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A Practical Approach Deriving Optimal Unit Hydrograph from Noisy Runoff in absence of Rainfall Data

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Abstract: As a procedure deriving UH (unit hydrograph), the root selection method necessitates only storm runoff data. However, this method must deal with the uncertainty related to the noise fluctuation of runoff ordinates and derive one optimal UH from many storms. This study proposes a procedure that applies the Savitzky-Golay filter to smooth the noise fluctuation of the runoff ordinates and uses the linear combination of UHs from individual storms to derive an optimal UH. The proposed method is applied to the storms of the Nenagh River basin in Ireland. The applicability of the Savitzky-Golay filter for smoothing the noise fluctuation of storm runoffs is examined by means of the Nash-Sutcliffe efficiency index. Furthermore, the root selection method is extended to also estimate IUHs. The results show that the adoption of the Savitzky-Golay filter improves the applicability of the root selection method and that the optimal UH predicts accurately the time-to-peak and peak discharge.

Keywords: root selection method; unit hydrograph; Savitzky-Golay filter; Nash-Sutcliffe index

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1. Introduction

The unit hydrograph (UH) concept is generally used for the analysis of rainfall-runoff relationship since it is practically convenient to describe linearly the basin response to rainfall input. Numerous methods have been proposed to find an UH from storm hydrographs with known rainfall inputs. However, one still needs an efficient method estimating the UH in absence of rainfall input. For unknown rainfall input in hydrograph analysis, several researches focused on separating a UH from the runoff hydrograph. De Laine [1] suggested the root matching method using the roots of the polynomial representing the Z-transform of a hydrograph of finite length. Turner et al. [2] proposed an alternative approach that is less sensitive to data quality than that of De Laine [1]. This approach called the root selection method used the Argand diagram to decide the shape of a resulting UH. Based on Turner et al. [2], Ojha et al. [3] applied scaling parameters to adjust the values of UH coordinates. To improve UH determination, Parmentier et al. [4] proposed a method that avoids the subjective selection of rainfall roots by identifying and controlling the time-dependent response components. Bruen et al. [5] extended the applicability of the root selection method in separating urbanization effects in runoff data. Based on the theory of response separation provided by the root selection method, Seong [6] suggested a novel method estimating the parameters of the Clark's IUH.

Though the root selection method has shown satisfactory results in determining a UH, the uncertainties in UH ordinates and the derivation of a representative UH based on multiple storm events remain still matters to be considered. De Laine [1] and Turner et al. [7] showed that the root selection method produces remarkably sensitive roots for the polynomial representing a single storm runoff hydrograph. This means that the uncertainties polluting the runoff hydrographs from different storms will affect sensitively each UH polynomial for the corresponding storm. To reduce such uncertainties, this study applies the five-point Savitzky-Golay filter to runoff data [8,9]. By

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applying the filter, the variation of UH ordinates extracted from different storms can be diminished within a narrow range so that the important UH parameters such as peak value and time-to-peak exhibit more stable values.

Even though the smoothing filter is applied, a UH based on individual storm event usually differs from one storm to another. Occasionally, the estimate of UH from a single event will yield unreliable result showing unsatisfactory value of goodness-of-fit index. Thus, this research suggests a method by which an optimum UH is derived from available multiple storm events. To do this, this research combines the UH polynomials from multiple storm events based on the linear combination theory. In addition, this research investigates the applicability of IUHs obtained from UHs resulting from the root selection by comparing them with existing IUHs obtained from a conventional method.

The validity of the proposed methodology is verified using the 22 storm records of the Nenagh River at Claianna, Ireland, which are adopted in the referred researches concerning the root selection method and investigating other general rainfall-runoff relationships [2,3,7,10,11].

2. Method

2.1 Smoothing of Runoff Data by the Savitzky-Golay Filter

Though runoff data can be collected by a proper way, there is still an uncertainty associated with natural randomness that might affect the root selection procedure. The uncertainty of runoff data appears frequently in the form of oscillations in the hydrograph. If rainfall data is available for rainfall-runoff analysis, this uncertainty can be eliminated or reduced by minimizing error techniques such as the least squares method or other regression methods. Moreover, the concurrent use of multiple storms in the convolution procedure would lower the uncertainties when deriving UH. However, in absence of rainfall data, the root selection method cannot consider the uncertainty of runoff data and will result in relatively higher variation of the UH ordinates obtained from different storms. In signal processing field, these oscillations from randomness are generally eliminated by data smoothing. Among the various smoothing techniques, the Savitzky-Golay filter is adopted in this research since it tends to preserve the peak heights of a given noisy signal. Such characteristics of the Savitzky-Golay filter makes it particularly adapted for deriving UH, especially when associated with the root selection method.

For a runoff data $y = \{q_1, \cdots, q_n\}$ consisting of n discharge ordinates, the Savitzky-Golay filter fits a polynomial of arbitrary order to the $2m+1$ data points surrounding the center for data smoothing. Chau [9] proposed the following expression involving a set of weights w_j for this polynomial:

$$q_i^* = \frac{1}{2m+1} \sum_{j=-m}^{m} w_j \, q_{i+j} \, , \tag{1}$$

where q_i^* denote the smoothed discharge ordinates and q_{i+j} are the original runoff ordinates, in which *i* and *j* are the running indices. It is known that the Savitzky-Golay filter uses the least-squares technique to find the weights w_j. For example, the best quadratic polynomial smoothing data to the surrounding five data values (m = 2 for 2m+1 = 5) is expressed in Equation (2):

$$q_i^* = \frac{1}{35}\left(-3\,q_{i-2} + 12\,q_{i-1} + 17\,q_i + 12\,q_{i+1} - 3\,q_{i+2}\right). \tag{2}$$

This study adopts Equation (2) to smooth the runoff because it involves the lesser number of runoff data points in the smoothing process. Equation (2) requires the first and last points to be treated. Here, an artificial extension of data by adding zeros is employed.

2.2 The Root Selection Method

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The linear discrete convolution relationship between a pulse response function being UH, an input being effective rainfall and an output being runoff is usually expressed as Equation (3) [6]:

$$y(nT) = \sum_{i=0}^{n} h(iT)\, x\big((n-i)T\big), \tag{3}$$

where T is the time interval for sampling; the output $y(nT)$ is the runoff sampled at $t = nT$; and, the input $x\big((n-1)T\big)$ is the effective rainfall between times $t = (n-1)T$ and nT. The pulse response function $h(iT)$ represents the ordinate value of T-period UH at time $t = iT, i = 0 \cdots n$. As for Equation (3), De Laine [1] proposed the root matching method to estimate the UH without using rainfall data. The root matching method uses the Z-transformation method for rainfall-runoff modelling in frequency domain. Thus, Equation (3) can be rewritten as:

$$Y(z^{-1}) = H(z^{-1}) X(z^{-1}), \tag{4}$$

where $Y(z^{-1})$ is a polynomial resulting from the Z-transform of direct runoff y; $X(z^{-1})$ is a polynomial representing the Z-transform of rainfall x; and, $H(z^{-1})$ is a polynomial indicating the Z-transform of UH. The root matching method uses the roots of the runoff polynomial $Y(z^{-1})$ to give $H(z^{-1})$. According to De Laine [1], from the results of multiple runoff analysis, the common roots in more than two runoff polynomials can be extracted because the common roots are independent of the rainfall and thus, the UH can be constructed using these common roots. However, Turner et al. [2] showed that the root matching method is sensitive to the errors in runoff data and proposed a new method finding the UH polynomial by considering the roots on the complex plot (or the Argand diagram) of the runoff polynomial. Based on the analysis using synthetic data and real runoff data, they found a specific feature of the roots reflecting UH on the Argand diagram. Furthermore, the authors showed that this feature was practically common to all runoff events in a same basin. Finally, Turner et al. [2] suggested a procedure (the root selection method) that could extract UH roots from the runoff roots on the Argand diagram and showed that the method gave a smaller average error in UH than the root matching method.

2.3 Determination of an Optimal UH for Multiple Storm Events

The root selection method has been successful in identifying UH roots from a runoff polynomial based on individual single runoff event. However, a practical procedure is still necessary to determine the optimal UH obtained from multiple runoff events. Accordingly, this study constructs an optimal runoff polynomial by selecting the optimal UH roots from a unique and integrated Argand diagram in which all runoff roots appear. The suggested method involves the following processes:

1 smoothening of fluctuation in ordinates of runoff hydrographs by the Savitzky-Golay filter based on large number of runoff data;

2 selection of the UH roots for each storm runoff data by conventional root selection procedure, and derivation of the individual single-event UH polynomial;

3 overlapping of the UH roots selected individually from different storm runoffs on the Argand diagram from one storm to another;

4 removal of eventual abnormal roots based on thick circular pattern that might indicate the uncertainty associated with natural randomness of runoff process;

5 reconstruction of the UH polynomial for each storm by considering the removed roots in step 4, and normalization of each coefficient of the polynomial by dividing each coefficient by the sum of all coefficients;

7 construction of an optimal UH polynomial by linear combination of the UH polynomials for storms considered to be a response to one unit of the effective rainfall for a specified duration time.

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The above-mentioned procedure can be formulated mathematically as follows. Consider k storm runoffs $\hat{y}=[y_1,\cdots,y_m,\cdots,y_k]$ in which y_m represents the direct runoff data for the mth storm event. Each ordinate of y_m becomes one coefficient of the runoff polynomial when applying the Z-transform to runoff y_m. The same notation applies for the UHs $\hat{h}=[h_1,\cdots,h_m,\cdots,h_k]$ and for the effective rainfall series $\hat{x}=[x_1,\cdots,x_m,\cdots,x_k]$ using the relationship given in Equation (3). Equation (4) is also generalized by introducing a subscript denoting the event number. The resulting convolution equation replacing Equation (4) is then:

$$Y_m(z^{-1})=H_m(z^{-1})X_m(z^{-1}),\quad m=1,\cdots,k\,. \tag{5}$$

Since this work uses the smoothed runoff obtained by the Savitzky-Golay filter instead of the raw runoff data $Y_m(z^{-1})$, Equation (5) is rewritten with the smoothed runoff $Y_m^*(z^{-1})$ as shown in Equation (6):

$$Y_m^*(z^{-1})=H_m(z^{-1})X_m(z^{-1}),\quad m=1,\cdots,k\,, \tag{6}$$

where $Y_m^*(z^{-1})=Z\left(y_m^*\right)$; and, y_m^* represents the smoothed runoff data for the mth storm. Considering UH polynomials for k storms $H_m(z^{-1})$ with $m=1,\cdots,k$, the coefficients of $H_m(z^{-1})$ do not indicate the ordinates of UH for storm number k but represent simply the relative values between the coefficients of $H_m(z^{-1})$. Therefore, the coefficients of $H_m(z^{-1})$ must be readjusted with the scale factors c_m such that the sum of the coefficients equals one unit of effective rainfall for a specific duration. After this adjustment, a combined UH polynomial can be derived by the linear combination of the $H_m(z^{-1})$, $m=1,\cdots,k$ using the scale factors c_m for the corresponding storm m as shown in Equation (7):

$$\bar{H}(z^{-1})=c_1H_1(z^{-1})+\ \cdots\ +c_kH_k(z^{-1})\,, \tag{7}$$

where $\bar{H}(z^{-1})$ is tentatively an optimal UH polynomial. The resulting optimal UH polynomial can be obtained by rescaling the coefficients to satisfy the definition of UH.

2.4 Parameters of Nash's IUH with Root Selection Method

A vital aspect of using IUH (instantaneous UH) in a hydrologic modeling is that the IUH is free from the difficulties related to the duration time of the effective rainfall. One widespread conceptual IUH model is the Nash's model [12]. The Nash's IUH follows a gamma probability density function as shown in Equation (8):

$$h(t)=\frac{1}{K\,\Gamma(n)}\left(\frac{t}{K}\right)^{n-1}\exp\left(-\frac{t}{K}\right), \tag{8}$$

where $h(\bullet)$ is the functional form of Nash's IUH; n and K are the model parameters to be estimated; and, $\Gamma(\bullet)$ is the gamma function. This work intends to find the IUH parameters from the UH obtained by root selection. For this purpose, this research applies the method of moments (MOM) to calculate the parameters of Nash's model. The use of MOM was proposed by Nash [12] in which the moments of the IUH are the moments of the input (effective rainfall series) and output (direct runoff). In this research, the parameters (n, K) for 21 storms in Nenagh basin are determined by MOM using input (one unit of the effective rainfall for a specified duration) and output functions (UH resulting from the root selection method).

3. Application and Result Assessment

3.1 Deriving UHs for Individual Storms (Single-Event Analysis)

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The site chosen to apply the proposed method estimating a multiple-event UH is Nenagh River basin with an area of 295 km², at Claianna, Ireland [10]. Twenty-two storm data with peak values varying from 18 to 51.6 m³/s on direct runoff s are considered. The corresponding rainfalls run between 2.91 and 11.61 mm. Both discharge and rainfall data were sampled at interval T = 3 hours. The flow chart of the proposed method is shown in Figure 1.

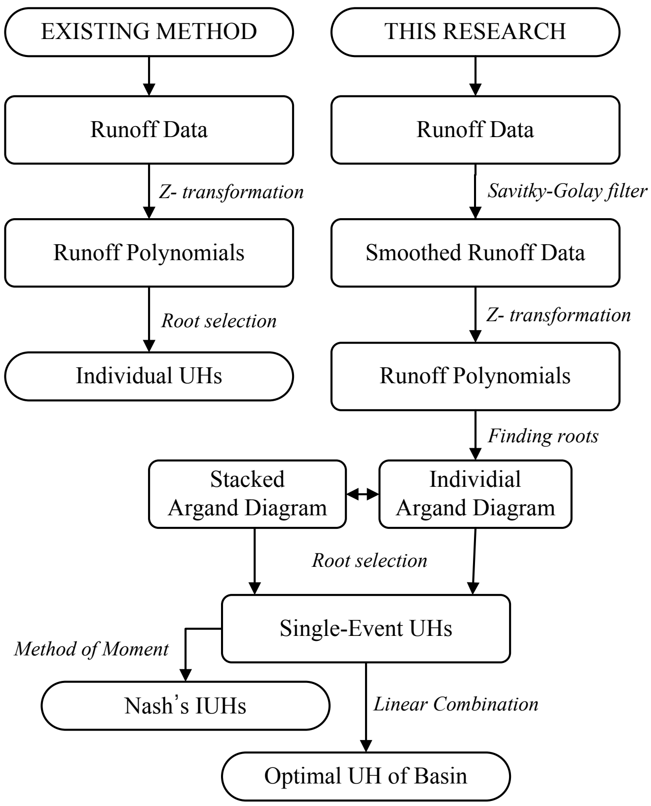

Figure 1. Flow chart of method proposed by this study in comparison with the conventional root selection method.

This research considers 22 storms as single runoff sample $\hat{y}=[y_1,\cdots\cdots,y_{22}]$, in which the last storm y_{22} is used to check the validity of the proposed method by showing runoff reproduction. The runoff hydrographs $\hat{y}$ corresponding to the 22 different storms are shown in the leftmost panel of each row in Figure 2.

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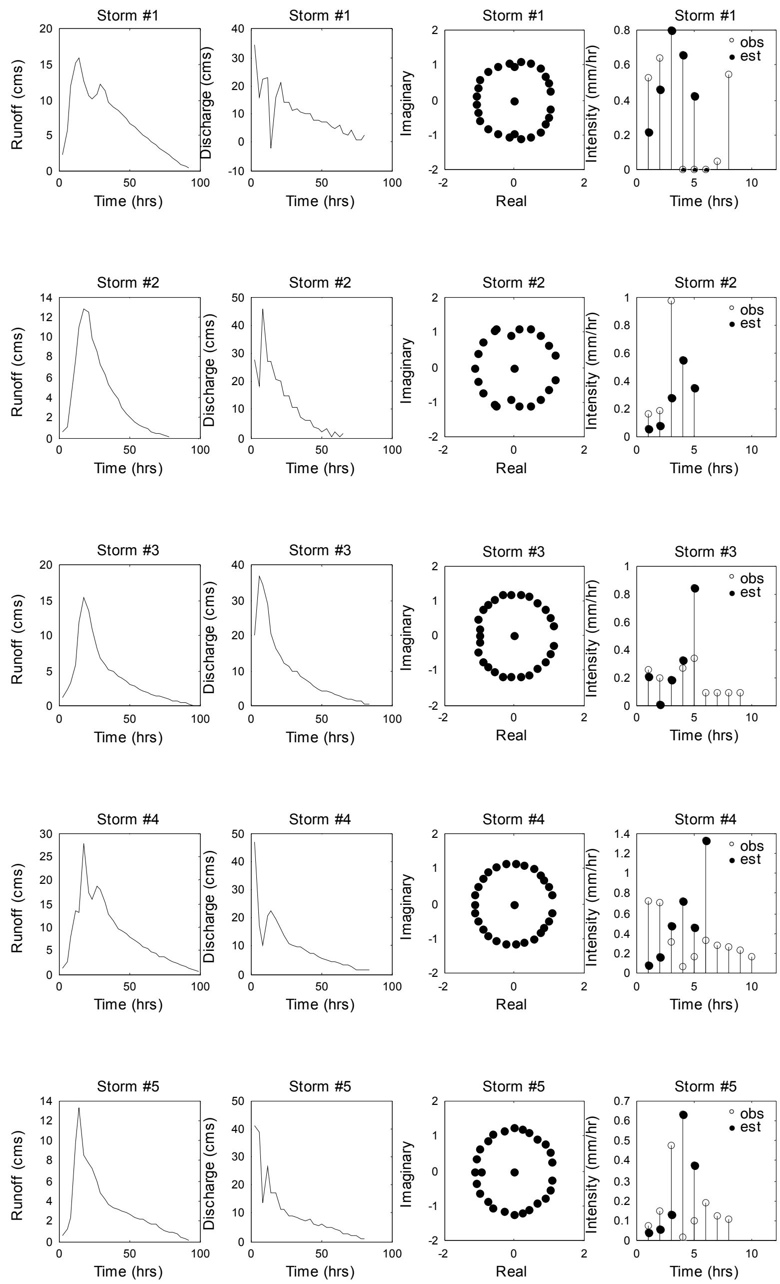

Figure 2. Observed runoff data (22 storms of Nenagh Basin) and the results of the root selection method applying each runoff: the panels from left to right of each row plot successively the runoff hydrograph, estimated unit hydrograph, Argand plot for selected roots, and hyetographs for both measured and estimated rainfalls

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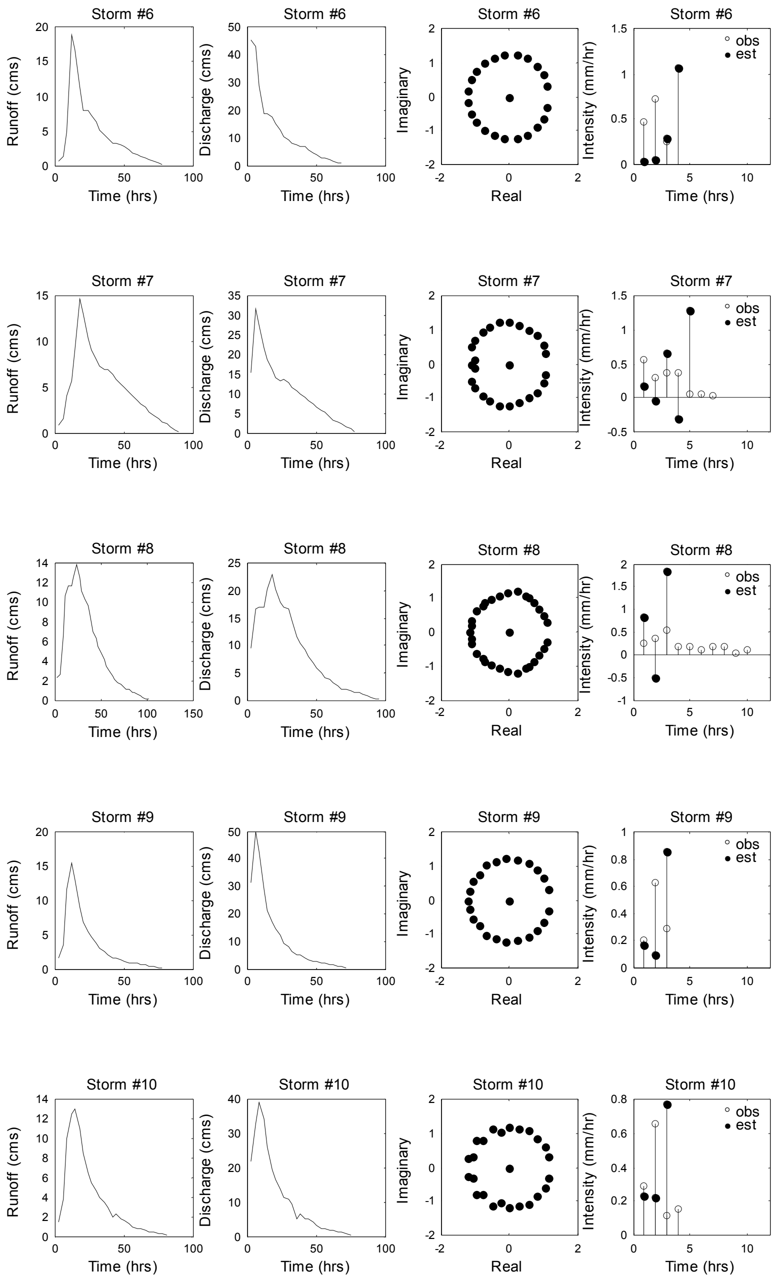

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186 **Figure 2.** (Continued)

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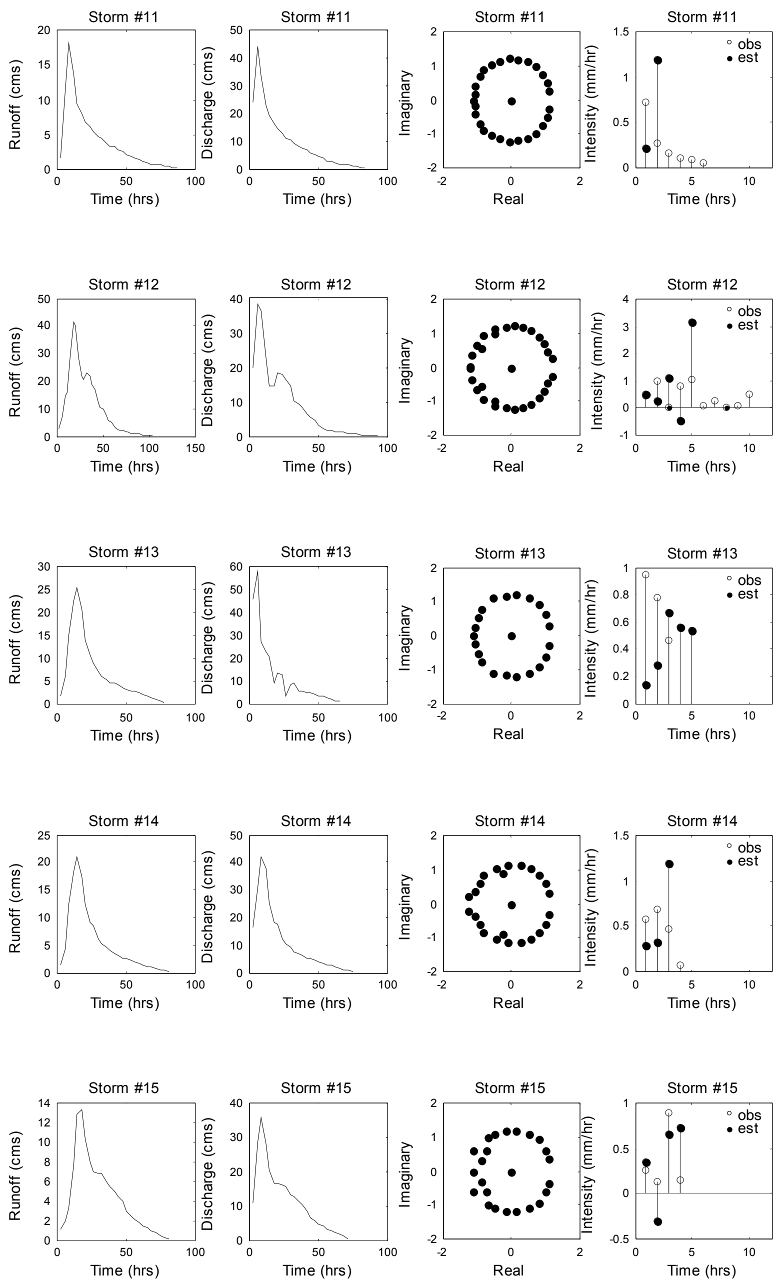

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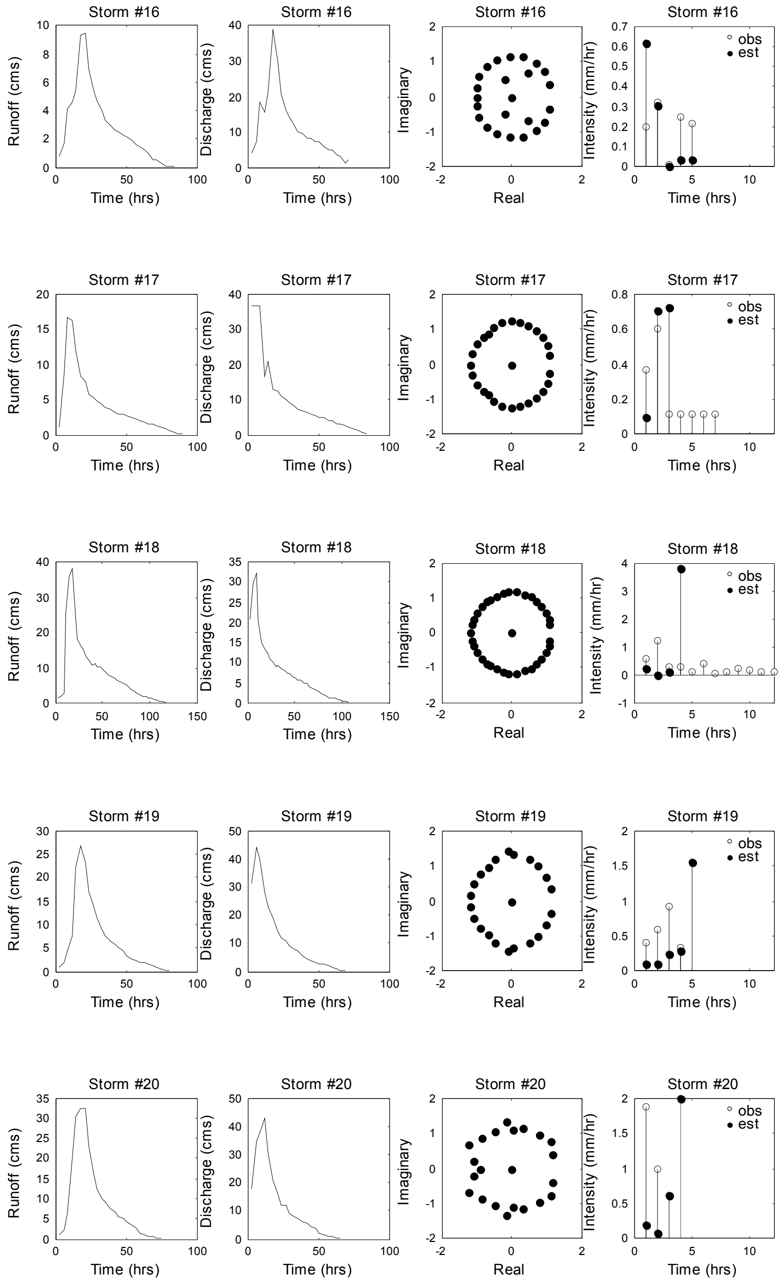

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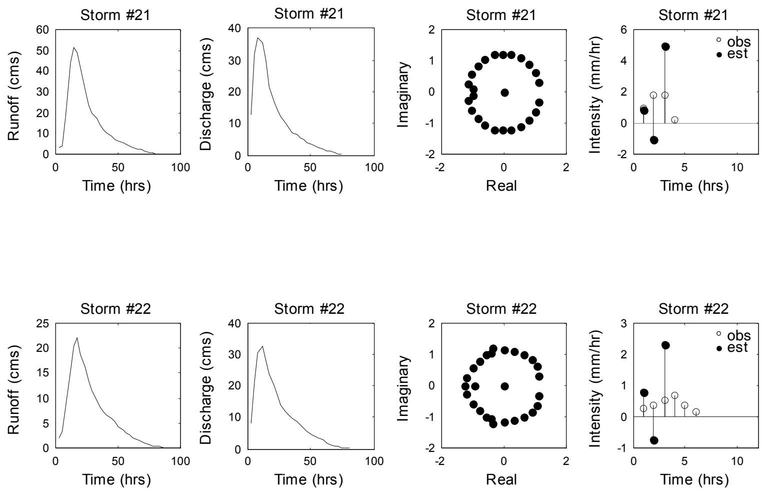

Figure 2. (Continued)

The UHs h_m $(m=1,\cdots,22)$ were derived individually from the 22 different storm events by the conventional root selection method. The resulting UHs and roots of $H_m(z^{-1})$ for corresponding storm events are indicated in the second and third panels from left of each row in Figure 2. It appears that the root patterns for many different storm events are fairly displaced from the regular pattern, resulting in fluctuations of ordinates in many UHs. Therefore, it could be supposed that the individual UHs are virtually subject to uncertainty related with the randomness. Each rightmost panel in the rows of Figure 2 shows the rainfall difference between the values from measured hyetograph and the roots from $X_m(z^{-1})$, $m=1,\cdots,22$. One sees the presence of inevitable and remarkable discrepancy in estimated rainfall in many storms. This indicates clearly that the uncertainty in runoff data affected the estimation of both rainfall and UHs.

To clarify the uncertainties in UHs, Figure 3(a) shows a plot in which the 22-individual storm UHs are stacked altogether, and Figure 3(b) plots the Argand diagrams of the corresponding Y_m $(m=1,\cdots,22)$. Figure 3(a) reveals the large variation between UHs, the significant oscillations of the ordinates values, and the abnormal ordinate values as well. Thus, if a representative UH is derived based on these UHs, this representative UH might be subjected to high variation related to uncertainties. Even though not all individual Argand diagrams show perfect circular pattern of runoff roots as shown in Figure 1, the stacked Argand diagrams in Figure 3(b) show explicit circular pattern of the runoff roots. Thus, stacking the runoff data permits to identify more easily the abnormal roots from the runoff roots.

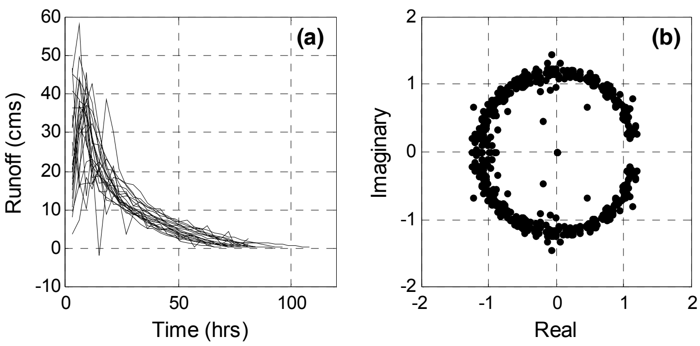

Figure 3. Applying root selection method in conventional way to 22 storms of Nenagh Basin: (**a**) estimated unit hydrographs by root selection method; (**b**) Argand plot for runoff polynomials.

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The Savitzky-Golay filtering was applied to the 22 runoffs. To produce single smoothed runoff y^*, the filter used adjacent five runoff ordinate values as introduced in Equation 2. Figure 4(a) shows the UH stacking the 22 different storms after the Savitzky-Golay filtering and Figure 4(b) indicates the Argand diagrams for corresponding Y_m^* $(m = 1, \cdots, 22)$.

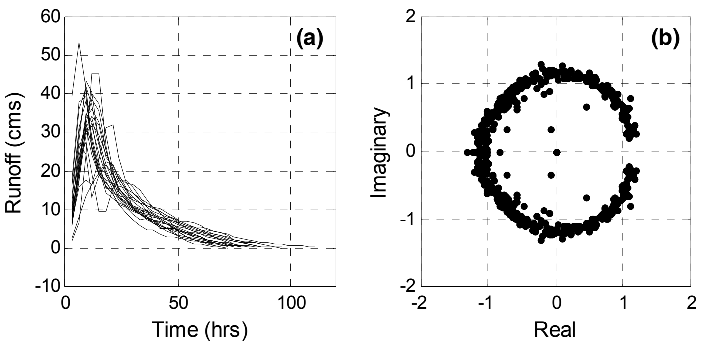

Figure 4. Effect of applying Savitzky-Golay filter to 22 storms of Nenagh Basin: (**a**) estimated unit hydrographs by root selection method; (**b**) Argand plot for runoff polynomials.

In Figure 4(a), one can observe that the variations among the ordinates of each UH are fairly reduced compared to Figure 3(a). Moreover, the overall pattern of UH root circles based on the filtered data shown in Figure 4(b) appears to be improved in the sense that the roots in the circular pattern gets closer together when using the Savitzky-Golay filter. The result shows that the Savitzky-Golay filter stabilizes the UH by reducing the uncertainty brought by the random error from the runoff data. Besides, compelling changes related with abnormal roots can be found. The abnormal points, positioned outside the circle in Figure 3(b), are repositioned inside the circle after smoothing as in Figure 4(b), while the abnormal points inside the circle virtually retain their position. However, this change of root pattern did not contribute in producing a better rainfall estimate.

3.2 Evaluation Using the Performance Index

In order to investigate the effect of the Savitzky-Golay filter more precisely, this study compared the performance of UHs with/without Savitzky-Golay filtering. Among the numerous goodness-of-fit criteria, this study selected the Nash-Sutcliffe model efficiency coefficient [13] as comparison index following the recommendation of ASCE [14]. The Nash-Sutcliffe index is defined as:

$$E = \left(1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2} \right), \tag{9}$$

in which y_i is the observed runoff; $\hat{y}_i$ is the modelled discharge; and, $\bar{y}$ is the average discharge at time t. The closer the Nash-Sutcliffe index to 1, the better the performance of the model. Specifically, a value of 0.9 for E indicates a very satisfactory model performance, a value between 0.8 and 0.9 indicates an acceptable model, and a value between 0.6 and 0.8 indicates unsatisfactory fitting results [15]. The applicability of UHs were compared using the Nash-Sutcliffe index for UH models with and without Savitzky-Golay filtering. Table 1 compares the Nash-Sutcliffe index yielded for the 22 storms of Nenagh basin. The last row in the table represents the average value for both UHs.

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Table 1. Comparison of Nash-Sutcliffe model efficiency coefficient for UHs with/without Savitzky-Golay filtering.

Event	Nash-Sutcliffe model efficiency coefficient	
	Without filter	With filter
1	.59	.84
2	.65	.97
3	.89	.88
4	.65	.77
5	.56	.76
6	-.01	.68
7	.58	.73
8	.85	.84
9	.88	.78
10	.96	.97
11	.89	.93
12	.88	.87
13	.09	.95
14	.93	.98
15	.88	.99
16	.73	.69
17	.85	.85
18	.48	.62
19	.27	.73
20	.09	.60
21	.91	.95
22	.93	.90
Avg.	.66	.83

The table apparently indicates that for most of the events, the UH model with filtering performs better than the UH without filtering. The overall index value clearly proves the applicability of Savitzky-Golay filtering as indicated by the average Nash-Sutcliffe index of 0.83 (> 0.8). This result shows that the UH model using Savitzky-Golay filtering can be recommended for the prediction of rainfall-runoff of the Nenagh basin at least.

3.3 Deriving of Optimal UH (Multiple-Event Analysis)

The 22 storms presented chronologically in Figure 2 were used for deriving an optimal UH. For this purpose, the first 21 storms were used to give an optimal UH estimate with careful selection of the abnormal roots on the basis of Figure 4. The last 22nd storm was used for validation of the prediction. Using each of the 21 single-event UHs $H_m(z^{-1})$, $m = 1, \cdots, 21$, a multiple-event UH $\bar{H}(z^{-1})$ was derived by the linear combination method as shown in Equation (7). The optimal UH $\bar{H}(z^{-1})$ is compared to a selection of $H_m(z^{-1})$, $m = 1, 2, 10, 16$ in Figure 5. In the figure, the UHs based on the selected storms vary from one storm event to another.

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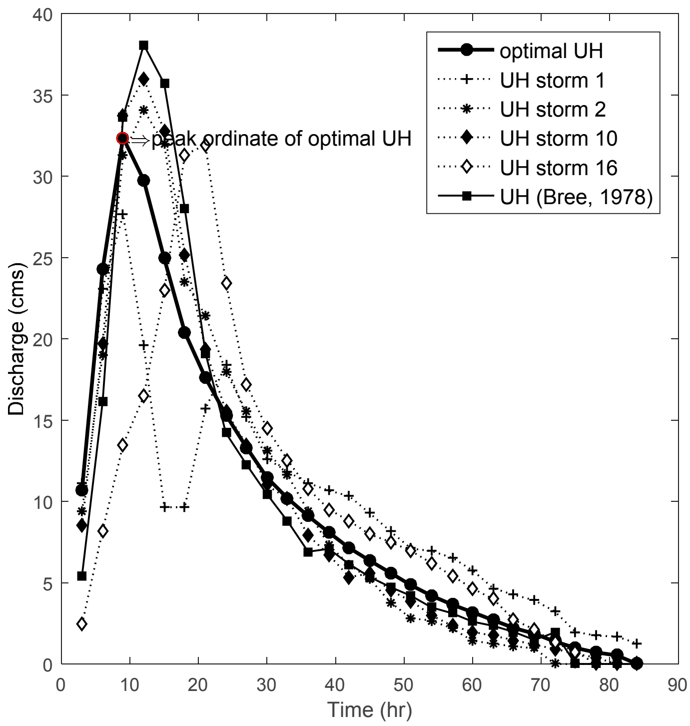

Figure 5. Comparison of UH ordinates obtained by this research (optimal UH) with those of selected events and Bree [10].

These selected storms were also used by Bree [10] to obtain a reliable UH when using the least squares method. According to Bree [10], the estimation of the UH from a single-event analysis did not result in a reliable UH. Besides, the combination of multiple storms produced more reliable UH by reducing the oscillations of UH ordinates. Similar result could be observed in this research. In Figure 5, one can find that the oscillations in the individual UHs $H_m(z^{-1})$ were virtually removed when the multiple-event analysis was applied. Thus, to calculate a design runoff in conjunction with a design rainfall, using multiple-event UH $\bar{H}(z^{-1})$ is preferable for better prediction.

3.4 Estimating Parameters of Nash's IUH

The parameters (n, K) for available storms in Nenagh basin were determined by MOM which uses the single-event UH for each storm and corresponding unit rainfall provided with 1 cm depth for 3-hours duration. For this calculation, a computer code serviced by Colorado State University [16] was used. Incidentally, Mohan and Vijayalakshmi [11] have carefully determined the Nash's parameters based on a typical MOM procedure using rainfall data. Thus, the parameter values given by Mohan and Vijayalakshmi [11] could provide proper reference values to be compared with those resulting from this research. Table 2 lists the parameters (n, K) for 21 storms obtained from these two different approaches.

Table 2. Comparison of Nash IUH's parameters of this research with those of Mohan and Vijayalakshmi [11].

Event	This research		Mohan and Vijayalakshmi (2008)	
	n	*k*	*n*	*k*
1	1.955	14.293	0.996	7.926
2	2.146	9.234	1.173	5.546

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3	1.640	14.068	0.917	6.888
4	1.655	15.523	0.891	8.569
5	1.606	15.439	0.929	6.915
6	1.540	12.417	1.204	5.745
7	1.893	12.961	1.101	8.003
8	2.206	12.491	0.863	7.670
9	1.078	13.456	1.310	3.907
10	1.858	10.854	1.241	4.629
11	1.389	14.527	1.204	4.772
12	1.867	11.700	0.632	8.018
13	1.583	12.389	1.228	5.251
14	1.827	11.436	1.230	4.999
15	2.151	10.578	1.151	6.230
16	2.882	9.240	1.099	6.098
17	1.401	16.180	1.099	5.388
18	1.545	18.801	0.781	9.308
19	1.709	10.538	1.163	5.692
20	2.167	8.443	1.195	5.924
21	2.293	9.365	1.201	5.172
Avg.	1.828	12.568	1.077	6.317
Sd.	0.394	2.659	0.178	1.457
$n \times k$	22.953		6.803	

Table 2 also indicates the statistical properties featuring the variation of the determined parameters. It can be seen that the result obtained in this study exhibit fairly higher values in both parameters than those of the reference values. However, the degree of variation of these values can be regarded as being low in view of the small values of the standard variations. The product values of n and K ($n \times K$) for all storms were considered to be constant with an average of 22.953, which is significantly higher than the reference value of 6.803. Thus, the IUH with the parameter values of this study would produce lower peak discharge and more delayed time-to-peak in comparison to the IUH given by Mohan and Vijayalakshmi [11]. However, it is evident that the IUH obtained from the root selection would be able to predict runoff more accurately by transferring a few number of UH roots into rainfall roots. Therefore, future work should focus on the efficiency of the selection procedure for UH roots.

3.5 Predicted Runoff Using the Optimal UH

Event 22 was used to verify the optimal UH $\hat{H}(z^{-1})$ estimated from 21 storms. Figure 6 compares the observed and predicted runoffs using the optimal UH. The single-event UH $H_{22}(z^{-1})$ derived by the root selection method is also plotted in Figure 6.

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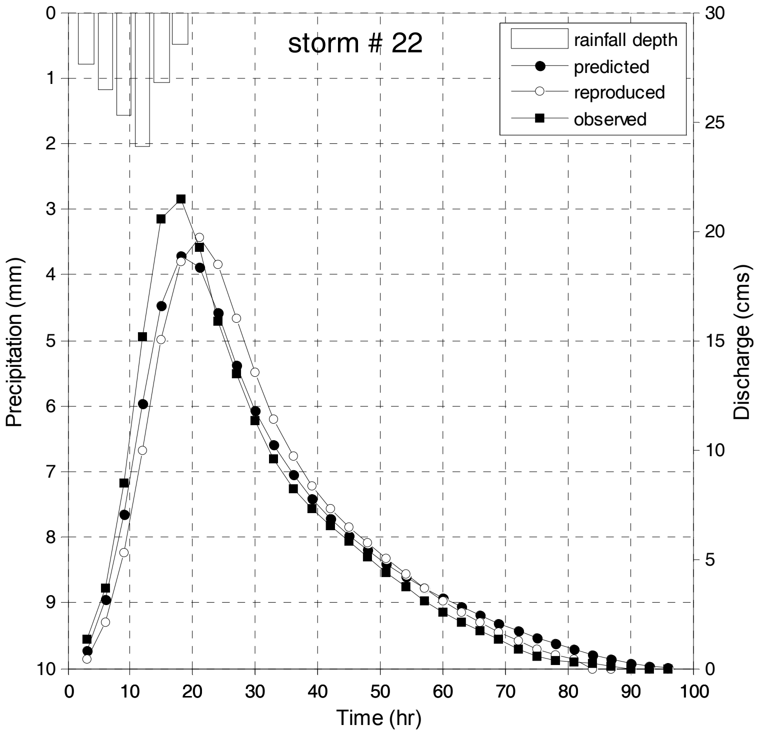

Figure 6. Comparison between the runoff predicted for 22nd storm using the optimal unit hydrograph obtained from 21 storms and the runoff reproduced using the single event unit hydrograph for 22nd event, and the observed runoff for 22nd storm event.

In Figure 6, there is no remarkable difference in the overall shape between the observed and predicted runoffs. The peak discharge of the predicted runoff agrees well with that of the observed one. However, the optimal UH resulted in a slightly larger value of the time-to-peak in the predicted runoff. This problem might be solved by selecting more abnormal roots from the runoff roots to reform the optimal UH to another optimal UH having smaller number of ordinates. However, this method involves more subjectivity in the root selection procedure.

Interestingly, the reproduced hydrograph using the single-event UH $H_{22}(z^{-1})$ shows larger deviation from the observed runoff compared to the predicted hydrograph using the optimal UH. This is mainly due to higher uncertainty associated with the determination of $H_{22}(z^{-1})$. This strongly suggests that using multiple-event UH along with runoff filtering would be applicable especially in absence of rainfall data.

4. Conclusion

The root selection method can be used for determining a UH when rainfall data are not available. The root selection method has shown successful results, but the uncertainty related to noise fluctuations in the resulting UH remains unclear yet. The determination of an optimal UH (multiple-event UH) based on available UHs individually obtained from many storm events (single-event UH) is an important problem for practical use. This study proposed an improved root selection method based on the smoothing technique using the Savitzky-Golay filter to reduce significantly the noise fluctuations in runoff data. The proposed method provided single-event UH for each storm with better applicability. This paper also focused on deriving the optimal UH by linear combination of the single-event UHs from different storms. In addition, determination of parameters of Nash's IUH model using single-event UH was also considered in this study. The proposed method was applied to twenty-two storm events for a basin of the Nenagh River at Claianna, Ireland.

In most storm events, the values of Nash-Sutcliffe index using the UHs obtained from the existing root selection method reached unacceptable level (less than 0.8). However, the values of Nash-Sutcliffe index using the UHs based on filtered runoff data appeared to be satisfactory with values higher than 0.8. This result proved that the application of the Savitzky-Golay filter in root selection method enables more accurate prediction by reducing the uncertainty associated with fluctuations of ordinates in runoff data.

The proposed method yielded a non-biased estimation of the optimal UH in the sense that the storms with larger runoff and the storms with smaller runoff equally contributed to the determination of the optimal UH. The fluctuations found in some single-event UHs were virtually removed in the optimal UH, and a validation result indicated that there was no remarkable difference between the optimal UH and the UH obtained by using rainfall data.

Using the root selection method along with the method of moments allowed the estimation of the parameters of Nash's IUH. However, it was shown that the IUH with these parameters would result in slightly lower peak values. An objective method to reduce the number of ordinates in UH is necessary.

To the knowledge of the author, the proposed root selection method in this research provided a fairly enhanced result in estimating UH for practical use even though few important problems remained to be solved. The proposed method can potentially be applied to other hydrological analysis even when using rainfall data.

5. **Patents**

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