Active Control of Edgewise Vibrations in Wind Turbine Blades Using Stochastic Disturbance Accommodating Control

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Abstract: Vibrations of blade and tower have important impact for wind turbine. This paper presents a active controller design to suppress blade edgewise vibrations under aerodynamic load and gravitational load. Treating the sum of aerodynamic load input in edgewise direction and gravitational load as unknown disturbance input, a stochastic disturbance accommodating control (SDAC) approach is proposed to design a controller which it utilizes a minimum-variance unbiased estimator (MVUE) to estimate both state and unknown input. The stability analysis proved that the proposed SDAC is bounded in mean square. In order to verify the performance of the minimum-variance unbiased estimator and the proposed SDAC, numerical simulations using Matlab/Simulink have been carried out for the National Renewable Energy Laboratory 5-MW wind turbine. Under the different circumstance which exists the random process and measure noise and noise free. It is shown that the estimation value by MVUE can tracking the real state and unknown input. The results are also compared to the traditional linear quadratic regulator (LQR) and show that the proposed stochastic disturbance accommodating control scheme can further reduce displacement in edgewise vibrations direction and the control strategy is more effective than the LQR.

Keywords: stochastic disturbance accommodating control; edgewise vibrations; minimum-variance unbiased estimator

1. Introduction

With the increasing size of wind turbine, the vibration problem is becoming increasingly important. Vibrations not only can cause structural or mechanical damage, decrease the lifetime of wind turbine[1]. On the other side, Ahlstrm[2] has shown that large blade deflections will cause the power drops off and influence on power production. As shown in[3], the mechanical vibration which directly grid-coupled induction generator can cause oscillations in the system voltage even cause the wind turbine trip out. The investigation show that the fatigue loads can be reduced by 20% – 40% which using individual pitch control[4]. But the speed of pitch actuator is limited by the weight of blade and the pitch control is coupling with speed or power regulation leading to practical engineering difficulties. Individual pitch actuation may not be well suited for load reduction[5]. It is necessary for that researchers to investigate the smart rotor control concept and pitch free control.

Smart rotor control for wind turbines is an innovative research area. By change the local aerodynamic property control load through a combination of sensors, actuators and controllers. Barlas and Kuik[6] provided a detailed summary of research in smart rotor control for wind turbines. WeiYu[7][8] using FAST code investigate the effect of smart rotor control using a deformable trailing edge flap for load reduction. The design of the smart rotor control depends on the aerodynamic control devices chosen. Lacker et al.[9] design a trailing edgeflaps for the fatigue load reductions and compared to individual pitch control by GH Bladed code. Further work by Lacker et al.[10], investigate smart rotor control approaches using trailing edgeflaps under the extreme loads. Because there have no accuracy analytical model about distributed aerodynamic control, the exist controller design is based on simple PI or modern control theory which the model achieved by system identification approach[11][12].
Due to the nacelle and blade have hollow nature, the damper install in side of blades can utility space which do not change the aerodynamic property and independent on speed or power regulation. The work by Staino and Basu[13] merge the active tendon control with passive pitch control design a dual control strategy decoupling vibration control from optimal power control. Previous work by many researchers, focused on the damping devices. The mitigation of wind turbine tower are mainly focused on passive dampers. Murtagh et al.[14][15] proposed a passive tuned mass damper(TMD) placed on the top of tower for reduction vibration. Through inverse fourier transform get the response of the coupled blades and tower. Lackner and Rotea[16] investigate the parametric study to determine optimal parameters of a passive TMD located in the nacelle of a offshore wind turbines. Dinh[17] investigates the use of single and multiple TMDs for passive control of side-side vibrations of tower. It shown that multiple TMDs can reduce 10% of the nacelle displacement more than single TMD. Tuned liquid column damper(TLCD) is another commonly damper. Colwell and Basu[18] investigate using TLCD for offshore wind turbine under the combination of wind and wave load. Another investigation using single and multiple TLCD has been done by Mensain and Leonardo[19], it is shown that use two TLCD were benefit over single TLCD. For suppressing blade vibration, many types semi-active and active control have been proposed which damper install in side of blades. Arrigan et al.[20] presented use the semi active tuned mass dampers to control flapwise vibrations. Staino et al.[21][22] proposed an active tendons mounted inside blade tip which give a variable active control force to mitigate edgewise vibration. Fitzgerald at al.[24] investigated active TMD for mitigating edgewise vibrations. Fitzgerald and Basu[25] further proposed a cable connected active TMD to suppress edgewise vibrations. However, both semi-active and active control solutions need relatively complicated controller configurations and some amount of power input. Due to the large centrifugal acceleration of the rotating blade, the TLCD with small mass ratios could effectively mitigate edgewise vibrations. Zhang et al. proposed a tuned liquid column TLCD equipped inside a rotating blade [26]. Basu et al.[27] investigate a circular TLCD for edgewise vibration.

All the paper above about blade vibration control using damper, the authors focus on the damper and control strategy using a state feedback LQ control and do not consider the process noise and measure noise which exist in practice. Moreover, only consider how to suppress vibration, not accommodating the load which aerodynamic load information is not using.

In this paper, we proposed a MVUE filter to estimate the unknown aerodynamic load and gravitational load, then designed a stochastic disturbance accommodating control under the measure noise circumstance. There also has other type of disturbance observer based control, paper[28] gives a review. Although, some researchers investigate the disturbance accommodating control for load mitigation or speed regulation of wind turbine[29][30][31][32]. But, the disturbance input is consider as a disturbance of wind speed and assume satisfy the waveform structure. We presents a stochastic disturbance accommodating controller which there is no prior information about unknown disturbance input. It utilizes a minimum-variance unbiased estimator to estimate both state and unknown input. It is proved that innovation is a white noise, then satisfied the separation theorem. Based on the principle of dynamic programming, a stochastic optimal control is achieved. Stochastic stability analysis by the stochastic Lyapunov style analysis shows the systems exponentially bounded in mean square and ensure the stability of the closed-loop system.

In order to verify the proposed control scheme, the NREL5-MW Wind Turbine is simulated using Matlab. A 5-DOF(degree of freedom) model which blade in-plane mode, tower side-side with one TMD DOF has been considered in this paper. The actuator is taking active tendons install inside of blade as point by Staino[21]. It is compared to the traditional linear quadratic regulator. Results show that the use of the proposed stochastic disturbance accommodating control scheme can precisely estimate the unknown input action on blade edgewise direction. Then significantly mitigate the vibration response of the blade through accommodating.
2. MODEL AND PROBLEM FORMULATION

Wind turbine is a complex system which blade, nacelle/tower and drivetrain are coupling with electrical. To get a model, we first define the coordinate. The motion of tower are described in the ground fixed coordinate $(X,Y,Z)$, this coordinate system is fixed in the support. The origin is intersection of center of the tower and the tower base connection to the support platform. X-axis point horizontally in the nominal downwind direction. Y-axis point to the left when looking in the nominal downwind direction. Z-axis point vertically up from the center of the tower. The motion of blades are described in the co-rotating coordinate $(x,y,z)$, this coordinate system is rotating with blade which the origin at the center of the hub. The x-axis point as same as X-axis, and z-axis point from the hub towards the blade tip.

2.1. Model of blade interaction with tower

The blade is assumed as a Bernoulli Euler cantilever beams, the mathematical model can be carried out based on energy formulation using Euler-Lagrange formulation. The blades are attached at the root to the tower/nacelle which rotate at a constant rotational speed $\Omega \text{rad/s}$ about the rotor hub’s horizontal axis. The blade have distribute mass $\mu(x)$ and distribute bending stiffness $EI(x)$ per unit length along the length $L$. The tower is modeled as Bernoulli cantilever beam. The dynamic coupling between the blade and the tower has been included through the motion of the nacelle. In practice, it is often suppress the tower vibration using passive TMD[15] or TLCD[19], in order to consider its effect, there is one DOF about TMD in the model at the top of the tower in side-side direction which the mass, stiffness and damper are $m_{tmd}, k_{tmd}$ and $c_{tmd}$.

In this paper, we focus on the estimate of the unknown aerodynamic load input and design accommodating control under the case of noise-free or exist random noise. The model described in this paper, we consider the blade vibrating in the first order fundamental mode. The 5DOF modal with first edgewise mode and tower side-side mode with a TMD inside was built in this paper, although there is more complicate 13DOF[27] or 24DOF[33] modal with coupling inplane and out plane vibration. A schema of the structural model is shown in Figure. 1

The azimuthal angle $\psi_j(t)$ of blade $j$ at the time instant $t$ is given by

$$\psi_j(t) = \psi_1(t) + (j-1)\frac{2\pi}{3}, \quad \psi_1(t) = \Omega t; \quad j = 1, 2, 3$$

(1)

The in-plane motions motions of blade $j$ are described by a modal expansion and given by

$$u_{j,\text{in}}(x,t) = \sum_{i=1}^{N} \phi_{j,\text{in}}(x)q_{j,\text{in}}(t)$$

(2)
Where \( u_j(x,t) \) are the total edgewise displacement and \( q_{ji} \) are modal blade co-ordinates describing the contents of mode number \( N \) in the motion of blade, \( \phi_i(x) \) are the mode shape functions, which be calculated by numerically. In order to reduce the number of states required for implementing the control a reduced order model which the fundamental mode has been derived for the system under consideration. As shown in [34], the reduced order system given through balance reduction, the \( H_\infty \) norm of error system is bounded.

Let \( T \) and \( V \) denote the kinetic and potential energy, respectively. Using the classical Euler-Lagrange method:

\[
L(q(t),\dot{q}(t)) = T(q(t),\dot{q}(t)) - V(q(t),\dot{q}(t))
\]  

The equations of motion of the system are given by

\[
\frac{d}{dt} \left( \frac{\partial L(q(t),\dot{q}(t))}{\partial \dot{q}_i(t)} \right) - \frac{\partial L(q(t),\dot{q}(t))}{\partial q_i(t)} = Q_{\text{ext},i}(t)
\]  

Where \( Q_{\text{ext},i}(t) \) is the external force acting on blade or tower.

The kinetic energy consists of three blade, nacelle/tower and the TMD. The total edgewise direction velocity in the rotating \((y,z)\) coordinate are

\[
v_{y,i} = \dot{u}_j \sin(\psi_j) - \Omega u_j(t)
v_{z,i} = \dot{u}_j \cos(\psi_j) + \Omega x + u_j(t)
\]  

The kinetic energy of TMD inside Tower can be modeled as

\[
T_{\text{tmd}} = \frac{1}{2} m_d \ddot{q}_d^2
\]  

The total kinetic energy of the system (the three blades and the tower with TMD) is

\[
T = \frac{1}{2} \sum_{j=1}^{3} T_{\text{blade}} + \frac{1}{2} T_{\text{nacelle}} + T_{\text{tmd}}
\]

\[
= \frac{1}{2} M_0 \dot{u}_4^2 + \frac{1}{2} \sum_{j=1}^{3} \int_0^L \mu(x) v_{y,i}^2 dx + \frac{1}{2} \sum_{j=1}^{3} \int_0^L \mu(x) v_{z,i}^2 dx + \frac{1}{2} m_d \ddot{q}_d^2
\]  

The potential energy consists of the potential energy caused by bending stiffness, centrifugal stiffness, gravitation and the potential energy of nacelle and TMD. The total potential energy is

\[
V = V_{\text{blade}} + \cdot V_{\text{nacelle}} + V_{\text{tmd}}
\]

\[
= \frac{1}{2} \sum_{j=1}^{3} \left( k_{e,ik} q_{jq_j} + k_{g,ik} \cos(\psi_j) + k_{w,ik} q_{j,k} + \frac{1}{2} k_4 u_j^2 + \frac{1}{2} k_d q_d^2 \right)
\]  

Where the stiffness is given by

\[
k_{e,ik} = \int_0^L E I(x) u''_j^2 dx
\]

\[
k_{e,ik} = \int_0^L F_c(x) u'_j dx = \Omega^2 \int_0^L \mu(\xi) \xi \ddot{q}_d dx
\]

\[
k_{g,ik} = \int_0^L F_g(x) u_j^2 dx
\]
Where $E$ is the Young’s modulus of elasticity of the material, $I(x)$ the second moment of area of the blade about the relevant axis and $u'(x), u''(x)$ respectively denote the first and the second derivative of the displacement. The centrifugal force on blade acting at the point $x$ from the hub is

$$F_c(x) = \Omega^2 \int_x^L \mu(\xi) \xi d\xi$$

(10)

The effect of the component of the gravitational force acting along the blade at a distance $x$ from the blade root is

$$F_{g,i}(x) = -\int_x^L \mu(\xi) g \cos(\psi_j) d\xi = -g \cos(\psi_j) \int_x^L \mu(\xi) d\xi$$

(11)

The generalized aerodynamic load on the blade $j$ for the $i$-th mode is computed as

$$Q_{ji} = \int_0^L p_j(x,t) \phi_i(x) dx$$

(12)

with $p_j(x,t)$ representing the variable wind load intensity along the blade length in the edgewise direction. The sideward motion is driven by the sideward aerodynamic force $Q$ and the generator torque $T_g$. If the tower is modelled by a prismatic beam, the generator torque can using a multiplication factor $3/2H$ for the bending moment loads[35].

The equation of motion for the system considered by Euler-Lagrangian equation is

$$\frac{d}{dt} \left( \frac{\partial T(q(t), ˙q(t))}{\partial ˙q(t)} \right) - \frac{\partial T(q(t), ˙q(t))}{\partial q(t)} + \frac{\partial V(q(t))}{\partial q(t)} = Q_{ext,i}(t)$$

(13)

$$\frac{d}{dt} \left( \frac{\partial T(q(t), ˙q(t))}{\partial ˙q_4(t)} \right) - \frac{\partial T(q(t), ˙q(t))}{\partial q_4(t)} + \frac{\partial V(q(t))}{\partial q_4(t)} = Q_4(t)$$

(14)

$$\frac{d}{dt} \left( \frac{\partial T(q(t), ˙q(t))}{\partial ˙q_d(t)} \right) - \frac{\partial T(q(t), ˙q(t))}{\partial q_d(t)} + \frac{\partial V(q(t))}{\partial q_d(t)} = Q_d(t)$$

(15)

The force on tower is

$$Q_4 = Q_{load} + \frac{3}{2H} T_g$$

(16)

The active control is modeled as an external force acting on each blade location $r$ and is given by

$$f_j = \int_0^L 2T_j \sin(\psi) \delta(x-r) \phi(r) dx$$

(17)

Where $r$ is the location of actuator.

Denoting the vector of the generalized coordinates of the system is

$$q(t) = \begin{pmatrix} q_1(t) \\ q_2(t) \\ q_3(t) \\ q_4(t) \\ q_d(t) \end{pmatrix}$$

(18)

The total generalized external force in the Euler Lagrange formulation is given by

$$Q_{ext}(t) = F + Q_{load} + Q_g + Q_e$$

(19)
The generalized control force vector $F$, the reduced generalized aerodynamic load $Q_{load}$ and the reduced generalized gravity load $Q_{g}$ can be derived from the corresponding quantities in the formulation including higher modes and can be written as

$$F(t) = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}, \quad Q_{load}(t) = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix}, \quad Q_g(t) = \begin{bmatrix} Q_{g,1} \\ Q_{g,2} \\ Q_{g,3} \end{bmatrix}, \quad Q_c(t) = \begin{bmatrix} 0 \\ 0 \\ \frac{3}{2\pi} T_S \end{bmatrix}$$ (20)

By defining the quantities

$$m_{1i} = \int_0^L \mu(x)\phi_i dx, m_{2i} = \int_0^L \mu(x)\phi_i^2 dx, m_4 = 3 \int_0^L \mu(x)\phi_i dx + M_0$$ (21)

The final second order matrix differential equation of blade in edgewise direction and tower in side-side direction can be written as

$$M(t)\ddot{q} + C(t)\dot{q} + K(t)q = Q_{ext}(t)$$ (22)

$$y = C_{col}(r)\dot{q} + C_{ov}(r)q$$ (23)

Where the matrices of the model is given by

$$M(t) = \begin{bmatrix} m_2 & 0 & 0 & m_1\cos(\psi_1) & 0 \\ 0 & m_2 & 0 & m_1\cos(\psi_2) & 0 \\ 0 & 0 & m_2 & m_1\cos(\psi_3) & 0 \\ m_1\cos(\psi_1) & m_1\cos(\psi_2) & m_1\cos(\psi_3) & m_4 & 0 \\ 0 & 0 & 0 & 0 & m_d \end{bmatrix}$$

$$C(t) = \begin{bmatrix} c_p & 0 & 0 & 0 & 0 \\ 0 & c_p & 0 & 0 & 0 \\ -2\Omega m_1\sin(\psi_1) & -2\Omega m_1\sin(\psi_2) & -2\Omega m_1\sin(\psi_3) & c_4 + c_d & -c_d \\ 0 & 0 & 0 & -c_4 & c_d \end{bmatrix}$$

$$K(t) = \begin{bmatrix} k_2 + k_w\cos(\psi_1) & 0 & 0 & 0 & 0 \\ 0 & k_2 + k_w\cos(\psi_2) & 0 & 0 & 0 \\ -\Omega^2 m_1\cos(\psi_1) & -\Omega^2 m_1\cos(\psi_2) & -\Omega^2 m_1\cos(\psi_3) & k_4 + k_d & -k_d \\ 0 & 0 & 0 & -k_4 & k_d \end{bmatrix}$$

Since the equation of motion contain periodic term about the azimuth angel of blades which caused by the blade rotation. Through multi-blade coordinate transformation(Coleman transformation) can cancel the effect of periodic term[36].

The transformation matrix is given by

$$q_{nr}(t) = P_{col}q(t), P_{col}u = \begin{bmatrix} 1 & \cos(\psi_1) & \sin(\psi_1) & 0 & 0 \\ 1 & \cos(\psi_2) & \sin(\psi_2) & 0 & 0 \\ 1 & \cos(\psi_3) & \sin(\psi_3) & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$ (24)
2.2. Aerodynamics torque

The aerodynamics torque input of the turbine can be described using air density $\rho$ and rotor disc area $A$ as:

$$T_{shaft} = \frac{1}{2} \rho \nu_{rot}^3 A C_p(\lambda, \beta) \Omega^{-1} \tag{25}$$

where $C_p$ is derived from look-up tables with tip speed ratio ($\lambda = \frac{R \omega}{v_{rot}}$) and pitch angle $\beta$.

2.3. Model of drive train

It is commonly using a 3rd order drive train model which is based on two rotating shafts connected through a gearbox with torsion spring constant $K_{shaft}$, viscous friction $B_{shaft}$, and gear ratio as:

$$\dot{\Omega} = \frac{1}{I_{rot}} (T_{shaft} - \phi K_{shaft} - \dot{\phi} B_{shaft})$$

$$\dot{\omega} = \frac{1}{I_{gen}} (-T_{gen} + \frac{1}{N}(\phi K_{shaft} + \phi B_{shaft}))$$

$$\dot{\phi} = \Omega - \frac{1}{N} \omega \tag{26}$$

where $\phi$ is the shaft torsion angle and $I_{gen}, I_{rot}$ are the inertias of the generator and rotor respectively.

3. SDAC

A detailed formulation of the SDAC for linear time invariant systems is presented in this section. Consider the linear discrete-time system of the following form:

$$x_{k+1} = Ax_k + Gd_k + Bu_k + w_k$$

$$y_k = Cx_k + v_k \tag{27}$$

where $x_k \in \mathbb{R}^m$ is the state vector, $u_k \in \mathbb{R}^n$ is the control input vector, $d_k \in \mathbb{R}^p$ is an unknown input vector, and $y_k \in \mathbb{R}^q$ is the measurement. The process noise $w_k \in \mathbb{R}^n$ and the measurement noise $v_k \in \mathbb{R}^q$ are assumed to be mutually uncorrelated, zero-mean, white random signals with known covariance matrices, $W_k = E[w_k w_k^T]$ and $V_k = E[v_k v_k^T]$, respectively. $W_k \geq 0, V_k \geq 0, x_0$ is independent of $v_k$ and $w_k$ for all $k$. It assumed that $(C_k, A_k)$ is observable and $(A_k, B_k)$ is controllable.

3.1. Disturbance Accommodating Control

The disturbance accommodating control tries to cancel out or minimize the effects of the disturbance input with using a control law as a superposition of control and disturbance accommodating components. The general control law is function of $I_k$ which the set $I_k \in \{x_0, \cdots, x_k, y_0, \cdots, y_k, u_0, \cdots, u_k\}$, given as

$$u_k = K I_k \tag{28}$$

In practice, the state variables $x_k$ and the disturbance variable $d_k$ cannot be measured directly. The realizable control law is:

$$u_k = -K_x \hat{x}_k - K_d \hat{d}_k \tag{29}$$

Where $K_x$ is the gain matrix given through optimal control. The disturbance gain $K_d$ set minimize the norm

$$\min_{K_d} \|BK_d - G\| \tag{30}$$
Solve this optimal problem, the gain matrix is given by

\[ K_d = B^+ G \]  

(31)

Where \( B^+ = B'(BB')^{-1} \) is the pseudo inverse of B.

The basic thought in DAC theory is that disturbances are described by an assumed-waveform model on state space form which the disturbance states \( x_{z,k} \) is updating through the known matrices \( A_d \) and \( C_d \) as Eq.(32)[29][30].

\[
\begin{align*}
\dot{x}_{z,k+1} &= A_d \hat{x}_k \\
\hat{d}_k &= C_d \hat{x}_{z,k}
\end{align*}
\]  

(32)

But the actual disturbances may be not the assumed-waveform form, even have no prior knowledge about unknown input. In next section, we design a input estimator without any information concerning the unknown input \( d_k \).

3.2. Minimum-variance unbiased input and state estimation

The objective of this paper is to design stochastic disturbances accommodating control which there is no prior knowledge about unknown \( d_k \). The unknown input \( d_k \) can be any type of signal.

The first problem is to design a globally optimal estimator of \( x_k \) and \( d_k \) given the sequence of measurements \( Y_k = \{y_0, y_1, \cdots, y_k\} \) without any information concerning the unknown input \( d_k \). In paper[38], Gillijns proposed an unbiased estimators in the sense of minimum-variance unbiased (MVU) over all linear unbiased estimators for system (27). In this paper, we take the similar form filter in paper[38]. We extend it to the control problem. Considering a recursive filter of the form:

\[
\begin{align*}
\hat{x}_{k|k-1} &= A \hat{x}_{k-1|k-1} + Bu_{k-1} \\
\hat{d}_{k-1} &= M_k (y_k - C_k \hat{x}_{k|k-1}) \\
\hat{x}^*_{k|k} &= \hat{x}_{k|k-1} + G \hat{d}_{k-1} \\
\hat{x}_{k|k} &= \hat{x}^*_{k|k} + L_k (y_k - C_k \hat{x}^*_{k|k})
\end{align*}
\]  

(33)-(36)

where \( M_k \in \mathbb{R}^{p \times q} \) and \( L_k \in \mathbb{R}^{m \times q} \) are gain matrix to be determined. Let \( \hat{x}_{k-1|k-1} \) be an unbiased estimate of \( x_{k-1} \), then \( \hat{x}_{k|k-1} \) is biased due to the unknown input in the true system. We want to get an unbiased estimator, the following two condition must satisfy.

**Lemma 1.** If \( \hat{x}_k \) is an unbiased state estimate, the gain matrix must satisfy the constraint

\[
\text{rank}(CG) = \text{rank}(G) = p
\]  

(37)

See the Appendix for the proof of Lemma 1. This condition is as same as in [37][38], but the proof is some different.

**Lemma 2.** Let \( \hat{x}_{k|k} \) be unbiased, then (33)-(34) is an unbiased estimator of \( \hat{d}_{k-1} \) if and only if \( M_k \) satisfies

\[ M_k CG = I \]  

(38)
An MVU input estimate is then obtained as follows by using some results from [38].

A straightforward calculation yields

\[ 
\hat{y}_k = y_k - C\hat{x}_{k|k-1} 
\]  

(39)

Defining

\[ 
\epsilon_k = C(A\hat{x}_{k-1} + w_{k-1}) + v_k 
\]  

(40)

The matrix \( M_k \) is given by the least-squares solution of (A4), satisfies (38). In order to satisfy the assumptions of the Gauss Markov theorem, the variance of \( \epsilon_k \) can be computed from the covariance matrices of the state estimator. An MVU estimator of \( d_k \) is then obtained by weighted least-squares estimation with weighting matrix \( (E[\epsilon_k\epsilon_k^T])^{-1} \). Denoting the variance of \( \epsilon_k \) by \( \tilde{R}_k \), \( P_{k|k} = E[\hat{x}_k\hat{x}_k^T] \) and defining the estimation error by

\[ 
\hat{x}_k = x_k - \hat{x}_{k|k} 
\]  

(41)

and the variance of error by

\[ 
P_{k|k} = E[\hat{x}_k\hat{x}_k^T] 
\]  

(42)

\[ 
P_{k|k-1} = \Lambda_{k-1} P_{k-1|k-1} \Lambda_{k-1}^T + W_{k-1} 
\]  

(43)

A straightforward calculation yields

\[ 
\tilde{R}_k = E[\epsilon_k\epsilon_k^T] = CP_{k|k-1}C^T + V_k 
\]  

(44)

An MVU input estimate is then obtained as follows by using some results from [38].

**Lemma 3.** [38] Let Assumption (37) hold, let \( \hat{x}_{k-1|k-1} \) be unbiased, let \( \tilde{R}_k \) be positive definite and let \( M_k \) be given by

\[ 
M_k = (F_k^T \tilde{R}_k^{-1} F_k)^{-1} F_k^T \tilde{R}_k^{-1} 
\]  

(45)

where \( F_k = C \Gamma \), then (34) is the MVU estimator of \( d_{k-1} \) given the innovation \( \tilde{y}_k \). The variance of the corresponding input estimate, is given by \( (F_k^T \tilde{R}_k^{-1} F_k)^{-1} \)

In paper [37], only state estimation is considered.

Gillijns and De Moor conclude that filter by [37] implicitly estimates the unknown input from the innovation by WLS estimation [38] and estimates the state is equivalent to [37].

**Lemma 4.** Let \( \hat{x}_{k-1|k-1} \) and \( \hat{d}_{k-1} \) be unbiased, then (35)(36) are unbiased estimators of \( \hat{x}_k \) for any value of \( K \).

Substituting (34) and (35) in (36), yields \( \hat{x}_{k|k} \)

\[ 
\hat{x}_{k|k} = A\hat{x}_{k-1|k-1} + Bu_{k-1} + L_k \hat{y}_k + (I - L_k C_k) \Gamma_{k-1} M_k \hat{y}_k 
\]  

(46)

Defining \( K_{e,k} = L_k + (I - L_k C_k) \Gamma_{k-1} M_k \), Eq.(46) is rewritten as

\[ 
\hat{x}_{k|k} = A\hat{x}_{k-1|k-1} + Bu_{k-1} + K_{e,k}(y_k - C_k \hat{x}_{k|k-1}) 
\]  

(47)

**Lemma 5.** Let \( M_k \) be given by (45), \( L_k \) by

\[ 
L_k = P_{k|k-1} \tilde{R}_k^{-1} 
\]  

(48)

then obtain the state update of similar to paper [37] except it exists a control term.
In [38], Gillijns proved Eq. (34)(36) are unbiased estimators minimizing the mean square error over the class of all linear unbiased estimates based on \( Y_k = \{ y_0, \ldots, y_k \} \) when \( M \) given by Eq. (45), \( L_k \) given by Eq. (48).

### 3.3. Stochastic optimal control with MVU estimator

The control problem is to find an output control law that minimizes the cost performance. The cost performance for controlling the system (27) is quadratic:

\[
J = E\left[ \sum_{k=0}^{N-1} E[(x_k^T Q_k x_k + u_{x_k}^T R_k u_{x_k}) | I_k] + E[x_N^T Q_N x_N | I_N] \right]
\]

Substituting \( \hat{x}_k = x_k - \bar{x}_k \) into (49) give us

\[
J = E\left[ \sum_{k=0}^{N-1} (\hat{x}_k^T Q_k \hat{x}_k) + u_{x_k}^T R_k u_{x_k} \right] + \bar{x}_N^T Q_N \bar{x}_N + \sum_{k=0}^{N} tr Q_k P_k
\]

from the orthogonal projection lemma, the minimum variance estimate \( \hat{x}_k \), will be independent of the estimation error \( \bar{x}_k \) and \( d_k \). In order to satisfy separation theorem, the state estimation innovation process, defined as

\[
z_k = y_k - C_k \hat{x}_{k|k}
\]

**Lemma 6.** \( z_k \) is a zero mean values white noise process with variance \( P_{z,k} = C(A P_{k-1} A^T + G(F_k^T V_k^{-1} P_k)^{-1} G^T + W_{k-1}) C^T + V_k \).

See the Appendix for the proof of Lemma 6.

Since \( z_k \) is a zero mean white noise, it is using dynamic programming recursive algorithm to solve the LQG optimal control problem.

**Theorem 1.** Consider the system (27) with estimator (36) and (34), the optimal control is

\[
\hat{u}_{k-1} = -(R_{k-1} + B^T S_k B)^{-1} B S_k A \hat{x}_{k-1} - K_d \hat{d}_{k-1}
\]

where \( S_k \) satisfy the Riccati equation

\[
S_{k-1} = Q_{k-1} + A^T S_k A - A^T S_k B (R_{k-1} + B^T S_k B)^{-1} B S_k A
\]

and the optimal cost criterion is

\[
J = \hat{x}_{0}^T S_{0} \hat{x}_{0} + \Pi_{0} + tr(S_{0} M_{0} C P_{z,0}^{-1} C M_{0}) + \sum_{k=0}^{N} tr Q_k P_k
\]

\[
+ tr(d_{0}^T (G - B K_d)^T Q_0 (G - B K_d) d_{0})
\]

See the Appendix for the proof of Theorem 1.

### 3.4. Stability Analysis

In this section we analyze the stability of the closed loop system. Given the estimator of the form Eq. (34)(36) and output feedback (29). Changing from \( \hat{x}_{k-1} \) and \( d_{k-1} \) to \( \bar{x}_{k-1} \) and \( \bar{d}_{k-1} \) given (41), the control input is

\[
u_{k-1} = -K_x(x_{k-1} - \bar{x}_{k-1}) - K_d(d_{k-1} - \bar{d}_{k-1})
\]

Now we can write the closed-loop dynamics as
\[
\begin{bmatrix}
    x_k \\
    \xi_k
\end{bmatrix} =
\begin{bmatrix}
    A - BK_x & B(K_x - K_dM_kCA) \\
    0 & (I - L_kC)(I - GM_k)A
\end{bmatrix}
\begin{bmatrix}
    x_{k-1} \\
    \xi_{k-1}
\end{bmatrix} +
\begin{bmatrix}
    (I - BK_dM_kC) & BK_dM_k \\
    (I - L_kC)(I - GM_kC) & -(I - L_kC)GM_k - L_k
\end{bmatrix}
\begin{bmatrix}
    w_{k-1} \\
    v_k
\end{bmatrix}
\tag{56}
\]

From the dynamical equations, since this is triangular, the eigenvalues are just those of \( A - BK_x \) together with those of \((I - L_kC)(I - GM_kC)A\). Thus the stability of the estimator and output feedback are independent. Now, it is clear that the system can be stabilized if choose some values of \( K_x, K_d, L_k, M_k \) satisfy some conditions.

**Theorem 2.** If there exist \( M, K \) satisfying (45) and (48), and if \((A, W^\frac{1}{2})\) is stability, then the variance \( P \) converges to a unique fixed point \( \bar{P} \) and the filter is stable.

\[ |\lambda_i[\hat{A}(M_k, L_k)]| < 1 \tag{57} \]

Where \( \hat{A}(M_k, L_k) = (I - L_kC)(I - GM_kC)A, \lambda_i \) denoting the eigenvalue. See the Appendix for the proof of Theorem 2.

Next, we give the stability of the closed loop system states.

**Theorem 3.** If there exist matrices \( R_k > 0, Q_k > 0 \) and \( M_k, K_d, K_d \) satisfying the condition (A25) in Appendix. The closed loop systems of (58) is exponentially bounded in mean square.

\[ x_k = (A - BK_x)x_{k-1} + B(K_x - K_dM_kCA)\xi_{k-1} + (I - BK_dM_kC)w_{k-1} + BK_dM_kv_k \tag{58} \]

See the Appendix for the proof of Theorem 3.

### 4. Application

In order to investigate the performance of MVUE for estimating unknown input action on blade which consist of aerodynamic load in edgewise direction and gravitational load, and verify the effectiveness of proposed SDAC. We simulate a wind turbine model in Matlab using the data from NREL-5WM reference wind turbine. The data of wind turbines are provided in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_0 )</td>
<td>1.412 \times 10^3</td>
<td>kg</td>
<td>( k_r )</td>
<td>6.623 \times 10^3</td>
<td>N.m(^{-1})</td>
</tr>
<tr>
<td>( m_1 )</td>
<td>3.049 \times 10^3</td>
<td>kg</td>
<td>( k_1 )</td>
<td>2.086 \times 10^3</td>
<td>N.m(^{-1})</td>
</tr>
<tr>
<td>( m_4 )</td>
<td>6.975 \times 10^3</td>
<td>kg</td>
<td>( k_2 )</td>
<td>47.25</td>
<td>N.m(^{-1})</td>
</tr>
<tr>
<td>( m_t )</td>
<td>6.975 \times 10^3</td>
<td>kg</td>
<td>( k_4 )</td>
<td>2.646 \times 10^6</td>
<td>N.m(^{-1})</td>
</tr>
<tr>
<td>( k_{tmd} )</td>
<td>2.594 \times 10^4</td>
<td>N.m(^{-1})</td>
<td>( \tau_{gen} )</td>
<td>0.1</td>
<td>s</td>
</tr>
<tr>
<td>( l_{gen} )</td>
<td>5341160</td>
<td>kg.m(^2)</td>
<td>( L_{rot} )</td>
<td>35444067</td>
<td>kg.m(^2)</td>
</tr>
<tr>
<td>( K_{shaf} )</td>
<td>867837000</td>
<td>N.m</td>
<td>( B_{shaf} )</td>
<td>6215000</td>
<td>N.m/(rad/s)</td>
</tr>
<tr>
<td>( L )</td>
<td>61.5</td>
<td>m</td>
<td>( \phi )</td>
<td>97:1</td>
<td></td>
</tr>
</tbody>
</table>

The modeshape is a six-order polynomials of distance \( \xi = r/L \) which calculated from wind turbine data using mode code. The first inplane modeshape is

\[ \phi_1(\xi) = -0.6952x^6 + 2.3760x^5 - 3.5772x^4 + 2.5337x^3 + 0.3627x^2 \tag{59} \]
The instantaneous wind is described as a mean wind $V_s$ and turbulence $V_t$.

$$ V(t) = \bar{V} + V_s(t) $$ (60)

The time series of turbulence is simulated by the with spectral representation (SR) method which was first applied for simulation multi-dimensional random process by Shinozuka[39]. The power spectral density (PSD) assumes the form

$$ S(\omega) = \begin{bmatrix} S_{11}(\omega) & S_{12}(\omega) & \ldots & S_{1N}(\omega) \\ S_{21}(\omega) & S_{22}(\omega) & \ldots & S_{2N}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ S_{N1}(\omega) & S_{N2}(\omega) & \ldots & S_{NN}(\omega) \end{bmatrix} $$ (61)

Where $\omega$ is frequency, $S_{jj}(\omega)$ is self spectrum, and $S_{jk}(\omega)$ is cross spectrum given by

$$ S_{jk}(\omega) = \text{Coh}_{jk} \sqrt{S_{jj}(\omega)S_{kk}(\omega)} $$ (62)

Where $\text{Coh}_{jk}$ is coherence. In this paper we use Davenport form:

$$ \text{Coh}_{jk} = \exp\left(-\frac{C d_{jk}}{\bar{V}}\right) $$ (63)

Where C is constant and $d_{jk} = r_j - r_k$ is distance between location j and location k. The time series can be simulated by[40]

$$ u(x,t) = \sqrt{2\Delta \omega} \sum_{n=1}^N \sum_{l=1}^{N_p} (H(\omega_{ml}) \cos(\omega_{ml}t + \theta_{jm} + \phi_{ml})) $$ (64)

Where $\Delta \omega = \omega / N, \omega_{ml} = l\Delta \omega + (m / N_p)\Delta \omega$ is the double indexing of frequencies, $\phi_{ml}$ is a uniform distribution random number in $[0 - 2\pi]$, $\theta_{jm} = r_j / V_s - r_m / V_s$ is time lag. $H(\omega_{ml})$ is defined by Cholesky’s decomposition of $S(\omega)$, given by

$$ S(\omega) = H(\omega)H^*(\omega)^T $$ (65)

The spectrum offered by Kaimal form expressed as

$$ \frac{f S_{vv}(H,f)}{\sigma^2} = \frac{4f L_t / \bar{V}}{(1 + 70.8f L_t / \bar{V})^{5/3}} $$ (66)

Where $L_t$ is the integral length scale, $\sigma$ is standard deviation which depends on the average wind $V_s$ and the turbulence intensity.

$$ \sigma = I_t \bar{V}_s $$ (67)

Where turbulence intensity is depends on the ground surface roughness and the height from ground.

In this paper, we simulated a 100s turbulence time series with turbulence intensity is 0.1s. The hub height is 90m. The wind turbine is operated in rated speed with rotor angular velocity $\Omega = 12.1\text{rpm}$. The mean wind is $12\text{m/s}$. The local aerodynamic loads in edgewise direction calculated using BEM (see more detail in[41]) and numerical integration along the blade. The aerofoil files contain lift curve and drag curve are given in[33]. The wind velocity corresponding to the hub height and the aerodynamic loads acting on blades and nacelle are shown in Figure. 2.
For the model in this work, the dynamic equation (22) has a state space realization. Since the control design only consider the blade dynamic. For simplicity, system state equation matrices are given as

$$ A = \begin{bmatrix} 0_{3 \times 3} & I_{3 \times 3} \\ -M_b^{-1}K_b & -M_b^{-1}C_b \end{bmatrix}, \quad B = \begin{bmatrix} 0_{3 \times 3} \\ M_b^{-1} \phi(r) \end{bmatrix}, \quad G = \begin{bmatrix} 0_{3 \times 3} \\ M_b^{-1} \end{bmatrix} $$

Where

$$ M(t) = \begin{bmatrix} m_2 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_2 \end{bmatrix}, \quad C(t) = \begin{bmatrix} c_b & 0 & 0 \\ 0 & c_b & 0 \\ 0 & 0 & c_b \end{bmatrix}, \quad K(t) = \begin{bmatrix} k_{b1} & 0 & 0 \\ 0 & k_{b2} & 0 \\ 0 & 0 & k_{b3} \end{bmatrix} $$

Where $k_{b1} = k_2 + k_w \cos(\psi_1), k_{b2} = k_2 + k_w \cos(\psi_2)$ and $k_{b3} = k_2 + k_w \cos(\psi_3)$.

Since the proposed SDAC is based on discrete state space equation, firstly we discrete the system as sample interval $T_s = 0.0004\text{s}$. The edgewise displacement on a real wind turbine, will not be directly measurable, and will need to be estimated from other sensor data. Many different sensors have been investigated for wind turbine health monitoring[5][35]. In this paper, we assume that the sensor is accelerometer sensor. The displacement and velocity were produced by double integrating and integrating the accelerometer signal[42]. To get the performance of MVUE and SDAC, different cases were considered. First one, we estimate the state and unknown load input under noise free circumstance; second case check the performance under random noise; finally, for the purpose to comparison, result of LQR control was shown. Since the control aim is to alleviate blade vibration and only three control input, in order to reduce the order of controller, control choose only three blade. The measure are given as displacement matrices and velocity matrices

$$ C = C_{\text{od}}(r)\dot{q} + C_{\text{ov}}(r)q $$

A proper choice of actuator and sensor location can improve the performance of control. For finding the optimal placement of sensors and actuators. We choose the location based on energy approach which is based on system observability and controllability[43]. The observability gramian matrix $W_o$ and controllability gramian matrix $W_c$ can be achieved by following Lyapunov equation.

$$ A'W_o + W_oA + C'C = 0 $$

$$ AW_c + W_cA' + BB' = 0 $$
The criteria of choosing actuator and sensor location is maximum the objective function

\[ J_{op} = \text{trace}(W)^{2N} \sqrt{\det(W)/\sigma(\lambda_i)} \]  

(71)

Where \( W \) is observability gramian matrix or controllability gramian matrix. \( N \) is the order of system, \( \sigma(\lambda_i) \) is the standard deviation of the eigenvalues \( \lambda_i \) of gramian matrix. The numerical results of the objective function are shown in Figure 3.

![Figure 3](image-url)  

(a) Numerical result of the objective function for observability gramian matrix. (b) Numerical result of the objective function for controllability gramian matrix.

As seen from figure 3, the maximum value of objective function are in tip. Then the location of actuator and sensor in this paper is in tip of each blade.

4.1. Case 1: MVUE under noise free

We first focus on the estimation problem and investigate the performance of the MVUE. The unknown input in this paper is the sum of aerodynamic load in edgewise direction and gravitational load. Since we want to check out the accuracy of estimation about known input, there is no control action in this case. Assuming that there is no random noise. First, we check the condition of Lemma 1. Since \( B = \begin{bmatrix} 0_{3 \times 3} & M_b^{-1}\phi(1) \end{bmatrix} \), calculating from \( CB = \begin{bmatrix} 0_{3 \times 3} & M_b^{-1}\phi(1) \end{bmatrix} \). We can get \( \text{rank}(CB) = \text{rank}(B) \), it satisfies the lemma 1.

In this simulation, the initial states are selected to be \( x_0 = [0.12; 0.17; 0.02; 2.49; 3.57; 0.53] \) which is a random sampled value under no control action. The real and the estimation of state and unknown load input which is the sum of aerodynamic load and gravitational load are shown in Figure 4. From the Figure, we can find that the estimated value tracking the real value both in state and unknown input. Figure 5 shows the error of estimation about blade 1, it is shown the estimation error is bounded. The RMS error of blade 1 tip displacement is 0.0014m and RMS error of unknown input is 0.2kN. Although the error of unknown aerodynamic input is fluctuating, but the mean value of estimation error of unknown input in three blades is -0.3624N, -1.3313N and 5.3673N, respectively. We can conclude that the results are according with the theory analysis which the estimator is in the sense of minimum variance and the error is bounded in mean square.
Figure 4. (a) State estimation: blade 1 tip displacement. (b) Unknown load input estimation of blade 1. (c) State estimation: blade 2 tip displacement. (d) Unknown load input estimation of blade 2. (e) State estimation: blade 3 tip displacement. (f) Unknown input load estimation of blade 3.
Next, we consider the case with random process and measurement noise.

4.2. Case 2: MVUE under random noise

In this simulation, we investigate the performance of MVUE under random process and measurement noise. Since the noise is random. In order to get a statistic analysis, we simulate 100 times and check the mean value of maximum displacement and RMS displacement and also the unknown force. The initial states are also selected to be $x_0 = [0.120.170.022.493.570.53]$. The noise covariance is selected to be $W = 0.001$ and $V = 0.001$, the initial process noise covariance is selected to be $0.9W$ and $1.1V$. The real value and the estimation of state and unknown input is shown in Figure 6. It is noticed that the estimation of states and disturbance tracking true values under the random process and measurement noise. The estimation error is bounded.

Figure 7 shows the error of estimation about blade 1. From the statistic analysis, the RMS error of blade 1 tip displacement is 0.0022m and RMS error of unknown input is 0.9kN. The mean value of estimation error of unknown input in blade 1 is -0.0124N and it is accord with the theory analysis which in the sense of minimum variance and the error bounded in mean square. The other two blades have the similar conclusion. Since the performance is affected by the variance of noise. If the variance is incorrect, the estimator may be have poor performance. For simplicity, in this paper, we only consider the systems under correct noise variance.

Through the simulation of case 1 and case 2, it can be verified that the minimum-variance unbiased estimator can estimating the state and unknown input accurately. In next case, we check the performance of the stochastic disturbance accommodating control which is based on the accuracy of the MVUE.
Figure 6. (a) State estimation of blade 1 tip displacement under random noise. (b) Unknown load input estimation of blade 1 under random noise. (c) State estimation of blade 2 tip displacement under random noise. (d) Unknown load input load estimation of blade 2 under random noise. (e) State estimation of blade 3 tip displacement under random noise. (f) Unknown load input estimation of blade 3 under random noise.
Figure 7. (a) The error of state estimation of blade 1 tip displacement under random noise. (b) The error of unknown load input estimation of blade 1 under random noise.

4.3. Case 3: Compared to LQR

In order to consider the effect of disturbance accommodating, in this case, we investigate the proposed control algorithm. An algorithm based on LQR controller has also been applied in this paper in order to compare with the results from SDAC. The control gains for the LQR have been chosen such that as same as SDAC for the aim of compare to the SDAC. We also take 100 times simulation to get results in statistical significance. Since the aim of control is to reduce the vibration of blades, set the weight $Q = I_{3 \times 3}, R = 10^{-10}I_{3 \times 3}$ (I is an identity matrix) which is as same as [21][24][25] for comparison.

The initial states are selected as to be $x_0 = [0.12; 0.17; 0.02; 2.49; 3.57; 0.53]$. The noise covariance is selected to be $W = 0.001; V = 0.001$ which is 10% of peak value displacement and speed, the initial process noise covariance is selected to be $0.9W; 1.1V$; The tip displacement in edgewise direction of the blade and active control force using LQR and SDAC is shown in Figure 8. In order to compared, we analysis the maximum value and RMS value of displacement and control force of each blade. It is noticed that SDAC shown a reduction of 94% compared to the uncontrolled case.

When compared with LQR, the mean value of 100 times simulations show there is a reduction of 60% maximum edgewise blade 1 tip displacement with reduced from 0.423m (LQR) to 0.167m (SDAC). The blade 2 tip displacement reduces from 0.428m (LQR) to 0.228m (SDAC) and The blade 2 tip displacement reduces from 0.446m (LQR) to 0.103m (SDAC), respectively.

The RMS active input force for blade 1 with the SDAC is 29.94kN and The RMS force for blade 1 with the LQR is 20.18kN. The result shown that the mean value of 100 times RMS displacement reduced 60% with increase 33% RMS value of control force for accommodating the aerodynamic load and gravitational load compared to the LQ. The SDAC shown significantly better reduce of vibration in edgewise direction.

We also give the nacelle displacement in Figure 9. Since there have no active control force action on the nacelle, the vibration is suppress by the passive TMD. It is shown that the nacelle vibration also reduced.
Figure 8. (a) Comparison of blade 1 tip displacement under random noise. (b) Comparison of active control force of blade 1 under random noise. (c) Comparison of blade 2 tip displacement under random noise. (d) Comparison of active control force of blade 2. (e) Comparison of blade 3 tip displacement under random noise. (f) Comparison of active control force of blade 3.
The power spectrum density (PSD) for blade 1 tip displacement is shown in Figure 10(a). It is noticed that under control action from both LQR and SDAC, the peak of the first edgewise mode frequency 1.08 Hz are reduced. A window of zoomed in 0.5 Hz is shown in Figure 10(b) for illustrating the reduction peak about 0.2 Hz which is the aerodynamic load frequency. As shown in figure, compared to under LQR control, the SDAC further reduces the peak around 0.2 Hz frequency. The other two blades have the similar conclusion.

5. Conclusions

The blade vibration problem, it caused by the aerodynamic load input in edgewise direction and gravitational load. Treating the sum of the aerodynamic load and gravitational load as disturbance input, then using the estimation information of load input can accommodating the disturbance and alleviating the vibration. This paper presents the formulation of a stochastic disturbance accommodating control which utilizes a minimum variance unbiased input and state estimation for simultaneously estimating the system states and the disturbance input from measurements. It is proved here that the innovation is a white noise, than a stochastic optimal control is established by separation theorem. The stochastic stability analysis conducted on the controlled system ensure the closed-loop system is stability in the sense of mean-square bounded. In order to investigate a general performance of the proposed SDAC scheme, simulations have been carried out on a wind turbine vibration control system. Results show that the using of MVUE can tracking the unknown load input when it exists random noise. Based on the estimate value, SDAC accommodating the effect of aerodynamic load and further reduce the displacement of the blade than LQR controller.
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Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

- DAC: Disturbance accommodating control
- SDAC: Stochastic disturbance accommodating control
- MVUE: Minimum-variance unbiased estimator
- BEM: Blade element moment
- PSD: Power spectrum density

Appendix

Proof of Lemma 1. Let \( \hat{x}_k \) is an unbiased state estimate, the estimate error must satisfy

\[
E[x_k - \hat{x}_{k|k}] = 0 \tag{A1}
\]

Substituting (27), (33)–(36) in (37), yields

\[
E[x_k - \hat{x}_{k|k}] = E[(A - L_k CA)\hat{x}_{k-1} + L_k Cw_{k-1} + L\nu_k + w_{k-1} + L_k CGd_{k-1} - Gd_{k-1} + G\hat{d}_{k-1} - L_k CG\hat{d}_{k-1}] = 0 \tag{A2}
\]

For this condition to hold for any value of \( d_{k-1} \) and \( \hat{d}_{k-1} \), since \( \hat{d}_{k-1} \) is obtained by the assumption that \( \hat{x}_k \) is an unbiased state estimate. The gain matrix must satisfy the constraint

\[
L_k CG - G = 0 \tag{A3}
\]

then \( \text{rank}(CG) = \text{rank}(G) = p \)

Proof of Lemma 2. It follows from (27) and (33) that

\[
\tilde{y}_k = CGd_{k-1} + e_k \tag{A4}
\]

Where \( e_k \) is given by

\[
e_k = C(A\hat{x}_{k-1} + w_{k-1}) + v_k \tag{A5}
\]

In \( e_k \), the control term is vanished which is the kind of [38]. Let \( \hat{x}_{k|k} \) be unbiased, then it follows from (A5) that \( E[e_k] = 0 \) and consequently from (39) that

\[
E[\tilde{y}_k] = CGd_{k-1} \tag{A6}
\]

Eq.(A6) indicates that an unbiased estimate of the unknown input \( d_{k-1} \) can be obtained from the innovation. From (34), \( MCG = l \)

Proof of Lemma 6. Defining \( \xi^*_k = x_k - \hat{x}^*_{k|k} \) and \( \tilde{d}_k = d_k - \hat{d}_k \), it follows from (27) to (33) and (34) that

\[
\tilde{d}_{k-1} = -M_k e_k \tag{A7}
\]

\[
\hat{x}^*_k = A_{k-1} \hat{x}_{k-1} + G_{k-1} \tilde{d}_{k-1} + w_{k-1} \tag{A8}
\]

We have

\[
z_k = C_k \xi^*_k = C_k (A_{k-1} \hat{x}_{k-1} + G_{k-1} \tilde{d}_{k-1} + w_{k-1}) + v_k \tag{A9}
\]
Since \( \tilde{x}_{k-1} \) and \( \hat{d}_{k-1} \) is unbiased, \( \tilde{w}_{k-1} \) and \( \nu_k \) is zero mean white noise, then \( E[z_k] = 0 \)

Denoting the variance of \( z_k \) by \( P_{z,k} \), a straightforward calculation yields

\[
P_{z,k} = E[z_k z_k^T] = C(A P_{z,k-1} A^T + G(F_k^T \bar{R}_k^{-1} F_k) )^{-1} G^T + W_{k-1}) C^T + V_k
\]

(A10)

**Proof of Theorem 1.** The estimation error is zero mean value innovations process, generated by (A9), since it is not influenced by the control law. Our control problem is minimize the cost function

\[
J = E[ \sum_{k=0}^{N-1} E[(x_k^T Q_k x_k + u_k^T R_k u_k)] I_k ] + E[x_k^T Q_N x_N | I_N]
\]

(A11)

Using \( x_k = \hat{x}_{k|k} + \hat{z}_k \), the SDAC problem becomes minimize the cost function

\[
J^* = E[ \sum_{k=0}^{N-1} (\hat{x}_k^T Q_k \hat{x}_k) + u_k^T R_k u_k ] + \hat{x}_{N|N}^T Q_N \hat{x}_N + \sum_{k=0}^{N-1} tr(P_k Q_k)
\]

subject to

\[
\hat{x}_{k|k} = A \hat{x}_{k-1|k-1} + Bu_{k-1} + C \hat{d}_{k-1} + L \tilde{z}_k
\]

(A13)

\[
= A \hat{x}_{k-1|k-1} + Bu_{x,k} + (G - BK_d) \hat{d}_{k-1} + L \tilde{z}_k
\]

Since \( \tilde{z}_k \) is a zero mean white noise. Solve this stochastic optimal control problem can be using the dynamic programming algorithm. The optimal return function at time \( k=N \) is

\[
\bar{J}_N = E[\hat{x}_N^T Q_N \hat{x}_N | I_N] = \hat{x}_N^T Q_N \hat{x}_N + \Pi_N
\]

(A14)

Using the backward equation, at \( k=N-1 \):

\[
\bar{J}_{N-1} = \min_{u \epsilon U} \hat{x}_{N-1}^T Q_{N-1} \hat{x}_{N-1} + \hat{u}_{N-1}^T R_{N-1} \hat{u}_{N-1} + E[\hat{x}_{N-1}^T Q_{N-1} \hat{x}_{N-1} | I_{N-1}]
\]

(A15)

Substitute (A13) into (A15):

\[
\bar{J}_{N-1} = \min_{u \epsilon U} \hat{x}_{N-1}^T (Q_{N-1} + A^T Q_N A) \hat{x}_{N-1} + 2 \hat{x}_{N-1}^T A^T Q_N Bu_{x,N} + tr(K_{N-1}^T Q_N K_N P_{z,N})
\]

\[
+ \hat{u}_{N-1}^T (R_{N-1} + B^T Q_N B) \hat{u}_{N-1} + tr(\hat{d}_{N-1}^T (G - BK_d)^T Q_N (G - BK_d) \hat{d}_{N-1})
\]

(A16)

By taking the partial derivative of (A16) with respect to \( u_{x,N-1} \) given the control law

\[
\hat{u}_{x,N-1} = -(R_{N-1} + B^T Q_N B)^{-1} B Q_N A \hat{x}_{N-1}
\]

(A17)

Substitute (A17) into (A16) get

\[
S_{N-1} = Q_{N-1} + A^T S_{N-1} A - A^T S_{N-1} B (R_{N-1} + B^T S_{N-1} B)^{-1} B S_{N-1} A
\]

(A18)

\[
\Pi_{N-1} = tr(Q_N M_N C P_{z,k}^{-1} C M_N + \Pi_N) + tr(\hat{d}_{N-1}^T (G - BK_d)^T Q_N (G - BK_d) \hat{d}_{N-1})
\]

(A19)

with the boundary conditions

\[
S_N = Q_N
\]

\[
\Pi_N = 0
\]

(A20)
At \( k = N-2 \), the control law is same formula as (A17), then the general control law is

\[
\dot{a} = -(R_k - B^T S_k B)^{-1} BS_k A \dot{x} - 1
\]

(A21)

and the optimal cost is

\[
J = \dot{x}_D^T S_0 \dot{x}_0 + \Pi_0 + tr(S_0 M_0 C P_{x,0}^{-1} M_0) + \sum_{k=0}^N tr Q_k P_k
\]

\[
+ tr(d_0^T (G - BK_d)^T Q_0 (G - BK_d) d_0)
\]

(A22)

\[
\dot{x}_D^T = (\dot{\alpha} - \beta) \dot{x}_D^T + \dot{\beta} + \dot{\gamma}
\]

Proof of Theorem 2. The proof of the following theorem is similar to Theorem 1, 2 given in[44]. We shall therefore state the theorem without proof.

Proof of Theorem 3. Let us use the Lyapunov candidate \( V_k = x_k^T S_k x_k \), which is the solution of the Lyapunov equation Eq. (53). We have

\[
\lambda_{\text{min}}(S_k)\|x_k\|^2 \leq V_k \leq \lambda_{\text{max}}(S_k)\|x_k\|^2
\]

We need an upper bound on \( E[V_{k+1}(x_{k+1})|x_k] \). From (53) we have

\[
E[V_{k+1}(x_{k+1})|x_k] = V_k = E[x_{k+1}^T S_{k+1} x_{k+1}] = x_k^T S_k x_k
\]

(A24)

\[
+ E[w_k^T W w_k] + E[v_k^T \dot{V} v_k]
\]

Where \( D = (K_k - K_d M_k C)^T B^T S_{k+1} B (K_k - K_d M_k C) \)

\( W = (I - BK_d M_k C)^T S_{k+1} (I - BK_d M_k C) \)

\( \bar{V} = M_k^T K_d ^T B^T S_{k+1} B K_d M_k \)

It can obtain \( \bar{P}, \bar{W} \) and \( \bar{V} \) is positive definite since \( S_{k+1} \) is positive definite. From (44), \( \bar{R} \) is positive definite since \( Q_R \) is positive definite. This lead to \( \bar{R} \) is bounded. So \( \bar{D}, \bar{W} \) and \( \bar{V} \) is bounded, and assume the upper bounded is \( \gamma \). Assuming

\[
d \leq \bar{D} \leq \bar{W} \leq \bar{W} \leq \bar{V} \leq \bar{V}.
\]

(A25)

\[
E[x_{k+1}^T \dot{D} x] = tr(x x_{k+1}^T \dot{D}) \leq \bar{d} tr(x x_{k+1}^T) \leq \bar{d} P_k
\]

(A26)

\[
E[w_k^T W w_k] = tr(w w^T W) \leq \bar{w} tr(w w^T) \leq \bar{w} W_k
\]

(A27)

\[
E[v_k^T \bar{V} v_k] = tr(v v^T \bar{V}) \leq \bar{v} tr(v v^T) \leq \bar{v} V_k
\]

(A28)

Setting

\[
\bar{w} W_k + \bar{v} V_k + \bar{d} P_k = \gamma
\]

(A29)

Next, we need an upper bound E[\( V_{k+1}(x_{k+1})|x_k \)], we have

\[
E[V_{k+1}(x_{k+1})|x_k] = V_k - x_k^T S_k x_k \leq (1 - \frac{\lambda_{\text{min}}(K_k^T R_k K_x + Q_k)}{\lambda_{\text{max}}(S_k)}) V_k
\]

(A30)

\[
< (1 - \frac{\mu}{\sigma}) V_k
\]

Where \( 0 < \mu < \lambda_{\text{min}}(K_k^T R_k K_x + Q_k) \lambda_{\text{max}}(S_k) < \sigma, \mu < \sigma \). Inserting into (A24), denoting \( \alpha = \frac{\mu}{\sigma} \) yielding

\[
E[V_{k+1}(x_{k+1})|x_k] - V_k < -\alpha V_k + \gamma
\]

(A31)
Therefore we are able to apply Lemma 8 in [45]. The closed loop systems in Eq.(56) is exponentially bounded in mean square. The proof of this theorem is complete.

References


