Preschoolers’ development of understanding zero

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While the knowledge about the development of understanding positive integers is rapidly growing, the development of understanding zero is not well-known. Here we tested several components of preschoolers’ understanding zero: whether they can use empty sets in numerical tasks, whether they can use empty sets soon after they understand the cardinality principle, whether they know what the word “zero” refers to, and whether they categorize zero as a number. The results show that preschoolers can handle empty sets in numerical tasks as soon as they understand the cardinality principle or even earlier, and some of them know that these sets are labeled as “zero.” However, they are unsure whether zero is a number. These results identify three components of knowledge about zero: operational knowledge, linguistic knowledge, and meta knowledge. To account for these results we propose that preschoolers might understand numbers as the properties of items or objects in a set. In this view, zero cannot be a number, because an empty set does not include any items, and the missing items cannot have any property, excluding also the number property. This model might explain why zero is handled correctly in numerical tasks, while it is not regarded to be a number.

Keywords: numerical cognition; zero; number status of zero; items based number representation

Highlights:

- Preschoolers can handle zero as soon as they can handle positive integers.
- However, preschoolers are not sure if zero is a number.
- Children may start to understand numbers as the property of set of items

1 Introduction

Children start to understand the use of symbolic exact numbers at around the age of 3 years (Wynn, 1990, 1992). Although many details about the development of understanding natural numbers are already known, the development of understanding zero is hardly known, and it is not integrated into the numerical cognition models. It is still largely unknown how zero is handled and understood in tasks where symbolic natural numbers are already used successfully by preschoolers. The main aim of the present study is to extend the description of the development of understanding zero and to consider its theoretical implications.

1.1 The lack of developmental models of understanding zero

Models about the development of numerical cognition mostly cannot specify how understanding zero is integrated into the more general numerical knowledge. As a starting point, in the infant literature, there is an agreement that nonsymbolic (such as arrays of dots, series of sounds or events, etc.) numerical information is probably processed by two representations (Feigenson et al., 2004; Piazza, 2010). Dominant models propose that in infants numerical information is handled by either the imprecise Approximate Number System (Feigenson, Dehaene, & Spelke, 2004; Piazza, 2010) or the visual attention related Object Tracking System (Feigenson et al., 2004). However, it is not straightforward whether any of these systems can handle zero. (See more details about how these models may or may not account for zero processing in the Supplementary material.)

The next important step in the development of number understanding is the understanding of the exact large symbolic numbers in preschoolers. Symbolic number means that the values are denoted by symbols, in preschoolers usually by number words, as opposed to the nonsymbolic quantities, such as arrays of visual objects or series of auditory events. According to the consensus of the literature, at around the age of 3 or 4 years, children start to understand the conceptual principles of number use, termed understanding the cardinality principle (Lipton & Spelke, 2006; Sarnecka & Carey, 2008; Wynn, 1990), with which they are able to handle exact symbolic numbers. Number knowledge in preschoolers is usually measured with the “give a number” task (or Give-N task) in which children are asked to give a specific number of objects from a pile of objects (Wynn, 1990, 1992). With this task one can determine what number understanding phase a child is in. First, preschoolers are in the pre-numeric phase, where although they know the counting list (the series of words “one-two-three”, etc.), they do not know the meaning of these words and they fail in the task. Second, children become subset-knowers, when they can give 1, 2, 3, or 4 items, but not when asked for more, even when they know how the counting list continues. Finally, preschoolers become cardinality-principle-knowers (CP-knowers), when they can give any amount of items that is in their known counting list. This last phase is believed to show the real understanding of exact symbolic numbers (Lipton & Spelke, 2006; Sarnecka & Carey, 2008; Wynn, 1990, 1992). (Although see some limitations of this description, for example, in K. Davidson et al., 2012; Le Corre, 2014; Le Corre & Carey, 2007; Sella & Lucangeli, 2020)

There is no consensus what representational changes occur when a preschooler understands the cardinality principle. Most models suppose that in some way the systems available in infancy might play a role in the first steps (Carey, 2004, 2009; Piazza, 2010), although it is not known entirely how these or other systems contribute to the understanding of symbolic exact numbers. Partly related to this problem, it is not known whether preschoolers understand symbolic zero when they understand the cardinality principle. Importantly, the few former works investigating preschoolers' symbolic zero understanding (see the next subsection) cannot be integrated into this framework, because those works have not investigated whether those preschoolers understood the cardinality principle or not.

1.2 Contradicting results about understanding zero in preschoolers

To our knowledge, there are only two studies investigating quantitatively the zero concept denoted symbolically in preschoolers (Bialystok & Codd, 2000; Wellman & Miller, 1986). Note again that the present work focuses on the processing of symbolic stimuli, because (a) investigating nonsymbolic zero involves many unresolved methodological issues, (b) in the recent years, several works revealed essential differences between symbolic and nonsymbolic number processing (e.g., Bulthé et al., 2015; Krajcsi et al., submitted, 2016; Noël & Rousselle, 2011; Schneider et al., 2017), which differences question whether symbolic stimuli are processed by the evolutionarily old, imprecise number representation, and (c) because CP-knowledge points beyond approximate number handling (Carey & Barner, 2019).[1] As we mentioned, none of these two studies is related to the current developmental models. Importantly, the conclusions of these two works contradict each other, whether handling zero is more difficult or not for preschoolers compared to handling positive integers. In this subsection, first, we briefly summarize the two studies, then we discuss the potential causes of the differing conclusions.

Wellman and Miller (1986) presented children between 3 and 7 years of age with the following tasks: (a) count items in sets (including empty sets), (b) name the smallest number they knew, (c) name Indo-Arabic symbols, and (d) compare numbers between 0 and 5 in Indo-Arabic notation. Their main finding was that the understanding and the use of zero were delayed compared to the use of positive numbers. In a detailed analysis the authors concluded that there are three typical behavioral patterns, or in their term, phases. In the first phase, the children could name the 0 symbol, although they did not understand the meaning of it. In the second phase, the children could count backward to zero, understanding that zero means nothing. Finally, in the third phase, the children knew that zero was the smallest number, and they could compare numbers even if one of the numbers was zero. The progress of the development was slow: 4-year-old children did not reach the first phase, and only 6-year-old children were able to reach the final phase.

However, a later study found evidence that understanding zero may be as uncomplicated as positive numbers for preschoolers. Bialystok and Codd (2000) investigated the understanding and

1There are a few other related former works that are not relevant from the viewpoint of the present work. First, some of the works cited in the main text also investigate children in their first school years. Still, we mostly focus on the preschool years, as our study investigates knowledge of zero in preschoolers around the time when the cardinality principle is acquired. Second, while several former works investigated the understanding in preschoolers, they study other aspects of this understanding not in line with the aims of the present study and their result is not conclusive for the present aims. While the Merritt & Brannon (2013) paper collects preschooler data about processing zero, the comparison task is not symbolic. Also, while Davidson (1992) uses numerical tasks with zero in preschoolers, those tasks alternate training tasks and questions, so it is not clear what is the effect of the training in the session. Finally, while Baroody, Lai, Li, & Baroody (2009) partly investigates symbolic arithmetical operations with zero, they do not report the results of positive number-only tasks, therefore, it is impossible to tell whether children have more difficulty with zero than with positive numbers, and they measure the number knowledge of the children with the Is it N task, which is considered to be an invalid measure of the number knowledge by the literature (Wynn, 1990, 1992).

spontaneous notation of positive integers, zero, and fractions in preschool children. They found that understanding zero is not harder than understanding positive integers, a conclusion that is not in line with the previously described study by Wellman and Miller (1986). In this work, children between 3 and 7 years of age were asked to give different amounts of cookies to puppets, and to make written notes about these amounts. The children had to recall that amount 20 minutes later, and again 2 weeks later. In both instances children were allowed to use the notes they had made. According to the results, children were able to solve the zero giving task. However, it is important to note that the instruction was not formed in the usual mathematical way, e.g., children were not told that "Give Big Bird zero cookies", instead they were instructed to "Give Big Bird no cookies for lunch." Children could make a note of the number zero as efficiently as making a note of the positive integers. Similarly, they could recall the correct number after 20 minutes be it either zero or positive integers. However, two weeks later unlike 5-year-old children, 3- and 4-year-old children were not able to recall the number zero as successfully as recalling positive integers. (The same result was found by Hughes, 1986, revealing that children can use notes for denoting zero, although quantitative results were not published for that study.)

The contradiction between the conclusions of the two studies (Bialystok & Codd, 2000; Wellman & Miller, 1986) might originate from methodological differences and from interpretational issues. Obviously, the two studies used completely different tasks, and it is possible that zero can be handled more easily in some tasks than in others. Yet there are some less trivial sources of differences. First, while Wellman and Miller (1986) suggest three phases of the development of understanding zero, actually, their data seem to reveal four phases. (Although the authors mentioned that the data might include some inconsistency, they insisted on the interpretation with three phases.) There is an extra phase between the second and the third phases: after successfully counting back to zero, preschoolers could compare numbers with zero, whereas they did not know that zero was the smallest number (see Table 1 in Wellman & Miller, 1986). This phase is actually paradoxical: While children know that zero is smaller than one, they think that one is the smallest number. What causes this dissociation in their knowledge about zero? As a possible explanation we hypothesize that children do not think that zero is a number. This possible apparent misconception is observable even in adults: In a study 15% of preservice elementary school teachers responded that zero is not a number (Wheeler & Feghali, 1983). Another possible explanation is that zero is not part of the counting list (which usually starts with "one"), and that is why children handle it differently (Merritt & Brannon, 2013). Both explanations would suggest that the meta-knowledge about the number status of zero could be independent of handling correctly the zero value. Consequently, one might think that children can understand zero sufficiently when they can compare zero correctly, but they do not yet know that zero should be categorized as a number. If this is the case, children could understand zero earlier than it was originally proposed by Wellman and Miller. We get back to this problem and to a more detailed list of possible explanations in the General discussion.

A second methodological problem that could cause different conclusions in the two studies is that the linguistic formulation of the tasks regarding zero might influence the performance. While the mathematical viewpoint suggests that zero is a number just like any other integers, and therefore zero should be used linguistically in the way other numbers are used, natural language mostly uses different linguistic forms to offer a statement about zero. For example, we usually do not say that "the car travels with zero kilometers per hour", instead, we say that "the car has stopped." Similarly, we do not say that "give zero cookies to Peppa Pig", but we say that "do not give any cookies to Peppa Pig" or "give no cookies to Peppa Pig." Thus, it is possible that using the mathematical language is harder for children than using the natural language, because the mathematical version is used less frequently. This could be hypothetically confirmed by the previous data: the Wellman and Miller (1986) study used the mathematical language and found difficulties in understanding zero, while the Bialystok and Codd (2000) study used the natural language and found no difficulties with handling zero. However, based only on these data one cannot be sure whether the language form really influenced the performance, because of the many other differences between the two studies.

1.3 Aims of the study

To create appropriate models, it is essential to have reliable data first. Because former studies drew contradicting conclusions about whether processing of the zero value is harder than processing positive values in preschoolers (Bialystok & Codd, 2000; Wellman & Miller, 1986), and because based only on former data one cannot specify the sources of those contradictions, the main aim of this work is to provide additional, more systematically collected data about the symbolic understanding of zero in preschoolers. This also means that our aims do not build upon the theoretical models, such as the Approximate Number System, the Object Tracking System, and so on (presented in the first part of the Introduction), but they intend to clarify the main phenomena and describe the development more precisely and extensively than previous works did (presented in the second part of the Introduction).

(Aim 1) Specifically, a more comprehensive range of tasks (giving a set, comparison, addition and subtraction) is used to investigate whether children can handle zero in numerical tasks as efficiently as they can handle positive integers. (Aim 2) We want to test the potential effect of language use (contrasting mathematical vs. natural language and investigating whether the number word "zero" is understood) on the performance in those tasks. (Aim 3) Additionally, to put these findings into the context of current models about understanding the cardinality principle, the present work investigates whether subset-knowers and cardinality-principle-knowers (as measured with the Give-N task) have a different level of understanding of zero if understanding zero is available at such an early age. (Aim 4) Finally, we want to investigate whether the extra phase we emphasized in the data on Wellman and Miller (1986) is reliably observable, i.e., whether at some point children can compare zero appropriately, while they still think that 0 is not a number.

2 Methods

2.1 Participants

Forty 3- and 4-year-old Hungarian preschoolers participated in the study, 20 boys and 20 girls, with mean age 4 years 0 months, range between 3 years 2 months and 5 years 1 month. Two preschools were involved in the data collection, one of them in the capital and the other one in a country town (20 preschoolers from each, 8 girls in the capital and 12 girls in country town). Both preschools mostly receive children from middle-class families. None of the preschoolers received formal training about handling zero in the preschools formerly.

2.2 Tasks

The tasks covered three main areas (Table 1). Note that the present tasks mainly investigate the handling of symbolic numbers. (1) Give-N task categorized children whether they are cardinality-principle-knowers (CP-knowers) or subset-knowers. (2) The previous Give-N task was also used with “zero” and “nothing” to see whether preschoolers can apply zero in that task. Additionally, comparison, addition and subtraction tasks measured whether preschoolers could use zero in operations as efficiently as they use positive integers. The same tasks also measured the effect of various linguistic versions of those tasks. (3) Meta-knowledge about numbers was measured to investigate whether children understood that zero was a number. See the order of the tasks at the end of the Methods section. Data collection for a single child took approximately 30 minutes.

Name of the task	**Short description**
Measuring number knowledge	
Give-N (positive numbers)	Give N balls to an agent.
Operations with zero	
Give-N (nothing and zero)	Give N balls to an agent.
Comparison	Choose the larger set.
Addition	Add two values.
Subtraction	Subtract one value from another.
Meta-knowledge about zero	
Smallest number	Name the smallest number.
Is it a number?	Specify whether different things are numbers or not.

Table 1. Summary of the tasks.

Tasks related to the aims. Various tasks provided information about various combinations of the present aims. Aim 1 (is zero more difficult to handle than positive integers for preschoolers) is measured with the Operations tasks, where the relevant contrast is whether zero-related operations show worse performance than the positive number related operations. Aim 2 (the role of the linguistic form in understanding zero) is measured across all tasks: Give-N task measures the interpretation of “zero” and “nothing” labels in that task, the operations tasks measure the effect of the mathematical and natural linguistic forms, and meta knowledge investigates again the “zero” and “nothing” labels in those contexts. The relevant contrast is whether different linguistic versions induce different performance level. Aim 3 (the role of number knowledge in terms of subset-knowers and CP-knowers) is measured in both the operations and meta-knowledge tasks where the relevant contrast is the comparison of the two number knowledge groups. Finally, Aim 4 (do preschoolers may lack meta-knowledge about zero when they can handle zero in operations) is investigated in the meta-knowledge tasks contrasted with the operations tasks.

Give-N task. In the Give-N task the children could see a pile of balls, and they had to give a specific number of balls to a toy bird. The task measures (a) whether the child understands the principle of cardinality, and (b) whether the child understands what the words “nothing” and/or “zero” refer to. Note that to our knowledge understanding zero and understanding positive numbers in this task has not been measured together, and theoretically it is possible that they are independent (i.e., the knowledge about positive numbers in itself cannot predict the knowledge about zero). Also, while the performance with positive numbers has been investigated and validated in great extent (for example see the seminal works of Wynn, 1990, 1992), the performance with zero in this task has not been studied yet. Importantly, zero-knowledge is measured with the same task and with the same criteria as positive integer knowledge, thus, this method could serve as an appropriate starting point to categorize children whether they know what “zero” refers to. The task is similar to the give-a-number task described by Wynn (1990) and by Bialystok and Codd (2000), the consensually accepted tool to measure the preschoolers’ cardinality-principle and number knowledge. At the end of the trial the experimenter explicitly asked the child whether she was ready with the task. This is essential in the zero-trials when the children do not have to give any items. In the task two versions of zero were applied. In the “natural” version of zero we utilized the form that is used in everyday language: “do not give any balls to the bird.” In the “mathematical” version we used the form that reflects the mathematical viewpoint: “give zero balls to the bird.” In Hungarian, the nouns after number words always use the singular form, e.g., “give zero ball” or “give two ball”, thus, the plural or singular form of the noun could not help the children to solve the task. (See the summary of the questions in all tasks in Hungarian and their English translations in the Appendix.) The following numbers were tested in the given order: 2, “natural” 0, 5, 3, “mathematical” 0, 4. The current order of the numbers was used to avoid that children could rely on the order of the increasing number words in the task. The whole number series was repeated twice. It has been argued that children solving the tasks with numbers 4 or larger than 4 show an understanding of the cardinality principle (Condry & Spelke, 2008; Sarnecka & Carey, 2008; Wynn, 1990, 1992). A child was categorized as knowing a number if both trials of that number were solved correctly, and if known numbers were not given as response for higher unknown numbers.[2] Following these results based on the knowledge of positive numbers, children were categorized as CP-knowers if they

2Number knowledge as measured with the Give a number task can be calculated with several methods. Here, we consider two alternative evaluation methods, and report that these methods gave the same results as the method we reported in the main text. (1) In most published works a number is judged to be known if correct response rate is not lower than 66%, while we used 100% threshold value here. Because we asked all numbers twice, the correct response could be only 100%, 50% or 0%. The 100% criterion would underestimate the child’s number knowledge compared to the usual 66% criterion, while the 50% criterion would overestimate it. Because around the limit of the child’s knowledge the correct response rate changes rapidly, the 100% and the 50% percent criteria give similar results. Given our analysis and exclusions, in most of our analysis the 50% criterion categorizes only 2 children as CP-knowers who were subset-knowers with the 100% criterion. We rerun all analysis with both criteria, and they gave the very same pattern of significant and non-significant results. For the sake of simplicity, we only present the results with the 100% criterion analysis here. (2) We also used an alternative Bayesian calculation method to specify whether someone is a subset-knower or a CP-knower (Negen et al., 2011), although to our knowledge the validity of this method has not been tested so far. It gave us a categorization result which was between the results of the previous 100% and 50% methods, thus, the results of this categorization are not presented here either.

could give 4 and 5 in both trials, otherwise they were categorized as subset-knowers. Independently of the previous categorization, children were categorized as nothing-givers if they correctly did not give anything in the “natural” 0 task in both trials, and zero-givers if they did not give anything in the “mathematical” 0 tasks in both trials. (Note that because the performance of the Give-N task with positive numbers is well known in the literature, children who successfully give a specific values are termed “knowers”, such as 1-knower, subset-knower, CP-knower. However, such knowledge in the literature is not available for zero, therefore to highlight the fact that it is not clear whether children solving this task with zero really understand some key features of zero, we term those children nothing- and zero-givers, instead of nothing- and zero-knowers, i.e., we refer to the task performance instead of the supposed knowledge of the child.)

Comparison. The aim of the task was to test whether the children knew the position of zero among other numbers, therefore, whether they are able to handle zero as efficiently as positive numbers (Aim 1), and to investigate whether the form of the task (natural verbal vs. mathematical verbal vs. non-verbal) influences their performance (Aim 2). Additionally, number knowledge groups identified by the Give-N task will be compared in the comparison task (Aim 3), and comparison operation performance will be contrasted with the meta-knowledge tasks (Aim 4). In the comparison task the children either could see two sets of objects or could hear two numbers, and had to choose the larger one. To test whether the children have problems with the verbal description of the task we used both a verbal and a non-verbal object version. In the object version two sets of balls were placed on the two opposite sides of the table, and the question was “On which side can you see more?” The number of the objects in a set was not named by the experimenter. In the case of the zero value the appropriate side of the table remained empty. In the verbal task no objects were used, and the question was “Which one is more, the x or the y?” (The Hungarian translation of “which one” does not include the word “one” in the question, thus, this part of the question could not confuse the children.) In the verbal condition the number 0 could be labeled either as “zero” (mathematical version) or “nothing” (natural version). If children understood the principle of cardinality measured by the Give-N task, the 3-2, 4-1, 2-4, 3-zero, 2-nothing, 1-5, zero-4, 1-3, 2-zero, nothing-4, 2-1, zero-1 number pairs (4 pairs with “zero”, 2 pairs with “nothing”, and 6 pairs with positive values only) were used in this order. Otherwise the 3-2, 2-1, 1-2, 1-zero, 2-nothing, 1-3, zero-2, 1-2, 2-zero, nothing-1 number pairs (3 pairs with “zero”, 2 pairs with “nothing”, and 5 pairs with positive values only) were applied in this order. The use of the two series for the two groups ensures that (a) the children see tasks only with a number range corresponding to their capability, while (b) it is possible to measure a relatively wide number range in the CP-knowers. Note that the two series do not prevent us from investigating the main questions, whether zero is processed as correctly as positive numbers or whether linguistic forms have an effect on performance, because no direct comparison of the two groups in the positive number performance is required, but comparison of tasks or conditions within the groups. Because in the object condition the “zero” and the “nothing” versions mean the same stimulus (missing objects), only trials with “zero” were tested.

Addition and subtraction. Like in the case of the comparison task, for the addition and subtraction tasks, the aim was to test whether children can handle zero as efficiently as positive numbers in arithmetic operations (Aim 1), whether the form of the task (natural verbal vs. mathematical verbal vs. non-verbal) influences their performance (Aim 2), whether number knowledge groups identified by the Give-N task perform differently in the arithmetic tasks (Aim 3), and whether zero handling in arithmetic task performance can be better than the meta-knowledge tasks performance (Aim 4). In the arithmetic tasks children had to add or subtract two numbers, and tell the results. To test whether the children had problems with the verbal description of the task, the tasks could be either in verbal form or shown with objects. In the verbal form either the “natural” or the “mathematical” form of zero could be used. In the object version, while the task was also explained in verbal form (in case of zero both in the “natural” and in the “mathematical” forms), the operands of the task were shown with arrays of balls, one operand on one side and the other operand on the other side of the table. In the case of zero value, the appropriate side of the table remained empty. If the child understood the principle of cardinality measured by Give-N task, the 1+1, 3+1, 2+0, 1+2, 0+3, 2+nothing, nothing+3; 2-1, 4-2, 3-0, 3-1, 2-0, 2-2, 3-nothing, 2-nothing tasks (4 tasks with “nothing”, 4 tasks with 0, and 7 tasks with positive values only) were used in this order, otherwise the 1+1, 2+1, 2+0, 1+2, 0+1, 2+nothing, nothing+1; 2-1, 2-0, 1-0, 1-1, 2-nothing, 1-nothing tasks (4 tasks with “nothing”, 4 tasks with 0, and 5 tasks with positive values only) were applied in this order. The motivation for the use of the two series was the same as in the comparison task. The task was embedded in a small story. In the addition task the following story was used: “The bird had x balls. The dog had y balls. How many balls do they have altogether when they play together?” In the subtraction task the following story was applied: “The bird had x balls. Then, the bird gave y balls to the dog as a present. How many balls was the bird left with?” In the “natural” linguistic version we applied forms that were mostly used in everyday speech, e.g., “The dog didn’t have any balls.” (In Hungarian it is not possible to use a version that is close to the English “Give no balls” version, because in Hungarian the predicate is negated, and the results are similar to a version as “Do not give none balls to the dog.”) In the “mathematical” linguistic version we used a form that reflected the mathematical viewpoint, e.g., “The dog had zero balls.”

One might ask whether the ”natural” versions of these tasks really measure numerical abilities or whether they measure some other abilities. For example, it is possible that children use a non-numerical concept of nothing to solve the tasks instead of the numerical concept of zero. However, it is important to see that for a preschooler both the concept of nothing and the concept of zero could be appropriate to solve numerical tasks with zero. At their age preschoolers can solve only a few numerical tasks: comparison, addition, subtraction (Levine et al., 1992), and in all of those tasks both the concept of nothing and the concept of zero would give the same correct result. Therefore, a correct or erroneous numerical task cannot differentiate in a simple way whether the concept of nothing or the concept of zero was used, and consequently, the question whether any of these concepts promote different strategies is not testable with the current methods.

Smallest number. The children were asked what the smallest number was. The aim of the task was to find out whether children regard zero as the smallest number, and whether the performance on this task strongly correlates with the operation tasks (Aim 4). The task was similar to the task utilized in the study of Wellman and Miller (1986). The task was applied as the first task of the session, so as not to influence the reply with the result of some of the tasks that might teach about the number zero. (See further details in the Order of the tasks part below.) However, because several other tasks could have had an effect on this knowledge, we repeated this task at the end of the session to see whether the knowledge about zero could be changed by the knowledge the children acquire in the testing process.

Is it a number? The children had to categorize whether numbers and other things were numbers or not. The children were verbally asked "is the … a number?" The aim of this task was to study explicitly whether children regarded zero as a number, and whether the performance on this task strongly correlates with the operation tasks (Aim 4). To explore whether children understood this categorization task, additional numerical and non-numerical words were used to validate the task. The following words were used in the following order: three, two, nothing, kitten, pop (sound), zero. Similar to the Smallest number task, this task was presented at the beginning of the session, and it was also repeated at the end of the session.

Order of the tasks. Because some tasks may provide information about the meaning and use of the number zero to the children, we set the order of the tasks based on their potential to modify the replies of the children in later tasks. Therefore, the tasks about the status of the zero were the first tasks. Additionally, tasks that include verbal and object version at the same time could teach the meaning of zero, thus we placed them at the end of the session. Thus, the following order of the tasks was applied: smallest number, is it a number, Give-N task, comparison, addition and subtraction verbal version, addition and subtraction object version, smallest number repeated, is it a number repeated.

3 Results and discussion

3.1 Groups based on number knowledge

Giving positive numbers. First, with the Give-N task we specified the number knowledge level of the children (subset-knowers vs. CP-knowers). This categorization was used in the following tasks to contrast children based on their number knowledge. Twenty children understood the cardinality principle (i.e., they could solve the Give-N tasks perfectly for numbers between 2 and 5), and 20 children did not reach this phase, being subset-knowers. Two of these latter children could not even solve the give 2 balls task, and because number 1 was not measured in the task, they could be either "one"-knowers or pre-number-knowers. Because we could not specify if these two children were subset-knowers or pre-number-knowers, we excluded them from further analysis. Among the remaining 18 subset-knowers there were 7 "two"-knowers, 10 "three"-knowers and 1 "four"-knower.

Giving "nothing" and "zero". Second, independent of the previous number-knowledge categorization, we specified whether children could solve the tasks involving the natural version of zero (nothing-givers) and the mathematical version of zero (zero-givers). Practically all children (96%) understood the natural version of give zero task ("do not give any balls"), the mathematical version ("give zero balls") proved to be more difficult (45% of the children could give "zero"). None of the nothing-givers and none of the zero-givers solved the task by adding zero accidentally for unknown numbers, because zero was never given when a positive number was asked by the experimenter. The difficulty with the mathematical version of zero could be rooted either in not knowing the word “zero”, or in the unusual and unnatural form of the task (i.e., give something which is nothing). However, some occasional note by the children showed that at least some of them did not know what the word “zero” refers to. Some examples of those notes are: “What does zero mean?”, “I cannot count up to zero”, “That would be too much for me”, “Zero is hundred”.

Relation of giving positive numbers and zero. Computationally, giving positive numbers and giving “nothing” or “zero” could be independent. The next analysis investigates whether empirical data support independence. The present data (see Figure 1) show that most children could solve the natural zero task, independent of whether they are subset-knowers or CP-knowers, meaning that giving “nothing” is independent whether a child is subset-knower or CP-knower. Additionally, while giving “zero” depends on the positive number knowledge (CP-knowers are more likely to successfully give zero than subset-knowers) giving zero is not strictly connected to the positive number knowledge: not all CP_knowers can give zero and some of the subset-knowers can successfully give zero. Note that this result is not likely to be some random noise. For example, in our sample CP-knowers could give all positive numbers (i.e., between 2 and 5) twice successfully, but they were unable to be successful with “zero”. It is also noteworthy that some of the subset-knowers even understood the word “zero.” All of the subset-knowers solving the mathematical version of the give zero task were “three”-knowers. A 2 (subset-knower group vs. CP-knower group as a between-subjects factor) × 2 (“natural” vs. “mathematical” version of zero task as a within-subjects factor) ANOVA on the proportion of correct responses confirmed statistically the previously described effects: a main effect of number knowledge group, $F(1,36)=7.59$, $p=0.009$, $\eta_p^2=0.174$ (better performance in the CP-knower group), and main effect of linguistic version, $F(1,36)=46.62$, $p<0.001$, $\eta_p^2=0.564$ (better performance with the “nothing” version) were found to be significant, and a tendency in the interaction, $F(1,36)=3.66$, $p=0.064$, $\eta_p^2=0.092$ was found (CP-knowers were more successful in giving “zero” than subset-knowers).

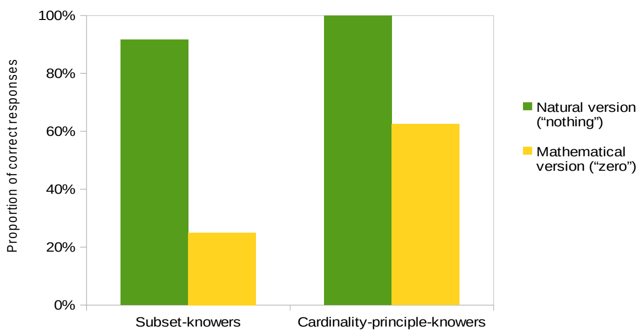

Figure 1. Proportion of correct responses for the natural and mathematical version of “give 0 objects” in subset-knowers and children understanding cardinality.

Groupings for the following analyses. Positive number knowledge (whether a child is a subset-knower or a CP-knower) will be used throughout the following tasks to investigate whether this knowledge influences understanding zero (Aim 3). Because giving “zero” is independent of the positive number knowledge, and because giving “zero” could reflect the ability to understand the label “zero”, children will be categorized also by this ability. A child was categorized as zero-giver, if the mathematical version (“zero”) of the task was solved correctly in both trials. We could not rely on the natural version (“nothing”) of the task, because this task seemed to be trivial for the preschoolers, resulting in a ceiling effect (Figure 1). These two orthogonal dimensions created four groups of children that were used in the subsequent analyses (Figure 2, this is practically the same information as in Figure 1, but shows the four groups more directly): 4 subset-knower and zero-giver (4.08 years mean age 0.26 SD), 14 subset-knower and zero-not-giver (3.8 years mean age, 0.4 SD), 12 CP-knower and zero-giver(4.33 years mean age, 0.55 SD), and 8 CP-knower and zero-not-giver (4.06 year mean age, 0.37 SD). A child is a zero-giver if she can solve correctly the mathematical versions of the give-zero tasks. Note that in the operations tasks, because of some missing data the subset-knower – zero-giver group is excluded, only 3 groups are compared. However, for the meta-knowledge tasks all the 4 groups will be analyzed. Also note that because there might be an interaction effect in positive number knowledge and giving zero, in the upcoming analyses the 4 (or 3) groups will be handled as four (or three) independent groups and not as 2×2 factors.

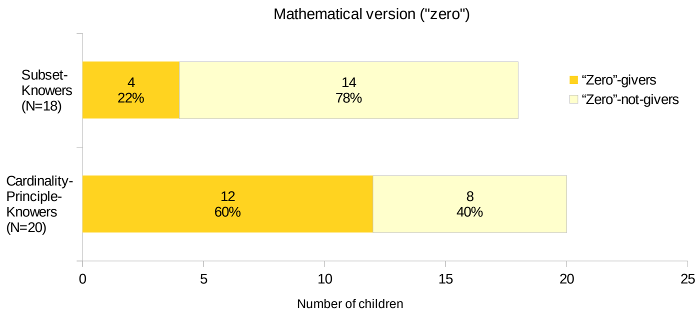

Figure 2. Number of children understanding the mathematical (“giving zero”) version of the zero task.

3.2 Operations with zero: comparison, addition and subtraction

These tasks investigated (1) whether the value of zero is more difficult to handle than handling positive integers, (2) the effect of language (a) contrasting verbal and nonverbal versions, which could reveal whether the children have either linguistic or conceptual problems with the zero value, and (b) contrasting the natural linguistic version (e.g., “add nothing to three”), and the mathematical linguistic version (e.g., “add zero to three”), (3) the differences between CP-knowers and subset-knowers.

Because of a data collection problem 2 out of 4 subset-knower and zero-giver children’s data were not available. Because the remaining data of the 2 children might be misleading due to extremely small sample size, this whole group was excluded from the analysis.

Results are presented as population estimations of the mean correct response proportions (Figure 3) and hypothesis tests of those values (Table 2). Different analyses were run for the three tasks (comparison, addition and subtraction – see rows in Figure 3 and Table 2) and for the object and verbal versions of the tasks (see columns in Figure 3 and Table 2). Within these analyses, number knowledge groups (x axes on Figure 3; subset-knower and zero-not-giver, CP-knower and zero-giver, CP-knower and zero-not-giver) and number types (different columns in Figure 3; positive values, zero and nothing, except in the comparison object version where nothing and zero distinction would not make sense) were used. Mixed ANOVAs applied number knowledge as between subject and number types as within subject factors.

Object version. First, in the object versions of the tasks, overall, handling zero was not more difficult, than handling positive values (left in Figure 3, and left in Table 2; Aim 1). (In the figures and in the appropriate ANOVAs the critical information is the main effect of number type (positive numbers vs. zero vs. nothing) reflecting an overall effect of zero, or the interaction between number type and number knowledge groups reflecting a group specific effect of zero. However, the main effect of groups is not relevant, because (a) differences between the groups only show whether some groups perform better in general, and not the relative difference between zero and positive integer related performance, and (b) the CP-knower and subset-knower groups received different tasks (see methods).) The only positive exception is that subtracting "nothing" was easier than subtracting positive numbers (see the operand type main effect in the object subtraction task), which might be reasonable, since in a relatively difficult subtraction task it is easier to do nothing than to subtract a positive value. Thus, these results show that handling zero is not harder than handling other numbers for preschoolers in the object version of the tasks. This means that preschoolers understood the value of zero conceptually. This shows that comparison, addition and subtraction operations with zero are available as soon as one understands positive integers. Note that the verbal variations of "nothing" and "zero" did not cause any significant effects, which is understandable in these tasks where the verbal form could be complementary to the object based presentation (Aim 2). Early ability to handle zero as efficiently as positive numbers in these operations is true for both CP-knowers and subset-knowers as reflected in the same pattern in both groups (Aim 3).

Verbal version. Second, in the purely verbal versions of the tasks, handling the zero value with the natural linguistic version ("nothing") was not more difficult than handling the positive values (right in Figure 3, and right in Table 2; note that the significant difference of the mathematical version ("zero") is not a relevant contrast here). Like in the case of the object version of the task, subtraction with "nothing" shows again a positive exception, as it is easier than subtraction with positive numbers. Again, these results show that understanding zero is available as soon as one understands positive integers (Aim 1). However, handling zero values with the mathematical linguistic version ("zero") revealed difficulties (Aim 2). Importantly, this difficulty was seen mainly in the zero-not-giver groups, but not in the zero-giver groups (see the interaction in the comparison ANOVA and the repeating pattern in the population estimations in all of the three tasks). This pattern means that in a trivial way, children who do not know what the word "zero" refers to cannot solve the tasks including that unknown word. This interpretation is confirmed by the fact that these tasks can be solved when the task is also supported by object demonstrations. Finally, the relative difficulty of handling zero and the linguistic form did not differ between subset-knowers and CP-knower, as reflected in the similar relative patterns across the number-knowledge groups (Aim 3).

To summarize, the verbal version of the operations tasks also confirms that preschoolers can handle zero as efficiently as positive numbers, with the trivial exception when a child does not know what the word “zero” refers to. Since the same tasks were solved efficiently in the object task, it confirms that the difficulty was linguistic in nature and not conceptual. Again, no difference between subset-knowers and CP-knowers could be found in those contrasts.

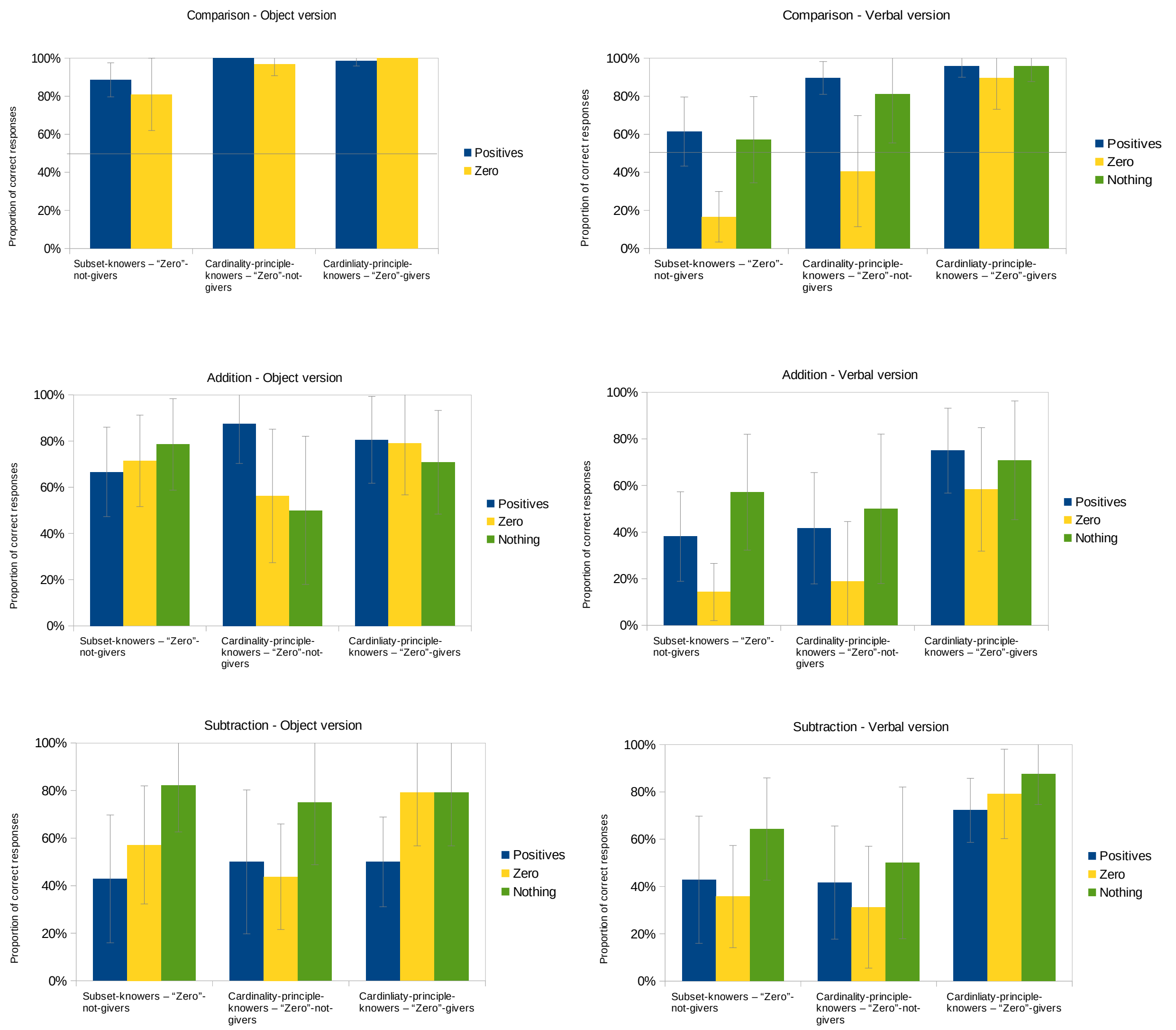

Figure 3. Proportion of correct responses in comparison (top), addition (middle) and subtraction (bottom) tasks in the non-verbal (left) and verbal (right) version with positive integers, zero and nothing (columns) within the different number knowledge groups (categories on x axes). Error bars show 95% confidence interval. Horizontal line at 50% in the comparison task shows the random choice level.

	Object version	Verbal version
Comparison	**Number type main effect**: ns **Group main effect**: $F(2,31)=4.13$, $p=0.026$, $\eta_p^2=0.211$; the subset-knowers showing worse performance than the other two groups, $ps<0.037$ **Interaction**: ns	**Number type main effect**: $F(2,62)=14.13$, $p<0.001$, $\eta_p^2=0.313$; the performance for "zero" was worse than for "nothing" or positives, both $ps<0.001$ **Group main effect**: $F(2,31)=18.95$, $p<0.001$, $\eta_p^2=0.55$; all groups differed from each other, all $ps<0.017$ **Interaction**: $F(4,62)=2.45$, $p=0.056$, $\eta_p^2=0.136$
Addition	**Operand type main effect**: ns **Group main effect**: ns **Interaction**: ns	**Operand type main effect**: $F(2,62)=5.91$, $p=0.004$, $\eta_p^2=0.016$; tasks with "zero" operands being harder to solve than the two other task types, both $ps<0.02$, LSD-test. **Group main effect**: $F(2,31)=5.14$, $p=0.012$, $\eta_p^2=0.249$; CP- and zero-giver group showing better performance than the other two groups, both $ps<0.018$. **Interaction**: ns
Subtraction	**Operand type main effect**: $F(2,62)=7.84$, $p=0.001$, $\eta_p^2=0.202$; the tasks with "nothing" being easier than the two other types of tasks, both $ps<0.003$ **Group main effect**: ns **Interaction**: ns	**Operand type main effect**: $F(2,62)=3.13$, $p=0.05$, $\eta_p^2=0.092$; the tasks with "nothing" operand being easier than the tasks with positive numbers or with zero, both $ps<0.05$ **Groups main effect**: $F(2,31)=5.92$, $p=0.007$, $\eta_p^2=0.276$; the CP- and zero-giver group showing a better performance than the other two groups, both $ps<0.007$ **Interaction**: ns

Table 2. ANOVA results of the operations tasks. All analysis were run on the correct response ratios. The following designs were applied: in object comparison version: 2 (number types: pairs with positive numbers vs. pairs with zero as a within-subjects factor) × 3 (number knowledge groups as a between-subjects factor); in verbal comparison version: 3 (number types: pairs with positive numbers vs. pairs with "zero" vs. pairs with "nothing" as a within-subjects factor) × 3 (number knowledge groups as a between-subjects factor); in addition and subtraction tasks: 3 (operand types: positive operands vs. zero vs. nothing as a within-subjects factor) × 3 (number knowledge groups as a between-subjects factor). Post-hoc tests are LSD tests.

Alternative interpretations. Some alternative interpretations of these data could be imagined. One may raise that when solving addition or subtraction tasks with zero, children cannot solve the task, but simply repeat the last operand, which sometimes could be the correct solution, e.g., in the task, 0+3, repeating the last operand is the correct result. However, our results are not in line with this possibility. First, if children did not know zero and only used this mechanical incorrect solution, they should have given incorrect responses in the comparison task, too, which was not the case. One may think that children could use the appropriate solution in comparison, and it is only addition and subtraction where they used the incorrect method. Still, this account cannot explain that if children know how addition and subtraction works (as seen in the positive operand tasks) and if they know how zero can be ordered relative to other values (as seen in the correct zero-comparison task), why they do not use these pieces of knowledge in a zero-arithmetic task. Second, in the present task, this last operand repeating strategy would give the correct result only in half of the present additions, e.g., for 0+3 the correct result can be given, while for 2+0 an incorrect result would be given, and none of the subtraction tasks can be solved, e.g., for 2-0 an incorrect result could be given (see the specific stimuli in the Tasks section). However, in the addition task the addition performance is higher than the 50% performance that could be expected by the alternative interpretation. Still, one might think that in those tasks children do not use the last operand, but they use the unknown operand. However, in other tasks (e.g., in the comparison tasks) the same children can handle zero appropriately, so it is less likely that they do not know zero. Additionally, this strategy would not solve the subtraction task at all, e.g., for the 2-0 task the response would be 0, which response pattern is not supported by the present data. As another potential incorrect heuristics is that children could compare the numbers instead of adding or subtracting them, which strategy could result in correct responses in the tasks that include zero. However, this strategy would result in incorrect responses in the tasks that include only positive numbers, which incorrect responses were not observable, thus, most preschoolers did not use this strategy either. Overall, if all tasks and results are considered at the same time, the most likely interpretation is that children handle zero on the same level as they handle positive numbers.

3.3 Meta-knowledge about zero

Smallest number. Most children thought that the smallest number was “one” (Table 3). Although some zero-givers proposed the zero as the smallest number, even most of the zero-givers thought that the smallest number was “one.” None of the children proposed a negative number as a reply, although in one of our pilot studies it happened once. A chi-square test on the proportion of responses (0; 1; other numbers; nothing; does not know) showed a significant effect, $\chi^2(4, N=38)=26.47$, $p<0.001$, reflecting high proportion of “one” responses. A chi-square test on the responses between the four number-knowledge groups revealed significant difference, $\chi^2(12, N=38)=23.5$, $p=0.024$, most probably reflecting the heterogeneous responses in the subset-knower and zero-not-giver group, and the relative uniform “one” responses in the other groups.

	Subset-knowers		Cardinality-principle-knowers	
	Zero-not-giver (N=14)	Zero-giver (N=4)	Zero-not-giver (N=8)	Zero-giver (N=12)
Zero	1 (7%)	1 (25%)		3 (25%)
One	2 (14%)	3 (75%)	6 (75%)	9 (75%)
Two	2 (14%)		1 (13%)	
Three	1 (7%)			
Five	1 (7%)			
Nothing	2 (14%)			
Does not know	5 (36%)		1 (13%)	

Table 3. Proportion of different replies to “what is the smallest number” in the four groups of number knowledge at the beginning of the session.

At the end of the session (after solving all other tasks) the smallest number task was asked again, repeating basically the very same pattern as at the beginning of the session.

When evaluating the possible dissociation of zero knowledge in the former comparison task and in the present smallest number task (Aim 4), we should consider zero-givers (because zero-not-givers typically could not name zero) and CP-knowers (because in the comparison task we had no subset-knowers within the zero-givers). The smallest number task is in a clear contradiction with the comparison task: while in the comparison task zero-giver CP-knower children knew that zero was smaller than one (83% correct response performance in the verbal version and 100% in the object version), they believed that one was the smallest number (only 25% of them believed that zero is the smallest number). This result repeats the pattern that can be seen in the data of Wellman and Miller (1986). One could argue that the children might have misunderstood the task. However, it is hard to imagine that they misunderstood the smallest number task, when CP-knower children understand conceptual properties of numbers, such as number words for large numbers with unknown position in the counting list are numbers (Lipton & Spelke, 2006), and they understand how set size change goes parallel with the counting list steps (Sarnecka & Carey, 2008). A possible resolution for this result is that preschoolers believed that zero did not belong to numbers. The next task tested more explicitly whether children regarded zero as a number.

Contrasting the subset-knowers and the CP-knowers in this task (Aim 3), on one hand, none of those groups considered zero to be the smallest number. On the other hand, subset-knowers gave more heterogeneous responses, which is in line with the fact that they had difficulties when solving the verbal comparison task (see Figure 3). Overall, while subset-knowers show a qualitatively different response in the Smallest number task compared to the CP-knowers, this difference is most probably related to their capability in comparison and not to their knowledge about zero.

Is it a number? Most groups thought that “two” and “three” were numbers, while “pop” (sound) and “kitten” were not numbers, although the subset-knower – zero-not-giver group showed an approximately random performance (Figure 4). This means that except for the latter group, preschoolers understood the task.

Critically, while the word “nothing” was evaluated as not a number, the status of the “zero” is more uncertain. This result is in line with the data that even some of the adults do not think that zero is a number (Wheeler & Feghali, 1983), and could be in line with our interpretation of the Smallest number task results above. A 4 (number knowledge groups as a between-subjects factor) × 6 (words to evaluate as a within-subjects factor) ANOVA on the proportion of correct responses (not on the proportion of the “yes” responses as displayed in Figure 4) revealed a main effect of group, $F(3,34)=3.008$, $p=0.044$, $\eta_p^2=0.21$, the subset-knower and zero-not-giver group showing poorer performance than the other three groups, all $ps<0.037$, and an interaction between the two factors, $F(15, 170)=2.12$, $p=0.011$, $\eta_p^2=0.158$. To find the source of the interaction, a similar ANOVA was run, excluding the subset-knower and zero-not-giver group. The 3 (groups) × 6 (words) ANOVA revealed only a main effect of words, $F(5,105)=3.34$, $p=0.008$, $\eta_p^2=0.137$. Post-hoc LSD-tests showed that the error rate for “zero” was higher than for “pop”, “kitten” and “nothing.” Thus, in the three groups evaluating “zero” was ambiguous, while the excluded subset-knower and zero-not-giver group showed random responses reflecting that they did not understand the task.

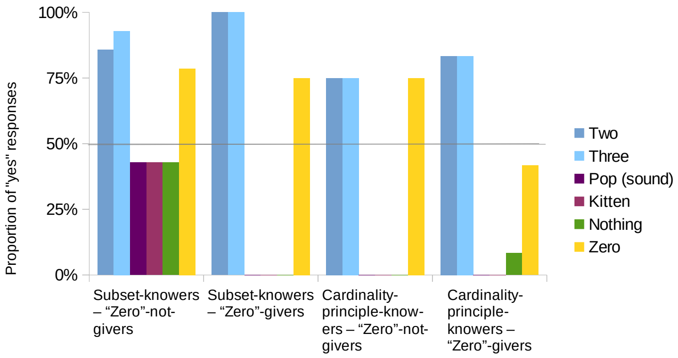

Figure 4. Proportion of “yes” responses in “is it a number” task at the beginning of the session. Horizontal line at 50% shows the random choice level.

Similar to the case of the smallest number task, at the end of the session (after solving all other tasks) “is it a number” task was asked again, and the pattern of the replies did not change.

The results here show that while the positive numbers are considered to be numbers by the preschoolers, they do not think that the word “nothing” is a number, and they are ambiguous about the numerosity of the word “zero.” (Note that here we ignore the results of the subset-knower – zero-not-giver group, because according to the data that group did not understand the task.) The ambiguity of the word “zero” is partly understandable in the zero-not-giver – CP-knower group, since they can only guess. However, the result is more informative in the zero-giver groups, since they could use the zero value correctly in numerical tasks even when the number was labeled as “zero.” How can these differences be interpreted? There could be different considerations that might influence the decision. First, all these words could be and were used by the preschoolers appropriately in numerical tasks (at least in the CP-knower and zero-giver group), thus, the difference between the number status of the words cannot be caused by the children’s numerical knowledge. Second, the word “nothing” can be used more generally in non-numerical cases, and additionally, unlike other number words, “nothing” cannot be used before nouns. These conceptual and linguistic differences might explain, why the word “nothing” is considered to be less numeric than the positive numbers and “zero.” Importantly, none of these numerical, conceptual and linguistic considerations could explain why the word “zero” is thought to be less numeric than positive numbers. Thus, we argue that the results reflect that preschoolers do not regard “zero” as a typical number. (Although the word “nothing” is also thought to be less numeric than positive numbers, one cannot tell how strong the mentioned conceptual and linguistic viewpoint influenced this decision, and consequently, how strong the real perceived number status of “nothing” influenced the decision.) This could mean that either (a) at least some of the children do not regard “zero” as a number, or (b) judging the numberness of a value on a continuous scale (e.g., prototypicality of the value as a number) “zero” is not considered as a typical number, or (c) most children are confused about the number status of zero, and they guess in this task. To summarize, this result is consistent with the idea that children do not think that “zero” is a typical number.

Regarding the connection between operational knowledge and meta-knowledge (Aim 4), the same difference can be observed here as in the Smallest number task: while these children (and especially the main interest of the zero-giver–CP-knower group) can handle zero in numerical operations, they are uncertain whether zero is a number. This finding is again in line with the extra phase we emphasized in the data on Wellman and Miller (1986): at a specific point preschoolers can handle numerical operations with zero, although they do not regard zero to be a typical number.

When contrasting the subset-knower and CP-knower groups, the only difference is that subset-knowers seem to be unsure about the meaning of the task, and seemingly subset-knower–zero-not-giver children gave responses around the 50% random level for all categories.

3.4 Reliability of the data

Forty preschoolers participated in the study, who were categorized into four groups in the analyses, which resulted in rather small groups. One might question how reliable our results can be. (1) Obviously, hypothesis tests take control of the small sample sizes. In other words, this means that some of the effects of interest are large enough to be significant even in this relatively small sample. (2) Moreover, a similar study was run before the study we reported here. The present study is the improved version of our first measurement. The main differences between our previous and the current measurements are that (a) the order of the tasks was modified, since in the present version all tasks that could potentially train the children were at the end of the session, (b) a few simple control tasks were removed, (c) there were minor improvements in the stimuli and instructions to have a better control over the stimulus properties and the verbal conditions, and (d) the children were older, instead of 3- and 4-year-old children, 4- and 5-year old children participated. Importantly, the pattern of the results is the same in the two studies: In the comparison and arithmetical tasks zero was handled at the same level as positive numbers in both the subset-knower and the CP-knower groups, and while children were unsure about the number status of zero. Two notable differences we found could be readily explained by the fact that the participants of the first measurement were older than the participant of the main measurement: the performance of the first measurement about the number status of zero was somewhat better than the performance of the sample in the present main study, and in the pilot study no zero-giver subset-knowers were found and there were only 6 subset-knower out of the 36 participants. Overall, while single measurements could be unreliable (Open Science Collaboration, 2015), and replications can be vital in obtaining reliable results, the present replication strengthens the argument that our findings are reliable. (3) Additionally, the Smallest number and the Is it a number tasks were repeated within the session, and as we have mentioned, the results showed the very same pattern, strengthening our claim that our results are reliable. (4) Also, the small size of the subset-knower, zero-giver group (4 children) cannot distort our result, because their data in the operation tasks was not used at all, and almost all of our results rely on the data of the other three groups. (5) Finally, the results cannot be seen as a set of type-I and type-II errors or random noise, because (a) similar and coherent patterns can be seen across independent tasks, (b) the results are coherent with many former reports, and (c) the results form a meaningful, coherent picture about the development of understanding zero. With such a large amount of tasks and statistical analyses it is highly improbable that small sample and related random variation could form such a coherent picture. Overall, these considerations strongly suggest that the present results are reliable and display real developmental patterns.

4 General discussion

Our results shed new light on several important aspects of the understanding of zero by preschoolers. First of all, our data demonstrate that preschoolers understand the handling of empty sets in a numerical context, and they can appropriately apply it in various numerical tasks, such as giving a set with zero items, comparison, addition and subtraction (Aim 1). Importantly, even subset-knowers can process empty sets appropriately: Their performance with empty sets was comparable with positive values if the mostly unknown word “zero” or the mathematical linguistic form was not used (part of Aim 3). This result is in contrast with the result of the Wellman and Miller (1986) study which suggested that handling zero is difficult for preschoolers, and the contradiction partly arises from the fact that in that study children potentially could not understand the word “zero” and the mathematical linguistic form (see also the discussion of Aim 2 below). Additionally, while Wellman and Miller (1986) propose that children understand zero only when they know that zero is the smallest number, we argue that handling zero in numerical tasks is a sufficient criterion, while knowing that zero is the smallest number is a knowledge that is irrelevant for evaluating preschoolers’ operational capabilities (see also the discussion of Aim 4 below).

Second, while preschoolers can handle empty sets, they have difficulties with the mathematical language (Aim 2). One component of this linguistic difficulty is the knowledge about the meaning of the word “zero.” The children who did not know the word “zero”, could not solve the tasks in which the number zero was denoted purely by this label, while they could solve the same task with different wording, suggesting that the difference is whether they know that the “zero” label refers to a concept they can use otherwise. Additionally, some children have explicitly noted, that they do not know the meaning of the word “zero.” As a second linguistic component, children had difficulty with the “mathematical” formulation of the tasks (e.g., “give zero balls to the bird”). Importantly, they could solve the task if (a) zero was denoted non-verbally, and (b) the “natural” linguistic form of the zero-related statements (e.g., “do not give any balls to the bird”) was used. This means that again, the difficulties described here are linguistic but not conceptual in nature. It is not surprising that children had problems with the mathematical formulations, since this form is mostly used for mathematical and more formal purposes, and preschoolers probably hear it infrequently. Our results also showed that these language related difficulties could be observed, independent of whether a child is subset-knower or CP-knower (part of Aim 3).

Third, children had problems with the number status of zero (Aim 4). First, they were unsure whether zero is a number. Second, they showed a contradiction: They thought that 0 is smaller than 1, but they thought that 1 was the smallest number (the same contradiction was apparent in the study of Wellman & Miller, 1986). This contradiction can be thought of as another reflection of not being sure if zero is a number: While 0 is smaller than 1 (“nothing” is less than anything else), 1 is the smallest number, because 0 is not a number. Again, this pattern was independent of whether a child is a subset-knower or a CP-knower, although subset-knowers had some difficulty even with understanding the meta-knowledge tasks.

Overall, these results identify several more or less independent components of the knowledge about zero. The first component is the operational knowledge, whether children can use zero in tasks, such as giving a set of objects, comparing values, adding and subtracting numbers. The second component is linguistic knowledge, including whether children know that the label “zero” is used to denote an empty set, and including whether they know the specific (and somewhat contradictory) mathematical formulation (such as “add zero balls to the rabbit”). The third component is meta knowledge: whether children know that zero is a number. The present results demonstrated that these three components do not necessarily appear at the same time. Also, our data demonstrate that these pieces of knowledge do not depend strictly on whether a child is a subset-knower or CP-knower.

4.1 Theoretical account of the development of zero

What representations or accounts can explain this more detailed picture of zero development? First of all, the linguistic effect is not surprising in a sense that linguistic knowledge about zero has an effect on how zero is handled in verbal tasks. What is more interesting from theoretical viewpoint is the different development of operational and meta-knowledge about zero. Why do children not think that zero is a typical number, while they can solve numerical tasks with that value correctly? Several explanations could offer a solution for this problem.

A first group of explanations might suggest that linguistic or cultural factors might cause the difference between zero and positive integers in some tasks and relatedly they might explain why operational and meta-knowledge about zero may dissociate. We argue that these explanations cannot account for the present results. (1) The special status of zero might be explained by its rare use in everyday language (Dehaene & Mehler, 1992). However, after understanding the cardinality principle children understand some common properties of numbers, even if that number is beyond the limit of their counting list (Lipton & Spelke, 2006). Critically, the frequencies of those numbers

are comparable to the frequency of zero. Consequently, the relatively low frequency of the word “zero” cannot explain why children do not regard zero as a number, because while a rare number (zero) is not regarded as number, other rare numbers (e.g., 20 or 40) are regarded as numbers.
(2) Another possible example of a linguistic explanation could suggest that zero is categorized incorrectly, because empty sets are handled linguistically in a different way than positive numbers: mostly the word “nothing” is used instead of “zero”, and “natural” sentences are used instead of the “mathematical” versions. However, zero-givers who are familiar with these linguistic forms are as unsure about the number status of zero as zero-not-givers, therefore, even if these rare linguistic forms are known, zero still can be thought as a non-number. Additionally, in a similar case of the number one, although children learn the distinction between one and many (i.e., other positive numbers) grammatically which helps them to learn the meaning of “one” (Carey, 2009), they still think that one is a number. Thus, in the case of this linguistic difference, the relevant value (one) is not categorized differently than larger natural numbers. (3) A final possible linguistic-cultural explanation could propose that a counting list usually starts with one, which could exclude zero from the set of numbers (Merritt & Brannon, 2013). However, at least in some cases this decision is not based on the counting list. For example, some adults also think that zero is not a number, and their justifications show that they consider a different information in this categorization: they say that zero is not a number, because “a number is the abstract value of the quantity of a set” or “it has no value” (Wheeler & Feghali, 1983). Additionally, it is unclear why this linguistic phenomenon would have an effect on categorization if the previous potentially stronger linguistic effects (first and second point of this paragraph) do not play a role in the categorization of zero. Importantly, none of these linguistic-cultural explanations can account for the fact that zero is handled differently only in some tasks, while it is handled similarly to the positive integers in some other tasks. To conclude, although we cannot exclude entirely some linguistic or cultural influences, it seems more likely that the cause of the dual nature of zero should be found somewhere else.

A second group of accounts supposes that there are representational causes why zero is not thought to be a typical number. (1) One can imagine that the representation that supports positive values cannot store the value of zero, hence, zero should be stored in a different system. The most frequently referred model suggests that semantic numerical processing is driven by the Approximate Number System (ANS) that represents numbers in an imprecise way (Feigenson et al., 2004; Merritt & Brannon, 2013; Moyer & Landauer, 1967). This system might include zero, although this ability can be debated (Dakin et al., 2011; Dehaene et al., 1993; Merritt & Brannon, 2013; Ramirez-Cardenas et al., 2016). (See more details about this system in the Supplementary material.) Some part of the results could be readily explained by this model: in the operation tasks, even if the stimuli were presented symbolically, the imprecise representation may handle the relatively small values used in the present experiment. However, it is not trivial how the model could account for the dual nature of zero. In the ANS framework, one might suppose that while the positive numbers and in some cases the zero value could be handled by this system (as in the comparison task), in some other cases the zero value should be handled by another system (as in the number status of zero task). However, it is not clear what this alternative system could be, and why the ANS sometimes could not handle zero. In other words, these suppositions are arbitrary, the additional details are created only to account for these new results, and they are not supported by any other phenomena. Overall, it is hard to give a coherent explanation how the ANS could account for the special status of zero. (2) Another possible explanation suggests that understanding numbers relies on the conceptual understanding of the items (e.g., objects) in a set. For example, Carey (2004, 2009) proposes that children induce the conceptual understanding of how counting can specify the size of a set relying on the set-templates in long term memory which templates are based on the activation of visual indexes. (It is important to note that it is not simply the visual index or Object Tracking System that supports the cardinality principle, because in itself the visual index is only a set of spatial indexes, which computationally is not capable to support conceptual understanding. Instead, some additional mechanisms, such as the set-templates in the long term memory, and other unspecified mechanisms are necessary to support the conceptual understanding of the cardinality principle.) Another model explaining why sign-value notation numbers (e.g., Roman notation) are easier to understand than place-value numbers (e.g, Indo-Arabic notation) proposes that sign-value notations are more similar to a supposed item-based number representation in which the powers (e.g., ones, tens, hundreds, etc. in a ten-based system) are denoted by objects or groups (Krajcsi & Szabó, 2012). In both of these latter models, numbers could be considered as the properties of items or objects in a set, and since in an empty set there are no items or objects, it would not make sense to talk about the property of the non-existing items, and accordingly, zero could not be a number. The case of a property for a missing item is similar to the paradox introduced by Lewis Carroll: In his story a smiling cat disappears at some point, while the smile of the cat remains visible. In a similar way, the property (here, the numerosity) of the non-existent items is not meaningful, and the lack of items cannot be described similar to a set of items. This model is also in line with the justifications of some adults about why they do not regard zero as a number, when they mostly state that numbers describe the items in a set (Wheeler & Feghali, 1983). Importantly, this view can explain why zero has a dual nature: while empty sets can be handled in the numerical tasks manipulating the objects or items, the status of zero is special, since it cannot represent the property of the items that are not present, but it represents the lack of those items. Therefore, we conclude that conceptual understanding of numbers as items or objects can explain why zero is handled in a special way.

4.2 Implications for practice

The results and the interpretation we presented here have some educational consequences. Teaching and understanding the properties of zero is difficult (Frobisher, 1999; Lichtenberg, 1972). Understanding some simple properties, such as the parity of zero or the number status of zero, could be problematic even for mathematics teachers (Hill et al., 2008; Wheeler & Feghali, 1983).

Based on our results it is possible to offer some educational recommendations for schools or even for preschools. At the same time, the efficiency of these recommendations might also validate and test our description of zero development. Our recommendations are based on the fact that several independent components of the knowledge about zero could be identified (operational knowledge, linguistic knowledge and meta knowledge) and those components develop in different pace. (1) Based on the present results about operational knowledge, children could handle empty sets as soon as they can handle positive numbers in basic numerical operations, thus, no further effort is needed to introduce this concept to them. (2) However, they are not familiar with the mathematical linguistic formulations of the problem, thus, one might teach these linguistic forms. (a) One linguistic component to teach is that the word “zero” means “nothing” or “doing nothing” in mathematical problem solving. (b) Another linguistic component to teach is the mathematical form of sentences, thus, we can recite both the “natural” and the “mathematical” version of the task, stressing that the two versions refer to the same thing: e.g., “do not give any balls to Suzy Sheep, or you can say, give zero balls to Suzy Sheep.” (3) Additionally, children do not know about the number status of zero, they are not sure that zero is a number. According to one possibility, one can explain that zero is also a number, because we can use it in counting and other mathematical tasks. However, this solution raises a problem. If our explanation for the present results is correct, then children are not sure that zero is a number, because they consider that number is a property of items, and zero describes the lack of items, a quantitatively different state. This “no items” state can handle empty sets in numerical tasks correctly, although it is not considered to be a number. Importantly, in the preschooler years (and in most educational systems in the first school years) there is no experience that could demonstrate that the “no items” state is actually a number. For example, most adults know that zero is a number, because it is only a step on the number line among the positive and negative integers, or because in written multi-digit calculations zero is handled in a similar way as the other digits. Still, preschoolers do not have any experience that could reveal the numerical nature of the “no items” state, because for example, they do not know negative integers, or they cannot make multi-digit calculations. Thus, although one can explain, that zero is a number, because we can use it in numerical tasks, still, this information cannot be justified in their experiences. Consequently, it seems more reasonable that it is unnecessary to give any education on this issue before the school years, and it could be better to teach about the number status of zero only when this information can be justified and which can be built up in a reasonable way. To sum up, (1) preschoolers already know how to handle empty sets in basic numerical operations, although (2) mathematical language of referring to empty sets can be taught, while (3) teaching the number status of zero is probably unnecessary at that age.

To summarize, preschoolers can handle zero on the same level as they handle positive integers. Although the linguistic form can cause difficulties for them, this is independent of their conceptual understanding. However, preschoolers are not sure whether zero is a number. This could be caused by the set-based representation of numbers: Numbers could be the properties of items in a set, while an empty set does not include any items, consequently, zero cannot be a number in this view.

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7 Appendix

All questions of the tasks in Hungarian and their English translation.

Hungarian version	**English translation**	**English translation reflecting the Hungarian structure more strictly**
	Give-N task	
Non-zero version: Adj a madárnak … golyót.	Give … balls to the bird.	Give to the bird … ball (*singular*).
Natural zero version: A madárnak semennyi golyót se adj.	Do not give any balls to the bird.	To the bird none ball (*singular*) do not give.
Mathematical zero version: Adj a madárnak nulla golyót.	Give zero balls to the bird.	Give to the bird zero ball (*singular*).
	Comparison	
Object version: Melyik oldalon van több?	On which side is there more?	On which side is there more?
Verbal version: Melyik a több, a … vagy a …?	Which one is more, the … or the …?	Which is the more, the … or the …?
	Addition and subtraction	
Addition: Mennyi golyójuk van összesen, ha együtt játszanak?	How many balls do they have altogether when they play together?	How many ball (*singular*) they have altogether, if together they play?
Subtraction: Mennyi golyó marad a madárnak?	How many balls was the bird left with?	How many ball (*singular*) has left for the bird?
	Smallest number	
Melyik a legkisebb szám, mi a legkevesebb?	What is the smallest number, what is the fewest one?	Which is the smallest number, what is the fewest?
	Is it a number?	
A … az szám?	Is the … a number?	The …, is it a number?

8 Supplementary material – How can infants handle zero?

In the infant literature there is an agreement that nonsymbolic numerical information (such as arrays of dots, series of sounds or events, etc.) is probably processed by two representations.[3] (1) The Approximate Number System (ANS) can represent numbers in an imprecise way. This number-specific representation stores the values noisily, and the larger the value is the noisier the representation is. This simple representation implements Weber's law, i.e., in its general form the performance of a number comparison task depends on the ratio of the two values: The larger the ratios of the values are, the better the performance is. In line with this Weber's law, it was found that infants are able to discriminate two values if the difference is larger than a specific ratio (Feigenson et al., 2004), which ratio (the Weber-fraction) improves with age (Piazza, 2010), therefore, ANS could be a representation that supports number-processing performance in infants. Note that in the recent years, there is also an increasing body of evidence questioning the supposed role of ANS in number processing, e.g., whether in nonsymbolic tasks, such as a dot comparison task, it is really the numerical property that influences the performance or alternatively correlating visual properties have a strong effect on the performance (Gebuis & Reynvoet, 2012), or in an alternative model symbolic number processing effects can be explained by a simple mechanism other than the ANS (Krajcsi et al., 2016). (2) As another system to account for infants' numerical performance, there is an Object Tracking System (OTS) or visual indexes that can represent objects up to a limit of 3 or 4 items, which system can implicitly store the number of those objects, and it is argued that some of the numerical performance of infants can be explained with this representation (Feigenson et al., 2004). While the largest part of the infant literature shows a consensus that the ANS and the visual index systems might be responsible for the numerical performance observed in infants, it is not clear whether these systems can represent zero.

Regarding the ANS, Merritt and Brannon (2013) investigated whether four-year-old preschoolers can handle zero in a nonsymbolic comparison task. (Note that although that work studied preschoolers, not infants, it directly investigates the role of the ANS in nonsymbolic number processing, similar to the number processing observed in infants.) They were looking for the distance effect (increasing error rate with smaller distance), which effect is considered to be the sign of the ratio-based ANS processing (Moyer & Landauer, 1967). While in the whole sample the distance effect was observed in the comparison task, suggesting the role of the ANS in handling zero, other aspects of the results suggest that it was not the ANS that processed zero. First, many children could not solve the task: Children, who performed relatively poorly in the positive number comparison task, performed randomly and did not show any distance effect with number pairs including zero. However, poor performance for positive numbers is surprising, because even much younger infants are sensitive to numerical information (Feigenson et al., 2004). Second, the performance with pairs including zero was worse than with pairs including one, however, the ANS would predict a better performance with the zero-pairs. The authors hypothesize that preschoolers

3Although the present work does not investigate the numerical abilities of infants, the preschooler numerical processing models rely on these infant models, and they are relevant in the interpretation of the current results.

initially might avoid zero, because for example, zero is not included in the usual counting sequence (Merritt & Brannon, 2013). Because it is supposed that the same ANS works in non-humans and humans across various ages with basically the same properties (Piazza, 2010), other studies might help us to specify whether ANS could store zero. Unfortunately, it is not easy to tell whether ANS can represent zero, even when investigated in adults or non-humans (Dehaene et al., 1993; Fischer & Rottmann, 2005; Merritt et al., 2009; Varma & Schwartz, 2011). One problem is that presenting a set with zero involves methodological issues, e.g., nothing is shown, with radically different perceptual properties compared to non-zero stimuli. A related problem is that presenting nonsymbolic empty sets opens up different interpretational possibilities about what information could be used to differentiate empty set from non-empty sets. Overall, the results about whether ANS may represent zero are inconclusive. While some authors argue that ANS can store zero (Merritt & Brannon, 2013), these results can be debated, and according to some models it would be impossible to store zero by the ANS (Wynn & Chiang, 1998), because for example, the ANS might rely on perceptual properties, and missing stimuli (i.e., a nonsymbolic empty set) cannot activate the appropriate features in the system (Dakin et al., 2011). It is also possible that even if zero is stored by the ANS, it requires additional supporting processes. For example, it was found that monkeys could solve number matching task with empty set, and cells tuned to the empty sets were found both in the ventral intraparietal area and in the prefrontal cortex, as it has been observed for positive numbers (Ramirez-Cardenas et al., 2016). However, only the cells in the prefrontal cortex showed the distance effect for zero, a sign of ANS processing, and not the intraparietal cells, although the ANS is supposed to be localized in the intraparietal area (Dehaene et al., 2003). Thus, it is not clear whether ANS could handle zero either in general, or specifically in infants or in preschoolers.

Some studies investigated more directly whether infants can represent the lack of objects. Six- and 8-month-old infants seemingly cannot maintain the representation of a lack of an object measured with violation-of-expectation paradigms: While they see the unexpected or “magical” disappearance of an object as an unexpected event, they do not see the “magical” appearance of an object as an unexpected event (Kaufman et al., 2003; Wynn & Chiang, 1998). On one hand, we can suppose that representing the lack of object is handled by the representation handling numerical information. In this case, because the numerical handling of small (i.e., less than 4 or 5 items) sets of objects is attributed to the visual indexes or to the OTS by default, and the ANS handles these small sets only in exceptional cases (Feigenson et al., 2004), the failure to detect a magical appearance supports the idea that it is the visual index or the OTS that cannot represent the lack of objects. On the other hand, it is also possible that representing lack of objects is handled by some other systems, representing other aspects of objects or locations, and these results cannot specify whether the ANS or the visual index can handle empty sets. To summarize, there is no consensus whether either the ANS or the visual index can represent zero, and there are hardly any data describing whether these or other systems can handle zero in infants.