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A Stability Criterion Model of Flexible Footbridges under Crowd-Induced Vertical Excitation

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Abstract: The excessive vibration caused by crowds walking across footbridges has attracted great public concerns in the past few decades. This paper presents, from considering the dynamic characteristics of the bipedal crowd model, a new stability limit criterion based on the bipedal excitation model. The stability limit can be used to estimate the upper boundary of crowd size. In addition, the dynamic stable performances of a structure, under a certain walking crowd size, can be predicted by the stability criterion. This proposed mechanism provides an alternative comprehension how crowd excite the excessive sway motion with a large-span structure.

Keywords: stability criterion; bipedal excitation; excessive vibration; dynamic performances.

1. Introduction

These structures with long-spans have been become prevalent all over the world and their dynamic vibrations are the crucial concern in its servicing period. A structure with the longer span is prone to trigger an excessive sway motion. The evidences from some vibrational accidents and investigations have showed how the bridge becomes flexible along with the extension of span (Newland, 2003; Fujino and Siringoringo, 2013). Some controlling techniques including the passive control (Soong and Spencer, 2002; Fan et al., 2010), active control (Nyawako and Reynolds, 2007) and semi-active control (Jalili, 2002) methods have been used to mitigate the effect of crowd-induced vibrations (Hudson and Reynolds, 2012). Moreover, some crowd-induced excited models (Matsumoto et al., 1978; Roberts, 2005; Piccardo and Tubino, 2008) are proposed to investigate the lateral resonance mechanism of flexible footbridges. Some further studies considering dynamic interaction between human and structure such as the kinetic crowd biomechanical model (Carroll et al., 2013) have been recommended to investigate the lateral vibration mechanism of

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footbridges. However, the investigations about the vertical structural vibration under crowd-induced excitations are rare. Zhou et al. (2006 and 2016) and Yang et al. (2013) studied the vertical dynamic characteristics of structure under a modeled human oscillator. Zivanovic (2015) reviewed the experimental and numerical developments of lightweight structures under human actions. Qin et al. (2013 and 2014) studied dynamic performances of footbridge under a walking biomechanical bipedal pedestrian model based on a constant walking energy level. However, the vertical vibrational stability of structure under dynamic crowds is none.

In this paper, a stability limit criterion with the vertical vibration is studied based on the crowd excitation mechanism with the modeled bipedal pedestrians (Qin et al, 2013). Firstly, the dynamic equilibrium equation of a structure is established by considering the vertical ground reaction force (GRF) between footholds and pavement. In addition, an assumed uniform distribution of the walking crowds is used to calculate the crowd size by employed the Taylor Expansion. Finally, the stability limit is identified and the upper boundary of crowd size for a stable vibration can be estimated according to the stability limit. Some parameters about the stability limit criterion are analyzed and a numerical example is applied to assess the stable state of a footbridge under dynamic walking crowds.

2. Dynamic excitation mechanism

A pedestrian, as the basic unit of a pedestrian flow, is modeled with the bipedal biomechanics model of mass-spring-dampers (Qin et al., 2013) as shown in Fig.1. The structure is simulated by a simply supported Euler-Bernoulli beam with a uniform section. L_B is the span length. EI means the flexural stiffness of the beam and $\overline{m}$ is the mass percent unit length along in the longitudinal direction. The left end of the beam is defined as the origin of a planar coordinate system x-0-y. The q^{th} $(q = 1, \cdots, \chi)$ pedestrian from the crowd size χ is simulated by the bipedal model with the lump mass $m^{(q)}$. $k_l^{(q)}$ and $k_t^{(q)}$ are the leading and trialing leg stiffness coefficients, respectively. $c_l^{(q)}$ and $c_t^{(q)}$ are the leading and trialing leg damping coefficients, respectively. $x_l^{(q)}$ and $x_t^{(q)}$ are the leading and trialing footholds positions in longitudinal direction, respectively. $x^{(q)}$ and $z^{(q)}$ are the longitudinal and vertical positions of pedestrian center of mass (CoM), respectively. $\theta_l^{(q)}$ means the intersection angle between the beam and leading leg; $\theta_t^{(q)}$ means the intersection angle between the beam and trialing leg. $F_l^{(q)}$ is the ground reaction force between the leading foothold and pavement along the leg axial. $F_t^{(q)}$ is the ground reaction force between the trialing foothold and pavement along the leg axial. Where, subscripts 'l' and 't' denote leading and trialing legs, respectively. There is an assumption that there is no slip between feet and ground. $z^{(q)}$ is the vertical displacement of CoM due to pedestrian-self vibration. $w^{(q)}$ is the vertical displacement of CoM due to structural vibration.

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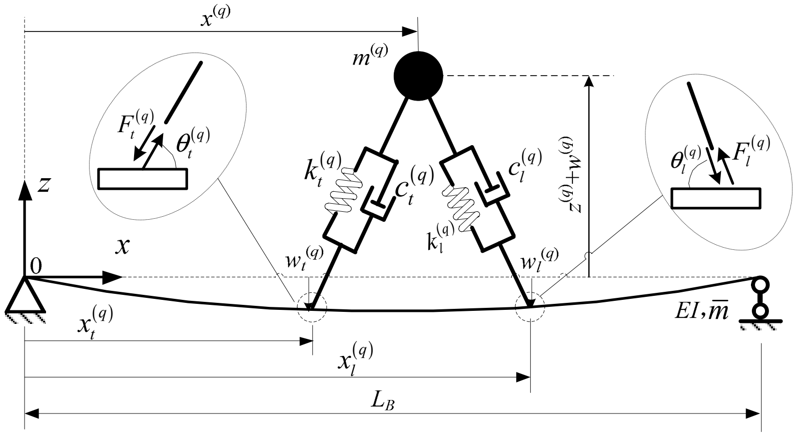

Fig.1. Crowd-structure interaction model

The dynamic equilibrium equation of the beam can be obtained as

$$EI\frac{\partial^4 w(x,t)}{\partial x^4}+\bar{m}\frac{\partial^2 w(x,t)}{\partial t^2}+c_s\frac{\partial w(x,t)}{\partial t}=\sum_{q=1}^{\chi}\delta\left(x-x_l^{(q)}\right)F_l^{(q)}\sin\theta_l^{(q)}+\sum_{q=1}^{\chi}\delta\left(x-x_t^{(q)}\right)F_t^{(q)}\sin\theta_t^{(q)} \tag{1}$$

where $\delta(\blacksquare)$ is the Dirac function; c_s is damping coefficient the beam. $w(x,t)$ is the vertical displacement of the beam at the position *x* and time point *t* and its expression is

$$w(x,t)=\mathbf{\Phi}(x)\mathbf{Y}(t)=\sum_{i=1}^{n}\phi_i(x)Y_i(t) \tag{2}$$

where $\phi_i(x)$is the modal function satisfying boundary condition and $Y_i(t)$is the generalized co-ordinate of the i^{th} mode for the beam; *n* is total modal number.

Defining $\phi_i(x)=\sin(i\pi x/L_B)$ and substituting the formula of $\phi_i(x)$ into the Eq. (1) yields

$$\begin{aligned}\bar{m}\sum_{i=1}^{n}\phi_i(x)\ddot{Y}_i(t)+c_s\sum_{i=1}^{n}\phi_i(x)\dot{Y}_i(t)+EI\sum_{i=1}^{n}\left(\frac{i\pi}{L_B}\right)^4\phi_i(x)Y_i(t)\\=\sum_{q=1}^{\chi}\delta\left(x-x_l^{(q)}\right)F_l^{(q)}\sin\theta_l^{(q)}+\sum_{q=1}^{\chi}\delta\left(x-x_t^{(q)}\right)F_t^{(q)}\sin\theta_t^{(q)}\end{aligned} \tag{3}$$

Multiplying $\phi_i(x)$ both sides of the Eq. (3) and integrating along the whole span of the beam, then divided by$L_B M_s/2$, one can obtains

$$\ddot{Y}_i(t)+2\xi_i\omega_i\dot{Y}_i(t)+\omega_i^2Y_i(t)=\frac{1}{M_i}\sum_{q=1}^{\chi}\left[\phi_i\left(x_l^{(q)}\right)F_l^{(q)}\sin\theta_l^{(q)}+\phi_i\left(x_t^{(q)}\right)F_t^{(q)}\sin\theta_t^{(q)}\right] \tag{4}$$

where $\omega_i=(i\pi/L_B)^2\sqrt{EI/\bar{m}}$ and ξ_i are the i^{th} circular frequency and damping ratio of structure, respectively; $c_s=2\xi_i\bar{m}\omega_i$ is structural damping coefficient, $M_i=\bar{m}L_B/2$ is modal mass.

The ground reaction forces from the leading and trialing footholds are obtained by

$$F_l^{(q)}=\left(L_0^{(q)}-L_l^{(q)}\right)k_l^{(q)}+\dot{L}_l^{(q)}c_l^{(q)} \tag{5-a}$$

$$F_t^{(q)}=\left(L_0^{(q)}-L_t^{(q)}\right)k_t^{(q)}+\dot{L}_t^{(q)}c_t^{(q)} \tag{5-b}$$

where $L_0^{(q)}$ is the relax length of leg; $L_l^{(q)}$ and $L_t^{(q)}$ are the lengths of leading and trialing legs, respectively; $\dot{L}_l^{(q)}$ and $\dot{L}_t^{(q)}$ are axial velocities of leading and trialing legs, respectively. The leading and trialing leg lengths can be calculated by

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$$L_l^{(q)} = \sqrt{\left(x^{(q)} - x_l^{(q)}\right)^2 + \left(z^{(q)} - w_l^{(q)}\right)^2} \tag{6-a}$$

$$L_t^{(q)} = \sqrt{\left(x^{(q)} - x_t^{(q)}\right)^2 + \left(z^{(q)} - w_t^{(q)}\right)^2} \tag{6-b}$$

where $w_l^{(q)}$ and $w_t^{(q)}$ are the vertical displacements of the beam at the leading and trialing footholds, respectively.

After derivation to the Eq. (6) by time, the axial velocities of legs are obtained by

$$\dot{L}_l^{(q)} = \frac{1}{L_l^{(q)}}\left[L_{lx}^{(q)}\dot{x}^{(q)} + L_{lz}^{(q)}\left(\dot{z}^{(q)} - \dot{w}_l^{(q)}\right)\right] \tag{7-a}$$

$$\dot{L}_t^{(q)} = \frac{1}{L_t^{(q)}}\left[L_{tx}^{(q)}\dot{x}^{(q)} + L_{tz}^{(q)}\left(\dot{z}^{(q)} - \dot{w}_t^{(q)}\right)\right] \tag{7-b}$$

where $L_{lx}^{(q)} = x^{(q)} - x_l^{(q)}$; $L_{tx}^{(q)} = x^{(q)} - x_t^{(q)}$; $L_{lz}^{(q)} = z^{(q)} - w_l^{(q)}$; $L_{tz}^{(q)} = z^{(q)} - w_t^{(q)}$.

The vertical dynamic equilibrium equation of the center of mass (CoM) according to the Fig. 2 can be obtained by

$$m^{(q)}\ddot{z}^{(q)} + m^{(q)}\ddot{w}^{(q)} + F_l^{(q)}\sin\theta_l^{(q)} + F_t^{(q)}\sin\theta_t^{(q)} = m^{(q)}\mathrm{g} \tag{8}$$

where g is the gravitational acceleration. $w^{(q)}$ is approximately estimated by the linear interpolation method as the Eq.(9)

$$w^{(q)} = \left(w_l^{(q)}L_{tx}^{(q)} - w_t^{(q)}L_{lx}^{(q)}\right)/L_s^{(q)} \tag{9}$$

where, $L_s^{(q)} = L_{tx}^{(q)} - L_{lx}^{(q)}$ is step length.

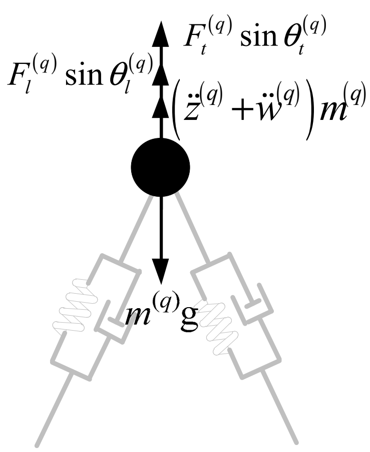

Fig.2. Vertical forcing diagram of CoM

Substituting the Eqs. (5) to (7) and (9) into the Eq. (8), and giving some operations, one obtains

$$m^{(q)}\sum_{i=1}^{n}\left(\frac{L_{tx}^{(q)}}{L_{s}^{(q)}}\phi_i\left(x_l^{(q)}\right)-\frac{L_{lx}^{(q)}}{L_{s}^{(q)}}\phi_i\left(x_t^{(q)}\right)\right)\ddot{Y}_i(t)-\sum_{i=1}^{n}\left(\sin\theta_l^{(q)}c_{lz}^{(q)}\phi_i\left(x_l^{(q)}\right)+\sin\theta_t^{(q)}c_{tz}^{(q)}\phi_i\left(x_t^{(q)}\right)\right)\dot{Y}_i(t)$$
$$=-\sin\theta_l^{(q)}\left[F_{sl}^{(q)}+c_l^{(q)}\frac{1}{L_l^{(q)}}\left(L_{lx}^{(q)}\dot{x}^{(q)}+L_{lz}^{(q)}\dot{z}^{(q)}\right)\right]-\sin\theta_t^{(q)}\left[F_{st}^{(q)}+c_t^{(q)}\frac{1}{L_t^{(q)}}\left(L_{tx}^{(q)}\dot{x}^{(q)}+L_{tz}^{(q)}\dot{z}^{(q)}\right)\right]+m^{(q)}\left(g-\ddot{z}^{(q}\right.$$

(10)

where the new variable symbols of the Eq. (10) are defined as:

$$c_{lz}^{(q)}=\frac{L_{lz}^{(q)}}{L_l^{(q)}}c_l^{(q)} \qquad c_{tz}^{(q)}=\frac{L_{tz}^{(q)}}{L_t^{(q)}}c_t^{(q)}$$
$$F_{sl}^{(q)}=\left(L_0^{(q)}-L_l^{(q)}\right)k_l^{(q)} \qquad F_{st}^{(q)}=\left(L_0^{(q)}-L_t^{(q)}\right)k_t^{(q)} \tag{11}$$

Generally, the contributions of vertical structural vibration to the ground reaction forces are relatively very tiny comparing with the contribution of pedestrian. Hence, the ground reaction forces can be approximately rewritten as

$$F_l^{(q)}\approx F_{sl}^{(q)}+c_l^{(q)}\frac{1}{L_l^{(q)}}\left(L_{lx}^{(q)}\dot{x}^{(q)}+L_{lz}^{(q)}\dot{z}^{(q)}\right)$$
$$F_t^{(q)}\approx F_{st}^{(q)}+c_t^{(q)}\frac{1}{L_t^{(q)}}\left(L_{tx}^{(q)}\dot{x}^{(q)}+L_{tz}^{(q)}\dot{z}^{(q)}\right) \tag{12}$$

The step length from pedestrian is generally also much less the footbridge span, i. e. $L_s^{(q)}\ll L_B$, so one can estimate

$$\begin{cases}\phi_i(x^{(q)})\approx\phi_i\left(x_l^{(q)}\right)\\ \phi_i(x^{(q)})\approx\phi_i\left(x_t^{(q)}\right)\end{cases} \tag{13}$$

Substituting the Eqs. (12) and (13) into the Eq. (10), then multiplying by $\phi_i(x^{(q)})/M_i$, and considering the integration of the crowd size χ, one can obtain

$$\frac{1}{M_i}\sum_{j=1}^{n}\sum_{q=1}^{\chi}m^{(q)}\phi_i(x^{(q)})\phi_j(x^{(q)})\ddot{Y}_j(t)-\frac{1}{M_i}\sum_{j=1}^{n}\sum_{q=1}^{\chi}\left(c_{lz}^{(q)}\sin\theta_l^{(q)}+c_{tz}^{(q)}\sin\theta_t^{(q)}\right)\phi_i(x^{(q)})\phi_j(x^{(q)})\dot{Y}_j($$
$$=-\frac{1}{M_i}\sum_{q=1}^{\chi}\phi_i(x^{(q)})\left(F_l^{(q)}\sin\theta_l^{(q)}+F_t^{(q)}\sin\theta_t^{(q)}\right)+\frac{1}{M_i}\sum_{q=1}^{\chi}\phi_i(x^{(q)})m^{(q)}\left(g-\ddot{z}^{(q)}\right)$$

(14)

Substituting the Eq. (13) into the Eq. (4), the dynamic equation of structure is obtained by

$$\ddot{Y}_i(t)+2\xi_i\omega_i\dot{Y}_i(t)+\omega_i^2Y_i(t)=\frac{1}{M_i}\sum_{q=1}^{\chi}\phi_i(x^{(q)})\left(F_l^{(q)}\sin\theta_l^{(q)}+F_t^{(q)}\sin\theta_t^{(q)}\right) \tag{15}$$

Combining the Eqs. (14) and (15) leads to

$$\ddot{Y}_i(t)+\frac{1}{M_i}\sum_{j=1}^{n}\sum_{q=1}^{\chi}m^{(q)}\phi_i(x^{(q)})\phi_j(x^{(q)})\ddot{Y}_j(t)+2\xi_i\omega_i\dot{Y}_i(t)$$
$$-\frac{1}{M_i}\sum_{j=1}^{n}\sum_{q=1}^{\chi}\left(c_{lz}^{(q)}\sin\theta_l^{(q)}+c_{tz}^{(q)}\sin\theta_t^{(q)}\right)\phi_i(x^{(q)})\phi_j(x^{(q)})\dot{Y}_j(t) \qquad +\omega_i^2Y_i(t)$$
$$=\frac{1}{M_i}\sum_{q=1}^{\chi}\phi_i(x^{(q)})m^{(q)}\left(g-\ddot{z}^{(q)}\right) \qquad (i,j=1,\cdots,n)$$

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(16)

The Eq. (16) is the dynamic equation of the structure under the walking crowd excitations. It is noted that the mass term of the equation includes the contribution from pedestrians' mass, which may make the structure become more flexible. In addition, the damping term of the equation includes the contributions from leg damping, which may influence the dissipating characteristics of the structure. This dynamic equation can take into account the contribution of human dynamic characteristics. In the following, the effect of crowd size on the structural dynamic performances would be explored for the model based on few practical assumptions.

3. Stability limit criterion

All pedestrians are assumed to be uniformly distributed on the footbridge deck as the Fig. 3 without an overlap. Assuming that all pedestrians have same lump mass m_p, leg length L_0, leg damping c_{leg} and step length L_s and their centers of mass (CoM) locates the mid-positions between footholds. Assuming that the bridge's fluctuate is much less than the vertical position of mass, i. e. $w_l^{(q)} \ll z^{(q)}$ and $w_t^{(q)} \ll z^{(q)}$. The only first structural modal is considered.

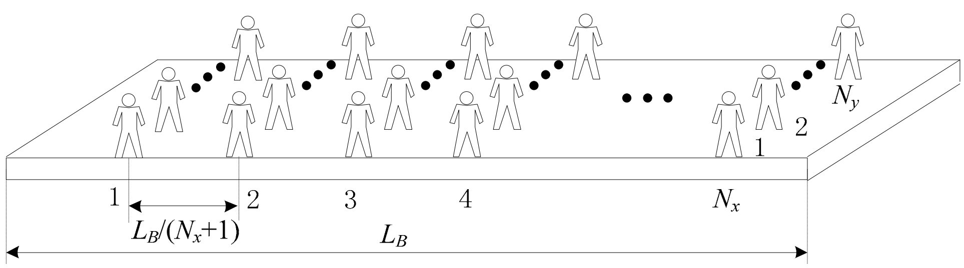

Fig. 3. The uniform arrange diagram of crowd on a footbridge

In the Fig. 3, N_x is the column number of pedestrians along the longitudinal direction; N_y means the row number of pedestrians along the lateral direction. Generally, the span of a footbridge is much larger than the lateral width. Therefore the column number N_x is also much larger than the row number N_y. $L_B/(N_x+1)$ means the distance between two neighbor longitudinal pedestrians. Based on the above assumptions, parameters can be obtained as following estimations

$$\chi = N_x N_y \tag{17}$$

$$\begin{aligned} \sin\theta_l^{(q)} \approx \sqrt{1-(L_s/2L_0)^2}; \quad \sin\theta_t^{(q)} \approx \sqrt{1-(L_s/2L_0)^2} \\ c_{lz}^{(q)} \approx c_{leg}\sqrt{1-(L_s/2L_0)^2}; \quad c_{tz}^{(q)} \approx c_{leg}\sqrt{1-(L_s/2L_0)^2} \end{aligned} \tag{18}$$

The most adverse situation to the footbridge that all pedestrians bounce with same circular frequency ω_p and excited fluctuation amplitude A_p is considered, i. e. $\ddot{z}^{(q)} = \omega_p^2 A_p \sin(\omega_p t)$. Y denotes the only considered generalized co-ordinate of the first mode so that the Eq. (16) can becomes following equation

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$$\left[1+\frac{N_y m_p}{M_i}\sum_{q=1}^{N_x}\sin^2\left(\frac{\pi q}{N_x+1}\right)\right]\ddot{Y}(t)+\left[2\xi\omega-\frac{2N_y c_{leg}\sqrt{1-(L_s/2L_0)^2}}{M_i}\sum_{q=1}^{N_x}\sin^2\left(\frac{\pi q}{N_x+1}\right)\right]\dot{Y}(t)$$
$$+\omega^2 Y(t)=\frac{N_y m_p}{M_i}\left(\mathrm{g}-\omega_p^2 A_p\sin(\omega_p \mathrm{t})\right)\sum_{q=1}^{N_x}\sin\left(\frac{\pi q}{N_x+1}\right) \quad (19)$$

where, ξ and ω are the first modal damping ratio and circular frequency of footbridge. The series of the Eq. (19) is simplified by employing the Taylor expansion so that

$$\begin{aligned}\sum_{q=1}^{N_x}\sin\left(\frac{\pi q}{N_x+1}\right) &\cong \frac{\pi q}{N_x+1}\sum_{q=1}^{N_x} q-\frac{\pi^3 q^3}{6(N_x+1)^3}\sum_{q=1}^{N_x} q^3\\ &=\frac{\pi}{2}N_x-\frac{\pi^3 {N_x}^2}{24(N_x+1)}\\ &=\frac{\pi}{2}N_x-\frac{\pi^3 {N_x}^2}{24N_x+o(N_x)}\\ &\cong\left(1-\frac{\pi^2}{12}\right)\frac{\pi}{2}N_x\end{aligned} \quad (20\text{-a})$$

$$\sum_{q=1}^{N_x}\sin^2\left(\frac{\pi q}{N_x+1}\right)\cong\left(\frac{\pi}{N_x+1}\right)^2\sum_{q=1}^{N_x} q-\frac{1}{3}\left(\frac{\pi}{N_x+1}\right)^3\sum_{q=1}^{N_x} q^4+\frac{1}{36}\left(\frac{\pi}{N_x+1}\right)^6$$
$$=\frac{\pi^2}{6}\frac{N_x(2N_x+1)}{N_x+1}-\frac{\pi^4}{90}\frac{N_x(2N_x+1)(3N_x^2+3N_x-1)}{(N_x+1)^3}+\frac{\pi^6}{1512}$$
$$=\frac{\pi^2}{6}\frac{2N_x^2+o(N_x)}{N_x+o(N_x)}-\frac{\pi^4}{90}\frac{6N_x^4+o(N_x)}{N_x^3+o(N_x)}+\frac{\pi^6}{1512}\frac{6N_x^6+o(N_x)}{N_x^5+o(N_x)}$$
$$\cong\left(\frac{1}{3}-\frac{\pi^2}{15}+\frac{\pi^4}{252}\right)\pi^2 N_x \quad (20\text{-b})$$

where $o(N_x)$ means the infinitesimal of higher order based on the assumption of $N_x \gg 1$.

Substituting Eq. (20) into the Eq. (19), the dynamic equation becomes

$$\ddot{Y}(t)+2\bar{\xi}\bar{\omega}\dot{Y}(t)+\bar{\omega}^2 Y(t)=\frac{F_p}{1+\alpha_m} \quad (21)$$

where $\bar{\omega}$ and $\bar{\xi}$ are respectively the frequency and damping ratio of structure including the walking crowd contribution.

$$\bar{f}=\frac{f}{\sqrt{1+\alpha_m}} \quad (22\text{-a})$$

$$\bar{\xi}=\frac{\xi-(\alpha_c/2\omega)}{\sqrt{1+\alpha_m}} \quad (22\text{-b})$$

where, f is structural fundamental frequency. α_m, α_c and F_p are mass, damping parameters and external force, respectively. Their expressions are listed as

$$\alpha_m=\left(\frac{1}{3}-\frac{\pi^2}{15}+\frac{\pi^4}{252}\right)\frac{\pi^2 m_p}{M_1}\chi \quad (23\text{-a})$$

$$\alpha_c=\left(\frac{1}{3}-\frac{\pi^2}{15}+\frac{\pi^4}{252}\right)\frac{2\pi^2 c_{leg}\sqrt{1-(L_s/2L_0)^2}}{M_1}\chi \quad (23\text{-b})$$

$$F_p=\frac{\pi m_p}{2M_1}\left(\mathrm{g}-\omega_p^2 A_p\sin(\omega_p \mathrm{t})\right)\left(1-\frac{\pi^2}{12}\right)\chi \quad (23\text{-c})$$

It is noted that the parameters α_m and α_c are positive related with crowd size χ in the Eqs. (23-a) to (23-b). The structural dynamic performances including frequency and damping in the Eq. (22) decreased under crowd action, which can be confirmed by measured results (Brownjohn, 1999). The larger crowd size can leads the lower frequency and damping performances. It is noted that the frequency is only related with the mass ratio $\chi m_p/M_1 = 2\chi m_p/M_s$ (M_s is the structural mass) between crowd and structure. This is due to that the part of the crowd increases mass of the whole crowd-structure system so that structure becomes more flexible. In addition, the Eq. (22-b) can explain how crowd trigger an instable phenomenon. This is due to that the crowd damping results in a negative contribution for structural dissipating performance. The larger crowd would leads to the lower damping performances. In addition, the damping contribution are related with the step length L_s and walking energy. According to the experimental reports of Hong et al. (2013), leg damping is positively linear related with the walking velocity. The damping model of the Eq. (22-b) opens a window about how understand the phenomenon that walking crowd influences the structural dissipating performances.

When the $\bar{\xi} = 0$, the structural response reaches a stable limit, which can be deduced by

$$2\xi\omega - \alpha_c = 0 \tag{24}$$

Substituting the Eq. (23-b) into the Eq. (24), the upper boundary χ_b of crowd size for stable response can be obtained by

$$\chi_b = \frac{\xi\omega M_1}{\pi^2\left(\frac{1}{3}-\frac{\pi^2}{15}+\frac{\pi^4}{252}\right)c_{leg}\sqrt{1-\left(\frac{L_S}{2L_0}\right)^2}} \tag{25}$$

The upper boundary χ_b is the maximum crowd size for keeping the stable response of structure. When crowd size is larger than the limit, structural damping ratio $\bar{\xi}$ would become negative and corresponding response would enlarge. It is noted that the maximum crowd size is related with the leg damping, step length and leg length. The larger step length can leads to the larger upper boundary. This is due to the smaller vertical component force with a larger step length. The larger leg damping would leads to the smaller crowd size limit. This boundary quantitatively gives the maximum carried crowd size with a certain large-span structure, which can provide references for preventing an excessive sway from crowd excitation.

The analytical solution of the Eq. (21) can be obtained as

$$\begin{aligned} Y(t) = {} & \exp\left(-\bar{\xi}\bar{\omega}t\right)\left(Y(0)\cos\omega_D t + \frac{\dot{Y}(0)+Y(0)\bar{\xi}\bar{\omega}}{\omega_D}\sin\omega_D t\right) \\ & -\left(1-\lambda^2\right)\beta\sin\left(\omega_p t\right) + 2\bar{\xi}\lambda\beta\cos\left(\omega_p t\right) + g\gamma \end{aligned} \tag{26-a}$$

$$\begin{aligned} \dot{Y}(t) = {} & -\bar{\xi}\bar{\omega}\exp\left(-\bar{\xi}\bar{\omega}t\right)\left(Y(0)\cos\omega_D t + \frac{\dot{Y}(0)+Y(0)\bar{\xi}\bar{\omega}}{\omega_D}\sin\omega_D t\right) \\ & + \exp\left(-\bar{\xi}\bar{\omega}t\right)\left[-Y(0)\omega_D\sin\omega_D t + \left(\dot{Y}(0)+Y(0)\bar{\xi}\bar{\omega}\right)\cos\omega_D t\right] \\ & -(1-\lambda^2)\beta\omega_p\cos\left(\omega_p t\right) - 2\bar{\xi}\lambda\beta\ \omega_p\sin\left(\omega_p t\right) \end{aligned}$$

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(26-b)

$$\ddot{Y}(t) = \left(\bar{\xi}\bar{\omega}\right)^2 \exp\left(-\bar{\xi}\bar{\omega}t\right)\left(Y(0)\cos\omega_D t + \frac{\dot{Y}(0)+Y(0)\bar{\xi}\bar{\omega}}{\omega_D}\sin\omega_D t\right)$$
$$-2\bar{\xi}\bar{\omega}\exp\left(-\bar{\xi}\bar{\omega}t\right)\left[-Y(0)\omega_D\sin\omega_D t + \left(\dot{Y}(0) + Y(0)\bar{\xi}\bar{\omega}\right)\cos\omega_D t\right]$$
$$+\exp\left(-\bar{\xi}\bar{\omega}t\right)\left[-Y(0)\omega_D^2\cos\omega_D t - \left(\dot{Y}(0)\omega_D + Y(0)\bar{\xi}\bar{\omega}\omega_D\right)\sin\omega_D t\right]$$
$$+(1-\lambda^2)\beta\omega_p^2\sin\left(\omega_p t\right) - 2\bar{\xi}\lambda\beta\omega_p^2\cos\left(\omega_p t\right) \quad (26\text{-c})$$

where, ω_D is the natural circular frequency of the crowd-structure system and it is obtained by

$$\omega_D = \bar{\omega}\sqrt{1-\bar{\xi}^2} \quad (27)$$

$$\beta = \frac{\lambda^2\gamma A_p}{\left(1-\lambda^2\right)^2+\left(2\bar{\xi}\lambda\right)^2}, \quad \gamma = \frac{\pi m_p}{2M_1(1+\alpha_m)}\left(1-\frac{\pi^2}{12}\right)\chi, \quad \lambda = \frac{\omega_p}{\bar{\omega}} \quad (28)$$

4. Numerical validation

A simply supported beam (Li et al., 2010) with the span of 42 meters is used to simulation. Its mass per meter length is $\bar{m} = 2.84\times10^3$ kg/m. The flexural stiffness is $EI = 9.78\times10^9\mathrm{Nm}^2$. The damping ratio is $\xi = 0.64\%$. The fundamental frequency of the beam is 1.653Hz. The parametric values of pedestrians are listed in the Tab. 1. The lump mass m_p and leg length L_0 is defined according to the experimental result of 10 healthy young adults (Hof et al., 2010). The step length L_s, step frequency f_p and walking velocity v_p are defined as the statistical mean value from 400 people walking on footbridges (Pachi and Ji, 2005). The leg stiffness k_{leg} and damping ratio ξ_{leg} are decided by referencing the experimental mean value from eight young healthy adults (Hong, et al., 2013). The amplitude A_p of CoM is defined according to the simulation of a HSI model (Qin et al., 2013).

Tab.1. The parameters of pedestrians

Symbols	Value	References
m_p	71.6kg	Hof et al., 2010
L_0	0.96m	
L_s	0.71m	Pachi and Ji, 2005
f_p	1.8Hz	
v_p	1.3m/s	
k_{leg}	13.69 v_p+1.587 (kN/m)	Hong et al., 2013
ξ_{leg}	0.045 v_p – 0.019	
A_p	0.045m	Qin et al., 2013

The maximum pedestrian number of stability limit is $\chi_b = 75$ with the Eq. (25). Three cases including 50, 75 and 100 pedestrians are used to calculate the acceleration responses showed by the Fig. 4. In the first case of the sub-figure (a), crowd size is less than the χ_b and the corresponding response is decreased then reaches stability.

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The decrease processing is due to the positive damping from crowd-structure system. In the second stable boundary case of the $\chi = \chi_b$, the damping of crowd structure system is zero and corresponding stable response only from the crowd excitation. In the third case of $\chi > \chi_b$, the response behaves an enlarged effect. This is due to that the oversize crowd induces a negative effect.

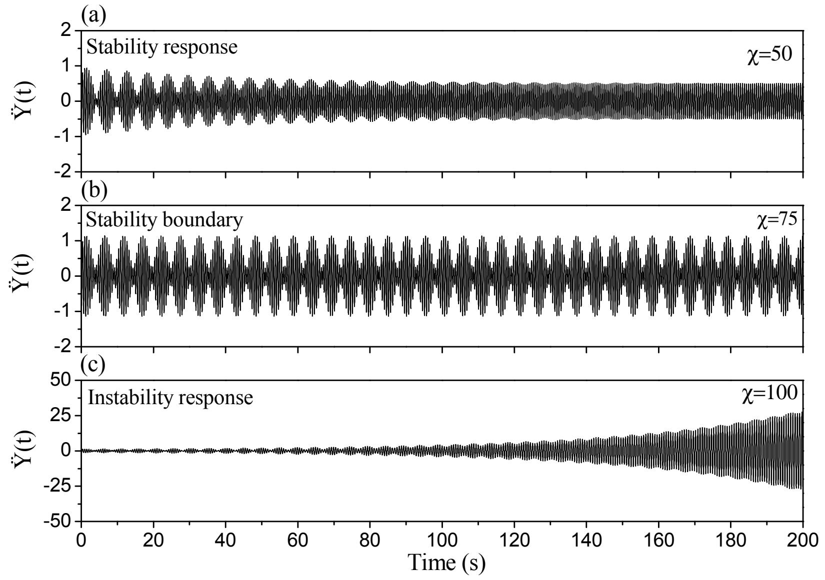

Fig. 4. The effect of crowd size on the dynamic response of the structure ($Y(0) = 0.1A_p$, $\dot{Y}(0) = 0$)

In the Fig. 5, all parameters of pedestrians are defined according to the Tab. 1, except for the walking velocity v_p. The walking velocity is changed from 1.0m/s to 1.5m/s. It is noted that the damping of structure under crowd is decreased along with the increase of crowd size. In addition, the damping is also decreased along with the increase of walking velocity. These show that the larger crowd size or faster speed can deteriorate the damping dissipating performance. The crowd size with stability limit is increased along with the lower walking speed. The effect of crowd size on the frequency of the structure is plotted in the Fig. 6. Except for the body mass m_p, other parameters of pedestrians are defined with the Tab. 1. The larger crowd size make the structure become more flexible because of its smaller frequency. In addition, the larger body mass also leads to the lower frequency. The crowd size χ_b with the Eq. (25) corresponding with the stability limit shows the upper boundary of structural dynamic stability is related with the structural damping, leg damping and walking gaits. The larger structural damping would support a larger crowd size. This is due to the better dissipating performance. However, the increase of leg damping leads to the decrease of the structure carrying capacity. The larger leg damping may be caused by the faster walking velocity (Hong et al., 2013), which tend to excite a more drastic response. However, these results are deduced according to the bipedal excitation mechanism. Some experiments or investigations about this excitation theory are needed in the future.

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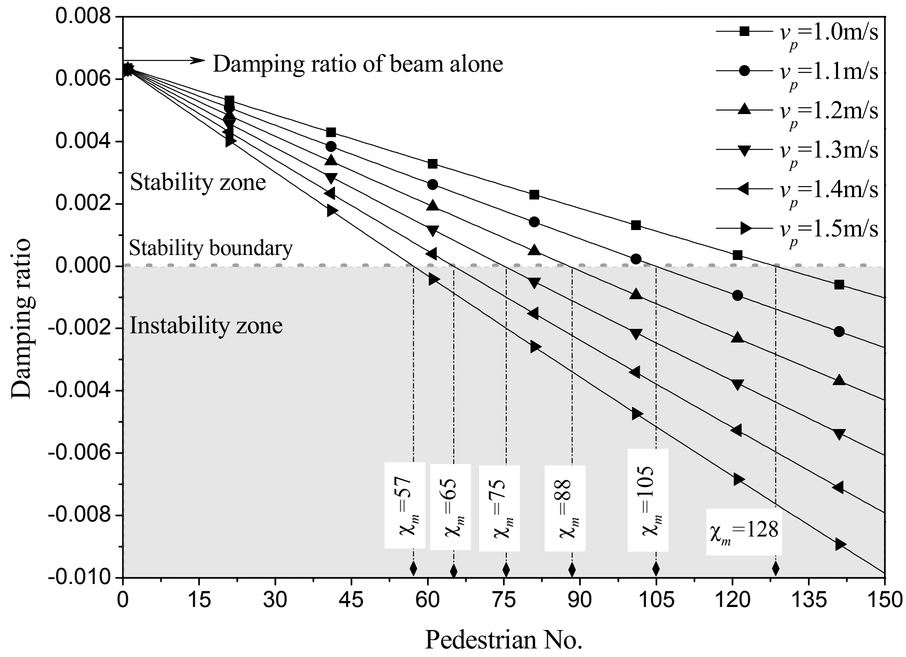

Fig. 5. The effect of crowd size on the damping of structure

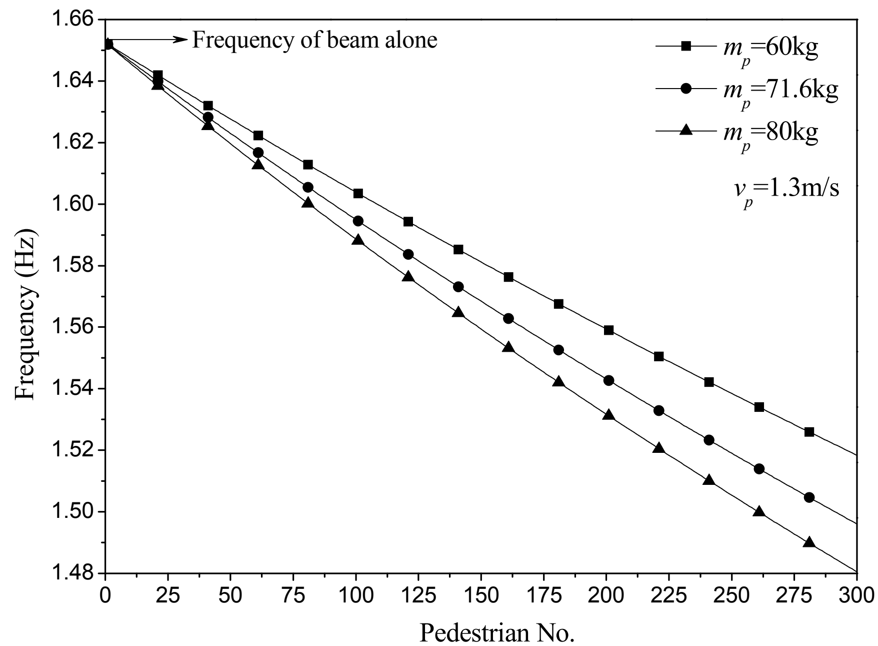

Fig. 6. The effect of crowd size on the frequency of structure

5. Conclusion

In this paper, a new excitation mechanism is proposed to motivate the vertical vibrational motion caused by crowds walking across slender footbridges, based on a bipedal walking model including the crowd dynamic characteristics. The footbridge is studied as a crowd excited dynamic system and a stability boundary limit identified, depending on the ratio between the structural and crowd damping, on the ratio

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between step and leg lengths. In addition, the frequency and damping of structure can also be identified and they are deteriorated by walking crowd. The excitation mechanism opens a window how walking crowd influence the vibrational performances of these large-span structures.

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