1 *Article*

2 # Compton Scattering of a Twisted Light

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7 **Abstract:** The variation of photonic orbital angular momentum at Compton scattering is
8 characterized. We determine scattering matrix of a twisted light based on the fundamental
9 conservation of orbital angular momenta. Numerical values for two different twisted light modes:
10 Laguerre Gaussian and Bessel Gaussian, are generated and illustrated. Our analysis indicate that
11 states of photonic orbital angular momentum are highly changeable at wide angle scattering but
12 more consistent at small angle scattering.

13 **Keywords:** Compton scattering; Twisted light; Orbital angular momentum
14

15 **1. Introduction**

16 Twisted light carries well-defined photonic orbital angular momentum (POAM). Its wave front
17 characterizes specific azimuthal phase according to POAM state [1].   Both photonic spin angular
18 momentum and POAM are separate components of total light angular momentum.

19 Compton scattering describes the change in linear momentum at elastic collisions between a photon
20 and an electron. The well-known shift in a scatter wave number reports certain change in the linear
21 momentum as well as in the energy of scattering photons [2]. However, a variation in POAM have
22 never been comprehensively described.

23 Several studies have been recently conducted to describe changes in POAM at Compton scattering
24 in ultra-relativistic considerations [3, 4].   A non-relativistic framework has been implemented in the
25 density matrix theory to inform the variation of POAM at Compton scattering [5].

26 Our study briefly analyzes a change in POAM of twisted light in Compton scattering by evaluating
27 the associated scattering matrix at semi classical physics framework. It illustrate the possibility for
28 POAM to vary through scattering by atomic electrons. It also emphasizes the conservation of total
29 angular momentum regards exchange POAM into massive electrons.

30 We evaluate in the next section analytical expression for scattering matrix of Compton scattering for
31 a twisted light. The expression is valid for any axial symmetric beam carries well defined POAM. In
32 the following section, two different beams for instance have been examined. Our numerical
33 calculations are then analyzed and discussed. The study is finally summarized.
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38 **2.   Conservation of OAM in Compton scattering**

39   The schematic diagram of Compton scattering in Fig. 1 identifies the scattered photonic wave number
40   (k) as well as POAM by primed parameters (l), while the associated wave number and orbital angular
41   momentum of the recoil electron is indicated by k_e and m_e, respectively. The scattering angle (θ) is
42   defined as the angle between the incident beam and the scattering direction.
43   Our analysis bases on evaluating semi classically the scattering matrix of Compton scattering to
44   illustrate the fundamental of exchange the orbital angular momentum between twisted light wave
45   and massive particle.
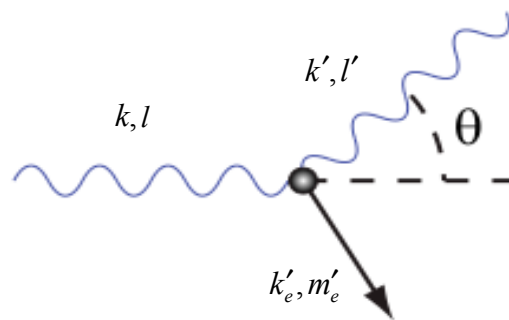46
47
48
49
50 k',l'
51   k,l
52 θ
53
54   k'_e, m'_e
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56
57   Fig.1: Schematic of Compton scattering with associated parameters.
58

59   Compton scattering matrix is determined as $S = \left\langle f' \left| e^{i(\vec{k}'-\vec{k}).\vec{r}} \right| f \right\rangle$ where: f', f represent scattered

60   and initial state of the system, $\vec{k}$, $\vec{k}'$ are incident and scattered wave vectors, and $\vec{r}$ is a position

61   vector [6]. The cylindrical coordinates are used in such a way the optical path is considered along z-
62   axis.
63   Both spatial electronic and photonic wave functions, Φ and Ψ respectively, are considered. Their
64   conjugates are indicated by superscript stars as well, Compton scattering matrix could be presented
65   as

$$S = \left\langle \Phi^*(\vec{r}')\Psi^*(\vec{r}') \left| e^{i(\vec{k}'-\vec{k}).\vec{r}} \right| \Phi(\vec{r})\Psi(\vec{r}) \right\rangle \tag{1}$$

67   Normalized wave functions in cylindrical coordinates are in used due to azimuthal symmetry of
68   twisted light. Generally; electronic and photonic wave functions could be expressed as:

69   $\Phi(\vec{r}) = P_m(\rho,z)e^{-im\phi}$, $\Psi(\vec{r}) = R_l(\rho,z)e^{-il\phi}$ where m,l are parameters indicate electronic state

70   and POAM state, respectfully. Therefore, Eq (1) is written explicitly as:

$$S = \left\langle P_{m'}^*(\rho',z')e^{i(m'+l')\phi} R_{l'}^*(\rho',z') \left| e^{i(\vec{k}'-\vec{k}).\vec{r}} \right| P_m(\rho,z)e^{-i(m+l)\phi} R_l(\rho,z) \right\rangle \tag{2}$$

72   The azimuthal symmetry of the spatial waves profiles splits exponential of azimuthal parameters out
73   as separate term as follow:

$$S = \left\langle P_{m'}^*(\rho',z')R_{l'}^*(\rho',z') \left| e^{i(\vec{k}'-\vec{k}).\vec{r}} \right| P_m(\rho,z)R_l(\rho,z) \right\rangle \left\langle e^{i(\Delta m+\Delta l)\phi} \right\rangle \tag{3}$$

75   where $\Delta m = m' - m$ and $\Delta l = l' - l$. The last angle bracket of Eq (3) is evaluated directly using Dirac
76   delta function:

$$\left\langle e^{i(\Delta m + \Delta l)\phi} \right\rangle = \delta(\Delta m + \Delta l) \tag{4}$$

77

78   Eq (4) reads that a decrease in POAM must be equivalent to an increase of electronic one: $\Delta m = -\Delta l$
79   . Consequently, Compton scattering matrix is not terminated as long as orbital angular momentum
80   exchanges between the photon and the electron, it must be conserved through scattering.
81   Evaluating Compton scattering matrix requires resolving the wave functions as well as middle
82   exponential term. Electronic radial wave function $P_m(\rho, z)$ is separable function, it is generally given

83   by:

$$P_m(\rho, z) = J_m(\kappa_e \rho) e^{ik_e z} \tag{5}$$

84

85   where J_m is Bessel function, κ_e and k_e are transverse and longitudal electronic pointing vector,
86   respectively. On the other hand, wave vector of collimated twisted light in general composites of two
87   components: orbital and longitudal [7]. It can be basically presented as

$$\vec{k} = \kappa \hat{\phi} + k \hat{z} \tag{6}$$

88

89   Hence, middle term in the scattering matrix, the exponential term in Eq.3, is simplified as follow:

$$\exp[(\vec{k}' - \vec{k}).\vec{r}] = \exp[(k' - k)z] \tag{7}$$

90

91   Eqs (4, 5 and 7) are implemented to evaluate scattering matrix as follow

$$S = \left\langle J_{m'}^*(\kappa_e' \rho') R_l^*(\rho', z') \middle| J_m(\kappa_e \rho) R_l(\rho, z) \right\rangle \ \delta(\Delta k_e + \Delta k) \tag{8}$$

92

93   Dirac delta term indicates that longitudinal component of the pointing vector, which represents the
94   linear momentum, is also exchanged between the electron and the photon. An increased in linear
95   momentum of the electron is equivalent to a decrease in linear momentum of the photon. It is
96   definitely consistent with well-known Compton`s formula [2]. Conservation of both linear and orbital
97   momentum emphasizes conservation of total momenta for twisted photons in Compton Scattering.
98   Our objective is to focus on the change of photonic orbital angular momentum in Compton scattering.
99   It leads evaluating scattering matrix regardless initial OAM state. It is computed for a relative OAM`s
100 change in zero order state "untwisted mode" i.e.:

$$S = \left\langle J_{\Delta l}^*(\kappa' \rho) R_{\Delta l}^*(\rho, z') \middle| J_0(\kappa \rho) R_0(\rho, z) \right\rangle \tag{9}$$

101

102

103 Conditions of Eq.3 and Eq.7 have been used. Indeed, It has been assumed there is no change in radial
104 parameter (ρ) through scattering, on the mean while axial parameter (z) depends only on the
105 scattering angle (θ). Thus, the scattering matrix is evaluated at certain axial distance.

106

107

108 **3. Results and Discussion**

109

Scattering matrix of Eq.9 is computed for two particular twisted light beams: *Bessel Gauss* and
110   *Laguerre Guassian (LG)*. The spatial wave function of LG is given by [8]:

111
$$R_l^{LG}(\rho, z) = \frac{C_p^l}{w(z)}\left[\frac{\sqrt{2}\rho}{w(z)}\right]^{|l|} \exp\left(-\frac{\rho^2}{w(z)^2}\right) L_p^{|l|}\left(\frac{2\rho^2}{w(z)^2}\right) \exp\left[\frac{ik\rho^2 z}{2(z^2 + z_R^2)} + iG_q^{|l|}(z)\right]$$

(10)

112   where C_p^l is a normalization constant, p is radial index, $w(z)$ is the beam width at distance z and

113   given by $w(z) = w_o\sqrt{1 + (z/z_R)^2}$,z_R is Rayleigh range which is defined as: $z_R = \pi w_o^2/\lambda$, $L_P^{|l|}(\bullet)$ is

114   associated LeGender polynomial, and $G_p^{|l|}(z) = -i(2p + |l| + 1)\tan^{-1}(z/z_R)$ is Gouy phase. It should be

115   noted that scattered axial parameter, z`, varies only through the scattered angle (θ). Our computations
116   bases on fixed z value.

117   Eqs.9 and 10 are used to evaluate scattering matrix of an x-ray of 1 nm wavelength and 1μm waist
118   width through wide scattering range $-\pi/2 < \theta < \pi/2$. It is assumed that there is no change in Gouy
119   phase through scattering.

120   The computed elements that associate for a certain change (Δl) in photonic OAM are determined and
121   plotted. Fig.2 illustrates Compton scattering matrix vs scattering angle for several changes in POAM.
122   The curves are generated by computing normalized scattering matrix on wide angle range for four
123   successive states change in POAM.

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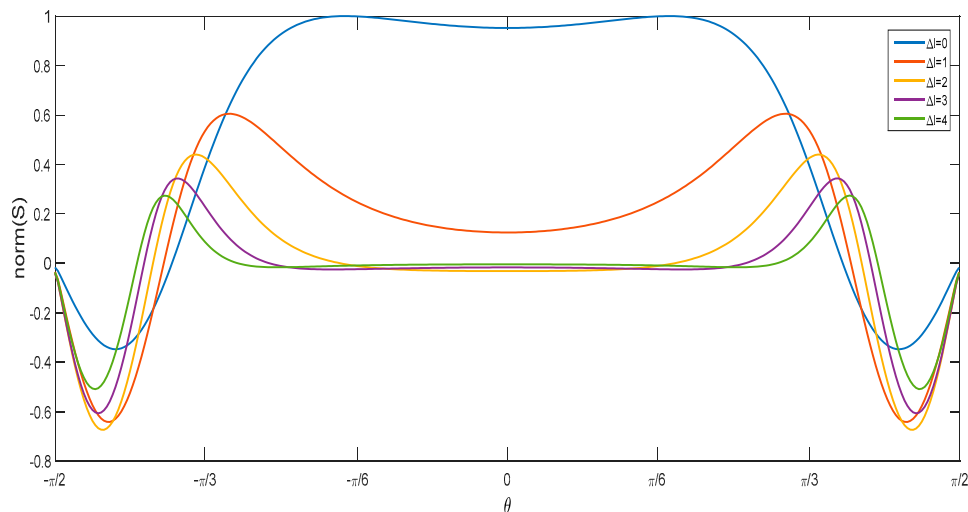
125   Fig.2 Normalized Compton Matrices vs scattering angle for LG beam at certain change in POAM

126   The normalized scattering elements shown in Fig.2 are very small for narrow range scattering. The
127   normalized scattering provides the probability of exchanging POAM which is minimum at small
128   scattering angle. However, a change by just one order state could happen with low possibility as
129   the curve of Δl=1 shows.

130   High possibilities of changing POAM states through scattering are represented by the peaks in Fig2.
131   However, scattering matrix elements are relatively small values for higher change in POAM, It
132   could be interpreted as a low possibility for a big change in POAM at Compton Scattering.

133 Negative-value elements in Fig.2 indicate flipping of OAM sates up, they inform that scattering in
134 wide angles causes flipping azimuthal phase up. Consequently, POAM states are twisted
135 "oppositely" at wide angles Compton-scattering. However; Scattering matrices are terminated at
136 right angle scattering due to the minimize differential cross section of Compton scattering [9].

137 Scattering Matrix is also computed for large changes in photonic OAM states and plotted in Fig.3
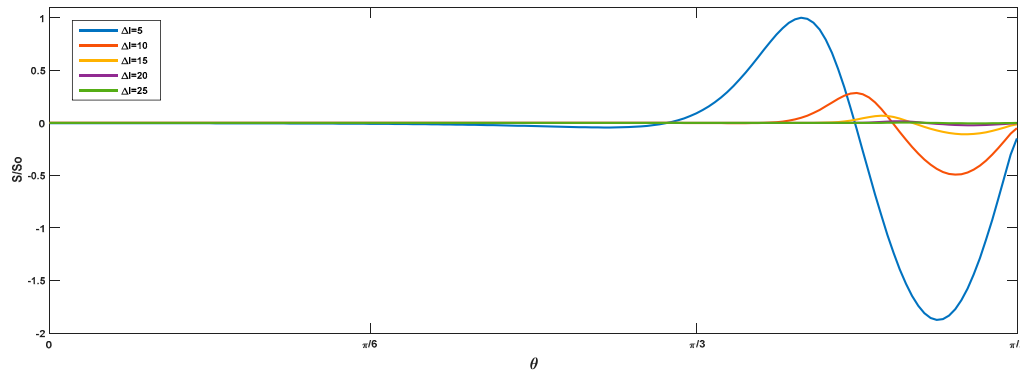
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139   Fig.3: Normalized Compton matrices vs scattering angle for LG beam at big changes in photonic OAM

140 Fig.3 confirms the low possibilities of scattering twisted photons with high changes in their POAM.
141 Indeed, it indicates Compton scattering at wide angles is more likely to occur with opposite twisted
142 orientation. Dominant negative peaks at Fig. 3 represent a tendency of large changes in photonic
143 OAM to be in opposite orientation.

144 The other twisted light beam that has been investigated is Bessel Gaussian beam which has spatial
145 wave given by [10]

146
$$R_l^{GB}(\rho,z) = A \frac{w_o}{w(z)} \exp\left[i\left(k - \frac{\kappa^2}{2k}\right)z - i\phi(z)\right] J_l\left[\frac{\kappa\rho}{(1+iz/z_R)}\right] \exp\left[\left(\frac{-1}{w(z)^2} + \frac{ik}{2R(z)}\right)\left(\rho^2 + \frac{\kappa^2}{k^2}z^2\right)\right]$$
(11)

147 where, A is normalization constant, $\phi(z) = \tan^{-1}(z/z_R)$ is the associated Gouy phase, and

148 $R(z) = z\sqrt{1 + (z_R/z)^2}$ is the radius of curvature. Eq. 11 has been implanted in, Eq. 9 to compute

149 the corresponding scattering matrix, same parameters: a nm wavelength

150 of an x-ray source, a μm beam waist and non-varying Gouy phase.

151 The associated scattering matrix is evaluated for first order changes in orbital angular as shown in
152 Fig. 4

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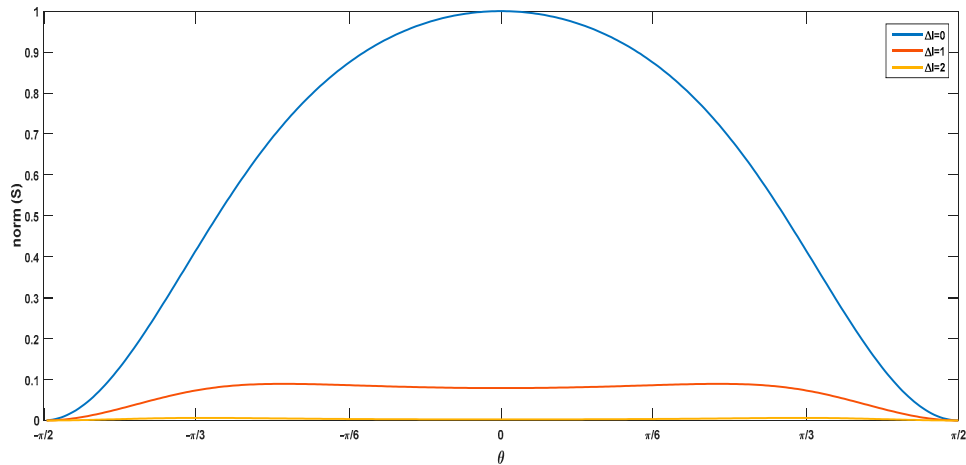
Fig.4: Normalized Compton matrices vs scattering angle for BG beam at few changes in photonic OAM

Scattering matrices of Bessel Gauss beam that illustrate in Fig. 4 change very slowly, relative high values peaks around scattering angle of π/3 compare with small values at small scattering angles, $|\theta| \leq \pi/6$. It likes to Laguerre Gaussian, Photonic OAM of Bessel Gauss beam is likely to invariant at small angle scattering, they just tends to vary in wide scattering, however; extreme minimum scattering elements is at right angle scattering where classical cross section is minimum.
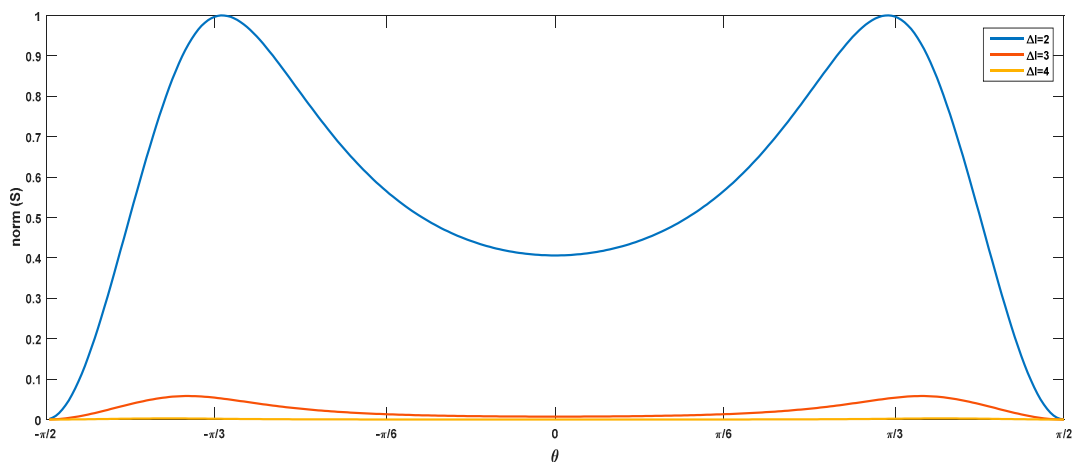
Fig.5: Normalized Compton matrices vs scattering angle for BG beam at certain changes in POAM

Scattering matrix is also computed for higher order changes in POAM of Bessel Gaussian mode as illustrated in Fig.5. The order is changed by varying associated azimuthal parameter to 2, 3 and 4, respectively.

Fig 5 shows emphasis the high possibility to exchange OAM at wide scattering compare with small angles scattering. POAM are more consistent in Compton scattering by small angles, although extreme lowest values at right angle scattering.

4. Conclusion

This study analyzes conservation of POAM at Compton scattering. We evaluated scattering matrix for twisted light that scatter by atomic electrons. Our analysis indicate POAM of a twisted light could be changed through Compton scattering. Specific numerical values are determined for

174   Compton scattering of Laguerre Gaussian and Bessel Gaussian light beams.   It has been reported
175   that POAM states are invariant at forward scattering as well as small angle scattering, they mainly
176   vary wide angle scattering.

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