Article

Active Control of Edgewise Vibrations in Wind Turbine Blades Using Stochastic Disturbance Accommodating Control

Cong Cong

School of Control and Computer Engineering, North China Electric Power University, Beijing 102206, China; congcong@ncepu.edu.cn; Tel.: +86-10-6177-2654.

Abstract: Vibrations of blade and tower have important impact for wind turbine. This paper presents a active controller design to suppress blade edgewise vibrations under aerodynamic load and gravitational load.Treating the sum of aerodynamic load input in edgewise direction and gravitational load as unknown disturbance input,a stochastic disturbance accommodating control(SDAC) approach is proposed to design a controller which it utilizes a minimum-variance unbiased estimator(MVUE) to estimate both state and unknown input. The stability analysis proved that the proposed SDAC is bounded in mean square.In order to verify the performance of the minimum-variance unbiased estimator and the proposed SDAC, numerical simulations using Matlab/Simulink have been carried out for the National Renewable Energy Laboratory 5-MW wind turbine.Under the different circumstance which exists the random process and measure noise and noise free. It is shown that the estimation value by MVUE can tracking the real state and unknown input. The results are also compared to the traditional linear quadratic regulator(LQR) and show that the proposed stochastic disturbance accommodating control scheme can further reduce displacement in edgewise vibrations direction and the control strategy is more effective than the LQR.

Keywords: stochastic disturbance accommodating control; edgewise vibrations; minimum-variance unbiased estimator

---

1. Introduction

With the increasing size of wind turbine,the vibration problem is becoming increasingly important.Vibrations not only can cause structural or mechanical damage,decrease the lifetime of wind turbine[1]. On the other side, Ahlstrm[2]has shown that large blade deflections will cause the power drops off and influence on power production.As shown in[3],the mechanical vibration which directly grid-coupled induction generator can cause oscillations in the system voltage even cause the wind turbine trip out.The investigation show that the fatigue loads can be reduced by $20\% - 40\%$ which using individual pitch control[4].But the speed of pitch actuator is limited by the weight of blade and the pitch control is coupling with speed or power regulation leading to practical engineering difficulties.Individual pitch actuation may not be well suited for load reduction[5].It is necessary for that researchers to investigate the smart rotor control concept and pitch free control.

Smart rotor control for wind turbines is an innovative research area.By change the local aerodynamic property control load through a combination of sensors, actuators and controllers.Barlas and Kuik[6] provided a detailed summary of research in smart rotor control for wind turbines.WeiYu[7][8] using FAST code investigate the effect of smart rotor control using a deformable trailing edge flap for load reduction.The design of the smart rotor control depends on the aerodynamic control devices chosen.Lacker et al.[9] design a trailing edgeflaps for the fatigue load reductions and compared to individual pitch control by GH Bladed code.Further work by Lacker et al.[10],investigate smart rotor control approaches using trailing edgeflaps under the extreme loads.Because there have no accuracy analytical model about distributed aerodynamic control,the exist controller design is based on simple PI or modern control theory which the model achieved by system identification approach[11][12].

Due to the nacelle and blade have hollow nature, the damper install in side of blades can utility space which do not change the aerodynamic property and independent on speed or power regulation.The work by Staino and Basu[13] merge the active tendon control with passive pitch control design a dual control strategy decoupling vibration control from optimal power control.Previous work by many researchers,focused on the damping devices.The mitigation of wind turbine tower are mainly focused on passive dampers. Murtagh et al.[14][15] proposed a passive tuned mass damper(TMD) placed on the top of tower for reduction vibration.Throgh inverse fourier transform get the response of the coupled blades and tower.Lackner and Rotea[16] investigate the parametric study to determine optimal parameters of a passive TMD located in the nacelle of a offshore wind turbines.Dinh[17] investigates the use of single and multiple TMDs for passive control of side-side vibrations of tower.It shown that multiple TMDs can reduce 10% of the nacelle displacement more than single TMD.Tuned liquid column damper(TLCD) is another commonly damper.Colwell and Basu[18] investigate using TLCD for offshore wind turbine under the combination of wind and wave load.Another investigation using single and multiple TLCD has been done by Mensain and Leonardo[19],it is shown that use two TLCD were benefit over single TLCD. For suppressing blade vibration, many types semi-active and active control have been proposed which damper install in side of blades. Arrigan et al.[20] presented use the semi active tuned mass dampers to control flapwise vibrations. Staino et al.[21][22]proposed an active tendons mounted inside blade tip which give a variable active control force to mitigate edgewise vibration.Fitzgerald at al.[24]investigated active TMD for mitigating edgewise vibrations.Fitzgerald and Basu[25] further proposed a cable connected active TMD to suppress edgewise vibrations.However,both semi-active and active control solutions need relatively complicated controller configurations and some amount of power input.Due to the large centrifugal acceleration of the rotating blade,the TLCD with small mass ratios could effectively mitigate edgewise vibrations.Zhang et al.proposed a tuned liquid column TLCD equipped inside a rotating blade [26].Basu et al[27] investigate a circular TLCD for edgewise vibration.

All the paper above about blade vibration control using damper,the authors focus on the damper and control strategy using a state feedback LQ control and do not consider the process noise and measure noise which exist in practice.Moreover, only consider how to suppress vibration ,not accommodating the load which aerodynamic load information is not using.

In this paper,we proposed a MVUE filter to estimate the unknown aerodynamic load and gravitational load,then designed a stochastic disturbance accommodating control under the measure noise circumstance.There also has other type of disturbance observer based control,paper[28]gives a review.Although,some researchers investigate the disturbance accommodating control for load mitigation or speed regulation of wind turbine[29][30][31][32].But,the disturbance input is consider as a disturbance of wind speed and assume satisfy the waveform structure.We presents a stochastic disturbance accommodating controller which there is no prior information about unknown disturbance input.It utilizes a minimum-variance unbiased estimator to estimate both state and unknown input.It is proved that innovation is a white noise,then satisfied the separation theorem.Based on the principle of dynamic programming,a stochastic optimal control is achieved.Stochastic stability analysis by the stochastic Lyapunov style analysis shows the systems exponentially bounded in mean square and ensure the stability of the closed-loop system.

In order to verify the proposed control scheme,the NREL5-MW Wind Turbine is simulated using Matlab. A 5-DOF(degree of freedom) model which blade in-plane mode, tower side-side with one TMD DOF has been considered in this paper.The actuator is taking active tendons install inside of blade as point by Staino[21].It is compared to the traditional linear quadratic regulator.Results show that the use of the proposed stochastic disturbance accommodating control scheme can precisely estimate the unknown input action on blade edgewise direction. Then significantly mitigate the vibration response of the blade through accommodating.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 1 May 2017 doi:10.20944/preprints201705.0017.v1

2. MODEL AND PROBLEM FORMULATION

Wind turbine is a complicate system which blade, nacelle/tower and drivetrain are coupling with electrical.To get a model,we first define the coordinate.The motion of tower are described in the ground fixed coordinate(X,Y,Z),this coordinate system is fixed in the support.The origin is intersection of center of the tower and the tower base connection to the support platform.X-axis point horizontally in the nominal downwind direction.Y-axis point to the left when looking in the nominal downwind direction.Z-axis point vertically up from the center of the tower.The motion of blades are described in the co-rotating coordinate(x,y,z),this coordinate system is rotating with blade which the origin at the center of the hub.The x-axis point as same as X-axis,and z-axis point from the hub towards the blade tip.

2.1. Model of blade interaction with tower

The blade is assumed as a Bernoulli Euler cantilever beams,the mathematical model can be carried out based on energy formulation using Euler-Lagrange formulation.The blades are attached at the root to the tower/nacelle which rotate at a constant rotational speed $\Omega rad/s$ about the rotor hub's horizontal axis.The blade have distribute mass $\mu(x)$ and distribute bending stiffness $EI(x)$ per unit length along the length L. The tower is modeled as Bernoulli cantilever beam.The dynamic coupling between the blade and the tower has been included through the motion of the nacelle.In practice, it is often suppress the tower vibration using passive TMD[15] or TLCD[19],in order to consider its effect,there is one DOF about TMD in the model at the top of the tower in side-side direction which the mass,stiffness and damper are m_{tmd},k_{tmd} and c_{tmd} .

In this paper,we focus on the estimate of the unknown aerodynamic load input and design accommodating control under the case of noise-free or exist random noise.The model described in this paper,we consider the blade vibrating in the first order fundamental mode.The 5DOF modal with first edgewise mode and tower side-side mode with a TMD inside was built in this paper,although there is more complicate 13DOF[27] or 24DOF[33] modal with coupling inplane and out plane vibration.A schema of the structural model is shown in Figure. 1

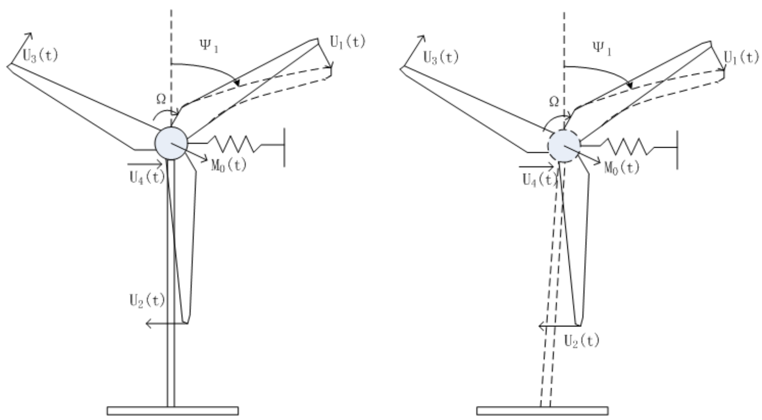


Figure 1. Wind turbine structure model for vibration.

The azimuthal angle $\psi_j(t)$ of blade j at the time instant t is given by

$$\psi_j(t) = \psi_1(t) + (j-1)\frac{2}{3}\pi, \quad \psi_1(t) = \Omega t; j = 1,2,3 \tag{1}$$

The in-plane motions motions of blade j are described by a modal expansion and given by

$$u_{j,in}(x,t) = \sum_{i=1}^{N} \phi_{j,in}(x) q_{j,in}(t) \tag{2}$$

Where $u_j(x,t)$ are the total edgewise displacement and q_{ji} are modal blade co-ordinates describing the contents of mode number N in the motion of blade.$\phi_i(x)$ are the mode shape functions,which be calculated by numerically.In order to reduce the number of states required for implementing the control a reduced order model which the fundamental mode has been derived for the system under consideration.As shown in [34],the reduced order system given through balance reduction,the H_∞ norm of error system is bounded.

Let T and V denote the kinetic and potential energy,respectively,Using the classical Euler-Lagrange method:

$$L(q(t),\dot{q}(t)) = T(q(t),\dot{q}(t)) - V(q(t),\dot{q}(t)) \tag{3}$$

The equations of motion of the system are given by

$$\frac{d}{dt}\left(\frac{\partial L(q(t),\dot{q}(t))}{\partial \dot{q}_i(t)} - \frac{\partial L(q(t),\dot{q}(t))}{\partial q_i(t)}\right) = Q_{ext,i}(t) \tag{4}$$

Where $Q_{ext,i}(t)$ is the external force acting on blade or tower.

The Kinetic energy consist of three blade,nacelle/tower and the TMD.The total edgewise direction velocity in the rotating (y,z) coordinate are

$$\begin{aligned} v_{y,j} &= \dot{u}_4 sin(\psi_j) - \Omega u_j(t) \\ v_{z,j} &= \dot{u}_4 cos(\psi_j) + \Omega x + \dot{u}_j(t) \end{aligned} \tag{5}$$

The kinetic energy of TMD inside Tower can be modeled as

$$T_{tmd} = \frac{1}{2} m_d \dot{q}_d^2 \tag{6}$$

The total kinetic energy of the system(the three blades and the tower with TMD)is

$$\begin{aligned} T &= \frac{1}{2}\sum_{j=1}^{3} T_{blade} + \frac{1}{2} T_{nacelle} + T_{tmd} \\ &= \frac{1}{2} M_0 \dot{u}_4^2 + \frac{1}{2}\sum_{j=1}^{3}\int_t^L \mu(x) v_{y,j}^2 dx + \frac{1}{2}\sum_{j=1}^{3}\int_t^L \mu(x) v_{z,j}^2 dx + \frac{1}{2} m_d \dot{q}_d^2 \end{aligned} \tag{7}$$

The potential energy consist of the potential energy caused by bending stiffness ,centrifugal stiffness, gravitation and the potential energy of nacelle and TMD.

The total potential energy is

$$\begin{aligned} V &= V_{blade} + +V_{nacelle} + V_{tmd} \\ &= \frac{1}{2}\sum_{j=1}^{3}(k_{e,ik} + k_{g,ik} cos(\psi_j) + k_{w,ik}) q_{ji} q_{jk} + \frac{1}{2} k_4 u_4^2 + \frac{1}{2} k_d q_d^2 \end{aligned} \tag{8}$$

Where the stiffness is give by

$$\begin{aligned} k_{e,ik} &= \int_0^L EI(x) u_j''^2 dx \\ k_{c,ik} &= \int_0^L F_c(x) u_j' dx = \Omega^2 \int_0^L [\int_x^L \mu(\xi)\xi d\xi] u_j'^2 dx \\ k_{g,ik} &= \int_0^L F_{g,j}(x) u_i'^2 dx \end{aligned} \tag{9}$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 1 May 2017 doi:10.20944/preprints201705.0017.v1

Where E is the Young's modulus of elasticity of the material, $I(x)$ the second moment of area of the blade about the relevant axis and $u_j'(x)$,$u_j''(x)$,respectively denote the first and the second derivative of the displacement. The centrifugal force on blade acting at the point x from the hub is

$$F_c(x) = \Omega^2 \int_x^L \mu(\xi)\xi d\xi \tag{10}$$

The effect of the component of the gravitational force acting along the blade at a distance x from the blade root is

$$F_{g,j}(x) = -\int_x^L \mu(\xi) g cos(\psi_j) d\xi = -g cos(\psi_j) \int_x^L \mu(\xi) d\xi \tag{11}$$

The generalized aerodynamic load on the blade j for the i-th mode is computed as

$$Q_{ji} = \int_0^L p_j(x,t)\phi_i(x)dx \tag{12}$$

with $p_j(x,t)$ representing the variable wind load intensity along the blade length in the edgewise direction The sideward motion is driven by the sideward aerodynamic force Q and the generator torque Tg.If the tower is modelled by a prismatic beam,the generator torque can using a multiplication factor 3/2H for the bending moment loads[35].

The equation of motion for the system considered by Euler-Lagrangian equation is

$$\frac{d}{dt}\left(\frac{\partial T(q(t),\dot{q}(t))}{\partial \dot{q}_{ij}(t)}\right) - \frac{\partial T(q(t),\dot{q}(t))}{\partial q_{ij}(t)} + \frac{\partial V(q(t))}{\partial q_{ij}(t)} = Q_{ext,i}(t) \tag{13}$$

$$\frac{d}{dt}\left(\frac{\partial T(q(t),\dot{q}(t))}{\partial \dot{q}_4(t)}\right) - \frac{\partial T(q(t),\dot{q}(t))}{\partial q_4(t)} + \frac{\partial V(q(t))}{\partial q_4(t)} = Q_4(t) \tag{14}$$

$$\frac{d}{dt}\left(\frac{\partial T(q(t),\dot{q}(t))}{\partial \dot{q}_d(t)}\right) - \frac{\partial T(q(t),\dot{q}(t))}{\partial q_d(t)} + \frac{\partial V(q(t))}{\partial q_d(t)} = Q_d(t) \tag{15}$$

The force on tower is

$$Q_4 = Q_{load} + \frac{3}{2H} T_g \tag{16}$$

The active control is modeled as an external force acting on each blade location r and is given by

$$f_j = \int_0^L 2T_j sin(\psi)\delta(x-r)\phi(r)dx \tag{17}$$

Where r is the location of actuator.

Denoting the vector of the generalized coordinates of the system is

$$q(t) = \begin{pmatrix} q_1(t) \\ q_2(t) \\ q_3(t) \\ q_4(t) \\ q_d(t) \end{pmatrix} \tag{18}$$

The total generalized external force in the Euler Lagrange formulation is given by

$$Q_{ext}(t) = F + Q_{load} + Q_g + Q_e \tag{19}$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 1 May 2017 doi:10.20944/preprints201705.0017.v1

The generalized control force vector F, the reduced generalized aerodynamic load Q_{load} and the reduced generalized gravity load Q_g can be derived from the corresponding quantities in the formulation including higher modes and can be written as

$$F(t) = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ 0 \\ 0 \end{bmatrix} Q_{load}(t) = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_d \end{bmatrix} Q_g(t) = \begin{bmatrix} Q_{g,1} \\ Q_{g,2} \\ Q_{g,3} \\ Q_{g,4} \\ Q_{g,d} \end{bmatrix} Q_e(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{3}{2H}T_g \end{bmatrix} \tag{20}$$

By defining the quantities

$$m_{1i} = \int_0^L \mu(x)\phi_i dx, m_{2i} = \int_0^L \mu(x)\phi_i^2 dx, m_4 = 3\int_0^L \mu(x)\phi_i dx + M_0 \tag{21}$$

The final second order matrix differential equation of blade in edgewise direction and tower in side-side direction can be written as

$$M(t)\ddot{q} + C(t)\dot{q} + K(t)q = Q_{ext}(t) \tag{22}$$

$$y = C_{od}(r)\dot{q} + C_{ov}(r)q \tag{23}$$

Where the matrices of the model is given by

$$M(t) = \begin{bmatrix} m_2 & 0 & 0 & m_1cos(\psi_1) & 0 \\ 0 & m_2 & 0 & m_1cos(\psi_2) & 0 \\ 0 & 0 & m_2 & m_1cos(\psi_3) & 0 \\ m_1cos(\psi_1) & m_1cos(\psi_2) & m_1cos(\psi_3) & m_4 & 0 \\ 0 & 0 & 0 & 0 & m_d \end{bmatrix}$$

$$C(t) = \begin{bmatrix} c_b & 0 & 0 & 0 & 0 \\ 0 & c_b & 0 & 0 & 0 \\ 0 & 0 & c_b & 0 & 0 \\ -2\Omega m_1sin(\psi_1) & -2\Omega m_1sin(\psi_2) & -2\Omega m_1sin(\psi_3) & c_4+c_d & -c_d \\ 0 & 0 & 0 & -c_4 & c_d \end{bmatrix}$$

$$K(t) = \begin{bmatrix} k_2+k_wcos(\psi_1) & 0 & 0 & 0 & 0 \\ 0 & k_2+k_wcos(\psi_2) & 0 & 0 & 0 \\ 0 & 0 & k_2+k_wcos(\psi_3) & 0 & 0 \\ -\Omega^2m_1cos(\psi_1) & -\Omega^2m_1cos(\psi_2) & -\Omega^2m_1cos(\psi_3) & k_4+k_d & -k_d \\ 0 & 0 & 0 & -k_4 & k_d \end{bmatrix}$$

Since the equation of motion contain periodic term about the azimuth angel of blades which caused by the blade rotataion.Through multi-blade coordinate transformation(Coleman transformation)can cancel the effect of periodic term[36].

The transformation matrix is given by

$$q_{nr}(t) = P_{col}q(t), P_{col}u = \begin{bmatrix} 1 & cos(\psi_1) & sin(\psi_1) & 0 & 0 \\ 1 & cos(\psi_2) & sin(\psi_2) & 0 & 0 \\ 1 & cos(\psi_3) & sin(\psi_3) & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{24}$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 1 May 2017 doi:10.20944/preprints201705.0017.v1

2.2. Aerodynamics torque

The aerodynamics torque input of the turbine can be described using air density ρ and rotor disc area A

$$T_{shaft} = \frac{1}{2} v_{rot}^3 \rho A C_p(\lambda, \beta) \Omega^{-1} \tag{25}$$

where C_p is derived from look-up tables with tip speed ration ($\lambda = \frac{R\omega}{v_{rot}}$) and pitch angle β.

2.3. Model of drive train

It is commonly using a 3rd order drive train model which is based on two rotating shafts connected through a gearbox with torsion spring constant K_{shaft}, viscous friction B_{shaft}, and gear ration

$$\begin{aligned} \dot{\Omega} &= \frac{1}{I_{rot}}(T_{shaft} - \phi K_{shaft} - \dot{\phi} B_{shaft}) \\ \dot{\omega} &= \frac{1}{I_{gen}}(-T_{gen} + \frac{1}{N}(\phi K_{shaft} + \dot{\phi} B_{shaft})) \\ \dot{\phi} &= \Omega - \frac{1}{N}\omega \end{aligned} \tag{26}$$

where ϕ is the shaft torsion angle and I_{gen}, I_{rot} are the inertias of the generator and rotor respectively.

3. SDAC

A detailed formulation of the SDAC for linear time invariant systems is presented in this section. Consider the linear discrete-time system of the following form:

$$\begin{aligned} x_{k+1} &= Ax_k + Gd_k + Bu_k + w_k \\ y_k &= Cx_k + v_k \end{aligned} \tag{27}$$

where $x_k \in R^m$ is the state vector,$u_k \in R^n$ is the control input vector, $d_k \in R^p$ is an unknown input vector, and $y_k \in R^q$ is the measurement. The process noise $w_k \in R^n$ and the measurement noise $v_k \in R^q$ are assumed to be mutually uncorrelated, zero-mean, white random signals with known covariance matrices, $W_k = E[w_k w_k^T]$ and $V_k = E[v_k v_k^T]$, respectively,$W_k \geq 0$,$V_k \geq 0$,x_0 is independent of v_k and w_k for all k.It assumed that (C_k,A_k)is observable and (A_k,B_k) is controllable.

3.1. Disturbance Accommoating Control

The disturbance accommodating control tries to cancel out or minimize the effects of the disturbance input with using a control law as a superposition of control and disturbance accommodating components.The general control law is function of I_k which the set $I_k \in \{x_0, \cdots, x_k, y_0, \cdots, y_k, u_0, \cdots, u_k\}$. given as

$$u_k = KI_k \tag{28}$$

In practice,the state variables x_k and the disturbance variable d_k cannot be measured directly.The realizable control law is:

$$u_k = -K_x \hat{x}_k - K_d \hat{d}_k \tag{29}$$

Where K_x is the gain matrix given through optimal control. The disturbance gain K_d set minimize the norm

$$\min_{K_d} \|BK_d - G\| \tag{30}$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 1 May 2017 doi:10.20944/preprints201705.0017.v1

Solve this optimal problem,the gain matrix is given by

$$K_d = B^+ G \tag{31}$$

Where $B^+ = B'(BB')^{-1}$ is the pseudo inverse of B.

The basic thought in DAC theory is that disturbances are described by an assumed-waveform model on state space form which the disturbance states $x_{z,k}$ is updating through the known matrices A_d and C_d as Eq.(32)[29][30].

$$\begin{aligned} \hat{x}_{z,k+1} &= A_d \hat{x}_k \\ \hat{d}_k &= C_d \hat{x}_{z,k} \end{aligned} \tag{32}$$

But the actual disturbances may be not the assumed-waveform form,even have no prior knowledge about unknown input.In next section, we design a input estimator without any information concerning the unknown input d_k.

3.2. Minimum-variance unbiased input and state estimation

The objective of this paper is to design stochastic disturbances accommodating control which there is no prior knowledge about unknown d_k. The unknown input d_k can be any type of signal.

The first problem is to design a globally optimal estimator of x_k and d_k given the sequence of measurements $Y_k = \{y_0, y_1, \cdots, y_k\}$ without any information concerning the unknown input d_k. In paper[38],Gillijns proposed an unbiased estimators in the sense of minimum-variance unbiased(MVU) over all linear unbiased estimators for system (27). In this paper,we take the similar form filer in paper[38].We extend it to the control problem. Considering a recursive filter of the form:

$$\hat{x}_{k|k-1} = A\hat{x}_{k-1|k-1} + Bu_{k-1} \tag{33}$$

$$\hat{d}_{k-1} = M_k(y_k - C_k \hat{x}_{k|k-1}) \tag{34}$$

$$\hat{x}^{\star}_{k|k} = \hat{x}_{k|k-1} + G\hat{d}_{k-1} \tag{35}$$

$$\hat{x}_{k|k-1} = \hat{x}^{\star}_{k|k} + L_k(y_k - C_k \hat{x}^{\star}_{k|k}) \tag{36}$$

where $M_k \in R^{p\times q}$ and $L_k \in R^{m\times q}$ are gain matrix to be determined. Let $\hat{x}_{k-1|k-1}$ be an unbiased estimate of x_{k-1}, then $\hat{x}_{k|k-1}$ is biased due to the unknown input in the true system.We want to get a unbiased estimator,the following two condition must satisfy .

Lemma 1. *If $\hat{x}_k$ is an unbiased state estimate,the gain matrix must satisfy the constraint*

$$rank(CG) = rank(G) = p \tag{37}$$

See the Appendix for the proof of Lemma 1. This condition is as same as in [37][38],but the proof is some different.

Lemma 2. *Let $\hat{x}_{k|k}$ be unbiased, then (33)-(34) is an unbiased estimator of $\hat{d}_{k-1}$ if and only if M_k satisfies*

$$M_k CG = I \tag{38}$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 1 May 2017 doi:10.20944/preprints201705.0017.v1

See the Appendix for the proof of Lemma 2. Firstly, we consider the estimation of the unknown input. Defining the innovation of unknown input estimation $\tilde{y}_k$ by

$$\tilde{y}_k = y_k - C\hat{x}_{k|k-1} \tag{39}$$

Defining

$$e_k = C(A\tilde{x}_{k-1} + w_{k-1}) + v_k \tag{40}$$

The matrix M_k is given by the least-squares solution of (A4), satisfies (38).In order to satisfy the assumptions of the Gauss Markov theorem,the variance of e_k can be computed from the covariance matrices of the state estimator.An MVU estimator of d_k is then obtained by weighted least-squares estimation with weighting matrix $(E[e_k e_k^T])^{-1}$.Denoting the variance of e_k by $\tilde{R}_k$,$P_{k|k} = E[\tilde{x}_k \tilde{x}_k^T]$ and defining the estimation error by

$$\tilde{x}_k = x_k - \hat{x}_{k|k} \tag{41}$$

and the variance of error by

$$P_{k|k} = E[\tilde{x}_k \tilde{x}_k^T] \tag{42}$$

$$P_{k|k-1} = A_{k-1} P_{k-1|k-1} A_{k-1}^T + W_{k-1} \tag{43}$$

A straightforward calculation yields

$$\tilde{R}_k = E[e_k e_k^T] = C P_{k|k-1} C^T + V_k \tag{44}$$

An MVU input estimate is then obtained as follows by using some results from[38].

Lemma 3. *[38] Let Assumption (37) hold, let $\hat{x}_{k-1|k-1}$ be unbiased, let $\tilde{R}_k$ be positive definite and let M_k be given by*

$$M_k = (F_k^T \tilde{R}_k^{-1} F_k)^{-1} F_k^T \tilde{R}_k^{-1} \tag{45}$$

whereF_k = CG,then (34) is the MVU estimator of d_{k-1} given the innovation $\tilde{y}_k$. The variance of the corresponding input estimate, is given by $(F_k^T \tilde{R}_k^{-1} F_k)^{-1}$

In paper[37],only state estimation is considered.

Gillijns and De Moor conclude that filter by [37] implicitly estimates the unknown input from the innovation by WLS estimation[38] and estimates the state is equivalent to [37].

Lemma 4. *Let $\hat{x}_{k-1|k-1}$ and $\hat{d}_{k-1}$ be unbiased,then (35)(36) are unbiased estimators of $\hat{x}_k$ for any value of K.*

Substituting (34) and (35) in (36), yields $\hat{x}_{k|k}$

$$\hat{x}_{k|k} = A\hat{x}_{k-1|k-1} + Bu_{k-1} + L_k\tilde{y} + (I - L_k C_k) G_{k-1} M_k \tilde{y} \tag{46}$$

Defining $K_{e,k} = L_k + (I - L_k C_k) G_{k-1} M_k$.Eq.(46) is rewritten as

$$\hat{x}_{k|k} = A\hat{x}_{k-1|k-1} + Bu_{k-1} + K_{e,k}(y_k - C_k \hat{x}_{k|k-1}) \tag{47}$$

Lemma 5. *Let M_k be given by (45),L_k by*

$$L_k = P_{k|k-1} C \tilde{R}_k^{-1} \tag{48}$$

then obtain the state update of similar to paper [37] except it exists a control term.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 1 May 2017 doi:10.20944/preprints201705.0017.v1

In [38],Gillijns proved Eq.(34)(36) are unbiased estimators minimizing the mean square error over the class of all linear unbiased estimates based on $Y_k = \{y_0, \cdots, y_k\}$ when M given by Eq.(45),L_k given by Eq.(48).

3.3. Stochastic optimal control with MVU estimator

The control problem is to find an output control law that minimizes the cost performance. The cost performance for controlling the system (27) is quadratic :

$$J = E[\sum_{k=0}^{N-1} E[(x_k^T Q_k x_k + u_{x,k}^T R_k u_{x,k})|I_k] + E[x_N^T Q_N x_N | I_N]] \tag{49}$$

Substituting $\hat{x}_k = x_k - \tilde{x}_k$ into (49) give us

$$J = E[\sum_{k=0}^{N-1} (\hat{x}_k^T Q_k \hat{x}_k) + u_{x,k}^T R_k u_{x,k}] + \hat{x}_N^T Q_N \hat{x}_N + \sum_{k=0}^{N} trQ_k P_k \tag{50}$$

from the orthogonal projection lemma,the minimum variance estimate $\hat{x}_k$, will be independent of the estimation error $\tilde{x}_k$ and $\tilde{d}_k$. In order to satisfy separation theorem,the state estimation innovation process,defined as

$$z_k = y_k - C_k \hat{x}^{\star}_{k|k} \tag{51}$$

Lemma 6. *z_k is a zero mean values white noise process with variance $P_{z,k} = C(AP_{k-1}A^T + G(F_k^T \tilde{V}_k^{-1} F_k)^{-1} G^T + W_{k-1})C^T + V_k$.*

See the Appendix for the proof of Lemma6.
Since z_k is a zero mean white noise.It is using dynamic programming recursive algorithm to solve the LQG optimal control problem.

Theorem 1. *Consider the system (27) with estimator (36) and (34),the optimal control is*

$$\bar{u}^{\star}_{k-1} = -(R_{k-1} + B^T S_k B)^{-1} B S_k A \hat{x}_{k-1} - K_d \hat{d}_{k-1} \tag{52}$$

Where S_k satisfy the Riccati equation

$$S_{k-1} = Q_{k-1} + A^T S_k A - A^T S_k B (R_{k-1} + B^T S_k B)^{-1} B S_k A \tag{53}$$

and the optimal cost criterion is

$$\begin{aligned} J =& \hat{x}_0^T S_0 \hat{x}_0 + \Pi_0 + tr(S_0 M_0 C P_{z,0}^{-1} C M_0) + \sum_{k=0}^{N} trQ_k P_k \\ &+ tr(\hat{d}_0^T (G - BK_d)^T Q_0 (G - BK_d) \hat{d}_0) \end{aligned} \tag{54}$$

See the Appendix for the proof of Theorem 1.

3.4. Stability Analysis

In this section we analyze the stability of the closed loop system. Given the estimator of the form Eq.(34)(36) and output feedback (29). Changing from $\hat{x}_{k-1}$ and $\hat{d}_{k-1}$ to $\tilde{x}_{k-1}$ and $\tilde{d}_{k-1}$,given (41),the control input is

$$u_{k-1} = -K_x(x_{k-1} - \tilde{x}_{k-1}) - K_d(d_{k-1} - \tilde{d}_{k-1}) \tag{55}$$

Now we can write the closed-loop dynamics as

$$\begin{bmatrix} x_k \\ \tilde{x}_k \end{bmatrix} = \begin{bmatrix} A - BK_x & B(K_x - K_d M_k CA) \\ 0 & (I - L_k C)(I - GM_k C)A \end{bmatrix} \begin{bmatrix} x_{k-1} \\ \tilde{x}_{k-1} \end{bmatrix} + \begin{bmatrix} I - BK_d M_k C & BK_d M_k \\ (I - L_k C)(I - GM_k C) & -(I - L_k C)GM_k - L_k \end{bmatrix} \begin{bmatrix} w_{k-1} \\ v_k \end{bmatrix} \tag{56}$$

From the dynamical equations,since this is triangular, the eigenvalues are just those of $A - BK_x$ together with those of $(I - L_k C)(I - GM_k C)A$. Thus the stability of the estimator and output feedback are independent. Now,it is clear that the system can be stabilized if choose some values of K_x, K_d, L_k, M_k satisfy some conditions.

Theorem 2. *If there exist M,K satisfying (45) and (48), and if $(A, W^{\frac{1}{2}})$ is stability,then the variance P converges to a unique fixed point $\bar{P}$ and the filter is stable.*

$$|\lambda_i[\tilde{A}(M_k, L_k)]| < 1 \tag{57}$$

Where $\tilde{A}(M_k, L_k) = (I - L_k C)(I - GM_k C)A$,$\lambda_i$ denoting the eigenvalue. See the Appendix for the proof of Theorem 2.

Next,we give the stability of the closed loop system states.

Theorem 3. *If there exist matrices $R_k > 0, Q_k > 0$ and M_k, K_k, K_d satisfying the condition (A25) in Appendix.The closed loop systems of (58) is exponentially bounded in mean square.*

$$x_k = (A - BK_x)x_{k-1} + B(K_x - K_d M_k CA)\tilde{x}_{k-1} + (I - BK_d M_k C)w_{k-1} + BK_d M_k v_k \tag{58}$$

See the Appendix for the proof of Theorem 3.

4. Application

In order to investigate the performance of MVUE for estimating unknown input action on blade which consist of aerodynamic load in edgewise direction and gravitational load, and verify the effectiveness of proposed SDAC.We simulate a wind turbine model in Matlab using the data from NREL-5WM reference wind turbine.The data of wind turbines are provided in Table 1.

Table 1. Parameters used in the 5-DOF model.

Parameter	Value	Unit	Parameter	Value	Unit
m_0	1.412×10^3	kg	k_e	6.623×10^3	$N.m^{-1}$
m_1	3.049×10^3	kg	k_1	2.086×10^3	$N.m^{-1}$
m_4	6.975×10^5	kg	k_2	47.25	$N.m^{-1}$
m_{tmd}	6.975×10^3	kg	k_4	2.646×10^6	$N.m^{-1}$
k_{tmd}	2.594×10^4	$N.m^{-1}$	τ_{gen}	0.1	s
I_{gen}	534.1160	$kg.m^2$	I_{rot}	35444067	$kg.m^2$
K_{shaft}	867837000	$N.m$	B_{shaft}	6215000	$N.m/(rad/s)$
L	61.5	m	N	97:1	

The modeshape is a six-order polynomials of distance $\bar{x} = r/L$ which calculated from wind turbine data using mode code.The first inplane modeshape is

$$\phi_1(\bar{x}) = -0.6952\bar{x}^6 + 2.3760\bar{x}^5 - 3.5772\bar{x}^4 + 2.5337\bar{x}^3 + 0.3627\bar{x}^2 \tag{59}$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 1 May 2017 doi:10.20944/preprints201705.0017.v1

The instantaneous wind is described as a mean wind V_s and turbulence V_t.

$$V(t) = \bar{V} + V_s(t) \tag{60}$$

The time series of turbulence is simulated by the with spectral representation (SR) method which was first applied for simulation multi-dimensional random process by Shinozuka[39].The power spectral density(PSD) assums the form

$$S(\omega) = \begin{bmatrix} S_{11}(\omega) & S_{12}(\omega) & \dots & S_{1N}(\omega) \\ S_{21}(\omega) & S_{22}(\omega) & \dots & S_{2N}(\omega) \\ \dots & \dots & \dots & \dots \\ S_{N1}(\omega) & S_{N2}(\omega) & \dots & S_{NN}(\omega) \end{bmatrix} \tag{61}$$

Where ω is frequence,$S_{jj}(\omega)$ is self spectrum,and $S_{jk}(\omega)$ is cross spectrum given by

$$S_{jk}(\omega) = Coh_{jk}\sqrt{S_{jj}(\omega)S_{kk}(\omega)} \tag{62}$$

Where Coh_{jk} is coherence.In this paper we use Davenport form:

$$Coh_{jk} = exp(-\frac{Cd_{jk}}{\bar{V}}) \tag{63}$$

Where C is constant and $d_{jk} = r_j - r_k$ is distance between location j and location k. The time series can be simulated by[40]

$$u(x,t) = \sqrt{2\Delta\omega}\sum_{m=1}^{j}\sum_{l=1}^{N}(H(\omega_{ml}))cos(\omega_{ml} - \vartheta_{jm} + \phi_{ml}) \tag{64}$$

Where $\Delta\omega = \omega/N$,$\omega_{ml} = l\Delta\omega + (m/N_p)\Delta\omega$ is the double indexing of frequencies,ϕ_{ml} is a uniform distribution random number in$[0 - 2\pi]$,$\vartheta_{jm} = r_j/V_s - r_m/V_s$ is time lag. $H(\omega_{ml})$ is defined by Cholesky's decomposition of $S(\omega)$,given by

$$S(\omega) = H(\omega)H^{\star}(\omega)^T \tag{65}$$

The spectrum offered by Kaimal form expressed as

$$\frac{fS_{vv}(H,f)}{\sigma^2} = \frac{4fL_t/v_s}{(1+70.8fL_t/v_s)^{5/3}} \tag{66}$$

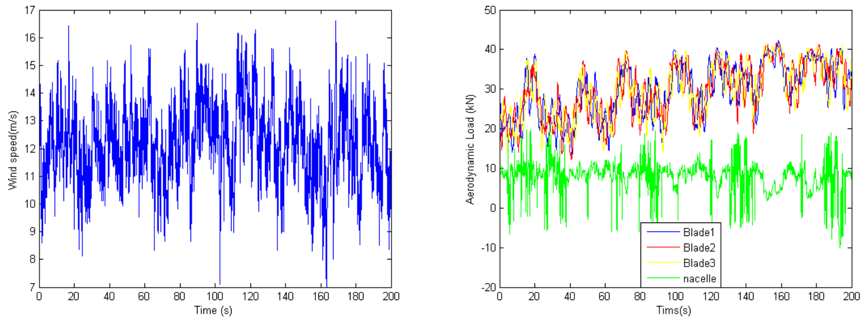
Where L_t is the integral length scale,σ is standard deviation which depends on the average wind V_s and the turbulence intensity.

$$\sigma = I_t v_s \tag{67}$$

Where turbulence intensity is depends on the ground surface roughness and the height from ground.

In this paper,we simulated a 100s turbulence time serious with turbulence intensity is 0.1s.The hub height is 90m. The wind turbine is operated in rated speed with rotor angular velocity $\Omega = 12.1rpm$.The mean wind is 12m/s. The local aerodynamic loads in edgewise direction calculated using BEM(see more detail in[41]) and numerical integration along the blade.The aerofoil files contain lift curve and drag curve are given in[33]. The wind velocity corresponding to the hub heightand the aerodynamic loads acting on blades and nacelle are shown in Figure. 2.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 1 May 2017 doi:10.20944/preprints201705.0017.v1



(a) **(b)**

Figure 2. (**a**) Wind velocity in hub height. (**b**) Aerodynamic load in edgewise direction action on blade and nacelle.

For the model in this work,the dynamic equation (22) has a state space realization.Since the control design only consider the blade dynamic.For simplicity,system state equation matrices are given as

$$A = \begin{bmatrix} 0_{3\times3} & I_{3\times3} \\ -M_b^{-1}K_b & -M_b^{-1}C_b \end{bmatrix}, B = \begin{bmatrix} 0_{3\times3} \\ M_b^{-1}\phi(r) \end{bmatrix}, G = \begin{bmatrix} 0_{3\times3} \\ M_b^{-1} \end{bmatrix}$$

Where

$$M(t) = \begin{bmatrix} m_2 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_2 \end{bmatrix}, C(t) = \begin{bmatrix} c_b & 0 & 0 \\ 0 & c_b & 0 \\ 0 & 0 & c_b \end{bmatrix}, K(t) = \begin{bmatrix} k_{b1} & 0 & 0 \\ 0 & k_{b2} & 0 \\ 0 & 0 & k_{b3} \end{bmatrix}$$

Where $k_{b1} = k_2 + k_w cos(\psi_1)$,$k_{b3} = k_2 + k_w cos(\psi_2)$ and $k_{b3} = k_2 + k_w cos(\psi_3)$.

Since the proposed SDAC is based on discrete state space equation,firstly we discrete the system as sample interval $Ts = 0.0004s$.The edgewise displacement on a real wind turbine, will not be directly measurable , and will need to be estimated from other sensor data.Many different sensors have been investigated for wind turbine health monitoring[5][35].In this paper ,we assume that the sensor is accelerometer sensor.The displacement and velocity was produced by double integrating and integrating the accelerometer signal[42].To get the performance of MVUE and SDAC, different cases was considered:First one,we estimate the state and unknown load input under noise free circumstance;second case check the performance under random noise;finally,for the purpose to comparison,result of LQR control was shown. Since the control aim is to alliviate blade vibration and only three control input,in order to reduce the order of controller,control choose only three blade.The measure are given as displacement matrices and velocity matrices

$$C = C_{od}(r)\dot{q} + C_{ov}(r)q \quad (68)$$

A proper choice of actuator and sensor location can improvement the performance of control.For finding the optimal placement of sensors and actuatros.We choose the location based on energy approach which is based on system observability and controllability[43].The observability gramian matrix W_o and controllability gramian matrix W_c can be achieved by following Lyapunov equation.

$$A'W_o + W_oA + C'C = 0 \quad (69)$$

$$AW_c + W_cA' + BB' = 0 \quad (70)$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 1 May 2017 doi:10.20944/preprints201705.0017.v1

The criteria of choosing actuator and sensor location is maximum the objective function

$$J_{op} = trace(W)\sqrt[2N]{det(W)}/\sigma(\lambda_i) \tag{71}$$

Where W is observability gramian matrix or controllability gramian matrix.N is the order of system,$\sigma(\lambda_i)$ is the standard deviation of the eigenvalues λ_i of gramian matric. The numerical results of the objective function are shown in Figure. 3.

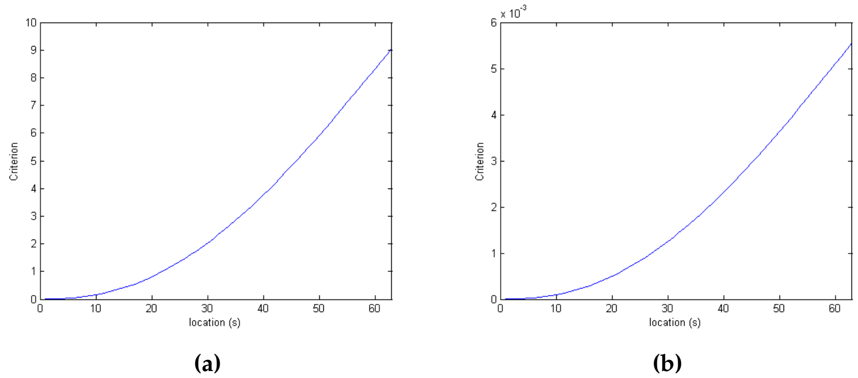


Figure 3. (**a**) Numerical result of the objective function for observability gramian matrix . (**b**) Numerical result of the objective function for controllability gramian matrix.

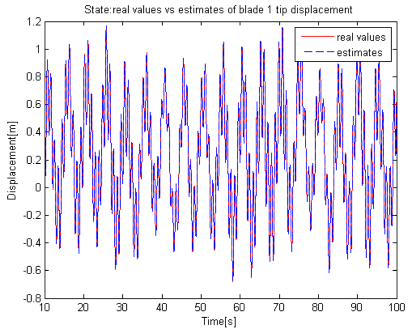
As seen from figure 3,the maximum value of objective function are in tip.Then the location of actuator and sensor in this paper is in tip of each blade.

4.1. Case 1:MVUE under noise free

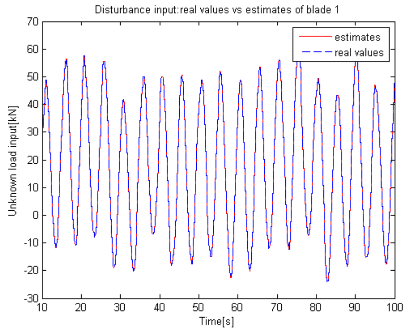
We first focus on the estimation problem and investigate the performance of the MVUE. The unknown input in this paper is the sum of aerodynamic load in edgewise direction and gravitational load. Since we want to check out the accuracy of estimation about known input,there is no control action in this case. Assuming that there is no random noise.First,we check the condition of Lemma 1. Since $B = \begin{bmatrix} 0_{3\times3} & M_b^{-1}\phi(1) \end{bmatrix}$, calculating from $CB = \begin{bmatrix} 0_{3\times3} & M_b^{-1}\phi(1) \end{bmatrix}$. We can get $rank(CB) = rank(B)$,it satisfies the lemma 1.

In this simulation,the initial states are selected to be $x_0 = [0.12; 0.17; 0.02; 2.49; 3.57; 0.53]$ which is a random sampled value under no control action.The real and the estimation of state and unknown load input which is the sum of aerodynamic load and gravitational load are shown in Figure 4. From the Figure,we can find that the estimated value tracking the real value both in state and unknown input. Figure 5 shows the error of estimation about blade 1,it is shown the estimation error is bounded.The RMS error of blade 1 tip displacement is 0.0014m and RMS error of unknown input is 0.2kN. Although the error of unknown aerodynamic input is fluctuating.But the mean value of estimation error of unknown input in three blades is -0.3624N,-1.3313N and 5.3673N,respectively.We can conclude that the results are according with the theory analysis which the estimator is in the sense of minimum variance and the error is bounded in mean square.

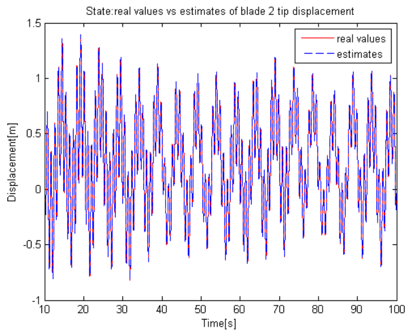
Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 1 May 2017 doi:10.20944/preprints201705.0017.v1



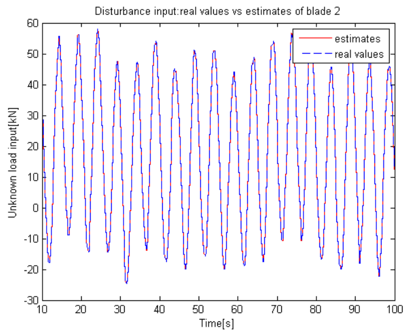
(a)



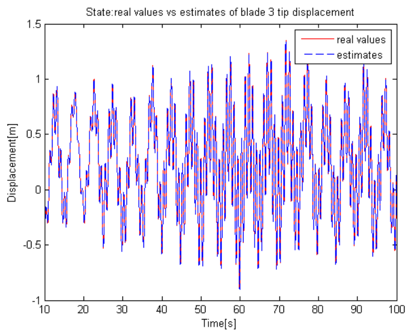
(b)



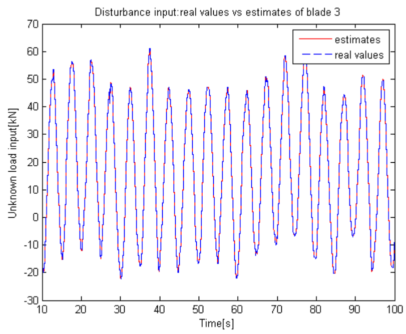
(c)



(d)



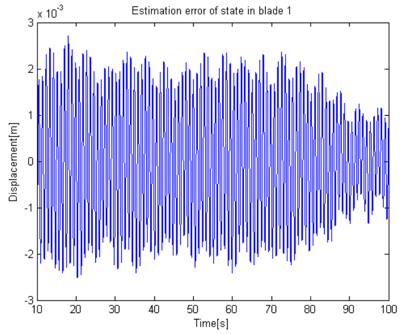
(e)



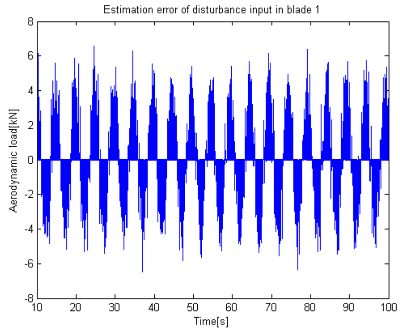
(f)

Figure 4. (**a**) State estimation:blade 1 tip displacement. (**b**) Unknown load input estimation of blade 1. (**c**) State estimation:blade 2 tip displacement. (**d**) Unknown load input estimation of blade 2.(**e**) State estimation:blade 3 tip displacement. (**f**) Unknown input load estimation of blade 3.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 1 May 2017 doi:10.20944/preprints201705.0017.v1



(a)



(b)

Figure 5. (**a**) The error of state estimation of blade 1 tip displacement. (**b**) The error of unknown input load estimation of blade 1.

Next,we consider the case with random process and measurement noise.

4.2. Case 2:MVUE under random noise

In this simulation,we investigate the performance of MVUE under random process and measure noise. Since the noise is random.In order to get a statistic analysis,we simulate 100 times and check the mean value of maximum displacement and RMS displacement and also the unknown force.The initial states are also selected to be $x_0 = [0.120.170.022.493.570.53]$ .The noise covariance is selected to be $W = 0.001$ and $V = 0.001$,the initial process noise covariance is selected to be $0.9W$ and $1.1V$.The real value and the estimation of state and unknown input is shown in Figure 6.It is noticed that the estimation of states and disturbance tracking true values under the random process and measure noise.The estimation error is bounded.

Figure 7 shows the error of estimation about blade 1.From the statistic analysis,the RMS error of blade 1 tip displacement is 0.0022m and RMS error of unknown input is 0.9kN.The mean value of estimation error of unknown input in blade 1 is -0.0124N and It is accord with the theory analysis which in the sense of minimum variance and the error bounded in mean square.The other two blades have the similar conclusion. Since the performance is affected by the variance of noise.If the variance is incorrect,the estimator may be have poor performance.For simplicity,in this paper,We only consider the systems under correct noise variance.

Through the simulation of case 1 and case 2,it can be verified that the minimum-variance unbiased estimator can estimating the state and unknown input accurately.In next case,we check the performance of the stochastic disturbance accommodating control which is based on the accuracy of the MVUE.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 1 May 2017 doi:10.20944/preprints201705.0017.v1

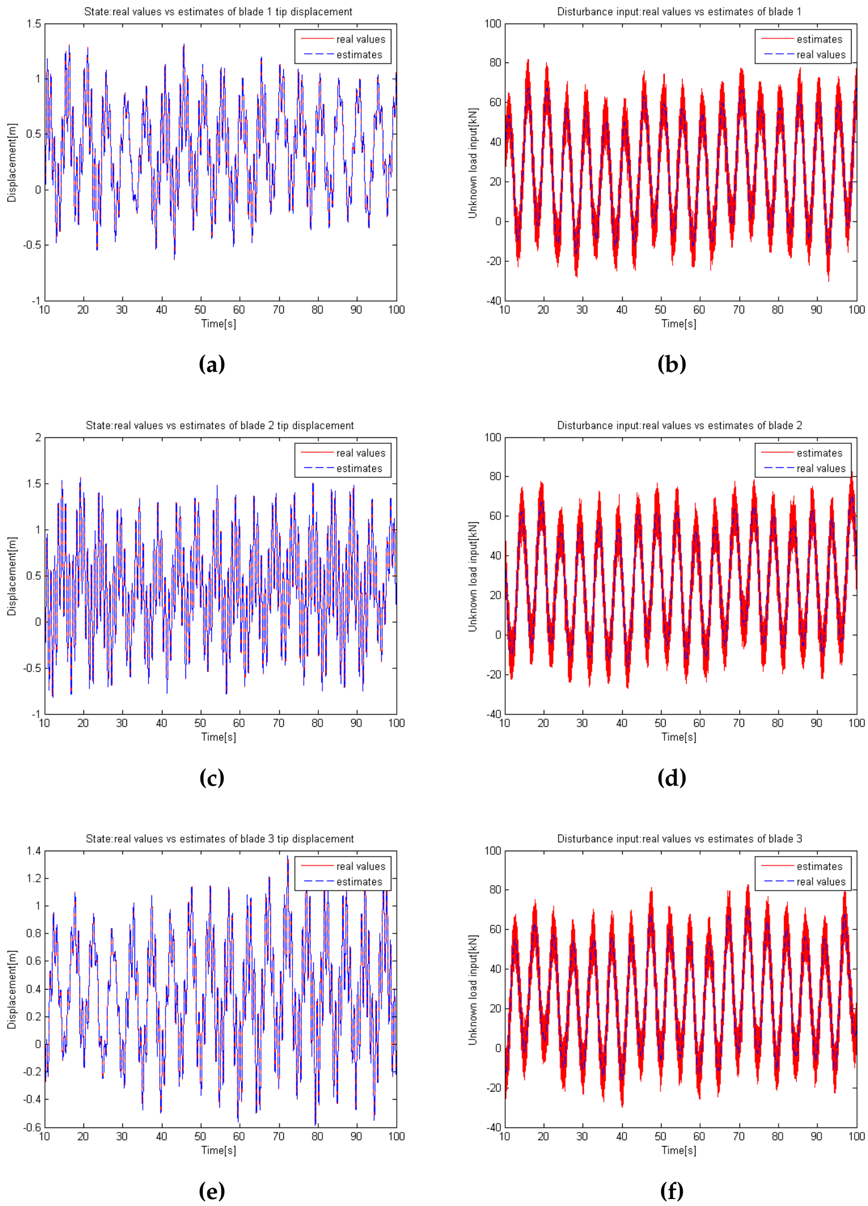
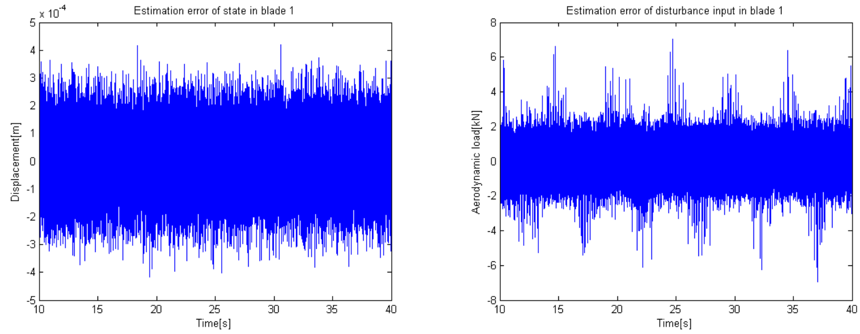


Figure 6. (**a**) State estimation of blade 1 tip displacement under random noise. (**b**) Unknown load input estimation of blade 1 under random noise. (**c**) State estimation of blade 2 tip displacement under random noise. (**d**) Unknown load input load estimation of blade 2 under random noise.(**e**) State estimation of blade 3 tip displacement under random noise. (**f**) Unknown load input estimation of blade 3 under random noise.

 doi:10.20944/preprints201705.0017.v1



(a) **(b)**

Figure 7. (**a**) The error of state estimation of blade 1 tip displacement under random noise. (**b**) The error of unknown load input estimation of blade 1 under random noise.

4.3. Case 3:Compared to LQR

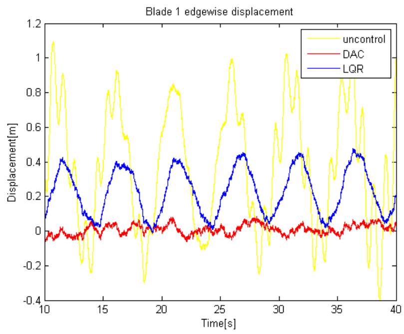
In order to consider the effect of disturbance accommodating, in this case,we investigate the proposed control algorithm.An algorithm based on LQR controller has also been applied in this paper in order to compare with the results from SDAC.The control gains for the LQR have been chosen such that as same as SDAC for the aim of compare to the SDAC.We also take 100 times simulation to get results in statistical significance.Since the aim of control is to reduce the vibration of blades, set the weight $Q = I_{3\times3}$,$R = 10^{-10} I_{3\times3}$(I is an identity matrix) which is as same as [21][24][25]for comparison.

The initial states are selected as to be $x_0 = [0.12; 0.17; 0.02; 2.49; 3.57; 0.53]$.The noise covariance is selected to be $W = 0.001; V = 0.001$ which is 10% of peak value displacement and speed,the initial process noise covariance is selected to be $0.9W; 1.1V$;.The tip displacement in edgewise direction of the blade and active control force using LQR and SDAC is shown in Figure 8.In order to compared,we analysis the maximum value and RMS value of displacement and control force of each blade.It is noticed that SDAC shown a reduction of 94% compared to the uncontrolled case.

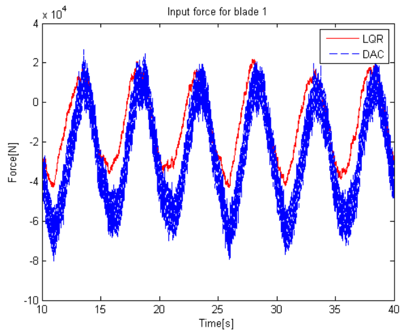
When compared with LQR,the mean value of 100 times simulations show there is a reduction of 60% maximum edgewise blade 1 tip displacement with reduced from 0. 423m(LQR) to 0.167m(SDAC).The blade 2 tip displacement reduces from 0.428m(LQR) to 0.228m(SDAC) and The blade 2 tip displacement reduces from 0.446m(LQR) to 0.103m(SDAC),respectively.

The RMS active input force for blade 1 with the SDAC is 29.94kN and The RMS force for blade 1 with the LQR is 20.18kN. The result shown that the mean value of 100 times RMS displacement reduced 60% with increase 33% RMS value of control force for accommodating the aerodynamic load and gravitational load compared to the LQ.The SDAC shown significantly better reduce of vibration in edgewise direction.

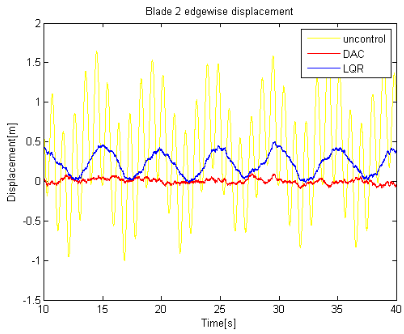
We also give the nacelle displacement in Figure 9.Since there have no active control force action on the nacelle,the vibration is suppress by the passive TMD.It is shown that the nacelle vibration also reduced.



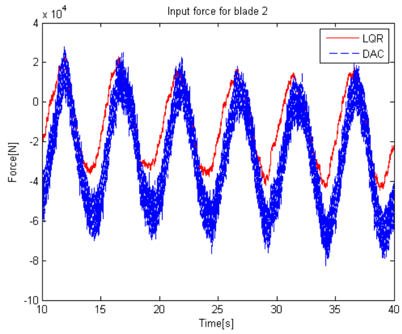
(a)



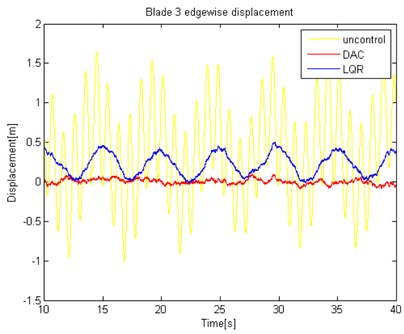
(b)



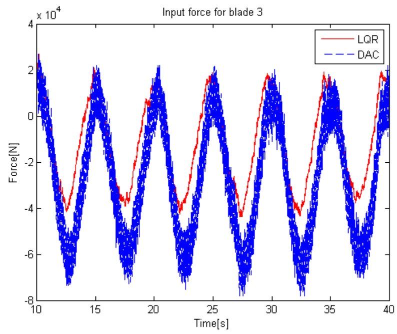
(c)



(d)



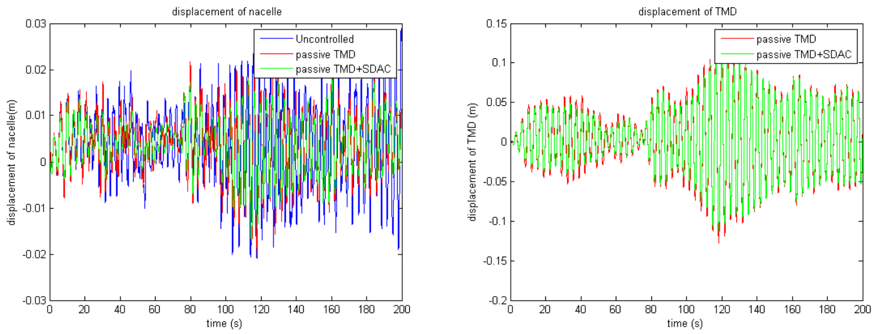
(e)



(f)

Figure 8. (**a**) Comparison of blade 1 tip displacement under random noise. (**b**) Comparison of active control force of blade 1 under random noise. (**c**) Comparison of blade 2 tip displacement under random noise. (**d**) Comparison of active control force of blade 2.(**e**) Comparison of blade 3 tip displacement under random noise. (**f**) Comparison of active control force of blade 3.

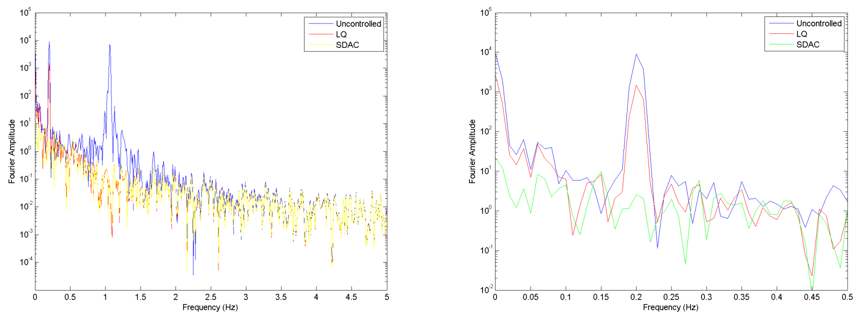
 doi:10.20944/preprints201705.0017.v1



(a) **(b)**

Figure 9. (**a**) Displacement of the nacelle. (**b**) Displacement of the TMD.

The power spectrum density(PSD) for blade 1 tip displacement is show in Figure 10(a).it is noticed that under control action from both LQR and SDAC the peak of the first edgewise mode frequency 1.08Hz are reduced .A window of zoomed in 0.5Hz is shown in Figure 10(b) for illustrating the reduction peak about 0.2Hz which is the aerodynamic load frequency.As shown in figure,compared to under LQR control,the SDAC further reduces the peak around 0.2Hz frequency.The other two blades have the similar conclusion.



(a) **(b)**

Figure 10. (**a**) Comparison of power spectrum density of blade 1 tip in frequency 0-5Hz. (**b**) Zoomed in 0-0.5Hz of power spectrum density of blade 1 tip.

5. Conclusions

The blade vibration problem,it caused by the aerodynamic load input in edgewise direction and gravitational load.Treating the sum of the aerodynamic load and gravitational load as disturbance input, then using the estimation information of load input can accommodating the disturbance and alleviating the vibration. This paper presents the formulation of a stochastic disturbance accommodating control which utilizes a minimum variance unbiased input and state estimation for simultaneously estimating the system states and the disturbance input from measurements.It is proved here that the innovation is a white noise,than a stochastic optimal control is established by separation theorem.The stochastic stability analysis conducted on the controlled system ensure the closed-loop system is stability in the sense of mean-square bounded.In order to investigate a general performance of the proposed SDAC scheme,simulations have been carried out on a wind turbine vibration control system. Results show that the using of MVUE can tracking the unknown load input when it exists random noise.Based on the estimate value ,SDAC accommodating the effect of aerodynamic load and further reduce the displacement of the blade than LQR controller.

Acknowledgments: This work was supported by the National Natural Science Foundation of China (61374084).

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

DAC	Disturbance accommodating control
SDAC	Stochastic disturbance accommodating control
MVUE	Minimum-variance unbiased estimator
BEM	Blade element moment
PSD	Power spectrum density

Appendix

Proof of Lemma 1. Let $\hat{x}_k$ is an unbiased state estimate,the estimate error must satisfy

$$E[x_k - \hat{x}_{k|k}] = 0 \tag{A1}$$

Substituting (27), (33)−(36) in (37),yields

$$\begin{aligned} E[x_k - \hat{x}_{k|k}] =& E[(A - L_kCA)\tilde{x}_{k-1} + L_kCw_{k-1} + Lv_k + w_{k-1} \\ & + L_kCGd_{k-1} - Gd_{k-1} + G\hat{d}_{k-1} - L_kCG\hat{d}_{k-1}] = 0 \end{aligned} \tag{A2}$$

For this condition to hold for any value of d_{k-1} and $\hat{d}_{k-1}$,since $\hat{d}_{k-1}$ is obtained by the assumption that $\hat{x}_k$ is an unbiased state estimate.The gain matrix must satisfy the constraint

$$L_kCG - G = 0 \tag{A3}$$

then $rank(CG) = rank(G) = p$ □

Proof of Lemma 2. It follows from (27) and (33) that

$$\tilde{y}_k = CGd_{k-1} + e_k \tag{A4}$$

Where e_k is given by

$$e_k = C(A\tilde{x}_{k-1} + w_{k-1}) + v_k \tag{A5}$$

In e_k,the control term is vanished which is the kind of [38]. Let $\hat{x}_{k|k}$ be unbiased, then it follows from (A5) that $E[e_k] = 0$ and consequently from (39) that

$$E[\tilde{y}_k] = CGd_{k-1} \tag{A6}$$

Eq.(A6) indicates that an unbiased estimate of the unknown input d_{k-1} can be obtained from the innovation.From (34),$MCG = I$ □

Proof of Lemma 6. Defining$\tilde{x}^\star_k = x_k - \hat{x}^\star_{k|k}$ and $\tilde{d}_k = d_k - \hat{d}_k$,it follows from (27) to (33) and (34) that

$$\tilde{d}_{k-1} = -M_ke_k \tag{A7}$$

$$\tilde{x}^\star_k = A_{k-1}\tilde{x}_{k-1} + G_{k-1}\tilde{d}_{k-1} + w_{k-1} \tag{A8}$$

We have

$$z_k = C_k\tilde{x}^\star_k = C_k(A_{k-1}\tilde{x}_{k-1} + G_{k-1}\tilde{d}_{k-1} + w_{k-1}) + v_k \tag{A9}$$

 doi:10.20944/preprints201705.0017.v1

Since $\tilde{x}_{k-1}$ and $\tilde{d}_{k-1}$ is unbiased, w_{k-1} and v_k is zero mean white noise,then $E[z_k] = 0$
Denoting the variance of z_k by $P_{z,k}$, a straightforward calculation yields

$$P_{z,k} = E[z_k z_k^T] = C(AP_{k-1}A^T + G(F_k^T \tilde{R}_k^{-1} F_k)^{-1} G^T + W_{k-1})C^T + V_k \tag{A10}$$

□

Proof of Theorem 1. The estimation error is zero mean value innovations process, generated by (A9),since it is not influenced by the control law.Our control problem is minimize the cost function

$$J = E[\sum_{k=0}^{N-1} E[(x_k^T Q_k x_k + u_{x,k}^T R_i u_{x,k})|I_k] + E[x_N^T Q_N x_N | I_N]] \tag{A11}$$

Using $x_k = \hat{x}_{k|k} + \tilde{x}_k$,the SDAC problem becomes minimize the cost function

$$\bar{J}^\star = E[\sum_{k=0}^{N-1} (\hat{x}_k^T Q_k \hat{x}_k) + u_{x,k}^T R_k u_{x,k}] + \hat{x}_N^T Q_N \hat{x}_N + \sum_{k=0}^{N-1} tr(P_k Q_k) \tag{A12}$$

subject to

$$\begin{aligned} \hat{x}_{k|k} &= A\hat{x}_{k-1|k-1} + Bu_{k-1} + G\hat{d}_{k-1} + L_k z_k \\ &= A\hat{x}_{k-1|k-1} + Bu_{x,k-1} + (G - BK_d)\hat{d}_{k-1} + L_k z_k \end{aligned} \tag{A13}$$

Since z_k is a zero mean white noise. Solve this stochastic optimal control problem can be using the dynamic programming algorithm.The optimal return function at time k=N is

$$\bar{J}_N^\star = E[\hat{x}_N^T Q_N \hat{x}_N | I_N] = \hat{x}_N^T Q_N \hat{x}_N + \Pi_N \tag{A14}$$

Using the backward equation, at k=N-1:

$$\bar{J}_{N-1}^\star = \min_{u \in U} \hat{x}_{N-1}^T Q_{N-1} \hat{x}_{N-1} + \hat{u}_{N-1}^T R_{N-1} \hat{u}_{N-1} + E[\hat{x}_N^T Q_N \hat{x}_N | \hat{x}_{N-1}] \tag{A15}$$

Substitute (A13) into (A15):

$$\begin{aligned} \bar{J}_{N-1}^\star = &\min_{u \in U} [\hat{x}_{N-1}^T (Q_{N-1} + A^T Q_N A)\hat{x}_{N-1} + 2\hat{x}_{N-1}^T A^T Q_N B u_{x,N-1} + tr(K_N^T Q_N K_N P_{z,N}) \\ &+ \hat{u}_{x,N-1}^T (R_{N-1} + B^T Q_N B)\hat{u}_{x,N-1}] + tr(\hat{d}_{N-1}^T (G - BK_d)^T Q_N (G - BK_d)\hat{d}_{N-1}) \end{aligned} \tag{A16}$$

By taking the partial derivative of (A16) with respect to $u_{x,N-1}$,given the control law

$$\bar{u}_{x,N-1}^\star = -(R_{N-1} + B^T Q_N B)^{-1} B Q_N A \hat{x}_{N-1} \tag{A17}$$

Substitute (A17) into (A16) get

$$S_{N-1} = Q_{N-1} + A^T S_N A - A^T S_N B (R_{N-1} + B^T S_N B)^{-1} B S_N A \tag{A18}$$

$$\Pi_{N-1} = tr(Q_N M_N C P_{z,k}^{-1} C M_N + \Pi_N) + tr(\hat{d}_{N-1}^T (G - BK_d)^T Q_N (G - BK_d)\hat{d}_{N-1}) \tag{A19}$$

with the boundary conditions

$$\begin{aligned} S_N &= Q_N \\ \Pi_N &= 0 \end{aligned} \tag{A20}$$

At k=N-2,the control law is same formula as (A17),then the general control law is

$$\bar{u}^{\star}_{x,k-1} = -(R_{k-1} + B^T S_k B)^{-1} B S_k A \hat{x}_{k-1} \tag{A21}$$

and the optimal cost is

$$\begin{aligned} J =& \hat{x}_0^T S_0 \hat{x}_0 + \Pi_0 + tr(S_0 M_0 C P_{z,0}^{-1} C M_0) + \sum_{k=0}^{N} tr Q_k P_k \\ &+ tr(\hat{d}_0^T (G - BK_d)^T Q_0 (G - BK_d) \hat{d}_0) \end{aligned} \tag{A22}$$

□

Proof of Theorem 2. The proof of the following theorem is similar to Theorem 1,2 given in[44].We shall therefore state the theorem without proof. □

Proof of Theorem 3. Let us use the Lyapunov candidate $V_k = x_k^T S_k x_k$,which $S_k > 0$ is the solution of the Lyapunov equation Eq.(53) .We have

$$\lambda_{min}(S_k)\|x_k\|^2 \leq V_k \leq \lambda_{max}(S_k)\|x_k\|^2 \tag{A23}$$

We need an bound on $E[V_{k+1}(x_{k+1})|x_k]$.From (53) we have

$$\begin{aligned} E[V_{k+1}(x_{k+1})|x_k] - V_k =& E[x_{k+1}^T S_{k+1} x_{k+1}|x_k] - x_k^T S_k x_k \\ =& x_k^T (E[K_x^T R_k K_x + Q_k]) x_k + E[\tilde{x}_k^T \bar{D} \tilde{x}] \\ &+ E[w_k^T \bar{W} w_k] + E[v_{k+1}^T \bar{V}_{k+1} v_{k+1}] \end{aligned} \tag{A24}$$

Where$\bar{D} = (K_x - K_d M_k C A)^T B^T S_{K+1} B (K_x - K_d M_k C A)$
$\bar{W} = (I - BK_d M_k C)^T S_{K+1} (I - BK_d M_k C)$
$\bar{V} = M_k^T K_d^T B^T S_{k+1} B K_d M_k$
It can obtain $\bar{P}$,$\bar{W}$ and $\bar{V}$ is positive definite since S_{k+1} is positive definite. From (44), $\tilde{R}$ is positive definite since Q,R is positive definite.This lead to $\tilde{R}$ is bounded. So $\bar{D}$,$\bar{W}$ and $\bar{V}$ is bounded,and assume the upper bounded is γ. Assuming

$$\underline{d} \leq \bar{D} \leq \bar{d}\underline{w} \leq \bar{W} \leq \overline{w}\underline{v} \leq \bar{V} \leq \bar{v}. \tag{A25}$$

$$E[\tilde{x}_k^T \bar{D} \tilde{x}] = tr(\tilde{x}\tilde{x}_k^T \bar{D}) \leq \bar{d} tr(\tilde{x}\tilde{x}_k^T) \leq \bar{d} P_k \tag{A26}$$

$$E[w_k^T \bar{W} w_k] = tr(ww^T \bar{W}) \leq \overline{w} tr(ww^T) \leq \overline{w} W_k \tag{A27}$$

$$E[v_k^T \bar{V} v_k] = tr(vv^T \bar{V}) \leq \bar{v} tr(vv^T) \leq \bar{v} V_k \tag{A28}$$

Setting

$$\overline{w} W_k + \bar{v} V_k + \bar{d} P_k = \gamma \tag{A29}$$

Next,we need an upper bound on $E[V_{k+1}(x_{k+1})|x_k]$,we have

$$\begin{aligned} E[V_{k+1}(x_{k+1})|x_k] = V_k - x_k^T S_k x_k \leq (1 - \frac{\lambda_{min}(K_x^T R_k K_x + Q_k)}{\lambda_{max}(S_k)}) V_k \\ < (1 - \frac{\mu}{\sigma}) V_k \end{aligned} \tag{A30}$$

Where $0 < \mu < \lambda_{min}(K_x^T R_k K_x + Q_k)$,$\lambda_{max}(S_k) < \sigma$,$\mu < \sigma$. Inserting into(A24),denoting $\alpha = \frac{\mu}{\sigma}$ yielding

$$E[V_{k+1}(x_{k+1})|x_k] - V_k < -\alpha V_k + \gamma \tag{A31}$$

 doi:10.20944/preprints201705.0017.v1

Therefore we are able to apply Lemma 8 in [45]. The closed loop systems in Eq.(56) is exponentially bounded in mean square. The proof of this theorem is complete. □

References

1. Larsen,T.J.; Hansen,A.M. Aeroelastic effects of large blade deflections for wind turbines. *The Science of Making Torque from Wind* **2004**, 238-246.
2. A.Ahlstrm,Influence of wind turbine flexibility on loads and power production. *Wind Energy* **2006**, *9*, 237-249.
3. Fadaeinedjad,R.; Moschopoulos,G.; Moallem,M. Investigation of voltage sag impact on wind turbine tower vibrations. *Wind Energy* **2008**, *11*, 351-375.
4. Bossanyi,E.A.The design of closed loop controllers for wind turbines.*Wind Energy* **2005**, *8*, 481-5.
5. Justin,K.Rice.; Michel,Verhaegen.Robust and distributed control of a smart blade.*Wind Energy* **2010**, *13*, 103-116.
6. Barlas,T.K.; Kuik, G.A.M.V. Review of state of the art in smart rotor control research for wind turbines.*Progress in Aerospace Sciences*, **2010**, *46*, 1-27.
7. Yu,W.;Zhang,M.M.;Xu,J.Z.Effect of Smart Rotor Control Using a Deformable Trailing Edge Flap on Load Reduction under Normal and Extreme Turbulence.*Energies*, **2012**, *5*, 3608-3626.
8. Zhang,M.M.; Bin Tan.; Jianzhong,Xu.Parameter study of sizing and placement of deformable trailing edge flap on blade fatigue load reduction.*Renewable Energy*, **2015**, *77*, 217-226.
9. Lackner,M.A.;van Kuik,G. A comparison of smart rotor control approaches using trailing edge flaps and individual pitch control.*Wind Energy*, **2010**, *13*, 117-134.
10. Lackner,M.A.;Kuik,G.A.M.V. The Performance of Wind Turbine Smart Rotor Control Approaches During Extreme Loads.*Journal of Solar Energy Engineering*, **2010**, *132*, 011008(1-8).
11. van Wingerden.;J,Hulskamp,A.;Barlas,T.;Houtzager,I.;Bersee,H.;Gijs van Kuik.; Verhaegen,M.Two Degree of Freedom Active Vibration Control of a Prototyped Smart Rotor.*Control Systems Technology IEEE Transactions on*, **2011**, *19*, 284-296.
12. Hulskamp,A.W.;van Wingerden,J.W.;Barlas,T.;Champliaud,H.;van Kuik,G.A.M.;Bersee,H.E.N.;Verhaegen,M. Design of a scaled wind turbine with a smart rotor for dynamic load control experiments.*Wind Energy*, **2011**, *14*, 339-354.
13. Staino,A.;Basu,B.Emerging trends in vibration control of wind turbines:a focus on a dual control strategy.*Philosophical Transactions of the Royal Society A:Mathematical,Physical and Engineering Sciences*, **2015**, *373*,20140069.
14. Murtagh,P.J.;Basu,B.;Broderick,B.Along-wind response of a wind turbine tower with blade coupling subjected to rotationally sampled wind loading.*Engineering Structures*, **2005**, *27*, 1209-1219.
15. Murtagh,P.J.;Ghosh,A.;Basu,B. Passive Control of Wind Turbine Vibrations Including Blade/Tower Interaction and Rotationally Sampled Turbulence.*Wind Energy*, **2008**, *11*, 305-317.
16. Matthew,A.;Lackner,M.A. Rotea.Passive structural control of offshore wind turbines.*Wind Energy*, **2011**, *14*, 373-388.
17. Dinh,V.;Basu,B.Passive control of floating offshore wind turbine nacelle and spar vibrations by multiple tuned mass dampers.*Structural Control and Health Monitoring*, **2015**, *22*, 152-176.
18. Colwell,S.;Basu,B.Tuned liquid column dampers in offshore wind turbines for structural control.*Engineering Structures*, **2009**, *31*, 358-369.
19. Mensah,A.F.;Duenas-Osorio,L.Improved reliability of wind turbine towers with tuned liquid column dampers(TLCDs).*Struct Safety*, **2014**, *47*, 78-86.
20. Arrigan,J.;Pakrashi,V.;Basu,B.;Nagarajaiah,S.Control of flapwise vibrations in wind turbine blades using semi-active tuned mass dampers.*Structural Control and Health Monitoring*, **2011**, *18*, 840-851.
21. Staino,A.;Basu,B.;Nielsen,S.R.K.Actuator control of edgewise vibrations in wind turbine blades.*Journal of Sound and Vibration*, **2012**, *331*, 1233-1256.
22. Staino,A.;Basu,B.Dynamics and control of vibrations in wind turbines with variable rotor speed.*Engineering Structures*, **2013**, *56*, 58-67.
23. Lackner,M.A.;Rotea,M.A.Structural control of floating wind turbines.*Mechatronics*, **2011**, *21*, 704-719.

24. Fitzgerald,B.;Basu,B.;Nielsen,S.R.K.Active tuned mass dampers for control of in-plane vibrations of wind turbine blades.*Structural Control and Health Monitoring*, **2013**, *20*, 1377-1396.
25. Fitzgerald,B.;Basu,B.Cable connected active tuned mass dampers for control of in-plane vibrations of wind turbine blades.*Journal of Sound and Vibration*, **2014**, *333*, 5980-6004.
26. Zhang,Z,;Basu,B,;Nielsen,S.R.K.Tuned liquid column dampers for mitigation of edgewise vibrations in rotating wind turbine blades.*Structural Control and Health Monitoring*, **2015**, *22*, 500-517.
27. Basu,B,;Zhang,Z,;Nielsen,S.R.K.Damping of edgewise vibration in wind turbine blades by means of circular liquid dampers.*Wind Energy*, **2016**, *19*, 213-226.
28. Wen-Hua Chen, Jun Yang, Lei Guo,et al.Disturbance Observer Based Control and Related Methods An Overview .*IEEE Transactions on Industrual electronics*, **2016**, *63*,1083-1095.
29. Stol,K.A.;Balas,M.J.Periodic Disturbance Accommodating Control for Blade Load Mitigation in Wind Turbines.*Journal of Solar Energy Engineering*, **2003**, *125*,379-385.
30. Frost,S.A.;Balas,M.J.;Wright,A.D. Direct adaptive control of a utility-scale wind turbine for speed regulation.*International Journal of Robust and Nonlinear Control*, **2009**, *19*,59-71.
31. Na Wang, Alan D. Wright, Kathryn E. Johnson.Independent Blade Pitch Controller Design for a Three-Bladed Turbine Using Disturbance Accommodating Control.*American Control Conference,2016.*
32. Irving P.Girsang, Haspreet S.Dhupia.Collective pitch control of wind turbines using stochastic disturbance accommodating control.*Wind Engineering*, **2013**, *37*,517-534.
33. Jonkman,J.M.;Butterfield,S.;Musial,W.;Scott,G. Definition of a 5-MW Reference Wind Turbine for Offshore System Development. *National Renewable Energy Laboratory,Technical Report NREL/TP-500-38060*, Golden,Colorado,2009.
34. Moheimani,S.O.R.;Pota,H.R.;Petersen,I.R.Spatial balanced model reduction for fexible structures. *Automatica*,**1999**, *35*,269-277.
35. Sørensen,B.F.;Lading,L.;Sendrup,P.;McGugan,M.;Debel,C.P.;Kristensen,O.J.;Larsen,G.;Hansen,A.M.; Rheinländer,J.;Rusborg,J.;Vestergaard,J.D.Fundamentals for remote structural health monitoring of wind turbine blades a preproject.Technical Report, RISØ,2002.
36. Bir,G.Multi-blade coordinate transformation and its application to wind turbine analysis,AIAA2008-1300.In:Proceedings of the 46th AIAA/ASME, Reno,2008.
37. Kitanidis,P.K.Unbiased minimum-variance linear state estimation.*Automatica*, **1987**, *23*,775-778.
38. Gillijns,S.;De Moor,B.Unbiased minimum variance input and state estimation for linear discrete time systems.*Automatica*, **2007**, *43*,111-116.
39. Shinozuka,M.;Deodatis,G.Simulation of Stochastic Processes by Spectral Representation. *Applied Mechanics Reviews*, **1991**, *44*,191-203.
40. Rossi,R.;Lazzari,M.;Vitaliani,R.Wind field simulation for structural engineering purposes. *International Journal for Numerical Methods in Engineering*, **2004**, *61*,738-763.
41. Hansen,M.O.L.The Classical Blade Element Momentum Method.In *Aerodynamics of Wind Turbines Second Edition*; M.O.L.Hansen; Earthscan London, UK, 2008; pp. 45-62.
42. Petersen,I.R.;Pota,H.R.Minimax LQG optimal control of a flexible beam.*Control Engineering Practice*, **2003**, *11*,1273-1287.
43. Pulthasthan,S.;Pota,H.R. The optimal placement of actuator and sensor for active noise control of sound-structure interaction systems.*Smart Materials and Structures*, **2008**, *17*,37001-37011.
44. Huazhen Fang,Raymond A. de Callafon.On the asymptotic stability of minimum-variance unbiased input and state estimation.*Automatica*, **2012**, *48*,3183-3186.
45. Li Xie.;Khargonekar,P.P.Lyapunov-based adaptive state estimation for a class of nonlinear stochastic systems.*Automatica*, **2012**, *48*,1423-1431.