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The Impact of the Cosmological Constant on the Newtonian Gravity

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Summary
By solving the weak field limit of Einstein's Field Equation including the Cosmological Constant, under the constraint of spherical isotropy, it is shown that, at large cosmological distance, the gravitational force exceeds the one that is predicted by Newton's gravity law, such that it corresponds with Milgrom's MOND hypothesis. However, the resulting prediction that, at extremely large distances, gravity with some spatial periodicity turns on-and-off into antigravity marks a decisive difference.

Keywords: Cosmological Constant; MOND; dark matter; antigravity

Introduction

It is well known that the weak field limit of Einstein's Field Equation corresponds with Newton's gravitation law. As I wish to discuss in this article, this is true as long as Einstein's Cosmological Constant is considered to be zero. This implies that a non-zero value of this constant modifies Newton' s law. Presently, a non-zero value of this constant is considered to be feasible, because it would explain the phenomenon that the Universe is expanding in acceleration rather than with a constant velocity such as presumed prior to 1998 [1,2]. It means that the Cosmological Constant embodies the "dark energy", which is seen as the true cause of this phenomenon [3]. If the associated modification of Newton's gravitation law would also be responsible for the excessive orbital velocity of stars at the far end of galaxies, it would be fair to state that the Cosmological Constant would embody "dark matter" as well. This raises the question in how far the modification of Newton's gravitation law due to the Cosmological Constant corresponds with the empirical modification of this law as proposed by Milgrom [4], known as MOND (Modified Newtonian Dynamics), as a substitute for the dark matter hypothesis for explaining the flat rotation curves of stars in galaxies. It is my aim to show that the gauge freedom in Einstein's Field Equation allows developing a theoretical basis for heuristic MOND, thereby revealing some unexpected properties and predictions. To do so, first an outline will be given of the line of thought, the details of which being addressed in an appendix. After that, a comparison will be given between the developed theoretical model for modified gravity and the view as usually presented in MOND.

The gravitational wave equation

Let us start by considering the gravitational wave equation as a consequence of the weak field limit of the Einsteinean Field Equation. The equation reads as,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad \text{with} \quad G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} . \tag{1}$$

where $T_{\mu\nu}$ is the stress-energy tensor, which describes the energy and the momenta of the source(s) and where $R_{\mu\nu}$ and R are respectively the so-called Ricci tensor and the Ricci scalar, which can be calculated if the metric tensor components $g_{\mu\nu}$ are known [5,6,7]. The quantity Λ is known as Einstein's Cosmological Constant. In the case that a particle under consideration is subject to a central force only, the time-space condition shows a spherical symmetric isotropy. This allows to read the metric elements g_{ij} from a simple line element that can be written as

$$\mathrm{d}s^2 = g_{tt}(r,t)\mathrm{d}q_0^2 + g_{rr}(r,t)\mathrm{d}r^2 + r^2\sin^2\vartheta\mathrm{d}\varphi^2 + r^2\mathrm{d}\vartheta^2 , \tag{2}$$

where $q_0 = \mathrm{i}ct$ and $\mathrm{i} = \sqrt{-1}$.

It means that the number of metric elements g_{ij} reduce to a few, and only two of them are time and radial dependent. A generalization of Schwarzschild's solution of Einstein's equation for empty space and $\Lambda = 0$, shown in the appendix of this paper, relates the metric components as,

$$g_{rr}g_{tt} = 1 . \tag{3}$$

Solving Einstein's equation under the weak field limit

$$g_{tt}(r,t) = 1 + h_\varphi(r,t) \text{, where } \left|h_\varphi(r,t)\right| << 1 , \tag{4}$$

under adoption of a massive source with pointlike distribution $T_{00} = Mc^2\delta^3(r)$, results in a wave equation with the format (see Appendix),

$$-\frac{1}{r}\frac{\partial^2}{c^2\partial t^2}(rh_\varphi) + \frac{1}{r}\frac{\partial^2}{\partial r^2}(rh_\varphi) = -\frac{8\pi GM}{c^2}\delta^3(r) , \tag{5}$$

Its stationary solution [8] is the well-known Newtonian potential,

$$\Phi = -\frac{MG}{r} \text{, where } h_\varphi = \frac{2\Phi}{c^2} . \tag{6}$$

where $h_\varphi = \frac{2\Phi}{c^2}$.

Eq. (5) is the equation of a wave that propagates in the direction of r with a velocity c. This equation is identical in format as Maxwell's wave equation for electromagnetism. It proves the causality of gravity.

Let us now memorize that Einstein derived his Field Equation by defining a covariant derivative after proper time $\tau' = \mathrm{i}c\tau$, such that that both the covariant derivatives of the Einstein tensor $G_{\mu\nu}$ and the energy-stress tensor $T_{\mu\nu}$ are zero, i.e.,

$$\frac{D}{dq_\mu}G_{\mu\nu} = \frac{D}{dq_\mu}T_{\mu\nu} = 0 . \tag{8}$$

Actually, this is a sum of covariant derivatives in Einstein notation, i.e.

$$\sum_{\mu=0}^{3}\frac{D}{dq_{\mu}}G_{\mu\nu}=\sum_{\mu=0}^{3}\frac{D}{dq_{\mu}}T_{\mu\nu}=0 \text{ for } \nu = 0,1,2,3, \quad (9)$$

From (8) it is concluded that

$$G_{\mu\nu}+B_{\mu\nu}=AT_{\mu\nu}, \text{ for } \mu = 0,1,2,3. \quad (10)$$

where A is a scalar constant and where $B_{\mu\nu}$ is a tensor with the particular property that its covariant derivative is zero. Furthermore, because of

$$\frac{D}{dq_{\mu}}g_{\mu\nu}=0. \quad (11)$$

i.e., because of the property the covariant derivatives of the metric tensor $g_{\mu\nu}$ are zero, we have,

$$B_{\mu\nu}=\Lambda g_{\mu\nu}, \quad (12)$$

where Λ is a scalar constant. As is well known and shown in the appendix once more, inclusion of this constant implies that under absence of massive sources, Einstein's equation can be satisfied if empty space is given up and is replaced by a space that behaves as a perfect liquid in thermodynamic equilibrium. In this condition the stress-energy tensor of space-time (described in Hawking-metric) without massive sources changes from $T_{\mu\mu}=0\rightarrow T_{\mu\mu}=-p\Lambda$, where $p=g_{\mu\nu}/8\pi G$, [9,10,11]. If in this fluid a massive pointlike source is inserted, the resulting wave equation under the weak field constraint is a modification of (5), such that

$$-\frac{\partial^2}{c^2\partial t^2}(r\Phi)+\frac{\partial^2}{\partial r^2}(r\Phi)+\lambda^2(r\Phi)=-r\frac{8\pi GM}{c^2}\delta^3(r), \quad (13)$$

where $\lambda^2=2\Lambda$.

This is shown in the appendix as well. From the perspective of classic field theory, a wave equation can be conceived as the result of an equation of motion derived under application of the action principle from a Lagrangian density L of a scalar field with the generic format

$$\mathrm{L}=-\frac{1}{2}\partial_{\mu}\Phi\partial^{\mu}\Phi+U(\Phi)+\rho\Phi, \quad (15)$$

where $U(\Phi)$ is the potential energy of the field and where $\rho\Phi$ is the source term. Comparing various fields of energy, we have,

$U(\Phi)=0$ for electromagnetism.

$U(\Phi) = -\lambda^2\Phi^2 / 2$ for this case,

$U(\Phi) = \lambda^2\Phi^2 / 2$ for the nuclear forces [12]. (16)

The non-trivial solutions of (14) in homogeneous format are, for the first case and the third case, respectively,

$$\Phi = \frac{\Phi_0}{\lambda r} \text{ and } \Phi = \Phi_0 \frac{\exp(-\lambda r)}{\lambda r}. \tag{17}$$

The first case applies to electromagnetism (for $\Phi_0 = Q\lambda / 4\pi\varepsilon_0$) and Newtonian gravity (for $\Phi_0 = -MG\lambda$). The third case applies to Proca's generalization of the Maxwellian field [13]. It reduces to the first case if $\lambda \to 0$, while keeping Φ_0 / λ constant. Generically, it represents a field with a format that corresponds with the potential as in the case of a shielded electric field (Debije [14]), as well with Yukawa's proposal [15] to explain the short range of the nuclear force.

Let us now consider the (unusual) second case. It can be readily verified from (14), and elaborated once more in the Appendix, that a non-trivial solution for this case is,

$$\Phi = \Phi_0 \frac{\cos\lambda r + \sin\lambda r}{\lambda r}. \tag{18}$$

In accordance with the concepts of classical field theory, the field strength can be established as the spatial derivative of the potential Φ. Identifying Φ_0 / λ as $-MG$ and λ as a range parameter, we may identify this field strength as a cosmological gravitational acceleration g. Let us compare this acceleration with the Newtonian one g_N. To do so more explicitly, we compare $g_N r^2$ with gr^2. The comparison is shown in figure 1.

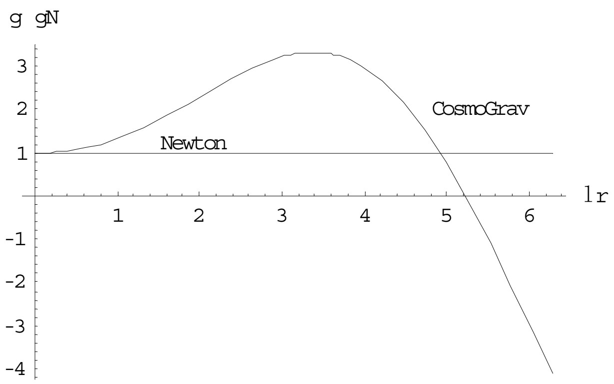

Figure 1: The cosmological gravity force compared with the Newtionian force

This figure shows that, for relative small values of r, the cosmological acceleration behaves similarly as the Newtonian one, but that its relative strength over the Newtonian one increases significantly for large values of r. This is a similar behavior as heuristically implemented in MOND. The effective range is determined by the parameter λ. It might

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therefore well be that the cosmological gravity force manifests itself only at cosmological scale. Let us consider its consequence.

Newtonian laws prescribe that the *transverse* velocity $v_\varphi(r)$ of a cosmic object revolving in a circular orbit with radius r in a gravity field is determined by

$$v_\varphi^2(r) = \frac{M(r)G}{r}. \tag{19}$$

where $M(r)$ is the amount of enclosed mass and where G is the gravitational constant. This relationship is often denoted as Kepler's third law. Curiously, like first announced by Vera Rubin [16] in 1975, the velocity curve of cosmic objects in a galaxy, such as, for instance, the Milky Way, appears being almost flat. It is tempting to believe that this can be due to a particular spectral distribution of the spectral density to compose $M(r)$. This, however, cannot be true, because $M(r)$ builds up to a constant value of the overall mass. And Kepler's law states in fact that a flat mass curve $M(r)$ is not compatible with a flat velocity curve. Figure 2 illustrates the problem.

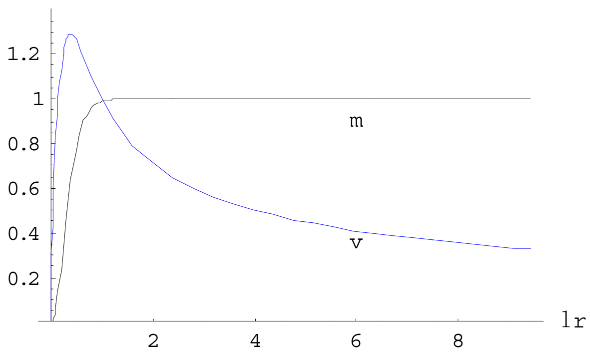

Figure 2. Incompatibility of a flat enclosed mass curve with a flat rotation curve.

It is one of the two: either the gravitational acceleration is, at cosmological distances, larger than the Newtonian one, or dark matter, affecting the mass distribution is responsible. Cosmological gravity as expressed by (18) may give the clue. Its effective range is determined by the parameter λ. It might therefore well be that the cosmological gravity force manifests itself only at cosmological scale. Figure 3 shows that under influence of this force, the rotation curves in the galaxy may assume a flat behavior indeed.

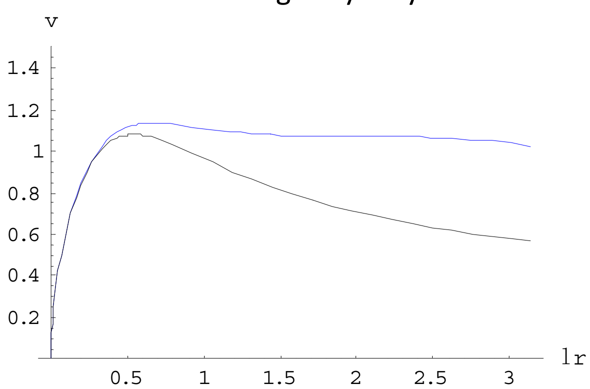

Figure 3: boost of the rotation curve under influence of cosmological gravity.

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This hypothetical cosmological gravity shows an intriguing phenomenon. Like shown in figure 4, at very far cosmological distance, the attraction of gravity is inversed into repulsion. There is some speculation reported in literature that such antigravity is required to explain the phenomenon of dark energy, responsible for the accelerated expansion of the universe [17]. Exploration of this phenomenon is a subject outside the scope of this article. It has to be noted that the solution (18) is not unique. There are more solutions possible by modifying the magnitude of $\sin\lambda r$ over $\cos\lambda r$. I have simply chosen here for the symmetrical solution. Cosmological observations would be required to obtain more insight in this. Such observations are required as well for establishing meaningful values for λ.

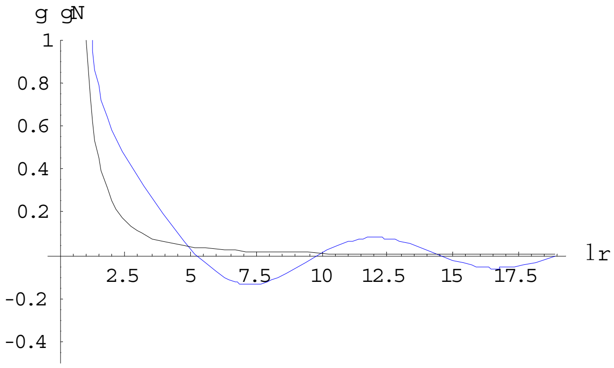

Figure 4: Inversion of the gravity force to antigravity at large cosmological distances. Black: Newtonian. Blue: Cosmological Gravity.

Comparison with MOND

It is instructive to compare this view on cosmological gravity with MOND. MOND is a heuristic approach based on a modification of the gravitational acceleration g such that

$$g = \frac{g_N}{\mu(x)}, \text{ with } x = g/a_0 \tag{20}$$

where $\mu(x)$ is an interpolation function, $g_N (= MG/r^2)$ the Newtonian gravitational acceleration and where a_0 is an empirical constant acceleration. The format of the interpolation function is not known, but the objectives of MOND are met by a simple function like [4,18]

$$\mu(x) = \frac{x}{\sqrt{1+x^2}}. \tag{21}$$

If $g/a_0 << 1$, such as happens for large r, (20) reduces to

$$g = \sqrt{a_0 g_N}. \tag{22}$$

Under this condition, the gravitational acceleration decreases as r^{-1} instead of r^{-2}. As a result, the orbital velocity curves as a function of r show up as flat curves.
Algebraic evaluation of (20) and (21) results into,

$$\frac{g}{g_N} = \sqrt{\frac{1+\sqrt{1+4k^2(\lambda r)^4}}{2}} \quad \text{with } k = \frac{a_0}{MG\lambda^2} \ . \tag{23}$$

This expression allows a comparison with the hypothesis as developed in this article. From (18), under consideration of $\Phi_0 = MG\lambda$,

$$\Phi = \Phi_0 \frac{\cos\lambda r + \sin\lambda r}{\lambda r} \rightarrow g = -\nabla\Phi = \frac{MG}{r^2}\{(1-\lambda r)\cos\lambda r + (1+\lambda r)\sin\lambda r\} \ , \tag{24}$$

hence

$$\frac{g}{g_N} = (1-\lambda r)\cos\lambda r + (1+\lambda r)\sin\lambda r \ . \tag{25}$$

As illustrated in figure 5, a pretty good fit between (23) and (25) is obtained if

$$k = \frac{a_0}{MG\lambda^2} = 2.5 \rightarrow a_0 = 2.5MG\lambda^2 \ . \tag{26}$$

Observations on various galaxies have shown that a_0 can be regarded as a galaxy-independent constant with a value about $a_0 \approx$ 1x10^{-10} m/s^2.

The implication of (28) is, that $a_0 \approx$ 1x10^{-10} m/s^2 is a second gravitational constant next to G. The two constants determine the range λ of the gravitational force in solar systems and galaxy systems as $\lambda^2 \approx 2a_0 / 5MG$, where M is the enclosed mass in those systems. Where this second gravitational quantity a_0 is a constant, this is apparently not true for the Einsteinean parameter Λ.

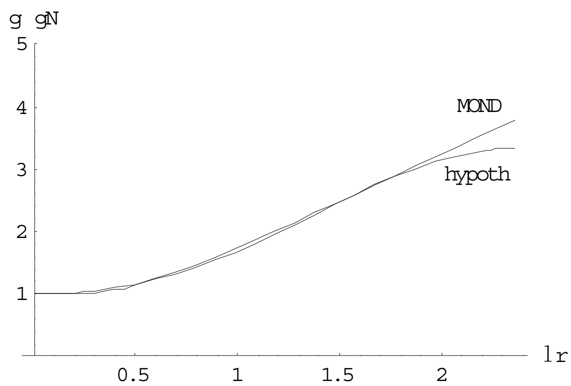

Figure 5: MOND's interpolation function compared with the theory as developed.

Discussion

In the description given in this article, gravity shows up as the disturbance of the equilibrium state of some fluid in space by a massive source. The fluid executes a negative pressure on the energetic flux from the source. It is the consequence of the adoption of a positive valued cosmological constant in Einstein's Field Equation. How to interpret the physical nature of this fluid is a still unanswered question. Empty space could be a dance of virtual particles that exist within Heisenberg's uncertainty interval. That view comes close to the challenging proposal as has been put forward by Verlinde [20], who regards space as a sea of such virtual particles and who relates the gravitational process with the disturbance of their entropy, from which Einstein's equation comes forward as an emergent result. Other authors explain the fluid as a result of gravitational vacuum polarization [21,22,23]. This hypothetical phenomenon is based upon the concept of virtual particle pairs that may exist within the uncertainty interval, such as put forward by Hawking in his studies on black holes [24]. Let us suppose, like Hajdukovic [22] did, that such pairs are elementary gravitational dipoles. In vacuum these dipoles have a random orientation. Their direction will be polarized under influence of a massive source. Similarly as in the case of electrical polarization of a dielectric material, the process can be characterized by a "charge" density $\rho_r(r)$ and a polarization vector $\mathrm{P_g}$. These are related as [25],

$$\rho_r(r) = -\nabla \cdot \mathbf{P_g}. \tag{27}$$

Identifying the charge density as the T_{tt} component of the stress-energy tensor, we know from the analysis as presented in this article that ρ_r is a negative constant decreasing with the mass M of the polarizing source. More particularly,

$$\rho_r = -p\Lambda = -\frac{c_o}{M}, \quad \text{with } c_0 = \frac{a_0}{10\pi G^2}. \tag{28}$$

Hence, from (27) and (28),

$$\rho_r = \frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\{r^2 P_g(r)\} = -\frac{c_0}{M}. \tag{29}$$

After integrating, we have

$$P_g(r) = -\frac{c_0}{3M}r + C_0. \tag{30}$$

Obviously C_0 is the maximum density of polarized dipoles. It occurs at $r=0$. For $r>0$ the number of polarized dipoles gradually decreases. That this decrease is less the larger the mass M of the polarizing source is not surprising. It is therefore not surprising as well that Milgrom's acceleration constant a_0 is the true constant of nature and that Einstein's Cosmological Constant decreases with M. This makes the gravitational process as described in this article akin to the Debye process of an electrically charged particle in a plasma. The difference, though, is in the sign

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of the pressure, which can be accommodated in the sign of the polarization vector $\mathbf{P}_g$. In the Debye process the resulting Coulomb field is suppressed ("screened"), because a charged source is surrounded by a (displacement) space charge with opposite sign. In the gravitational process the field is enhanced, because the massive source is surrounded by a (displacement) space charge with the same sign. These processes are characterized by a range parameter λ. In the gravitational case, λ is closely related with the cosmological constant as $\lambda^2 = 2\Lambda$. This view fits extremely well with the theory developed in this article. The disclaimer, though, is the problem how to relate the virtual pair with known quantum mechanical particles.

The gravitational model adopted in this article is isotropic and spherically symmetric. It therefore applies to solar and galaxy systems. The concept may apply to the universe as whole. In that case a somewhat different description is required, because, according to Friedmann's view, the universe has to be conceived as an equi-temporal plane without a center. The gravitational model as developed in this article can be harmonized with the heuristic Milgrom's MOND hypothesis for galaxies, which is supported by overwhelming experimental evidence from observations. This harmonization requires to equate $\lambda^2 (= 2\Lambda) \approx 2a_0 / 5MG$, where M is the enclosed mass in those systems and where Milgrom's acceleration constant $a_0 \approx$ 1x10^{-10} m/s^2 shows up as a true second gravitational constant next to G. This is not in conflict with Einstein's theory, because the cosmological constant is not necessarily a constant of nature. As pointed out before, it is a scalar constant that therefore does not show a dependence on space-time coordinates. It may depend on physical attributes (like M).

Conclusion

It has been shown that the weak field limit solution of Einstein's Field Equation with inclusion of the Cosmological Constant, under the constraint of spherical isotropy, produces a gravitational wave equation with an underlying Lagrangian density in a format that resembles the scalar part of Proca's generalization of the Maxwellian one. For electromagnetism, Proca's "mass term" is zero, for nuclear (Yukawa) forces the "mass term" is positive, for gravity the "mass term" is negative. As a consequence, the electromagnetic field potential decays as $1/r$, the nuclear potential decays more aggressively as $\exp(-\lambda r)/r$ and the gravity potential decays less aggressively as $(\cos \lambda r + \sin \lambda r)/r$. Effectively, the gravity potential remains the Newton one in our common world, but is different at cosmological scale. This property explains the cosmological phenomenon that is usually assigned to dark matter. Because of the match in results, the developed model can be regarded as an underlying theory for the heuristic MOND approach, albeit that the prognosis that, at very large cosmological distances, gravity periodically turns on-and-off into antigravity marks a decisive difference. It is shown in this article that the range determining parameter λ is related with a second gravitational constant $a_0 \approx$ 1x10^{-10} m/s^2 next to G. The two constants determine the range λ of the gravitational force in solar systems and galaxy systems as $\lambda^2 \approx 2a_0 / 5MG$, where M is the enclosed mass in those systems. So, where this

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second gravitational quantity a_0 seems to be a constant of nature, this is not true for the Einsteinean parameter Λ, which appears being just a scalar constant, i.e., being independent of space-time coordinates. The theory as developed in this article gives an adequate explanation for the galaxian phenomenon of flat rotation curves and for the cosmological phenomenon that our universe is expanding in acceleration, such as predicted by Friedmann's law, under influence of a positive value of Einstein's cosmological parameter.

APPENDIX : THE GRAVITATIONAL WAVE EQUATION

The objective in this appendix is to derive the weak field limit of the gravitational wave equation with inclusion of the Cosmological Constant. This objective implies that we have to solve Einstein's Field Equation for a spherically symmetric space-time metric that is given by the line element (2),

$$\mathrm{d}s^2 = g_{tt}(r,t)\mathrm{d}q_0^2 + g_{rr}(r,t)\mathrm{d}r^2 + r^2\sin^2\vartheta\mathrm{d}\varphi^2 + r^2\mathrm{d}\vartheta^2\,, \tag{A-1}$$

where $q_0 = \mathrm{i}ct$.

Note: The space-time ($\mathrm{i}ct$, r, ϑ,φ) is described on the basis of the "Hawking" metric (+,+,+,+). The components $g_{\mu\mu}$ compose the metric tensor $g_{\mu\nu}$, which determine the Ricci tensor $R_{\mu\nu}$ and the Ricci scalar R. These quantities play a decisive role in Einstein's Field Equation, which reads as

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \quad \text{with} \quad G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\,. \tag{A-2}$$

In a space without massive sources, the Einstein Field Equation under this symmetric spherical isotropy, reduces to a simple set of equations for the elements $R_{\mu\mu}$ of the Ricci tensor,

$$R_{tt} - \frac{1}{2}Rg_{tt} + \Lambda g_{tt} = 0\,; \qquad R_{rr} - \frac{1}{2}Rg_{rr} + \Lambda g_{rr} = 0\,; \tag{A-3a,b,c,d}$$

$$R_{\vartheta\vartheta} - \frac{1}{2}Rg_{\vartheta\vartheta} + \Lambda g_{\vartheta\vartheta} = 0\,; \quad R_{\varphi\varphi} - \frac{1}{2}Rg_{\varphi\varphi} + \Lambda g_{\varphi\varphi} = 0.$$

Let us proceed by considering the Ricci scalar. It is defined generically as

$$R = \sum_{\mu=0}^{3}\sum_{\nu=0}^{3} g^{\mu\nu}R_{\mu\nu}\,. \tag{A-4}$$

In spherical symmetry the matrices contain diagonal elements only, so that (A-4) reduces to

$$R = \sum_{\mu=0}^{3} g^{\mu\mu}R_{\mu\mu}\,. \tag{A-5}$$

This result can be applied to (A-3). Multiplying the first one with $g^{00}(=g^{tt})$, the second one with g^{11}, etc., and subsequent addition results of the terms $\mu=1,2,3$ gives, (A-6),

$$\sum_{\mu=1}^{3} g^{\mu\mu}R_{\mu\mu}-\frac{3}{2}R+3\Lambda=-g^{tt}R_{tt}+\sum_{\mu=0}^{3} g^{\mu\mu}R_{\mu\mu}-\frac{3}{2}R+3\Lambda=-g^{tt}R_{tt}-\frac{1}{2}R+3\Lambda=0$$

so that $g^{tt}R_{tt}=-\frac{1}{2}R+3\Lambda$. (A-7)

Repeating this recipe for $g_{\mu\mu}(=g^{\mu\mu})$, we have for reasons of symmetry

$$g^{\mu\mu}R_{\mu\mu}=-\frac{1}{2}R+3\Lambda. \tag{A-8}$$

Note that the subscripts and superscripts 00, 11 ,22, and 33 are, respectively, identical to $tt, rr, \vartheta\vartheta$ and $\varphi\varphi$. Applying this result to Einstein's equation set gives,

$$2g^{\mu\mu}R_{\mu\mu}-2\Lambda=\frac{8\pi GT_{\mu\mu}g^{\mu\mu}}{c^4}, \tag{A-9}$$

such that after multiplication by g_{tt}, we have

$$2R_{\mu\mu}-2g_{\mu\mu t}\Lambda=\frac{8\pi GT_{\mu\mu}}{c^4}, \tag{A-10}$$

Let us proceed under the condition of the absence of massive sources ($T_{\mu\mu}=0$) and let us consider the Ricci tensor component R_{tt} under use of the results shown in Table A-1, obtained by a calculation shown later in this Appendix. Note: g' and g'' means differentiation, respectively double differentiation of g into r; $\dot{g}$ and $\ddot{g}$ means differentiation, respectively double differentiation of g into t. Multiplying (A-3a) by $1/g_{tt}$ and (A-3b) by $1/g_{rr}$ gives,

$$\frac{R_{tt}}{g_{tt}}-\frac{1}{2}R+\Lambda=0 \text{ and } \frac{R_{rr}}{g_{rr}}-\frac{1}{2}R+\Lambda=0, \tag{A-11}$$

which, after subtraction and under use of the expressions in Table A-1 results into.

$$-\frac{1}{r}\frac{1}{g_{rr}}\left(\frac{g'_{rr}}{g_{rr}}+\frac{g'_{tt}}{g_{tt}}\right)=0, \tag{A-12}$$

hence

$$\frac{g'_{rr}}{g_{rr}}+\frac{g'_{tt}}{g_{tt}}=0\,, \qquad \text{(A-13)}$$

which can be integrated to (the Schwarzschild condition),

$$g_{rr}g_{tt}=1\,. \qquad \text{(A-14)}$$

This, in turn, gives

$$\frac{\dot{g}_{rr}}{g_{rr}}+\frac{\dot{g}_{tt}}{g_{tt}}=0\,, \qquad \text{(A-15)}$$

Using (A-13), (A-15) and the Table A-1 values on R_{tt} gives

$$R_{tt}=\frac{1}{g_{rr}}\left(-\frac{1}{2}g''_{tt}-\frac{1}{r}g'_{tt}-\frac{\ddot{g}_{rr}}{2c^2}\right)=\frac{1}{g_{rr}}\left(-\frac{1}{2r}\frac{\partial^2(rg_{tt})}{\partial r^2}+\frac{1}{2c^2}\frac{\partial^2 g_{tt}}{\partial t^2}\right). \qquad \text{(A-16)}$$

Hence, from (A-10) and (A-16) ,

$$\frac{2}{g_{rr}}\left(-\frac{1}{2r}\frac{\partial^2(rg_{tt})}{\partial r^2}+\frac{1}{2c^2}\frac{\partial^2 g_{tt}}{\partial t^2}\right)-2\Lambda g_{tt}=\frac{8\pi G T_{tt}}{c^4}\,, \qquad \text{(A-17)}$$

or, equivalently,

$$-\frac{1}{r}\frac{\partial^2(rg_{tt})}{\partial r^2}+\frac{1}{c^2}\frac{\partial^2 g_{tt}}{\partial t^2}-2\Lambda=\frac{8\pi G T_{tt}}{c^4}g_{rr}\,, \qquad \text{(A-18)}$$

Applying the well-known conditions,

$\Lambda = 0$ (no cosmological constant),

$g_{tt}(r,t)=1+h_\varphi(r,t)$, where $|h_\varphi(r,t)|<<1$ (the weak field limit)

$T_{tt}=Mc^2\delta^3(r)$, (pointlike massive source) (A-19a,b,c)

yields the proper wave equation

$$\frac{\partial^2(rh_\varphi)}{\partial r^2}-\frac{1}{c^2}\frac{\partial^2 rh_\varphi}{\partial t^2}=-\frac{8\pi GM}{c^2}r\delta^3(r)\,, \qquad \text{(A-20)}$$

which results in the static regime to

$$\frac{1}{r}\frac{\partial^2(rh_\varphi)}{\partial r^2}=-\frac{8\pi GM}{c^2}\delta^3(r)\,. \qquad \text{(A-21)}$$

This is similar to Poisson's equation,

$$\nabla^2\Phi = \frac{1}{r}\frac{\partial^2(r\Phi)}{\partial r^2} = -4\pi G\rho = -4\pi GM\delta^3(r)\,, \tag{A-22}$$

the solution of which is the Newtonian potential,

$$\Phi = -\frac{GM}{r}\ [\mathrm{m^2s^{-2}}]. \tag{A-23}$$

Comparing (A-20) with (A-22) gives the equivalence

$$h_\varphi = \frac{2\Phi}{c^2}\,. \tag{A-24}$$

Let us now consider the case $\Lambda \neq 0$ under absence of a massive source. Obviously, (A-18) is only satisfied if the influence of the cosmological constant is counter balanced by the hypothetical source

$$T_{tt} = -p\Lambda\,,\ \text{where}\ p = \frac{c^4}{4\pi G}\ \text{and}\ g_{rr} = 1\,. \tag{A-25}$$

Note the factor 2 difference with [11], which can be traced back to the (overlooked) Λ dependence in R.

Because all four members of the Einstein set (A-10) have to be satisfied, we have, under consideration of (A-10) and Table A=1,

$$T_{\mu\mu} = -p\Lambda\ \text{and}\ g_{\mu\nu} = 1\,. \tag{A-26}$$

This particular stress-energy tensor with equal diagonal elements corresponds with the one for a perfect fluid in thermodynamic equilibrium [21]. So, where empty space corresponds with virtual sources $T_{\mu\mu} = 0$, the fluidal space corresponds with virtual sources $T_{\mu\mu} = -p\Lambda$. Insertion of a massive pointlike source in this fluid and modifying (A-17) by adding the virtual sources, gives

$$\frac{2}{g_{rr}}\left(-\frac{1}{2r}\frac{\partial^2(rg_{tt})}{\partial r^2} + \frac{1}{2c^2}\frac{\partial^2 g_{tt}}{\partial t^2}\right) - 2\Lambda g_{tt} = \frac{8\pi G T_{tt}}{c^4} - 2\Lambda\,. \tag{A-27}$$

Under the weak field limit condition, this equation evaluates to

$$\frac{\partial^2(rh_\varphi)}{\partial r^2} - \frac{1}{c^2}\frac{\partial^2 rh_\varphi}{\partial t^2} + 2\Lambda rh_\varphi = -\frac{8\pi GM}{c^2}r\delta^3(r)\,. \tag{A-28}$$

Obviously, this is a proper wave function. After redefining the scalar constant Λ as

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$$\Lambda = \frac{\lambda^2}{2}, \qquad \text{(A-29)}$$

It is written as

$$\frac{\partial^2 (rh_\varphi)}{\partial r^2} - \frac{1}{c^2}\frac{\partial^2 rh_\varphi}{\partial t^2} + \lambda^2 rh_\varphi = -\frac{8\pi GM}{c^2} r\delta^3(r). \qquad \text{(A-30)}$$

If $\Lambda < 0$, we have under static conditions, a similarity with Helmholtz' equation with the screened Poisson's equation, the solution of which is Yukawa's potential,

$$\Phi = \frac{GM}{r}\exp(-\lambda r), \qquad \text{(A-31)}$$

which reduces to Poisson's one for $\lambda \to 0$.

If $\Lambda > 0$, we have under static conditions, a similarity with Helmhotz' equation [19] with a characteristic solution,

$$\Phi = \frac{GM}{r}\{\cos\lambda r + \sin\lambda r\}. \qquad \text{(A-32)}$$

This solution reduces to Poisson's one for $\lambda \to 0$ as well.

This is the weak field limit solution of Einstein's Equation if one does not take the validity of Poisson's equation of gravity for granted, but adopts Helmholtz equation instead under an appropriate choice of the Cosmological Constant.

Table A1: metric tensor and Ricci tensor

metric tensor	Ricci tensor
$g_{tt} \equiv g_{00}$	$R_{tt} = -\frac{1}{2}\frac{g''_{tt}}{g_{rr}} - \frac{\ddot{g}_{rr}}{2c^2 g_{rr}} + \frac{g'_{tt}}{4g_{rr}}(\frac{g'_{rr}}{g_{rr}} + \frac{g'_{tt}}{g_{tt}}) - \frac{\dot{g}_{rr}}{4c^2 g_{rr}}(\frac{\dot{g}_{rr}}{g_{rr}} + \frac{\dot{g}_{tt}}{g_{tt}}) - \frac{1}{r}\frac{g'_{tt}}{g_{rr}}$
$g_{rr} \equiv g_{11}$	$R_{rr} = -\frac{1}{2}\frac{g''_{tt}}{g_{tt}} - \frac{\ddot{g}_{rr}}{2c^2 g_{tt}} + \frac{g'_{tt}}{4g_{tt}}(\frac{g'_{rr}}{g_{rr}} + \frac{g'_{tt}}{g_{tt}}) - \frac{\dot{g}_{rr}}{4c^2 g_{tt}}(\frac{\dot{g}_{rr}}{g_{rr}} + \frac{\dot{g}_{tt}}{g_{tt}}) + \frac{1}{r}\frac{g'_{rr}}{g_{rr}}$
$g_{\vartheta\vartheta} \equiv g_{22} = r^2$	$R_{\vartheta\vartheta} = 1 + \frac{r}{2g_{rr}}(\frac{g'_{rr}}{g_{rr}} - \frac{g'_{tt}}{g_{tt}}) - \frac{1}{g_{rr}}$
$g_{\varphi\varphi} \equiv g_{33} = r^2\sin^2(\vartheta)$	$R_{\kappa\varphi} = \sin^2(\vartheta) R_{\vartheta\vartheta}$

Calculation of the Ricci tensor

The Ricci tensor is described in expanded form by

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$$R_{ij} = \sum_{k=0}^{3} \left(\frac{\partial \Gamma_{ij}^{k}}{\partial q_k} - \frac{\partial \Gamma_{ik}^{k}}{\partial q_j}\right) + \sum_{i=0}^{3}\sum_{k=0}^{3} \left(\Gamma_{ij}^{i}\Gamma_{ki}^{k} - \Gamma_{ik}^{i}\Gamma_{ji}^{k}\right). \qquad \text{(A-33)}$$

The Christoffel symbols Γ_{ij}^{k} represent functions of the metric elements, such that

$$\Gamma_{ij}^{k} = \frac{1}{2}\sum_{m=0}^{3} g^{km} \left\{\frac{\partial g_{jm}}{\partial q_i} + \frac{\partial g_{im}}{\partial q_j} - \frac{\partial g_{ji}}{\partial q_m}\right\}. \qquad \text{(A-34)}$$

Under symmetric spherical isotropy, only diagonal terms remain, so that the expression reduces to

$$R_{ii} = \sum_{k=0}^{3} \left(\frac{\partial \Gamma_{ii}^{k}}{\partial q_k} - \frac{\partial \Gamma_{ik}^{k}}{\partial q_i}\right) + \sum_{i=0}^{3}\sum_{k=0}^{3} \left(\Gamma_{ii}^{i}\Gamma_{ki}^{k} - \Gamma_{ik}^{i}\Gamma_{ii}^{k}\right), \qquad \text{(A-35)}$$

and the Christoffel symbols reduce to

$$\Gamma_{ij}^{k} = \frac{1}{2 g_{kk}} \left\{\frac{\partial g_{kj}}{\partial q_i} + \frac{\partial g_{ki}}{\partial q_j} - \frac{\partial g_{ji}}{\partial q_k}\right\}, \qquad \text{(A-36)}$$

such that only three different forms remain,

$$\Gamma_{kk}^{k} = \frac{1}{2 g_{kk}} \frac{\partial g_{kk}}{\partial q_k}; \quad \Gamma_{ii}^{k} = -\frac{1}{2 g_{kk}} \frac{\partial g_{ii}}{\partial q_k} \ (k \neq i) \text{ and } \Gamma_{ik}^{k} = \Gamma_{ki}^{k} = \frac{1}{2 g_{kk}} \frac{\partial g_{kk}}{\partial q_i}. \qquad \text{(A-37)}$$

Table A2: Christoffel elements and affine connections of the isotropic non-rotating metric

Γ_{tt}^{t}	Γ_{tr}^{t}		
Γ_{rt}^{t}	Γ_{rr}^{t}		

Γ_{tt}^{r}	Γ_{tr}^{r}		
Γ_{rt}^{r}	Γ_{rr}^{r}		
		$\Gamma_{\vartheta\vartheta}^{r}$	
			$\Gamma_{\varphi\varphi}^{r}$

		$\Gamma_{r\vartheta}^{\vartheta}$	
	$\Gamma_{\vartheta r}^{\vartheta}$		
			$\Gamma_{\varphi\varphi}^{\vartheta}$

			$\Gamma_{r\varphi}^{\varphi}$
			$\Gamma_{\vartheta\varphi}^{\varphi}$
	$\Gamma_{\varphi r}^{\varphi}$	$\Gamma_{\varphi\vartheta}^{\varphi}$	

Table A2 shows the Christoffel elements different from zero, where

$$\Gamma^t_{tt} = \frac{1}{2ic}\frac{\dot{g}_{tt}}{g_{tt}} \quad \Gamma^t_{rr} = -\frac{1}{2ic}\frac{\dot{g}_{rr}}{g_{tt}} \quad \Gamma^t_{tr} = \Gamma^t_{rt} = \frac{1}{2}\frac{g'_{tt}}{g_{tt}}$$

$$\Gamma^r_{rr} = \frac{1}{2}\frac{g'_{rr}}{g_{rr}} \quad \Gamma^r_{tt} = -\frac{1}{2}\frac{g'_{tt}}{g_{rr}} \quad \Gamma^r_{rt} = \Gamma^r_{tr} = \frac{1}{2ic}\frac{\dot{g}_{rr}}{g_{rr}}$$

$$\Gamma^r_{\vartheta\vartheta} = -\frac{r}{g_{rr}} \quad \Gamma^r_{\varphi\varphi} = -\frac{r\sin^2\vartheta}{g_{rr}}$$

$$\Gamma^\vartheta_{r\vartheta} = \Gamma^\vartheta_{\vartheta r} = \frac{1}{r} \quad \Gamma^\vartheta_{\varphi\varphi} = -\sin\vartheta\cos\vartheta$$

$$\Gamma^\varphi_{r\varphi} = \Gamma^\varphi_{\varphi r} = \frac{1}{r} \quad \Gamma^\varphi_{\vartheta\varphi} = \Gamma^\varphi_{\varphi\vartheta} = \cot\vartheta \qquad \text{(A-38)}$$

Application of (A-38) on (A-33) gives the Ricci tensor as listed in Table A1.

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