A New Safety Assessment Method of Type-2 Prediction Sets Based on Credibility Degree of Data
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Abstract: A safety assessment method of type-2 prediction sets based on credibility degree is proposed as to the problem of misjudgment problem in the process of assessment which fails to reflect the credibility degree of data. Firstly, define the type-2 prediction sets and its relevant concepts on the basis of possibility distribution and credibility distribution, and subsequently propose the calculation method of similarity measure and its center between type-2 prediction sets. Secondly, establish joint distribution of disparate data on the basis of assessment requirements and safety rating scale of disparate data, and by using weighted fusion method fuse the disparate data on the distribution of type-2, respectively. Finally, compute the similarity measure between the fusion distribution and its safety rating scale distribution, determine the center of mass position in the joint plane scale and determine safe level. The experiment results show that the method proposed in this paper used in tailings dam’s safety determination is effective to reduce the granularity of evaluation results to avoid the misjudgment problems, and also facilitates the accurate judge of dam safety level.

Key words: safety assessment; credibility degree; TYPE-2 prediction sets; similarity measure

1. Introduction

Predicted data of experts and sensor monitoring data reflect the running ability of actual project from different aspects[1,2], such as large buildings, public facilities, precision equipment. It is an important guarantee to carry out the safety assessment by coupling up these two modes to avoid a major catastrophic accident. The current assessment methods[3][4][5] [6][7][8] usually start from the simple premise that uncertainty of data is considered as randomness or ambiguity or possibility and so on, respectively, and use the probability theory, fuzzy set theory and possibility theory to determine the safety level. Those methods have a strong pertinent and extensive application value in specific uncertainty environment. However, the main purpose of the safety assessment is not only to judge the research object in time after the accident or the device has malfunctioned, but aims at predicting its future development tendency through the event of small abnormalities. At the same time, it’s impossible to obtain a large amount of experimental data for some specific evaluation objects which can't make lots of destructive test, such as tailings dam, slopes, aerospace equipment, etc. That is to say, it is a pressing problem how to predict work status or ability of the specific evaluation objects by small sample data within a certain time or period in the future. Only solve the above problem at an appropriate time can we make it possible to prevent the occurrence of an accident and prevent further development and take measures to effectively avoid accidents. The possibility distribution is a quantitative description of the degree of difficulty for event which is given by zadeh in the possibility of space. It emphasizes the degree difficulty of the elements appearance when the event occurs, and describes the occurrence degree of particular element in condition of event occurrence. Besides, the possibility distribution have some advantages such as needing less sample information and small amount of calculation [9,10,11]. Therefore, the possibility
distribution is more suitable to measure the uncertainty and decision processing of data than the probability distribution and fuzzy distribution in later fusion.

The credibility of the data is another important factor that affects the actual engineering assessment in addition to uncertainty, which reflects the difference in the capabilities of the data source, such as expert experience, comprehension, and sensor performance, operational conditions, etc. Ignoring its role in the safety assessment will result in misjudgement of the safety level. Therefore, how to reflect the credibility and uncertainty of the data in an assessment of real project is an important channel to ensure that the assessment results are comprehensive and highly effective.

The idea of type-2 fuzzy sets [12,13,14,15] can offer reference to describe the credibility and uncertainty of data in this paper, which uses the primary and secondary membership functions to represent the ambiguity of the elements in the fuzzy set and the ambiguity of the principal membership function respectively. Therefore, we propose a method to convert the type-1 distribution into type-2 distribution which can reflect the credibility of data in this paper, and describe the credibility and uncertainty of the data respectively through the credibility distribution function and the possibility distribution function to improve the universality and resolve the misjudgment of safety level.

In this paper we propose a new type-2 predictive sets safety assessment method based on the credibility degree of data, which uses the similarity matrix and center of mass matrix between predictive sets to determine the relationship between the data and the safety scale in the assessment of heterogeneous data. And the assessment result is determined by the location of the center of mass in the Projection plane of joint distribution scale.

This paper is organized as follows. Section II defines the related concepts of type-2 prediction sets based on the possibility distribution and credibility distribution, and the similarity measure between type-2 prediction sets. Section III presents the new safety assessment method based on credibility degree of data. One illustrative example of tailings dam is considered in Section IV, where the result that is obtained and discussed. Conclusions are given in Section V.

2 Type-2 prediction sets and its similarity measure

2.1 Type-2 prediction sets

In order to elicit the type-2 predictive sets, the definitions of possibility distribution and credibility distribution are introduced firstly, specifically:

**Definition 2.1** (Possibility Distribution [9,10]) Assuming that \( (U,F(U),\Pi) \) is the likelihood space, \( X \) is a value variable on the finite universe \( U \), and \( A \in F(U), \) if the fuzzy constraint associated with \( A \) and \( X \) is \( RX \), the possibility distribution \( \Pi_x \) associated with the \( X \) variable is:

\[
\Pi_x = R(X)
\] (1)

Let \( \pi_x \) be the possibility distribution function of \( \Pi_x \), The possibility that the variable \( X \) takes value \( x \) is as follows:

\[
\pi_x(x) = \text{Poss}(X = x)
\] (2)

It describes the size of the \( x \) value that appears.

**Definition 2.2** (Possibility Trace): Assuming that \( A \) is any of the type-2 of predictive sets on
finite universe $X$, then the union of any point $x$ on $\tilde{A}$ in $X$ and the Cartesian product between the point likelihood distributions will form a planar region, which is called possibility of trace of $\tilde{A}$, can be expressed as:

$$FOP(\tilde{A}) = \bigcup_{x \in X} x \times J_x$$ (3)

When $\tilde{A}$ is a continuous field, then

$$FOP(\tilde{A}) = \bigcup_{x \in X} x \times [\inf(\pi_x), \sup(\pi_x)]$$ (4)

The Footprint of Possibility (FOP) and the correlation plot is as shown in Figure 1.

![Figure 1 FOP of the type-2 predictive sets](image)

**Definition 2.3** (Credibility distribution): Assuming that $(U, F(U), \Pi)$ is the possibility space, $X$ is a value variable on the finite field $U$, $\pi_x(x)$ is the possibility distribution function of the variable $X$, and $f : x \times \pi_x(x) \rightarrow [0,1]$ is a set of value maps, then for $\forall x \in X$,

$$Con(FOP) = f(x \times \pi_x(x))$$ (5)

It is the credibility distribution of $\pi_x(x)$, as shown in Figure 2, which describes the credibility of the corresponding possibility degree of the $X$ value variable.

![Figure 2 Credibility distribution](image)

**Definition 2.4** (Type-2 Predictive Sets): Assuming that $U$ is a finite universe, and $\tilde{A}$ is any set of predictions on its value variable $X$, then when $\tilde{A}$ satisfies:
\[ \tilde{A} = \{(x, \pi_\alpha(x)) : f_\alpha(x, \pi_\alpha(x)) \in J_s \subseteq [0,1], f_\alpha(x, \pi_\alpha(x)) \in [0,1], x \in X\} \]  

(6)

In this case, we say that \( \tilde{A} \) is a type-2 predictive set in finite universe \( U \).

Where \( \pi_\alpha(x) \) is a possibility distribution function and \( f_\alpha(x, \pi_\alpha(x)) \) is a credibility distribution function. When \( f_\alpha(x, \pi_\alpha(x)) = 1 \), the type-2 predictive sets is called the interval type-2 predictive sets.

When the finite field \( X \) and \( J_s \) are continuous or discrete distributions, the type-2 predictive sets \( \tilde{A} \) can be expressed as

\[
\tilde{A} = \begin{cases} 
\int_{x \in X} \int_{\pi_\alpha(x) \in J_s} \frac{f_\alpha(x, \pi_\alpha(x))}{(x, \pi_\alpha(x))}, & J_s \subseteq [0,1] \quad \text{continuous} \\
\sum_{x \in X} \sum_{\pi_\alpha(x) \in J_s} \frac{f_\alpha(x, \pi_\alpha(x))}{(x, \pi_\alpha(x))}, & J_s \subseteq [0,1] \quad \text{discrete} 
\end{cases}
\]  

(7)

According to Definition 2.3 and 2.4, the type-2 predictive set graphics corresponding to Figure 2 can be drawn, as is shown in Figure 3.

![Figure 3 Graphic schematics of type-2 predictive sets](image)

2.2 Similarity measure

**Definition 2.5** (\( \alpha \)-plane of Type-2 Predictive Sets) If \( \tilde{A} \) is a type-2 prediction sets, \( \alpha \in [0,1] \) is a constant, then \( \tilde{A}_\alpha \) is said to be a \( \alpha \)-plane of type-2 predictive sets, if it has the following form:

\[
\tilde{A}_\alpha = \{(x, \pi_\alpha(x)), x, \pi_\alpha(x) \geq \alpha \forall \pi_\alpha(x) \in [0,1] \} = \alpha \int_{\forall \pi_\alpha(x) \in [0,1]} \int \{(x, \pi_\alpha(x)) \mid \pi_\alpha(x) \}
\]  

(8)

For each \( \alpha \in [0,1] \), \( \tilde{A}_\alpha \) can be transformed into a special two-type predictive set [16,17], which can be presented as follows:
\[ R_{\alpha} = \frac{\alpha}{A_{\alpha}} \quad \forall x \in X, u \in [0,1] \]  

(9)

That is to say, the value of the reliability distribution function of \( R_{\alpha} \) is \( \alpha \). At this time, the type-2 predictive sets \( \tilde{A} \) can be expressed as

\[ \tilde{A} = \bigcup_{\alpha \in [0,1]} R_{\alpha} = \bigcup_{\alpha \in [0,1]} \frac{\alpha}{A_{\alpha}} = \sup_{\alpha \in [0,1]} \left[ \frac{\alpha}{A_{\alpha}} \right] \]  

(10)

The similarity measure between ordinary fuzzy sets could be extended. A series of similar measures \( s_j(\tilde{A}_\alpha, \tilde{B}_\alpha) \) between different sets can be obtained when \( \alpha \in [0,1] \) is difference, we give the similarity measure of the type-2 predictive set on the basis of the literature [16].

\[ s_j(\tilde{A}_\alpha, \tilde{B}_\alpha) = \frac{\sum_{i=1}^{N} \min[\sup(\pi_{\alpha_{ij}}(x_i)), \sup(\pi_{\beta_{ij}}(x_i))] + \sum_{i=1}^{N} \max[\inf(\pi_{\alpha_{ij}}(x_i)), \inf(\pi_{\beta_{ij}}(x_i))] \times s_j(\tilde{A}_\alpha, \tilde{B}_\alpha)}{\sum_{i=1}^{N} \max[\sup(\pi_{\alpha_{ij}}(x_i)), \sup(\pi_{\beta_{ij}}(x_i))] + \sum_{i=1}^{N} \max[\inf(\pi_{\alpha_{ij}}(x_i)), \inf(\pi_{\beta_{ij}}(x_i))] \times \alpha} \]  

(11)

Where \( \alpha_j \) represents the \( j = 1, 2, \cdots, M \) value of \( \alpha \), and \( \alpha_j \) must contain 1 for each value.

Combining equations (10) and (11), we can give a similar measure between \( \tilde{A} \) and \( \tilde{B} \).

\[ S_j(\tilde{A}, \tilde{B}) = \bigcup_{\alpha \in [0,1]} \frac{\alpha_j}{s_j(\tilde{A}_\alpha, \tilde{B}_\alpha)} \]  

(12)

Since the operator \( \bigcup \) represent the upper bound or the maximum value, \( S_j(\tilde{A}, \tilde{B}) \) can be not reflect the size of the similar measure in the case of each \( \alpha_j \) value. The following gives \( \overline{S}_j(\tilde{A}, \tilde{B}) \) to reflect all \( s_j(\tilde{A}_\alpha, \tilde{B}_\alpha) \) and \( \alpha_j \) information, it is the center of mass \( S_j(\tilde{A}, \tilde{B}) \) :

When \( s_j(\tilde{A}_\alpha, \tilde{B}_\alpha) \) is a monotonic function of \( \alpha \)

\[ \overline{S}_j(\tilde{A}, \tilde{B}) = \frac{\sum_{j=1}^{N} \alpha_j \times s_j(\tilde{A}_\alpha, \tilde{B}_\alpha)}{\sum_{j=1}^{N} \alpha_j} \]  

(13)

Otherwise,

\[ \overline{S}_j(\tilde{A}, \tilde{B}) = \frac{\sum_{\alpha \in S} \alpha_j \times s_j(\tilde{A}_\alpha, \tilde{B}_\alpha)}{\sum_{\alpha \in S} \alpha_j} \]  

(14)

Where \( S \) is the subset of the remainder after removing the upper bound of \( \alpha_j \).

3 The method of safety assessment

Data have two characteristics either from the sensors or the expert system in the safety assessment:
first, the randomness and fuzziness of data on the value of uncertainty which mainly result from various factors; and second, the reliability of data source on the differences of the data reliability which mainly result from the experience, understanding and comprehensive judgement ability of expert or the working state of monitoring sensor. Therefore, only when they are unified in data processing can we obtain the reliable evaluation results and make it convenient for administrator to implement the emergency control measures. In the meantime, administrator can analyze their differences and the changing trends to select the optimum solution according to the evaluation results of different reliability distribution function, and provide convenience for selecting the reliability distribution function in safety assessment.

The safety assessment method proposed in this paper mainly includes three aspects: (i) determine the heterogeneous data safety rating scale on the basis of the safety rules, and provide the safe level determination basis for the assessment decision; (ii) measure the assessment data using type-2 Projection plane sets and obtain the fusion distribution of all kinds of data by weight fusion; (iii) determine the relationship between the fusion distribution and safety rating scale distribution, then give the final result. The safety assessment schematic is shown in Figure 4.

3.1 Decision basis for safety levels

Determine the safety level standards of the expert system and sensors monitor system based on system safety rules, respectively, and sensors monitoring levels are A, B, C, D, and the expert levels are I, II, III, IV. Because of the scale is as a judge on the basis of the grade the interval type 2 prediction rating scale to describe all types of data safety, the reliability distribution function should be 1. In this paper, taking the Gaussian distribution as an example, so the safety level of sensors monitoring and expert are shown in figure 5 (a) and (b).

From Fig.5 we can obtain the joint distribution scale and its Projection plane of expert and sensor
monitoring system, as is shown in Figure 6, which is used as a proof to judge the safety level where 
x-coordinate is the rating scale of sensor monitoring system and y-coordinate is the rating scale of 
expert system and z-coordinate is the joint possibility degree.

![Figure 5 Safety level scales of sensors monitoring and expert system](image)

![Figure 6 Joint distribution scales and its Projection plane](image)

3.2 Specific implementation steps

Two types of results should be integrated in the determination of a system safety level, that 
is 

$$SL = f(SL_{\text{monitor}}, SL_{\text{state}})$$

Where 

$SL$ is the final decision result of the system, $SL_{\text{monitor}}$ is the result 
of the determination of the sensor monitoring level, $SL_{\text{state}}$ is the result of the judgment of the expert 

system level, and $f(\cdot)$ is the decision function, which is determined by Table 1. When the similarity 
measure center satisfies Rule 1, the system state is at the safety level; When Rule 2 is met, the 

system status is at blue warning level; When Rule 3 is met, the system status is at yellow warning 
level, and the supervision of dam body and related facilities operation should be strengthened; When 

Rule 4 is met, the system status is at red warning level.

Table 1 system safety level to determine the basis

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Rule 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>Rule 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td></td>
<td>Rule 3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
IV Rule 4

If the similarity measurement center does not satisfy Table 1, it shows that there is a conflict between the two kinds of judgment results. Next, we calculate the center of gravity in the joint distribution scale projection plane

\[ CG = \{ CG_I, CG_B, CG_C, CG_D \} \]

and then the distance between the center of mass position and the center of gravity of each safety level would be determined, then we could determine the system safety level according to the shortest distance method.

Specific implementation steps are as follows:

Step 1: Determine the system safety level scale according to Section 3.1, establish a joint distribution, and determine the area of each safety level Projection plane area.

Step 2: determine the participating data for assessment under each level of safety level in expert system and its weight, expressed as \( \{ I_i \}_{i=1,2,...,n} \) and \( \{ w_{E1}, w_{E2}, \cdots, w_{En} \} \), respectively; determine the safety level under each level of safety level in sensors monitoring system and its weight, expressed as \( \{ M_j \}_{j=1,2,...,m} \) and \( \{ w_{S1}, w_{S2}, \cdots, w_{Sm} \} \), respectively. Then the safety level are represented with type-2 prediction sets, namely

\[
\tilde{I}_\text{Expert System} = \{ \tilde{I}_1, \tilde{I}_2, \cdots, \tilde{I}_n \} \text{ and } \tilde{M}_\text{sensor monitoring} = \{ \tilde{M}_1, \tilde{M}_2, \cdots, \tilde{M}_m \}.
\]

Step 3: Synthesize the possibility distribution function and credibility distribution function for amount of participating data using the weighted fusion method, and obtain the fusion distribution of expert system and sensors monitoring system, respectively.

\[
\begin{align*}
\tilde{I}_c & = \sum_{i=1}^{n} w_{Ei} \times \tilde{I}_i \\
\tilde{M}_c & = \sum_{j=1}^{m} w_{Si} \times \tilde{M}_j
\end{align*}
\]

Step 4: Using the formulas (11) ~ (14) to calculate the similarity measure and its center of mass between the fusion distribution and security level scale in section 3.1, then establish the corresponding matrix

\[
\begin{align*}
S_{RI} & = [S_j(\tilde{I}_c, \tilde{I}_{R1}), S_j(\tilde{I}_c, \tilde{I}_{R2}), S_j(\tilde{I}_c, \tilde{I}_{R3}), S_j(\tilde{I}_c, \tilde{I}_{R4})] \\
\overline{S}_{RI} & = [\overline{S}_j(\overline{I}_c, \overline{I}_{R1}), \overline{S}_j(\overline{I}_c, \overline{I}_{R2}), \overline{S}_j(\overline{I}_c, \overline{I}_{R3}), \overline{S}_j(\overline{I}_c, \overline{I}_{R4})]
\end{align*}
\]

Where \( \overline{I}_{R1}, \overline{I}_{R2}, \overline{I}_{R3}, \overline{I}_{R4} \) are the security level scale of expert data which are represented with type-2 prediction sets, respectively; \( S_j(\overline{I}_c, \overline{*}) \) and \( \overline{S}_j(\overline{I}_c, \overline{*}) \) are the similarity measure and center of mass between the fusion distribution of expert system data and corresponding security level scale.

\[
\begin{align*}
S_{RM} & = [S_j(\tilde{M}_c, \tilde{M}_{R1}), S_j(\tilde{M}_c, \tilde{M}_{R2}), S_j(\tilde{M}_c, \tilde{M}_{R3}), S_j(\tilde{M}_c, \tilde{M}_{R4})] \\
\overline{S}_{RM} & = [\overline{S}_j(\overline{M}_c, \overline{M}_{R1}), \overline{S}_j(\overline{M}_c, \overline{M}_{R2}), \overline{S}_j(\overline{M}_c, \overline{M}_{R3}), \overline{S}_j(\overline{M}_c, \overline{M}_{R4})]
\end{align*}
\]

Where \( \tilde{M}_{R1}, \tilde{M}_{R2}, \tilde{M}_{R3}, \tilde{M}_{R4} \) are the security level scale of sensors monitoring data which are represented with type-2 prediction sets, respectively; \( S_j(\tilde{M}_c, \overline{*}) \) and \( \overline{S}_j(\overline{M}_c, \overline{*}) \) are the similarity
measure and center of mass between the fusion distribution of sensors monitoring data and corresponding security level scale.

Step 5: Determine the specific location of the center of mass in the projective plane of security level scale according to the similarity matrix and the center of mass matrix in Step 4, and confirm safe level based on the rule in table 1. If the rules in table 1 aren’t met, then go to Step 6.

Step 6: Determine the center of gravity position of security level scale projective plane, and calculate the distance to the center of gravity position and the center of mass point location of assessment data, then decide the security level by the method of the shortest distance.

4 Case studies

In this section, we take an iron ore tailings dam as an example to illustrate the proposed method in this paper. This tailings dam is located in the northeast of a county town in Shanxi, which is still in the middle of the designed service life and the height of primary dam and fill dam are 350m and 500m respectively.

Based on the safety level standards, the four level reference scales of the expert and sensor monitoring system of reference are respectively determined by expert group according to the method in section 3.1, as is shown in table 2, and the joint distribution and its projection plane are displayed in Figure 7 (a) and (b). The data in Table 2 and its subsequent data are represented according to the following rules in order to make them easy to understand: the type-2 prediction sets data of Gaussian is represented by expectation and variance; type-2 prediction sets data of Triangle is represented in the form of possibility distribution.

Table 2 Reference security level scale

<table>
<thead>
<tr>
<th>Safety level scale of sensors monitoring</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Possibility distribution</td>
<td>(0.21,0.090)</td>
<td>(0.42,0.088)</td>
<td>(0.68,0.085)</td>
<td>(0.85,0.095)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Safety level scale of expert system</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Possibility distribution</td>
<td>(0.06,0.18,0.38)</td>
<td>(0.28,0.45,0.58)</td>
<td>(0.50,0.65,0.84)</td>
<td>(0.75,0.88,1.0)</td>
</tr>
</tbody>
</table>

In this paper, we determine the indicators of participate evaluation are reservoir water level, phreatic line, length of dry and beach dam displacement in the sensor monitoring system, and the indicators of participate evaluation are leakage control facility, flood draining facility, drainage well facility and dam appearance (such as surface crack, depression, etc.) in the expert system. On the basis of preprocessing and analysis, we can determine the data in form of type-2 prediction sets and
the weights, respectively, as is shown in table 3 and 4.

<table>
<thead>
<tr>
<th>Indicator of participate evaluation</th>
<th>reservoir water level</th>
<th>phreatic line</th>
<th>length of dry displacement</th>
<th>beach dam displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Possibility distribution</td>
<td>(0.58, 0.08; 0.62, 0.02)</td>
<td>(0.56, 0.06; 0.62, 0.03)</td>
<td>(0.60, 0.10; 0.62, 0.01)</td>
<td>(0.62, 0.07; 0.64, 0.02)</td>
</tr>
<tr>
<td>Credibility distribution</td>
<td>(0.60, 0.04; 0.040)</td>
<td>(0.59, 0.07; 0.025)</td>
<td>(0.61, 0.05; 0.030)</td>
<td>(0.63, 0.08; 0.040)</td>
</tr>
<tr>
<td>weight coefficient</td>
<td>0.45</td>
<td>0.26</td>
<td>0.17</td>
<td>0.12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Indicators of participate evaluation</th>
<th>leakage control facility</th>
<th>flood draining facility</th>
<th>drainage well facility</th>
<th>dam appearance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Possibility distribution</td>
<td>(0.38, 0.59; 0.66)</td>
<td>(0.40, 0.58; 0.64)</td>
<td>(0.38, 0.58; 0.60)</td>
<td>(0.42, 0.57; 0.66)</td>
</tr>
<tr>
<td>Credibility distribution</td>
<td>(0.40, 0.92; 0.030)</td>
<td>(0.42, 0.95; 0.020)</td>
<td>(0.39, 0.96; 0.010)</td>
<td>(0.45, 0.97; 0.015)</td>
</tr>
<tr>
<td>weight coefficient</td>
<td>0.28</td>
<td>0.23</td>
<td>0.19</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Using formula (15) we can fuse the possibility distribution and credibility distribution of the indicator information of participate evaluation and obtain the comprehensive distribution. Using formula (10)–(12) we can calculate similarity measure between the comprehensive distribution and safety level scale in Table 2, and establish the similarity measure matrix as follows:

\[
S_{ij} = [0.3265, 0.7420, 0.5360, 0.2945]
\]

\[
S_{ji} = [0.4625, 0.7520, 0.4750, 0.2180]
\]

Then the center of mass matrix is as follows:

\[
\overline{\mathbf{x}} = [0.0175, 0.2450, 0.1745, 0.0275]
\]

\[
\overline{\mathbf{y}} = [0.0568, 0.4270, 0.3542, 0.0357]
\]

According the similarity measure and center of mass matrix, we can obtain the coordinate position of evaluation test data is at \(x_{\text{centroid}} = 0.500\), \(y_{\text{centroid}} = 0.575\). Using \((x_{\text{centroid}}, y_{\text{centroid}}) = (0.500, 0.575)\) we can find out the location point A of the evaluation test data in the projective plane of security level scale, as is shown in Figure 8(a). The location point A shows that the state of tailings dam is safe and should be at the blue alert level. Without regard to the support credibility, the location is an uncertainty region B, as is shown in Figure 8(a), which is not provide decision basis for direct decision. So that, necessary to adopt other methods to determine the safety level, such as clustering procedure, nearest-neighbour method. The distance of uncertainty region B center point and four safety levels are shown in Table 5 whether the supremum or infimum of FOP.

<table>
<thead>
<tr>
<th>Participate evaluation</th>
<th>A and I</th>
<th>B and II</th>
<th>C and III</th>
<th>D and IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supremum of FOP</td>
<td>0.5449</td>
<td>0.1628</td>
<td>0.0707</td>
<td>0.4036</td>
</tr>
<tr>
<td>Infimum of FOP</td>
<td>0.5590</td>
<td>0.2283</td>
<td>0.1082</td>
<td>0.3941</td>
</tr>
<tr>
<td>Average value</td>
<td>0.55195</td>
<td>0.19555</td>
<td>0.08945</td>
<td>0.39885</td>
</tr>
</tbody>
</table>
Form Table 5 we know the safety levels are yellow alert level whether by the supremum or infimum of FOP to calculate, and that shows the dam and leakage control facility need to take reinforcement and maintenance measure. This result is completely contradictory to those reflect credibility situation, and after expert team of dam design confirm, the result by type-2 prediction sets is more consistent with the actual situation.

In order to further illustrate the advantage of this paper proposed, Figure 9 gives the relationship between location point A and uncertainty region B. It includes: the different position of location point A in uncertainty region B reflects the credibility distribution trend of test data in corresponding support sets, and gradually decreases around location point A. If the location point A and the outside curve of uncertainty region B is overlaps, then the credibility degree of FOP supremum is biggest. If the location point A and the inside curve of uncertainty region B is overlaps, then the credibility degree of FOP infimum is biggest. In other words, the position of location point A is determined by credibility distribution function. So the method in this paper can also provide convenience for scholars to study the comparison and analysis of evaluation results in different credibility distribution function.

5 Conclusion

This paper has carried out the following research work:
(i) Type-2 prediction sets is proposed in order to reflect the credibility and uncertainty of data. The data handler will represent the data to three-dimensional structure with credibility distribution and
(ii) Type-2 prediction sets is applied to the analysis of the safety assessment with heterogeneous data. A new method of type-2 prediction sets is designed to solve the misjudgment problem, in which the similarity measure and center of mass are used to determine the relationship between test data and safety level scale, and then give the assessment results through position of test data in the joint projection scale. The particle size of result in this paper is even smaller than in some literatures, so it is more conducive for policy makers to make a decision;

(iii) One case study of tailings dam has been shown to illustrate the proposed method. The comparisons between the results of the new presented method and other existing method are offered. As a result, the proposed method can be applied to solve the assessment of credibility situation. In addition, it can provide convenience for scholars to select an ideal alternative of credibility distribution function through the analysis and comparison of the results in different credibility distribution function.

The next step of work will be mainly concentrated in two aspects: One is to study the different role of areas, perimeter and center of gravity of FOP for seeking a better calculation method of similarity measure. The other is to study the determination method of credibility distribution function and its influence on assessment results to provide convenience for policymakers to master different assessment results and the change trend in the future, and obtain the optimal solution to provide evidence for how to reasonably select the credibility distribution.

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