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The Time Domain Transition Radiation as a Vehicle to Probe the Connection between the Elementary Charge, Heisenberg's Uncertainty Principle and the Size of the Universe

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Abstract: Starting with the well-established field equations that describe the radiation fields of lightning return strokes, the radiation fields generated when a charged particle is incident on or moving away from a perfectly conducting plane are obtained. These fields are known in the literature as transition radiation. The field equations derived thus are used to evaluate the energy, momentum and the action associated with the transition radiation. The results show that for a charged particle moving with speed v, the longitudinal momentum associated with the transition radiation is approximately equal to E/c for values of $1-v/c$ smaller than about 10^{-3} where E is the total radiated energy and c is the speed of light in free space. The action of the transition radiation, defined as the product of the energy dissipated and the duration of the emission, increases as $1-v/c$ decreases and, for an electron, it becomes equal to $h/4\pi$ when $v=c-v_m$ where v_m is the speed pertinent to the lowest possible momentum associated with a particle confined inside the universe and h is the Planck constant. Combining these results with Heisenberg's uncertainty principle, an expression that predicts the value of the elementary charge is derived.

Keywords: transition radiation; Heisenberg's uncertainty principle; electronic charge; elementary charge; size of the universe

1. Introduction

In several papers published recently in this journal, Cooray and Cooray [1, 2, 3] have analysed the radiation fields generated by a travelling wave antenna and the momentum and action associated with these fields. Combining the results obtained with Heisenberg's uncertainty principle an expression for the elementary charge based on other universal parameters such as the Bohr radius and the radius of the universe was derived. In the travelling wave antenna, a current pulse is assumed to propagate from one end of the antenna to the other end with the speed of light. The assumption that the speed of propagation of the current pulse is equal to the speed of light in free space makes the analysis valid only for a current pulse propagating along a transmission line and not for the propagation of a charged particle in vacuum. In the travelling wave antenna, it is the electromagnetic field that propagates with the speed of light while the charged particles in the conductor are propagating with the drift speed, which is very much less than the speed of light. The question is whether a similar analysis can be conducted with material charged particles and derive results similar to those obtained in previous publications [1, 2, 3]. Important point here is that the speed of propagation of the charged particle can never be equal to the speed of light (even though it can reach speeds infinitesimally close to the speed of light), as in the case of a current pulse propagating along a conductor. Indeed, as is shown later the radiated energy goes to infinity as the speed of the particle becomes equal to the speed of light. In evaluating the radiation fields generated by moving charged particles, this fact has to be taken into

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account in the analysis. The procedure that is being followed here is identical to the one that was used in the previously mentioned publications. The starting point of this analysis can be the electromagnetic field equations pertinent to one of the models that was constructed to evaluate the electromagnetic fields from lightning return strokes.

One of the most popular models of return strokes is the transmission line model [4]. In this model, a current pulse of finite duration is assumed to propagate with constant speed, which is less than the speed of light, and without attenuation along a straight vertical line. In a recent publication Cooray and Cooray [5] showed that the field equations pertinent to this model can be obtained by treating the propagating current pulse as a movement of an equivalent charge emanating from a conducting plane (the ground in the case of lightning) and moving with constant speed. Thus, these field equations also describe the electromagnetic fields generated when a charged particle moving with constant speed is incident on or moving out from a conducting plane. The radiation fields generated by such events are known in the literature as transition radiation. The existence of transition radiation was first predicted by Ginzberg and Frank [6], and later analysed in more details by others [7, 8]. In general, transition radiation is produced when a charged particle, moving with a certain speed, crosses a boundary between two mediums with different dielectric constants. The spectrum of the radiation depends on the dielectric constants of the two media. The same type of radiation is produced when a charged particle is ejected from or impinged on a vacuum-conductor interface. Here we will concentrate on the latter case.

This study, in combination with the studies reported in the publications [1], [2] and [3], completes the analysis of the features of classical electromagnetic radiation, both in time and frequency domain, and their consequences when the charge associated with the propagating current pulses approaches the elementary charge.

2. Electromagnetic fields of transmission line model

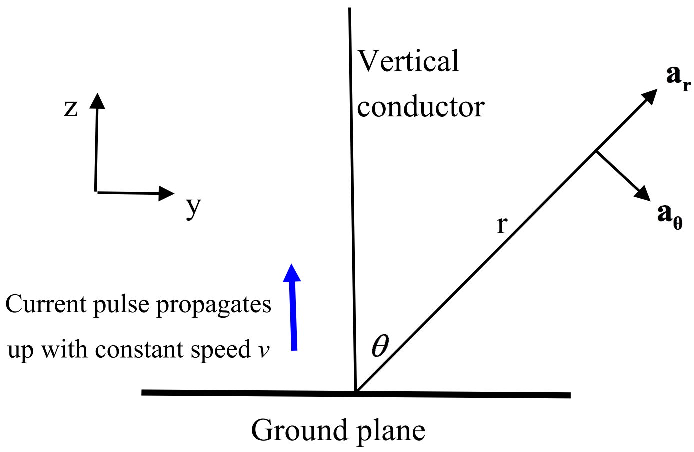

Figure 1: The geometry relevant to the derivation of field equations pertinent to the transmission line model.

In the transmission line model a current pulse is assumed to propagate with uniform speed from ground level along a vertical path with a constant speed v [4]. The geometry relevant to the calculation is shown in Figure 1. The radiation field generated by the emanation of the current pulse from ground is given by [5]

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$$\mathbf{E}(t)=\frac{i(t-r/c)v\sin\theta}{4\pi\varepsilon_0 c^2 r}\left\{\frac{1}{1-\beta\cos\theta}+\frac{1}{1+\beta\cos\theta}\right\}\mathbf{a_\theta} \tag{1}$$

where $\beta = v/c$. Note that $\mathbf{a_\theta}$ is an unit vector in the direction of increasing θ (see Figure 1). The first term inside the bracket is generated by the emission of the current pulse from the conducting plane and the second term is generated by its image in the perfectly conducting ground plane. The Poynting vector associated with this radiation field is given by [5]

$$\mathbf{S}(t)=\frac{i(t-r/c)^2\beta^2\sin^2\theta}{4\pi^2\varepsilon_0 c r^2}\frac{1}{\left(1-\beta^2\cos\theta^2\right)^2}\mathbf{a_r} \tag{2}$$

These fields are also identical to those derived previously by Krider [9]. The radiation field expressions remain the same if a negative current pulse propagates with constant speed v along the channel and incident on the ground. This is the case with the electromagnetic fields generated by the termination of negative corona currents in return strokes when they reach the ground end of the return stroke channel [5].

3. The transition radiation

As mentioned earlier, the transition radiation is generated when a charged particle moving with constant speed is ejected or incident on a perfectly conducting ground. Let us represent the movement of the charged particle as a propagating current pulse. The current pulse shape is related to the way in which the charge is distributed across the charged particle. For a point charged particle the current pulse will be represented by a Dirac delta function. The emission of a positively charged particle from a perfectly conducting ground plane can then be represented, as in the case of the return stroke, by a positive current pulse moving out from the ground plane. The incidence of a negatively charged particle on the ground plane can be represented, as in the case of corona current pulses in return stroke models [5], by a negative current pulse moving with constant speed and incident on the ground plane. Here, the incidence of a negatively charged particle, $-q$, on a perfectly conducting ground plane is considered. The geometry relevant to the analysis is shown in Figure 2. If the movement of the charged particle with uniform speed v is represented by a current pulse $i(t)$ then the radiation field generated during the encounter is given by Equation (1) and the corresponding Poynting vector is given by Equation (2).

In the literature, the analysis of transition radiation is usually carried out in frequency domain but here we will confine our analysis to the time domain. Moreover, in analysing the transition radiation the charge is usually considered to be concentrated on a point particle but in the present study we represent it as having a spatial distribution so that the movement of the charge can be represented by a current pulse of finite duration. Of course, when the duration of the current pulse approaches zero (i.e. when it is represented by a Dirac delta function) it reduces to the movement of a point charge.

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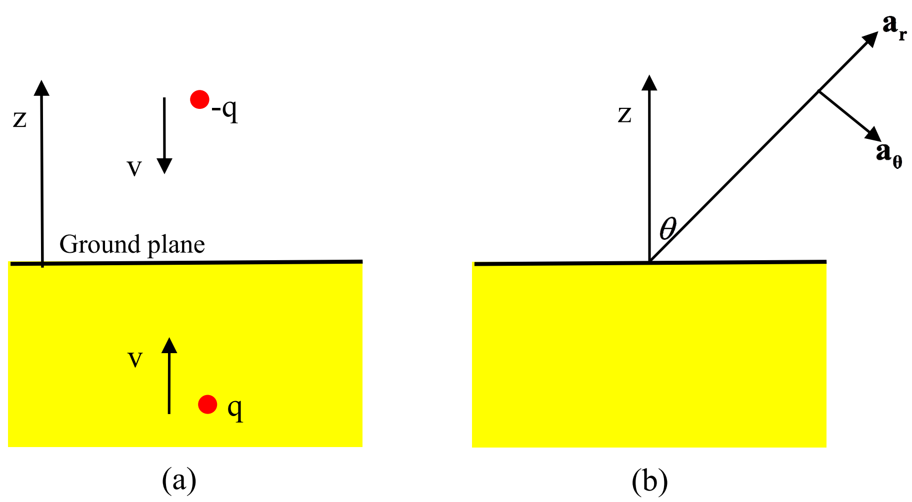

Figure 2: (a) A charged particle moves along the negative z-axis towards the ground plane. (b) The definition of parameters pertinent to the expression derived for the electric radiation field. In the diagram $\mathbf{a}_r$ and $\mathbf{a}_\theta$ are unit vectors in the radial and increasing θ directions.

Let us consider a negatively charged particle propagating with speed v and incident on a perfectly conducting ground plane. The direction of motion of the charged particle is along the negative z-axis and the conducting plane is located on the x-y plane (Figure 2a). The results to be derived would be identical if a positively charged particle moving out of the conducting plane with speed v is considered instead. The process of interaction between the charged particle and the conducting plane is represented by a Gaussian current pulse that moves along the negative z-axis towards the plane and incident on it. Transition radiation is produced during the interaction of the current pulse with the conducting plane.

In the analysis the Gaussian current pulse is represented by

$$i(t) = i_0 e^{-t^2/2\sigma^2} \tag{3}$$

In the above equation i_0 is a constant having units of Amperes, t is the time and σ is a parameter that controls the width of the Gaussian current pulse. The charge, q, associated with this current pulse is given by

$$q = i_0 \tau \tag{4}$$

In Equation (4), τ is the duration of the current pulse and it is given by

$$\tau = \sqrt{2\pi\sigma^2} \tag{5}$$

Thus, the current pulse can also be written as

$$i(t) = \frac{q}{\sqrt{2\pi\sigma^2}} e^{-t^2/2\sigma^2} \tag{6}$$

Observe that as $\sigma \to 0$ the current pulse reduces to a Dirac delta function. Usually the field expression for transition radiation is given by assuming that the current pulse is a Dirac delta function. However, as shown in reference [10] this will lead to physical inconsistencies when calculating the total energy associated with the radiation.

3.1 Expression for the total energy dissipated by transition radiation

One can see from Equation (2) that the energy dissipated by transition radiation per unit solid angle is given by

$$\frac{\Delta W}{\Delta\Omega}=\frac{i(t-r/c)^2\beta^2\sin^2\theta}{4\pi^2\varepsilon_0 c}\frac{1}{\left(1-\beta^2\cos\theta^2\right)^2} \tag{7}$$

Performing the Fourier transformation, the energy dissipated per unit solid angle per unit frequency is given by [11]

$$\frac{\Delta W}{\Delta\Omega\Delta\omega}=\frac{\beta^2\sin^2\theta}{2\pi^2\varepsilon_0 c}\frac{\left|A(\omega)\right|^2}{\left(1-\beta^2\cos\theta^2\right)^2} \tag{8}$$

where $A(\omega)$ is the Fourier spectrum of $i(t-r/c)$. This is the traditional equation that is being used in analysing the transition radiation [6, 7, 8]. In the limit when the current is represented by a Delta function $\left|A(\omega)\right|=q$ and the spectrum becomes flat and independent of frequency. Unfortunately, in this case the total energy dissipated by all frequencies goes to infinity. This shows that representing a point particle by a Delta function and at the same time to assume that the plane is perfectly conducting leads to physical inconsistencies. One solution is to appeal to the quantum nature of the radiation and to cut off the integral at an upper frequency limit given by ω_c. The order of magnitude of this frequency can be obtained by noting that the energy associated with a photon of a given frequency ω is equal to $h\omega/2\pi$ and the total energy dissipated as transition radiation cannot be larger than the total kinetic energy of the particle. This is reasonable because the particle cannot radiate more energy than its kinetic energy. This in turn indicates that, in order to avoid physical inconsistencies, the duration of the current waveform should not be smaller than about $2\pi/\omega_c$. Fortunately, as one can see later, the results to be obtained in this paper are independent of the duration of the current pulse. Now, confining our analysis to the time domain, the total energy lost by the charged particle as transition radiation is given by

$$\Delta U=\frac{\left(\int_{-\infty}^{\infty} i(t-r/c)^2\,dt\right)\beta^2}{4\pi^2\varepsilon_0 c}\int_0^{\pi/2}\int_0^{2\pi}\frac{\sin^3\theta\, d\theta\, d\varphi}{\left(1-\beta^2\cos\theta^2\right)^2} \tag{9}$$

Performing the spatial integral we obtain

$$\Delta U=\frac{\left(\int_{-\infty}^{\infty} i(t-r/c)^2\,dt\right)\beta^2}{2\pi\varepsilon_0 c}\frac{1}{4}\left[\frac{(1+\beta^2)}{\beta^3}\ln\left(\frac{1+\beta}{1-\beta}\right)-\frac{2}{\beta^2}\right] \tag{10}$$

Note that if $(1-\beta)<<1$, i.e. $\beta\approx 1$, the above expression reduces to

$$\Delta U=\frac{\left(\int_{-\infty}^{\infty} i(t-r/c)^2\,dt\right)\beta^2}{4\pi\varepsilon_0 c}\left[\ln\left(\frac{2}{1-\beta}\right)-1\right] \tag{11}$$

3.2. Expression for the momentum transported by the transition radiation

Consider the radiation moving in a radial direction associated with the angle θ. The flux of momentum transported by the radiation field moving in that direction is given by

$$\mathbf{M}(t)=\frac{i(t-r/c)^2\beta^2\sin^2\theta}{4\pi^2\varepsilon_0 c^2 r^2}\frac{1}{\left(1-\beta^2\cos\theta^2\right)^2}\mathbf{a_r} \tag{12}$$

Due to symmetry only the z-component of this momentum is not zero and it is given by

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$$\Delta p_z(t) = \frac{i(t-r/c)^2 \beta^2 \sin^2\theta}{4\pi^2\varepsilon_0 c^2 r^2} \frac{\cos\theta}{\left(1-\beta^2\cos\theta^2\right)^2} \tag{13}$$

Thus, the total momentum transported by the radiation in the z-direction is given by

$$\Delta P_z = \frac{\left(\int_{-\infty}^{\infty} i(t-r/c)^2\,dt\right)\beta^2}{4\pi^2\varepsilon_0 c^2} \int_0^{\pi/2}\int_0^{2\pi} \frac{\sin^3\theta\cos\theta\,d\theta\,d\varphi}{\left(1-\beta^2\cos\theta^2\right)^2} \tag{14}$$

After performing the spatial integral the above equation can be written as

$$\Delta P_z = \frac{\left(\int_{-\infty}^{\infty} i(t-r/c)^2\,dt\right)\beta^2}{2\pi\varepsilon_0 c^2}\left[\frac{1}{2\beta^2}\ln\frac{1}{\beta}+\frac{1}{2}\ln\frac{\beta}{1-\beta}-\frac{1}{2\beta}\right] \tag{15}$$

Note that if $(1-\beta) << 1$, i.e. $\beta \approx 1$, the above expression for the momentum reduces to

$$\Delta P_z = \frac{\left(\int_{-\infty}^{\infty} i(t-r/c)^2\,dt\right)}{4\pi\varepsilon_0 c^2}\left[\ln\frac{1}{1-\beta}-1\right] \tag{16}$$

Comparison of Equation (16) with Equation (11) shows that when $|\ln(1-\beta)| \gg \ln 2$ the vertical component of the momentum transported by the transition radiation reduces to

$$\Delta P_z \approx \Delta U / c \tag{17}$$

This indicates that when $|\ln(1-\beta)| \gg \ln 2$ the radiation, and hence the energy transported, is directed mostly along the z-axis. It is of interest to observe that the above condition is rarely satisfied in the case of lightning return strokes because the speed of propagation of the current pulse is considerably less than the speed of light.

3.3. The action associated with the transition radiation

The action associated with the transition radiation is defined as the product of the duration over which the transition radiation is generated and the total energy transported by it. In order to evaluate this quantity let us first perform the time integral associated with the Equation 10 with the expression for the current given by Equation (6). Once this is done the result is given by

$$\Delta U = \frac{q^2\beta^2}{16\pi^{3/2}\varepsilon_0 c\sigma}\left[\frac{(1+\beta^2)}{\beta^3}\ln\left(\frac{1+\beta}{1-\beta}\right)-\frac{2}{\beta^2}\right] \tag{18}$$

Then the action, A, associated with the transition radiation is given by

$$A = \tau\Delta U = \frac{\tau q^2\beta^2}{16\pi^{3/2}\varepsilon_0 c\sigma}\left[\frac{(1+\beta^2)}{\beta^3}\ln\left(\frac{1+\beta}{1-\beta}\right)-\frac{2}{\beta^2}\right] \tag{19}$$

Substituting for τ from Equation 5 we obtain

$$A = \frac{\sqrt{2}q^2\beta^2}{16\pi\varepsilon_0 c}\left[\frac{(1+\beta^2)}{\beta^3}\ln\left(\frac{1+\beta}{1-\beta}\right)-\frac{2}{\beta^2}\right] \tag{20}$$

Observe that the action is independent of the duration of the pulse. Note also that in the case when $\beta < 1$ and $(1-\beta) \ll 1$ the Equation (20) reduces to

$$A = \frac{\sqrt{2}q^2}{8\pi\varepsilon_0 c}\left[\ln\left(\frac{1+\beta}{1-\beta}\right)-1\right] \tag{21}$$

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Note that the action goes to infinity when $\beta \to 1$. In other words, the action becomes infinity as the speed of the particle reaches the speed of light. The next question that one has to answer is the following: How close does the speed of a particle can approach the speed of light? In the next section we will show that the speed of the particle cannot reach a speed which is arbitrarily close to the speed of light but there is an upper bound to the speed which is less than the speed of light.

4. The upper bound of the speed that can be achieved by a particle confined in the current universe

All particles in nature are restricted by a maximum speed, which is the speed of light in free space. So in principle, any particle can reach a speed which is less than but infinitesimally close to the speed of light. This means that there is no upper limit to the exponent k which is defined by the expression

$$(1-\beta) = 10^{-k} \tag{22}$$

That is, the value of the parameter k is unbounded. On the other hand Heisenberg's uncertainty principle dictates that both the momentum and the location of a particle cannot be measured to infinite accuracy. That is, if the speed of the particle is well defined, i.e. the momentum is well defined, there is a large uncertainty in the location of the particle. When one confines the particle to a given volume, the maximum uncertainty associated with the location of the particle is specified and that in turn determines, through the Heisenberg's uncertainty principle, the minimum uncertainty with which the momentum of the particle, and hence the speed of the particle, can be specified. As the size of the region that confines the particle decreases the uncertainty associated with its speed increases. This fact is utilized in text books to illustrate that an electron cannot be confined inside a nucleus because its speed and its energy becomes so large that it will escape from the confinement. In the same way, the uncertainty principle makes it impossible for us to define the momentum and the speed of a particle located inside the universe to an infinite accuracy. Thus, in any experiment, real or gedanken, carried out inside the universe, the speed of a particle confined inside the universe cannot be specified to an accuracy better than the one dictated by the uncertainty principle. Consider a particle of mass m moving with a defined speed v along the z-axis. Let us denote the diameter of the universe by D_u. Since the particle is confined inside the universe, the uncertainty in the location of the particle along the x and y directions is D_u. This is the case because the particle has to be somewhere inside the universe. This makes the momentum of the particle in the x and y direction greater than zero. Using the quantum mechanical theory of particle in a box one can show that the minimum speed associated with the particle along the x and y axis, say v_{m}, is given by the equation

$$D_u m v_m \approx h/2 \tag{23}$$

One can use non-relativistic momentum in deriving the above equation because for known masses this speed is extremely small in comparison to the speed of light. Thus the minimum speed along the x and y axis is given by

$$v_{\mathrm{m}} \approx \frac{h}{2D_u m} \tag{24}$$

This, of course, is an order of magnitude estimation based on the assumption that the universe can be approximated by a box with equal dimensions. In the case of an electron this speed is given by (replacing h/m from the expression for the Bohr radius)

$$v_{\mathrm{m}e} = \frac{\pi c \alpha a_0}{D_u} \tag{25}$$

In Equation (25), α is the fine structure constant and a_0 is the Bohr radius. Since the limiting speed in free space is the speed of light, the maximum speed $v_{\max}$ that can be realized by a particle moving along the z-axis is given by

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$$c = \sqrt{v_{\max}^2 + 2v_m^2} \tag{26}$$

For speeds $v_{\max}$ comparable to the speed of light, the above equation gives

$$v_{\max} = c(1 - v_m / c) \tag{27}$$

Note that in deriving Equation 27 we have assumed that $(1 + v_m / c) \approx 1$. From this and from (24) one can conclude that the minimum value of $(1-\beta)$ that can be realized by a particle confined inside the universe is given by

$$(1-\beta)_{\min} = \frac{h}{2mcD_u} \tag{28}$$

In the case of an electron the minimum value of $(1-\beta)$ is given by

$$(1-\beta)_{\min} = \pi\alpha a_0 / \mathrm{D}_u \tag{29}$$

It is of interest to note that an electron moving with this maximum speed will have an energy on the order of 10^{16} GeV.

5. Upper limit of the action associated with the transition radiation generated by a charged particle

Due to confinement inside the universe, the minimum value of $(1-\beta)$ that can be realized by a particle is given by Equation 28 and for an electron it is given by Equation 29. Thus, the maximum action associated with the transition radiation, A_m, is obtained by substituting the expression for $(1-\beta)_{\min}$ given in Equation 28 into Equation 21. This is given by

$$A_m = \frac{\sqrt{2}q^2}{8\pi\varepsilon_0 c}\left[\ln\left(\frac{4mD_u c}{h}\right) - 1\right] \tag{30}$$

Since the first term inside the bracket is much larger than unity one can write this as

$$A_m = \frac{\sqrt{2}q^2}{8\pi\varepsilon_0 c}\left[\ln\left(\frac{4mD_u c}{h}\right)\right] \tag{31}$$

In the case of an electron the maximum action associated with the transition radiation, A_{em}, is given by

$$A_{me} = \frac{\sqrt{2}e^2}{8\pi\varepsilon_0 c}\left[\ln\left(\frac{2D_u}{\pi\alpha a_0}\right)\right] \tag{32}$$

If we substitute numerical values to the parameters in the above equation we find that

$$A_{me} \approx h / 4\pi \tag{33}$$

This shows that if one imposes the condition that $A \geq h/4\pi$, the magnitude of the charge generating the transition radiation has to satisfy the condition $q \geq e$. In other words, if the smallest action that could be measured is limited to $h/4\pi$ then the smallest charge that could be detected using the transition radiation as a vehicle is the elementary charge. That is, $A \geq h/4\pi \rightarrow q \geq e$. A result identical to this was obtained also for the radiation generated by dipoles and antennas in references [1], [2] and [3]. In those studies, the parameter q is the charge associated with the current pulse propagating along the antenna. Possible reasons for the inequality given above is discussed in the next section.

6. Discussion

The results presented earlier, namely, that the maximum action of the transition radiation associated with the elementary charge is of the order of $h/4\pi$, can be explained by appealing again to the uncertainty principle. Consider a charged particle moving with speed v along the z-axis towards the perfectly conducting plane. Its speed is well defined but its location along the z-axis is known only to

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an accuracy comparable to the size of the universe. At a certain time the charged particle will approach the conducting plane and just at the moment it strikes the plane it gives rise to the transition radiation. Let us attempt to detect the location of the charged particle just before it hits the plane using transition radiation. The transition radiation allows the location of the charged particle only to an accuracy of about τ / v. This is the uncertainty in distance over which the transition radiation is emitted. During this process the momentum of the charged particle will become uncertain by an amount $\Delta U / c$, the longitudinal momentum that is being lost to the transition radiation. These two quantities should satisfy the uncertainty principle, and therefore,

$$\tau v \frac{\Delta U}{c} \geq \frac{h}{4\pi} \tag{34}$$

For values of v close to the speed of light it reduces to equation

$$\tau \Delta U \geq \frac{h}{4\pi} \tag{35}$$

Another way to look at the problem is to utilize the fact that the origin of a radiation pulse can be located only to an accuracy of its effective wavelength. In the present case the duration of the radiation field is τ and the effective wavelength associated with it is about τc. Thus the uncertainty in the location of the origin of transition radiation is about τc which, when used in the uncertainty principle, will give rise to Equation (35). Now, substituting relevant parameters to the left side of Equation (35) rearranging the terms we obtain

$$q^2 \geq \frac{\sqrt{2} h \varepsilon_0 c}{\ln\left[\frac{1}{1-\beta}\right]} \tag{36}$$

Since the smallest value of $(1-\beta)$ is denoted by $(1-\beta)_{\min}$, the smallest value of the charge is given when $(1-\beta) = (1-\beta)_{\min}$. Making this substitution in Equation 36 we find that the smallest value of the charge that can be associated with the transition radiation is given by (after substituting for $(1-\beta)_{\min}$ from Equation 28)

$$q^2 \approx \frac{\sqrt{2} h \varepsilon_0 c}{\ln\left(\frac{4 m D_u c}{h}\right)} \tag{37}$$

If we substitute for the mass m in Equation (37) the rest mass of an electron, the above equation should give the smallest value of the charge that is associated with a particle having that rest mass. From this we find that the elementary charge, i.e. the charge associated with the electron, is given by

$$q^2 \approx \frac{\sqrt{2}\, h \varepsilon_0 c}{\ln\left(\frac{2 D_u}{\pi \alpha a_0}\right)} \tag{38a}$$

This equation also predicts that the fine structure constant is given by

$$\alpha \approx \frac{1}{\sqrt{2}\, \ln\left(\frac{4 m_e c D_u}{h}\right)} \tag{38b}$$

All the parameters in the above equations are known. A relationship which is qualitatively similar to the one given in Equation (38a) was derived previously by Cooray and Cooray [3] by studying the electromagnetic radiation fields of long antennas. Now, the exact size of the universe is not known but the diameter of the visible universe is about 8.8 x 10^{26} m. If we substitute these parameters in Equation (38a) we obtain $q \approx \pm 1.66 \times 10^{-19}$ C. The value obtained for $1/\alpha$ is 128. In this respect it is interesting to note that in a recent study it was shown that the real universe is at least 250 times larger than the

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observable universe [12]. If we accept this value we obtain $q \approx \pm 1.61 \times 10^{-19}$ C and $1/\alpha = 135$. Exact fits to the elementary charge (i.e. 1.602×10^{-19} C) and $1/\alpha$ (i.e. 137) are obtained when the size of the universe is about 800 times the visible universe.

A strict interpretation of Equations (38a) and (38b) shows that the elementary charge or the fine structure constant decreases slowly as the universe expands. Actually, discussion on whether the fundamental constants of nature can change with the age of the universe is still going on in the current literature and no conclusions on this topic can be made at the present time. On the other hand, recall that the Heisenberg's uncertainty principle will give only an order of magnitude estimation of the smallest detectable charge. Thus, Equations (38a) and (38b) should be considered as order of magnitude estimates. Indeed, since the diameter of the universe appears inside the logarithmic term the variation of this parameter does not influence the value of the charge or the fine structure constant significantly. The size of the universe corresponding to any age of the universe can easily be calculated using the variation of scale factor with time [11]. For example, ten billion light years ago or when the universe was 3.77 billion years old the size of the universe was about a factor 0.42 smaller. If we plug this into the equation (38a) (and assuming that the current size of the universe is 800 times larger than the visible universe) we will obtain 1.609 x 10^{-19} C for the elementary charge. The value we get when the universe was 380,000 years old, i.e. when the universe became transparent and atoms are formed, is 1.66 x 10^{-19} C. The matter dominated epoch of the universe started when it was 47,000 years old and the value for the elementary charge we get for that age is 1.66 x 10^{-19} C. But, at these times the consistency and the temperature of the universe is very different to the present one and it is questionable whether the simple considerations made in deriving the above equations are valid at all. However, these numbers show that the predicted variation of the elementary charge is very small and all the numbers given above are of the same order of magnitude.

As an exercise, let us treat the elementary charge, fine structure constant and the Planck constant as absolute universal constants that do not change as the universe aged and consider the predictions of equations 38a and 38b. The first obvious solution is that the diameter of the universe is a constant i.e. the universe is stationary. This conclusion is against the current scientific wisdom. The other solution is to treat the Bohr radius as a parameter which is directly proportional to the diameter of the universe i.e. to treat D_u / a_0 as a universal constant. This can be realized in two ways while keeping the elementary charge and the fine structure constant as universal constants. The first solution is to treat the mass of the electron as a variable that decreases with the growth of the universe. In the other solution, the mass of the electron remains constant but the speed of light has to be inversely proportional to the diameter of the universe while the impedance of free space, $\sqrt{\mu_0 / \varepsilon_0}$, remains constant. In this case, the speed of light will have a high value close to the beginning of the universe and decreases as the universe age. In this respect, it is of interest to point out that there are cosmologies where the speed of light is assumed to decrease with the age of the universe and they are known as Variable Speed of Light (VSL) cosmologies [13]. The important point here is that the variable speed in our case is not an assumption but a consequence of the assumption that the fine structure constant and the elementary charge are universal constants.

Finally, it is important to stress that the calculations presented in this paper are based purely on classical electrodynamics and these results motivate a thorough analysis of the same problem using quantum electrodynamics.

7. Conclusions

Starting from electromagnetic fields pertinent to the transmission line model of the lightning return stroke with a current propagation speed less than the speed of light, expressions for the time domain

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fields and the associated energy, momentum and action of transition radiation generated when a charged particle is incident on a perfectly conducting ground plane are derived. The field expressions show that the energy, momentum and the action depend on the charge and the speed of propagation of the charged particles. After showing that there is an upper limit, which is less than the speed of light in free space, to the maximum speed that can be achieved by the charged particle, the action of the transition radiation fields pertinent to this limiting speed is used to estimate the elementary charge and the fine structure constant by appealing to the Heisenberg's uncertainty principle. The estimated charge and the fine structure constant lie within a few percent of the elementary charge if the size of the universe is assumed to be equal to the size of the observable universe.

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References

1. Cooray, V. and G. Cooray, (2016), On the remarkable features of the lower limits of charge and the radiated energy of antennas as predicted by classical electrodynamics. Atmosphere, 7, 64.
2. Cooray, V. and G. Cooray, (2016), On the Momentum transported by the radiation field of a long transient dipole and time energy uncertainty principle. Atmosphere, *7*, 151.
3. Cooray, V. and G. Cooray, (2017), On the Action of the Radiation Field Generated by a Traveling-Wave Element and Its Connection to the Time Energy Uncertainty Principle, Elementary Charge and the Fine Structure Constant, Atmosphere, 8, 46.
4. Uman, M.A. and D.K. McLain, (1969), Magnetic field of lightning return stroke, J. Geophys. Res., vol. 74, pp. 6899–6910.
5. Cooray, V., G. Cooray, (2010), The Electromagnetic fields of an accelerating charge: Applications in lightning return-stroke models. IEEE Proc., *52*, 944–955.
6. Ginzburg, V. L. and I. M. Frank, (1946), J. Eng. Phys. and Thermo-physics, 16, 1-15.
7. Ter-Mikaelian, M. L., (1972), High-energy electromagnetic processes in condensed media, Wiley-Interscience, New York.
8. Dolgoshe, B., Transition radiation detectors, (1993), Nuclear Instruments and Methods in Physics Research, A326, 434-469.
9. E. P. Krider, E. P., (1992), On the electromagnetic fields, Poynting vector and peak power radiated by lightning return strokes, J. Geophys. Res., vol. 97, pp. 15913–15917.
10. Cannon, N. J., (2000), Fields of particles and beams exiting a conductor, Progress In Electromagnetics Research, PIER 28, 147–183.
11. Jackson, J. D., (1975), Classical Electrodynamics, John Wiely & sons, New York.
12. M. Vardanyan, M., R. Trotta , J. Silk, (2011), Applications of Bayesian model averaging to the curvature and size of the Universe, Monthly Notice of the Royal Astronautical society Lett 413, 1, L91-L95.
13. https://en.wikipedia.org/wiki/Chronology_of_the_universe