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QUASIRECOGNITION BY PRIME GRAPH OF THE GROUPS $^2D_{2n}(q)$ WHERE $q < 10^5$

HOSSEIN MORADI[1,A] & MOHAMMAD REZA DARAFSHEH[2,B] & ALI IRANMANESH[1,C]

Abstract. Let G be a finite group. The prime graph $\Gamma(G)$ of G is defined as follows: The set of vertices of $\Gamma(G)$ is the set of prime divisors of $|G|$ and two distinct vertices p and p' are connected in $\Gamma(G)$, whenever G has an element of order pp'. A non-abelian simple group P is called recognizable by prime graph if for any finite group G with $\Gamma(G) = \Gamma(P)$, G has a composition factor isomorphic to P. In [4] proved finite simple groups $^2D_n(q)$, where $n \neq 4k$ are quasirecognizable by prime graph. Now in this paper we discuss the quasirecognizability by prime graph of the simple groups $^2D_{2k}(q)$, where $k \geq 9$ and q is a prime power less than 10^5.

1. Introduction

Let G be a finite group. By $\pi_e(G)$ we denote the set of elements orders of G. For an integer n we define $\pi(n)$ as the set of prime divisors of n , and we set $\pi(G)$ for $\pi(|G|)$. The prime graph of the Gruenberg-Kegel graph of G is denoted by $\Gamma(G)$ and is graph with vertices set $\pi(G)$ in which two distinct vertices p and q are joined by an edge if and only if $pq \in \pi(G)$, and in this case we will write $p \sim q$.

A subset of vertices of $\Gamma(G)$ is called an independent subset of $\Gamma(G)$ if its vertices are pairwise nonadjacent. Denote by $t(G)$ the maximal number of primes in $\pi(G)$ pairwise nonadjacent in $\Gamma(G)$. Also we denote by $t(2, G)$ the maximal number of vertices in the independent sets of $\Gamma(G)$ containing 2. A finite nonabelian simple group P is called quasirecognizable by prime graph, if every finite group G with $\Gamma(G) = \Gamma(P)$ has a composition factor isomorphic to P. Also P is called recognizable by prime graph if $\Gamma(G) = \Gamma(P)$ implies that $G \cong P$.

In [3] and [5], finite groups with the same prime graph as $\Gamma(PSL(2, q))$, where q is a prime power, are determined. In [7, 8, 9] finite groups with the same prime graph as $\Gamma(L_n(2))$, $\Gamma(U_n(2))$, $\Gamma(D_n(2))$, $\Gamma(^2D_n(2))$ and $\Gamma(^2D_{2k}(3))$ are obtained. Also in [10] it is proved that if p is a prime less than 1000 and for suitable n, the finite sinple groups $L_n(p)$, $U_n(p)$ are quasirecognizable by prime graph. Now as the main result of this paper, we prove the following theorem:

Main Theorem. *The finite simple group $^2D_{2k}(q)$, where $k \geq 9$ and $q < 10^5$ is quasirecognizable by prime graph.*

Throughout this paper, all groups are finite and by a simple group we mean nonabelian simple groups. All further unexplained notations are standard and the reader is refered to [2].

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2. Preliminary Results

Lemma 2.1. [16, Theorem 1] *Let G be a finite group with $t(G) \geq 3$ and $t(2,G) \geq 2$. Then the following hold:*

(1) *there exists a finite nonabelian simple group S such that*

$$S \leq \bar{G} = G/K \leq Aut(S)$$

for the maximal normal soluble subgroup K of G.

(2) *for every independent subset ρ of $\pi(G)$ with $|\rho| \geq 3$ at most one prime in ρ divides the product $|K| \cdot |\bar{G}/S|$. In particular, $t(S) \geq t(G) - 1$.*

(3) *one of the following holds:*
   (a) *every prime $r \in \pi(G)$ nonadjacent to 2 in $\Gamma(G)$ does not divide the product $|K| \cdot |\bar{G}/S|$; in particular, $t(2,S) \geq t(2,G)$;*
   (b) *there exists a prime $r \in \pi(K)$ nonadjacent to 2 in $\Gamma(G)$; in which case $t(G) = 3$, $t(2,G) = 2$, and $S \cong A_7$ or $L_2(q)$ for some odd q.*

Remark 2.2. In Lemma 2.1, for every odd prime $p \in \pi(S)$, we have $t(p,S) \geq t(p,G) - 1$.

If q is a natural number, r is an odd prime and $(q,r) = 1$, then by $e(r,q)$ we denote the smallest natural number m such that $q^m \equiv 1 \pmod r$. Given an odd q, put $e(2,q) = 1$ if $q \equiv 1 \pmod 4$ and put $e(2,q) = 2$ if $q \equiv -1 \pmod 4$. Using Fermat's little theorem we can see that if r is an odd prime such that $r \mid (q^n - 1)$, then $e(r,q) \mid n$.

Lemma 2.3. [20, Proposition 2.5] *Let $G = D_n^{\varepsilon}(q)$, where q is power of prime p. Define*

$$\eta(m) = \begin{cases} m, & \text{if } m \text{ is odd;} \\ m/2, & \text{otherwise.} \end{cases}$$

Suppose r, s are odd primes and $r,s \in \pi(D_n^{\varepsilon}(q)) \setminus \{p\}$. Put $k = e(r,q)$, $l = e(s,q)$, and $1 \leq \eta(k) \leq \eta(l)$. Then r and s are non-adjacent if and only if $2\eta(k) + 2\eta(l) > 2n - (1 - \varepsilon(-1)^{k+l})$ and k and l satisfy the following condition:

$$\frac{l}{k} \text{ is not an odd integer,}$$

and, if $\varepsilon = +$, then the chain of equalities:

$$n = l = 2\eta(l) = 2\eta(k) = 2k$$

is not true.

Lemma 2.4. [20, Proposition 2.3] *Let G be one of simple groups of Lie type, $B_n(q)$ or $C_n(q)$, over a field of characteristic p. Define*

$$\eta(m) = \begin{cases} m, & \text{if } m \text{ is odd;} \\ m/2, & \text{otherwise.} \end{cases}$$

Let r, s be odd primes with $r,s \in \pi(G) \setminus \{p\}$. Put $k = e(r,q)$ and $l = e(s,q)$, and suppose that $1 \leq \eta(k) \leq \eta(l)$. Then r and s are non-adjacent if and only if $\eta(k) + \eta(l) > n$, and k, l satisfy to:

$$\frac{l}{k} \text{ is not an odd natural number.}$$

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Lemma 2.5. [19, Proposition 2.1] *Let $G = L_n(q)$, where q is power of prime p. Let r and s be odd primes and $r, s \in \pi(G) \setminus \{p\}$. Put $k = e(r, q)$ and $l = e(s, q)$ and assume that $2 \leq k \leq l$. Then r and s are nonadjacent if and only if $k + l > n$ and k does not divide l.*

Lemma 2.6. [19, Proposition 2.2] *Let $G = U_n(q)$, where q is power of prime p. Define*

$$\nu(m) = \begin{cases} m, & m \equiv 0 \pmod 4; \\ m/2, & m \equiv 2 \pmod 4; \\ 2m, & m \equiv 1 \pmod 2). \end{cases}$$

Let r and s be odd primes and $r,\ s \in \pi(G) \setminus \{p\}$. Put $k = e(r, q)$ and $l = e(s, q)$ and suppose that $2 \leq \nu(k) \leq \nu(l)$. Then r and s are nonadjacent if and only if $\nu(k) + \nu(l) > n$ and $\nu(k)$ does not divide $\nu(l)$.

For every Lemmas 2.3 and 2.6, simultaneously we define the following function:

$$\nu'(m) = \begin{cases} m, & \varepsilon = +; \\ \nu(m), & \varepsilon = -. \end{cases}$$

which we will use in the proofs. We note that a prime r with $e(r, q) = m$ is called a primitive prime divisor of $q^m - 1$ (obviously, $q^m - 1$ can have more than one primitive prime divisor).

Lemma 2.7. *(Zsigmondy's theorem)* [22] *Let p be a prime and let n be a positive integer. Then one of the following holds:*

(1) *there is a primitive prime p' for $p^n - 1$, that is , $p' \mid (p^n - 1)$ but $p' \nmid (p^m - 1)$, for every $1 \leq m < n$,*
(2) *$p = 2$, $n = 1$ or 6,*
(3) *p is a Mersenne prime and $n = 2$.*

3. Proof of the Main Theorem

Throughout this section, we suppose that $D := {}^2D_{2k}(p^\alpha)$ where $k \geq 9$, $2 < p^\alpha < 10^5$ and G is a finite group such that $\Gamma(G) = \Gamma(D)$. We denote a primitive prime divisors of $q^i - 1$ by r_i and a primitive prime divisors of $q'^i - 1$ by r'_i, where $q' \neq q$.

By [19, Tables 4, 6, 8], we deduce that $t(D) \geq 14$ and $t(2, D) \geq 2$. Therefore $t(G) \geq 14$ and $t(2, G) \geq 2$. Now by Lemma 2.1, it follows that there exists a finite nonabelian simple group S such that

$$S \leq \bar{G} := G/K \leq \mathrm{Aut}(S)$$

where K is the maximal normal solvable subgroup of G. Also, $t(S) \geq t(G) - 1$ and $t(2, S) \geq t(2, G)$ by Lemma 2.1. Therefore $t(S) \geq 13$ and $t(2, S) \geq 2$. On the other hand, by [19, Tables 2, 9], if S is isomorphic to a sporadic or an exceptional simple group of Lie type, then $t(S) \leq 12$. This implies that S is not isomorphic to any sporadic and any exceptional simple group of Lie type.

In the sequel, we consider each possibility for S.

Lemma 3.1. *S is not isomorphic to any alternating group.*

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Proof. Suppose that $S \cong A_m$, where $m \geq 5$. Since $t(S) \geq 13$, Lemma 2.1, we get that $m \geq 61$ and so $\{47, 59\} \subseteq \pi(S)$.

Case 1. Let $\{47, 59\} \nsubseteq \pi(q^2-1)$, where $q = p^\alpha < 10^5$. Therefore we get that $e(47, q) \geq 23$ or $e(59, q) \geq 29$. . $\{47, 59\} \subseteq \pi(S)$. Hence G contains an element $a \in A_m$, such that $e(a, q) \geq 23$, which implies that $n \geq 24$. Now we have $\min\{t(47, G), t(59, G)\} \geq 19$. Hence, according to Remark 2.2, $\min\{t(47, S), t(59, S)\} \geq 18$ in S. On the other hand, 47 is not connected to the prime numbers in the interval $[m-46, m]$ in the prime graph of A_m and similarly, 59 is not connected to the prime numbers in the interval $[m-58, m]$, in the prime graph of A_m. But these intervals contains at most 16 prime numbers, and this implies that $t(S) < t(G) - 1$, a contradiction.

Case 2. Let $\{47, 59\} \subseteq \pi(q^2-1)$, where $q = p^\alpha < 10^5$. Using GAP, we get that:
$q \in A := \{11093, 21713, 27259, 28201, 38351, 38821, 39293, 44839, 55931, 61007, 66553, 93811, 99829\}$.

First let $q \in A \setminus \{21713\}$. Then we have $e(23, q) \geq 11$ and so similarly to Case 1, we get that $t(23, G) - 1 > t(23, S)$, a contradiction.

Let $q = 21713$. Since $e(19, q) = 18$, again similarly to Case 1, we get that a contradiction. □

Lemma 3.2. *If S is isomorphic to a classical simple group of Lie type over a field of characteristic p, then $S \cong D$.*

Proof. Let S be a nonabelian simple group of Lie type over GF(q'), $q' = p^\beta$. By the hypothesis,

$$S \leq \bar{G} := G/N \leq \mathrm{Aut}(S),$$

where N is the maximal normal solvable subgroup of G. In the sequel, we denote by r_i, a primitive prime divisor of $q^i - 1$ and denote by r'_i, a primitive prime divisor of $q'^i - 1$. We remark that $\{p, r_{2n}\} \subseteq \pi(S)$ and $|\rho(p, G) \cap \pi(S)| \geq 3$ by Lemma 2.1.

Now we consider the following cases:

Case 1. Let $r_{2n-2} \in \pi(S)$. Also let p_1 and p_2 be two primitive prime divisors of $p^{(2n-2)\alpha} - 1$ and $p^{2n\alpha} - 1$, respectively. So we may assume that p_1 and p_2 are r_{2n-2} and r_{2n}, respectively. This implies that $\{r_{2n-2}, r_{2n}\} \subseteq \pi(S)$. Thus r_{2n-2} is a primitive prime divisor of $q'^s - 1$ and r_{2n} is a primitive prime divisor of $q'^t - 1$, where $s = e(r_{2n-2}, p^\beta)$ and $t = e(r_{2n}, p^\beta)$. It follows that $(2n-2)\alpha \mid s\beta$ and $2n\alpha \mid t\beta$. On the other hand, using Zsigmondy's theorem, we conclude that $t\beta \leq 2n\alpha$ and so $t\beta = 2n\alpha$. Also since $2n < 2(2n-2)$, we have $s\beta = (2n-2)\alpha$ and $s < t$.

Now we consider each possibility for S, separately. If $\rho(p, S) = \{r'_i \mid i \in I\} \cup \{p\}$, then using the results in [19], each $r'_j \in \pi(S)$, where $j \notin I$, is adjacent to p in $\Gamma(S)$.

Subcase 1.1. Let $S \cong L_m(q')$. By [19, Proposition 2.6], we see that each prime divisor of $|L_m(q')|$ is adjacent to p, except r'_m and r'_{m-1}. Hence $\rho(p, S) = \{p, r'_{m-1}, r'_m\}$. Therefore p_1 and p_2 are some primitive prime divisors of $q'^m - 1$ and $q'^{m-1} - 1$. Since $s < t$, we conclude that $m = t$ and $m - 1 = s$. Hence $2n\alpha = m\beta$ and $(2n-2)\alpha = (m-1)\beta$. Consequently, we get that $\beta = 2\alpha$ and $m = n$, that is $S \cong L_n(p^{2\alpha})$. Then S has a maximal torus of order $(p^{2n\alpha} - 1)/((p^{2\alpha} - 1)(n, p^{2\alpha} - 1))$, say T. Obviously, $r_n, r_{2n} \in \pi(T)$.

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Therefore $r_n \sim r_{2n}$ in $\Gamma(L_n(p^{2\alpha}))$, whereas $r_n \nsim r_{2n}$ in $\Gamma(G)$, by Lemma 2.3, which is a contradiction.

Subcase 1.2. Let $S \cong U_m(q')$. If $m = 3$, then $\rho(p, S) = \{p, r'_1 \neq 2, r'_6\}$ and so $s = 1$ and $t = 6$, hence $(2n-2)\alpha = \beta$ and $2n\alpha = 6\beta$. Therefore $n = 6/5$, a contradiction.
If $m \equiv 0 \pmod 4$, then $\rho(p, S) = \{p, r'_{2m-2}, r'_m\}$ and so $s = m$ and $t = 2m-2$, hence $(2n-2)\alpha = m\beta$ and $2n\alpha = (2m-2)\beta$. Then $n = (2m-2)/(m-2)$ and so $n = 3$, a contradiction.
If $m \equiv 3 \pmod 4$, then $\rho(p, S) = \{p, r'_{(m-1)/2}, r'_{2m}\}$. Therefore $s = (m-1)/2$ and $t = 2m$. Hence $(2n-2)\alpha = (m-1)\beta/2$ and $2n\alpha = 2m\beta$. Now easy computation shows that it is impossible.
If $m \equiv 1, 2 \pmod 4$, then similarly to the above discussion, we get a contradiction.

Subcase 1.3. Let $S \cong B_m(q')$ or $C_m(q')$. Since $t(p, S) \geq 3$, using [19, Tables 4], we get that m is odd. In this case $\rho(p, S) = \{p, r'_m, r'_{2m}\}$. Hence $s = m$ and $t = 2m$ and so $(2n-2)\alpha = m\beta$ and $2n\alpha = 2m\beta$, which implies that $n = 2$, a contradiction.

Let $S \cong {}^2D_m(q')$, where m is odd. Since $\rho(p, S) = \{p, r'_{2m-2}, r'_{2m}\}$, we conclude that $(2n-2)\alpha = (2m-2)\beta$ and $2n\alpha = 2m\beta$ and so $m = n$, which is impossible, since n is even.

Similarly, we can prove that $S \ncong {}^2D_m(q')$, where m is even and $S \ncong D_m(q')$.

Case 2. Let $r_{2n-2} \notin \pi(S)$. Hence $r_{n-1} \in \pi(S)$. Let p_1 and p_2 be as r_{n-1} and r_{2n}, respectively. Therefore r_{n-1} and r_{2n} are primitive prime divisors of $q'^s - 1$ and $q'^t - 1$, respectively, where $s = e(r_{n-1}, p^\beta)$ and $t = e(r_{2n}, p^\beta)$. Now we conclude that $(n-1)\alpha \mid s\beta$ and $2n\alpha \mid t\beta$. On the other hand, using Zsigmondy's theorem, we conclude that $t\beta \leq 2n\alpha$ and so $t\beta = 2n\alpha$. If $s\beta > (n-1)\alpha$, then using Zsigmondy's theorem we conclude that $s\beta = (2n-2)\alpha$, which implies that $r_{2n-2} \in \pi(S)$, which is a contradiction. Hence we suppose that $s\beta = (n-1)\alpha$.

Subcase 2.1. Let $S \cong L_m(q')$, where $q' = p^\beta$. We know that $\rho(p, S) = \{p, r'_{m-1}, r'_m\}$. Hence $t = m$ and $s = m-1$ and $2n\alpha = m\beta$ and $(n-1)\alpha = (m-1)\beta$. These equations imply that $m = 2 - 2/(n+1)$, which is impossible.

Subcase 2.2. Let $S \cong {}^2D_m(q')$, where m is odd. We note that $\rho(p, S) = \{p, r'_{2m-2}, r'_{2m}\}$ and so $(n-1)\alpha = (2m-2)\beta$ and $2n\alpha = 2m\beta$ and so $m = 2-2/(n+1)$, which is impossible.

Subcase 2.3. Let $S \cong {}^2D_m(q')$, where m is even. Since $\rho(p, S) = \{p, r'_{m-1}, r'_{2m-2}, r'_{2m}\}$, we get that $2n\alpha = 2m\beta$ and $(n-1)\alpha = (2m-2)\beta$ or $(n-1)\alpha = (m-1)\beta$. If $(n-1)\alpha = (2m-2)\beta$, then we get that $m = 2 - 2/(n+1)$, which is impossible. Hence $(n-1)\alpha = (m-1)\beta$, which implies that $m = n$ and $\alpha = \beta$, and so $S \cong D$, which is a contradiction, since $r_{2n-2} \notin \pi(S)$.

We can use a similar proof for groups $U_m(q')$, $B_m(q')$, $C_m(q')$ and $D_m(q')$ and get a contradiction. We omit the proof for convenience. □

Lemma 3.3. *If S is isomorphic to a classical simple group of Lie type over a field of characteristic $p' \neq p$, then $S \ncong D$.*

Proof. Let S be isomorphic to a classical simple group of Lie type over a field with q' elements, where $q' = p'^\beta$. Using [19, Table 4], $t(p', S) \leq 4$ and so Lemma 2.1 implies that $t(p', G) \leq 5$. On the other hand, by Lemma 2.3, we deduce that if $r \in \pi(G) \setminus \{r_1, r_2, r_3, r_4, r_6\}$, then $t(r, G) > 5$. Hence $p' \in \{r_1, r_2, r_3, r_4, r_6\}$ and so $p' \mid (q^2+1)(q^6-1)$.

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Consider $r'_3 \in \pi(S) \subseteq \pi(G)$ and $3 = e(r'_3, q') \leq e(r'_3, p')$. By Lemmas 2.3, 2.4, 2.5 and 2.6 we get that for each classical simple group of Lie type S, we have $t(r'_3, S) \leq 6$. On the other hand using Remark 2.2, we have $t(r'_3, G) \leq t(r'_3, S) + 1$ and so $t(r'_3, G) \leq 7$. We note that $r'_3 \in \pi(S) \subseteq \pi(G) = \pi(^2D_{2k}(q))$. Hence, by Lemma 2.3, it follows that $e(r'_3, q) \leq 10$. Since $t(S) \geq 13$, we conclude that $r'_i \in \pi(S)$, where $2 \leq i \leq 10$ and we have similar argument for r'_i, $2 \leq i \leq 10$ and $e(r'_i, q) \leq 2i + 4$.

Hence according above discussion, if $p' \in \pi((q^2+1)(q^6-1))$, then the following condition holds:

If $r'_i \in \pi(p'^i - 1)$, then $e(r'_i, q) \leq 2i + 4$, where $2 \leq i \leq 10$.

Using GAP, we get that the above condition holds only for $q = 54251$, where $p' = 2$. Since $t(S) \geq 13$, we conclude that $r'_{13} \in \pi(S)$. If $p' = 2$ and $q = 54251$, then $r'_{13} = 8191$ and so $e(8191, q) = 1365$, which contradicts to Remark 2.2. Therefore by the above argument, we get that S is not isomorphic to any classical simple group of Lie type over a field of characteristic $p' \neq p$. $\square$

Using the Classification Theorem of finite simple groups and Lemmas 3.1, 3.2 and 3.3, we get that the finite simple group $^2D_{2k}(q)$, where $k \geq 9$ and $q < 10^5$ is quasirecognizable by prime graph.

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1)Department of Mathematics, Tarbiat Modares University, Tehran, Iran, 2) School of mathematics, statistics and computer, College of science, University of Tehran, Tehran, Iran

E-mail address: A) moradi61hh@yahoo.com, B) darafsheh@ut.ac.ir, C) Iranmana@yahoo.com