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Article

Quantum Tunneling Radiation from Loop Quantum Black Holes and the Information Loss Paradox

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Abstract: In this work, we present some results related with the issue of the Loop Quantum Black Holes (LQBH) thermodynamics by the use of the tunneling radiation formalism. The information loss paradox is also discussed in this context, where we have considered the influence of back reaction effects.

Keywords: loop quantum black hole; tunneling radiation; back-reaction; information recovery

1. Introduction

Starting from the Hawking demonstration that black holes can radiate thermally in the seventies [1], some labor has been done in order to understand the black hole evaporation phenomenon. In this sense several methods have been developed in order to calculate the temperature and entropy of black holes [2–5]. However, some questions about the black hole evaporation process remain open until now, as the information loss paradox [6,7] and the issue about the origin of the black hole entropy [8].

Among the methods that have been developed in order to understand black hole evaporation, more recently, a semiclassical method has been constructed upon the interpretation of Hawking radiation as a tunneling process across the black hole horizon [9–11]. The basic idea is that the Hawking flux, observed at infinity, has its origin in positive energy particles created just inside the horizon which could tunnel through it quantum mechanically. The tunneling approach is specially interesting in order to calculate black hole temperature since it provides a dynamical model to the black hole emission process. In this way, the tunneling approach turn out to be very useful when one wishes to incorporate back-reaction effects in order to describe black hole evaporation. In addition, even though calculations in the tunneling formalism are straightforward and relatively simple, they are robust in the sense that they can be applied to a wide variety of spacetimes [12–25].

Tunneling formalism has contributed also to the discussion of the black hole information loss paradox, even at the semiclassical level. In this way, Parikh [26]demonstrated, at first, that a nonthermal spectrum could be found out when one interpret the black hole emission process as a tunneling phenomena. However, no information recovery was obtained from the Parikh analysis. Such analysis was used also by Arzano et al [27] where quantum gravity effects was considered. However, after, Zhang [28] demonstrated that the Parikh argument needed to be rectified. In this way, by the use of a statistical argument, Zhang demonstrated that in the view of the tunneling approach, some information could be recovered by black holes during its evaporation process.

On the other hand, in the framework of black hole evaporation, it is expected that quantum gravity effects must have a crucial role, specially in the last stages of black hole evaporation.

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In this way, additional investigations taking into account quantum gravity contributions to the black hole emission process have been done by considering noncommutative geometry, Generalized Uncertainty Principle (GUP), as well as as well as Loop Quantum Gravity and string theory scenarios [29–38], where the information loss issue have been also considered.

In this work, at first, we shall revise the results of [39] where the Hamilton-Jacobi version of the tunneling formalism can be used to investigate how quantum gravity effects could influence in the emission process by a black hole. In order to do this we shall investigate the thermodynamic properties of Loop Quantum Black Holes [40,41], which corresponds to a quantum corrected black hole solution that appears in the context of Loop Quantum Gravity. In this way, the temperature and entropy of this kind of black hole are calculated by the use of the tunneling method. These first results presented in [39] replicate those found in the references [41–43], where other methods have been used. In this way, it can be demonstrated that the quantum tunneling formalism can be successfully applied to address the thermodynamics of LQBHs, opening a way for a whole range of applications. Among the possible applications, in the present paper, we shall investigate the possibility of information recovery through the calculation of the correlations between consecutive modes emitted during the LQBH evaporation. In this case, the results of [39], which were based on the Parikh approach has been rectified by the use of the Zhang treatment. In order to perform the two last tasks, back reactions effects will be taken into account.

This paper is organized as follows. In section (2), we revise the main features of the LQBHs scenario. In section (3), we revise the use of the tunneling formalism to calculate the temperature and entropy of LQBHs, and how back reactions effects can be included, in section (4). In section (5), we shall address the information loss problem in the LQBH scenario by the use of the tunneling formalism. The last section is devoted to remarks and conclusions.

2. Loop Quantum Black Holes

Efforts in order to find out black holes solutions in the context of loop quantum gravity have been done by several authors [44–57]. In this work, we shall investigate the thermodynamics of a particular solution called self-dual solution which was obtained by the use of loop quantum cosmology quantization techniques to Schwarzschild scenario [40]. Such solution possess, among its most interesting features, the resolution of the black hole singularity, which in this context is replaced by a wormhole connecting our universe to a new asymptotic flat region.

The self-dual black hole, in this way, represents a quantum gravitationally corrected Schwarzschild solution described by the following metric

$$ds^2 = -G(r)c^2dt^2 + F(r)^{-1}dr^2 + H(r)d\Omega^2 \ , \tag{1}$$

with

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2 \ . \tag{2}$$

In the equation (1), the metric functions are given by

$$G(r) = \frac{(r - r_+)(r - r_-)(r - r_*)}{r^4 + a_0^2} \ , \tag{3}$$

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$$F(r) = \frac{(r-r_+)(r-r_-)r^4}{(r+r_*)^2(r^4+a_0^2)} \,, \tag{4}$$

and

$$H(r) = r^2 + \frac{a_0^2}{r^2} \,, \tag{5}$$

where

$$r_+ = \frac{2Gm}{c^2} \;\; ; \;\; r_- = \frac{2Gm}{c^2}P^2 \,.$$

In the loop black hole scenario, we have the presence of two horizons - an event horizon and a Cauchy horizon. Moreover, r_* is defined as

$$r_* = \sqrt{r_+ r_-} = \tfrac{2Gm}{c^2}P \,, \tag{6}$$

where P is the polymeric function [40]

Moreover, $a_0 = \frac{A_{min}}{8\pi}$, where A_{min} represents the minimal value of area in Loop Quantum Gravity, and the mass parameter m is related with the ADM mass by

$$m = M(1+P) \,. \tag{7}$$

The LQBH spacetime possess the interesting property of self-duality. This property is given by a symmetry present in the metric (1). In this way, the LQBH metric is invariant by the transformations $\tilde{r} = a_0/r$ and $\tilde{t} = tr_*^2/a_0$, with $\tilde{r}_\pm = a_0/r_\mp$. Such transformations connect the description of an outside to an inside observer, where the first see a black hole with mass m described by the metric (1) while the inside observer see a black hole with mass $1/m$ described by the dual metric. From the self-duality property, in a different way from the classical Schwarzschild solution, the LQBH scenario allow black holes to have a mass smaller than the Planck mass [40].

Another interesting feature of LQBH scenario comes from the fact that, in the metric (1), r is only asymptotically the radial coordinate. It is because $g_{\theta\theta}$ is not given by r^2 but by $H(r)$. In this way, the physical radial coordinate, defined in order to measure the proper circumferential distance, is given by

$$R = \sqrt{r^2 + \frac{a_0^2}{r^2}} \,. \tag{8}$$

From the expression above, one can see that, in the limit of $r \to 0$, we shall have another asymptotically flat Schwarzschild region rather than a singularity. Such new region corresponds to a wormhole whose dimensions are the order of the Planck length. The wormhole throat is described by the Kantowski-Sachs spacetime [58].

The thermodynamical properties of LQBH can be obtained from the metric (1). In fact, the Bekenstein-Hawking temperature T_{BH} is related with the surface gravity κ which is given by

$$\kappa^2 = -g^{\mu\nu}g_{\rho\sigma}\nabla_\mu\chi^\rho\nabla_\nu\chi^\sigma = -\frac{1}{2}g^{\mu\nu}g_{\rho\sigma}\Gamma^\rho_{\mu 0}\Gamma^\sigma_{\nu 0} \,, \tag{9}$$

where, in the expression above, $\chi^\mu = (1,0,0,0)$ is identified as a timelike Killing vector and $\Gamma^\mu_{\sigma\rho}$ are the connections coefficients.

In this way, from the metric (1), we obtain

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$$T_H = \frac{\hbar}{2\pi c}\kappa = \frac{\hbar c^3}{4\pi G k_B} \frac{(2m)^3(1-P^2)}{[(2m)^4 + a_0^2]} \ . \tag{10}$$

As we can observe, the temperature above agree with the classical Hawking temperature in the large mass limit. On the other hand, it goes to zero form $m \to 0$.

The entropy of LQBH is obtained from the usual thermodynamical relation $S_{BH} = \int c^2 dm/T(m)$, which give us

$$S = \frac{4\pi k_B c^3}{\hbar G} \frac{(1+P)^2}{(1-P^2)} \left[\frac{16m^4 - a_0^2}{16m^2}\right] \ . \tag{11}$$

Further investigations about LQBH have been performed in order to calculate the gravitational wave spectrum from this kind of black holes [59] as well as its gravitational its lensing [60]. Moreover, the entropy-area relation that appears in the context of LQBH has been used in order to derive, based on a thermodynamical argument, quantum corrected bounce-type Friedmann equations [61], in agreement with the standard loop quantum cosmology [62].

As we can see, the LQBH metric brings quantum gravity corrections to the black hole thermodynamical properties like temperature and entropy. Such corrections could induce modifications in the way how black hole evaporates. In the following, we shall use the quantum tunneling formalism in order to address the thermodynamical properties of LQBHs. The information loss problem is also addressed in this context.

3. Quantum tunneling radiation from loop quantum black holes

In 2000, Parikh and Wilczec [9], following previous discussions by Krauss and Wilczec [63–65], developed the first tunneling method in order to describe the black hole evaporation process, named null geodesic method. After, in 2005, Angheben [16] at al presented a alternative description to the black hole tunneling process based in a Hamiltonian-Jacobi ansatz, consisting in an extension of the complex path analysis developed by Padmanabham et al [25,66–68].

By the use of the Hamilton-Jacobi method introduced by Angheben et al, the thermodynamical properties of LQBHs have been investigated by Silva and Brito [39], where the inclusion of back reaction effects and the information loos problem have been addressed. However, the discussion about the information loss problem done in [39] has been based on an approach introduced by Parikh [10] which has been rectified by Zhang [28]. In this way, we shall, at first, revise the results of [39] related to the calculation of LQBH temperature and entropy and the inclusion of back reaction effects. In the section (5), we shall rectify the results of [39] related to the issue of information recovery, by the of Zhang approach.

In this way, we have that, near the event horizon, one could reduce the description of the particle emission by a black hole to a 2-dimensional theory [69,70], where the metric corresponds to the $(t-r)$ sector of the original metric since its angular part is red-shifted away in this limit. Therefore, the near-horizon metric becomes:

$$ds^2 = -G(r)c^2dt^2 + F(r)^{-1}dr^2 \ . \tag{12}$$

In addition, in the near-horizon limit, the effective potential vanishes and there are no grey-body factors.

Now, let us consider the Klein-Gordon equations

$$\hbar^2 g^{\mu\nu}\nabla_\mu\nabla_\nu\phi - m^2c^2\phi = 0 \ \ , \tag{13}$$

which, by the application of the metric (12) give us

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$$-\frac{1}{c^2}\partial_t^2\phi+\Lambda\partial_r^2\phi+\frac{1}{2}\Lambda'\partial_r\phi-\frac{m^2c^2}{\hbar^2}G(r)\phi=0\,, \tag{14}$$

where $\Lambda = F(r)G(r)$

In this point, we shall take the standard WKB ansatz:

$$\phi(r,t)=e^{-\frac{i}{\hbar}I(r,t)}. \tag{15}$$

Such WKB approximation is justified by the fact that, when the outgoing wave is traced back towards the horizon, its wavelength, as measured by a local fiducial observer is ever-increasingly blue shifted and the point particle interpretation can be allowed [9].

In this way, by the use of the WKB ansatz (15), one obtains the relativistic Hamilton-Jacobi equation, in the limit of $\hbar \to 0$:

$$\frac{1}{c^2}(\partial_t I)^2-\Lambda(\partial_r I)^2-m^2c^2G(r)=0\,. \tag{16}$$

We shall seek for a solution in the form:

$$I(r,t)=-\omega t+W(r)\,. \tag{17}$$

in a way that we obtain

$$W=\int\frac{dr}{\Xi}\sqrt{\frac{1}{c^2}\omega^2-m^2c^2G}\,, \tag{18}$$

where $\Xi = \Lambda^{1/2}$

In this point, we shall adopt the proper spatial distance

$$d\sigma=\frac{dr^2}{\Xi(r)}\,, \tag{19}$$

where, by taking the near horizon approximation, we obtain

$$\Xi(r)=\Xi'(r_H)(r-r_H)+\ldots\,. \tag{20}$$

In this way, we find that

$$\sigma=2\frac{\sqrt{r-r_H}}{\Xi'(r_H)}\,, \tag{21}$$

where $0<\sigma<\infty$.

In terms of the proper spatial distance, we obtain for the spatial part of the action I

$$\begin{aligned} W &= \frac{2}{\Xi'(r_H)}\int\frac{d\sigma}{\sigma}\sqrt{\frac{1}{c^2}\omega^2-\frac{\sigma^2}{4}m^2c^2G'(r_H)\Xi'(r_H)} \\ &= \frac{2\pi i\omega}{\Xi'(r_H)c}+\text{real contribution}\,. \end{aligned} \tag{22}$$

In this way, the tunneling probability of the emission of a particle with energy ω will be given by

$$\Gamma\simeq Exp[-\frac{2}{\hbar}ImI]=Exp\Big\{-\frac{\pi G}{c^3\hbar}\frac{[(2m)^4+a_0^2]}{m^3(1-P^2)}\omega\Big\}\,. \tag{23}$$

Now, assuming a Boltzmann form, $\Gamma \sim e^{-\beta\omega}$ for the emission probability above, where β is the inverse temperature $\beta = 1/k_B T_H$, we obtain the LQBH temperature as:

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$$T_H = \frac{\omega}{ImI} = \frac{\hbar c^3}{4\pi G k_B} \frac{(2m)^3(1-P^2)}{[(2m)^4 + a_0^2]} \ , \tag{24}$$

which coincides with the former expression (10) found out in the references [41–43].

From the expression for LQBH temperature one obtains for the entropy:

$$S = \frac{4\pi k_B c^3}{\hbar G} \frac{(1+P)^2}{(1-P^2)} \left[\frac{16m^4 - a_0^2}{16m^2} \right] . \tag{25}$$

In this way, from the results above, we have that the tunneling formalism is straightforward in order to calculate the LQBH thermodynamical properties. Such results pave the way for a whole of applications, some of which we shall address in the following sections.

4. Back reaction effects

Based on the results obtained in the last section, which demonstrated that the tunneling approach is appropriate to calculate the thermodynamical properties of LQBHs, following the results of [39], in this section we shall show how back reaction effects can be introduced in the description of its evaporation process. By taking into account such self-gravitational effects, in this way, one can refine the description of LQBH thermodynamics, mainly in the quantum gravitational regime. It is because back reaction effects must be taken into account in the late stages of black hole evaporation, where the usual framework for the emission process will loose its validity [9–11,25,64,71–79].

In this way, in the Hamilton-Jacobi formalism, back reaction effects can be introduced when one takes the action I to be given by the following relation [80]

$$I = -\frac{i}{2} \frac{\hbar}{k_B} [S(M - \omega) - S(M)] \ , \tag{26}$$

where M is identified as the black hole ADM mass.

In the case of LQBHs, we shall take, for practical purposes, the changing in the mass parameter m related with the black hole ADM mass through the equation (7). Therefore, we shall consider that a reduction in the black hole ADM mass will correspond to a reduction of $\epsilon = \omega(1+P)^2$ in the mass parameter m.

Let us consider the following relation:

$$\begin{aligned} I &= -\frac{i}{2} \frac{\hbar}{k_B} [S(m - \varepsilon) - S(m)] \\ &= -\frac{4\pi c^3}{G} \frac{i(1+P)^2}{(1-P^2)} \varepsilon(\varepsilon - 2m) \left[1 + \frac{a_0^2}{16m^2(m-\varepsilon)^2} \right] \ . \end{aligned}$$

Consequently, we shall have, for the probability of the black hole emit a quanta with energy ϵ, when back-reaction effects are taken into account:

$$\Gamma(\varepsilon) = Exp \left\{ \frac{4\pi G}{c^3 \hbar} \frac{(1+P)^2}{(1-P^2)} \varepsilon(\varepsilon - 2m) \left[1 + \frac{a_0^2}{16m^2(m-\varepsilon)^2} \right] \right\} \ . \tag{27}$$

In the next section, we shall apply these results in order to investigate the possibility to have some correlation between the quanta emitted by a LQBH due to quantum gravity corrections present in this scenario, when back reaction effects are considered.

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5. Information recovery from LQBHs

In order to answer the question if some information could be recovered during the LQBH evaporation, we shall analyze the correlation function between two modes consecutively emitted by a LQBH, considering the presence of back reaction effects. Such correlation function is given by:

$$C(\varepsilon_1+\varepsilon_2;\varepsilon_1,\varepsilon_2) = ln[\Gamma(\varepsilon_1+\varepsilon_2)] - ln[\Gamma(\varepsilon_1)\Gamma(\varepsilon_2)] \ . \tag{28}$$

It has been initially demonstrated by Parikh and Wilczek [9] that non-thermal corrections to the black hole radiation spectrum can be obtained when one take into account back reaction effects. However, Parikh at first demonstrated that, in the classical treatment by the aforecited authors, no statistical correlation between the quanta emitted by a black hole has been found out [10]. Such treatment was after followed by Arzano et al [27]. On the other hand, based upon standard statistical methods, by distinguish statistical dependence or independence of sequential emissions, Zhang demonstrated that a statistical correlation can be established between the quanta emitted by a black hole [28]. Such correlation is given by

$$C(\varepsilon_1+\varepsilon_2;\varepsilon_1,\varepsilon_2) = \frac{8\pi G}{c^3\hbar}\varepsilon_1\varepsilon_2 \ . \tag{29}$$

In this way, by the use of the Zhang approach, we shall refine the results of [39] which was construct upon the Parikh and Wilczek argument. Therefore, considering the quantum gravity corrections from LQBHs, using the equation (27), the correlation function between two consecutive modes with energies ε_1 and ε_2 will be given by:

$$\begin{aligned} C(\varepsilon_1+\varepsilon_2;\varepsilon_1,\varepsilon_2) = & \frac{8\pi(1+P)^2}{(1-P)^2}\frac{G}{c^3\hbar}\varepsilon_1\varepsilon_2 + \frac{\pi a_0^2(1+P)^2}{4(1-P^2)m^2}\frac{G}{c^3\hbar}\left\{\frac{(\varepsilon_1+\varepsilon_2)(\varepsilon_1+\varepsilon_2-2m)}{(m-\varepsilon_1-\varepsilon_2)^2}\right. \\ & - \left.\left[\frac{\varepsilon_1(\varepsilon_1-2m)}{(m-\varepsilon_1)^2}+\frac{\varepsilon_2(\varepsilon_2-2m)}{(m-\varepsilon_2)^2}\right]\right\} \ , \end{aligned} \tag{30}$$

where the semiclassical term found out by [28] appears, unless the polymeric function, with a quantum gravity correction which comes from the LQBH metric.

In the figure (1) the correlation functions for a classical and for a LQBH are compered. As we can observe, the quantum gravity contributions to the black hole evaporation process from the LQBH can, when compered with a classical black hole, relieve in a more substantial way the information loss problem. Such effects become more evident when the black hole approaches the Planck scale, in the final stages of the emission process.

6. Conclusions and Remarks

In this work, at first, we have revised the results of [34] related with the investigation of the LQBH thermodynamics by the use of the Hamilton-Jacobi version of the tunneling formalism. At first, we have shown that the results found out in the references [41–43] for the LQBH temperature and entropy can be reproduced by the use of the tunneling method, in a way that such method can be, in fact, reliably applied in order to address LQBH thermodynamics. We have present also the results related with the inclusion of back reaction effects in the description of LQBH evaporation process. Such effects are important in order to understand the thermodynamical dynamics of black holes during the last stages of their evaporation, where quantum gravity becomes important and the usual thermodynamical approach to such phenomena fails.

Finally, we have addressed the possibility of recovery of some information during the LQBH evaporation process, mainly during the its last stages. The results of the present work have revealed that, due to the quantum gravity corrections present in LQBH metric (1), the modes emitted during its evaporation process are related by a non-thermal correlation function with a quantum gravity

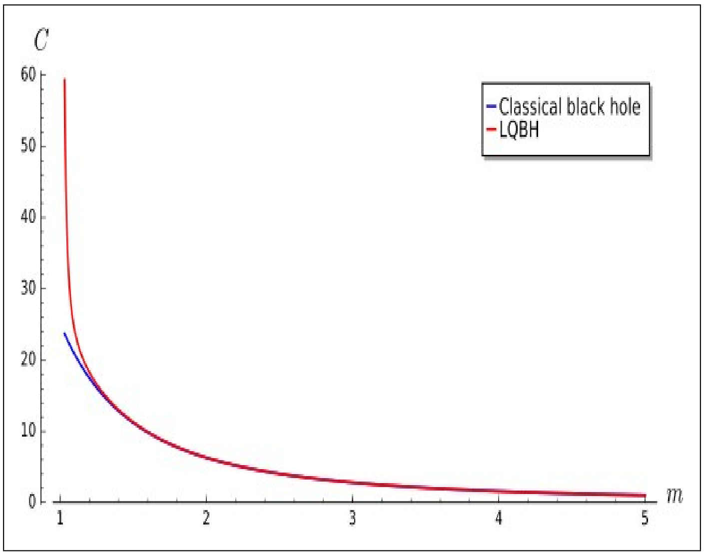

Figure 1. The correlation functions for a classical and a LQBH. The results point to a substantial contribution to information recovery from LQBHs front classical black holes. We have considered $\omega m = 0.2$ (the peak of the emission spectrum [81–83])

contribution. Consequently, some information can be recovered during LQBH evaporation process. The results presented in this paper, related with the calculation of the correlation functions, which have been calculated upon the Zhang argument [28], rectify the results of [34], based on the Parikh argument [10].

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