The granularity of gravity

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Summary
It is shown that Verlinde's hypothetical concepts of entropic gravity can be successfully applied to the author's previous work on the modification of the Newtonian gravity by Einstein's Cosmological Constant. It allows a theoretical justification for Milgrom's empirical acceleration constant, thereby revealing some fundamental differences with Verlinde's conclusions.

Keywords: entropic gravity; Cosmological Constant; quantum gravity; dark matter

Introduction

This article is meant as a supplement to a previous study [1]. In particular as an extension of its discussion paragraph. It is my aim to give some view on the granularity of the gravitational space, in which I will borrow some concepts from Verlinde's "entropic gravity" [2,3,4,5]. I have decided to separate the issue from my previous article, because of the need to accept the somewhat hypothetical, if not speculative, nature of these concepts. That concepts are needed is beyond doubt, because otherwise there are no means to give an explanation for the negative pressure executed by the spatial fluid that must be present in vacuum for explaining a positive value of the Cosmological Constant in Einstein's Field Equation. Such a value is required to remove the anomaly of particular cosmological phenomena, like the stellar rotation curves in galaxies and the accelerated expansion of the universe. In my previous article, it has straightforwardly been derived that in a gravitational system with a central mass M in vacuum, the Cosmological Constant, while independent of space-time coordinates, amounts to $\Lambda = \lambda^2/2 = a_0/5MG$, where a_0 ($\approx 10^{-10}$ m/s^2) is Milgrom's acceleration constant [6,7,8] and G the gravitational constant.

Satisfying Einstein's Field equation in vacuum under absence of a massive source under the condition of a positive Cosmological Constant Λ requires the presence of a background energy [9]. Under inclusion of the massive source, the background energy will of course still be there and seems to show up as polarized dipoles [1], which would explain the $1/M$ dependency of the Cosmological Constant. It is my aim trying to harmonize the dipole modeling of the spatial fluid with Verlinde's entropic gravity concept. The first step is pointing out that this concept can be interpreted as the modeling of the spatial fluid by an elastic glassy medium and, secondly, that elasticity can be modeled in terms of dipoles.

Entropic gravity

Let me start by giving a synopsis of Verlinde's theory. This might be useful, because Verlinde's articles are written in the formalism of the string theory and are therefore hardly accessible for non-experts (like me). Nevertheless, it will appear to be possible to extract the fundamentals and to reformulate these in a less abstract terminology. The gravitational model that I wish to describe applies to a pointlike massive source in vacuum (I prefer to use the word vacuum over empty space, because of the obvious reason that we cannot escape from the spatial fluid appearance in Einstein's Field Equation with Cosmological Constant). The massive source is an abstraction of some (baryonic) matter with a volume mass density $\rho_B(r)$ present within some sphere with radius R, such that $\rho_B(r) = 0$ for $r \geq R$. This baryonic mass executes a gravitational force F_N on test particles with mass m beyond the sphere, which in the weak field limit of Einstein's equation coincides with the Newtonian law that at the higher abstraction level is derived from Poisson's equation,

$$\frac{1}{r^2}\frac{d}{dr}(r^2\frac{d\Phi}{dr}) = -4\pi G\rho_B(r) \rightarrow r^2\frac{d\Phi}{dr} = -G\int_0^R \rho_B(r)4\pi r^2 dr, \text{ such that} \quad (1)$$

$$F_N = -m\frac{d\Phi}{dr} = m\frac{G}{r^2}\int_0^R \rho_B(r)4\pi r^2 dr\,. \quad (2)$$

Beyond the radius of the sphere, the mass distribution within the sphere is irrelevant. This implies that the value of the integral can simply be indicated as the encapsulated baryonic mass M . Even so, if desired, one might replace the volume mass distribution by an equivalent surface mass distribution $\sigma_D(r)$ on the shell of the sphere. The ultimate abstraction is, to model the encapsulated baryonic mass as a pointlike massive source that can be modelled as a Schwarzschild black hole with mass M and a radius $R_S = 2MG/c^2$, where c is the speed of light in vacuum.

Doing so, the properties of the black hole will appear on the shell of the sphere that encapsulates the baryonic mass. This view is known as "the holographic principle". One of these properties is the entropy of the black hole (hence the entropy of the encapsulated mass). The origin for assigning this property is the question of how many molecules are contained within the radius of a black hole. The answer is: as many as degrees of freedom for the energy of the molecules are available. The (dimensionless) informatics entropy S is related with the logarithm of this number. Bekenstein [10] theorized that

$$S = \frac{Ac^3}{4G\hbar}, \text{ where } A = 4\pi R_S^2, \quad (3)$$

and Hawking [11] subsequently argued that entropy should go hand in hand with temperature. He found,

$$T_H = \frac{\hbar c^3}{8\pi GMk_B}\,. \quad (4)$$

In fact, he derived this result by a similar approach as Unruh [12] derived for the temperature experienced by a detector at constant acceleration $\mathbf{a}$ in vacuum as

$$k_B T = \frac{1}{2\pi}\frac{\hbar|\mathbf{a}|}{c}\,. \quad (5)$$

Unruh's result coincides with Hawking's result if $\mathbf{a}$ is taken as the Newtonian acceleration at the black hole's radius R_S , i.e., if

$$|\mathbf{a}| = \frac{MG}{R_S^2}\,. \quad (6)$$

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In these expressions $\hbar$ and k_B, respectively are Planck's (reduced) constant and Boltzmann's constant.

In his entropic gravity concept, Verlinde heuristically associates these properties with the Newtonian gravity. He does so by hypothesizing an amount of $N = 4S$ virtual molecules on the holographic screen and assigning an energy $k_B T/2$ to each of these. To me, it makes more sense to adopt $N = S$ in conjunction with $2kT$ instead. Initially, it will give the same result. Later in this article it will be shown why this second option, apart from its rationale, is the better one. In both cases, it enables identifying the energy equivalent M' (= Mc^2) of the encapsulated mass as,

$$M' = 2Nk_B T = 4\frac{Ac^3}{G\hbar}k_B T . \tag{7}$$

Subsequently eliminating N from (3) and (7), gives,

$$T = \frac{MG\hbar}{2\pi k_B c}\frac{c^4}{4M^2G^2} = \frac{\hbar c^3}{8\pi GMk_B} . \tag{8}$$

Curiously, this just gives the Hawking temperature! To associate these concepts with gravity, Verlinde states that, as soon as a massive test particle approaches the holographic screen at a distance equal to its (reduced) Compton wave length, it will be unified with the virtual molecules on the screen. This unification takes place under influence of a force derived from the second law of thermodynamics, which subsequently is identified as the gravitational force. Quantitatively, this process is analyzed as follows. From thermodynamics we have the increase of (non-dimensionless) physical entropy by an amount ΔS under supplying an energy amount ΔE, such that

$$\Delta E = T\Delta S . \tag{9}$$

The unification of a test particle wit a (reduced) Compton wave length $\Delta x = \hbar / mc$, is equivalent with a supplied energy $\Delta E = F\Delta x$, where F is the force that displaces the test particle over a distance Δx. Verlinde attributes the origin of the unifying force from the temperature of the holographic screen by reversing Unruh's argumentation, by stating that the force is the result of the screen's temperature, rather than the opposite. Hence,

$$\Delta E = F\frac{\hbar}{mc} , \tag{10}$$

where from (5),

$$F = 2\pi k_B T\frac{mc}{\hbar} . \tag{11}$$

From (9) , (10) and (11) we get,

$$\Delta E = 2\pi k_B T = T\Delta S , \tag{12}$$

such that the change in entropy as a consequence of the unification appears being,

$$\Delta S = 2\pi k_B \ . \tag{13}$$

From (11) and (6), and considering that Poisson's law allows a transport of the mass properties of the black hole (with radius R_S) to the screen that encapsulates the massive sources (with radius r), it follows readily ,

$$F = \frac{mMG}{r^2}, \tag{14}$$

in which we recognize Newton's law.

This is an interesting result, because it gives an entropic view on gravity. However, it is not emergent in the sense that it is derived from "first principles". The reason is the presence of a loop hole in the derivation. Basically, the derivation relies upon Poisson's equation, which says that the mass distribution of encapsulated baryonic mass is irrelevant for the force that is experienced for a test particle beyond the encapsulation. Moreover, Hawking's temperature expression (4) relies on the application of the Newtonian law, which can be seen from replacing the generic acceleration $\mathbf{a}$ in Unruh's expression by the Newtonian g at the radius $R_S = 2MG/c^2$. It is therefore not surprising that Newton's law pops up, because it was already there. Obviously, Verlinde's derivation contains some heuristic assumptions and numerical fixings, such as a logarithmic measure $N = 4S$ virtual particles in conjunction with an energy $kT/2$ (or, equivalently, $N = S$ in conjunction with $2kT$), and the reduced Compton wave length for the unification. Nevertheless, this entropic view is a challenging picture of gravity, which might give a clue for solving outstanding problems.

The successful generalization of the entropy of a black hole to the entropy of encapsulated mass gives a lead to the idea for conceiving mass as a manifestation of entropy. One might even play, like Verlinde does, with the idea that entropy is the cradle of matter and that conversion from entropy into matter would be possible. Inspired by this, Verlinde proposes a modification of the Newtonian gravity, which could possibly give an explanation for the unsolved gravitational problems. In this theory, the holographic screen (short for the screen that encapsulates baryonic matter), does not longer solely possess the entropy properties of a black hole. Instead, the entropy is built up by the entropy of a black hole (with mass equal to the mass of the encapsulated matter) and an additional entropy subtracted from the vacuum between the black hole and the cosmological horizon. This implies that Verlinde hypothesizes that, in spite of the absence of matter, entropy can be assigned to the vacuum. By generalizing Bekenstein's entropy of a (massive) black hole S as given by (3), Verlinde assigns an entropy $S_D(r)$ to a (mass less) sphere in vacuum with radius r, to the amount of

$$S_D(r) = \frac{r}{L}\frac{c^3 A(r)}{4G\hbar}, \tag{15}$$

where L is the cosmological horizon. This assignment is based upon a threefold hypothesis. Verlinde supposes (a) that the entropy of the vacuum is bound by the cosmological horizon L, (b) that this en tropy has a uniform volume distribution and (c) that the entropy at the cosmological horizon is given by the same formula as Bekenstein's expression for the Schwarzschild black hole. Subsequently, Verlinde hypothesizes that baryonic matter put into the vacuum subtracts entropy from the vacuum. The subtracted entropy has an energy equivalent (and therefore a mass equivalent). The subtracted energy is beneficial to the baryonic matter, which, as a consequence, is virtually increased. Because change of entropy implies a change of volume, the vacuum volume shrinks. Because such a shrink is not without a resistance, an amount of elastic energy is involved. Therefore, it must be possible to calculate the virtual increase of baryonic mass from the elastic energy from the volume shrink caused by putting matter into the vacuum. From this point we could try to follow Verlinde's analysis. It is my aim in this article to compare and to discuss the entropic view with the dipole view as developed in previous work [13,1]. This needs a re-interpretation of the vacuum's entropy as well as a physical interpretation of the gravitational dipole.

Entropy of the vacuum

For a better understanding of the relationship between the vacuum's entropy and the entropy of a black hole, it might be instructive to visualize the entropy of a black hole in a way as suggested by Susskind [14]. He proposed to consider the black hole's mass M as a sum of N elementary amounts ΔM , brought in or radiated off, by bosons. To this end, the (reduced) Compton wavelength of ΔM must equate the periphery $2\pi R_S$ of the black hole, such that

$$2\pi R_S = \frac{\hbar}{c\Delta M} \rightarrow \Delta M = \frac{\hbar}{2c\pi R_S} \rightarrow$$

$$S = N = \frac{M}{\Delta M} = \frac{2c\pi R_S}{\hbar} M = \frac{2c\pi R_S}{\hbar} (\frac{2MG}{c^2}) \frac{c^2}{2G} = \frac{\pi c^3}{\hbar G} R_S^2 = \frac{c^3}{4\hbar G} A. \tag{16}$$

The result of this simplistic view nicely corresponds with the results of the rigid analysis as originally performed by Hawking [11] and repeated by others [15]. These analyses give the correct answer to the question how to handle the Compton wavelength (reduced or non reduced) and on the correct value of the numerical proportionality factor related with the area A (which was unknown in Bekenstein's conceptual set up). Where the entropy expression for the black hole is an *area law*, Verlinde's entropy expression (15) for the vacuum basically is a *volume law*. Rather than counting the number of elementary massive amounts ΔM that are actual present, it counts the number of elementary massive amounts ΔM that can be maximally comprised by the vacuum. This maximum is bound by the "emerging" area of the cosmological horizon. Entropy is a measure of information, not more, not less. Information presupposes a carrier, being signals or circuits, like in Shannon's case, or massive energies, like in Bekenstein's case. Verlinde's entropy assignment to the vacuum, implicitly presupposes a carrier as well. To be meaningful, the entropic virtual masses must represent some mass less physical energetic entities. Let us follow Susskind's model and identify elementary carriers of the entropy. According to Verlinde's hypothesis, the amount N of these carriers on the holographic screen is given by,

$$N = S = \frac{r}{L}\frac{c^3 A(r)}{4G\hbar} . \tag{17}$$

The difference ΔN between the amount of virtual masses on the holographic screen and the screen shifted by an amount Δr amounts to

$$\Delta N = \frac{r}{L}\{\frac{c^3 A(r+\Delta r)}{4G\hbar} - \frac{c^3 A(r)}{4G\hbar}\} = \frac{r}{L}\frac{c^3}{4G\hbar}\{4\pi(r+\Delta r)^2 - 4\pi r^2\} = 2\pi \frac{r^2}{L}\frac{c^3}{G\hbar}\Delta r . \tag{18}$$

The volume difference ΔV between the two screens is,

$$\Delta V = \frac{4}{3}\pi(r+\Delta r)^3 - \frac{4}{3}\pi r^3 \approx 4\pi r^2 \Delta r . \tag{19}$$

The volume density of the entropic carriers ρ_e therefore amounts to

$$\rho_e = \frac{\Delta N}{\Delta V} = \frac{c^3}{G\hbar}\frac{2\pi r^2}{4\pi r^2 L} = \frac{c^3}{2G\hbar L}\,. \tag{20}$$

Dipole interpretation

So far, the entropic carrier is an abstract concept without a physical interpretation. Let us proceed by trying to relate this entropic carrier with the gravitational dipole like proposed in the author's study on the impact of the Cosmological Constant on the Newtonian gravity [1]. It may seem that a gravitational dipole concept, in the sense of a bond between a positive mass and a negative mass, violates physics, because a negative mass is not a viable concept. It showed up, however, in a model that explains the weak limit solution of Einstein's equation with a positive Cosmological Constant. As is well known, such a solution forces viewing the vacuum as a fluidal space with a negative pressure (corresponding with a positive background mass density). A close inspection will reveal (see appendix in [1]) that this solution is nothing else but the result of a modulation of the background energy density, due to a disturbance caused by the insertion of a central mass M . This allows to conceive the disturbance as gravitational dipoles on the pedestal of the background energy density. What may seem as a negative mass in the gravitational dipole is a dip in the background energy. This makes the gravitational dipole a valid concept. This gravitational dipole modelling associated with the Cosmological Constant has resulted into an expression for the dipole moment density P_{g0} as [1],

$$P_{g0} = \frac{a_0}{20\pi G} \tag{21}$$

where a_0 is Milgrom's acceleration constant.

Let us hypothesize a relationship between the gravitational dipole with its dipole moment density (21) and the entropic carrier with its volume density (20). This requires the assignment of some physical meaning to the entropic carrier. We may call it matter, without further précising. We may say now, that in entropic gravity, a cosmological object (be it a galaxy or the cosmos itself) is considered as a sphere that contains some matter with a uniformly distributed volume density. This sphere expands elastically such that the volume density remains uniformly distributed, while decreasing linearly with the radius of the expanding horizon. As a result, the matter content within the sphere increases quadratic while expanding. In Einsteinean gravity (with Cosmological Constant), the cosmological object is considered as a sphere with an infinite radius that contains some fluid with a uniformly distributed energetic volume density. In this Einsteinean view, the numerical value of the background energy contained in the fluid is irrelevant and can be discarded, because gravity shows up as a disturbance in the energetic distribution. The gravitational dipoles show up as grains in the thermodynamic equilibrium state of this fluid. Although these grains are ultimately (in the stationary state of the solution of the gravitational wave equation) unidirectional polarized, effectively they are only unidirectional polarized within the cosmological horizon and randomly polarized beyond (because, as long as there is a cosmological horizon, there is no stationary state for the solution of the gravitational wave equation).

This observation allows to conceive the elastic expansion of the entropic fluid as the polarization wave in the Einsteinean fluidal space.

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The size of the grains

The volume size V_g of the entropic matter grains follows readily from (19) and (20) as

$$V_g = \frac{\Delta V}{\Delta N} = 2\frac{G\hbar L}{c^3}. \qquad (22)$$

Assuming that the vacuum shows a packing density κ of spherical grains with radius R_g, we get

$$R_g = (\kappa \frac{3G\hbar L}{2\pi c^3})^{1/3}. \qquad (23)$$

Equating κ with the optimum spherical packing density (Kepler's conjecture)

$$\kappa = \pi/(3\sqrt{2}) \qquad (24)$$

and considering that $L = ct_L$ ($t_L = 13.5$ Gyear), R_g is calculated as,

$$R_g = 2.27 \text{ x}10^{-15} \text{ m}. \qquad (25)$$

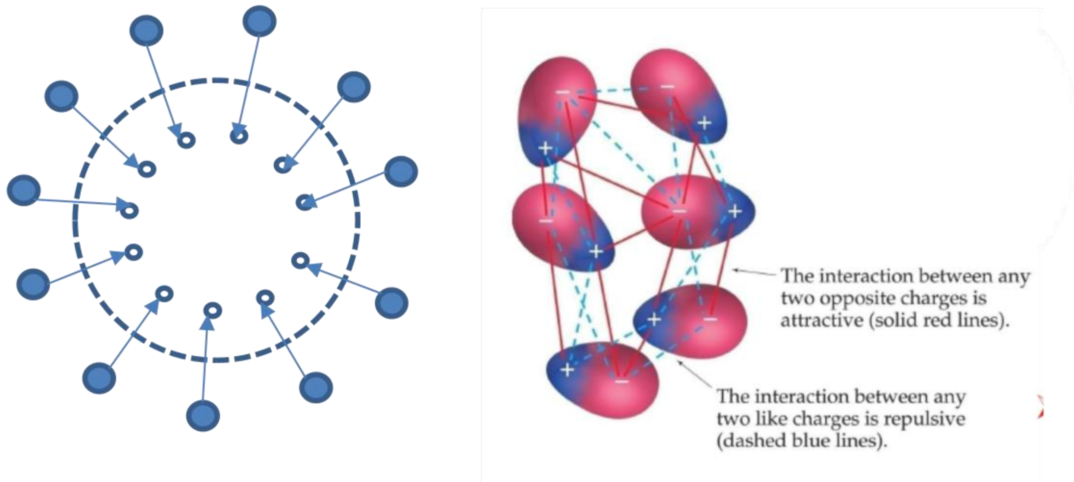

Fig.1. Left: Modelling the entropy of the vacuum as dipoles across the holographic screens up to the cosmological screen. Right: *dipole-dipole interaction in an elastic medium* (from:T.L. Brown, H.E. LeMay, B.E. Bursten, Chemistry, The Central Science, 10th. Ed, Chapter 10).

To justify our hypothesis, these entropic matter grains are the carriers of the gravitational dipoles. The entropic matter grains have a certain size (23,25) and the gravitational dipoles have a certain strength. That the size evolves over time, because of its dependency on the cosmological horizon L, is not necessarily a show stopper for maintaining an invariant value for dipole moment density. This is motivated by the following consideration. Let us assign a dipole strength p_d to the gravitational dipole. Hence, N grains make a volume V with a dipole moment Np_d. The dipole moment density is,

$$\frac{Np_d}{V} = \frac{Np_d}{NV_g} = P_{g0} \to p_d = P_{g0}V_g, \tag{26}$$

where P_{g0} is given by (21). The structure of the dipole is unknown and perhaps irrelevant. In its most simple representation the dipole is a structure with two elementary kernels of energy, with mass equivalents m_d and some spacing d in between. The two kernels are bound together as a result of an equilibrium of forces between the repelling force from the two kernels and the confinement force from the surrounding grains, such as illustrated in figure 1. The dipole strength of the entropic grains follows from (26), (24) and (21) as

$$p_d = \frac{a_0}{20\pi G}\frac{2G\hbar L}{c^3} = \frac{a_0\hbar L}{10\pi c^3}. \tag{27}$$

Because there is no reason why the dipoles with strength (27) in the entropic grains with size (22) would be different for different galaxies, it is fair to conclude that that Milgrom's acceleration constant a_0 is the same for all galaxies. This is an important theoretical result.

Comparison

It has been shown so far that it is possible to harmonize the gravitational dipole view on gravity with Verlinde's entropy assignment to the vacuum. This has been possible by conceiving the elastic motion of Verlinde's entropic matter as the gravitational wave that polarizes the grains in the thermodynamic equilibrium state of spatial energetic fluid that must exist to allow a positive Cosmological Constant in Einstein's Field Equation. As already shown in previous work [1], the Cosmological Constant does not seem being a true constant of nature. Instead, Milgrom's acceleration constant a_0, related with the Cosmological Constant via physical variables, appears being the invariant one and the same for all galaxies. The theory as developed so far in this article, does not allow a derivation of its numerical value. It just shows up as a second gravitational constant next to G. This is the consequence from adopting Einstein's Field Equation as an axiom. As shown so far in this article, this is not in conflict with the entropic principles as brought forward by Verlinde.

Verlinde, though, gives a different energetic matter interpretation to the entropy without taking the Cosmological Constant into account. Doing so, he established a numerical value for a_0 by theory, namely,

$$a_0 = c^2/6L, \tag{28}$$

which is close to Milgrom's empirical value. Unlike the Einsteinean result, the value decreases under expansion of the cosmological horizon. Moreover, Verlinde modifies the gravitational acceleration constant to

$$g(r) = g_N(r) + \sqrt{a_0 g_N(r)}, \tag{29}$$

where $g_N(r)$ is the Newtonian one.

Verlinde claims a match of this result with Milgrom's hypothesis, which is confirmed by a wealth of cosmological observations on galaxies [16]. To my opinion Verlinde's conclusion here is incorrect. Milgrom's equation is different, nl.,

$$g = \frac{g_N}{\mu(x)} \text{, with } x = g / a_0 \text{ where } \mu(x) = \frac{x}{\sqrt{1+x^2}} . \tag{30}$$

This implies that, at small r, Milgrom's gravitational acceleration coincides with the Newtonian one. This is not true for (29). Probably, the difference is sufficiently significant for being checked by observations. The author's result, on the other hand, coincides for small r with the Newtonian one, but is different from Milgrom's one at extreme cosmological distance. These two differences (the discrepancy with the Newtonian gravity at small r and the prognosis that at extreme cosmological distance the gravitational force is subject to a spatial periodicity [1]) are the result of a different interpretation of the entropic matter in Verlinde's theory and in the one brought forward in this article.

As a final remark I would like to note that, in spite of the fact that the concept of temperature has been leading to the entropic vision, in retrospect it does not play an essential role. In a way, this is satisfying, because it enables to maintain temperature as a macrostate of molecules and gravity as a force with an origin at a baryonic level below the molecular one.

Conclusion

Interpreting Verlinde's vacuum entropy as quantum mechanical bosons allows conceiving the vacuum as a fluid of gravitational dipoles subject to polarization under influence of a baryonic source. Consequently, the vacuum behaves similarly as the gravitational equivalent of a dielectrum showing the equivalent of a displacement charge, thereby effectively enhancing the strength of the baryonic source.

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