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Article

Accelerated Benders' Decomposition for Integrated Forward/Reverse Logistics Network Design under Uncertainty

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Abstract: In this paper, a two-stage stochastic programming modelling is proposed to design a multi-period, multistage, and single-commodity integrated forward/reverse logistics network design problem under uncertainty. The problem involves both strategic and tactical decision levels. The first stage deals with strategic decisions, which are the number, capacity, and location of forward and reverse facilities. At the second stage tactical decisions such as base stock level as an inventory policy is determined. The generic introduced model consists of suppliers, manufactures, and distribution centers in forward logistic and collection centers, remanufactures, redistribution, and disposal centers in reverse logistic. The strength of proposed model is its applicability to various industries. The problem is formulated as a mixed-integer linear programming model and is solved by using Benders' Decomposition (BD) approach. In order to accelerate the Benders' decomposition, a number of valid inequalities are added to the master problem. The proposed accelerated BD is evaluated through small-, medium-, and large-sized test problems. Numerical results reveal that proposed solution algorithm increases convergence of lower bound and upper bound of BD and is able to reach an acceptable optimality gap in a convenient CPU time.

Keywords: integrated forward/reverse logistics network; accelerated benders' decomposition; two-stage stochastic programming

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1. Introduction

The main purpose of Supply Chain Management (SCM) is to integrate entities including suppliers, manufacturers, distribution centers, and retailers in order to acquire raw materials, transform raw materials to finished products and distribute products to customers in an efficient way [1]. Achieving success in supply chain management involves decisions relating to flow of information, products, and funds. Above-mentioned decisions fall into three levels; those are supply chain design, - planning, and –operations [2]. In general, a Supply Chain Network Design (SCND) problem includes long-term decisions (strategic level) such as facilities' location, number, capacity level, and technology selection; mid-term decisions (tactical level) that usually contain the production quantity and the volume of transportation between entities; and finally short-term decisions (operational level) where all material flows are scheduled based on decisions made in the two other levels [3].

Over the last decade, the intensity of environmental regulations and guarantee commitments lead manufactures to adopt activities associated with returned product, such as collection, recovery, remanufacturing, refurbishing, and disposal of used products that generally called Reverse Logistics (RL) [4]. RL literature is divided in two groups; those which considered forward and reverse flows simultaneously and those that fully concentrate on reverse flows. Actually the integrated forward

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and reverse flow networks, such as Closed-Loop Supply Chain (CLSC), have more complexity in design and planning.

Many researchers have investigated supply networks design in deterministic environment. In comparison with forward supply chains that consider uncertainties in customers demand, price, and resource capacity levels, RL operations are confronted with a higher degree of uncertainty such as collection rates, availability of recycled production inputs, disposal and recycling rates[5]. Nevertheless, the majority of studies assume that the operational characteristics and design parameters of RL networks are deterministic.

In recent years, a number of reviewing papers have been published on reverse logistics. Ackali et al [6] presented a critical review on RL and Integrated Forward/Reverse Logistic Network (IFRLN) problems, and discussed the main characteristic of models and solution methods proposed in the literature. Chanintrakul et al. [7] reviewed open loop and closed-loop supply chain models with considering the impact of uncertainty in recent researches. They argued the fact that few researches deals with demand and return uncertainty in terms of quality and quantity. And moreover, tactical decisions should be resolved along with strategic decisions in which previous researches have not effectively investigated.

In the context of RL various models have been developed in the last decade (e.g. [8-10]). For integrated forward/reverse logistic network design one of the first stochastic models was presented by Listes [11] and later Listes et al. [12]. The model explores one echelon forward network combined with two echelon reverse network. The uncertainty is handled in a stochastic formulation by means of discrete alternative scenarios. Matthew et al. [13] studies a network design problem for carpet recycling in the US where supply and demand parameters were stochastic. Later Salema et al. [14] extended the Fleischmann's model [15] to a capacitated multi-product stochastic CLSC applied to an office document company in Spain.

Most of articles in stochastic IFRLN literature are single-period (e.g. [16-21]). Lee et al. [22] introduce a multi-period, multi-product dynamic location and allocation model under demand uncertainty. To solve the model an integrated sampling Average Approximation (SAA) method with a simulated annealing (SA) algorithm is developed.

The literatures that studied stochastic IFRLN network design problem considering inventory policy are few. Lieckens et al. [23] extends a closed-loop supply Mixed-Integer Linear Programming (MILP) model combined with queuing characteristics using a G/G/m model which increase the dynamic aspects like Lead Time and inventory position of the basic model. Since combining RL with queuing model intensifies the computational complexity of the model, they restrict to a single-level, single-product network design problem that covers a single-period. The new MINLP is solved with the differential evolution technique (DE). El-Sayed et al. [24] proposed a MILP multi-period, multi-echelon forward and reverse logistic network design model under uncertainty. The problem is formulated to maximize the total expected profit under risk. To achieve a generic model of CLSC authors incurred various costs such as transportation, materials, remanufacturing, recycling, disposal, non-utilized capacity, storage, shortage, recycling, and inventory holding cost.

To structure the literature review specifically on closed loop supply chain and integrated forward/reverse logistic network design problem under uncertainty, we give a systematic review of existing studies presented in Table 2. To facilitate the structure of Table 2, characteristics of networks are coded and demonstrated in Table 1. As shown in Table 2, most of the papers are those that are single-period and single-product. A few papers solve their model with exact optimization approach where utilizing commercial solvers are more common.

Table 1

Modeling approach codes

Category	Detail	Code	Category	Detail	Code
Model objectives	Cost minimization	CM	**Features of model**	**Period**	
	Profit maximization	PM		Single-period	S
	Responsiveness	R		Multi-period	M
	Quality	Q		**Facility capacity**	
	Other	OT		Uncapacitated	U
Features of model	**Stochastic parameters**			Capacitated	C
	Quantity of demand	D		Capacity expansion	CE
	Quantity of returns	R		Single sourcing	SS
	Quality of returns	RQ	**Model**	Mixed Integer Linear Programming	MILP
	Recovery rate	RR		Mixed Integer Non-Linear Programming	MINLP
	Recovery cost	RC	**Decision variables of model**	Inventory decisions	I
	Transportation cost	TC		Facility capacity	Fc
	Lead time	LT		Demand satisfaction	D
	Income	In		Transportation values	TV
	Other	OT		Location/allocation	LA
	Product commodity			Transportation mode selection	TM
	Single-commodity	S	**Solution methodology**	Technology selection	TS
	Multi-commodity	M		Exact solution method	EX
				Heuristic solution method	HE

Table 2

Summary of Stochastic integrated forward/reverse logistic network design

Ref.	**Model obj.**	**Stoch. param.**	**Product com.**	**Period**	**Facility cap.**	**Model**	**D.V.**	**Sol. method**	**Solution approach**
[11]	PM	R	S	S	C	MILP	TV,LA	EX	B&C
[13]	PM	D	M	M	C	MILP	TV, LA, Fc, TM	--	AIMMS
[12]	PM	R, In	M	S	C	MILP	TV, LA	--	CPLEX
[25]	PM	D,R	S	S	C	MILP	TV, LA,SS	EX	Integer L-Shape Method
[14]	CM	TC, D, R	M	S	C	MILP	TV, LA, D	--	CPLEX
[23]	PM	LT	S	S	C	MINLP	TV, LA, Fc, I	HE	Differential Evaluation (DE)
[22]	CM	D,R	M	M	C	MILP	TV, LA	HE	SAA with SA

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[17]	CM, OT	TC,R, OT	M	S	C	MILP	TV, LA, TS	--	CPLEX10
[21]	CM	TC, D, R, RQ	S	S	C	MILP	TV, LA	--	LINGO
[18]	PM	D, R	S	S	C	MILP	TV, LA	--	CPLEX
[20]	CM	D,R	M	S	C	MILP	TV, LA	EX	SAA with CPLEX
[16]	CM	RQ	S	S		MILP	LA	EX	SAA
[26]	CM, R, Q	D, R , RC, OT	M	S	C,SS	MILP	TV, LA, Fc	--	Commercial Solver
[19]	CM	D,R	S	S	C	MILP	TV, LA	--	CPLEX
[24]	PM	D,R	S	M	C	MILP	TV, LA, I	--	XpressSp
Our paper	CM	D,R	S	M	C	MILP	TV, LA, I	EX	Accelerated BD

In this paper, we first develop a MILP model for a multi-period, single-product, and capacitated integrated forward/reverse logistic network design. Due to uncertainty of various parameters in real problems, demand and return quantity of products are considered to be stochastic. The model is formulated with a two-stage stochastic programming approach. In the first stage, the strategic decisions are determined, which are the number, capacity, and location of collection, plants and distribution centers as well as amount of wholesale contract. Tactical decisions are made in the second stage (e.g. base stock level). We utilize Latin Hypercube Sampling to make scenarios from input data by considering correlations between each market. The model is solved with an accelerating Benders' Decomposition (BD) approach. Numerical tests investigate the power of accelerated BD in handling with uncertainty and solving the problem with an acceptable optimal gap.

In summary, Major contributions to this research are: (1) designing a new multi-period integrated forward/reverse logistic network design amenable for forward and reverse flow in integrated scheme (2) taking tactical decisions into account by considering an inventory policy for distribution centers and raw material stocks (3) Applying risk pooling strategy as well as push/pull mechanism to the model (4) Solving the introduced two stage stochastic programming model with an accelerate BD where some valid inequalities are added to the master problem equations in order to avoid infeasibility of problem solution space.

The remainder of the paper is organized as follows. In the next section, we present a mathematical formulation of proposed IFRLN design. The solution method is introduced in section3, followed by analysis of computational result in section4. Finally, in section5, we conclude by reviewing contributions of this research and offer some issues for future researches.

2. Problem definition

2.1. Model description

The general structure of proposed IFRLN is illustrated in Figure 1. In forward direction, the new product is manufactured in plants by raw material provided from different suppliers, i.e. whole sale contract, spot market, and recycled material. The product is conveyed from plants to customers through distribution centers within certain safety stock level. In backward direction, returned product is transferred from product sellers to collection centers for testing and inspecting. After classification, returned product is conveyed to distribution centers, remanufacturing plants, and disposals with respect to amount of repair. In any circumstances, the remanufactured product is transferred to second market customers through certain distribution centers. The model is proposed with generic nature, but it can encompass various industries such as digital, equipment, and vehicle

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industries. As a matter of fact, the model is more appropriate for industries with high return rate of products where these products can be selling up later as refurbished products in second markets.

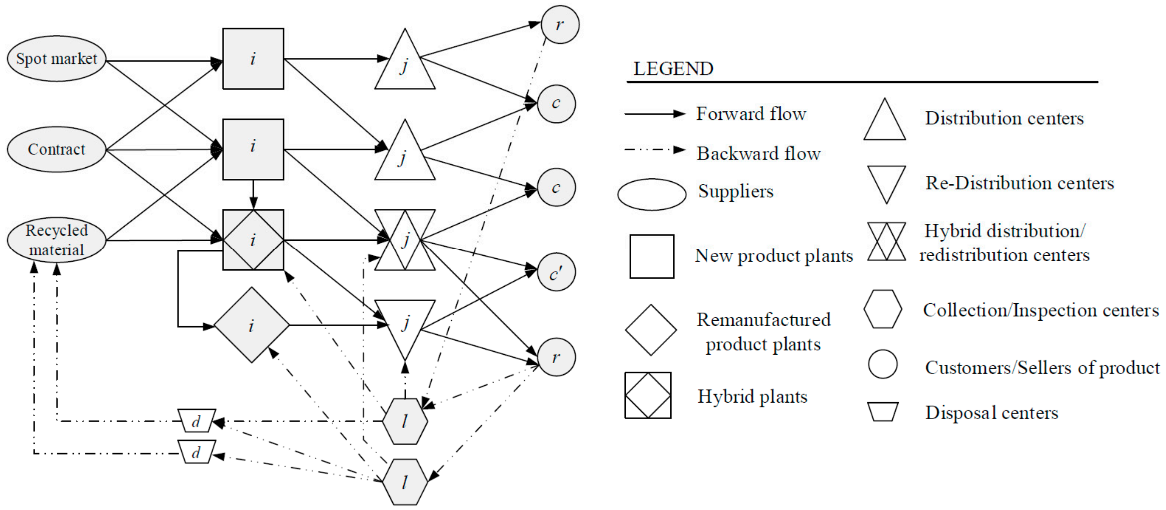

Figure 1. The proposed integrated forward/reverse logistic network model consisting suppliers, manufacturers, distribution centers, collection/inspection centers and disposal centers.

The introduced model is a multi-stage, multi-period, capacitated, single commodity IFRLN under uncertainty. Our specifications of model are listed as below:

- The periodic review policy is used for the distribution centers and manufactures in which the inventory levels are reviewed at certain intervals and the appropriate orders are placed after each review. The inventory level of raw material should meet a specific amount in each period. The production and shipment from the manufacturers to the distribution centers take place to raise the inventory level of distribution centers to the base-stock level (S) at the beginning of each period. This concept is referred to as the push strategy in the related literature. On the other hand, 6 the customer demands are met with the inventory kept by the distribution centers. The customers only place the orders to the distribution centers. This system is known as a pull-based system.
- A Hybrid concept for production plants is considered. Due to fact that Locating manufacture and remanufacture plants in a same potential place will reduce fixed costs, we are interested in locating hybrid plants.
- In distribution centers, risk pooling strategy is considered where both new and remanufactured product is held simultaneously. The ''risk-pooling'' strategy is as an efficient ways to manage demand uncertainty, for which inventory needs to be centralized at distribution centers (DC's) arriving to a convenient service levels. Each DC use base stock level inventory policy to satisfy demands from retailers as well as safety stock to cope with the variability of the customer demands at retailers to achieve ''risk-pooling'' benefits.
- As mentioned above, inventory level of raw material should meet a specific amount in each period. To this aim, raw material is provided through wholesale contract, spot market and recycled material. Wholesale contract is a long term agreement with suppliers to convey certain proportion of raw material in the beginning of each period. If amount of provided raw material by wholesale contract and recycled material do not meet the base stock level in each period, shortage of raw material compensates with buying from spot market but in higher price.

To specify the study scope, assumptions and limitations in the proposed model formulation are as follows.

- A single-product, multi-stage, multi-period supply chain network is given.
- We assume there are a finite set of facilities, i.e. manufacturers and distribution centers, should be opened.
- There is no limitation on the capacity of the material flow through the network.

- We face with the uncertainty on the demand of the customers to the distribution centers and return of used products to collection centers.
- Transportation cost is linearly dependent on the distance between stages.
- Distribution centers and raw material stock at manufactures incur inventory holding costs at the end of each period.
- All of the returned products must be collected, but, shortage is allowed for satisfying the demands of second market's customers.
- Customers' locations are known and fixed.

2.2. Model formulation

According to Birge et al. [27], in a stochastic optimization model the decisions could be taken in two stages. In the first stage, strategic decisions are determined as here-and-now decisions that should be made before the demand and return realization and the tactical decisions should be made in the secondstage as wait and see decisions. Moreover, the second-stage in our model considers multi-periods in which the tactical costs can be efficiently captured. This would be advantageous specifically for those supply chain networks whose demands differ from one period to another period. The following notations are used for the mixed integer linear programming (MILP) of the proposed model:

Sets:

I	Set of potential manufacturer locations $i, i' \in I$
J	Set of potential distribution center locations $j \in J$
T	Set of periods in planning horizon $t, k \in T$
C	Set of customers for new product $c \in C$
C'	Set of customers for used product $c' \in C'$
l	Set of potential collection center locations $l \in L$
D	Set of disposal locations $d \in D$
R	Set of seller product $r \in R$
S	Set of scenarios $s \in S$

Parameters, constants, and coefficients:

F_i^M	Fixed cost of locating manufacturer at location *i*
F_i^{RM}	Fixed cost of locating remanufacturer at location *i*
F_j^{Dc}	Fixed cost of locating distribution center for new product at location *j*
$F_j^{Dc'}$	Fixed cost of locating distribution center for used product at location *j*
F_l^{Cl}	Fixed cost of locating collection center at location *l*
s_i^P	Saving cost of locating a hybrid manufacture/ remanufacture facility at location *i*

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s_j^{Dcs}	Saving cost of locating a hybrid distribution center facility at location j
Vc_i^{M}	Cost for capacity of manufacturer i per unit of product
Vc_i^{RM}	Cost for capacity of remanufacturer i per unit of product
Vc_j^{Dc}	Cost for capacity of distribution center j per unit of new product
$Vc_j^{Dc'}$	Cost for capacity of distribution center j per unit of used product
Vc_l^{Cl}	Cost for capacity of collection center l per unit of returned product
Cap_i^{Max-M}	Maximum available capacity of manufacturing at location i
Cap_i^{Max-RM}	Maximum available capacity of remanufacturing at location i
Cap_j^{Max-Dc}	Maximum available capacity for new product at distribution center j
$Cap_j^{Max-Dc'}$	Maximum available capacity for second hand product at distribution center j
Cap_l^{Max-Cl}	Maximum available capacity of collection center at location l
Cap_i^{Max-P}	Maximum available capacity for production facilities at location i
$Cap_j^{Max-Dcs}$	Maximum available capacity for distributing center facilities at location j
Tc_{ij}^{M-Dc}	Cost of transporting per unit of product between manufacturer p and distribution center j
Tc_{jc}^{Dc-Cu}	Cost of transporting per unit of new product between distribution center j and customer c
$Tc_{jc'}^{Dc'-Cu'}$	Cost of transporting per unit of used product between distribution center j and customer cu'
Tc_{rl}^{Sr-Cl}	Cost of transporting per unit of product between seller r and collection center l
Tc_{ld}^{Cl-Di}	Cost of transporting per unit of product between collection center l and disposal d
Tc_{di}^{Di-M}	Cost of transporting per unit of recycled product between disposal d and manufacture i
$Tc_{lj}^{Cl-Dc'}$	Cost of transporting per unit of product between collection center l and distribution center j
Tc_{li}^{Cl-M}	Cost of transporting per unit of product between collection center l and manufacture i

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$Tc_{ii'}^{M-Rm}$	Cost of transporting per unit of product between manufacture i and remanufacture i'
Ic_{j}^{Dc}	Cost of holding per unit of inventory in distribution center j
Ic_{i}^{M}	Cost of holding per unit of inventory in manufacture i
D_{cst}^{Cu}	Product demand of customer c in scenario s at period t
Rs_{rts}	Product returns of seller r in scenario s at period t
$\Pr_{s}$	Probability of scenario s
BOM	The quantity of raw material needed for one unit of a product
C_{sm}	Cost of buying raw material from spot market
β	Rate of raw materials shipped from disposal center to raw material stock
λ	Rate of new product shipped from manufacture centers to distribution centers
γ_1	Rate of product shipped from collection centers to distribution centers
γ_2	Rate of product shipped from collection centers to disposal centers
M	A large number
N_t	Number of periods

Decision variables:

x_{i}^{M}	Binary variable equals to 1 if a manufacturer is located at location i, 0 otherwise
x_{i}^{RM}	Binary variable equals to 1 if a remanufacturer is located at location i, 0 otherwise
y_{j}^{Dc}	Binary variable equals to 1 if a distribution center for new product is located at location j, 0 otherwise
$y_{j}'^{Dc}$	Binary variable equals to 1 if a distribution center for used product is located at location j, 0 otherwise
x_{ii}^{P}	Binary variable equals to 1 if a manufacture and remanufacture located at location i, 0 otherwise
y_{j}^{Dcs}	Binary variable equals to 1 if a new product distribution center and used product distribution center located at location j, 0 otherwise
z_{l}^{Cl}	Binary variable equals to 1 if a collection center is located at location l, 0 otherwise
W^{C}	Quantity committed in wholesale contract

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r_{it}^{M}	Quantity committed in contract to manufacture *i* at period *t*
sm_{ist}^{M}	Quantity bought from spot market for manufacture *i* in scenario *s* at period *t*
qp_{ist}^{M}	Quantity of production in manufacture *i* in scenario *s* at period *t*
c_{i}^{M}	Capacity of manufacture *i*
c_{i}^{RM}	Capacity of remanufacture *i*
c_{j}^{Dc}	Capacity of distribution center *j* for new product
$c_{j}^{Dc'}$	Capacity of distribution center *j* for used product
c_{l}^{Cl}	Capacity of collection center *l*
b_{j}^{Dc}	Base-stock level of distribution center *j* at the beginning of each period
b_{i}^{M}	Base-stock level of manufacture *i* at the beginning of each period
inv_{ist}^{M}	Inventory level of manufacture *i* at the end of period *t* in scenario *s*
inv_{jst}^{Dc}	Inventory level of distribution center *j* for new products at the end of period *t* in scenario *s*
$inv_{jst}^{Dc'}$	Inventory level of distribution center *j* for second market products at the end of period *t* in scenario *s*
f_{ijst}^{M-Dc}	flow of production in manufacture *i* transported to distribution center *j* at period *t* in scenario *s*
f_{dist}^{Di-M}	flow of material from disposal *d* transported to manufacture *i* at period *t* in scenario *s*
$f_{ijst}^{RM-Dc'}$	flow of remanufactured product in remanufacture *i* transported to distribution center *j* in scenario *s* at period *t*
$f_{ii'st}^{M-Rm}$	flow of production in manufacture *i* transported to remanufacture i' in scenario *s* at period *t*
f_{list}^{Cl-Rm}	flow of returned product from collection center *l* transported to remanufacture *i* in scenario *s* at period *t*
$f_{ljst}^{Cl-Dc'}$	flow of returned product from collection center *l* transported to distribution center *j* at period *t* in scenario *s*
f_{ldst}^{Cl-Di}	flow of returned product from collection center *l* transported to disposal *d* at period *t* in scenario *s*

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f_{jcst}^{Dc-Cu}	flow of new product from distribution center j transported to customer c at period t in scenario s
$f_{jc'st}^{Dc-Cu'}$	flow of used product from distribution center l transported to customer c' at period t in scenario s
f_{rlst}^{Sr-Cl}	Flow of returned product from sellers r transported to collection center l at period t in scenario s

It should be noted that the uncertain demand and return in our mathematical formulation is introduced by ζ. ζ_s is a given realization of uncertain parameters and E_ζ represents the expected value with respect to ζ.

According to [27] the actual value of ζ becomes known in the second stage in which recourse decisions can be calculated. Therefore, decisions related to the first-stage are made by taking the future uncertain effects into account. These effects are measured by the recourse function, $Q(x,w,b)=E_\zeta\left(Q\left(x,w,b,\zeta^s\right)\right)$, where $Q(x,w,b)$ is the value of the second-stage for a given realization of the demand and return.

$$\min w=\sum_i x_i^M F_i^M+\sum_i x_i^{RM} F_i^{RM}+\sum_j y_j^{Dc} F_j^{Dc}+\sum_j y_j^{Dc'} F_j^{Dc'}+\sum_l z_l^{Cl} F_l^{Cl}+\sum_i c_i^M Vc_i^M+\sum_i c_i^{RM} Vc_i^{RM}$$
$$+\sum_j c_j^{Dc} Vc_j^{Dc}+\sum_j c_j^{Dc'} Vc_j^{Dc'}+\sum_l c_l^{Cl} Vc_l^{Cl}+W^c MN_t-\sum_i x_i^P s_i^P-\sum_j y_j^{Dcs} s_j^{Dcs}+Q(x,w,b) \tag{1}$$

Subject to:

$$c_i^M \le x_i^M \times \left(Cap_i^{Max-M}\right) \qquad \forall i\in I \tag{2}$$

$$c_i^{RM} \le x_i^{RM} \times \left(Cap_i^{Max-RM}\right) \qquad \forall i\in I \tag{3}$$

$$x_i^M + x_i^{RM} \ge 2\times x_i^P \qquad \forall i\in I \tag{4}$$

$$c_i^M + c_i^{RM} \le \left(Cap_i^{Max-P}\right)\times x_i^P \qquad \forall i\in I \tag{5}$$

$$x_i^M + x_i^{RM} \le x_i^P + 1 \qquad \forall i\in I \tag{6}$$

$$c_j^{Dc} \le y_j^{Dc}\left(Cap_j^{Max-Dc}\right) \qquad \forall j\in J \tag{7}$$

$$c_j^{Dc'} \le y_j^{Dc'}\left(Cap_j^{Max-Dc'}\right) \qquad \forall j\in J \tag{8}$$

$$y_j^{Dc} + y_j^{Dc'} \ge 2\times y_j^{Dcs} \qquad \forall j\in J \tag{9}$$

$$c_j^{Dc} + c_j^{Dc'} \le \left(Cap_j^{Max-Dcs}\right)y_j^D \qquad \forall j\in J \tag{10}$$

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$$y_j^{Dc} + y_j^{Dc'} \le y_j^{Dcs} + 1 \qquad \forall j \in J \tag{11}$$

$$c_l^{Cl} \le z_l^{Cl} \times Cap_l^{Max-Cl} \qquad \forall l \in L \tag{12}$$

$$b_j^{Dc} \le c_j^{Dc} \qquad \forall j \in J \tag{13}$$

$$W^C = \sum_i r_{it}^M \qquad \forall t \in T \tag{14}$$

where $Q(x,w,b)$ bring the solution of the following second-stage problem:

$$Min\, Q(x,w,b) = E_\zeta\left(Q(x,w,b,\zeta^s)\right) = \sum_s \Pr_s \begin{pmatrix} \sum_t\sum_i\sum_j f_{ijst}^{M-Dc} Tc_{ij}^{M-Dc} + \sum_t\sum_d\sum_i f_{dist}^{Di-M} Tc_{di}^{Di,M} \\ + \sum_t\sum_i\sum_j f_{ijst}^{Rm-Dc'} Tc_{ij}^{RM-Dc'} + \sum_t\sum_i\sum_{i'} f_{ii'st}^{M-Rm} Tc_{ii'}^{M-Rm} \\ + \sum_t\sum_i\sum_l f_{list}^{Cl-RM} Tc_{li}^{Cl-RM} + \sum_t\sum_l\sum_d f_{ldst}^{Cl-Di} Tc_{ld}^{Cl-Di} \\ + \sum_t\sum_l\sum_j f_{ljst}^{Cl-Dc'} Tc_{lj}^{Cl-Dc'} + \sum_t\sum_j\sum_c f_{jcst}^{Dc-Cu} Tc_{jc}^{Dc-Cu} \\ + \sum_t\sum_j\sum_{c'} f_{jc'st}^{Dc'-Cu'} Tc_{jc'}^{Dc'-Cu'} + \sum_t\sum_r\sum_l f_{rlst}^{Sr-Cl} Tc_{rl}^{Sr-Cl} \\ + \sum_i sm_{ist}^M C_{sm} + \sum_i inv_{ist}^M Ic_i^M + \sum_j inv_{jst}^{Dc} Ic_j^{Dc} + \sum_j inv_{jst}^{Dc'} Ic_j^{Dc} \end{pmatrix} \tag{15}$$

Subject to:

$$b_i^M = \sum_d\sum_{k=1}^t f_{disk}^{Di-P} + \sum_{k=1}^t r_{ik}^M + \sum_{k=1}^t sm_{isk}^M - \sum_{k=1}^{t-1} BOM \times qp_{isk}^M \qquad \forall t \in T, \forall i \in I, \forall s \in S \tag{16}$$

$$b_j^{Dc} = \sum_{k=1}^t\sum_i f_{ijsk}^{M-Dc} - \sum_{k=1}^{t-1}\sum_c f_{jcsk}^{Dc-Cu} \qquad \forall t \in T, \forall j \in J, \forall s \in S \tag{17}$$

$$inv_{ist}^M = \sum_d\sum_{k=1}^t f_{disk}^{Di-P} + \sum_{k=1}^t r_{ik}^M + \sum_{k=1}^t sm_{isk}^M - \sum_{k=1}^t BOM \times qp_{isk}^M \qquad \forall t \in T, \forall i \in I, \forall sc \in Sc \tag{18}$$

$$inv_{jst}^{Dc} = \sum_{k=1}^t\sum_i f_{ijsk}^{M-Dc} - \sum_{k=1}^t\sum_c f_{jcsk}^{Dc-Cu} \qquad \forall t \in T, \forall j \in J, \forall s \in S \tag{19}$$

$$inv_{jst}^{Dc'} = \sum_i\sum_{k=1}^t f_{ijsk}^{RM-Dc'} + \sum_l\sum_{k=1}^t f_{ljsk}^{Cl-Dc'} - \sum_{c'}\sum_{k=1}^t f_{jc'sk}^{Dc'-Cu'} \qquad \forall t \in T, \forall j \in J, \forall s \in S \tag{20}$$

$$b_i^M \ge BOM \times qp_{ist}^M \qquad \forall t \in T, \forall i \in I, \forall s \in S \tag{21}$$

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$$\sum_{k=1}^{t} BOM \times qp_{isk}^{M} \leq \sum_{d}\sum_{k=1}^{t} f_{disk}^{Di-P} + \sum_{k=1}^{t} r_{ik}^{M} + \sum_{k=1}^{t} sm_{isk}^{M} \qquad \forall t \in T, \forall i \in I, \forall s \in S \tag{22}$$

$$qp_{ist}^{M} \leq c_{i}^{M} \qquad \forall t \in T, \forall i \in I, \forall s \in S \tag{23}$$

$$\sum_{j} f_{ijst}^{RM-Dc'} \leq C_{i}^{RM} \qquad \forall t \in T, \forall i \in I, \forall s \in S \tag{24}$$

$$\sum_{r} f_{rlst}^{Sr-Cl} \leq c_{l}^{Cl} \qquad \forall t \in T, \forall l \in L, \forall s \in S \tag{25}$$

$$\sum_{i} f_{ijst}^{RM-Dc'} + \sum_{l} f_{ljst}^{Cl-Dc'} \leq c_{j}^{Dc'} \qquad \forall t \in T, \forall j \in J, \forall s \in S \tag{26}$$

$$\sum_{k=1}^{t}\sum_{i} f_{ijsk}^{RM-Dc'} + \sum_{k=1}^{t}\sum_{l} f_{ljsk}^{Cl-Dc'} - \sum_{k=1}^{t-1}\sum_{c'} f_{jc'sk}^{Dc'-Cu'} \leq c_{j}^{Dc'} \qquad \forall t \in T, \forall j \in J, \forall s \in S \tag{27}$$

$$\sum_{j} f_{ijst}^{RM-Dc'} = \sum_{l} f_{list}^{Cl-Rm} + \sum_{i} f_{ii'st}^{M-Rm} \qquad \forall t \in T, \forall i \in I, \forall s \in S \tag{28}$$

$$\sum_{k=1}^{t}\sum_{i} f_{ijsk}^{M-Dc} \geq \sum_{k=1}^{t}\sum_{c} f_{jcsk}^{Dc-Cu} \qquad \forall t \in T, \forall j \in J, \forall s \in S \tag{29}$$

$$\sum_{j} f_{jcst}^{Dc-Cu} \geq D_{cst} \qquad \forall t \in T, \forall c \in C, \forall s \in S \tag{30}$$

$$Rs_{rts} = \sum_{l} f_{rlst}^{Sr-Cl} \qquad \forall t \in T, \forall r \in R, \forall s \in S \tag{31}$$

$$\lambda \times qp_{ist}^{M} = \sum_{j} f_{ijst}^{M-Dc} \qquad \forall t \in T, \forall i \in I, \forall s \in S \tag{32}$$

$$(1-\lambda) qp_{ist}^{M} = \sum_{i'} f_{ii'st}^{M-Rm} \qquad \forall t \in T, \forall i \in I, \forall s \in S \tag{33}$$

$$\sum_{k=1}^{t}\sum_{c'} f_{jc'sk}^{Dc'-Cu'} = \sum_{k=1}^{t}\sum_{i} f_{ijsk}^{RM-Dc'} + \sum_{k=1}^{t}\sum_{l} f_{ljsk}^{Cl-Dc'} \qquad \forall t \in T, \forall j \in J, \forall s \in S \tag{34}$$

$$\sum_{j} f_{ljst}^{Cl-Dc'} = \gamma_{1} \sum_{r} f_{rlst}^{Sr-Cl} \qquad \forall t \in T, \forall l \in L, \forall s \in S \tag{35}$$

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$$\sum_{i} f_{list}^{Cl-Rm} = \gamma_2 \sum_{r} f_{rlst}^{Sr-Cl} \qquad \forall t \in T, \forall l \in L, \forall s \in S \tag{36}$$

$$\sum_{d} f_{ldst}^{Cl-Di} = (1-\gamma_1-\gamma_2) \sum_{r} f_{rlst}^{Sr-Cl} \qquad \forall t \in T, \forall l \in L, \forall s \in S \tag{37}$$

$$\beta \sum_{l} f_{ldst}^{Cl-Di} = \sum_{i} f_{dist}^{Di-M} \qquad \forall t \in T, \forall d \in D, \forall s \in S \tag{38}$$

Relation (1) is the objective function that minimizes the sum of the first-stage costs and the expected second-stage costs. The first-stage costs represent the costs of locating and capacity of the manufacturers, remanufactures, distribution centers for new and used products and collection centers along with wholesale contract amount and base stock level. Finally, saving costs of locating hybrid facilities are subtracted from the above-mentioned objective function. The second-stage objective function, i.e. Relation (15), includes two types of costs: firstly, the transportation costs, and secondly, the inventory holding costs.

Constraints (2-6), (7-11) and (12) ensure that the capacity restrictions for each production plants, distribution center facilities, and collection centers respectively. Constraints (4-6) deal with the hybrid strategy of locating manufacturing and remanufacturing plants. Constraint (13) guarantees that the capacity of each distribution center should be greater than base-stock level amount. Relation (14) assures that the amount of raw material provided to every manufactures in each period by wholesale contract should be equal to wholesale contract amount. Relations (16-20) are balance constraints that calculate base stock level at the beginning and inventory level at the end of each period. To be more specific relation (16) shows base stock level of each plant is equal with amount of raw material transported from all disposals, bought from spot market and assigned from wholesale contract in each period. These constraints refer to the push-based strategy concept in aforementioned mathematical formulation.

Relation (18) assesses Inventory of each plant in each scenario and period equals to sum of input raw materials subtracted from quantity of material used in production in that period. Relation (19) calculates the inventory level at the end of period *t* by subtracting the total output flow of new product to the customers in scenario s from all input flows to each distribution center until period t. Constraint (23) assures that products are not produced more than manufacturers' capacities in each scenario and period, while constraint (24) assures that used products will not carry more than the capacity of its DCs. Constraint (29) ensures the demands of all retailers are satisfied in scenario *s* at period *t*. relation (30) show that used product quantity in DCs is equal to customer's demand of it in each period and scenario. Rests of the constraints are mostly flow constraints between stages and facilities.

3. A Benders' decomposition-based solution algorithm

Benders' Decomposition (BD) algorithm is a classical solution approach for combinatorial optimization problems, which was firstly presented to solve MILP problems by Benders[28] . This method is one of wide commonly used techniques in the SCND problems (see for example[2] ,[29], [30]). In CLSC literature, Üster et al. [31] explore a multi-product network design problem and solve the model using Benders' Decomposition where multiple Benders' cuts are generated.

Benders' algorithm decomposes the main problem into two parts. The first part, called master problem (MP), solves a relaxed version of the problem to obtain values for a subset of the variables. The second part, called subproblem (SP), obtains the values of remaining variables while fixing variables of master problem, and utilizes these to generate cuts for the MP. The MP and SP are solved iteratively until the algorithm is converged. It should be note that there are two types of cuts:

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feasibility cut and optimality cut. Feasibility cut is added to the MP when the SP becomes infeasible, otherwise optimality cut is needed to be embedded in the MP.

BD is computationally very time-consuming if a large number of scenarios are used to characterize the randomness. To face with this problem in stochastic optimization problems, various techniques of accelerating Benders' decomposition have been proposed in recent decade. Research has mainly focused on either reducing the number of integer relaxed master problems being solved or on accelerating the solution of the relaxed master problem. In fact these techniques commonly generated stronger lower bounds and promoted faster convergence opposed to the classical Benders' approach. Multi-cut [32], local branching [33], valid inequalities [34, 35], alpha covering-bundling cuts, Magnanti [36], and combination of Meta heuristic approaches [37] are the most popular accelerating BD techniques. None of these approaches are a generic solution to accelerate BD and they mostly deal with very limited and specific problems.

In this paper, due to the nature of our problem, we apply valid inequalities to accelerate Benders' decomposition algorithm for solving the developed optimization problem.

Valid inequalities are some constraints that should be added to MP constraints. These constraints can strengthen the LP relaxation of the problem. They can also improve convergence of lower and upper bound by helping the relaxed MP to find close to optimal solutions. Indeed, because the iterative algorithm is initialized from empty subset s of extreme rays and extreme points, the relaxed MP initially contains only the integrality constraints. As a result, several iterations must be performed before enough information is transferred to the MP. Introducing valid inequalities in the MP can thus dramatically reduce the number of cuts that will have to be generated from extreme points and extreme rays of the dual SP polyhedron.

A pseudo-code of the proposed Benders' decomposition algorithm is presented as follows:

Benders' decomposition algorithm

Step 0. Initialization

i. $Z_0^{Upper} = +\infty$.

ii. $Z_0^{Lower} = -\infty$.

iii. $k = 0$.

iv. Solve the initial master problem to obtain

$$\left\{ c_i^{RM}, c_i^{M}, c_j^{Dc}, c_j^{Dc'}, b_j^{Dc}, b_i^{M}, c_l^{Cl}, w^c \right\}.$$

While ($Z_k^{Upper} - Z_k^{Lower} > \varepsilon$)

Step 1. Solving the sub-problems

For each $s \in S$

Solve the sub-problems by determined

$$\left\{ \hat{c}_i^{RM}, \hat{c}_i^{M}, \hat{c}_j^{Dc}, \hat{c}_j^{Dc'}, \hat{b}_j^{Dc}, \hat{b}_i^{M}, \hat{c}_l^{Cl}, \hat{w}^c \right\}$$

End for

Step 2. Updating the lower and upper bounds

i. $Z_k^{Upper} = \sum_{s \in S} \Pr_s \left(Z_{s,k}^{SP} \right) + f + \gamma \left[\xi + \frac{1}{1-\alpha} \left(\sum_{s \in S} \Pr_s \times \mu_s \right) \right]$

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ii. $$Z_k^{Lower} = \sum_{s\in S} \Pr_s \theta_s + f + \gamma\left[\xi + \frac{1}{1-\alpha}\left(\sum_{s\in S}\Pr_s \times \mu_s\right)\right]$$

Step 3. Solving the master problem

i. Add optimality cuts to the master problem for each scenario.

$$\theta^s \ge Z^{SP}_{s,k} + \sum_t \pi^{w^c}_{tsk} \times \left(w^c - \hat{w}^c_{sck}\right) + \sum_i \pi^{c_i^{RM}}_{isk} \times \left(c_i^{RM} - \hat{c}^{RM}_{isk}\right) + \sum_i \pi^{c_i^{M}}_{isk} \times \left(c_i^{M} - \hat{c}^{M}_{isk}\right)$$
$$+ \sum_j \pi^{c_j^{Dc'}}_{jsk} \times \left(c_j^{Dc'} - \hat{c}^{Dc'}_{jsk}\right) + \sum_j \pi^{c_j^{Dc}}_{jsk} \times \left(c_j^{Dc} - \hat{c}^{Dc}_{jsk}\right) + \sum_j \pi^{b_j^{Dc}}_{jsk} \times \left(b_j^{Dc} - \hat{b}^{Dc}_{jsk}\right) +$$
$$\sum_i \pi^{b_i^{M}}_{isk} \times \left(b_i^{M} - \hat{b}^{M}_{isk}\right) + \sum_l \pi^{c_l^{Cl}}_{lsk} \times \left(c_l^{Cl} - \hat{c}^{Cl}_{lsk}\right)$$

ii. k=k+1.

iii. Solve the master problem to obtain

$$\left\{c_i^{RM}, c_i^{M}, c_j^{Dc}, c_j^{Dc'}, b_j^{Dc}, b_i^{M}, c_l^{Cl}, w^c\right\}.$$

End while

3.1. Valid inequalities

As mentioned, we have added some valid inequalities equations to MP constraints in order to improve the convergence rate by hopefully reducing the associated feasible solution of MP. Using these valid inequalities reduces solution space of MP and avoids infeasibility of SP solution in each iterations. As a result only an optimal cut is generated to apply to MP. In our problem, following constraints can be added to the MP to ensure the feasibility of the sub-problems:

$$\sum_j b_j^{Dc} \ge \sum_C D_{cts} \qquad \forall t\in T, \forall s\in S \qquad (39)$$

$$\left(\sum_i b_i^{M}\right) / BOM \ge \left(\sum_j b_j^{Dc}\right) / \lambda \qquad (40)$$

$$\sum_l c_l^{Cl} \ge \sum_r Rs_{rts} \qquad \forall t\in T, \forall s\in S \qquad (41)$$

$$\sum_i c_i^{M} \ge \left(\sum_j b_j^{Dc}\right) / \lambda \qquad (42)$$

$$\sum_i c_i^{RM} \ge \gamma_2 \times \sum_r Rs_{rts} + \left(\sum_j b_j^{Dc}\right) \times \left((1-\lambda)/\lambda\right) \qquad \forall t\in T, \forall s\in S \qquad (43)$$

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$$\sum_j c_j^{Dc'} \ge \gamma_2 \times \sum_r Rs_{rts} + \left(\sum_j b_j^{Dc}\right) \times ((1-\lambda)/\lambda) + \gamma_1 \times \sum_r Rs_{rts} \qquad \forall t \in T, \forall s \in S \qquad (44)$$

$$b_i^M / BOM \le c_i^M \qquad \forall i \in I \qquad (45)$$

Constraint (39) guarantees that total base stock level of all DCs should be greater than or equal to the summation of customers' demand in each period and scenario. Constraint (40) indicates the relation between base stock level of manufacturers and DCs. Like constraint (39), constraint (41) guarantees that summation of returned products from all sellers cannot exceed total capacity of all collection centers. Constraints (42-45) addressing the relation between facilities capacities and base stock levels. For instance constraint (45) illustrates that capacity of each manufacturer must be at least equal to provided new product.

Lemma 1. *Adding Constraint (39) to the mathematical formulation has no effect on the optimal value of the objective function.*

Proof of Lemma 1. When the feasible solution for the addressed problem is available, the Constraints (17) and (30) are satisfied. Therefore, we can rewrite these constraints for the first period as follows:

$$b_j^{Dc} = \sum_{k=1}^{t}\sum_i f_{ijsk}^{M-Dc} - \sum_{k=1}^{t-1}\sum_c f_{jcsk}^{Dc-Cu} \rightarrow \sum_j b_j^{Dc} = \sum_i\sum_j f_{ijsk}^{M-Dc} - \sum_c\sum_j f_{jcsk}^{Dc-Cu} \quad \forall s \in S, \forall t \in T \text{ (I)}$$

$$\sum_j f_{jcst}^{Dc-Cu} \ge D_{cst} \rightarrow \sum_c\sum_j f_{jcst}^{Dc-Cu} \ge \sum_c D_{cst} \quad \forall s \in S, \forall t \in T \text{ (II)}$$

(I) and (II) lead to constraint $\sum_j b_j^{Dc} \ge \sum_C D_{cts}$. Since we show that Constraint (39) is constructed using the constraints of the SP, adding it to the mathematical formulation do not change the feasible space. So, the optimal value of the objective function remains unchanged. □

Lemma 2. *Adding constraint (40) to the mathematical formulation has no effect on the optimal value of the objective function.*

Proof of Lemma 2. The same as proof of Lemma 1, when the feasible solution for the addressed problem is available, the constraints (21) and (32) are satisfied. Therefore, we can rewrite these constraints as follows:

$$b_i^M \ge BOM \times qp_{ist}^M \rightarrow \sum_i b_i^M \ge BOM \times \sum_i qp_{ist}^M$$

$$\left(\sum_i b_i^M\right) / BOM \ge \sum_i qp_{ist}^M \quad \forall s \in S, \forall t \in T \text{ (I)}$$

$$\lambda \times qp_{ist}^M = \sum_j f_{ijst}^{M-Dc} \rightarrow \lambda \times \sum_i qp_{ist}^M = \sum_j\sum_i f_{ijst}^{M-Dc}$$

$$\sum_i qp_{ist}^M = \left(\sum_j\sum_i f_{ijst}^{M-Dc}\right) / \lambda \quad \forall s \in S, \forall t \in T \text{ (II)}$$

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Since $b_j^{Dc} = \sum_{k=1}^{t}\sum_i f_{ijsk}^{M-Dc} - \sum_{k=1}^{t-1}\sum_c f_{jcsk}^{Dc-Cu}$ obviously it can be inferred that $b_j^{Dc} \geq \sum_{k=1}^{t}\sum_i f_{ijsk}^{M-Dc}$.

(I) and (II) lead to constraint $\left(\sum_i b_i^M\right) / BOM \geq \left(\sum_j b_j^{Dc}\right) / \lambda$. Since we show that Constraint (41) is constructed using the constraints of the SP, adding it to the mathematical formulation do not change the feasible space. So, the optimal value of the objective function remains unchanged. □

Lemma 3. *Adding constraint (41) to the mathematical formulation has no effect on the optimal value of the objective function.*

Proof of Lemma 3. The same as proof of previous Lemmas, when the feasible solution for the addressed problem is available, the constraints (25) and (31) are satisfied. Therefore, we can rewrite these constraints as follows:

$$\sum_r f_{rlst}^{Sr-Cl} \leq c_l^{Cl} \rightarrow \sum_l \sum_r f_{rlst}^{Sr-Cl} \leq \sum_l c_l^{Cl} \quad \forall s \in S, \forall t \in T \quad \text{(I)}$$

$$Rs_{rts} = \sum_l f_{rlst}^{Sr-Cl} \rightarrow \sum_r Rs_{rts} = \sum_r \sum_l f_{rlst}^{Sr-Cl} \quad \forall s \in S, \forall t \in T \quad \text{(II)}$$

(I) and (II) lead to constraint $\sum_l c_l^{Cl} \geq \sum_r Rs_{rts}$. Since we show that Constraint (41) is constructed using the constraints of the SP, adding it to the mathematical formulation do not change the feasible space. So, the optimal value of the objective function remains unchanged. □

4. Computational results

To evaluate the performance of the proposed Benders' decomposition algorithm in terms of the solution quality, we performed some numerical experiments on a set of randomly generated problem instances. The algorithm was implemented in GAMS using CPLEX solver. All experiments were run with an Intel Pentium IV dual core 2.1 GHz CPU PC at 1 GB RAM under a Microsoft Windows XP environment.

4.1. Data generation for parameters and settings

The required data for random generation of problem instances drawn from the probability distributions and equations are shown in Table 3. Afterward, using the generated parameters, twelve problem instances with different sizes are constructed. Table 4 specifies the features of problem instances used to evaluate proposed solution approach.

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Table 3

Nominal values of the model parameters. For most of the parameters a uniform distribution is utilized. For demand and return an autoregressive time series (AR) is used.

Parameter	Range	Parameter	Range
F_i^{M}	~Uniform (1000000, 4000000)	$Tc_{ii'}^{M-Rm}$	~ Uniform (10, 25)
F_i^{RM}	~Uniform (500000,1500000)	Tc_{li}^{Cl-M}	~ Uniform (10, 20)
F_j^{Dc}	~Uniform (500000,2500000)	Ic_j^{Dc}	~ Uniform (20, 25)
$F_j^{Dc'}$	~Uniform (400000, 600000)	Ic_i^{M}	~ Uniform (30, 40)
F_l^{Cl}	~Uniform (300000,900000)	D_{cts}^{Cu}	$AR(1): D_{cu,t,sc}^{Cu} = \alpha + \beta_1 D_{cu,t-1,sc}^{Cu} + \varepsilon_{cu,t,sc}$
Vc_i^{M}	~Uniform (1000, 1800)		α ~ Uniform (20, 40)
Vc_i^{RM}	~Uniform(2000,2800)		β_i ~ Uniform (0.15, 0.2)
Vc_j^{Dc}	~Uniform (1500, 3000)		$\varepsilon_{cu,t,sc}$ ~ N(0, Uniform (20, 35))
$Vc_j^{Dc'}$	~Uniform (900,1500)		$D_{cu,t-1,sc}^{Cu}$ ~ Uniform (30, 50)
Cap_j^{Max-Dc}	~Uniform (7000, 15000)	Rs_{rts}	$AR(1): Rs_{sr,t,sc} = \alpha + \beta_1 Rs_{sr,t-1,sc} + \varepsilon_{cu,t,sc}$
$Cap_j^{Max-Dc'}$	~Uniform (1000, 2000)		α ~ Uniform (10, 20)
Cap_l^{Max-Cl}	~Uniform (1000, 5000)		β_i ~ Uniform (0.15, 0.2)
Tc_{ij}^{M-Dc}	~Uniform (10,30)		$\varepsilon_{cu,t,sc}$ ~ N(0, Uniform (10, 25))
Tc_{jc}^{Dc-Cu}	~Uniform (15, 30)		$Rs_{sr,t-1,sc}$ ~ Uniform (20, 30)
$Tc_{jc'}^{Dc'-Cu'}$	~Uniform (10, 30)	M	60
Tc_{ld}^{Cl-Di}	~Uniform (20, 35)	β	0.7
Tc_{di}^{Di-M}	~Uniform (10, 30)	λ	0.95
Tc_{rl}^{Sr-Cl}	~Uniform (15, 30)	γ_1	0.4
$Tc_{lj}^{Cl-Dc'}$	~Uniform (10, 20)	γ_2	0.4

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Table 4

Characteristics of test problems. 4 test cases are generated for each small, medium, and large test problems. Each test case has a specific distinction to the other cases.

Size of test problems	ID	i	j	l	C	C'	r	d	S	T
Small	1	4	8	8	10	15	10	2	20	12
	2	4	8	8	10	15	10	2	40	12
	3	5	10	10	12	15	12	2	20	12
	4	5	10	10	12	15	12	2	40	12
Medium	5	8	18	12	18	15	15	2	20	12
	6	8	18	12	18	15	15	2	40	12
	7	10	20	12	20	15	15	2	20	12
	8	10	20	12	20	15	15	2	40	12
Large	9	15	40	30	40	15	20	2	20	12
	10	15	40	30	40	15	20	2	40	12
	11	20	60	40	60	15	20	2	20	12
	12	20	60	40	60	15	20	2	40	12

As shown in table 4, in order to investigate performance of accelerated BD, test problems vary in size. These size leads to better understanding of accelerated BD power versus classic BD. In large scale problems as number of binary variables increases, solving the model with BD become more time consuming. Table 5, demonstrate the number of binary and continues variables of generated test problems.

Table 5

Number of variables and constraints in each test problem

ID	**Number of Variables**		**No. of constraints**	**No. of scenarios**
	Binary	**Continues**		
1	44	117,213	35,116	20
2	44	234,333	70,156	40
3	55	169,316	43,532	20
4	55	338,516	86,972	40
5	90	358,747	67,586	20
6	90	717,307	135,026	40
7	102	433,183	75,682	20
8	102	866,143	151,364	40
9	195	1,439,176	143,584	20
10	195	2,877,976	287,167	40
11	280	2,750,921	202,516	20
12	280	5,501,321	405,032	40

Test problems are solved with accelerated BD, classic BD, and CPLEX solver. We limit the solving time to 3h and BD iterations to 40 for small scale problems where for medium size, time was limited to 5h and BD iterations to 70 and for large scale problems the time limit was 10h and the BD iteration was 100. If a solution approach reached any of mentioned-limitation, the solution process should be stopped. Table 6 illustrates the optimality gap and CPU time of solving each test problem

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with these methods. Accelerated BD, solve the large scale problems better than classic BD with acceptable optimality gap. In small scale problems the difference is not considerable. CPLEX only solve four small scale test problems in an admissible time.

Table6

A comparison of proposed accelerated BD to classic BD and CPLEX for small, medium, and large size test problems.

Accelerated BD		Classic BD		CPLEX		
CPU(s)	**Optimality gap**	**CPU(s)**	**Optimality gap**	**CPU(s)**	**Optimality gap**	**ID**
320.64	0.8197	330.12	4.2310	210	0	1
642.61	0.4826	645.56	7.3141	721.18	0	2
393.76	0.5528	400.50	11.8911	400.50	0	3
780.02	0.8998	779.74	15.0164	>1h	--	4
1268.44	1.3446	1312.51	11.4512	2751.16	0	5
2618.37	1.5875	2669.98	14.7121	>5h	--	6
1540.67	2.6123	1591.56	15.1241	>5h	--	7
3089.33	3.4303	3090.12	16.0195	>5h	--	8
5009.21	4.9106	5093.42	15.9184	>10h	--	9
10121.71	7.2837	10274.84	17.4120	>10h	--	10
7021.13	6.2287	7421.12	18.1027	>10h	--	11
14011.87	8.5850	14573.69	19.8193	>10h	--	12

By comparing proposed accelerate BD with classic BD, one can realize that valid inequalities cause a faster convergence of lower and upper bound. Moreover, classic BD is initialized from empty subset s of extreme rays and extreme points where valid inequalities cause an initial value for lower bound of accelerated BD and lead to faster convergence of the upper and lower bounds.

5. Conclusions

In today's competitive business environment, the design and management of an integrated forward/ reverse supply chain network is one of the most important and difficult problems that managers encounter. To this aim, we propose a generic multi-stage, multi-period, single commodity and capacitated IFRLN design. To deal with uncertainty, demand of products (new and recovered product) and return of product from resellers are considered as stochastic parameters. Moreover we consider push/pull strategy and risk pooling strategy in the model. To solve the proposed two-stage stochastic programming model, Benders' Decomposition approach is used. Due to slow convergence of lower and upper bound in large scale problems, a number of valid inequalities are applied to master problem. Test problem results represents that accelerated BD have dominant optimality gap in comparison with classic BD in acceptable CPU time.

In the context of IFRLN a few papers solve their model with exact approaches specially BD. We believe this paper provides a good starting point in this research area.

It is suggested to extend the model for multi-commodity configuration. There are other stochastic parameters that are appropriate to consider in the model such as quality of products, raw material price, return rate, and recoverable rate of products. We propose base stock level as inventory policy where other non-linear inventory policy such as (S,S) and (R,Q) policies can investigate through the extended model. Moreover, since the refurbished and new products should have different prices, we believe taking pricing policies and guarantee regulations into account, will be the major future research area.

In the context of solution approach, other accelerating approaches of BD such as Lagrangian Relaxation (LR) or Meta Heuristics can be applied and verify the differences of these methods.

References

1. D. Simchi-Levi, P. Kaminsky, and E. Simchi-Levi, Designing and Managing the Supply Chain–Concepts, strategies and case studies, 2edt McGraw-Hill Irwin, 2007.
2. H. Mohammadi Bidhandi, R. Mohd. Yusuff, M. M. H. Megat Ahmad, and M. R. Abu Bakar, "Development of a new approach for deterministic supply chain network design," European Journal of Operational Research, vol. 198, pp. 121-128, 2009.
3. A. Amiri, "Designing a distribution network in a supply chain system: Formulation and efficient solution procedure," European Journal of Operational Research, vol. 171, pp. 567-576, 2006.
4. M. A. Ilgin and S. M. Gupta, "Environmentally conscious manufacturing and product recovery (ECMPRO): A review of the state of the art," J Environ Manage, vol. 91, pp. 563-91, Jan-Feb 2010.
5. M. Biehl, E. Prater, and M. J. Realff, "Assessing performance and uncertainty in developing carpet reverse logistics systems," Computers & Operations Research, vol. 34, pp. 443-463, 2007.
6. E. Akçalı, S. Çetinkaya, and H. Üster, "Network design for reverse and closed-loop supply chains: An annotated bibliography of models and solution approaches," Networks, vol. 53, pp. 231-248, 2009.
7. P. Chanintrakul, A. E. Coronado Mondragon, C. Lalwani, and C. Y. Wong, "Reverse logistics network design: a state-of-the-art literature review," International Journal of Business Performance and Supply Chain Modelling, vol. 1, pp. 61-81, 2009.
8. K. Lieckens and N. Vandaele, "Multi-level reverse logistics network design under uncertainty," International Journal of Production Research, vol. 50, pp. 23-40, 2012.
9. S. A. Alumur, S. Nickel, F. Saldanha-da-Gama, and V. Verter, "Multi-period reverse logistics network design," European Journal of Operational Research, 2012.
10. C. L. Liao and M. Y. Jiang, "Reverse Logistics Network Design with Recovery Rate Taken into Account," Industrial Engineering Journal/Gongye Gongcheng, vol. 14, pp. 47-51, 2011.
11. O. Listes, " A decomposition approach to a stochastic model for supply and return network design," Econometric Institute Reports, vol. 43, p. 27, 2002.
12. O. Listeş and R. Dekker, "A stochastic approach to a case study for product recovery network design," European Journal of Operational Research, vol. 160, pp. 268-287, 2005.
13. J. R. MATTHEW, J. C. Ammons, and J. N. DAVID, "Robust reverse production system design for carpet recycling," IIE Transactions, vol. 36, pp. 767-776, 2004.
14. M. I. G. Salema, A. P. Barbosa-Povoa, and A. Q. Novais, "An optimization model for the design of a capacitated multi-product reverse logistics network with uncertainty," European Journal of Operational Research, vol. 179, pp. 1063-1077, 2007.
15. M. Fleischmann, P. Beullens, J. M. Bloemhof-Ruwaard, and L. N. Van Wassenhove, "THE IMPACT OF PRODUCT RECOVERY ON LOGISTICS NETWORK DESIGN," Production and Operations Management, vol. 10, pp. 156-173, 2001.
16. M. Chouinard, S. D'Amours, and D. Aït-Kadi, "A stochastic programming approach for designing supply loops," International Journal of Production Economics, vol. 113, pp. 657-677, 2008.
17. M. C. Fonseca, Á. García-Sánchez, M. Ortega-Mier, and F. Saldanha-da-Gama, "A stochastic bi-objective location model for strategic reverse logistics," Top, vol. 18, pp. 158-184, 2009.
18. S. S. Kara and S. Onut, "A two-stage stochastic and robust programming approach to strategic planning of a reverse supply network: The case of paper recycling," Expert Systems with Applications, vol. 37, pp. 6129-6137, 2010.

19. S. S. Kara; and S. Onut, "A stochastic optimization approach for paper recycling reverse logistics network design under uncertainty," International Journal of Environment Science Technology, vol. 7, p. 15, 2010.
20. D.-H. Lee, M. Dong, and W. Bian, "The design of sustainable logistics network under uncertainty," International Journal of Production Economics, vol. 128, pp. 159-166, 2010.
21. M. S. Pishvaee, F. Jolai, and J. Razmi, "A stochastic optimization model for integrated forward/reverse logistics network design," Journal of Manufacturing Systems, vol. 28, pp. 107-114, 2009.
22. D.-H. Lee and M. Dong, "Dynamic network design for reverse logistics operations under uncertainty," Transportation Research Part E: Logistics and Transportation Review, vol. 45, pp. 61-71, 2009.
23. K. Lieckens and N. Vandaele, "Reverse logistics network design with stochastic lead times," Computers & Operations Research, vol. 34, pp. 395-416, 2007.
24. M. El-Sayed, N. Afia, and A. El-Kharbotly, "A stochastic model for forward–reverse logistics network design under risk," Computers & Industrial Engineering, vol. 58, pp. 423-431, 2010.
25. O. Listeş, "A generic stochastic model for supply-and-return network design," Computers & Operations Research, vol. 34, pp. 417-442, 2007.
26. M. Ramezani, M. Bashiri, and R. Tavakkoli-Moghaddam, "A new multi-objective stochastic model for a forward/reverse logistic network design with responsiveness and quality level," Applied Mathematical Modelling, 2012.
27. J. R. Birge and F. Louveaux, Introduction to Stochastic Programming. New York: Springer, 1997.
28. J. F. Benders, "Partitioning procedures for solving mixed-variables programming problems," Numerische mathematik, vol. 4, pp. 238-252, 1962.
29. T. Santoso, S. Ahmed, M. Goetschalckx, and A. Shapiro, "A stochastic programming approach for supply chain network design under uncertainty," European Journal of Operational Research, vol. 167, pp. 96-115, 2005.
30. S. MirHassani, C. Lucas, G. Mitra, E. Messina, and C. Poojari, "Computational solution of capacity planning models under uncertainty," Parallel Computing, vol. 26, pp. 511-538, 2000.
31. H. Üster, G. Easwaran, E. Akçali, and S. Çetinkaya, "Benders decomposition with alternative multiple cuts for a multi-product closed-loop supply chain network design model," Naval Research Logistics, vol. 54, pp. 890-907, 2007.
32. J. R. Birge and F. V. Louveaux, "A multicut algorithm for two-stage stochastic linear programs," European Journal of Operational Research, vol. 34, pp. 384-392, 1988.
33. W. Rei, J. F. Cordeau, M. Gendreau, and P. Soriano, "Accelerating Benders decomposition by local branching," INFORMS Journal on Computing, vol. 21, pp. 333-345, 2009.
34. H. D. Sherali and B. M. P. Fraticelli, "A modification of Benders' decomposition algorithm for discrete subproblems: An approach for stochastic programs with integer recourse," Journal of Global Optimization, vol. 22, pp. 319-342, 2002.
35. G. K. D. Saharidis, M. Boile, and S. Theofanis, "Initialization of the Benders master problem using valid inequalities applied to fixed-charge network problems," Expert Systems with Applications, vol. 38, pp. 6627-6636, 2011.
36. T. L. Magnanti and R. T. Wong, "Accelerating Benders decomposition: Algorithmic enhancement and model selection criteria," Operations Research, vol. 29, pp. 464-484, 1981.
37. C. A. Poojari and J. E. Beasley, "Improving benders decomposition using a genetic algorithm," European Journal of Operational Research, vol. 199, pp. 89-97, 2009.