doi:10.20944/preprints201708.0041.v2

The Granularity of Gravity

Engel Roza

Stripperwei 1, 5551 ST Valkenswaard, The Netherlands
Email: engel.roza@onsbrabantnet.nl

Abstract: It is shown that Verlinde's hypothetical concepts of entropic gravity can be successfully applied to the author's previous work on the modification of the Newtonian gravity by Einstein's Cosmological Constant. It allows an assessment by theory for Milgrom's empirical acceleration constant, thereby revealing some fundamental differences with Verlinde's conclusions.

Keywords: entropic gravity; Cosmological Constant; quantum gravity; dark matter

Introduction

This article is meant as a supplement to a previous study [1]. In particular as an extension of its discussion paragraph. It is my aim to give some view on the granularity of the gravitational space, in which I will borrow some concepts from Verlinde's "entropic gravity" [2,3,4,5]. I have decided to separate the issue from my previous article, because of the need to accept the somewhat hypothetical, if not speculative, nature of these concepts. That concepts are needed is beyond doubt, because otherwise there are no means to give an explanation for the negative pressure executed by the spatial fluid that must be present in vacuum for explaining a positive value of the Cosmological Constant in Einstein's Field Equation. Such a value is required to remove the anomaly of particular cosmological phenomena, like the stellar rotation curves in galaxies and the accelerated expansion of the universe. In my previous article, it has straightforwardly been derived that in a gravitational system with a central mass M in vacuum, the Cosmological Constant, while independent of space-time coordinates, amounts to $\Lambda = \lambda^2 / 2 = a_0 / 5MG$, where a_0 ($\approx 10^{-10}$ m/s^2) is Milgrom's acceleration constant [6,7,8] and G the gravitational constant.

Satisfying Einstein's Field equation in vacuum under absence of a massive source under the condition of a positive Cosmological Constant Λ requires the presence of a background energy [9]. Under inclusion of the massive source, the background energy will of course still be there and seems to show up as polarized dipoles [1], which would explain the $1/M$ dependency of the Cosmological Constant. It is my aim trying to harmonize the dipole modeling of the spatial fluid with Verlinde's entropic gravity concept. The first step is pointing out that this concept can be interpreted as the modeling of the spatial fluid by an elastic glassy medium and, secondly, that elasticity can be modeled in terms of dipoles.

Entropic gravity

Let me start by giving an synopsis of Verlinde's theory. This might be useful, because Verlinde's articles are written in the formalism of the string theory and are therefore hardly accessible for non-experts (like me). Nevertheless, it will appear to be possible to extract the fundamentals and to reformulate these in a less abstract terminology. The gravitational model that I wish to describe applies to a pointlike massive source in vacuum (I prefer to use the word vacuum over empty space, because of the obvious reason that we cannot escape from the spatial fluid appearance in Einstein's Field Equation with Cosmological Constant). The massive source is an abstraction of some (baryonic) matter with a volume mass density $\rho_B(r)$ present within some sphere with radius R, such that $\rho_B(r) = 0$ for $r \geq R$. This baryonic mass executes a gravitational force F_N on test particles with mass m beyond the sphere, which in the weak field limit of Einstein's equation coincides with the Newtonian law that at the higher abstraction level is derived from Poisson's equation,

$$\frac{1}{r^2}\frac{d}{dr}(r^2\frac{d\Phi}{dr}) = -4\pi G\rho_B(r) \rightarrow r^2\frac{d\Phi}{dr} = -G\int_0^R \rho_B(r)4\pi r^2 dr \text{, such that} \quad (1)$$

$$F_N = -m\frac{d\Phi}{dr} = m\frac{G}{r^2}\int_0^R \rho_B(r)4\pi r^2 dr\,. \quad (2)$$

Beyond the radius of the sphere, the mass distribution within the sphere is irrelevant. This implies that the value of the integral can simply be indicated as the encapsulated baryonic mass M. Even so, if desired, one might replace the volume mass distribution by an equivalent surface mass distribution $\sigma_D(r)$ on the shell of the sphere. The ultimate abstraction is, to model the encapsulated baryonic mass as a pointlike massive source that can be modelled as a Schwarzschild black hole with mass M and a radius $R_S = 2MG/c^2$, where c is the speed of light in vacuum.

Doing so, the properties of the black hole will appear on the shell of the sphere that encapsulates the baryonic mass. This view is known as "the holographic principle". One of these properties is the entropy of the black hole (hence the entropy of the encapsulated mass). The origin for assigning this property is the question of how many molecules are contained within the radius of a black hole. The answer is: as many as degrees of freedom for the energy of the molecules are available. The (dimensionless) informatics entropy S is related with the logarithm of this number. Bekenstein [10] theorized that

$$S = \frac{Ac^3}{4G\hbar}\text{, where } A = 4\pi R_S^2, \quad (3)$$

and Hawking [11] subsequently argued that entropy should go hand in hand with temperature. He found,

$$T_H = \frac{\hbar c^3}{8\pi GMk_B}\,. \quad (4)$$

In fact, he derived this result by a similar approach as Unruh [12] derived for the temperature experienced by a detector at constant acceleration $\mathbf{a}$ in vacuum as

$$k_B T = \frac{1}{2\pi}\frac{\hbar|\mathbf{a}|}{c}\,. \quad (5)$$

Unruh's result coincides with Hawking's result if $\mathbf{a}$ is taken as the Newtonian acceleration at the black hole's radius R_S, i.e., if

$$|\mathbf{a}| = \frac{MG}{R_S^2}\,. \quad (6)$$

In these expressions $\hbar$ and k_B, respectively are Planck's (reduced) constant and Boltzmann's constant.

In his entropic gravity concept, Verlinde heuristically associates these properties with the Newtonian gravity. He does so by hypothesizing an amount of $N = 4S$ virtual molecules on the holographic screen and assigning an energy $k_B T/2$ to each of these. To me, it makes more sense to adopt $N = S$ in conjunction with $2kT$ instead. Initially, it will give the same result. Later in this article it will be shown why this second option, apart from its rationale, is the better one. In both cases, it enables identifying the energy equivalent M' ($= Mc^2$) of the encapsulated mass as,

$$M' = 2Nk_B T = 4\frac{Ac^3}{G\hbar}k_B T. \tag{7}$$

Subsequently eliminating N from (3) and (7), gives,

$$T = \frac{MG\hbar}{2\pi k_B c}\frac{c^4}{4M^2G^2} = \frac{\hbar c^3}{8\pi GMk_B}. \tag{8}$$

Curiously, this just gives the Hawking temperature! To associate these concepts with gravity, Verlinde states that, as soon as a massive test particle approaches the holographic screen at a distance equal to its (reduced) Compton wave length, it will be unified with the virtual molecules on the screen. This unification takes place under influence of a force derived from the second law of thermodynamics, which subsequently is identified as the gravitational force. Quantitatively, this process is analyzed as follows. From thermodynamics we have the increase of (non-dimensionless) physical entropy by an amount ΔS under supplying an energy amount ΔE, such that

$$\Delta E = T\Delta S. \tag{9}$$

The unification of a test particle wit a (reduced) Compton wave length $\Delta x = \hbar / mc$, is equivalent with a supplied energy $\Delta E = F\Delta x$, where F is the force that displaces the test particle over a distance Δx. Verlinde attributes the origin of the unifying force from the temperature of the holographic screen by reversing Unruh's argumentation, by stating that the force is the result of the screen's temperature, rather than the opposite. Hence,

$$\Delta E = F\frac{\hbar}{mc}, \tag{10}$$

where from (5),

$$F = 2\pi k_B T\frac{mc}{\hbar}. \tag{11}$$

From (9) , (10) and (11) we get,

$$\Delta E = 2\pi k_B T = T\Delta S, \tag{12}$$

such that the change in entropy as a consequence of the unification appears being,

$$\Delta S = 2\pi k_B . \tag{13}$$

From (11) and (6), and considering that Poisson's law allows a transport of the mass properties of the black hole (with radius R_S) to the screen that encapsulates the massive sources (with radius r), it follows readily ,

$$F = \frac{mMG}{r^2}, \tag{14}$$

in which we recognize Newton's law.

This is an interesting result, because it gives an entropic view on gravity. However, it is not emergent in the sense that it is derived from "first principles". The reason is the presence of a loop hole in the derivation. Basically, the derivation relies upon Poisson's equation, which says that the mass distribution of encapsulated baryonic mass is irrelevant for the force that is experienced for a test particle beyond the encapsulation. Moreover, Hawking's temperature expression (4) relies on the application of the Newtonian law, which can be seen from replacing the generic acceleration $\mathbf{a}$ in Unruh's expression by the Newtonian g at the radius $R_S = 2MG/c^2$. It is therefore not surprising that Newton's law pops up, because it was already there. Obviously, Verlinde's derivation contains some heuristic assumptions and numerical fixings, such as a logarithmic measure $N = 4S$ virtual particles in conjunction with an energy $kT/2$ (or, equivalently, $N = S$ in conjunction with $2kT$), and the reduced Compton wave length for the unification. Nevertheless, this entropic view is a challenging picture of gravity, which might give a clue for solving outstanding problems.

The successful generalization of the entropy of a black hole to the entropy of encapsulated mass gives a lead to the idea for conceiving mass as a manifestation of entropy. One might even play, like Verlinde does, with the idea that entropy is the cradle of matter and that conversion from entropy into matter would be possible. Inspired by this, Verlinde proposes a modification of the Newtonian gravity, which could possibly give an explanation for the unsolved gravitational problems. In this theory, the holographic screen (short for the screen that encapsulates baryonic matter), does not longer solely possess the entropy properties of a black hole. Instead, the entropy is built up by the entropy of a black hole (with mass equal to the mass of the encapsulated matter) and an additional entropy subtracted from the vacuum between the black hole and the cosmological horizon. This implies that Verlinde hypothesizes that, in spite of the absence of matter, entropy can be assigned to the vacuum. By generalizing Bekenstein's entropy of a (massive) black hole S as given by (3), Verlinde assigns an entropy $S_D(r)$ to a (mass less) sphere in vacuum with radius r, to the amount of

$$S_D(r) = \frac{r}{L}\frac{c^3 A(r)}{4G\hbar}, \tag{15}$$

where L is the cosmological horizon. Subsequently, Verlinde hypothesizes that baryonic matter put into the vacuum subtracts entropy from the vacuum. The subtracted entropy has an energy equivalent (and therefore a mass equivalent). The subtracted energy is beneficial to the baryonic matter, which, as a consequence, is virtually increased. Because change of entropy implies a change of volume, the vacuum volume shrinks. Because such a shrink is not without a resistance, an amount of elastic energy is involved. Therefore, it must be possible to calculate the virtual increase of baryonic mass from the elastic energy from the volume shrink caused by putting matter into the vacuum.

Note that conceiving the vacuum as an elastic medium is not much different from conceiving the vacuum as a spatial fluid that allows the positive Cosmological Constant in Einstein's Equation. The energy equivalent of Verlinde's vacuum entropy is not much different from the vacuum energy contained in the spatial fluid. We may go a step further by noting that subtracting energy from one side of the holographic screen (the vacuum) and adding it to the other side (the matter) can be modelled as the creation of dipoles. This is illustrated in figure 1.

It will be clear that assigning entropy to vacuum is quite a step. To distinguish the entropy of the vacuum from the entropy of the black hole, one might probably say that, where the black hole's entropy can be considered as a countable ensemble of fermionic (baryonic) energy levels, the vacuum's entropy can be considered as a countable ensemble of bosonic energy levels.

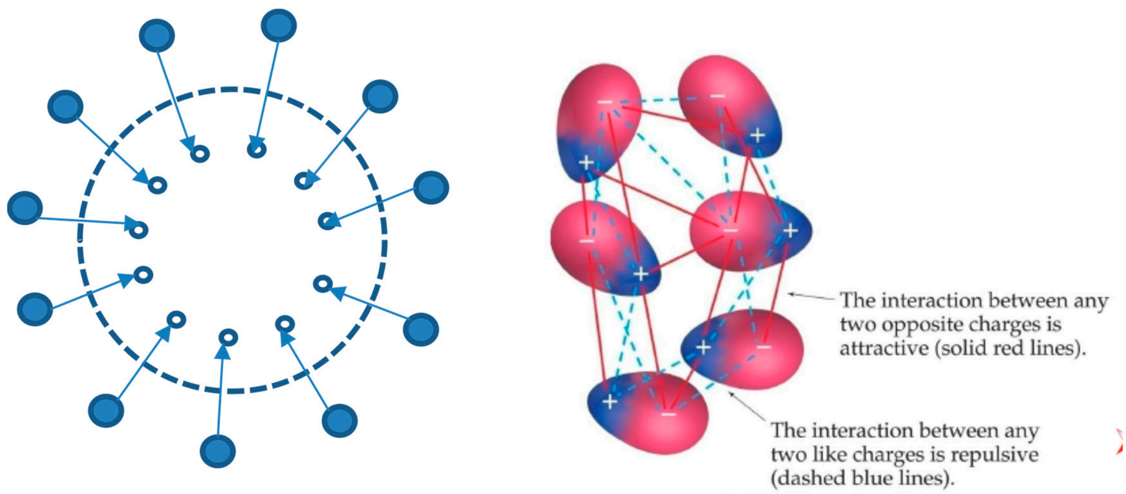

Fig.1. Left: Dipole view of the conversion of entropy outside the cosmological screen into matter within the screen. Right: *dipole-dipole interaction in an elastic medium* (from:T.L. Brown, H.E. LeMay, B.E. Bursten, Chemistry, The Central Science, 10th. Ed, Chapter 10).

Dipole interpretation

From this point we could try to follow Verlinde's analysis. Let me admit that I am unable to do so. Let me therefore try to harmonize the dipole view [13,1] straight from this point. If there is a reason why a particle beyond the holographic screen unifies with the screen under influence of a entropic force triggered by the screen's temperature, there will be no reason why a particle within the screen should not do the same, thereby leaving a "hole" in the encapsulated mass. The combination of the two acts as a dipole with a positive energy kernel and a negative energy kernel. According to the theory as described before, the spacing will be two (reduced) Compton lengths. We proceed now as follows. Eqs. (15) and (3) allow to establish the number of dipoles N that cross the cosmological screen, because

$$N = S = \frac{c^3 A(L)}{4G\hbar}. \tag{16}$$

The strength p_d of a dipole, consisting of elementary kernels of energy with mass equivalent m_d and spaced by two reduced Compton lengths $d = 2\hbar / m_d c$, amounts to

$$p_d = m_d d = 2\frac{\hbar}{c}. \tag{17}$$

The difference ΔN between the amount of dipoles on the cosmological screen and the screen shifted by an amount ΔL amounts to

$$\Delta N = \frac{c^3 A(L+\Delta L)}{4G\hbar} - \frac{c^3 A(L)}{4G\hbar} = \frac{c^3}{4G\hbar}\{4\pi(L+\Delta L)^2 - 4\pi L^2\} = 2\pi L\frac{c^3}{G\hbar}\Delta L. \tag{18}$$

The volume difference ΔV between the two screens is,

$$\Delta V = \frac{4}{3}\pi(L+\Delta L)^3 - \frac{4}{3}\pi L^3 \approx 4\pi L^2 \Delta L. \tag{19}$$

The dipole moment density P_{g0} therefore amounts to

$$P_g = \frac{p_d \Delta N}{\Delta V} = 2\frac{\hbar}{c}\frac{c^3}{G\hbar}\frac{2\pi L}{4\pi L^2} = \frac{c^2}{GL}. \tag{20}$$

Replacing the cosmological scale L (Hubble length) by the acceleration scale $a_L = c^2 / L$, we have

$$P_g = \frac{a_L}{G}. \tag{21}$$

This result allows a comparison with the dipole modelling of the Cosmological Constant, which resulted into [1],

$$P_{g0} = \frac{a_0}{20\pi G} \tag{22}$$

where a_0 is Milgrom's acceleration constant. Equating (21) allows calculating this constant from theory as,

$$a_0 = 20\pi a_L. \tag{23}$$

Equating the Hubble length as $L = ct_L$ and $t_L \approx 13.5$ Gyear, we get from (23),

$$a_0 = 1.2 \times 10^{-10} \text{ m/s}^2. \tag{24}$$

This is the very same value as Milgrom's empirical one [13].

Comparison

Verlinde's conclusions are somewhat different. Instead of (23), Verlinde concludes,

$$a_0 = a_L / 6. \tag{25}$$

This is substantially different and less close to Milgrom's value. Moreover, Verlinde modifies the gravitational acceleration constant to

$$g(r) = g_N(r) + \sqrt{a_0 g_N(r)}, \tag{26}$$

where $g_N(r)$ is the Newtonian one.

Verlinde claims a match of this result with Milgrom's hypothesis, which is confirmed by a wealth of cosmological observations on galaxies. To my opinion Verlinde's conclusion here is incorrect. Milgrom's equation is different, nl.,

$$g = \frac{g_N}{\mu(x)}, \text{ with } x = g/a_0 \text{ where } \mu(x) = \frac{x}{\sqrt{1+x^2}}. \tag{27}$$

This implies that at small r Milgrom's gravitational acceleration coincides with the Newtonian one. This is not true for (26). Probably, the difference is sufficiently significant for being checked by observations. The author's result, on the other hand, coincides for small r with the Newtonian one, but is different from Milgrom's one at extreme cosmological distance.

In view of this, it seems to be fair giving support to Hajdukovic's view [14] that the vacuum is filled by a fluid consisting of grains that are structured as confined dipoles with a matter kernel and an antimatter kernel, spaced by twice their Compton wave lengths.

The size of the grains

The volume size V_g of the dipole grains follows readily from (18) and (19) as

$$V_g = \frac{\Delta V}{\Delta N} = 2\frac{G\hbar L}{c^3}. \tag{28}$$

Assuming that the vacuum shows a packing density κ of spherical grains with radius R_g, we get

$$R_g = (\kappa \frac{3G\hbar L}{2\pi c^3})^{1/3}. \tag{29}$$

Equating κ with the optimum spherical packing density (Kepler's conjecture)

$$\kappa = \pi/(3\sqrt{2}) \quad (30)$$

and considering that $L = ct_L\,(t_L = 13.5\,\text{Gyear})$, R_g is calculated as,

$$R_g = 1.6 \text{ x}10^{-14} \text{ m.} \quad (31)$$

Each of these grains contains a gravitational dipole with strength $2\hbar/c$. It is tempting to relate the grain size and the dipole strength with mass values attributed to the two poles. At this stage, I prefer not doing so. It has to do with the suggestive relationship between the gravitational dipole and the nuclear mesons. Both type of constructs are configurations of a kernel with positive energy and a kernel of negative energy in an equilibrium of a repelling force (in particle physics due to Yukawa's $\exp(-\lambda r)/\lambda r$ potential) and an attracting confinement force (in particle physics the gluing colour force). The assignment of mass values is troubled by the problem to distinguish binding energy and vibration energy from bare massive energy. More research is needed to put these correspondences in perspective.

Conclusion

Interpreting Verlinde's vacuum entropy as quantum mechanical bosons allows conceiving the vacuum as a fluid of gravitational dipoles subject to polarization under influence of a baryonic source. Consequently, the vacuum behaves similarly as the gravitational equivalent of a dielectrum showing the equivalent of a displacement charge, thereby effectively enhancing the strength of the baryonic source.

References

[1] E. Roza, doi:10.20944/preprints201705.0164.v3 (2017)
[2] E. Verlinde, JHEP 1104, 029 ; arXiv: 1001.0785v1 [hep-th] (2011)
[3] E. Verlinde, arXiv:1611.02269v2 [hep-th] (2016)
[4] A. McCoss, Journ. of Quantum Inf. Sc, 7, 67 (2017)
[5] M.M. Brouwer *et al*., Monthly Notices Royal Astronomical Soc., Vol. 466, Issue 3, 2547 (2017)
[6] M. Milgrom, The Astrophysical Journal, 270, 365 (1983)
[7] M. Milgrom, Can. J. Phys. ,93, 126 (2015)
[8] S. McGaugh, Can. J. Phys., 93, 250 (2015)
[9] A. Einstein, Preuss. Akad. Wiss, Berlin (Math. Phys.), 142 (1917)
[10] J.D. Bekenstein, Phys. Rev. D7, 2333 (1973)
[11] S.W. Hawking, Comm. Math. Phys. 43, 199 (1975)
[12] W.G. Unruh, Phys. Rev. D 14, 870 (1976)
[13] http://www.scholarpedia.org/article/The_MOND_paradigm_of_modified_dynamics
[14] D. Hajdukovic, Astrophysics and Space Science, 334, vol.2, 215 (2011)