

Article

# Bézier Curve Modeling for Neutrosophic Data Problem

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**Abstract:** In this paper, a geometric model is introduced for Neutrosophic data problem for the first time. This model is based on neutrosophic sets and neutrosophic relations. Neutrosophic control points are defined according to these points, resulting in neutrosophic Bézier curves.

**Keywords:** neutrosophic sets; neutrosophic logic; bézier curve

## 1. Introduction

While today's technologies are rapidly developing, the contribution of mathematics is fundamental and leading the science. In particular, the developments in geometry are not only modeling the mathematics of the objects but also being geometrically modeled in most abstract concepts. What is the use of these abstract concepts in modeling? In the future of science, there will be artificial intelligence. For the development of this technology, many branches of science work together and especially the topics such as logic, data mining, quantum physics, machine learning come to the forefront. Of course, the place where these areas can cooperate is the computer environment. Data can be transferred in various ways. One of them is to transfer the data as a geometric model. The first method that comes to mind in terms of a geometric model is the Bézier technique. Although this method is generally used for curve and surface designs, it is used in many disciplines ranging from the solution of differential equations to robot motion planning.

The embodied state of the adventure of obtaining meaning and mathematical results from uncertainty states (fuzzy) was begun by Zadeh [1]. Fuzzy sets proposed by Zadeh provided a new dimension to the concept of classical sets. Atanassov introduced intuitionistic fuzzy sets dealing with membership and non-membership degrees [2]. Neutrosophy was proposed by Smarandache as a mathematical application of the concept neutrality [3]. Neutrosophic set concept is defined with membership, non-membership and indeterminacy degrees. Neutrosophic set concept is separated from intuitionistic fuzzy set by the difference as follow: intuitionistic fuzzy sets are defined by degree of membership and non-membership degree and, uncertainty degrees by the 1- (membership degree plus non-membership degree), while degree of uncertainty are considered independently of the degree of membership and non-membership in neutrosophic sets. Here, membership, non-membership and uncertainty (indeterminacy) degrees can be judged according to the interpretation in the spaces to be used, such as truth and falsity degrees. It depends entirely on subject space (discourse universe). In this sense, the concept of neutrosophic set is the solution and representation of the problems with various fields.

Recently, geometric interpretations of data that uncertain truth were presented by Wahab and friends [4–7]. They studied geometric models of fuzzy and intuitionistic fuzzy data and gave fuzzy interpolation and Bézier curve modeling. In this paper, we consider a geometric modeling of Neutrosophic data.

## 2. Preliminaries

In this section, we will first give some fundamental definitions dealing with Bézier curve and Neutrosophic sets (elements). We will then introduce the new definitions needed to form a *Neutrosophic Bézier curve*.

**Definition 2.1.** Let  $P_i, (i = 0, 1, 2, \dots, n), P_i \in E^3$  be the set of points. A Bézier curve with degree  $n$  is defined by

$$B(t) = \sum_{i=0}^n B_i^n(t) P_i, t \in [0, 1] \quad (2.1)$$

where  $B_i^n(t) = \sum_{i=0}^n \binom{n}{i} (1-t)^{n-i} t^i$  and  $P_i$  are the Bernstein polynomial function and the control points, respectively. Notice that there are  $(n+1)$ -control points for a Bézier curve with degree  $n$ . Because  $n$ -interpolation is done with  $(n+1)$ -control points [8–11].

**Definition 2.2.** Let  $E$  be a universe and  $A \subseteq E$ .  $N = \{(x, T(x), I(x), F(x)) : x \in A\}$  is a neutrosophic element where  $T_p = N \rightarrow [0, 1]$  (membership function),  $I_p = N \rightarrow [0, 1]$  (indeterminacy function) and  $F_p = N \rightarrow [0, 1]$  (non-membership function).

**Definition 2.3.** Let  $A^* = \{(x, T(x), I(x), F(x)) : x \in A\}$  and  $B^* = \{(y, T(y), I(y), F(y)) : y \in B\}$  be neutrosophic elements.  $NR = \{(x, y), T(x, y), I(x, y), F(x, y)) : (x, y) \in A \times B\}$  is a neutrosophic relation on  $A^*$  and  $B^*$ .

## 3. Neutrosophic Bézier Model

**Definition 3.1.** NS of  $P^*$  in space  $N$  is NCP and  $P^* = \{P_i^*\}$  where  $i = 0, \dots, n$  is a set of NCPs where there exists  $T_p = N \rightarrow [0, 1]$  as membership function,  $I_p = N \rightarrow [0, 1]$  as indeterminacy function and  $F_p = N \rightarrow [0, 1]$  as non-membership function with

$$T_p(P^*) = \begin{cases} 0 & \text{if } P_i \notin N \\ a \in (0, 1) & \text{if } P_i \tilde{\in} N \\ 1 & \text{if } P_i \in N \end{cases} \quad F_p(P^*) = \begin{cases} 0 & \text{if } P_i \notin N \\ c \in (0, 1) & \text{if } P_i \tilde{\in} N \\ 1 & \text{if } P_i \in N \end{cases}$$

$$I_p(P^*) = \begin{cases} 0 & \text{if } P_i \notin N \\ e \in (0, 1) & \text{if } P_i \tilde{\in} N \\ 1 & \text{if } P_i^- \in N \end{cases}$$

Bézier Neutrosophic curves are generated based on the control points from one of  $TC = \{(x, y, T(x, y))\}$ ,  $IC = \{(x, y, I(x, y))\}$  and  $FC = \{(x, y, F(x, y))\}$  sets. Thus, there will be three different neutrosophic Bézier curve models for a neutrosophic relation and variables  $x$  and  $y$ . A neutrosophic control point relation can be defined as a set of  $n+1$  points that shows a position and coordinate of a location and is used to described three curve which are denoted by

$$NR_{p_i} = \{NR_{p_0}, NR_{p_1}, \dots, NR_{p_n}\}$$

and can be written as

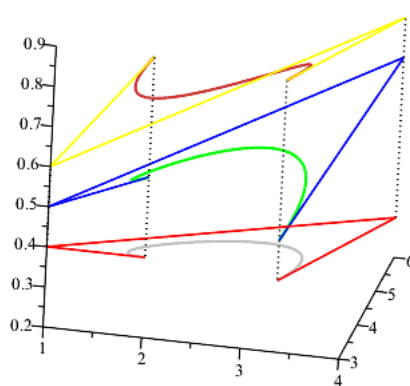
$$\{(x_0, y_0), T(x_0, y_0), I(x_0, y_0), F(x_0, y_0)), \dots, ((x_n, y_n), T(x_n, y_n), I(x_n, y_n), F(x_n, y_n))\}$$

in order to control the shape of a curve from a neutrosophic data.

**Table 1.** A neutrosophic data example

Point	Truth degree	Indeterminacy degree	Falsity degree
(2,3)	0.6	0.4	0.7
(1,3)	0.5	0.6	0.2
(4,6)	0.7	0.5	0.3
(3,5)	0.3	0.2	0.7

The Neutrosophic data in Table.1 is illustrated Fig.1 as a running example.

**Figure 1.** Neutrosophic Bézier curves for data in Table 1.

#### 4. Conclusion and Future Work

Visualization or geometric modeling of data plays an important role in data mining, databases, stock market, economy, and stochastic processes. In this article, we used the Bézier technique for visualizing neutrosophic data. This model is suitable for statisticians, data scientists, economists and engineers. Furthermore, the differential geometric properties of this model can be investigated as in [8] for classification of neutrosophic data. On the other hand, transforming the images of objects into neutrosophic data is an important problem [12]. In our model, the curve and the data can be transformed into each other by the blossoming method, which can be used in neutrosophic image processing. This and similar applications can be studied in the future.

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