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Flow Field and Energy Dissipation of Stepped Spillways with Different Horizontal Face Angles

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Abstract: Energy dissipation is one of the most important factors in choosing stepped spillways. However, few studies are about energy dissipation with different horizontal face angles. In this paper, the realizable k- ϵ turbulence model was used to study the flow field and energy dissipation rates of different stepped spillways with five horizontal face angles in the skimming flow regions. It shows that the free water surfaces and energy dissipation rates change with horizontal face angles and the energy dissipation rates initially decrease and then increase as the horizontal face angle increases. And the horizontal face angles cause the great changes of flow field.

Keywords: numerical simulation; horizontal face angle; energy dissipation rates; stepped spillway

1. Introduction

Stepped spillways are widely used in hydraulic engineering, as an energy dissipation structure for their better energy dissipation rates than that of in the smooth spillway (Anwar and Dermawan, 2011). Energy dissipation rate is an important factor in choosing energy dissipation structure, therefore, the energy dissipation in stepped spillways has been a research focus. Chanson (1994) studied the effect of flow regimes on energy dissipation in stepped chutes. Felder and Chanson

(2011) studied the energy dissipation in stepped spillway with nonuniform step heights. Shahheydari et al. (2015) studied the effect of discharges and slopes on energy dissipation by using Flow3D software. Barani et al. (2005), Mero and Mitchell (2016) studied the energy dissipation over various forms of stepped spillways. Guenther et al. (2013), Morovati et al. (2016) studied the energy dissipation of pooled stepped spillways. Tabari and Tavakoli (2016) studied the effect of different parameters such as step height, number of steps, step length and unit discharges on energy dissipation in stepped spillway by using numerical method.

In conclusion, most studies about energy dissipation rates nearly all focus on flow regimes, step size, slope and so on, whereas there is few studies on body type with various horizontal face angles. In this paper, flow field and energy dissipation rates of stepped spillways in skimming flow region with different horizontal face angles were studied. The energy dissipation rates with different horizontal face angles were obtained. And these results can be used in choosing a stepped spillway with better energy dissipation rates.

2. Numerical model

2.1 Creation of the numerical model

Fig. 1 shows the layout of the numerical model. It consists of a pressed slope section, a smooth section, a transitional section, a stepped section, and a tail water section. The width of stepped spillways $B = 40$ cm, the inlet height $h = 12$ cm, and the outlet height of the pressed slope section is 8 cm. In stepped section, the step sizes of different stepped spillways were the same (6 cm height and 12 cm length) and the only difference was the horizontal face angles, here, five horizontal face angles ($\theta = -30^\circ, -15^\circ, 0^\circ, 15^\circ, \text{ and } 30^\circ$) were studied. The tail water section is directly connected to the stepped section. There are 56 steps in each stepped spillway, which is named #1 to #56.

Fig. 1 Layout of the numerical model

2.2 Governing equations

The realizable k - ϵ turbulence model was presented by Shih et al. (1995) because it is useful for simulating stepped flow (Qian et al. 2009; Duangrudee et al. 2014). And the air-water interface was tracked by the volume of fluid (VOF) method. The equations of turbulent kinetic energy and its dissipation rate are as follows:

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho u_j k)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + G_k + G_b - \rho \epsilon - Y_M + S_k \quad (1)$$

$$\frac{\partial(\rho \epsilon)}{\partial t} + \frac{\partial(\rho u_j \epsilon)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_\epsilon} \right) \frac{\partial \epsilon}{\partial x_j} \right] + \rho C_1 S_\epsilon - \rho C_2 \frac{\epsilon^2}{k + \sqrt{\nu \epsilon}} + C_{1\epsilon} \frac{\epsilon}{k} C_{3\epsilon} G_b + S_\epsilon \quad (2)$$

$$\text{Here, } C_1 = \max[0.43, \frac{\eta}{\eta + 5}], \quad \eta = \frac{S k}{\epsilon}, \quad S = \sqrt{2 S_{ij} S_{ij}}, \quad S_{ij} = 0.5 \left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right)$$

$$\rho = \alpha_w \rho_w + (1 - \alpha_w) \rho_a, \quad \mu = \alpha_w \mu_w + (1 - \alpha_w) \mu_a$$

Where α_w is the volume fraction of water; ρ_w and ρ_a are the densities of water and air, respectively; μ_w and μ_a are the viscosities of water and air, respectively; $C_2 = 1.9$, $\sigma_k = 1.0$, $\sigma_\epsilon = 1.2$ are the empirical constants.

2.3 Boundary Conditions

The water inlet was set as velocity-inlet condition and its velocity was depending on the unit discharges (q). The outlet boundary was treated as pressure-outlet condition, where atmospheric pressure was assumed. For wall boundary, no-slip velocity condition was used and the standard wall

function was used to deal with the near-wall regions. And air boundary was set as pressure-inlet condition, where atmospheric pressure was assumed.

2.4 Validation model

In order to test the accuracy of numerical values, physical model experiments were made in the State Key Laboratory of Hydraulic and Mountain River Engineering, Sichuan University, Chengdu. The model consisted of an upper water tank, a stepped spillway including transitional section and stepped section, a tail water section, a measuring weir and a reservoir. The size of the stepped spillway and the unit discharges were identical to that of in the numerical model. Two horizontal face angles ($\theta=15^\circ, 30^\circ$) were chosen in the physical model experiments. And the validation model is shown in fig. 2.

Fig. 2 Layout of the validation model

Table 1, 2 are the comparison of pressures between numerical and physical values on horizontal surface and vertical surface, respectively. Here, X represents the distance of the pressure detecting points on horizontal surface from the step's inner edge, L represents the length of the step; Y represents the distance of the pressure detecting points on vertical surface from the step's lower edge, H represents the height of the step. Table 3 is the comparison of energy dissipation rates between numerical and physical values with various unit discharges. From tables 1 to 3, the maximum error of the pressure on horizontal surface is 7.94%, the maximum error of the pressure on the vertical surface is 7.41% and the maximum error of the energy dissipation rates is 6.7%. Although there are some errors, the accuracies are enough.

Table 1 Comparison of pressures between numerical and physical values on horizontal surface ($q=0.489\text{m}^2/\text{s}$, $\theta=30^\circ$)

X/L	P ₁ (m)			P ₂ (m)			P ₃ (m)		
	physical	numerical	error (%)	physical	numerical	error (%)	physical	numerical	error (%)
0.08	-7.02	-7.58	7.94	8.02	8.58	6.95	13.39	13.34	-0.37
0.17	-9.48	-9.93	4.76	3.80	4.04	6.23	12.70	13.12	3.34
0.25	-12.72	-13.18	3.56	1.40	1.46	4.15	12.80	13.11	2.43
0.33	-15.48	-14.45	-6.65	3.42	3.22	-5.76	14.05	13.30	-5.35
0.42	-11.04	-11.83	7.19	8.21	8.46	3.14	14.24	13.67	-4.01
0.50	-2.20	-2.37	7.45	15.08	15.23	1.03	14.99	14.09	-6.01
0.58	11.67	12.42	6.41	22.48	21.64	-3.72	15.17	14.35	-5.43
0.67	31.65	32.87	3.85	27.98	26.28	-6.08	14.44	14.22	-1.53
0.75	51.92	52.66	1.41	30.63	28.53	-6.86	14.57	13.65	-6.30
0.83	61.08	63.15	3.39	31.24	28.86	-7.62	13.40	12.64	-5.68
0.92	60.57	57.25	-5.47	27.32	25.72	-5.88	11.85	11.19	-5.59

Table 2 Comparison of pressures between numerical and physical values on vertical surface ($q=0.489\text{m}^2/\text{s}$, $\theta=30^\circ$)

Y/H	P ₁ (m)			P ₂ (m)			P ₃ (m)		
	physical	numerical	error(%)	physical	numerical	error(%)	physical	numerical	error(%)
0.83	-23.94	-25.70	7.35	-3.31	-3.53	6.86	8.16	7.65	-6.21
0.67	-11.58	-12.33	6.54	-1.49	-1.55	4.35	8.68	8.49	-2.14
0.50	-10.48	-11.25	7.41	-2.84	-2.64	-7.09	8.90	9.03	1.39
0.33	-9.12	-9.47	3.83	1.49	1.60	6.95	10.71	10.39	-3.01
0.17	-7.61	-7.26	-4.48	8.95	9.56	6.82	11.92	12.37	3.70

Table 3 comparison of energy dissipation rates between numerical and physical values with various unit discharges

case		$\theta=30^\circ$			$\theta=15^\circ$		
q(m ² /s)	physical value	numerical value	error(%)	physical value	numerical value	error(%)	
0.313	75.21	79.56	5.78	69.40	68.42	-1.41	
0.425	73.21	75.39	2.98	65.62	66.65	1.57	
0.489	70.14	74.84	6.70	64.38	66.06	2.61	
0.552	69.24	73.56	6.24	64.11	65.97	2.90	
0.600	67.76	70.24	3.66	60.45	62.86	3.99	

3. Numerical results and analysis

3.1 streamlines and free water surface

Fig. 3 shows the streamlines and the free water surfaces of different shaped steps on one step numbered #43 for example. In this figure, the free water surface was across the width. It can be seen that: 1) when $\theta = 0^\circ$, the streamlines are parallel to the axial plane, so the free water surface is horizontal; 2) when $\theta > 0^\circ$, the streamlines are not parallel to the axial plane and they are from sidewalls to axial plane, so the water flow makes collisions at near the axial plane, as a result, the free water surface is not horizontal, and it is higher near the axial plane. But the body of stepped spillway with $\theta = 30^\circ$ changes larger than that of in stepped spillway with $\theta = 15^\circ$, so the collision of water flow at near the axial plane is more intense, as a result, the change of free water surface is more intense; 3) when $\theta < 0^\circ$, the streamlines are also not parallel to the axial plane and they are

from axial plane to sidewalls, and the water flow from axial plane makes collisions with sidewalls, as a result, the free water surface is not horizontal, and the free water surface is higher near the sidewalls, forming wall pressing flow. But the body of stepped spillway with $\theta = -30^\circ$ changes larger than that of in stepped spillway with $\theta = -15^\circ$, so the collision of water flow at near the sidewalls is more intense, as a result, the change of free water surface is more intense.

Fig. 3 streamlines and free water surfaces of different shaped steps (#43) (left: streamlines; right: free water surface)

3.2 energy dissipation rate

As a result of the conservation of upstream and downstream energy, the ratio of the energy loss to the upstream energy is defined as the energy dissipation rate:

$$\eta = \frac{\Delta E}{E_1} \times 100\% = \frac{E_1 - E_2}{E_1} \times 100\% \quad (3)$$

where E_1 , E_2 are the total energy in the beginning and ending of stepped section, respectively;

$E_1 = \Delta h + v_1^2/(2g)$, $E_2 = v_2^2/(2g)$; where Δh is the difference in height between the two

sections; v_1 and v_2 are the average velocities in the two sections.

Fig. 4 shows the energy dissipation rates change with various unit discharges and horizontal face angles. It can be seen that: 1) the energy dissipation rates in all shaped stepped spillways decrease with the increasing of unit discharges but the decrease rate is larger in the stepped spillway with $\theta = 0^\circ$; 2) at a given unit discharge, as the angle increases, the energy dissipation rate initially decreases and then increases; the energy dissipation rate is the lowest in the stepped spillway with $\theta = 0^\circ$.

Fig. 4 energy dissipation rates change with various unit discharges and horizontal face angles

3.3 turbulence kinetic energy and dissipation rate

Fig. 5 shows the turbulence kinetic energy and the ratio of turbulence kinetic energy dissipation rates and its turbulence kinetic energy change with different unit discharges and horizontal face angles. In this figure, k and ε mean the maximum turbulence kinetic energy and the maximum turbulence kinetic energy dissipation rate in corresponding calculation conditions, respectively; ε/k means the ratio of turbulence kinetic energy dissipation rates and its turbulence kinetic energy, which can reflect the changes of energy dissipation rates. As we all know that when the unit discharge is larger and the more complex of the structure causing by horizontal face angles, the fluctuating velocity is larger. Then the turbulent kinetic energy and the turbulence kinetic energy

dissipation rate will be larger.

We can see that from fig. 5(a), in all shaped stepped spillways, ε/k decreases as the increasing of unit discharges. It means that the increase of turbulence kinetic energy dissipation rate is less than that of in turbulent kinetic energy as the increasing of unit discharges, so the energy dissipation rates decrease as the increasing of unit discharges.

From fig. 5(b), we can know that at a given unit discharge, with increasing angles, the ε/k initially decreases and then increases; the minimum ε/k is observed at $\theta = 0^\circ$, and they are slightly smaller in a stepped spillway with $\theta > 0^\circ$ than that of in a stepped spillway with $\theta < 0^\circ$ for equal absolute values of the angles, so with increasing horizontal face angles, the energy dissipation rates initially decrease and then increase at all unit discharges and they are slightly smaller in a stepped spillway with $\theta > 0^\circ$ than that of in a stepped spillway with $\theta < 0^\circ$ for equal absolute values of the angles, which is shown in fig. 4.

Fig. 5 turbulence kinetic energy and the ratio of turbulence kinetic energy dissipation rates and turbulence kinetic energy change with different unit discharges and horizontal face angles

4. Conclusions

In this paper, through studying the variations of the energy dissipation rates and flow field with different horizontal face angles, some conclusions can be drawn as follows:

1) the free water surface changes with the horizontal face angles, when $\theta > 0^\circ$, the free water surface is higher near the axial plane; when $\theta < 0^\circ$, the free water surface is higher near the sidewalls; and the absolute values of the angles are larger, the fluctuation of free water surface will be larger;

2) the energy dissipation rate increases with the absolute values of horizontal face angles and decreases as the unit discharge increases;

In conclusion, the horizontal face angles of stepped spillway change the flow field and the flow direction, which produces some unique characteristics, such as unique vortex structures, which can cause better energy dissipation. These results will be useful in choosing a better stepped spillway for energy dissipation.

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Notation:

- B model width
- h inlet height
- θ horizontal face angle
- q unit discharge
- η energy dissipation rate
- E_1 total energy in the first step of a stepped section
- Δh difference in height between two sections
- E_2 the total energy at a section below the stepped section
- ΔE difference in energy between two sections
- v_1, v_2 average velocities in two sections, respectively
- k the maximum turbulence kinetic energy;
- ε the maximum turbulence kinetic energy dissipation rate;
- ε/k the ratio of turbulence kinetic energy dissipation rates and its turbulence kinetic energy

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