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Article

A New Kind of Aesthetics – The Mathematical Structure of the Aesthetic

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Abstract: This paper proposes a new approach to investigation into the aesthetics. Specifically, it argues that it is possible to explain the aesthetic and its underlying dynamic relations with axiomatic structure (the octahedral axiom derived category) based on contemporary mathematics – namely, category theory – and through this argument suggests the possibility for discussion about the mathematical structure of the aesthetic. If there was a way to describe the structure of aesthetics with the language of mathematical structures and mathematical axioms – a language completely devoid of arbitrariness – then we would make possible a synthetical argument about the essential human activity of "the aesthetics", and we would also gain a new method and viewpoint on the philosophy and meaning of the act of creating a work of art and artistic activities. This paper presents one hypothesis as a first step in constructing the science of dynamic generative aesthetics based on axiomatic functionalism, which is in turn based on a new interdisciplinary investigation into the functional structure of aesthetics.

Keywords: aesthetics; mathematical structure; category theory; natural intelligence

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1. Introduction

This paper focuses on a new kind of aesthetics, which is made possible by defining the mathematical structure of aesthetics. By aesthetics here we mean various academic approaches to contemplation about the question of how to define aesthetics – a question about the essence and criteria of what defines the aesthetic sense evoked by nature and the fine arts, as well as aesthetic value [1].

The term "aesthetics", originally derived from the Greek "aisthesis" meaning "sensory perception", took on a new meaning when it was appropriated by the German philosopher Alexander Gottlieb Baumgarten, who redefined the concept to refer to the "lower perception" of senses or sensibility rather than the "high perception" of reason or rationality, and coined the term "aesthetica/Aesthetik" – the science of perception through one's senses (as opposed to one's intellect) [2]. The "aesthetics" prior to that, regardless of whether it was "the science of sensitivity" or "the philosophy of beauty" was studied exclusively based on the correlation between subject and object, and scholars were attempting to describe the aesthetic experience that comprises the aesthetic objectively, as a universal phenomenon. But such subject = object relations are too simplistic. For example, this approach does not evidently include the aesthetic context (such as history or environment) necessary to recognize or contemplate the aesthetic. As Van Maanen [3] points out, a great effort has been made to incorporate the background of aesthetic value both within the academic study of aesthetics and outside of it. Therefore, it has become common sense to think about aesthetic value from the perspective of its production and consumption, or the marketplace/group called the "art world" [4]. However, so

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far it has almost always been the case that such contextualistic background has only been studied from the humanistic approach of semiotics and linguistics (for instance Alfred Gell's "Art and Agency: An Anthropological Theory" [5], or the sociology of fine art, such as T.J. Clark's "The Conditions of Artistic Creation" [6]).

Our approach in this research differs from post-Baumgarten's "science of sense", which mostly focuses on "how humans sense and perceive the aesthetic", as well as from the approach of analytical aesthetics [7] formed in the 20th century, which employs the methodology of analytical philosophy. First and foremost, we focus on the structure of how the aesthetic is formed (the ontology of aesthetics, or, what makes the aesthetic aesthetic). With that notion as a fundamental principle, we suggest a new kind of aesthetics established upon mathematics and different from the traditional aesthetics that has been approached as a certain kind of typology that analyses a set of examples in an inductive, and not deductive, way. By creating a distinct model of aesthetics' structure through the use of mathematical axioms we inclusively describe not only the subjects and the objects necessary to provide for the phenomenon of the aesthetic, and not only the relation between these subjects and objects, but also the environment surrounding them and the temporal changes resulting from interactions between various factors. If we could demonstrate that the structure of the aesthetic conforms to the mathematical structure that reflects the context of the aesthetic and the progress of time, we could define the dynamic generative aesthetics that is constantly undergoing change, and give birth to a new interdisciplinary investigation into aesthetics, which is the main objective of this research.

2. Method of Research

In our research, we use mathematics as a language to describe the structure of aesthetics. There are various fields of mathematics, and to be specific we are using the contemporary mathematics that has rapidly developed since the beginning of the 20th century – namely, topology, homology theory and category theory. All of these are characterized by mutually connecting algebra and geometry, and because of that enabling us to operate in an abstract way with objects of interest and their interrelations. In other words, category theory, when making an inquiry into a particular object, addresses not the concrete content of that object, but the structure transferred from the nature of its container. By doing that we make it possible to operate with abstract matters while preserving their structure and with the relationships between these matters. This point is rooted in the mathematical concepts of "homomorphism" – a morphism that retains its structure, and "quotient group"[1] – a quotient set consisting of equivalent classes. The elucidation of functional structures through physical interpretation of category theory has further progressed through the contributions of this paper's co-authors, Hori and Naruse. By "function" here we mean "the emergence of various selectable physical aspects resulting from certain non-trivial processes" – a phenomenon stemming from its relation to the environment system. With this definition in mind we argue that "intelligence" can be defined as "finding the value in having alternative options and choosing the consequence of function" [8] [9] [10]. Following these academic works, our research suggests that aesthetics is a type of function that emerges when the function's conditions are fulfilled [11]. In other words, we presume that aesthetics arises by way of "the emergence of various selectable aesthetic conditions resulting from certain non-trivial processes" and through "the act of finding the value in having these alternative options and the act of aesthetic judgement", and we further elaborate about the structure of aesthetics.

3. Elementary Process of the Aesthetic

First of all, in addition to the relation (1) between the viewer in traditional aesthetics as the subject P and the work of art as the object Q, we introduce the following elements – the "hidden state" X that

[1] All the elements of G can be divided without redundancy using normal subgroup H of Group G.

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enabled such relation, and the image of hidden state Y. Using the concept of a multiplicative system in category theory, we describe this relation as shown in (2).

$$P \longleftrightarrow Q \tag{1}$$

$$\begin{tikzcd} & X \arrow[dl, Rightarrow, "t"'] \arrow[dr, "v"] & \\ P \arrow[dr, "u"'] & & Q \arrow[dl, Rightarrow, "s"] \\ & Y & \end{tikzcd} \tag{2}$$

The relation of X to P, Q and of Y to P, Q correspond to the product (3) and the coproduct (4) respectively. The square-shaped diagram formed by X, P, Q, Y shown in (2) is called a "commutative diagram" [12].

$$\begin{tikzcd} & X \arrow[dl, "t"'] \arrow[d, dashed] \arrow[dr, "v"] & \\ P & P \otimes Q \arrow[l] \arrow[r] & Q \end{tikzcd} \tag{3}$$

$$\begin{tikzcd} P \arrow[r] \arrow[dr, "u"'] & P \oplus Q \arrow[d, dashed] & Q \arrow[l] \arrow[dl, "s"] \\ & Y & \end{tikzcd} \tag{4}$$

There is a variety of options regarding observer P who possesses an aesthetic mindset, and the aesthetic object or artwork Q which possesses an aesthetic element(s). Similarly, the choice between representative morphisms t and s (double lined arrow) is arbitrary, and belongs to multiplicative system of morphisms, i.e. one can describe the process in a different but equivalent manner based on any set of representative morphisms belonging to the multiplicative system. The morphisms v and u are referred to, respectively, as the right and left quotient morphisms of $P \to Q$ based on the representatives t and s.

This relationship is ruled by the hidden state X. In other words, X is the overall state of non-trivial aesthetics co-created by the observer and the aesthetic object, which can be understood as an aesthetic concept. Under such circumstances the relation between observer P and the artwork Q can be inferred from the aesthetic state Y as an image of X.

4. Introduction of the Environmental Condition

One of the special characteristics of the aesthetic model proposed in our paper is that in addition to the factors forming the elementary process of aesthetic we also introduce the environmental condition surrounding these factors. Here by "environment" we mean diverse social, political, cultural, and technological context of consciously recognizing aesthetics and sharing aesthetic experience.

First, we introduce $M = \mathrm{Ker}(u)$, the kernel of $u : X \to Q$, and M here is defined by morphism $k : M \to X$ mapped to 0 (equivalent class) by u. M is the generation factor of the aesthetic object Q, and because M in Q is injected into 0 and not tangible, we can call M the environment of Q. This means that the multifarious works of art Q such as paintings, sculptures, music and poetry and their observers P emerge through the aesthetic concept X from certain factors M generating an aesthetic object.

Additionally, we introduce $F = \mathrm{Cok}(s)$, the cokernel of $s : Q \to Y$. The cokernel F here is defined as the quotient space $Y/\mathrm{Im}(s)$ derived from the image of s as seen from the Y codomain of s. In other words, F is the result of the analysis of the aesthetic state and also the result of categorizing the

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aesthetic state Y created by the elements of the aesthetic object Q where elements Q are employed for such categorization. Since the elements Q become the criteria for categorization, they become intangible, and therefore we can also call this the environment of Q. That is, F is the result of analyzing the aesthetic state Y evoked by aesthetic objects like works of art and nature with each quality and characteristic of the objects Q.

The introduction of these two environmental conditions into diagram (2) can be depicted as (5).

$$\begin{tikzcd}
M \arrow[dr, "k"] & & \\
& X \arrow[dl, Rightarrow, "t"'] \arrow[dr, "v"] & \\
P \arrow[dr, "u"'] & & Q \arrow[dl, Rightarrow, "s"] \\
& Y \arrow[dl, Rightarrow, "l"] & \\
F & &
\end{tikzcd} \tag{5}$$

P	observer
ine Q	aesthetic object
ine X	aesthetic concept
ine Y	aesthetic state
ine M	generation factor of aesthetic object
ine F	analysis results of aesthetic state

Such sequences of $M \to X \to Q$ and $Q \to Y \to F$ are called short exact sequences. Short exact sequence (6) is the exact sequence where $\mathrm{Im}(i) = \mathrm{Ker}(j)$ (image of i corresponds with the kernel of j), i is a monomorphism, and j is an epimorphism [2]. In such circumstances, $C = B/A$ or in other words C is a cokernel of B. A is determined by 0 at the left end which is not A. The 0 at the right end indicates that A is the reference to categorize C.

$$0 \longrightarrow A \xrightarrow{i} B \xrightarrow{j} C \longrightarrow 0 \tag{6}$$

The short exact sequence means that the diverse content of the aesthetic concept X is transfered to the aesthetic (real) object Q as the "equivalent", and is divided into "equivalence classes" that are included in M (generation factors of the aesthetic object). The identical relation binds Y (an aesthetic state created as a diverse expansion of an aesthetic object) and F (analysis results of aesthetic state categorized by Q). Moreover, if you interpret the morphism $v : X \to Q$ as the appreciation of aesthetics (a morphism $u : P \to Y$ from the observer to the aesthetic state), the diverse content of aesthetic concept X can be abstracted in that process, and we can derive a clear classification, $Q = X/M$. The factors M, that cannot be categorized directly, are identified based on classification of Q.

5. Introduction of Time

Aesthetics changes and develops not only in the context of environment conditions, but also dynamically along the time axis, or in other words, as time passes. This is why it is essential that

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[2] If the sequence $0 \to A \to B$ is exact at A, that map is a monomorphism (injective, or one-to-one). If the sequence $B \to C \to 0$ is exact at C, that map is an epimorphism (surjective, or onto).

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following the introduction of the environment condition we also introduce time into our model. For that we use the concept of the complex in homological algebra. Consequently, when you line up the objects one after another such a chain of objects results in a complex $C^\bullet = (C^n, \partial^n)$ consisting from objects C^n mutually connected with a 0 homomorphism ($\partial^n \partial^{n-1} = 0$), which in turn can expressed as an indefinite diagram (7).

$$C^\bullet = \cdots \longrightarrow C^{n-1} \xrightarrow{\partial^{n-1}} C^n \xrightarrow{\partial^n} C^{n+1} \xrightarrow{\partial^{n+1}} C^{n+2} \xrightarrow{\partial^{n+2}} \cdots \tag{7}$$

Then "time" is generated from the duration necessary for the short exact sequences (6) to reach equilibrium. The stepwise "histories" of time evolution recorded in a n-th cohomology of $C^\bullet$, $H^n(C^\bullet) = \mathrm{Ker}(\partial^n)/\mathrm{Im}(\partial^{n-1})$ where a complex $C^\bullet$ is a container of local features. The diagram of category that includes this cohomology evidently receives a non-trivial influence from such environmental conditions as society or technology – conditions not limited to the observer and the artwork directly connected to aesthetics.

In category theory, a complex is also an object, for morphism $f : C^\bullet \to D^\bullet$ ($f^n : C^n \to D^n$) is defined by $f^{n-1}\partial^n = \partial^n f^n$. It is possible to construct a short exact sequence structure in a complex, too. So, a chain-wise exact sequence of the complex $0 \to A^\bullet \to B^\bullet \to C^\bullet \to 0$ retains the short exact sequence $0 \to A^n \to B^n \to C^n \to 0$ at all degrees, and each of the complexes A^n, B^n, C^n contains histories $H^n(A^\bullet), H^n(B^\bullet), H^n(C^\bullet)$ and provides the basic diagram that includes generative time structure.

Using snake lemma in homological algebra, we derive a long exact sequence of cohomology (8) from the chain-wise exact sequence of complex that is constructed exclusively by homomorphisms [13].

$$\cdots \longrightarrow H^n(A^\bullet) \longrightarrow H^n(B^\bullet) \longrightarrow H^n(C^\bullet) \longrightarrow H^{n+1}(A^\bullet) \longrightarrow \cdots \tag{8}$$

This allows us to conclude that backed up by diagrams of clear complexes for each chain, the environmental condition (cohomology), a shared fundament supporting the chain of objects, preserves the spatiotemporal chain that includes the invisible non-trivial matters and events. Here we call the morphism $H^n(C^\bullet) \to H^{n+1}(A^\bullet)$ a translation morphism, or a characteristic arrow.

6. Triangulated Category and Octahedron Structure

When the arbitrary morphism of category has image and coimage, and both image and coimage become isomorphic, we call it the "exact category". The function of the morphism f of the exact category is, after the relation of coimage and image has become isomorphic $\mathrm{Coim}(f) \sim \mathrm{Im}(f)$, to totally annihilate all the matters prior to that and form the matters that will continue from that point on.

Thus, we make it possible to discuss a chain of matters from the homeomorphic perspective, and if such a chain is a split exact sequence, we can represent a diagram of long exact sequence of cohomology as a direct conjunction of complexes (9) using the homotopy equivalence of category.

When a triangular diagram is constructed in the sense of derived category, it is called the "triangulated category" [13]. At this time, by ignoring the future homology of the environment, it is possible to equate C^n with the history of C^n containing the cohomology of the past.

$$\cdots \longrightarrow C^{n-1} \rightsquigarrow A^n \longrightarrow B^n \longrightarrow C^n \rightsquigarrow A^{n+1} \longrightarrow \cdots \tag{9}$$

The "morphism raising a complex one level higher" $R \rightsquigarrow P[1]$ incorporating a wave-shaped arrow is a characteristic morphism (arrow) of the long exact sequence depicted in diagram (8). The formation of a triangular scheme of complexes is called a triangulated category, and while it changes the "histories" it is a simple and clear way to represent the evolving but structure-preserving object(s).

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We then replace the object of commutative diagram (5) with the complex, and expand it using this triangulated category. First, if we add the result of analysis of the aesthetic state $F^{\bullet}$ from the previous step interconnected with translation morphisms, and $M^{\bullet}$, the generation factor of the aesthetic object from the later step, we get the relation expressed in diagram (10).

$$
\begin{tikzcd}
M^{\bullet} \arrow[dr] & & F^{\bullet} \arrow[dl, rightsquigarrow] \\
& X^{\bullet} \arrow[dl, Rightarrow] \arrow[dr] & \\
P^{\bullet} \arrow[dr] & & Q^{\bullet} \arrow[dl, Rightarrow] \\
& Y^{\bullet} \arrow[dl, Rightarrow] \arrow[dr, rightsquigarrow] & \\
F^{\bullet} & & M^{\bullet}
\end{tikzcd}
\tag{10}
$$

Moreover, by supplementing the model with morphism $Q^{\bullet} \to F^{\bullet}$, $M^{\bullet} \to Q^{\bullet}$ which leads to nature, and characteristic arrow $F^{\bullet} \rightsquigarrow P^{\bullet}$, $P^{\bullet} \rightsquigarrow M^{\bullet}$, we get the diagrams (11) and (12).

$$
\begin{tikzcd}
M^{\bullet} \arrow[dr] & & F^{\bullet} \arrow[dl, rightsquigarrow] \\
& X^{\bullet} \arrow[dl, Rightarrow] \arrow[dr] & \\
P^{\bullet} \arrow[uu, rightsquigarrow] \arrow[dr] & & Q^{\bullet} \arrow[dl, Rightarrow] \arrow[uu, Rightarrow] \\
& Y^{\bullet} \arrow[dl, Rightarrow] \arrow[dr, rightsquigarrow] & \\
F^{\bullet} \arrow[uu, rightsquigarrow] & & M^{\bullet} \arrow[uu]
\end{tikzcd}
\tag{11}
$$

$$
\begin{tikzcd}
P^{\bullet} \arrow[dr] & & Q^{\bullet} \arrow[dl, Rightarrow] \\
& Y^{\bullet} \arrow[dl, Rightarrow] \arrow[dr, rightsquigarrow] & \\
F^{\bullet} \arrow[uu, rightsquigarrow] \arrow[dr, rightsquigarrow] & & M^{\bullet} \arrow[uu] \arrow[dl] \\
& X^{\bullet} \arrow[dl] \arrow[dr, Rightarrow] & \\
Q^{\bullet} \arrow[uu, Rightarrow] & & P^{\bullet} \arrow[uu, rightsquigarrow]
\end{tikzcd}
\tag{12}
$$

If you overlay $M^{\bullet}$ and $F^{\bullet}$, which belong to different stratums with different environments, as a result you get an octahedron consisting of one commutative diagram and four triangulated categories (13).

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$$\begin{tikzcd}
& & X^\bullet \arrow[drr] \arrow[ddll, Rightarrow] & & \\
F^\bullet \arrow[urr, rightsquigarrow] \arrow[d, rightsquigarrow] & & & & Q^\bullet \arrow[llll, Rightarrow] \arrow[ddll, Rightarrow] \\
P^\bullet \arrow[rrrr, rightsquigarrow] \arrow[drr] & & & & M^\bullet \arrow[u] \arrow[uull] \\
& & Y^\bullet \arrow[uull, Rightarrow] \arrow[urr, rightsquigarrow] & &
\end{tikzcd} \tag{13}$$

$P^\bullet$	observer
ine $Q^\bullet$	aesthetic object
ine $X^\bullet$	aesthetic concept
ine $Y^\bullet$	aesthetic state
ine $M^\bullet$	generation factor of aesthetic object
ine $F^\bullet$	analysis result of aesthetic state

7. Aesthetic Activity and Time Evolution

This octahedron corresponds to one of the important consequences of derived category called an "octahedron axiom". If we look more closely at these four triangulated categories, they correspond respectively to "aesthetic contemplation", "artwork criticism", "artwork production" and "aesthetic experience", with each of these being part of aesthetic activity as elements of the formation aesthetics, which are mutually connected and shape a mathematical model that progresses over time.

Aesthetic contemplation
generation factor of aesthetic object $M^\bullet$ $\to$ aesthetic concept $X^\bullet$ $\to$ observer $P^\bullet$ $\to$ generation factor of aesthetic object $M^\bullet[1]$

Artwork criticism
aesthetic concept $X^\bullet$ $\to$ aesthetic object $Q^\bullet$ $\to$ analysis result of aesthetic state $F^\bullet$ $\to$ aesthetic concept $X^\bullet[1]$

Artwork production
generation factor of aesthetic object $M^\bullet$ $\to$ aesthetic object $Q^\bullet$ $\to$ aesthetic state $Y^\bullet$ $\to$ generation factor of aesthetic object $M^\bullet[1]$

Aesthetic experience
observer $P^\bullet$ $\to$ aesthetic state $Y^\bullet$ $\to$ analysis result of aesthetic state $F^\bullet$ $\to$ observer $P^\bullet[1]$

By describing relations of aesthetic activities through these four exact sequences shown in Table1, the mutual connection of the objects (complexes) through the braid structure of octahedron [13][14] becomes evident.

Finally, we go back to the elementary process of the aesthetic (2) at the beginning again. Assuming that (P, Q) is a simple sum of two objects, $X \to (P, Q) \to Y$ has the meaning of factor X, various phenomena (P, Q) expanded from X, and Y that categorized by X, just like a short exact sequence. Furthermore, referring to (F, M) introduced as the environmental condition (5), the set (F, M) of the environmental condition which is all except for those surrounded by the commutative diagram supports the structure of time evolution of the elementary process of aestheric. In that process, (P, Q) captures the singularity as the aesthetic of the environmental condition step by step, and acquires its non-trivial history. Each time it acquires the singularity the dimension of cohomology increases (the dimension of homology decreases) then its structure becomes simpler. This is one expression of Mayer-Vietoris sequences [15]. It shows that an unknown aesthetic concept X is gradually elucidated

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Table 1. Braid structure as aesthetic activities

				Artwork criticism		**Aesthetic experience**			
				0		0			
				↓		↓			
0	→	generation factor of aesthetic object	→	aesthetic concept	→	observer	→	0	**Aesthetic contemplation**
		↓		↓		↓			
0	→	generation factor of aesthetic object	→	aesthetic object	→	aesthetic state	→	0	**Artwork production**
				↓		↓			
				analysis result of aesthetic state	→	analysis result of aesthetic state			
				↓		↓			
				0		0			

by specifying its nature according to the aesthetic state Y generated by the aesthetic object Q and observer P (14).

$$\begin{tikzcd}
 & P^\bullet \arrow[dr] & & & P^\bullet[1] \arrow[dr] & \\
X^\bullet \arrow[ur, Rightarrow] \arrow[dr] & & Y^\bullet \arrow[r, rightsquigarrow] & X^\bullet[1] \arrow[ur, Rightarrow] \arrow[dr] & & Y^\bullet[1] \\
 & Q^\bullet \arrow[ur, Rightarrow] & & & Q^\bullet[1] \arrow[ur, Rightarrow] &
\end{tikzcd} \tag{14}$$

8. Conclusions

Mathematician Saunders MacLane has stated in his work "Categories for the Working Mathematician" (a textbook in category theory that is regarded as the premier and classic authority on the subject) [12]: "A category (distinguished from a metacategory) will mean any interpretation of the category axioms within set theory". This paper extends this statement and attempts the interpretation of category axioms into aesthetics.

Mathematician André Weil analyzed the data about kinship relations in primitive society collected and organized by anthropologist Claude Lévi-Strauss, and concluded that there is a mathematical structure of "group" evidently present in those relations [16]. This achievement points at the possibility and usefulness of applying a mathematical approach to humanities studies such as anthropology. Our research takes into account such prior works and points out that derived category in category theory and octahedron category (which is the mathematical structure that describes the way "it has to be") might be the base of the structure of aesthetics – an essential activity for human being.

There has been a number of attempts to apply mathematics or science to aesthetics from various perspectives. Older works and topics include "aesthetics measure" by George David Birkhoff [17], "information aesthetics" by Abraham Moles [18] Frieder Nake [19], Max Bense [20] and Hiroshi Kawano [21], and the "algorithmic theory of beauty" by Jürgen Schmidhubern [22]. Among more recent developments we can name, for instance, "neuroaesthetics" that discusses the process of beauty representation inside the human brain [23] [24].

What remains different between prior works and our research is the core purpose. The former attempts to apply methods and knowledge of mathematics and other sciences such as information theory or semiotics to "measuring" beauty and materializing this process, while our approach focuses on describing the structure (function) of the aesthetic with axioms. In other words, on expressing the structure of the aesthetic with mathematical structure itself. As mentioned in the beginning of this paper, the purpose of our research is to describe generative aesthetics, that dynamically changes as the result of interaction with the environment, through employing mathematical axioms that express the structure of the aesthetic, and to execute an interdisciplinary investigation based on this notion. As Arthur Danto points out in his research [4] [25], the value and meaning of the aesthetic is not the universal and unchanging "nature" of the phenomenon, but something relative to and depending on the context of its environment – certain time periods, ethnic groups, society or culture. Therefore, it seems essential to describe the structure of the aesthetic axiomatically, in its true state of being generated together with its environment, and express through axioms this mechanism generated by the dynamism of aesthetic contemplation, artwork criticism, artwork production, and aesthetic experience – all of these forming "the aesthetics", which is exactly what our model aims to achieve. As a result, we offer a new model of aesthetics called axiomatic functionalism.

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