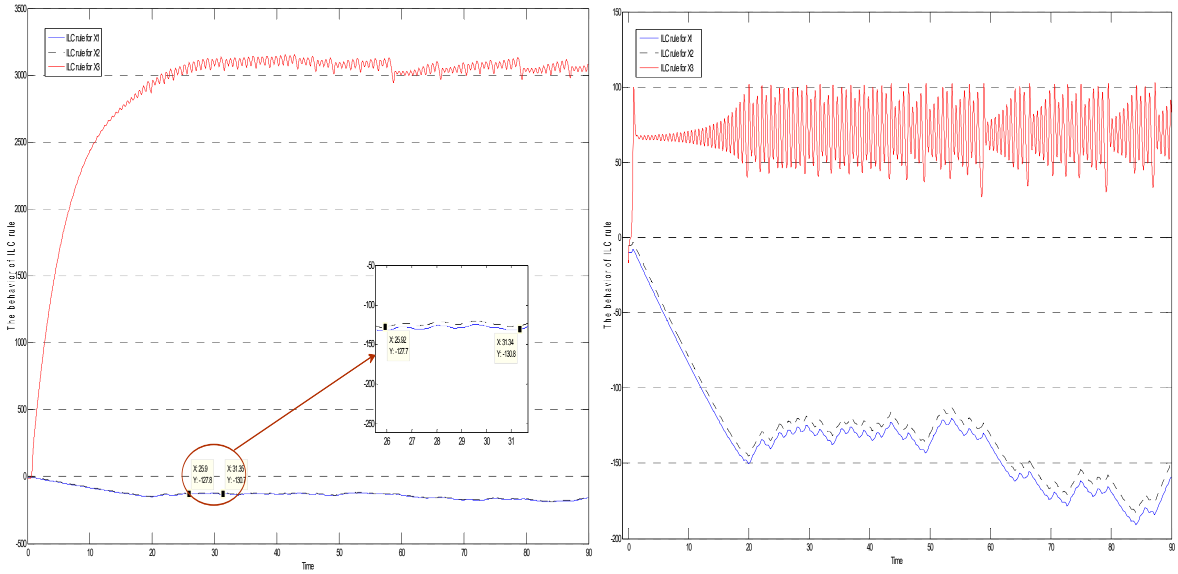


Figure 4. The brhavior of the iterative learning control law.



(**a**) (**b**)

Figure 5. The different behaviors of ILC with the different matrix ***B*1** is list as: (**a**) ***B*1** is identity matrix; (b) $B1 = [0.1, 0.1, 0.92]$.

The behavior of the derivative of Lyapunov function demonstrates in Figure 6 and the how many time step of derivation Lyapunov function is less than zero or negative in the table 1, but the number is not relative to the different parameters μ in equation (10). The obvious phenomenon is to regard the ILC rule to find an appropriate linear combination in equation (6) and the negative derivative of Lyapunov function in equation (10). Lyapunov function is positive and non-increasing function by the conditions of lemma 2 and corollary and the demonstration in Figure 6 has approached the consequences and proved theorem 1. The curve in the Figure 7 demonstrates the behavior of Learning operator of ILC rule by using the formula in equation (12) in which the operator decreases rapidly to stable and verifies the equation (14) is held.

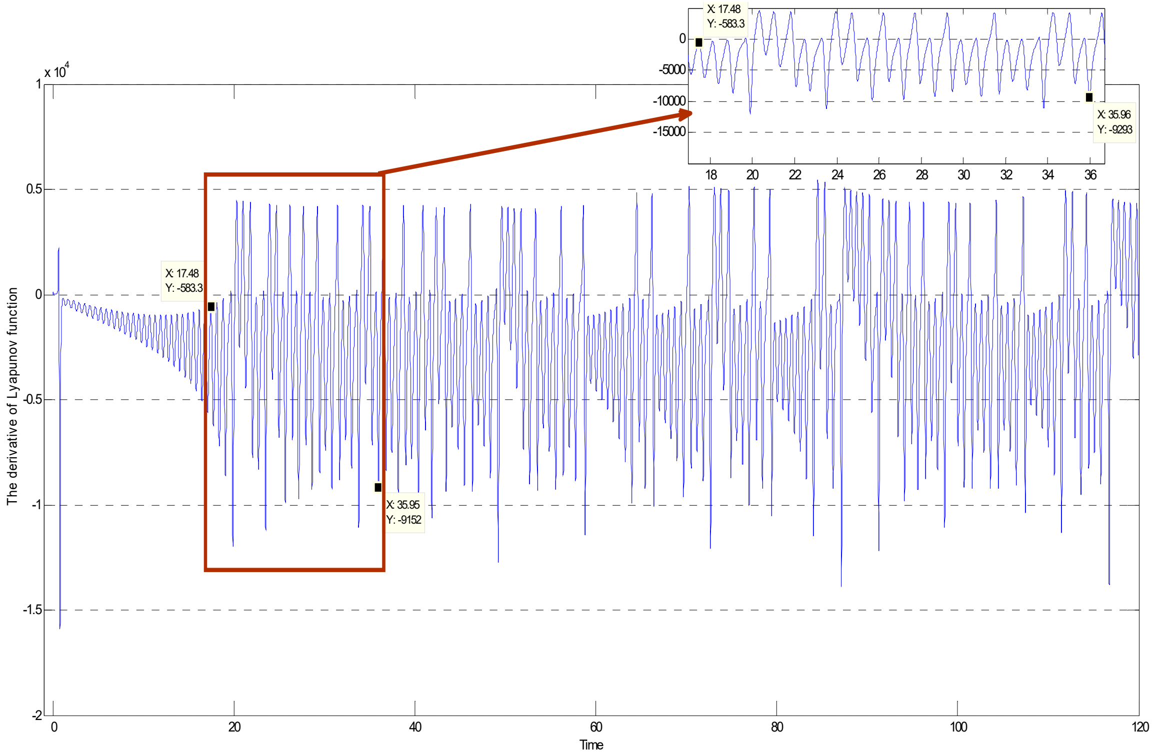


Figure 6. The behavior of the derivative of Lyapunov function.

Table 1. The number of negative Lyapunov function with different values of μ.

Value of μ	Negative	Positive	Total
1	9623	2377	12000
3	9637	2377	12000
10	9637	2373	12000
100	9629	2371	12000

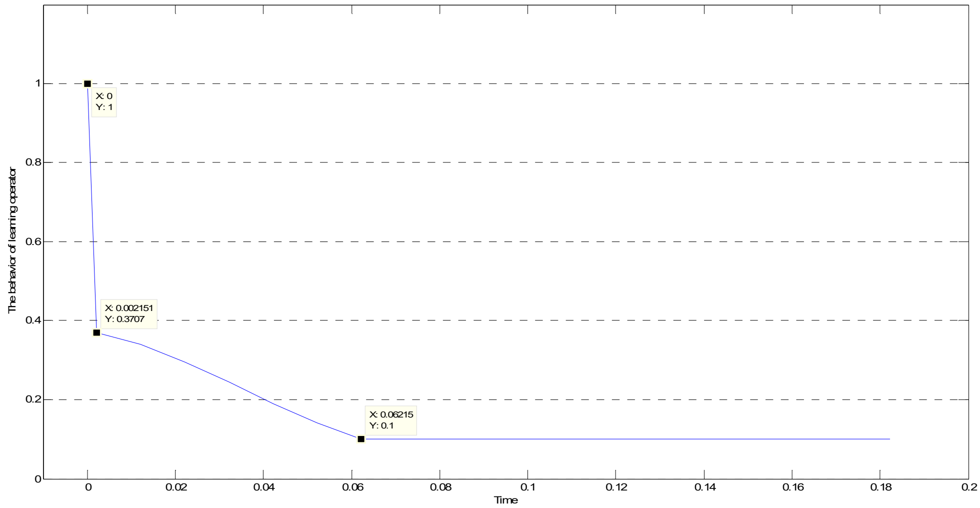


Figure 7. The behavior of the ILC rule..

4. Conclusions

This research exhibited the design of iterative learning controller and the results of simulation with example to prove the mathematical theory of the chaotic system synchronization via iterative learning control law. The demonstrations verified the mathematical theory as possible to approximate the synchronization between systems. The ILC method is a convenient method to trace the trajectory of systems, but it is not perfect tracking for all situations. In addition, the iterative learning control law should be conditionally dependent on the system and would not be unique to the specific system. It is a significant challenge to find coefficient matrices, which are the combination of previous ILC law and trajectory error in this research, respectively. The ILC method could be use for the non-linear system with time-delay and couple to adjust the learning control law and the process should also applies to adaptive control, sliding mode control, and fuzzy control. The primary research is essential for the tracking systems, such as robotic systems, secure communication systems, image identification systems, and many others, which are part of the future developments and applications.

Author Contributions: “Chun-Kai Cheng conceived and designed the simulation; Chun-Kai Cheng performed the simulation; Chun-Kai Cheng analyzed the data; Chun-Kai Cheng and Paul C.-P. Chao wrote the paper.”

Conflicts of Interest: The authors declare no conflict of interest.

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