

Article

# Modeling and Dynamics of a MDOF Isolation System

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**Abstract:** This study analyzes the modeling and dynamics of a novel passive in Multi-Degree-of-Freedom (MDOF) vibration isolation platform which can achieve significant isolation effect. Symmetrical Scissor-Like structures (SLSs) are utilized in the proposed MDOF isolation platform as the supporting and isolation elastic components. Based on the mathematical modeling and theoretical analysis of the MDOF vibration isolation system with SLSs, the effect of structural parameter and joint friction on stiffness and damping properties is investigated. It is shown that due to geometric relations within the SLSs, the natural frequencies can be reduced via adjusting structural parameters of the SLS for different direction vibration isolation. Theoretical and experimental results show that the SLS isolation platform can achieve much better loading capacity and vibration isolation performance simultaneously by only using linear passive components because of the MDOF adjustable stiffness property. Therefore, with low costing and energy consumption, the proposed novel isolation platform can provide the improvement of vibration suppression in various engineering practices.

**Keywords:** MDOF isolation platform; geometrical nonlinearity; vibration suppression, adjustable stiffness property

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## 1. Introduction

It is required high stability and significant isolation performance for isolation system because it isolates the vibration to protect the instruments and equipment for many vibration environments. For different applications, different structures and control methods are carried out for different vibration suppression mechanisms, for example, energy transferring between different vibration modes [1-2], vibration suppressing with absorbers [3-5], or utilizing semi-active/active vibration control methods etc. [6-8]. In most cases in vibration isolation, better isolation effectiveness can be obtained by using elements with smaller restoring forces which result in smaller natural frequency of the system, especially for microgravity environment in aerospace engineering [9-11].

For various vibration isolation purposes, Quasi-zero-stiffness (QZS) vibration isolation systems for one direction vibration have been extensively studied to improve working environment and provide better background for aerospace devices and precision instruments [12-20]. In order to induce adjustable stiffness property with sufficient loading capacity, a structure with springs called Scissor-Like structure (SLS) is proposed. The most obvious advantage of the vibration isolation system with SLSs is that the natural frequency and nonlinear stiffness coefficients are dependent on the structural parameters, which could realize high static and low dynamic property. Considering the advantages of the structure with SLSs, the SLSs are utilized to construct a novel MDOF vibration isolation system. The proposed isolation system has nonlinear stiffness and damping characteristics in six directions, which are all adjustable and can achieve superior vibration isolation using only pure linear and passive elements in the system with a simple and flexible installation structure.

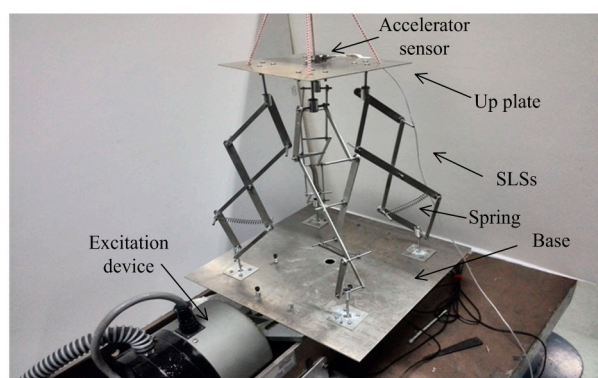
In the literature, the techniques of vibration isolation for multi-direction excitation with excellent isolation performance over larger frequency region have always been a hot and hard research topic. For the MDOF vibration isolation platform, active controllers are the chief method [9-11, 21-27]. The key point of the mechanism of active control in MDOF isolator is to generate anti-vibration forces in different directions by actuators and sensors in the designed system. The researches of MDOF isolation system are focused on the controllers and control strategies. Different

designs and the analysis of the dynamics of parallel mechanism are studied [21-27], for example, hybrid magnet consisting of electromagnet and permanent magnets is used as the active actuators in Refs. 21 and 22. Although active controllers can actively isolate vibration and timely control vibration, considering the energy required by active actuators, passive isolation techniques provide higher stability without any external power [28].

This paper proposed a MDOF isolation platform using SLSs as the elastic components. Then the natural frequencies and vibration responses are studied to show the isolation effectiveness of the proposed system. The superior vibration isolation can be achieved using only pure linear and passive elements in the system with a simple and flexible installation structure because of the adjustable stiffness. This system could provide an effective solution to many engineering problems for excellent MDOF vibration suppression and sustainable development for microgravity environment. The paper is organized as follows. The prototype of the isolation platform with SLSs is carried out and its mathematical modeling is obtained in Section 2. In Section 3, the responses of the isolation system obtained by Harmonic Balance Method (HBM) [28] and experiment are conducted, and the isolation effectiveness of the proposed isolation system is compared to the one using linear springs as isolator. A conclusion is drawn at the last section.

## 2.2 The prototype and modeling of the proposed isolation platform

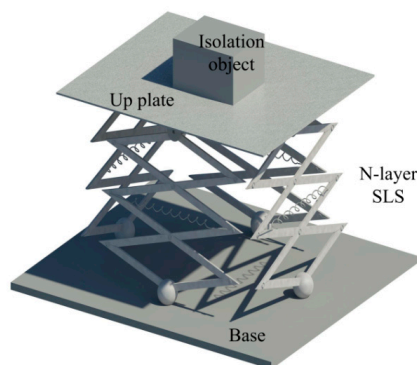
### 2.1 Experimental prototype



**Figure 1.** Experimental prototype of the MDOF isolation system with SLSs.

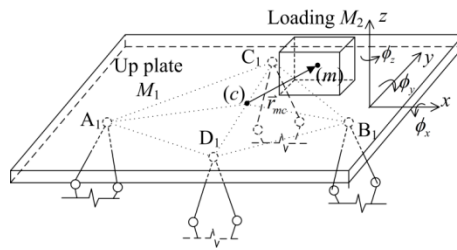
Figure 1 is the experimental prototype of the proposed MDOF isolation system with SLSs. As shown in Figure 1, the up plate of the isolation platform is connected with the base by four scissor-like structures (SLSs). The SLS is consisted of several connecting rods and a linear spring is assembled in one layer in each SLS.

### 2.2 Structural diagram



**Figure 2.** The structural diagram of the MDOF isolation system

Figure 2 is the structural diagram of the isolation system. Considering the isolation platform is proposed for protecting an instrument (the isolation object) which is put on arbitrary position on the up plate, the location of the isolation object and the connecting point of the SLS are shown in Figure 3.



**Figure 3.** Geometry of the up plate and the isolation object  $M_2$  and the coordinates.

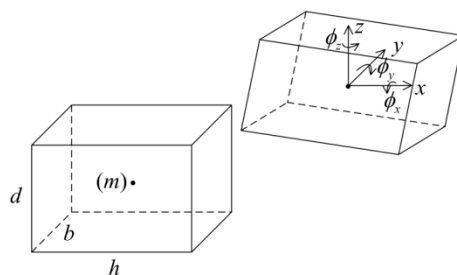
In Figure 3, Point (c) is the geometrical and mass center of the up plate  $M_1$  and Point (m) is the center of isolation object with mass  $M_2$ . The vector the origin of the up plate (c) and the isolation object (m) is as  $\mathbf{r}_{mc} = \{L_1, L_2, L_3\}$ . The absolute motion of the platform  $\mathbf{x} = \{x, y, z, \phi_x, \phi_y, \phi_z\}$  are chosen as generalized coordinates.

### 2.3 Modeling

Lagrange principle is used to obtain the model of the isolation platform. The mass of connecting rods and joints in SLSs are neglected because their weights are much smaller compared to the up plate, base and isolation object. The kinetic energy consists of the up plate and the isolation object  $M_2$ , which are set as  $T_1$  and  $T_2$ . The velocity of the up plate is the velocity at the center (c), thus the kinetic energy  $T_1$  is as

$$T_1 = \frac{1}{2}M_1(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{1}{2}J_{xx}\dot{\phi}_x^2 + \frac{1}{2}J_{yy}\dot{\phi}_y^2 + \frac{1}{2}J_{zz}\dot{\phi}_z^2 \quad (1)$$

For the isolation object  $M_2$  whose center has distance to the center of the center of the up plate, its velocity contains the translational velocity around Point (c) and the rotational velocity around the center Point (m). Figure 4 is the motions of the isolation object  $M_2$  which represents the isolation object (e.g. precise instruments etc.).



**Figure 4.** The motion of the object  $M_2$  on the up plate.

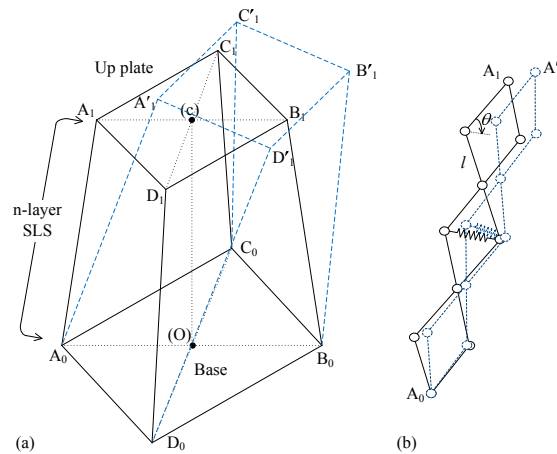
Figure 4 is the deflections of the isolation object  $M_2$  on the up plate whose moments of inertia in different rotational direction are defined as  $J_{xxm}$ ,  $J_{yy m}$  and  $J_{zzm}$ . Figure 5 is the deflection of one of the SLS in the isolator. From Figure 4, the kinetic energy  $T_2$  of the isolation object is as

$$T_2 = \frac{1}{2}M_2 \left[ \left( \dot{x} + \dot{\phi}_y L_3 - \dot{\phi}_z L_2 \right)^2 + \left( \dot{y} - \dot{\phi}_x L_3 + \dot{\phi}_z L_1 \right)^2 + \left( \dot{z} + \dot{\phi}_x L_2 - \dot{\phi}_y L_1 \right)^2 \right] + \frac{1}{2}J_{xxm}\dot{\phi}_x^2 + \frac{1}{2}J_{yy m}\dot{\phi}_y^2 + \frac{1}{2}J_{zzm}\dot{\phi}_z^2 \quad (2)$$

Therefore, the kinetic energy  $T$  of the system is as

$$T = T_1 + T_2 = \frac{1}{2}M_1(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{1}{2}J_{xx}\dot{\phi}_x^2 + \frac{1}{2}J_{yy}\dot{\phi}_y^2 + \frac{1}{2}J_{zz}\dot{\phi}_z^2 + \frac{1}{2}M_2\left[\left(\dot{x} + \dot{\phi}_y L_3 - \dot{\phi}_z L_2\right)^2 + \left(\dot{y} - \dot{\phi}_x L_3 + \dot{\phi}_z L_1\right)^2 + \left(\dot{z} + \dot{\phi}_x L_2 - \dot{\phi}_y L_1\right)^2\right] + \frac{1}{2}J_{xxm}\dot{\phi}_x^2 + \frac{1}{2}J_{yym}\dot{\phi}_y^2 + \frac{1}{2}J_{zzm}\dot{\phi}_z^2 \quad (3)$$

The potential energy  $V$  of the system is from the deformation of the springs in SLSs. The deflections of SLSs in the system are shown in Figure 5. The points connecting by SLS in the base are defined as  $A_0$ ,  $B_0$  and  $C_0$ ; the points in the up plate are defined as  $A_1$ ,  $B_1$  and  $C_1$ ; and the points in the up plate for deflection are defined as  $A'_1$ ,  $B'_1$  and  $C'_1$ .



**Figure 5.** Deflection of the isolation system. (a) is the location of up plate and SLSs, and (b) is the deflection of one SLS.

Figure 5 is the deflections of the MDOF vibration isolation system and each SLS. From Figure 5 (a), it can be seen that the points  $A_0$ ,  $B_0$ ,  $C_0$  and  $D_0$  make a quadrilateral, and  $A_1$ ,  $B_1$ ,  $C_1$  and  $D_1$  are also make a quadrilateral. From Figure 5 (b) which is the deflection of the SLS, the original length of the springs in SLS is as  $2l\cos\theta$ , and the length of the springs can be obtained by  $\|A_0A'_1\|$ . Because the absolute motions of up plate at center  $\mathbf{x}=\{x, y, z, \phi_x, \phi_y, \phi_z\}$  are chosen as generalized coordinate where  $x$ ,  $y$  and  $z$  are the absolute translational motions and  $\phi_x$ ,  $\phi_y$  and  $\phi_z$  are absolute rotational motions, the location of point  $A'_1$ ,  $B'_1$  and  $C'_1$  can be obtained by coordinate transformation matrix. Due to the six-excitation excitations from base which are as  $\mathbf{x}_e=\{x_e, y_e, z_e, \phi_{xe}, \phi_{ye}, \phi_{ze}\}$ , the relative motions of the up plate at center are as  $\hat{\mathbf{x}}=\{\hat{x}, \hat{y}, \hat{z}, \hat{\phi}_x, \hat{\phi}_y, \hat{\phi}_z\}$ . The transformation matrixes in the three rotational directions are as  $\mathbf{R}_x$ ,  $\mathbf{R}_y$  and  $\mathbf{R}_z$ , thus the rotational transformation matrix is defined as  $\mathbf{R}$  which is dot product of the three transformation matrixes.

$$\mathbf{R} = \mathbf{R}_x \cdot \mathbf{R}_y \cdot \mathbf{R}_z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \hat{\phi}_x & \sin \hat{\phi}_x \\ 0 & -\sin \hat{\phi}_x & \cos \hat{\phi}_x \end{bmatrix} \cdot \begin{bmatrix} \cos \hat{\phi}_y & 0 & -\sin \hat{\phi}_y \\ 0 & 1 & 0 \\ \sin \hat{\phi}_y & 0 & \cos \hat{\phi}_y \end{bmatrix} \cdot \begin{bmatrix} \cos \hat{\phi}_z & \sin \hat{\phi}_z & 0 \\ -\sin \hat{\phi}_z & \cos \hat{\phi}_z & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4)$$

$$= \begin{bmatrix} \cos \hat{\phi}_y \cos \hat{\phi}_z & \cos \hat{\phi}_y \sin \hat{\phi}_z & -\sin \hat{\phi}_y \\ \sin \hat{\phi}_x \sin \hat{\phi}_y \cos \hat{\phi}_z - \cos \hat{\phi}_x \sin \hat{\phi}_z & \cos \hat{\phi}_x \cos \hat{\phi}_z + \sin \hat{\phi}_x \sin \hat{\phi}_y \sin \hat{\phi}_z & \sin \hat{\phi}_x \cos \hat{\phi}_y \\ \cos \hat{\phi}_x \sin \hat{\phi}_y \cos \hat{\phi}_z + \sin \hat{\phi}_x \sin \hat{\phi}_z & -\sin \hat{\phi}_x \cos \hat{\phi}_z + \cos \hat{\phi}_x \sin \hat{\phi}_y \sin \hat{\phi}_z & \cos \hat{\phi}_x \cos \hat{\phi}_y \end{bmatrix}$$

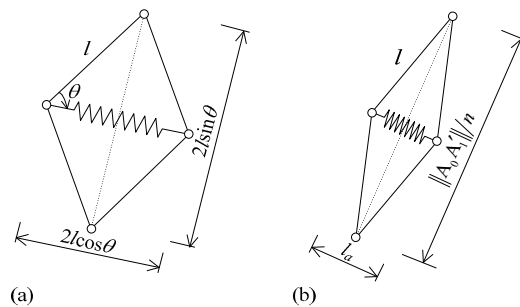
where  $\mathbf{R}_x$ ,  $\mathbf{R}_y$  and  $\mathbf{R}_z$  are rotational transformation matrix in  $\phi_x$ ,  $\phi_y$  and  $\phi_z$  directions.

From Figure 5, assuming the coordinate of point (c) is  $\{0, 0, 0\}$ , for the generalization of the results, the vector  $\mathbf{cA}_0$ ,  $\mathbf{cB}_0$ ,  $\mathbf{cC}_0$  and  $\mathbf{cD}_0$  are assumed as  $\mathbf{cA}_0=\{x_{a0}, y_{a0}, z_{a0}\}$ ,  $\mathbf{cB}_0=\{x_{b0}, y_{b0}, z_{b0}\}$ ,  $\mathbf{cC}_0=\{x_{c0}, y_{c0}$ ,

$z_{c0}$  and  $\mathbf{cD}_0 = \{x_{d0}, y_{d0}, z_{d0}\}$ , and the vector  $\mathbf{cA}_1$ ,  $\mathbf{cB}_1$ ,  $\mathbf{cC}_1$  and  $\mathbf{cD}_1$  are assumed as  $\mathbf{cA}_1 = \{x_{a1}, y_{a1}, z_{a1}\}$ ,  $\mathbf{cB}_1 = \{x_{b1}, y_{b1}, z_{b1}\}$ ,  $\mathbf{cC}_1 = \{x_{c1}, y_{c1}, z_{c1}\}$  and  $\mathbf{cD}_1 = \{x_{d1}, y_{d1}, z_{d1}\}$ . Therefore, the vector  $\mathbf{A}_0\mathbf{A}'_1$ ,  $\mathbf{B}_0\mathbf{B}'_1$ ,  $\mathbf{C}_0\mathbf{C}'_1$  and  $\mathbf{D}_0\mathbf{D}'_1$  can be obtained by the translational motion matrix and rotational transformation matrix as

$$\begin{cases} \mathbf{A}_0\mathbf{A}'_1 = \mathbf{cA}'_1 - \mathbf{cA}_0 = \{\hat{x}, \hat{y}, \hat{z}\} + \mathbf{R} \cdot \mathbf{cA}_1 - \mathbf{cA}_0 \\ \mathbf{B}_0\mathbf{B}'_1 = \mathbf{cB}'_1 - \mathbf{cB}_0 = \{\hat{x}, \hat{y}, \hat{z}\} + \mathbf{R} \cdot \mathbf{cB}_1 - \mathbf{cB}_0 \\ \mathbf{C}_0\mathbf{C}'_1 = \mathbf{cC}'_1 - \mathbf{cC}_0 = \{\hat{x}, \hat{y}, \hat{z}\} + \mathbf{R} \cdot \mathbf{cC}_1 - \mathbf{cC}_0 \\ \mathbf{D}_0\mathbf{D}'_1 = \mathbf{cD}'_1 - \mathbf{cD}_0 = \{\hat{x}, \hat{y}, \hat{z}\} + \mathbf{R} \cdot \mathbf{cD}_1 - \mathbf{cD}_0 \end{cases} \quad (5)$$

From Eq. (5), the deflection of the axis of SLS is obtained, and then the deformations of springs in SLS can be obtained by the analysis of the shape of SLS. Figure 6 is the deflection of one layer of the SLS.



**Figure 6.** The deflection of one layer in the SLS. (a) is original shape, and (b) is deformed shape.

From Figure 6 (a), it can be seen that the original length of springs in the SLS is  $2l \cos \theta$ . For deflection, the lengths of the springs in the three SLS are defined as  $l_a$ ,  $l_b$ ,  $l_c$  and  $l_d$  which can be obtained by triangular relationship shown in Figure 6 (b) which is as

$$\begin{cases} l_a^2 + (\|\mathbf{A}_0\mathbf{A}'_1\|/n)^2 = 4l^2 \\ l_b^2 + (\|\mathbf{B}_0\mathbf{B}'_1\|/n)^2 = 4l^2 \\ l_c^2 + (\|\mathbf{C}_0\mathbf{C}'_1\|/n)^2 = 4l^2 \\ l_d^2 + (\|\mathbf{D}_0\mathbf{D}'_1\|/n)^2 = 4l^2 \end{cases} \quad (6)$$

The potential energy  $V$  of the system is consisted of the elastic energy of the four springs in SLSs which can be expressed as

$$\begin{aligned} V &= \frac{1}{2}k(l_a - 2l \cos \theta)^2 + \frac{1}{2}k(l_b - 2l \cos \theta)^2 + \frac{1}{2}k(l_c - 2l \cos \theta)^2 + \frac{1}{2}k(l_d - 2l \cos \theta)^2 \\ &= \frac{1}{2}k \left( \sqrt{4l^2 - \frac{\|\mathbf{A}_0\mathbf{A}'_1\|^2}{n^2}} - 2l \cos \theta \right)^2 + \frac{1}{2}k \left( \sqrt{4l^2 - \frac{\|\mathbf{B}_0\mathbf{B}'_1\|^2}{n^2}} - 2l \cos \theta \right)^2 \\ &\quad + \frac{1}{2}k \left( \sqrt{4l^2 - \frac{\|\mathbf{C}_0\mathbf{C}'_1\|^2}{n^2}} - 2l \cos \theta \right)^2 + \frac{1}{2}k \left( \sqrt{4l^2 - \frac{\|\mathbf{D}_0\mathbf{D}'_1\|^2}{n^2}} - 2l \cos \theta \right)^2 \end{aligned} \quad (7)$$

Considering the air damping of the system, the dynamic equation can be obtained by Lagrange principle as

$$\frac{d}{dt} \left[ \frac{\partial(T-V)}{\partial \dot{\mathbf{x}}} \right] - \frac{\partial(T-V)}{\partial \mathbf{x}} = -\mathbf{C} \cdot (\dot{\mathbf{x}} - \dot{\mathbf{x}}_e) \quad (8)$$

where  $T$  is the kinetic energy and  $V$  is the potential energy shown as Eq. (3) and Eq. (7), The mass matrix  $M$  of the system is as

$$M = \begin{bmatrix} M_1 + M_2 & 0 & 0 \\ 0 & M_1 + M_2 & 0 \\ 0 & 0 & M_1 + M_2 \\ 0 & -M_2 L_3 & M_2 L_2 \\ M_2 L_3 & 0 & -M_2 L_1 \\ -M_2 L_2 & M_2 L_1 & 0 \\ 0 & M_2 L_3 & -M_2 L_2 \\ -M_2 L_3 & 0 & M_2 L_1 \\ M_2 L_2 & -M_2 L_1 & 0 \\ J_{xc} + J_{ym} + M_2 (L_2^2 + L_3^2) & -M_2 L_1 L_2 & -M_2 L_1 L_3 \\ -M_2 L_1 L_2 & J_{yc} + J_{zm} + M_2 (L_1^2 + L_3^2) & -M_2 L_2 L_3 \\ -M_2 L_1 L_3 & -M_2 L_2 L_3 & J_{zc} + J_{zm} + M_2 (L_1^2 + L_2^2) \end{bmatrix} \quad (9)$$

After first-order Taylor series expand, the stiffness matrix  $K$  is as

$$K = \begin{bmatrix} \frac{k(l_2 - l_1)^2}{2l^2 n^4 \cos^2 \theta} & 0 & 0 & 0 & \frac{kl_1(l_1 - l_2)H}{2l^2 n^4 \cos^2 \theta} & 0 \\ 0 & \frac{k(l_2 - l_1)^2}{2l^2 n^4 \cos^2 \theta} & 0 & \frac{kl_1(l_2 - l_1)H}{2l^2 n^4 \cos^2 \theta} & 0 & 0 \\ 0 & 0 & \frac{kH^2}{l^2 n^4 \cos^2 \theta} & 0 & 0 & 0 \\ 0 & \frac{kl_1(l_2 - l_1)H}{2l^2 n^4 \cos^2 \theta} & 0 & \frac{kl_1^2 H^2}{2l^2 n^4 \cos^2 \theta} & 0 & 0 \\ \frac{kl_1(l_1 - l_2)H}{2l^2 n^4 \cos^2 \theta} & 0 & 0 & 0 & \frac{kl_1^2 H^2}{2l^2 n^4 \cos^2 \theta} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (10)$$

where  $H = \sqrt{4n^2 l^2 \sin^2 \theta - (l_2 - l_1)^2}$ , and matrix  $C$  is the damping matrix of the system as

$$C = \begin{bmatrix} c_1 & \dots & 0 \\ & c_2 & \\ & & c_3 & \vdots \\ \vdots & & & c_4 & \\ & & & & c_5 \\ 0 & \dots & & & & c_6 \end{bmatrix} \quad (11)$$

where  $c_i$  ( $i=1, \dots, 6$ ) are damping coefficients in the six coordinates.

Because of the assumptions that the amplitudes of motions are sufficiently small, each term in the dynamic equation can be expanded by Taylor series and higher order terms can be neglected. Therefore, the dynamic equation of the system is as

$$M\ddot{x} + Kx + C\dot{x} = Kx_e + C\dot{x}_e \quad (12)$$

where  $M$  is the mass matrix,  $K$  is the stiffness matrix and  $C$  is the damping matrix, respectively, and  $x$  is the motion vector and  $x_e$  is the base excitation vector.

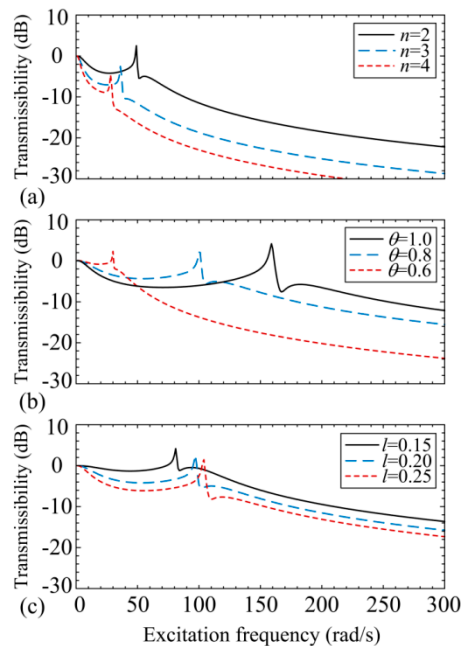
### 3. Isolation effect

#### 3.1. Solutions of system

The isolation effect of the proposed MDOF isolation platform with SLSs reflected by displacement transmissibility is based on the dynamic equation Eq. (12) and the responses for different structural parameters can be obtained by solving the linearized dynamic equations. When the excitation is harmonic excitation as  $x_e = e_i \cos \omega t$ , the responses could be set as  $x = a_i \cos(\omega t + \varphi)$ . Then, the displacement transmissibility  $T_i$  (dB) could be defined as the ratio between the amplitude of response and the excitation in respective direction as

$$T_i = 10 \log_{10} \left( \frac{|a_i|}{|e_i|} \right) \quad (i = x, y, z, \phi_x, \phi_y, \phi_z) \quad (13)$$

where  $a_i$  is the amplitude of response and  $e_i$  is the amplitude of harmonic excitation in different directions. From the dynamic equation Eq. (12) and mass, stiffness and damping matrices, it reveals that the values of components of the dynamic equations of each freedoms can be adjusted by changing the values of the structural parameters  $n$ ,  $\theta$  and  $l$ . The isolation effects for different structural parameters are shown in Figure 7.



**Figure 7.** Isolation effect in  $x$  direction for different structural parameters (a) different  $n$ ; (b) different  $\theta$ ; (c) different  $l$ .

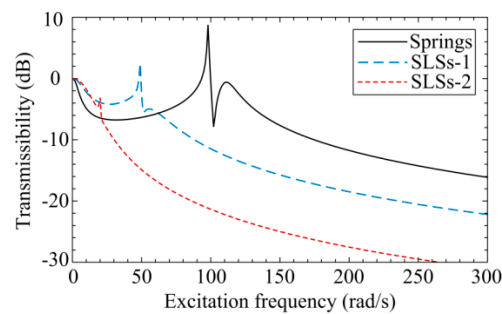
From Figure 7, it can be seen that firstly the natural frequency where has largest peak is reduced by increasing  $n$ , or decreasing  $\theta$  and  $l$ , which verifies the analysis of natural frequencies in pervious section. Secondly, the value of the peak is reduced by increasing  $n$  and  $l$  while changing  $\theta$  has no obvious effect on the value of the peak, and thirdly the response of the amplitudes are lower for larger  $n$  and  $l$ , or smaller  $\theta$ . Also, it demonstrates that the value of the displacement transmissibility (dB) in the frequency range from zero to the first resonant peak could be controlled less than zero and the displacement transmissibility is always below zero after the first resonant peak for larger  $n$  or  $l$  and smaller  $\theta$ . Therefore, it can be concluded that the isolation effectiveness in a broad frequency



band of the proposed isolation platform with SLSs can be improved by adjusting the structural parameters  $n$ ,  $\theta$  and  $l$  easily.

### 3.2. Comparison with normal spring-mass isolation platform

The isolation effectiveness of the isolation platform with SLSs and springs are compared which is shown in Figure 8. For two cases as  $n=2$ ,  $\theta=\pi/4$ ,  $l=0.2$  (case 1) and  $n=3$ ,  $\theta=\pi/6$ ,  $l=0.2$  (case 2), the performances of isolation platform with springs and SLSs are obtained and shown in Figure 8.

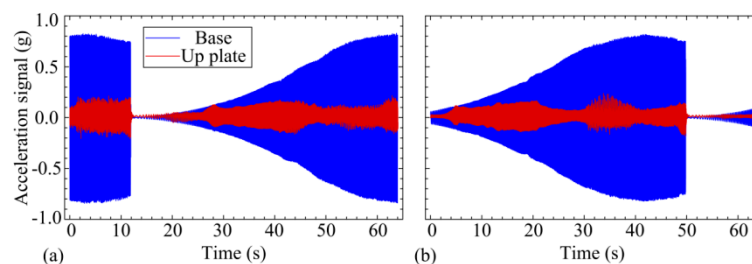


**Figure 8.** The comparison of the isolation effectiveness in  $x$  direction between the platform with springs and SLSs for  $n=2$ ,  $\theta=\pi/4$  and  $l=0.2$  (case 1) and  $n=3$ ,  $\theta=\pi/6$  and  $l=0.2$  (case 2).

From Figure 8, it can be seen that the isolation platform with springs has two obvious resonant peaks are about 100 and 110  $\text{rad}\cdot\text{s}^{-1}$  expect zero. While for the two cases of the isolation platform using SLSs, the fundamental natural frequency is much smaller than the one using springs. For case 1 as  $n=2$ ,  $\theta=\pi/4$  and  $l=0.2$ , the first resonant peak is at about 50  $\text{rad}\cdot\text{s}^{-1}$ , and for case 2 as  $n=3$ ,  $\theta=\pi/6$  and  $l=0.2$ , the first resonant peak is at 25  $\text{rad}\cdot\text{s}^{-1}$ . Also, the values of displacement transmissibility for the two cases of the platform with SLSs are much smaller than the case with springs in a broad frequency domain.

### 3.3 Isolation effectiveness

In order to obtain the natural frequency of the proposed MDOF isolation platform and its isolation effectiveness, random excitation is proposed on the base and the response of the experimental prototype. The frequency domain of the random excitation is from 0 to 100 Hz. The structural parameters of the experimental prototype are as  $n=2$ ,  $l=0.1$ ,  $l_2=0.285$ ,  $l_1=0.105$ ,  $M_2=0.22$ ,  $M_1=0$ ,  $L_1=L_2=L_3=0$ , and other structural parameters  $\theta$  and  $k_l$  could be adjusted easily for different isolation requirements. In Figure 9, two cases of isolation effectiveness are shown as  $\theta$  and  $k_l$  are as  $\theta=\pi/3$ ,  $k_l=900$ .

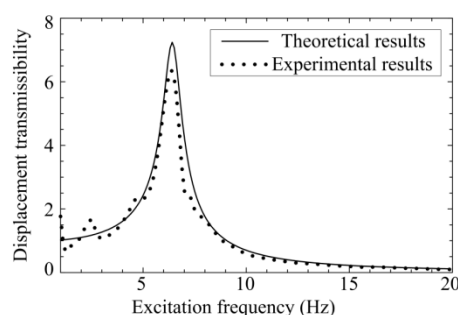


**Figure 9.** The isolation effectiveness for different structural parameters. (a) for  $\theta=\pi/3$  and  $k_l=900$ ; (b)  $\theta=2\pi/5$  and  $k_l=450$ .

Figure 10 shows the comparison between the theoretical results and experimental results for



the periodic excitation with  $z_c=0.03$ ,  $x_c=y_c=\phi_{xc}=\phi_{yc}=\phi_{zc}=0$ ,  $\theta=\pi/3$  and  $k_l=900$ . The signals of the base and the up plate measured are acceleration signals and the acceleration signals are calculated to displacement signals by dividing the square of frequency since the vibrations are periodic motions.



**Figure 10.** The displacement transmissibility for vertical direction vibration for  $\theta=\pi/3$ ,  $k_l=900$  and  $z_c=0.03$ .

From Figure 10, the natural frequency of the proposed isolation platform could be reduced by adjusting the structural parameters of the SLSs for different structural parameters. When utilizing four springs with  $k_l=900$  in a normal spring-damping isolator, the vertical-direction natural frequency is as  $\sqrt{\frac{4 \times 900}{0.22}} = 128 \text{ rad/s} \approx 20.38 \text{ Hz}$ , while for utilizing the SLSs the natural frequency could be reduced to about 6.4 Hz as shown in Figure 10 for  $\theta=\pi/3$ ,  $k_l=900$ . Therefore, using SLSs instead of springs could improve the isolation effectiveness, based on the fact that the natural frequency is reduced and the effective isolation range is increased.

#### 4. Conclusions

This study proposed a MDOF vibration isolation platform with four symmetrical scissor-like structures (SLSs), which is designed for microgravity environments such as protecting instruments and isolation devices in aerospace stations. Theoretical modeling, analysis and comparison studies demonstrated its advantages and versatility in vibration isolation/control. The advantages of this isolation platform are listed as follow,

(a) By designing the structural parameters of SLSs, the proposed MDOF isolation platform can achieve smaller natural frequency and much larger effective isolation frequency range compared with normal spring-mass isolator;

(b) Because of the stiffness property with small linear coefficients in different directions of the proposed isolation platform, the system becomes an improved MDOF adjustable isolation platform;

(c) The adjustable stiffness coefficients in different directions and isolation effectiveness of the isolation platform with SLSs are verified by experiment prototype and experimental results.

Further studies will focus on the effect of active control for a MDOF vibration isolation system using the scissor-like structures (SLSs) and the combined effect of control parameters and nonlinear coefficients on isolation performance.

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