Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 10 July 2017 doi:10.20944/preprints201707.0015.v1

Article

Multiple Attribute Decision Making (MADM) in Proportional-Stop-Loss Reinsurance

Jiexuan Wang

National University of Singapore, Department of Industrial and Systems Engineering Management;

Correspondence: wang.jiexuan@u.nus.edu; Tel.: +65-8506-8476

Abstract: This article addresses reinsurance decision making process, which involves the insurance company and the reinsurance company, and is negotiated through reinsurance intermediaries. The article proposes a decision flow to model the reinsurance design and selection process. In contrast to existing literature on pure proportional reinsurance or stop-loss reinsurance, this article focuses on the combination into Proportional-Stop-loss reinsurance design which better addresses interest of both parties. In terms of methodology, the significant contribution of the study is to incorporate Multiple Attribute Decision Making (MADM) into modelling the reinsurance selection. The Multi-Objective Decision Making (MODM) model is applied in designing reinsurance alternatives. Then MADM is applied to aid insurance companies in choosing the most appropriate reinsurance contract. To illustrate the feasibility of incorporating intelligent decision supporting system in reinsurance market, the study includes a numerical case study using simulation software @Risk in modeling insurance claims, and programming in MATLAB to realize MADM. Managerial implications could be drawn from the case study results. More specifically, when choosing the most appropriate reinsurance, insurance companies should base their decision on multiple measurements instead of single-criteria decision making models for their decisions to be more robust.

Keywords: multi-attribute decision making; reinsurance; proportional reinsurance; non-proportional reinsurance; TOPSIS

---

1. Introduction

1.1 Background

Reinsurance is generally known as “the insurance for insurance”. Following similar concepts and principles as insurance, it provides financial compensation to insurance companies for the risk of large losses. The reinsured (or “insurance companies”) buys reinsurance from the reinsurer (or “reinsurance companies”) in exchange for loss limitation, revenue protection and free up capital. In recent years, reinsurance has grown in both market value and diversity due to global trends such as the global climate change, increase in insurance mega losses, volatility in equity markets and emerging risks such as terrorism. Regardless of the financial size, an insurance company rarely retains all of their risk. Thus it is of interest to understand the decision making process of reinsurance contract, which in reality is usually done with the negotiation intermediaries, i.e. the reinsurance brokers. Typically, there are two categories of reinsurance decisions, both of which will be addressed in this study:

- Optimal reinsurance form under given criteria;
- Given the reinsurance form of decision, decide on the reinsurance parameters. (e.g. optimal retention portion for proportional reinsurance, optimal retention limit for stop-loss reinsurance, etc.)

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 10 July 2017 doi:10.20944/preprints201707.0015.v1

The common forms of reinsurance are shown in Figure A1 and their complete definitions are included in Appendix A. This study focuses on treaty reinsurance which covers an entire portfolio with multiple single risks. Both facultative and treaty reinsurance could be further broken down into proportional reinsurance and non-proportional reinsurance (Carter 1979). Early research has shown that under variance risk measurement with fixed premium, stop-loss contract is the optimal reinsurance form for reinsurance buyers ((Borck 1960) and (Hürlimann 2011)), whereas that quota-share best addresses the interest of the reinsurer (Vajda 1962). Clearly, there would be conflicts of choice between two parties. Thus, this study addresses a combinational form of proportional and stop-loss treaty reinsurance (See Section 2.2), following the definition by Samson and Thomas (1985). Quota-share reinsurance and stop-loss reinsurance could be considered as special cases of Proportional-Stop-Loss reinsurance. In deciding the optimal reinsurance parameters, this study attempts to utilize Multiple Attribute Decision Making (MADM) which improves from previous literatures on single criterion.

1.2 Papaer Development

The paper is organized as follows. Section 2 recalls recent research that this study is built upon. Section 3 develops the decision flow based on the form of Proportional-Stop-loss Reinsurance and determines the optimal reinsurance parameters using Multi-Objective Decision Making (MODM). Section 4 includes a numerical case study, which models claims using @Risk and implements MADM for buyer's selection using MATLAB. Section 5 discusses the contributions, limitations, further directions and concludes the study.

1.3 Literature Review

Decision analysis models on single criterion have been extensively discussed both for the reinsurer (see Appendix B.1) in structuring reinsurance and for the reinsured (see Appendix B.2) for evaluating and selecting the most appropriate reinsurance product (Samson and Thomas 1985). Only recently did researchers begin to look into the cooperative behavior of both parties to reach a joint-party optimality (see Appendix B.3). In addition, recent growth of promising decision analysis based on multiple criteria has ignited sparks in the reinsurance field of study (See Appendix B.2.4). Complete review of existing decision-making methodologies in reinsurance is included in this study in Appendix B. In particular, this study is developed upon three recent researches ((Bazaz and Najafabadi 2015), (Bulut Karageyik and Şahin 2017) and (Payandeh-Najafabadi and Panahi-Bazaz 2017)) which focus on Multi-Attribute Decision Making (MADM) and Proportional-Stop-loss Reinsurance.

Basak and David (2015) first proposes to use MADM to the problem of selecting optimal reinsurance level under competing criteria. In choosing the input alternatives, they use ruin probability as a constraint, i.e. the insurance company should not have a probability of ruin greater than 1%. Loss distribution was modeled as the translated gamma process and the reinsurance forms considered was pure proportional and pure stop-loss reinsurance. The study also includes comparison with single criterion decision making and concludes that MADM is extremely insightful for selecting optimal reinsurance.

Later, Basak and Sule (2017) improves on the research to include Value-at-Risk (VaR) measurement into consideration, specifically targeting at optimal retention level in excess-of-loss reinsurance design. Key measurement criteria are expected profit, expected shortfall, finite time ruin probability and variance of risk. By comparing and contrasting different MADM techniques, the authors safely conclude that under the case of reinsurance where correlation between measurements are low enough, different MADM techniques will generate similar optimal retention level.

However, both studies were focusing either pure proportional reinsurance or pure non-proportional reinsurance, with neither considering the combination of both. The only comprehensive discussion of proportional-stop-loss reinsurance up-to-date is conducted by Hürlimann (2017), which took a viewpoint from both the insurer and the reinsurer. However, the limitation of this research is

 doi:10.20944/preprints201707.0015.v1

that it has a narrow focus on only VaR measurement to arrive at a closed-form determination of optimal reinsurance.

To the best of my knowledge, there is no previous research conducting analysis of proportional-stop-loss reinsurance based on multiple measurement schemes and considering decisions from both parties. Thus, this study serves the purpose of filling this gap. More specifically, Section 3 will incorporate reinsurance into the trade procedure of deals done between two parties. Decision making models with single measurement of variance of risk will be used in modeling reinsurer offerings and MADM model will be used for best reinsurance selection by insurance company.

2. Materials and Methods

2.1 Decision Flow

As reinsurance decision making involves the seller party (the reinsurance company, or the reinsurer) and the buyer party (the insurance company, or the reinsured), it could be safely viewed as a two-sided trade process, which involves negotiation between the selling and buying party. Furthermore, reinsurance deals could adapt to established two-sided trade matching models P1(Figure 1) or P2 (Figure 2) with the existence of a broker (Liang 2014).

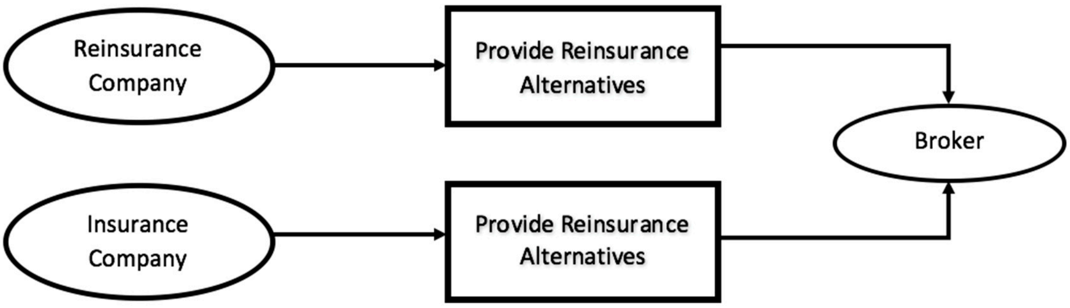

Figure 1. Trade procedure P1 (Liang, 2014)

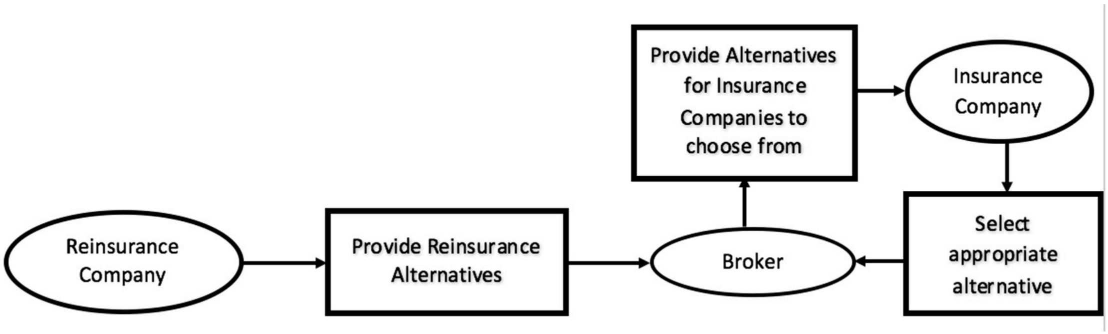

Figure 2. Trade procedure P2 (Liang, 2014)

For the first trade procedure P1, both reinsurer and the reinsured exchange information through the broker. Previous researches have suggested to model P1 using two-sided cooperative game with incomplete information (See (Borch 1960) and (Wang 2003)), while little literature has discussed about the reinsurance deals settled under procedure P2. Under P2, the seller (the reinsurer) will provide several plans for the buyer to choose from. However, noticing the prevalence of procedure P2 in reinsurance trading practice, this study attempts to model reinsurance scenario under P2 by using MODM in providing reinsurance alternatives and by using MADM in selecting appropriate reinsurance design for the reinsured. Thus, the research design develops as follows in Flowchart 3

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 10 July 2017 doi:10.20944/preprints201707.0015.v1

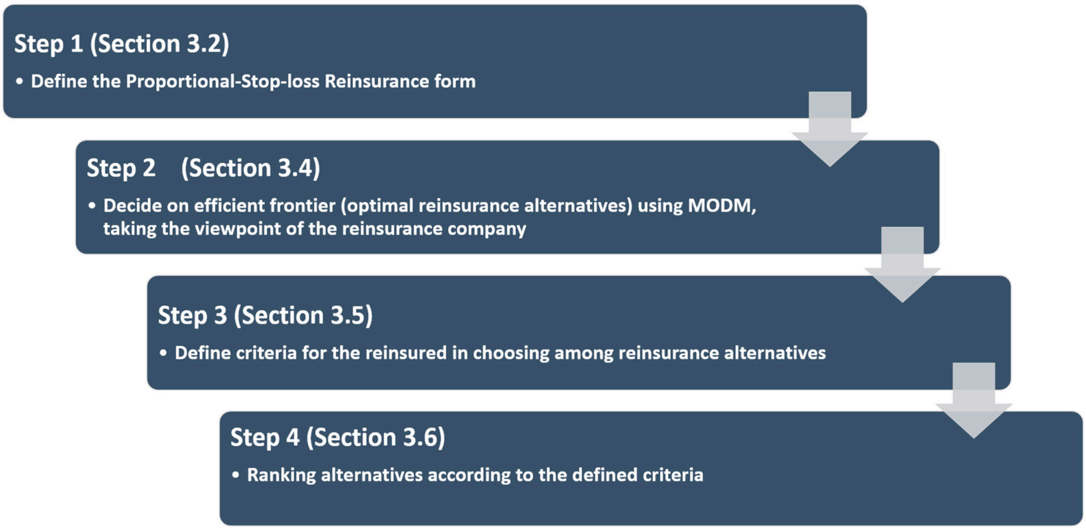

Figure 3. Research development flowchart

2.2 The Proportional-Stop-Loss Reinsurance Model

The paper discusses Proportional-Stop-Loss reinsurance adopting definitions from (Samson and Thomas 1985), (Hürlimann 2011) and (Payandeh-Najafabadi and Panahi-Bazaz 2017). Under which, given a single loss of X and a reinsurance arrangement with parameters (a, M) the reinsurer is bonded to pay a claim amount of:

$$X_r = a(X-M)_+ \Leftrightarrow X_r = \begin{cases} 0 & if X \leq M \\ a(X-M) & if X > M \end{cases} \quad (1)$$

where a is the fraction ceded to the reinsurer and M is the retention limit. The reinsured will pay the rest of the claim $X_i = X - X_r$. When M=0, the Proportional-Stop-Loss model becomes the classical quota-share reinsurance model, and when a=1, it becomes the classical stop-loss reinsurance model.

2.4 Variable Definition

To ensure the consistency of notations in this paper, we define the key variables as Table 3.1. Almost all definitions follow previous literature, and necessary elaborations will be given in later sections.

Table 1. Key variable definitions

Variable	Variable Explanation
t	the time period of one contract, in our case study $t = 1$;
N	the number of claims incurred in period t (during one contract);
$W(t)$	the wealth holding by insurance company at time t;
ζ	the loading factor of reinsurance premium paid reinsurer;
θ	the loading factor of the premium paid to the reinsured;
X	the claim amount of one single loss;
X_i	the claim amount payable by insurance company (reinsured);
X_r	the claim amount payable by reinsurance company (reinsurer);
$S(t)$	the aggregate loss of an insurance portfolio;
$S_i(t)$	the aggregate claim (loss) incurred to insurance company (reinsured);
$S_r(t)$	the aggregate claim (loss) incurred to reinsurance company (reinsurer);

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 10 July 2017 doi:10.20944/preprints201707.0015.v1

$F_S(X)$	the cumulative distribution function of S;
$\overline{F_S} = 1 - F_S(X)$	the survival distribution function of S;
(a, M)	the Proportional-Stop-Loss reinsurance parameter, $X_r = a(X - M)_+$;
c	the total premium per unit time;
c_i	the premium gained by the insurance company;
c_r	the premium payable to the reinsurer;
ES_α	the expected shortfall with a confidence level of α;
$PROFIT_i$	the expected profit gained by insurance company;
$\psi(i)$	the ruin probability of insurance company's wealth $U(t)$;
$U_i(t)$	the utility of insurance company at the end of period t;

2.5 Providing Alternatives Using MODM

Considering the reinsurance practices and following previous research on joint-party reinsurance problem, this study attempts to model the reinsurer pricing objectives under Value-at-Risk (VaR) measurement. We attempt to formulate a model maximizing the reinsurer expected profit while minimizing the variance of profit. In deciding a reinsurance design, the reinsurer needs to specify the premium and the arrangement of reinsurance claim amount, in other words, the reinsurance premium loading factor ζ and the reinsurance design parameter (a, M). Under the Expected Value Premium Principle, insurance premium must be at least greater than expected individual loss (Bulut Karageyik and Şahin 2017). Thus, the bi-objective model is formulated as:

$$\begin{array}{cc} \max\limits_{a\in[0,1],M\geq 0,\zeta\in[1,0]} & c_r \cdot t - S_r \\ \min\limits_{a\in[0,1],M\geq 0,\zeta\in[1,0]} & Var[c_r \cdot t - S_r] \\ subject.to. & M > ln(\zeta/\theta) \\ & \zeta \geq \theta \end{array} \tag{2}$$

where S_r is defined as the aggregated claim of loss (compounded from individual loss X_r), c_r is the premium paid to reinsurance company per unit time, defined according to expected value premium principle. (Formulas in Section 2.6.1).

Clearly, there is conflict between two objectives and there is no single design of (ζ, a, M) that could achieve all objectives. The closed-form optimality derivations (Hürlimann 2011) are omitted and the optimal solution would be an efficient frontier analyzed in closed form. The optimal pairs will satisfy:

$$\zeta = e^{M/\lambda} \cdot \theta \text{ subject to: } \zeta \geq \theta \tag{3}$$

Note that for an increasing ceding level a, the reinsurer risk and expected profit will both increase proportionally, thus the reinsurer preference will be ambiguous for different ceding portion a while fixing the pair of (ζ, M). This is in line with (Payandeh-Najafabadi and Panahi-Bazaz 2017) which suggests that optimal design (a, M, ζ) depend on the loss distribution (in our case, λ) but not on the market premium (θ), and does not depend on the portion retained (a). Thus, it would be flexible for reinsurance company to select an appropriate ceding portion a given their risk appetite and their financial capability (which is often not necessarily known by the broker). In Section 4, we will briefly discuss the resulting effects of choosing different ceding portion a, based on numerical case study.

Thus, the alternatives provided by the reinsurance firm will be in the form of (a, M, ζ). These are inputing alternatives we will use to apply MADM. For illustration, Figure 6 shows the optimal pairs of (ζ, M) given other parameters in the case study.

2.6 Calculating Decision Criteria

Now we need to define the selection criteria for reinsurance design. In this study, we are concerned with expected profit, expected shortfall, ruin probability and expected utility as selection criteria. All of them are calculated taking the viewpoints of reinsurance buyers (the reinsured).

2.6.1 Expected Profit of Insurance Company (the Reinsured) $\boldsymbol{PROFIT_i}$

In general, the expected profit of the reinsured is calculated as the difference between the insurer's income and the claims paid to the policyholders. Net premium gained by the insurance company is calculated under the Expected Value Premium Principle, defined as:

$$c^{*}\text{=Total Premium Income -Reinsurance Premium} \\ =(1+\theta)E[S]-(1+\zeta)E[S_r] \tag{4}$$

The net insurance profit after considering the reinsurance arrangement is

$$Profit_i = c^{*} - E[S_i] \tag{5}$$

Our objective is to maximize the expected profit of the insurance company.

2.6.2 Expected Shortfall ES_{α}

Expected shortfall is calculated under Value at Risk (VaR) measurement. VaR given a confidence level of $\alpha\epsilon(0,1)$ is defined as the smallest l such that the probability of loss $L < l^{*}$ is at least α (Bazaz and Najafabadi 2015), i.e.

$$VaR_{\alpha}(l^{*}) = min(l^{*}\epsilon\mathbb{R}: Pr(L \leq l^{*}) \geq \alpha) \tag{6}$$

Expected shortfall is the financial risk measurement to investigate market risk of the portfolio. It is calculated as the expected value of tail distribution of VaR_{α} as follows:

$$ES_{\alpha}(L) = \frac{1}{1-\alpha}\int_{\alpha}^{1} VaR_u(L)du \tag{7}$$

An increase in retention level M will cause the insurer's liability to insurance policyholders to increase, and thus ES will increase accordingly. In contrast, a larger ceding portion of a will release insurer from burden and thus will decrease the amount of liability held by the reinsured. Our objective is to find the optimal (a,M) pair that could minimize the expected shortfall of insurance company.

2.6.3 Ruin Probability

The ruin probability criterion is based on definitions of finite time ruin probability measurement. The insurer's asset is represented as $W(t)$ and is defined by:

$$W_i(t) = w_i(0) + c^{*}\cdot t - S_i(t) \tag{8}$$

In equation (8) c^{*} is the net premium income per unit time gained by the insurance company, and S(t) is the aggregate claim amount up to time t, which is calculated by:

$$S_i(t) = \sum_{i=1}^{N(t)} X_i \tag{10}$$

The finite time ruin probability, $\psi(w_0,t)$, is given as:

$$\psi(w_0,t) = Pr(W(s) < 0) \text{ for some s}, 0 < s \leq t \tag{11}$$

In our study, the ruin probability is approximated through simulation study as the closed form ruin probability for compounding exponential loss distribution under Proportional-Stop-Loss

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 10 July 2017 doi:10.20944/preprints201707.0015.v1

reinsurance design is hard to obtain. Our objective is to minimize the ruin probability of $\psi(w,t)$ such that the insurance company would be less likely to bankrupt if there is a large loss incurred.

2.6.4 Expected Utility

To address the utility theory used in vast literature on reinsurance optimization (Samson and Thomas 1983), the utility function of the reinsured is defined as exponential utility function, which assumes constant absolute risk aversion:

$$U_i^t(W_i(t)) = -e^{-kW_i(t)} \quad (12)$$

In reality, utility function may have much more complexity and may be different for different insurance companies. However, as long as the value of utility could be obtained in numeric value, decision could be made through MADM. In deciding the optimal reinsurance alternative, one of our objectives is to maximize the expected utility of the insurance company.

2.7 Selecting the Best Alternative Using MADM

In Section 2, we reviewed decision analysis techniques on reinsurance decisions under single measurement. In order to model the decision of the reinsurance purchasing party (the insurance company or the reinsured) under multiple measurement criteria, this study adopts Multi-Criteria Decision Making techniques. In particular, the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), reviewed by previous research (Bazaz and Najafabadi 2015) as the most popular MADM technique and most suitable for pure numerical criteria, is applied to the reinsurance selection problem. Furthermore, suggested by (Bulut Karageyik and Şahin 2017), the correlation between criteria in reinsurance problem is small enough to return similar results from different TOPSIS methodologies, thus in this study we choose the classical TOPSIS method to support our analysis.

Following similar definitions of TOPSIS in previous study (Bazaz and Najafabadi 2015), (Ameri Sianaki 2015) and (Bulut Karageyik and Şahin 2017), we briefly describe the steps of applying the method as follows. This study attempts to implement the TOPSIS decision supporting system by storing reinsurance alternatives in Excel and processing the input matrices with MATLAB code. Part of the MATLAB code was developed with reference to previous efforts by Amari (Ameri Sianaki 2015), and was revised accordingly to serve the needs of this study. Below is the complete procedure of conducting TOPSIS.

1. Formulate decision matrix D with m alternatives $A_1, A_2, \ldots, A_m$ and n decision criteria $C_1, C_2, \ldots, C_n$. The attribute value of A_i on C_j for $i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$ is represented as d_{ij}
2. Calculate weight of the criteria using entropy technique as follows:

$$q_{ig} = \frac{d_{ig}}{x_{1g} + x_{2g} + .. x_{mg}}; \forall g \epsilon \{1,2,\ldots,c\}$$

$$\Delta_g = -k \sum q_{ig} \cdot log_2(q_{ig}); \forall g \epsilon \{1,2,\ldots,c\}$$

$$d_g = 1 - \Delta_g, w_g = \frac{d_g}{(d_1 + \ldots + d_g)} \quad (13)$$

$$w_g{}' = \frac{\lambda_g \cdot w_g}{\lambda_1 \cdot w_1 + \lambda_2 \cdot w_2 + \ldots + \lambda_c \cdot w_c}$$

3. Normalize the decision matrix using the following formula:

$$r_{ij} = \frac{d_{ij}}{\sqrt{\sum_{i=1}^{m} d_{ij}^2}} \quad (14)$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 10 July 2017 doi:10.20944/preprints201707.0015.v1

One may notice that by scaling the criteria (multiplying a constant to d_{ij}), the decision would not change; However, it will not necessarily return same decision for different utility functions that generate same decision under expected utility measurement as adding a constant to d_{ij} in the r_{ij} formula will change the resulting r_{ij}.

4. Calculate the weighted normalized decision matrix by using normalized decision matrix parameter r_{ij} and weight vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ to return the weighted normalized decision matrix parameter $V_{ij} = \omega_j \cdot r_{ij}$. If criteria are given same weight, $\omega_1 = \omega_2 = \ldots = \omega_n = \frac{1}{n}$.
5. Compute the vectors of positive ideal solutions and the negative ideal solutions, denoted by

$$S^+ = (S_1^+, S_2^+, S_3^+, \ldots, S_n^+)$$
$$S^- = (S_1^-, S_2^-, S_3^-, \ldots, S_n^-) \quad (15)$$

6. Calculate the distance between each alternative and the positive and negative ideal points. The distance between alternative A_i and the positive ideal points is

$$D_i^+ = \sqrt{\sum_{j=1}^{n} (V_{ij} - S_j^+)^2}, \text{for } i = 1,2,\ldots,m; \quad (15)$$

The distance between alternative and the negative ideal solutions are:

$$D_i^- = \sqrt{\sum_{j=1}^{n} (V_{ij} - S_j^-)^2}, \text{for } i = 1,2,\ldots,m; \quad (16)$$

7. Calculate the relative closeness coefficient of each alternative represented as:

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-}, C_i \epsilon [0,1] \quad (17)$$

8. Rank the alternatives according to C_i. The alternative with higher C_i value is preferred over lower C_i alternatives.

A graphical representation of TOPSIS is shown in Figure 4. Each blue ball represents one available alternative. The red ball represents the negative ideal solution and the green ball represents the positive ideal solution. The blue ball that is relatively near to the green ball and away from the red ball would be the best alternative amongst all. Given at least 4 selecting criteria, it would be hard to visualize the alternatives in 3-dimensional space, thus the calculation of distance is coded using MATLAB.

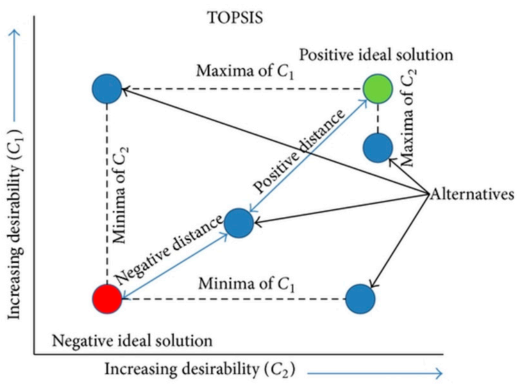

Figure 4. Graphical representation of TOPSIS (Chauhan and Vaish 2013)

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 10 July 2017 doi:10.20944/preprints201707.0015.v1

3. Case study

Following the decision flow in Section 2 this study models the reinsurance deal procedure as illustrated in Figure 5. We will choose a loss distribution model in Section 3.1, generate reinsurance offering alternatives in Section 3.2, tabulate decision matrix in Section 3.3 and finally apply MADM in selecting from alternatives in Section 3.4.

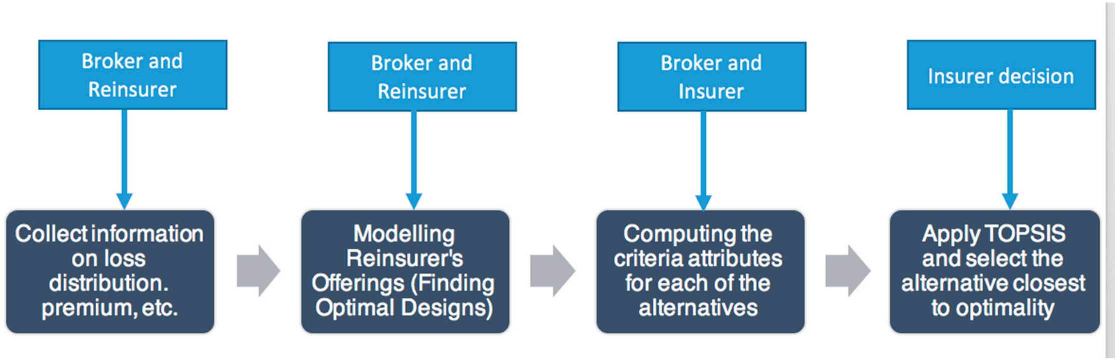

Figure 5. Case study flowchart in modeling real-world reinsurance deals

3.1 Loss Distribution Modeling

Under treaty reinsurance which covers the entire line of insurance business handled by the insurance company, the aggregated claim S is a compounding distribution of N single risks or claims incurred during time period t.

Following the majority of research study on modeling loss or claim, such as (Samson and Thomas 1985), (Bazaz and Najafabadi 2015), (Payandeh-Najafabadi and Panahi-Bazaz 2017) and (Bulut Karageyik and Şahin 2017), our case study chooses to model individual claim as exponential loss model with parameter $\mu = 100$, i.e.

$$Pr(X = x*) = \mu e^{-\mu x^*} \tag{18}$$

The occurrence of claim follows Poisson distribution with mean $\lambda = 10$, i.e.

$$Pr(N(t) = n) = \frac{(\lambda^k)e^{-\lambda}}{k!} \tag{19}$$

Thus, S is the compounding distribution of $N(t)$ identical, independently distributed risk each with distribution X.

In this case study, we set $t = 1$, $u\,(0) = 1500$ and original insurance premium parameter $\theta = 0.1$.

3.2 Generating Alternatives from Viewpoints of Reinsurers

Following Section 4.2, we define the case by setting the portion ceded as a=0.6, a=0.75, a=0.9 and a=1 respectively in assessing the differences in results when retention level is changed. The optimal pair (ζ, M) (Figure 6 with ζ on x-axis and M on y-axis) are sought by grid search as solving Formula 3.2 in mathematics form may not be succinct.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 10 July 2017 doi:10.20944/preprints201707.0015.v1

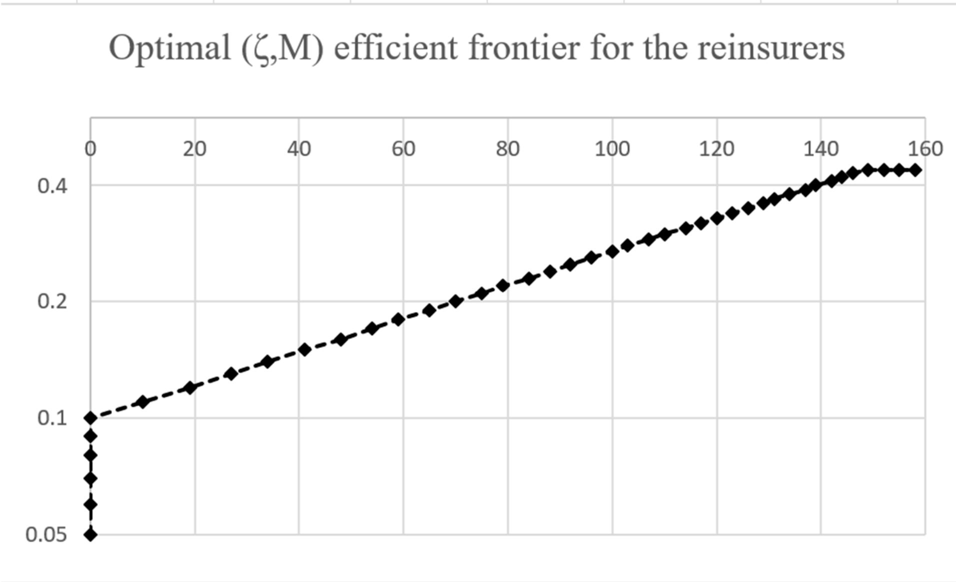

Figure 6. Optimal Pairs of reinsurance design given constant ceding portion a

When $\zeta < 0.1$, constraint $\zeta < \theta$ will be violated; all points to the right and below of the efficient frontier is deemed as inferior to the points on the efficient frontier. In managerial terms, the reinsurance designs with parameters to the southeast of the efficient frontier will cause the reinsurance company to likely generate less profit while suffering from a larger risk.

3.3 Constructing decision matrix

For the first trial, we fixed the ceding portion at a=0.6 and select 35 pairs of the optimal (ζ, M) as the reinsurance design parameters for TOPSIS alternatives. Profit for insurance company and expected utility after claims are calculated using theoretical mean of the random distributions. However, given the claim process as compounding exponential loss with Poisson occurrence, ruin probability and expected shortfall at 95% confidence level are hard to obtain in analytic terms. Thus, by using Monte Carlo Simulation with 100,000 iterations, loss and claim is modeled as exponential value with Poisson occurrence and $ES_{0.95}$ and ruin probability $\psi(a, M)$ are calculated in Excel as follows:

A	B	C	D	E	F	G	H	I	J	K
Theo U_i(t)	1538.37786	1538.97666	1540.0545	1541.43174	1542.92874	1545.26406	1548.4377	1551.4317	1554.06642	1558.43766
theta	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
zeta	0.1	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19
u	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
t	1	1	1	1	1	1	1	1	1	1
a	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
M	0	10	19	27	34	41	48	54	59	65
N	10	10	10	10	10	10	10	10	10	10
X	100	100	100	100	100	100	100	100	100	100
Xi	40	46	51.4	56.2	60.4	64.6	68.8	72.4	75.4	79
Xr	60	54	48.6	43.8	39.6	35.4	31.2	27.6	24.6	21
S	996.5980549	996.5980549	996.5980549	996.5980549	996.5980549	996.5980549	996.5980549	996.5980549	996.5980549	996.5980549
(Si)	399.2	459.08	512.972	560.876	602.792	644.708	686.624	722.552	752.492	788.42
Sr	598.8	538.92	485.028	437.124	395.208	353.292	311.376	275.448	245.508	209.58
U(t)	1538.37786	1538.97666	1540.0545	1541.43174	1542.92874	1545.26406	1548.4377	1551.4317	1554.06642	1558.43766
value-at-risk	75.18886993	98.1306547	99.16616383	99.48271089	75.21509366	74.64912875	74.45636394	74.40275484	54.06642042	58.43766042
c*	437.5778604	498.0566604	553.0265004	602.3077404	645.7207404	689.9720604	735.0617004	773.9837004	806.5584204	846.8576604
Var(Si)	14719.36	19466.3536	24304.97522	29056.38462	33561.61274	38391.40274	43545.75462	48222.0953	52301.19794	57414.7036
PROFITr	59.88	59.2812	58.20336	56.82612	55.32912	52.9938	49.82016	46.82616	44.19144	39.8202
Var (Sr)	33118.5600	26826.0336	21729.0872	17648.8806	14426.4447	11528.5707	8955.2586	7007.8873	5567.2299	4057.0236
ES (0.95)	670.1016	705.6377	730.6881	763.1634	783.3937	812.4188	836.4823	855.6909	886.3125	893.6415
Psi (u.t)	0.087312252	0.086452339	0.086887538	0.089444258	0.089566722	0.089115785	0.089900436	0.090491325	0.092109158	0.091589813
PROFITi	38.37786042	38.97666042	40.05450042	41.43174042	42.92874042	45.26406042	48.43770042	51.43170042	54.06642042	58.43766042
Expected Utility	0.142588906	0.142640246	0.142732651	0.142850709	0.142979015	0.143179133	0.143451014	0.143707427	0.143933006	0.144307132

Figure 7. Screenshot of criteria calculation using Monte Carlo Simulation with @Risk

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 10 July 2017 doi:10.20944/preprints201707.0015.v1

By retrieving criteria value and reformatting in the tabulated workbook for processing TOPSIS, the decision matrix is built as shown in Figure 8 and is ready for processing using MATLAB coded developed in Section 2.

	A	B	C	D	E	F	G	H	I
1	Alternatives					ES0.95	Ruin Probability	PROFITi	Expected Utility
2	Alternatives	a	M	zeta	riteri	c1	c2	c3	c4
3	A1		0	0.1		670.1016436	0.087312252	38.37786042	0.142588906
4	A2		10	0.11		705.6377247	0.086452339	38.97666042	0.142640246
5	A3		19	0.12		730.688082	0.086887538	40.05450042	0.142732651
6	A4		27	0.13		763.1634052	0.089444258	41.43174042	0.142850709
7	A5		34	0.14		783.3936523	0.089566722	42.92874042	0.142979015
8	A6		41	0.15		812.4187793	0.089115785	45.26406042	0.143179133
9	A7		48	0.16		836.4822701	0.089900436	48.43770042	0.143451014
10	A8		54	0.17		855.6908783	0.090491325	51.43170042	0.143707427
11	A9		59	0.18		886.3125475	0.092109158	54.06642042	0.143933006
12	A10		65	0.19		893.6414999	0.091589813	58.43766042	0.144307132
13	A11		70	0.2		910.8349333	0.091607518	62.32986042	0.14464012
14	A12		75	0.21		935.2232616	0.092231426	66.82086042	0.145024176
15	A13		79	0.22		936.9221891	0.092097565	70.59330042	0.145346649
16	A14		84	0.23		957.8482411	0.092074192	76.22202042	0.145827574
17	A15		88	0.24		976.5046079	0.092530559	81.01242042	0.146236659
18	A16		92	0.25		992.2091456	0.09389913	86.28186042	0.146686426
19	A17		96	0.26		1007.423898	0.093290762	92.03034042	0.147176811
20	A18	0.6	100	0.27		1011.69034	0.09339054	98.25786042	0.147707743
21	A19		103	0.28		1043.980452	0.094080032	98.25786042	0.147707743
22	A20		107	0.29		1097.52394	0.095797522	98.25786042	0.147707743
23	A21		110	0.3		1123.563791	0.094323856	98.25786042	0.147707743
24	A22		114	0.31		1170.495524	0.093745244	98.25786042	0.147707743
25	A23		117	0.32		1205.441455	0.094158638	98.25786042	0.147707743
26	A24		120	0.33		1231.082995	0.094099739	98.25786042	0.147707743
27	A25		123	0.34		1255.11953	0.09403522	98.25786042	0.147707743
28	A26		126	0.35		1279.785066	0.093324499	98.25786042	0.147707743
29	A27		129	0.36		1299.283714	0.092930586	98.25786042	0.147707743
30	A28		131	0.37		1353.999274	0.094426982	98.25786042	0.147707743
31	A29		134	0.38		1372.454608	0.094210057	98.25786042	0.147707743
32	A30		137	0.39		1382.934637	0.094673436	98.25786042	0.147707743
33	A31		139	0.4		1422.224375	0.09415124	98.25786042	0.147707743
34	A32		142	0.41		1444.980799	0.092483131	98.25786042	0.147707743
35	A33		144	0.42		1477.163227	0.095391474	98.25786042	0.147707743
36	A34		146	0.43		1512.923601	0.094778562	98.25786042	0.147707743
37	A35		149	0.44		1531.323057	0.093241787	98.25786042	0.147707743
38	criteria Sign range	1				-1.00	-1	1	1
39	W(Lambda)					1.00	1	1	1

Figure 8. Build decision matrix for preparation of TOPSIS

3.4 Selecting alternative using TOPSIS

With above alternatives as inputting decision matrix, we could observe that if we only consider minimizing expected shortfall and ruin probability, the profit or expected utility will be exceptionally low. The color scale shows intuitively this conflict with red representing smallest value and green representing largest value. We define the weight vector of criteria as all equals (equal values in blue cells Range F39:I39). Calling the Matlab function built earlier in Section 2 by executing the following code:

```
topsis (decisionMakingMatrix,lambdaWeight,criteriaSign)
```

we could get normalized weight matrix, the identified ideal solutions and the distance between each alternative and the ideal optimality. All these results are stored in Excel sheet "TOPSIS OUTPUT Variables" with selection ranking results shown in Figure 9.

Alternative	Alt1	Alt2	Alt3	Alt4	Alt5	Alt6	Alt7	Alt8	Alt9	Alt10	Alt11	Alt12
D+	0.0569751	0.056478	0.0555945	0.0545868	0.05343051	0.0517104	0.0492624	0.0469969	0.0455135	0.0419515	0.0393043	0.0367563
D-	0.0693894	0.0665287	0.0645277	0.0619595	0.06041675	0.0582922	0.0567964	0.0558353	0.0540705	0.0548093	0.0549429	0.0551283
C	0.5491211	0.5408545	0.5371836	0.5316299	0.53068253	0.5299168	0.5355181	0.5429752	0.5429638	0.5664413	0.5829654	0.5999735
	18	22	23	25	26	27	24	19	20	15	13	11

Alternative	Alt13	Alt14	Alt15	Alt16	Alt17	Alt18	Alt19	Alt20	Alt21	Alt22	Alt23	Alt24
D+	0.0339859	0.0312588	0.0296431	0.0283441	0.0278169	0.0275222	0.0301239	0.034438	0.036536	0.0403173	0.0431329	0.0451989
D-	0.056861	0.0585793	0.0603648	0.0629628	0.0662408	0.0707038	0.069195	0.0668414	0.0657687	0.0639637	0.0627341	0.0618979
C	0.6258991	0.6520543	0.6706616	0.6895734	0.70425705	0.7198073	0.6966954	0.6599706	0.6428709	0.6133785	0.5925748	0.5779624
Rank	9	7	5	4	2	1	3	6	8	10	12	14

Alternative	Alt25	Alt26	Alt27	Alt28	Alt29	Alt30	Alt31	Alt32	Alt33	Alt34	Alt35
D+	0.0471355	0.0491228	0.0506938	0.0551023	0.05658928	0.0574337	0.0605993	0.0624327	0.0650258	0.067907	0.0693894
D-	0.061167	0.0604723	0.0599641	0.0587391	0.05839528	0.058216	0.0576492	0.0573983	0.057142	0.0569944	0.0569751
C	0.5647793	0.5517793	0.541887	0.5159732	0.50785324	0.5033824	0.487526	0.4789934	0.4677338	0.4563152	0.450879
Rank	16	17	21	28	29	30	31	32	33	34	35

Figure 9. Ranking of the alternatives based on TOPSIS

From the TOPSIS output, we could identify that Alternative 18 (0.27,100) is the best choice followed closely by Alternative 17 (0.26,96). Alternative 19 (0.28,103) is not far away as the third best alternative identified. Alternative 35 (0.44, 149) is the furthest from ideal solutions. Multiple trials were tested fixing a at a=0.75 (Trial 2), a=0.9 (Trial 3) and a=1 (Trial 4). Trial 4 resembles pure Excess-of-loss Reinsurance to compare and contrast decision differences under different ceding portion in Proportional-Stop-loss Reinsurance design. Results of these trials are included in Appendix C. From the result, we could draw insightful managerial implications.

4. Managerial Implications

The result shows several interesting findings:

1. The best alternative suggested by TOPSIS not necessarily optimize any one single criterion, rather, it has an overall highest ranking due to its relative weighted closeness to all four criteria. In reality, if reinsurance is chosen merely according to expected profit, the insurance company may suffer from high probability of financial crisis. On the other hand, if the decision merely considers constraining higher shortfalls, insurance company may look bad on their profit and loss statement due to low profit earned.
   By increasing the ceding amount from a=0.6 to a=0.75,0.9 and 1 result from Trial 1,2,3,4 suggests that the ranking of alternatives is different when parameters are changed. When ceding portion are fixed at relatively lower level (such as a=0.6 to a=0.75) the best alternative to choose will have the retention limit equaling to mean value of loss. Thus, if the given reinsurance parameters (either a, θ or M) are altered, it is recommended for insurance company to evaluate again the reinsurance plans instead of extrapolating conclusions from previous experiences.
2. In addition, Trial 4 with a=1 is modeling excess-of-loss reinsurance form where $X_r = MAX(0, X - M) = 1 * (0, X - M)_+$. Accordingly, result from Trial 4 are in correspondence with previous knowledge on excess-of-loss reinsurance. Under excess-of-loss reinsurance, the best form is given at $M = M_{max}$, which is in correspondence with Section 2 in Payandeh-Najafabadi & Panahi-Bazaz (Payandeh-Najafabadi and Panahi-Bazaz 2017).
3. In each trial, the Alternative 1 M = 0 simulates the scenario of pure proportional reinsurance. Trial 5 attempts to model different retention level under proportional reinsurance ($M = X$) with fixed reinsurance premium loading factor θ. The result shows that given same premium loading factor, retention level of 0.6 would be most preferable.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 10 July 2017 doi:10.20944/preprints201707.0015.v1

	A	B	C	D	E	F	G	H	I	J	K
1	Alternatives					ES0.95	Ruin Probability	PROFITi	Expected Utility	Rank	C
2	Alternatives	a	M	zeta	riteri	c1	c2	c3	c4		
3	A1	0.1	0	0.15		4046.645008	0.079521227	88.27786042	0.146856731	7	0.307501
4	A2	0.2	0	0.15		3537.668564	0.074630029	76.30186042	0.145834394	6	0.287943
5	A3	0.3	0	0.15		3093.458715	0.079440396	62.32986042	0.14464012	5	0.2594
6	A4	0.4	0	0.15		2747.712124	0.08434825	46.36186042	0.14327319	3	0.21561
7	A5	0.5	0	0.15		2206.42891	0.08018247	28.39786042	0.141732783	2	0.142364
8	A6	0.6	0	0.15		1767.175006	0.090638388	8.437860424	0.140017971	1	0.004457
9	A7	0.7	0	0.15		1306.695578	0.090656182	-13.51813958	0.13812772	4	0.219885
10	A8	0.8	0	0.15		877.5800343	0.093619193	-37.47013958	0.136060889	8	0.460288
11	A9	0.9	0	0.15		373.9148882	0.105017214	-63.41813958	0.133816229	9	0.721277
12	A10	1	0	0.19		91.36213958	0.108302642	-91.36213958	0.13139238	10	0.988854
38	criteria Sign range	1				-1.00	-1	1	1		
39	W(Lambda)					1.00	1	1	1		

Figure 10. Trial 5 Pure proportional reinsurance selection

4. By setting (a, M) to (0, 0), we could also model the scenario of no reinsurance. The results show that with no reinsurance, the expected shortfall of insurance company will be significantly higher than all other alternatives, and the ruin probability will be higher as well. This suggests that insurance company without reinsurance is more likely to bankrupt if large losses are incurred. As a compensation, the expected profit and utility will increase by a small amount for the insurance company due to high profit from insurance premium and low probability of large losses. However, note the high ruin probability which suggests a much higher risk of bankrupting, the insurance company will often seek for reinsurance to keep ruin probability low.
5. Furthermore, through the simulation process, the variance and profitability of the reinsurer are also being observed and calculated (as could be seen from Figure 10. The result was in correspondence with our previous argument that by scaling the ceding portion a to larger values, both the variance and the profitability of the reinsurer will increase, suggesting that there is a trade-off between high profit and high risk of large losses. Thus, this supports our previous assumption that the reinsurer is ambiguous towards design that only differs in parameter a.

5. Conclusions

5.1 Contributions

The research has the following contributions:

1. To the best of our knowledge, this is the first theoretical study using MADM to approach Proportional-Stop-loss reinsurance model, though there is small amount of recent studies using MADM in designing either pure proportional or pure stop-loss reinsurance contract;
2. It is one of the few studies taking a non-discriminatory position considering both the insurance and the reinsurance company in designing optimal reinsurance contract, and the study made significant contribution by incorporating existing MODM models and promising MADM model in one decision flow to arrive at robust decision for reinsurance design;
3. This study demonstrates the feasibility to incorporate intelligent decision supporting system in reinsurance deal-making. As observed by the author through industry experiences, @Risk has grown its popularity recently for actuarial study in modeling risk and claims. The prototype of TOPSIS implemented through Matlab suggests that a software of multi-criteria decision support would be promising.
4. As previous research suggested (Bazaz and Najafabadi 2015), MADM is not likely to address finding of optimal type of reinsurance. However, with the generic formulation of Proportional-Stop-loss Reinsurance, we would be able to model proportional reinsurance and stop-loss reinsurance as special cases of Proportional-stop-loss, thus the choice between proportional and

non-proportional reinsurance using MADM could be possible under this formulation of reinsurance.

5.2 Limitations

There are still some limitations for this research, specifically in the following aspects:

1. In terms of the scope of study, due to time and resource constraint, the study only considers proportional-stop-loss treaty reinsurance, while basing the decision process on ruin probability, CVaR and expected utility criteria. Other types of reinsurance and decision measurements have not been elaborated and tested upon.
2. In terms of methodology, this study attempts to utilize the simulation software @Risk to model the loss and claim distribution and to use numerical TOPSIS model in modeling decisions from the insurance company, without reaching to a close-form solution. Thus, the conclusions were drawn based on simulation result rather than robust theoretical derivation.
3. In terms of model implementation, due to resource constraint, this study only includes a numerical made-up case instead of existing cases to conduct archival research in addressing the decision process in the reinsurance purchase decisions.

5.3 Future direction

In reality, the trade contracts will usually go through lengthy negotiations with broking firms acting as intermediaries, thus, empirical study with cases from existing broking firms may be more realistic and practical in addressing the usefulness of this decision framework. In addition, behavioral study of both reinsurer and the reinsured would be of great importance to suggest whether they are rationale players in the reinsurance market.

Furthermore, it would be promising for mathematical and quantitative researchers to look into the closed-form optimization for Proportional-Stop-Loss under each single measurement. As pure proportional or stop-loss reinsurance could be regarded as special cases of Proportional-Stop-Loss Reinsurance could, this will reconcile existing mathematical models on either side and help in calculating the precise decision matrix for MADM analysis.

Supplementary Materials: The following are available online at www.mdpi.com/link, Figure S1: title, Table S1: title, Video S1: title.

Acknowledgments: The author is grateful to Associate Professor Poh Kim Leng for his suggestions and guidance throughout the research process.

Conflicts of Interest: The author declares no conflict of interest.

Appendix A. Forms of reinsurance in the market

Table A1. Common forms of reinsurance in the market

QS(Quota Share) Treaty	Under quota share treaty, a reinsurer is bound to share a fixed proportion of every risk accepted by the ceding company. Usually, 20-80 is set as according to the “Pareto Rule”;
Surplus Treaty	Under surplus treaty, the reinsured could decide on which line or risk is to be cede to the treaty. It will also create a fixed proportion after ceding and is accounted quarterly.
Facultative/ Obligatory Treaty	This treaty reinsurance is a mixture between facultative and treaty reinsurance which is automatically arranged but provides the option to cede risk.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 10 July 2017 doi:10.20944/preprints201707.0015.v1

Excess of loss Treaty	The insurer assumes whole responsibility for any loss up to a predetermined level, and the reinsurer agrees to play 100% of the exceeding amount above the line. Usually a ceiling will also be set and insurer has to assume responsibility above the ceiling. An excess of loss treaty is often structured in layers. Each layer will attract reinsurers depending on their underwriting policy, which differs according to their respective risk and return.
Stop loss/Aggregate excess of loss	The stop loss/aggregate excess of loss works similar to excess of loss treaty, but the limits are expressed as percentages known as "loss ratios".
Reinstatement	The amount of cover provided by an excess of loss contract is not usually unlimited but restricted by reinstatement provisions. In the event of great loss and that the amount of cover is used up by the loss, a reinstatement could be purchased to increase more layers This treaty reinsurance is a mixture between facultative and treaty reinsurance which is automatically arranged but provides the option to cede risk.

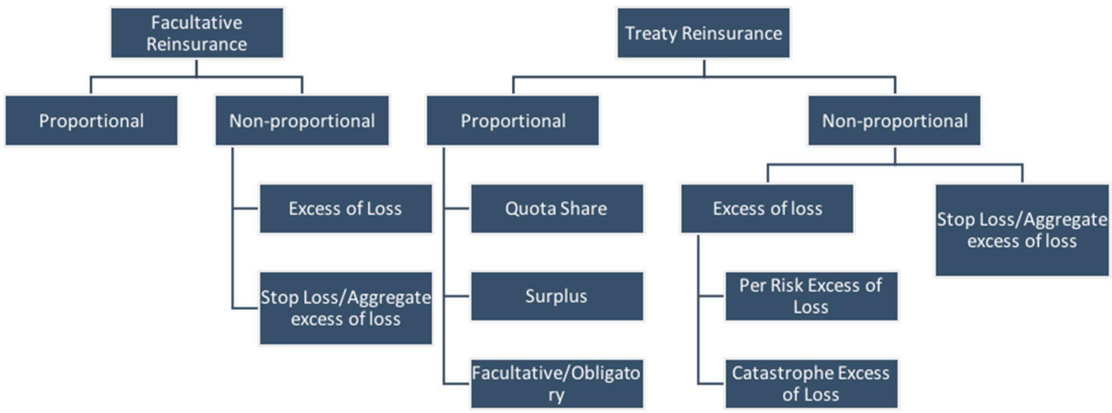

Figure A1. Classification of reinsurance forms

Appendix B. Reviewed Methodologies

B.1 Decision making for underwriters

Core pricing considerations include (WillisRe 2013): (1) market conditions and realities, (2) return on capital, (3) commissions, (4) expenses, (5) level of certainty, (6) probability and severity of loss. The commonly adopted methodologies to approximate prices are experience rating and exposure rating.

B.1.1 Experience rating.

According to Clark (1996), experience rating pricing starts with compiling historical experience on the reinsurance coverage. The pricing authorities then adjust subject premiums to future levels. Historical losses are then adjusted to future levels and loss cost rate is calculated to take account of expenses, commissions, return on capital and market conditions

B.1.2 Exposure rating.

Exposure rating, however, rely more on current exposure in predicting future losses. This technique relates premium to likely losses. To achieve this, underwriter needs to set up risk profiles of the portfolio in bands. A "first loss curve" (WillisRe 2013) could be used to estimate the split of premium required between the deductible and cover of any insured or reinsured risk. The first loss curves are tailored for each portfolio under examination and assumes a common form of Gamma, Lognormal, etc.

B.1.3 Pareto model.

According to WillisRe (2013), Pareto model is the most widely used model in reinsurance and could be used in conjunction with experience and exposure rating to estimate risk premiums for excess of loss treaties with high deductibles, where loss experience is insufficient and could be sometimes misleading. However, some criticize the classical Pareto model of its deficiency in modelling loss frequency (Fackler n.d.). The classic Pareto model either fits the small-to-medium risk or large risk well but not both at the same time. Fackler (Fackler n.d.) discusses the generalization and extension of Pareto Model to improve its fitness of loss distribution.

B.1.4 Stochastic pricing

In traditional actuarial pricing models, models used to approximate expected losses from specific occurrences are usually deterministic based on specific loss occurrences (WillisRe 2013). Recent research has been approaching the model using stochastic (probabilistic) approach taking account of the probability attached to various outcomes. Stochastic modeling is considered to sit in-between experience and exposure methods in that assumptions could depend on either loss history or current exposure or a combination of both. Stochastic models generally provide a better approximation than simple deterministic calculations.

B.2 Decision making for ceding companies

Insurance company usually choose to purchase appropriate reinsurance in the hope of achieving: (1) Increased diversity and financial flexibility in balance sheet; (2) Increased sustainability of operating performance, especially in accident years; and (3) Diversified business profile in enterprise risk management. For an insurer to choose the most appropriate reinsurance design regarding type (as in Appendix A) and parameters, quantitative theories have been developed and applied to aid the decision making process.

B.2.1 Operations research.

Since the seminal work of Borch (1960) (Borck 1960), optimal insurance/reinsurance problems have been studied extensively (Cheung et al. 2014). Recently, because of the promising risk measures such as value at risk (VaR) and conditional value at risk (CVaR) in quantifying financial and insurance risks, the study of risk measure based optimal reinsurance problems has attracted great attention, such as (Cai et al. 2008). Not only stochastic models based on current situation was discussed (Zeng and Luo 2013a), dynamic programming has also been utilized to model multi-period optimization in reinsurance, such as (Chunxiang and Li 2015) and (Guan and Liang 2014).

B.2.2 Utility theory and decision modelling.

Friefelder, as cited in Samson and Thomas (1983), suggests that mean-variance approach which most operations research models were based have obvious weakness and that utility theory is the best method in determining property and liability insurance rates. Samson and Thomas (1983) (Samson and Thomas 1983) first approaches the topic of decision making in reinsurance using utility theory, formulation of which suggests that the choice of an appropriate utility function is important to decision making process. They later on suggest using decision analysis tools such as decision tree

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 10 July 2017 doi:10.20944/preprints201707.0015.v1

structure in modelling ceding company's buying quantity of excess-of-loss treaty reinsurance (Samson & Thomas, 1985). A rough reproduction of the simulation model using DPL software is shown in Figure B1. This model could be further improved using continuous distribution in modelling loss profile. This model illustrates the feasibility of using decision analysis in choosing whether to buy Excess-of-loss reinsurance and the quantity of reinstatement to buy.

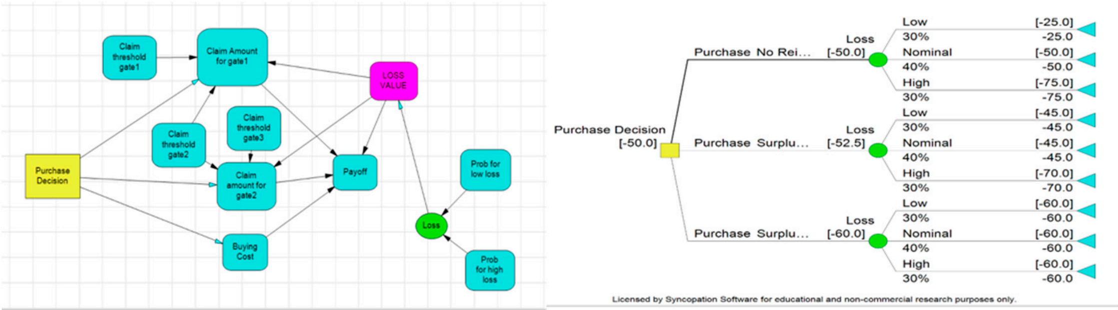

Figure B1. Screenshot of excess of loss decision model using DPL

B.2.3 Options theory.

Smith and Nau (1995) discussed thoroughly how option pricing techniques could be used to simplify investment decision making when risk-hedging options are available. Given that reinsurance itself is a risky project, Brotman (1987) suggests that reinsurance treaty arrangement and futures market share similar characteristics and that hedging strategies using options to buy or sell reinsurance could meet special need of insurance companies.

B.2.4 Multi-attribute decision making.

All the reinsurance optimization models in previous sections mostly aid in decision making using single criteria. However, Basak and David (Bulut Karageyik and Dickson 2016) provides new insights to approach optimal retention level in proportional quota share reinsurance using multi attribute decision making (MADM). In their discussion, "released capital, expected profit and expected utility of resulting wealth" are the competing criteria of choice, and compare the results with that of single attribute decision making. Following this study, using MADM in determining optimal retention level for proportional reinsurance has been extensively evaluated by Basak and Sule (Bulut Karageyik and Şahin 2017). The authors use measures of Expected Shortfall, expected profit and ruin probability taking the perspective of reinsurance buyer, and they conclude that as relationship between criteria is too small, there is no significant difference in results using different MADM techniques such as between TOPSIS-Euclidean and TOPSIS-Mahalanobis.

B.3 Optimal reinsurance considering both parties

Recently, intriguing models started to address the promising and practical issue of reaching a "win-win" policy, such as (Hürlimann 2011) and (Bazaz and Najafabadi 2015) taking views from both cedent and reinsurer. and applying Conditional tail expectation (cVaR, CTE) in deciding optimal stop-loss limits that maximizes joint party profit while controlling joint party risks. Dimitrova & Kaishev (Dimitrova and Kaishev 2010) based their analysis on maximizing joint survival probability to get the efficient frontier reinsurance design. Other mathematical models such as game theory could also be applied to reinsurance negotiation as demonstrated by Zeng and Luo (2013) in seeking for a Pareto Optimal proportional reinsurance policy. A recent research (Payandeh-Najafabadi and Panahi-Bazaz 2017) looks into the joint party optimal reinsurance design with combination of proportional and stop-loss reinsurance contract. By using Bayesian estimator, the research seeks to achieve maximum ending wealth for both party. We could see that under this category, almost all existing studies are using at most two decision criteria.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 10 July 2017 doi:10.20944/preprints201707.0015.v1

Appendix C. TOPSIS Trials #2, #3 and#4

	A	B	C	D	E	F	G	H	I
1	Alternatives					ES0.95	Ruin Probability	PROFITi	Expected Utility
2	Alternatives	a	M	zeta	riteri	c1	c2	c3	c4
3	A1		0	0.1		834.7716436	0.087312252	23.40786042	0.141304401
4	A2		10	0.11		855.1880247	0.086452339	24.15636042	0.141368672
5	A3		19	0.12		866.495922	0.086887538	25.50366042	0.141484347
6	A4		27	0.13		886.6509352	0.089444258	27.22521042	0.141632132
7	A5		34	0.14		896.0279323	0.089566722	29.09646042	0.141792739
8	A6		41	0.15		913.9902293	0.089115785	32.01561042	0.142043226
9	A7		48	0.16		926.7813101	0.089900436	35.98266042	0.142383515
10	A8		54	0.17		936.2594183	0.090491325	39.72516042	0.142704417
11	A9		59	0.18		958.7374075	0.092109158	43.01856042	0.142986713
12	A10		65	0.19		955.9915499	0.091589813	48.48261042	0.143454861
13	A11		70	0.2		964.7269333	0.091607518	53.34786042	0.14387149
14	A12		75	0.21		980.5075116	0.092231426	58.96161042	0.144351965
15	A13		79	0.22		975.2753291	0.092097565	63.67716042	0.144755355
16	A14		84	0.23		987.3092011	0.092074192	70.71306042	0.145356885
17	A15		88	0.24		998.7799679	0.092530559	76.70106042	0.145868492
18	A16		92	0.25		1007.179146	0.09389913	83.28786042	0.146430906
19	A17		96	0.26		1014.968778	0.093290762	90.47346042	0.147044026
20	A18	**0.75**	100	0.27		1011.69034	0.09339054	98.25786042	0.147707743
21	A19		103	0.28		1043.980452	0.094080032	98.25786042	0.147707743
22	A20		107	0.29		1097.52394	0.095797522	98.25786042	0.147707743
23	A21		110	0.3		1123.563791	0.094323856	98.25786042	0.147707743
24	A22		114	0.31		1170.495524	0.093745244	98.25786042	0.147707743
25	A23		117	0.32		1205.441455	0.094158638	98.25786042	0.147707743
26	A24		120	0.33		1231.082995	0.094099739	98.25786042	0.147707743
27	A25		123	0.34		1255.11953	0.09403522	98.25786042	0.147707743
28	A26		126	0.35		1279.785066	0.093324499	98.25786042	0.147707743
29	A27		129	0.36		1299.283714	0.092930586	98.25786042	0.147707743
30	A28		131	0.37		1353.999274	0.094426982	98.25786042	0.147707743
31	A29		134	0.38		1372.454608	0.094210057	98.25786042	0.147707743
32	A30		137	0.39		1382.934637	0.094673436	98.25786042	0.147707743
33	A31		139	0.4		1422.224375	0.09415124	98.25786042	0.147707743
34	A32		142	0.41		1444.980799	0.092483131	98.25786042	0.147707743
35	A33		144	0.42		1477.163227	0.095391474	98.25786042	0.147707743
36	A34		146	0.43		1512.923601	0.094778562	98.25786042	0.147707743
37	A35		149	0.44		1531.323057	0.093241787	98.25786042	0.147707743
38	criteria Sign range	1				-1.00	-1	1	1
39	W(Lambda)					1.00	1	0.5	0.5
40									

Figure C1. Trial 2 Screen-shot of crucial TOPSIS output parameters

Alternative	Alt1	Alt2	Alt3	Alt4	Alt5	Alt6	Alt7	Alt8	Alt9	Alt10	Alt11	Alt12
D+	0.1320813	0.1307614	0.1283854	0.1253516	0.12205232	0.116908	0.1099147	0.1033174	0.0975231	0.0878841	0.0793125	0.0694342
D-	0.0170302	0.0165837	0.01667	0.0171408	0.01849387	0.0214132	0.0266617	0.0322606	0.0373297	0.0464293	0.0546185	0.0641677
C	0.1142111	0.1125501	0.1149215	0.120293	0.13158572	0.1548078	0.1952147	0.2379488	0.2768181	0.345679	0.4078107	0.4802904
	34	35	33	32	31	30	29	28	27	26	25	24

Alternative	Alt13	Alt14	Alt15	Alt16	Alt17	Alt18	Alt19	Alt20	Alt21	Alt22	Alt23	Alt24
D+	0.0611182	0.0487488	0.0382502	0.0267505	0.01442573	0.0043257	0.0051152	0.0064243	0.0070609	0.0082083	0.0090627	0.0096896
D-	0.0723485	0.0845284	0.094939	0.1064393	0.1190163	0.1326909	0.1326177	0.1325065	0.132457	0.1323756	0.1323214	0.1322851
C	0.5420717	0.63423	0.712813	0.7991551	0.89189515	0.9684294	0.9628616	0.953759	0.9493909	0.9416129	0.9359003	0.9317513
Rank	23	22	21	20	17	1	2	3	4	5	6	7

Alternative	Alt25	Alt26	Alt27	Alt28	Alt29	Alt30	Alt31	Alt32	Alt33	Alt34	Alt35
D+	0.0102773	0.0108803	0.011357	0.0126948	0.01314599	0.0134022	0.0143628	0.0149192	0.015706	0.0165803	0.0170302
D-	0.1322538	0.1322244	0.1322031	0.1321524	0.13213841	0.1321311	0.1321082	0.1320982	0.1320879	0.1320821	0.1320813
C	0.9278946	0.9239697	0.9208903	0.9123575	0.9095155	0.9079096	0.901941	0.8985211	0.8937302	0.8884699	0.8857891
Rank	8	9	10	11	12	13	14	15	16	18	19

Figure C2. Trial 2 Screen-shot of crucial TOPSIS output parameters

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 10 July 2017 doi:10.20944/preprints201707.0015.v1

	A	B	C	D	E	F	G	H	I
1	Alternatives					ES0.95	Ruin Probability	PROFITi	Expected Utility
2	Alternatives	a	M	zeta	Criteri	c1	c2	c3	c4
3	A1		0	0.1		988.2699744	0.085088195	8.437860424	0.140017971
4	A2		10	0.11		999.1228241	0.088673751	9.336060424	0.140095211
5	A3		19	0.12		1002.263826	0.088121149	10.95282042	0.140234225
6	A4		27	0.13		992.671412	0.0883292	13.01868042	0.140411823
7	A5		34	0.14		996.8261654	0.089295076	15.26418042	0.140604822
8	A6		41	0.15		969.8431979	0.089787781	18.76716042	0.140905813
9	A7		48	0.16		1018.170562	0.092399405	23.52762042	0.141314684
10	A8		54	0.17		970.0579878	0.089894564	28.01862042	0.141700233
11	A9		59	0.18		1003.176245	0.089720096	31.97070042	0.142039373
12	A10		65	0.19		990.9022899	0.089280019	38.52756042	0.142601742
13	A11		70	0.2		1042.748026	0.089877328	44.36586042	0.14310217
14	A12		75	0.21		947.1232568	0.08922491	51.10236042	0.143679225
15	A13		79	0.22		979.2784188	0.089342529	56.76102042	0.144163651
16	A14		84	0.23		999.4414069	0.093276822	65.20410042	0.144885935
17	A15		88	0.24		1001.186805	0.096410473	72.38970042	0.145500165
18	A16		92	0.25		1006.397288	0.094699571	80.29386042	0.146175309
19	A17		96	0.26		971.5311893	0.090507333	88.91658042	0.146911221
20	A18	0.9	100	0.27		1036.796887	0.091251777	98.25786042	0.147707743
21	A19		103	0.28		1025.100632	0.09641785	98.25786042	0.147707743
22	A20		107	0.29		1092.039887	0.096444365	98.25786042	0.147707743
23	A21		110	0.3		1095.142832	0.096424704	98.25786042	0.147707743
24	A22		114	0.31		1173.781477	0.098787514	98.25786042	0.147707743
25	A23		117	0.32		1214.496515	0.098542238	98.25786042	0.147707743
26	A24		120	0.33		1208.626958	0.098701899	98.25786042	0.147707743
27	A25		123	0.34		1215.531859	0.09889659	98.25786042	0.147707743
28	A26		126	0.35		1273.365626	0.094173914	98.25786042	0.147707743
29	A27		129	0.36		1281.506042	0.090339338	98.25786042	0.147707743
30	A28		131	0.37		1327.804543	0.095786712	98.25786042	0.147707743
31	A29		134	0.38		1383.018954	0.099597592	98.25786042	0.147707743
32	A30		137	0.39		1394.675135	0.092995609	98.25786042	0.147707743
33	A31		139	0.4		1362.567173	0.09055014	98.25786042	0.147707743
34	A32		142	0.41		1452.76873	0.094773467	98.25786042	0.147707743
35	A33		144	0.42		1421.889442	0.091291142	98.25786042	0.147707743
36	A34		146	0.43		1431.78405	0.090361098	98.25786042	0.147707743
37	A35		149	0.44		1543.429196	0.09393016	98.25786042	0.147707743
38	criteria Sign range	1				-1.00	-1	1	1
39	W(Lambda)					1.00	1	1	1

Figure C3. Trial 3 Screen-shot of crucial TOPSIS output parameters

Alternative	Alt1	Alt2	Alt3	Alt4	Alt5	Alt6	Alt7	Alt8	Alt9	Alt10	Alt11	Alt12
D+	0.1811699	0.1793585	0.1760976	0.1719304	0.16740133	0.1603351	0.1507347	0.1416747	0.1337043	0.1204785	0.108706	0.095114
D-	0.0056236	0.0058032	0.0074685	0.0107931	0.01484024	0.0216293	0.0308979	0.0399196	0.0477808	0.0609492	0.072645	0.0862672
C	0.0301062	0.0313411	0.0406857	0.0590679	0.08143167	0.1188654	0.1701119	0.2198285	0.2632766	0.3359422	0.4005768	0.4756127
Rank	35	34	33	32	31	30	29	28	27	26	25	24

Alternative	Alt13	Alt14	Alt15	Alt16	Alt17	Alt18	Alt19	Alt20	Alt21	Alt22	Alt23	Alt24
D+	0.0837009	0.0666725	0.0521798	0.036239	0.0188433	0.0009097	0.0007959	0.0014709	0.0015023	0.0022985	0.0027103	0.002651
D-	0.0976365	0.1146316	0.1291095	0.1450376	0.16243114	0.1812421	0.1812455	0.1812271	0.1812263	0.1812081	0.1812001	0.1812012
C	0.5384244	0.6322614	0.712174	0.8000902	0.89605096	0.9950056	0.9956281	0.9919487	0.9917786	0.9874744	0.985263	0.985581
Rank	23	22	21	20	19	2	1	3	4	5	7	6

Alternative	Alt25	Alt26	Alt27	Alt28	Alt29	Alt30	Alt31	Alt32	Alt33	Alt34	Alt35
D+	0.0027209	0.0033049	0.0033867	0.0038564	0.00441623	0.004533	0.0042076	0.0051215	0.0048084	0.0049085	0.0060394
D-	0.1811999	0.1811901	0.1811889	0.1811826	0.18117673	0.1811757	0.1811787	0.1811718	0.1811736	0.181173	0.1811695
C	0.9852061	0.9820869	0.9816515	0.9791591	0.97620477	0.9755909	0.9773038	0.9725085	0.9741461	0.9736219	0.9677397
Rank	8	9	10	11	13	14	12	17	15	16	18

Figure C4. Trial 3 Screen-shot of crucial TOPSIS output parameters

 doi:10.20944/preprints201707.0015.v1

	A	B	C	D	E	F	G	H	I
1	Alternatives					ES0.95	Ruin Probability	PROFITi	Expected Utility
2	Alternatives	a	M	zeta	riteri	c1	c2	c3	c4
3	A1		0	0.1		1098.049974	0.085088195	-1.542139576	0.13915928
4	A2		10	0.11		1098.823024	0.088673751	-0.544139576	0.139245188
5	A3		19	0.12		1092.802386	0.088121149	1.252260424	0.1393998
6	A4		27	0.13		1074.996432	0.0883292	3.547660424	0.139597319
7	A5		34	0.14		1071.915685	0.089295076	6.042660424	0.139811963
8	A6		41	0.15		1037.557498	0.089787781	9.934860424	0.1401467
9	A7		48	0.16		1078.369922	0.092399405	15.22426042	0.140601391
10	A8		54	0.17		1023.770348	0.089894564	20.21426042	0.141030124
11	A9		59	0.18		1051.459485	0.097200962	24.60546042	0.141407232
12	A10		65	0.19		1032.46899	0.085280019	31.89086042	0.142032523
13	A11		70	0.2		1078.676026	0.089877328	38.37786042	0.142588906
14	A12		75	0.21		977.3127568	0.08722491	45.86286042	0.143230438
15	A13		79	0.22		1004.847179	0.085934253	52.15026042	0.143768954
16	A14		84	0.23		1019.082047	0.093276822	61.53146042	0.144571825
17	A15		88	0.24		1016.037045	0.096410473	69.51546042	0.145254526
18	A16		92	0.25		1016.377288	0.094699571	78.29786042	0.146004869
19	A17		96	0.26		976.5611093	0.088507333	87.87866042	0.146822672
20	A18	1	100	0.27		1036.796887	0.091251777	98.25786042	0.147707743
21	A19		103	0.28		1025.100632	0.08641785	98.25786042	0.147707743
22	A20		107	0.29		1092.039887	0.096444365	98.25786042	0.147707743
23	A21		110	0.3		1095.142832	0.096424704	98.25786042	0.147707743
24	A22		114	0.31		1173.781477	0.098787514	98.25786042	0.147707743
25	A23		117	0.32		1214.496515	0.098542238	98.25786042	0.147707743
26	A24		120	0.33		1208.626958	0.088701899	98.25786042	0.147707743
27	A25		123	0.34		1215.531859	0.09889659	98.25786042	0.147707743
28	A26		126	0.35		1273.365626	0.094173914	98.25786042	0.147707743
29	A27		129	0.36		1281.506042	0.090339338	98.25786042	0.147707743
30	A28		131	0.37		1327.804543	0.095786712	98.25786042	0.147707743
31	A29		134	0.38		1383.018954	0.099597592	98.25786042	0.147707743
32	A30		137	0.39		1394.675135	0.092995609	98.25786042	0.147707743
33	A31		139	0.4		1362.567173	0.09055014	98.25786042	0.147707743
34	A32		142	0.41		1452.76873	0.094773467	98.25786042	0.147707743
35	A33		144	0.42		1421.889442	0.091291142	98.25786042	0.147707743
36	A34		146	0.43		1431.78405	0.090361098	98.25786042	0.147707743
37	A35		149	0.44		1543.429196	0.09393016	98.25786042	0.147707743
38	criteria Sign range	1				-1.00	-1	1	1
39	W(Lambda)					1.00	1	1	1

Figure C5. Trial 4 Screen-shot of crucial TOPSIS output parameters

Alternative	Alt1	Alt2	Alt3	Alt4	Alt5	Alt6	Alt7	Alt8	Alt9	Alt10	Alt11	Alt12
D+	0.2122375	0.2101156	0.2062963	0.2014154	0.1961105	0.1878342	0.1765874	0.1659774	0.1566397	0.1411504	0.1273582	0.1114441
D-	0.0021764	0.0004588	0.0038411	0.0087105	0.01401154	0.0222867	0.0335335	0.0441442	0.0534823	0.0689743	0.0827689	0.0986859
C	0.0101505	0.0021788	0.0182791	0.0414537	0.06668285	0.106066	0.1595914	0.210089	0.2545299	0.3282541	0.3938992	0.4696421
	34	35	33	32	31	30	29	28	27	26	25	24
Alternative	Alt13	Alt14	Alt15	Alt16	Alt17	Alt18	Alt19	Alt20	Alt21	Alt22	Alt23	Alt24
D+	0.0980792	0.078138	0.0611706	0.0425172	0.02221186	0.0025705	0.0024108	0.0021459	0.0020171	0.0017849	0.0016121	0.0014852
D-	0.1120561	0.1320054	0.1489836	0.1676596	0.18803338	0.210105	0.2101051	0.2101054	0.2101057	0.2101065	0.2101072	0.2101078
C	0.5332568	0.6281682	0.7089253	0.7977074	0.89435261	0.9879134	0.988656	0.9898898	0.9904909	0.9915761	0.9923858	0.9929807
	23	22	21	20	19	18	17	16	15	14	13	12
Alternative	Alt25	Alt26	Alt27	Alt28	Alt29	Alt30	Alt31	Alt32	Alt33	Alt34	Alt35	
D+	0.0013663	0.0012443	0.0011479	0.0008772	0.0007859	0.0007341	0.0005397	0.0004272	0.0002679	9.105E-05	6.183E-06	
D-	0.2101085	0.2101092	0.2101098	0.2101118	0.21011258	0.210113	0.2101148	0.2101159	0.2101176	0.2101196	0.2101207	
C	0.9935391	0.9941126	0.9945665	0.9958425	0.99627358	0.9965186	0.997438	0.997971	0.9987265	0.9995669	0.9999706	
	11	10	9	8	7	6	5	4	3	2	1	

Figure C6. Trial 4 Screen-shot of crucial TOPSIS output parameters

References

1. Ameri Sianaki, Omid. 2015. "Intelligent Decision Support System for Energy Management in Demand Response Programs and Residential and Industrial Sectors of the Smart Grid." PhD thesis, Curtin University.
2. Bazaz, Ali Panahi, and Amir T Payandeh Najafabadi. 2015. "An Optimal Reinsurance Contract from Insurer's and Reinsurer's Viewpoints." *Applications & Applied Mathematics* 10 (2).

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 10 July 2017 doi:10.20944/preprints201707.0015.v1

3. Borch, Karl. 1960. "Reciprocal Reinsurance Treaties Seen as a Two-Person Co-Operative Game." *Scandinavian Actuarial Journal* 1960 (1-2). Taylor & Francis: 29–58.
4. Borck, Karl. 1960. *An Attempt to Determine the Optimum Amount of Stop Loss Reinsurance.*
5. Bulut Karageyik, Başak, and David C.M. Dickson. 2016. "Optimal reinsurance under multiple attribute decision making." *Annals of Actuarial Science* 10 (01): 65–86. doi:10.1017/S1748499515000123.
6. Bulut Karageyik, Başak, and Şule Şahin. 2017. "Determination of the Optimal Retention Level Based on Different Measures." *Journal of Risk and Financial Management* 10 (1). Multidisciplinary Digital Publishing Institute: 4.
7. Cai, Jun, Ken Seng Tan, Chengguo Weng, and Yi Zhang. 2008. "Optimal Reinsurance Under Var and Cte Risk Measures." *Insurance: Mathematics and Economics* 43 (1). Elsevier: 185–96.
8. Carter, R. L. 1979. *Reinsurance*. Brentford [Eng.]: Kluwer Pub. in association with the Mercantile; General Reinsurance Co.
9. Chauhan, Aditya, and Rahul Vaish. 2013. "Fluid Selection of Organic Rankine Cycle Using Decision Making Approach." *Journal of Computational Engineering* 2013. Hindawi Publishing Corporation.
10. Cheung, K. C., K. C. J. Sung, S. C. P. Yam, and S. P. Yung. 2014. "Optimal Reinsurance Under General Law-Invariant Risk Measures." *Scandinavian Actuarial Journal* 2014 (1): 72–91.
11. Chunxiang, A., and Zhongfei Li. 2015. "Optimal Investment and Excess-of-Loss Reinsurance Problem with Delay for an Insurer Under Heston's Sv Model." *Insurance, Mathematics & Economics* 61: 181.
12. Clark, D.R. 1996. "Basics of reinsurance pricing." *CAS Study Note*, 41–43.
13. Dimitrova, Dimitrina S., and Vladimir K. Kaishev. 2010. "Optimal joint survival reinsurance: An efficient frontier approach." *Insurance: Mathematics and Economics* 47 (1): 27–35. doi:10.1016/j.insmatheco.2010.03.006.
14. Fackler, M. n.d. "Reinventing Pareto: Fits for both small and large losse." http://www.actuaries.org/ASTIN/Colloquia/Hague/Papers/Fackler.pdf .
15. Guan, Guohui, and Zongxia Liang. 2014. "Optimal Reinsurance and Investment Strategies for Insurer Under Interest Rate and Inflation Risks." *Insurance, Mathematics & Economics* 55: 105–15.
16. Hürlimann, Werner. 2011. "Optimal Reinsurance Revisited–point of View of Cedent and Reinsurer." *Astin Bulletin* 41 (02). Cambridge Univ Press: 547–74.
17. Liang, Haiming. 2014. "Research on Methods for Two-Sided Trade Matching Decision-Making Based on the Broker." PhD thesis, Northeastern University.
18. Payandeh-Najafabadi, Amir T, and Ali Panahi-Bazaz. 2017. "An Optimal Combination of Proportional and Stop-Loss Reinsurance Contracts from Insurer's and Reinsurer's Viewpoints." *arXiv Preprint arXiv:1701.05450*.
19. Samson, Danny, and Howard Thomas. 1983. "Reinsurance Decision Making and Expected Utility." *The Journal of Risk and Insurance* 50 (2): 249–64. http://www.jstor.org/stable/252352.
20. ———. 1985. "Decision analysis models in reinsurance." *European Journal of Operational Research* 19 (2): 201–11. doi:10.1016/0377-2217(85)90173-0.
21. Smith, James E., and Robert F. Nau. 1995. "Valuing Risky Projects: Option Pricing Theory and Decision Analysis." http://pubsonline.informs.org/doi/abs/10.1287/mnsc.41.5.795.
22. Vajda, Stefan. 1962. "Minimum Variance Reinsurance." *Astin Bulletin* 2 (02). Cambridge Univ Press: 257–60.
23. Wang, Gang. 2003. "Reinsurance Optimization Models [d]." PhD thesis, Hunan University.
24. WillisRe. 2013. "APTP13 - Understanding Reinsurance."
25. Zeng, Xudong, and Shangzhen Luo. 2013a. "Stochastic Pareto-optimal reinsurance policies." *Insurance: Mathematics and Economics* 53 (3): 671–77. doi:10.1016/j.insmatheco.2013.09.006.
26. ———. 2013b. "Stochastic Pareto-optimal reinsurance policies." *Insurance: Mathematics and Economics* 53 (3): 671–77. doi:10.1016/j.insmatheco.2013.09.006.