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The Granularity of Gravity

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Summary
It is shown that Verlinde's hypothetical concepts of entropic gravity can be successfully applied to the author's previous work on the modification of the Newtonian gravity by Einstein's Cosmological Constant. It allows an assessment by theory for Milgrom's empirical acceleration constant, thereby revealing some fundamental differences with Verlinde's conclusions.

Keywords: entropic gravity; Cosmological Constant; quantum gravity; dark matter

Introduction

This article is meant as a supplement to a previous study [1]. In particular as an extension of its discussion paragraph. It is my aim to give some view on the granularity of the gravitational space, in which I will borrow some concepts from Verlinde's "entropic gravity" [2,3,4,5]. I have decided to separate the issue from my previous article, because of the need to accept the somewhat hypothetical, if not speculative, nature of these concepts. That concepts are needed is beyond doubt, because otherwise there are no means to give an explanation for the negative pressure executed by the spatial fluid that must be present in vacuum for explaining a positive value of the Cosmological Constant in Einstein's Field Equation. Such a value is required to remove the anomaly of particular cosmological phenomena, like the solar rotation curves in galaxies and the accelerated expansion of the universe. In my previous article, it has straightforwardly been derived that in a gravitational system with a central mass M in vacuum, the Cosmological Constant, while independent of space-time coordinates, amounts to $\Lambda = \lambda^2/2 = a_0/5MG$, where a_0 ($\approx 10^{-10}$ m/s^2) is Milgrom's acceleration constant [6,7,8] and G the gravitational constant.

Satisfying Einstein's Field equation in vacuum under absence of a massive source under the condition of a positive Cosmological Constant Λ requires the presence of a background energy [9]. Under inclusion of the massive source, the background energy will of course still be there and seems to show up as polarized dipoles [1], which would explain the $1/M$ dependency of the Cosmological Constant. It is my aim trying to harmonize the dipole modeling of the spatial fluid with Verlinde's entropic gravity concept. The first step is pointing out that this concept can be interpreted as the modeling of the spatial fluid by an elastic glassy medium and, secondly, that elasticity can be modeled in terms of dipoles.

Entropic gravity

Let me start by giving an synopsis of Verlinde's theory. This might be useful, because Verlinde's articles are written in the formalism of the string theory and are therefore hardly accessible for non-experts (like me). Nevertheless, it will appear to be possible to extract the fundamentals and to reformulate these in a less abstract terminology. The gravitational model that I wish to describe applies to a pointlike massive source in vacuum (I prefer to use the word vacuum over empty space, because of the obvious reason that we cannot escape from the spatial fluid appearance in Einstein's Field Equation with Cosmological Constant). The massive source is an abstraction of some (baryonic) matter with a volume mass density $\rho_B(r)$ present within some sphere with radius R, such that $\rho_B(r) = 0$ for $r \geq R$. This baryonic mass executes a gravitational force F_N on test particles with

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mass m beyond the sphere, which in the weak field limit of Einstein's equation coincides with the Newtonian law that at the higher abstraction level is derived from Poisson's equation,

$$\frac{1}{r^2}\frac{d}{dr}(r^2\frac{d\Phi}{dr}) = -4\pi G\rho_B(r) \rightarrow r^2\frac{d\Phi}{dr} = -G\int_0^R \rho_B(r)4\pi r^2 dr \text{, where} \tag{1}$$

$$F_N = -m\frac{d\Phi}{dr} = m\frac{G}{r^2}\int_0^R \rho_B(r)4\pi r^2 dr\,. \tag{2}$$

Beyond the radius of the sphere, the mass distribution within the sphere is irrelevant. This implies that the value of the integral can simply be indicated as the encapsulated baryonic mass M. Even so, if desired, one might replace the volume mass distribution by an equivalent surface mass distribution $\sigma_D(r)$ on the shell of the sphere. The ultimate abstraction is, to model the encapsulated baryonic mass as a pointlike massive source that can be modelled as a Schwarzschild black hole with mass M and a radius $R_S = 2MG/c^2$, where c is the speed of light in vacuum.

Doing so, the properties of the black hole will appear on the shell of the sphere that encapsulates the baryonic mass. This view is known as "the holographic principle". One of these properties is the entropy of the black hole (hence the entropy of the encapsulated mass). The origin for assigning this property is the question of how many molecules are contained within the radius of a black hole. The answer is: as many as degrees of freedom for the energy of the molecules are available. The (dimensionless) informatics entropy S is related with the logarithm of this number. Bekenstein [10] theorized that

$$S = \frac{Ac^3}{4G\hbar}\text{, where } A = 4\pi R_S^2, \tag{3}$$

and Hawking [11] subsequently argued that entropy should go hand in hand with temperature. In a similar approach as Unruh [12] derived for the temperature experienced by a detector at constant acceleration in vacuum, he found,

$$T_H = \frac{\hbar c^3}{8\pi GMk_B}\,. \tag{4}$$

In his entropic gravity concept, Verlinde heuristically associates these properties with the Newtonian gravity. He does so by hypothesizing an amount of $N = 4S$ virtual molecules on the holographic screen and assigning an energy $k_BT/2$ to each of these (k_B is Boltzmann's constant). This enables to identify the energy equivalent M' of the encapsulated mass as,

$$M' = \frac{1}{2}Nk_BT = 2\frac{Ac^3}{G\hbar}k_BT\,. \tag{5}$$

Subsequently eliminating N from (3) and (5), gives,

$$T = \frac{MG\hbar}{2\pi k_B c}\frac{c^4}{4M^2G^2} = \frac{\hbar c^3}{8\pi GMk_B}\,. \tag{6}$$

Curiously, this just gives the Hawking temperature! To associate these concepts with gravity, Verlinde states that, as soon as a massive test particle approaches the holographic screen at a distance equal to its (reduced) Compton wave length, it will be unified with the virtual molecules on the screen. This unification takes place under influence of a force derived from the second law of thermodynamics, which subsequently is identified as the gravitational force. Quantitatively, this process is analyzed as follows. From thermodynamics we have the increase of (non-dimensionless) physical entropy by an amount ΔS under supplying an energy amount ΔE, such that

$$\Delta E = T\Delta S\,. \tag{7}$$

The unification of a test particle with a (reduced) Compton wave length $\Delta x = \hbar / mc$, is equivalent with a supplied energy $\Delta E = F\Delta x$, where F is the force that displaces the test particle over a distance Δx. Therefore,

$$\Delta E = F\frac{\hbar}{mc}. \tag{8}$$

From (5) and (8) we get,

$$\Delta E = \frac{1}{2}k_B T(\Delta N) = T\Delta S\,, \tag{9}$$

such that the elementary change in entropy seems to be given by

$$\Delta S = k_B / 2\ . \tag{10}$$

Verlinde modifies this heuristically into

$$\Delta S = 2\pi k_B\ . \tag{11}$$

Hence, from (8) –(11)

$$F = 2\pi k_B T\frac{mc}{\hbar}\ . \tag{12}$$

From (6) and (5), and considering that Poisson's law allows a transport of the mass properties of the black hole (with radius R_S) to the screen that encapsulates the massive sources (with radius r), it follows readily ,

$$F = \frac{mMG}{r^2}, \tag{13}$$

in which we recognize Newton's law.

This is an interesting result, because it gives an entropic view on gravity. However, it is not emergent in the sense that it is derived from "first principles". The reason is the presence of a loop hole in the derivation. Basically, the derivation relies upon Poisson's equation, which says that the mass distribution of encapsulated baryonic mass is irrelevant for the force that is experienced for a test particle beyond the encapsulation. It is therefore not surprising that Newton's law pops up. Moreover, Verlinde's derivation contains quite some heuristic assumptions and numerical fixings,

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such as a logarithmic measure $N = 4S$ for virtual particles, the reduced Compton wave length for the unification and the elementary (physical) entropy quantity $\Delta S = 2\pi k_B$. Nevertheless, this entropic view is a challenging picture of gravity, which might give a clue for solving outstanding problems.

The successful generalization of the entropy of a black hole to the entropy of encapsulated mass gives a lead to the idea for conceiving mass as a manifestation of entropy. One might even play, like Verlinde does, with the idea that entropy is the embryo of matter and that conversion from entropy into matter would be possible. Inspired by this, Verlinde proposes a modification of the Newtonian gravity, which could possibly give an explanation for the unsolved gravitational problems. In this theory, the holographic screen (short for the screen that encapsulates baryonic matter), does not longer solely possess the entropy properties of a black hole. Instead, the entropy is built up by the entropy of a black hole (with mass equal to the mass of the encapsulated matter) and an additional entropy subtracted from the vacuum between the black hole and the cosmological horizon. This implies that Verlinde hypothesizes that, in spite of the absence of matter, entropy can be assigned to the vacuum. By generalizing Bekenstein's entropy of a (massive) black hole S as given by (3), Verlinde assigns an entropy $S_D(r)$ to a (mass less) sphere in vacuum with radius r, to the amount of

$$S_D(r) = \frac{r}{L}\frac{c^3 A(r)}{4G\hbar}, \tag{14}$$

where L is the cosmological horizon. Subsequently, Verlinde hypothesizes that baryonic matter put into the vacuum subtracts entropy from the vacuum. The subtracted entropy has an energy equivalent (and therefore a mass equivalent). The subtracted energy is beneficial to the baryonic matter, which, as a consequence, is virtually increased. Because change of entropy implies a change of volume, the vacuum volume shrinks. Because such a shrink is not without a resistance, an amount of elastic energy is involved. Therefore, it must be possible to calculate the virtual increase of baryonic mass from the elastic energy from the volume shrink caused by putting matter into the vacuum.

Note that conceiving the vacuum as an elastic medium is not much different from conceiving the vacuum as a spatial fluid that allows the positive Cosmological Constant in Einstein's Equation. The energy equivalent of Verlinde's vacuum entropy is not much different from the vacuum energy contained in the spatial fluid. We may go a step further by noting that subtracting energy from one side of the holographic screen (the vacuum) and adding it to the other side (the matter) can be modelled as the creation of dipoles. This is illustrated in figure 1.

Dipole interpretation

From this point we could try to follow Verlinde's analysis. Let me admit that I am unable to do so. Let me therefore try to harmonize the dipole view [13,1] straight from this point. We proceed as follows. Eqs. (14) and (3) allow to establish the number of dipoles N that cross the cosmological screen, because

$$N = 4S = \frac{c^3 A(L)}{G\hbar}. \tag{15}$$

The strength p_d of a dipole, consisting of elementary kernels of energy with mass equivalent m_d and spaced by their (reduced) Compton length $d = \hbar / m_d c$, amounts to

$$p_d = m_d d = \frac{\hbar}{c}. \tag{16}$$

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The difference ΔN between the amount of dipoles on the cosmological screen and the screen shifted by an amount ΔL amounts to

$$\Delta N = \frac{c^3 A(L+\Delta L)}{G\hbar} - \frac{c^3 A(L)}{G\hbar} = \frac{c^3}{G\hbar}\{4\pi(L+\Delta L)^2 - 4\pi L^2) = 8\pi L \frac{c^3}{G\hbar}\Delta L \,. \tag{18}$$

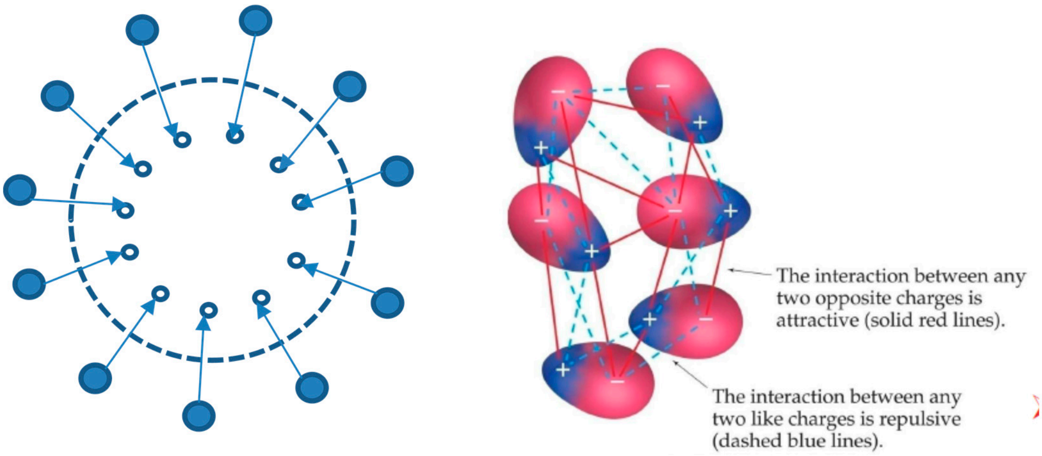

Fig.1. Left: Dipole view of the conversion of entropy outside the cosmological screen into matter within the screen. Right: *dipole-dipole interaction in an elastic medium* (from:T.L. Brown, H.E. LeMay, B.E. Bursten, Chemistry, The Central Science, 10th. Ed, Chapter 10).

The volume difference ΔV between the two screens is,

$$\Delta V = \frac{4}{3}\pi(L+\Delta L)^3 - \frac{4}{3}\pi L^3 \approx 4\pi L^2 \Delta L \,. \tag{19}$$

The dipole moment density P_{g0} therefore amounts to

$$P_g = \frac{p_d \Delta N}{\Delta V} = \frac{\hbar}{c}\frac{c^3}{G\hbar}\frac{8\pi L}{4\pi L^2} = 2\frac{c^2}{GL} \,. \tag{20}$$

Replacing the cosmological scale L (Hubble length) by the acceleration scale $a_L = c^2 / L$, we have

$$P_g = 2\frac{a_L}{G} \,. \tag{21}$$

This result allows a comparison with the dipole modelling of the Cosmological Constant, which resulted into [1],

$$P_{g0} = \frac{a_0}{20\pi G} \tag{22}$$

where a_0 is Milgrom's acceleration constant. Equating (21) allows calculating this constant from theory as,

$$a_0 = 40\pi a_L \,. \tag{23}$$

Equating the Hubble length as $L = ct_L$ and $t_L \approx$ 13.5 Gyear, we get from (23),

$$a_0 = 2.4 \times 10^{-10} \text{ m/s}^2. \tag{24}$$

This is pretty close to Milgrom's empirical value.

Comparison

Verlinde's conclusions are somewhat different. Instead of (23), Verlinde concludes,

$$a_0 = a_L / 6\,. \tag{25}$$

This is substantially different and less close to Milgrom's value. Moreover, Verlinde modifies the gravitational acceleration constant to

$$g(r) = g_N(r) + \sqrt{a_0 g_N(r)}\,, \tag{26}$$

where $g_N(r)$ is the Newtonian one.

Verlinde claims a match of this result with Milgrom's hypothesis, which is confirmed by a wealth of cosmological observations on galaxies. To my opinion Verlinde's conclusion here is incorrect. Milgrom's equation is different, nl.,

$$g = \frac{g_N}{\mu(x)}\,, \text{ with } x = g/a_0 \text{ where } \mu(x) = \frac{x}{\sqrt{1+x^2}}\,. \tag{27}$$

This implies that at small r Milgrom's gravitational acceleration coincides with the Newtonian one. This is not true for (26). Probably, the difference is sufficiently significant for being checked by observations. The author's result, on the other hand, coincides for small r with the Newtonian one, but is different from Milgrom's one at extreme cosmological distance.

In view of this, it seems to be fair giving support to Hajdukovic's view [13] that the vacuum is filled by a fluid consisting of grains that are structured as confined dipoles with a matter kernel and an antimatter kernel, spaced by their Compton wave length.

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