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Peer-reviewed version available at *Symmetry* **2017**, *9*, 124; doi:10.3390/sym9070124

Article

Forecasting Based on High-Order Fuzzy-Fluctuation Trends and Particle Swarm Optimization Machine Learning

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Abstract: Most of existing fuzzy forecasting models partition historical training time series into fuzzy time series and build fuzzy-trend logical relationship groups to generate forecasting rules. The determination process of intervals is complex and uncertainty. In this paper, we present a novel fuzzy forecasting model based on high-order fuzzy-fluctuation trends and the fuzzy-fluctuation logical relationships of the training time series. Firstly, we compare each data with the data of its previous day in historical training time series to generate a new fluctuation trend time series(FTTS). Then, fuzzify the FTTS into fuzzy-fluctuation time series(FFTS) according to the up, equal or down range and orientation of the fluctuations. Since the relationship between historical FFTS and the fluctuation trend of future is nonlinear, Particle Swarm Optimization (PSO) algorithm is employed to estimate the required parameters. Finally, use the acquired parameters to forecast the future fluctuations. In order to compare the performance of the proposed model with that of the other models, we apply the proposed method to forecast the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) time series datasets. The experimental results and the comparison results show that the proposed method can be successfully applied in stock market forecasting or such kinds of time series. We also apply the proposed method to forecast Shanghai Stock Exchange Composite Index (SHSECI) to verify its effectiveness and universality.

Keywords: Fuzzy forecasting, fuzzy-fluctuation trend, particle swarm optimization, fuzzy time series, fuzzy logical relationship

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1. Introduction

In stock market, it is well known that historic time series imply the fluctuation rules and can be used to forecast the future of its fluctuation trends. In 1993, Song and Chissom proposed the fuzzy time series forecasting model [25-27]. Since then, researchers have proposed various fuzzy time series forecasting models and employed them to predict stock market[3, 5-6, 21], electricity load demand[13, 22], project cost[11], and the enrollment at Alabama University[14, 24], etc. In order to improve the accuracy of the forecasting model, some researchers combine fuzzy and non-fuzzy time series heuristic optimization methods for stock market forecasting [1, 19-20, 30].

Most of these fuzzy time series models follow the basic steps as Chen(1996) proposed[4]:

Step 1: Define the universe U and the number and length of the intervals.

Step 2: Fuzzify the historical training time series into fuzzy time series.

Step 3: Establish fuzzy logical relationships(FLR) according to the historical fuzzy time series and generate forecasting rules based on fuzzy logical groups(FLG).

Step4: Calculate the forecast values according to the FLG rules and the right-hand side(RHS) of the forecasted point.

Concerning the determination of suitable intervals, various proposals provide different methods, e.g. same

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length method [4], unequal length method [28], distribution and average-based length method [16], GEM-based partitioning method [30], etc. Some authors even employ PSO techniques to determine the length of the intervals [6]. In fact, addition to the determination of intervals, the definition of the universe of discourse also has an effect on the accuracy of forecasting results. In these models, min data value, max data value and two suitable positive numbers must be determined to make a proper bound of the universe of discourse.

Concerning the establishment of fuzzy logical relationships, in order to reflect the recurrence and the weights of different FLR in the forecasting rules, Yu(2005) used chronologically-determined weight in the defuzzification process[29]. Cheng et al. (2008) used the frequencies of different right-hand sides (RHS) of FLG rules to determine the weight of each LHS[9]. Many other researchers proposed different defuzzification method based on Cheng's method [5-8, 13].

In this paper, we present a novel method to forecast the fluctuation of stock market. Unlike existing models, the proposed model is based on the fluctuation values instead of the exact values of the time series. Firstly, we calculate the fluctuation for each data by comparing it with the data of its previous day in historical training time series to generate a new fluctuation trend time series(FTTS). Then, fuzzify the FTTS into fuzzy-fluctuation time series(FFTS) according to the up, equal or down range of each fluctuation data value. Since the relationship between historical FFTS and future fluctuation trends is nonlinear, Particle Swarm Optimization (PSO) algorithm is employed to estimate the required parameters. At last, use these acquired parameters to forecast future fluctuations.

The remaining content of this paper is organized as follows: Section 2 introduces some preliminaries of fuzzy-fluctuation time series based on Song and Chissom's fuzzy time series [25-27]. Section 4 introduces the process of PSO machine learning method. Section 4 describes a novel approach for forecasting based on high-order fuzzy-fluctuation trends and PSO heuristic learning process. In Section 5, the proposed model is used to forecast the stock market using TAIEX datasets from 1997 to 2005 and SHSECI from 2007 to 2015. Conclusions and potential issues for future research are summarized in Section 6.

2. Preliminaries

Song and Chissom[25-27] combined fuzzy set theory with time series and presented the following definitions of fuzzy time series. In this section, we will extend fuzzy time series to fuzzy-fluctuation time series (FFTS) and propose the related concepts.

Definition 1. Let $L=\{l_1,l_2,...,l_g\}$ be a fuzzy set in the universe of discourse U, it can be defined by its membership function, $\mu_L:U\rightarrow[0,1]$, where $\mu_L(u_i)$ denotes the grade of membership of u_i, $U=\{u_1,u_2,...u_i,...,u_l\}$.

The fluctuation trends of a stock market can be expressed by a linguistic set $L=\{l_1,l_2,...,l_g\}$, e.g., let g=3, $L=\{l_1,l_2,l_3\}$ ={*down, equal, up*}. The element l_i and its subscript i is strictly monotonically increasing [15], so the function can be defined as follows: $f:l_i=f(i)$. To preserve all of the given information, the discrete $L=\{l_1,l_2,...,l_g\}$ also can be extended to a continuous label $\overline{L}=\{l_a\,|\,a\in R\}$, which satisfies the above characteristics.

Definition 2. Let $X(t)(t=1,2,...,T)$ be a time series of real numbers, where T is the number of the time series. $Y(t)$ is defined as a fluctuation time series, where $Y(t)=X(t)-X(t-1),(t=2,3,...,T)$. Each element of $Y(t)$ can be represented by a fuzzy set $S(t)(t=2,3,...,T)$ as defined in Def 1. Then we called time series $Y(t)$ is fuzzified into a fuzzy-fluctuation time series (FFTS) $S(t)$.

Definition 3. Let $S(t)\ (t=2,3,...,T)$ be a FFTS. If $S(t)$ is determined by $S(t-1),S(t-2),...,S(t-n)$,then the fuzzy-fluctuation logical relationship is represented by

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$$S(t) \leftarrow S(t-1), S(t-2), \ldots, S(t-n) \quad (1)$$

and it is called the nth-order fuzzy-fluctuation logical relationship (FFLR) of the fuzzy-fluctuation time series, where $S(t)$ is called the left-hand side(LHS) and $S(t-n),\ldots,S(t-2)S(t-1)$ is called the right-hand side(RHS) of the FFLR. This model can be considered as an equivalent of Auto Regressive model of AR(n) defined in eq. (2).

$$\bar{S}(t) = \phi_1 S(t-1) + \phi_2 S(t-2) +,\ldots,+ \phi_n S(t-n) + \varepsilon_t \quad (2)$$

where $\phi_k (k = 1,2,\ldots,n)$ represented the portion of $S(t-k)$ for calculating the forecast is ϕ_k, ε_t is the calculate error, $\bar{S}(t)$ is introduced to preserve more information, as described in Def 1.

3. Pso-based machine learning method

In this paper, particle swarm optimization (PSO) is employed to estimate parameters in Eq.(2). PSO method was introduced as an optimization method for continuous nonlinear functions [18]. It is a stochastic optimization technique, which is similar to social models such as birds flocking or fish schooling. During the optimization process, particles are distributed randomly in the design space and their location and velocities are modified according to their personal best and global best solutions. Let $m+1$ represents the current time step, $x_{i,m+1}$, $v_{i,m+1}$, $x_{i,m}$, $v_{i,m}$ indicate the current position, current velocity, previous position and previous velocity of particle i, respectively. The position and velocity of particle i are manipulated according to the following equations:

$$x_{i,m+1} = x_{i,m} + v_{i,m+1} \quad (3)$$

$$v_{i,m+1} = w \times v_{i,m} + c_1 \times Rand() \times (p_{i,m} - x_{i,m}) + c_2 \times Rand() \times (p_{g,m} - x_{i,m}) \quad (4)$$

where w is an inertia weight which determines how much the previous velocity is preserved[23], c_1 and c_2 are the self-confidence coefficient and social confidence coefficient, respectively, $rand() \in [0,1]$ is a random number, $p_{i,m}$ and $p_{g,m}$ are the personal best position found by particle i and global best position found by all particles in the swarm up to time step m, respectively.

Let the design space is defined by $[x_{min}, x_{max}]$. If the position of particle i exceeds the boundary, then $v_{i,m+1}$ is modified as follows[12]:

$$x_{i,m+1} = \begin{cases} x_{\max} - (0.5 \times rand() \times (x_{\max} - x_{\min})), & if \quad x_{i,m+1} > x_{\max} \\ x_{\min} + (0.5 \times rand() \times (x_{\max} - x_{\min})), & if \quad x_{i,m+1} < x_{\min} \end{cases} \quad (5)$$

4. A novel forecasting model based on high-order fuzzy-fluctuation trends

In this paper, we propose a novel forecasting model based on high-order fuzzy-fluctuation trends and PSO machine learning algorithm. In order to compare the forecasting results with other researchers' work [2,3,5,7,10,17,29,30], authentic Taiwan Stock Exchange(TAIEX 1999) is employed to illustrate the forecasting process. The data from January to October are used as training time series and the data from November to December are used as testing dataset. The basic steps of the proposed model are as follows:

Step 1: Construct FFTS for historical training data

For each element $X(t)(t = 1,2,\ldots,T)$ in historical training time series, its fluctuation trend is determined by $Y(t) = X(t) - X(t-1), (t = 2,3,\ldots,T)$. According to the range and orientation of the fluctuations, $Y(t)(t = 2,3,\ldots,T)$ can be fuzzified into a linguistic set {down, equal, up}. Let len be the whole mean of all elements in the fluctuation time series $Y(t)(t = 2,3,\ldots,T)$, define $u_1 = [-\infty, -len/2)$, $u_2 = [-len/2, len/2)$, $u_3 = [len/2, +\infty]$, then $Y(t)(t = 2,3,\ldots,T)$ can be fuzzified into a fuzzy-fluctuation time series

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$S(t)(t = 2,3,...,T)$. It is also can be extended to a continuous labeled time series $\bar{S}(t)(t = 2,3,...,T)$, which preserves the accurate original information of $Y(t)(t = 2,3,...,T)$. For example, let *len=85*, *X(1)=6152.43*, *X(2)=6199.91*, then *Y(2)=47.48*, *S(2)=3*, $\bar{S}(2) \approx 2.5586$. On the other hand, based on the previous data *X(1)* and the accurate fuzzy number $\bar{S}(2)$, *X(2)* can be obtained by: $X(1) + len \times (\bar{S}(2) - 2)$, that is $6152.43 + (2.5588 - 2) \times 85 \approx 6199.91$.

Step 2: Establish nth-order FFLRs for the forecasting model

According to Eq.(2), each $\bar{S}(t)(t \geq n+2)$ can be represented by its previous *n* days' fuzzy-fluctuation number. Therefore, the total of FFLRs for historical training data is *pn=T-n-1*.

Step 3: Determine the parameters for the forecasting model based on PSO machine learning algorithm

In this paper, PSO method is employed to determine the parameters $\phi_k (k = 1,2,...,n)$ and a general error ε in Eq.(2). The personal best position and global best position are determined by minimizing the root of the mean squared error (RMSE) in the training process.

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n} (forecast(t) - actual(t))^2}{n}} \tag{6}$$

where *n* denotes the number of values forecasted, *forecast(t)* and *actual(t)* denote the forecasting value and actual value at time *t* in the training process, respectively. For determined $\phi_k (k = 1,2,...,n)$ and ε, the forecast value at time *t* is as follows:

$$forecast(t) = actual(t-1) + len \times (\phi_1 S(t-1) + \phi_2 S(t-2) +,...,+ \phi_n S(t-n) + \varepsilon - 2) \tag{7}$$

The pseudo-code for PSO-based machine learning algorithm is shown in Fig.1.

PSO-based machine learning algorithm for the training process:

INPUT: *X*: training time series, containing *T* cases, denoted as *X[1], X[2],..., X[i]..., X[T]*.

S: a fuzzy-fluctuation time series of training data, containing *T-1* cases, denoted as *S[2], S[3],..., S[i]..., S[T]*.

n: the number of *n*th-order.

itern: the number of iterations.

x_{min}, x_{max}: lower and upper bounds of space.

w, c_1, c_1: parameters described in Eq.(3) and Eq.(4).

OUPUT: *Φ[k]* and ε: parameters for the forecasting model, *k=1,2,...n*.

1. Initialize the position and velocity for each particle *i*, like following:

   pn:=T-1-n; /* the number of particles. */

   For i:=1 to pn

   For j:=1 to n

2. Calculate the fitness value for each particle *i* according to Eq.(6).

   Set *x[pbest]* to current *x[i]* for each particle.

   Locate the global best fitness value *x[gbest]* and set *Φ[k]* and ε to corresponding *x[gbest]*.

3. for *m:=1* to *itern* loop

   For each particle *i*

   Calculate particle velocity according to Eq.(3).

   Update particle position according to Eq.(4) and Eq.(5)

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If the fitness value is better than the best fitness value *x[pbest]* of particle *i* in history
Set current value as the new *x[pbest]* for particle *i*
Locate current global best fitness value, if it is better than the *x[gbest]* in history
Set current global best fitness value as the new *x[gbest]*, and set *Φ[k]* and *ε* to *x[gbest]*.
4. Output *Φ[k]* and *ε*

Fig. 1. Pseudo-code of PSO-based machine learning algorithm

Step 4: Forecast test time series

For each data in the test time series, its future number can be forecasted according to Eq.(7), based on the observed data point *X(t-1)*, its n-order fuzzy-fluctuation trends and the parameters generated from the training dataset.

5. Empirical analysis

A. Forecasting TAIEX

Many researches use TAIEX1999 as an example to illustrate their proposed forecasting methods [2,3,5,7,10,17,29,30]. In order to compare the accuracy with their models, we also use TAIEX1999 to illustrate the proposed method.

[Step 1] Calculate the fluctuation trend for each element in the historical training dataset of TAIEX1999. Then, use the whole mean of the fluctuation numbers of training dataset to fuzzify the fluctuation trends into FFTS as shown in Table I.

Table I Historical Training Data and Fuzzified fluctuation data of TAIEX1999

date	TAIEX	Fluctuation	fuzzified	date	TAIEX	Fluctuation	fuzzified	date	TAIEX	Fluctuation	fuzzified
1/5/1999	6152.43	-	-	4/17/1999	7581.5	114.68	3	7/26/1999	7595.71	-128.81	1
1/6/1999	6199.91	47.48	3	4/19/1999	7623.18	41.68	2	7/27/1999	7367.97	-227.74	1
1/7/1999	6404.31	204.4	3	4/20/1999	7627.74	4.56	2	7/28/1999	7484.5	116.53	3
1/8/1999	6421.75	17.44	2	4/21/1999	7474.16	-153.58	1	7/29/1999	7359.37	-125.13	1
1/11/1999	6406.99	-14.76	2	4/22/1999	7494.6	20.44	2	7/30/1999	7413.11	53.74	3
1/12/1999	6363.89	-43.1	1	4/23/1999	7612.8	118.2	3	7/31/1999	7326.75	-86.36	1
1/13/1999	6319.34	-44.55	1	4/26/1999	7629.09	16.29	2	8/2/1999	7195.94	-130.81	1
1/14/1999	6241.32	-78.02	1	4/27/1999	7550.13	-78.96	1	8/3/1999	7175.19	-20.75	2
1/15/1999	6454.6	213.28	3	4/28/1999	7496.61	-53.52	1	8/4/1999	7110.8	-64.39	1
1/16/1999	6483.3	28.7	2	4/29/1999	7289.62	-206.99	1	8/5/1999	6959.73	-151.07	1
1/18/1999	6377.25	-106.05	1	4/30/1999	7371.17	81.55	3	8/6/1999	6823.52	-136.21	1
1/19/1999	6343.36	-33.89	2	5/3/1999	7383.26	12.09	2	8/7/1999	7049.74	226.22	3
1/20/1999	6310.71	-32.65	2	5/4/1999	7588.04	204.78	3	8/9/1999	7028.01	-21.73	2
1/21/1999	6332.2	21.49	2	5/5/1999	7572.16	-15.88	2	8/10/1999	7269.6	241.59	3
1/22/1999	6228.95	-103.25	1	5/6/1999	7560.05	-12.11	2	8/11/1999	7228.68	-40.92	2
1/25/1999	6033.21	-195.74	1	5/7/1999	7469.33	-90.72	1	8/12/1999	7330.24	101.56	3
1/26/1999	6115.64	82.43	3	5/10/1999	7484.37	15.04	2	8/13/1999	7626.05	295.81	3
1/27/1999	6138.87	23.23	2	5/11/1999	7474.45	-9.92	2	8/16/1999	8018.47	392.42	3
1/28/1999	6063.41	-75.46	1	5/12/1999	7448.41	-26.04	2	8/17/1999	8083.43	64.96	3
1/29/1999	5984	-79.41	1	5/13/1999	7416.2	-32.21	2	8/18/1999	7993.71	-89.72	1
1/30/1999	5998.32	14.32	2	5/14/1999	7592.53	176.33	3	8/19/1999	7964.67	-29.04	2
2/1/1999	5862.79	-135.53	1	5/15/1999	7576.64	-15.89	2	8/20/1999	8117.42	152.75	3

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2/2/1999	5749.64	-113.15	1	5/17/1999	7599.76	23.12	2	8/21/1999	8153.57	36.15	2
2/3/1999	5743.86	-5.78	2	5/18/1999	7585.51	-14.25	2	8/23/1999	8119.98	-33.59	2
2/4/1999	5514.89	-228.97	1	5/19/1999	7614.6	29.09	2	8/24/1999	7984.39	-135.59	1
2/5/1999	5474.79	-40.1	2	5/20/1999	7608.88	-5.72	2	8/25/1999	8127.09	142.7	3
2/6/1999	5710.18	235.39	3	5/21/1999	7606.69	-2.19	2	8/26/1999	8097.57	-29.52	2
2/8/1999	5822.98	112.8	3	5/24/1999	7588.23	-18.46	2	8/27/1999	8053.97	-43.6	1
2/9/1999	5723.73	-99.25	1	5/25/1999	7417.03	-171.2	1	8/30/1999	8071.36	17.39	2
2/10/1999	5798	74.27	3	5/26/1999	7426.63	9.6	2	8/31/1999	8157.73	86.37	3
2/20/1999	6072.33	274.33	3	5/27/1999	7469.01	42.38	2	9/1/1999	8273.33	115.6	3
2/22/1999	6313.63	241.3	3	5/28/1999	7387.37	-81.64	1	9/2/1999	8226.15	-47.18	1
2/23/1999	6180.94	-132.69	1	5/29/1999	7419.7	32.33	2	9/3/1999	8073.97	-152.18	1
2/24/1999	6238.87	57.93	3	5/31/1999	7316.57	-103.13	1	9/4/1999	8065.11	-8.86	2
2/25/1999	6275.53	36.66	2	6/1/1999	7397.62	81.05	3	9/6/1999	8130.28	65.17	3
2/26/1999	6318.52	42.99	3	6/2/1999	7488.03	90.41	3	9/7/1999	7945.76	-184.52	1
3/1/1999	6312.25	-6.27	2	6/3/1999	7572.91	84.88	3	9/8/1999	7973.3	27.54	2
3/2/1999	6263.54	-48.71	1	6/4/1999	7590.44	17.53	2	9/9/1999	8025.02	51.72	3
3/3/1999	6403.14	139.6	3	6/5/1999	7639.3	48.86	3	9/10/1999	8161.46	136.44	3
3/4/1999	6393.74	-9.4	2	6/7/1999	7802.69	163.39	3	9/13/1999	8178.69	17.23	2
3/5/1999	6383.09	-10.65	2	6/8/1999	7892.13	89.44	3	9/14/1999	8092.02	-86.67	1
3/6/1999	6421.73	38.64	2	6/9/1999	7957.71	65.58	3	9/15/1999	7971.04	-120.98	1
3/8/1999	6431.96	10.23	2	6/10/1999	7996.76	39.05	2	9/16/1999	7968.9	-2.14	2
3/9/1999	6493.43	61.47	3	6/11/1999	7979.4	-17.36	2	9/17/1999	7916.92	-51.98	1
3/10/1999	6486.61	-6.82	2	6/14/1999	7973.58	-5.82	2	9/18/1999	8016.93	100.01	3
3/11/1999	6436.8	-49.81	1	6/15/1999	7960	-13.58	2	9/20/1999	7972.14	-44.79	1
3/12/1999	6462.73	25.93	2	6/16/1999	8059.02	99.02	3	9/27/1999	7759.93	-212.21	1
3/15/1999	6598.32	135.59	3	6/17/1999	8274.36	215.34	3	9/28/1999	7577.85	-182.08	1
3/16/1999	6672.23	73.91	3	6/21/1999	8413.48	139.12	3	9/29/1999	7615.45	37.6	2
3/17/1999	6757.07	84.84	3	6/22/1999	8608.91	195.43	3	9/30/1999	7598.79	-16.66	2
3/18/1999	6895.01	137.94	3	6/23/1999	8492.32	-116.59	1	10/1/1999	7694.99	96.2	3
3/19/1999	6997.29	102.28	3	6/24/1999	8589.31	96.99	3	10/2/1999	7659.55	-35.44	2
3/20/1999	6993.38	-3.91	2	6/25/1999	8265.96	-323.35	1	10/4/1999	7685.48	25.93	2
3/22/1999	7043.23	49.85	3	6/28/1999	8281.45	15.49	2	10/5/1999	7557.01	-128.47	1
3/23/1999	6945.48	-97.75	1	6/29/1999	8514.27	232.82	3	10/6/1999	7501.63	-55.38	1
3/24/1999	6889.42	-56.06	1	6/30/1999	8467.37	-46.9	1	10/7/1999	7612	110.37	3
3/25/1999	6941.38	51.96	3	7/2/1999	8572.09	104.72	3	10/8/1999	7552.98	-59.02	1
3/26/1999	7033.25	91.87	3	7/3/1999	8563.55	-8.54	2	10/11/1999	7607.11	54.13	3
3/29/1999	6901.68	-131.57	1	7/5/1999	8593.35	29.8	2	10/12/1999	7835.37	228.26	3
3/30/1999	6898.66	-3.02	2	7/6/1999	8454.49	-138.86	1	10/13/1999	7836.94	1.57	2
3/31/1999	6881.72	-16.94	2	7/7/1999	8470.07	15.58	2	10/14/1999	7879.91	42.97	3
4/1/1999	7018.68	136.96	3	7/8/1999	8592.43	122.36	3	10/15/1999	7819.09	-60.82	1
4/2/1999	7232.51	213.83	3	7/9/1999	8550.27	-42.16	2	10/16/1999	7829.39	10.3	2
4/3/1999	7182.2	-50.31	1	7/12/1999	8463.9	-86.37	1	10/18/1999	7745.26	-84.13	1

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4/6/1999	7163.99	-18.21	2	7/13/1999	8204.5	-259.4	1	10/19/1999	7692.96	-52.3	1
4/7/1999	7135.89	-28.1	2	7/14/1999	7888.66	-315.84	1	10/20/1999	7666.64	-26.32	2
4/8/1999	7273.41	137.52	3	7/15/1999	7918.04	29.38	2	10/21/1999	7654.9	-11.74	2
4/9/1999	7265.7	-7.71	2	7/16/1999	7411.58	-506.46	1	10/22/1999	7559.63	-95.27	1
4/12/1999	7242.4	-23.3	2	7/17/1999	7366.23	-45.35	1	10/25/1999	7680.87	121.24	3
4/13/1999	7337.85	95.45	3	7/19/1999	7386.89	20.66	2	10/26/1999	7700.29	19.42	2
4/14/1999	7398.65	60.8	3	7/20/1999	7806.85	419.96	3	10/27/1999	7701.22	0.93	2
4/15/1999	7498.17	99.52	3	7/21/1999	7786.65	-20.2	2	10/28/1999	7681.85	-19.37	2
4/16/1999	7466.82	-31.35	2	7/22/1999	7678.67	-107.98	1	10/29/1999	7706.67	24.82	2
4/17/1999	7581.5	114.68	3	7/23/1999	7724.52	45.85	3	10/30/1999	7854.85	148.18	3

[Step 2] Based on the FFTS from January 5, 1999 to October 30 shown in Table I, establish nth-order FFLRs for the forecasting model. For example, suppose $n=6$, following FFLRs of FFTS can be generated:

$$\begin{aligned}
\bar{S}(7) &= 1.082 = \phi_1+\phi_2+2\phi_3+2\phi_4+3\phi_5+3\phi_6+\varepsilon_7 \\
\bar{S}(8) &= 4.5091 = \phi_1+\phi_2+\phi_3+2\phi_4+2\phi_5+3\phi_6+\varepsilon_8 \\
&\vdots \\
\bar{S}(221) &= 3.7433 = 2\phi_1+2\phi_2+2\phi_3+2\phi_4+3\phi_5+\phi_6+\varepsilon_{221}
\end{aligned} \tag{8}$$

[Step 3] Replace each error ε_t in Eq.(8) with one and the same ε. Let the number of iterations $itern=100$, the inertia weight $w=0.7298$, self-confidence coefficient and social confidence coefficient $c_1=c_2=1.4962$, use PSO algorithm listed in Fig.1 to determine the parameters $\phi_k (k=1,2,...,n)$ and ε. In the PSO process, each element in the generalized Eq.(8) is a particle and their personal best and global best positions are determined by the RMSE of actual values and forecast values. The obtained global best parameters are shown in Table II.

Table II Global Best Parameters Obtained Using PSO for Training Dataset

ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	E	RMSE
-0.1638	0.0803	0.1372	-0.0321	0.0433	0.2546	1.4408	115.73

[Step 4] Use the obtained global best parameters in Table II to forecast the test dataset from November 1, 1999 to December 30. For example, the forecasting value of the TAIEX on November 8, 1999, is calculated as follows:

Firstly, according to the fuzzy-fluctuation trends (2,1,1,1,2,1) and the parameters in Table II, the forecasted continuous labeled fuzzy-fluctuation number is :

$$2\times(-0.1638)+0.0803+0.1372-0.0321+2\times0.0433+0.2546+1.4408=1.6398$$

Then, the forecasted fluctuation from current value to next value can be obtained by defuzzify the fluctuation fuzzy number:

$$(1.6398-2)\times 85=-30.62$$

Finally, the forecasted value can be obtained by current value and the fluctuation value:

$$7376.56-30.62=7345.94$$

The other forecasting results are shown in Table III and Fig.2.

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Table III Forecasting Results from November 1,1999 to December 30, 1999

Date	Actual	Forecast	$(\text{Forecast-actual})^2$	Date	Actual	Forecast	$(\text{Forecast-actual})^2$
11/1/1999	7814.89	7869.35	2965.89	12/1/1999	7766.20	7705.59	3673.57
11/2/1999	7721.59	7825.35	10766.14	12/2/1999	7806.26	7790.48	249.01
11/3/1999	7580.09	7704.00	15353.69	12/3/1999	7933.17	7824.29	11854.85
11/4/1999	7469.23	7573.21	10811.84	12/4/1999	7964.49	7967.96	12.04
11/5/1999	7488.26	7460.24	785.12	12/6/1999	7894.46	7965.87	5099.39
11/6/1999	7376.56	7468.50	8452.96	12/7/1999	7827.05	7897.62	4980.12
11/8/1999	7401.49	7345.94	3085.80	12/8/1999	7811.02	7806.25	22.75
11/9/1999	7362.69	7400.03	1394.28	12/9/1999	7738.84	7823.68	7197.83
11/10/1999	7401.81	7379.30	506.70	12/10/1999	7733.77	7701.12	1066.02
11/11/1999	7532.22	7410.86	14728.25	12/13/1999	7883.61	7718.38	27300.95
11/15/1999	7545.03	7553.82	77.26	12/14/1999	7850.14	7921.86	5143.76
11/16/1999	7606.20	7569.42	1352.77	12/15/1999	7859.89	7862.87	8.88
11/17/1999	7645.78	7631.90	192.65	12/16/1999	7739.76	7857.12	13773.37
11/18/1999	7718.06	7667.91	2515.02	12/17/1999	7723.22	7750.49	743.65
11/19/1999	7770.81	7750.58	409.25	12/18/1999	7797.87	7733.15	4188.68
11/20/1999	7900.34	7800.66	9936.10	12/20/1999	7782.94	7815.10	1034.27
11/22/1999	8052.31	7936.55	13400.38	12/21/1999	7934.26	7781.74	23262.35
11/23/1999	8046.19	8079.43	1104.90	12/22/1999	8002.76	7953.13	2463.14
11/24/1999	7921.85	8072.42	22671.32	12/23/1999	8083.49	8060.46	530.38
11/25/1999	7904.53	7908.83	18.49	12/24/1999	8219.45	8119.70	9950.06
11/26/1999	7595.44	7912.20	100336.90	12/27/1999	8415.07	8246.57	28392.25
11/29/1999	7823.90	7576.21	61350.34	12/28/1999	8448.84	8462.94	198.81
11/30/1999	7720.87	7823.06	10442.80	Root Mean Square Error(RMSE)			99.31

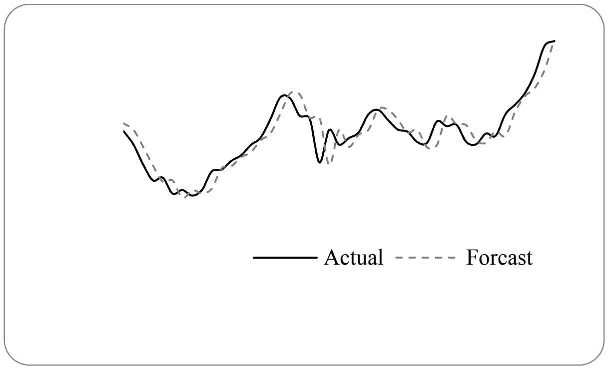

Fig.2 Forecasting Results from November 1,1999 to December 30, 1999

The forecasting performance can be assessed by comparing the difference between the forecasted values and the actual values. The widely indicators used in time series models comparisons are mean squared error (MSE), root of the mean squared error (RMSE), mean absolute error (MAE), mean percentage error (MPE), etc. These indicators are defined by Eqs. (9)–(12):

$$MSE = \frac{\sum_{t=1}^{n}(forecast(t) - actual(t))^2}{n} \quad (9)$$

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Peer-reviewed version available at Symmetry 2017, 9, 124; doi:10.3390/sym9070124

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n}(forecast(t) - actual(t))^2}{n}} \quad (10)$$

$$MAE = \frac{\sum_{t=1}^{n}\left|(forecast(t) - actual(t))\right|}{n} \quad (11)$$

$$MPE = \frac{\sum_{t=1}^{n}\left|(forecast(t) - actual(t))\right| / actual(t)}{n} \quad (12)$$

where *n* denotes the number of values forecasted, *forecast(t)* and *actual(t)* denote the predicted value and actual value at time *t*, respectively. As to the proposed method for 6th-order, the MSE, RMSE, MAE, MPE are 9862.33, 99.31, 75.22, 0.01, respectively.

In order to compare the forecasting results with different parameters such as the number *n* of the nth-order and the element number *g* of linguistic set used in the fluctuation fuzzifying process, different experiments under different parameters were carried out. Each kind of experiments was repeated 30 times. The forecasting errors of the averages for the experiments are shown in Table IV and Table V.

Table IV Comparison of Forecasting Errors for Different nth-order(*g*=3)

n	1	2	3	4	5	6	7	8	9	10
RMSE	109.04	105.47	103.04	102.96	101.92	99.12	99.59	99.6	98.75	99

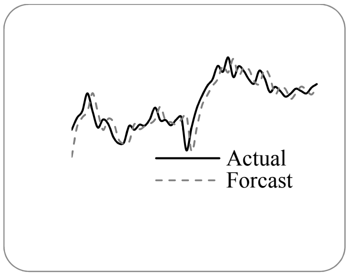
Table V Comparison of Forecasting Errors for Different Linguistic Set (*n=6*)

G	3	5	7	none
RMSE	99.12	101.67	105.82	128.97

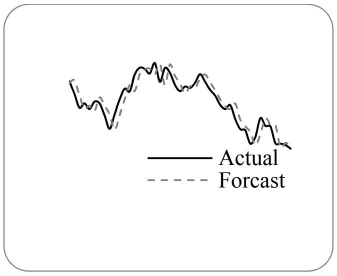
In table V, *g=3* represents that the linguistic set is {Down, Equal, Up}, *g=5* means {Greatly down, Slightly down, Equal, Slightly up, Greatly up}, *g=7* means { Very Greatly down, Greatly down, Slightly down, Equal, Slightly up, Greatly up, Very Greatly up }, and "none" means that the fluctuation values won't be fuzzified at all.

From Table IV and Table V, we can see that the RMSEs are lower when *n* is equal to six or more. As to parameter *g*, obviously, fuzzified fluctuation trends perform better than none fuzzified ones, and it is proper to let *g=3*.

Let *n=6* and *g=3*, employ the proposed method to forecast the TAIEX from 1997 to 2005. The forecasting results and errors are shown in Fig. 3 and Table VI.



(1997)



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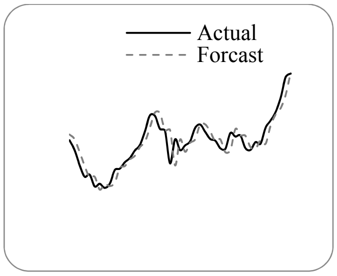

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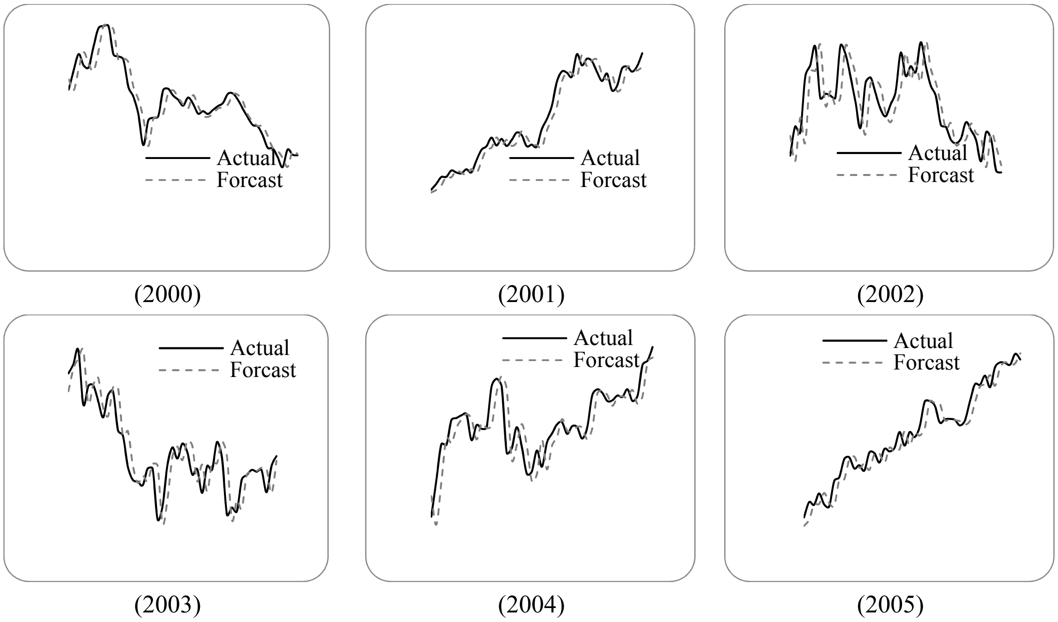

Fig. 3. The stock market fluctuation for TAIEX test dataset(1997-2005)

Table VI RMSEs of forecast errors for TAIEX 1997 to 2005

Year	1997	1998	1999	2000	2001	2002	2003	2004	2005
RMSE	143.60	115.34	99.12	125.70	115.91	70.43	54.26	57.24	54.68

The following Table VII shows a comparison of RMSEs for different methods for forecasting TAIEX1999. From this table, we can see that the performance of proposed method is acceptable. The greatest advantage of the proposed method is that it put forward a method relying completely on machine learning mechanism. Though RMSEs of some of the other methods outperform the proposed method, they often need to determine complex discretization partitioning rules or use adaptive expectation model to justify the final forecasting results. The method proposed in this paper is more simply and easily to be realized by a computer program completely.

Table VII A Comparison of RMSEs for Different Methods for Forecasting TAIEX1999

Methods	RMSE
Yu ‘s Method(2005)[29]	145
Hsieh et al. ‘s Method(2011)[17]	94
Chang et al. ‘s Method(2011)[2]	100
Cheng et al. ‘s Method(2013)[10]	103
Chen et al. ‘s Method(2013)[7]	102.11
Chen & Chen ‘s Method(2015) [5]	103.9
Chen & Chen ‘s Method(2015) [3]	92
Zhao et al. ‘s Method(2016)[30]	110.85
The Proposed Method	99.12

B. Forecasting SHSECI

The SHSECI (Shanghai Stock Exchange Composite Index) is the most famous stock market index in China. In the following, we apply the proposed method to forecast the SHSECI from 2007 to 2015. For each year, the authentic datasets of historical daily SHSECI closing prices from January to October are used as the training data, the datasets from November to December are used as the testing data. The forecasting results and the RMSEs of forecast errors are shown in Fig. 4 and Table VIII, respectively.

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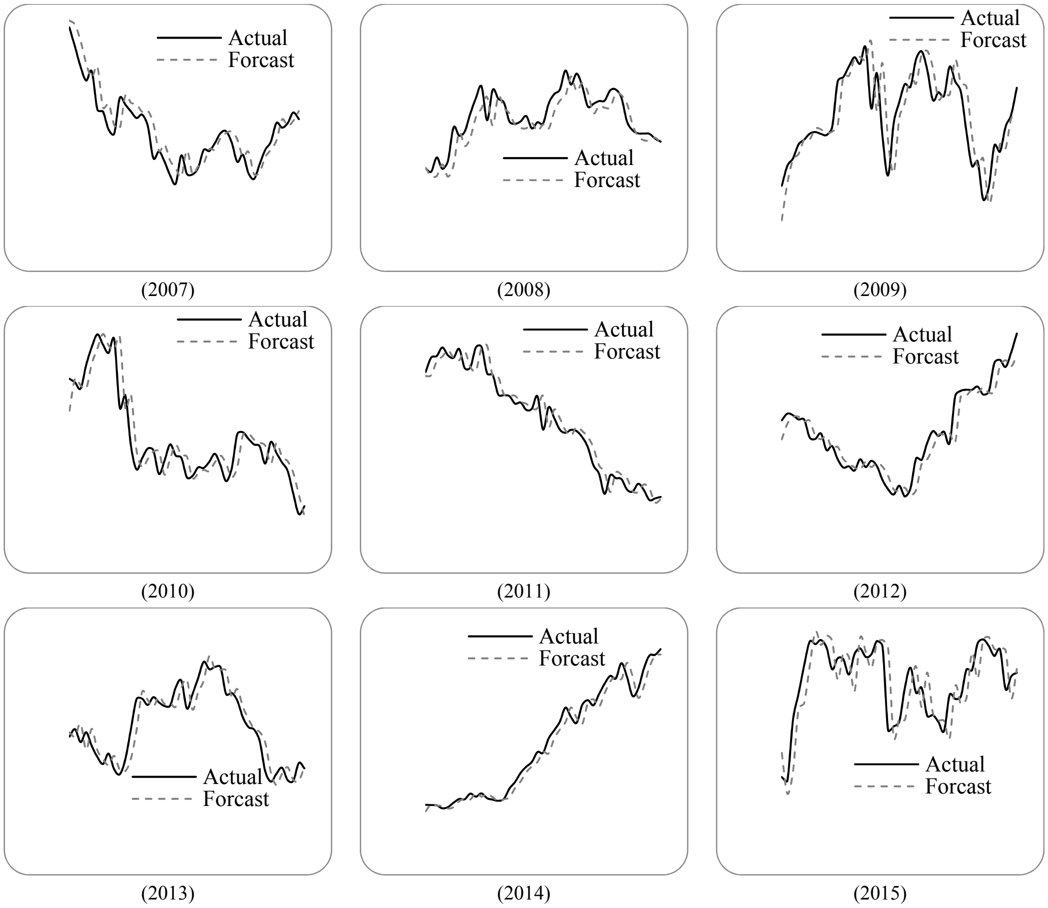

Fig. 4. The stock market fluctuation for SHSECI test dataset(2001-2015)

Table VIII RMSEs of forecast errors for SHSECI from 2007 to 2015

Year	2007	2008	2009	2010	2011	2012	2013	2014	2015
RMSE	113.11	55.28	49.59	45.73	28.45	25.05	19.86	41.44	59.5

From Fig.4. and Table VIII, we can see that the proposed method can successfully predict the stock market.

6. Conclusion

In this paper, a novel forecasting model is proposed based on high-order fuzzy-fluctuation logical trends and PSO machine learning method. The proposed method is based on the fluctuations of the time series. PSO method is employed to looking for the best parameters to minimize the RMSE for historical training dataset. Experiments shows that these parameters generated from training dataset can be successfully used for future dataset as well. In order to compare the performance with that of other methods, we take TAIEX1999 as an example. We also forecasted TAIEX1997-2005 and SHSECI 2007-2015 to verify its effectiveness and universality. In the future, we will consider other factors which might affect the fluctuation of the stock market, such as the trade volume, the beginning value, the end value, etc. We also will consider the influence of other stock markets, such as the Dow Jones, the NASDAQ, the M1b and so on.

Acknowledgments: This work was supported by the National Natural Science Foundation of China under Grants (71471076), the Fund of Ministry of education of Humanities and Social Sciences (14YJAZH025), the Fund of China Nation Tourism Administration (15TACK003), the Natural Science Foundation of Shandong Province (ZR2013GM003) , and the Foundation program of Jiangsu University(16JDG005).

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