MATLAB and MS Excel codes for the shown approximations with estimation of error before and after optimization.
(Re-Reynolds number, RelEpsilon-Relative roughness; in MS Excel: 'A1: Re and B1: RelEpsilon')

1. Brkić approximation [Appr. 1]

Brkić, D. (2011). "An explicit approximation of the Colebrook equation for fluid flow friction factor." Petrol. Sci. Tech., 29(15), 1596-1602. http://dx.doi.org/10.1080/10916461003620453

Before optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
A1 = log(Re./(1.816.*log(1.1.*Re./log(1+1.1.*Re))));
Lambda = (-2.*log10(2.18.*A1./Re+RelEpsilon./3.71)).^-2;
```

MS Excel:

=LN(A1/(1.816*LN(1.1*A1/LN(1+1.1*A1))))

=-2*LOG10((B1/3.71)+(2.18*C1/A1)) cell C1

=POWER(1/D1,2) cell D1

After optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
B1 = log(Re./(2.479.*log(1.1.*Re./log(1+1.1.*Re))));
Lambda = (-2.013.*log10(2.261.*B1./Re+RelEpsilon./3.71)).^-2;
```

MS Excel:

=LN(A1/(2.479*LN(1.1*A1/LN(1+1.1*A1)))) cell C1

=-2.013*LOG10((B1/3.71)+(2.261*C1/A1)) cell D1

=POWER(1/D1,2)

Table 1. Specific errors before and after genetic optimization; Brkić approximation [Appr. 1]

	Before	After
Maximal relative error δ_{max}	2.2065%	1.2868%
Average error δ_{avr}	0.4125%	0.8860%
Mean square error δ_{MSE}	$3.3662\cdot10^{-8}$	$1.3650\cdot10^{-7}$

Medium level of error before and after optimization; $10^{-8}<\delta_{MSE}<5\cdot10^{-6}$ according to:
-Winning, H.K., Coole, T., 2013. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow Turbulence Combust. 90(1), 1-27. https://dx.doi.org/10.1007/s10494-012-9419-7
Error estimated according to methodology from:
-Brkić, D., 2011. Review of explicit approximations to the Colebrook relation for flow friction. J. Petrol. Sci. Eng. 77(1), 34-48. https://dx.doi.org/10.1016/j.petrol.2011.02.006

2. Brkić approximation [Appr. 2]

Brkić, D. (2011). "An explicit approximation of the Colebrook equation for fluid flow friction factor." *Petrol. Sci. Tech.*, 29(15), 1596-1602. http://dx.doi.org/10.1080/10916461003620453

Before optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
A2 = log(Re./(1.816.*log(1.1.*Re./log(1+1.1.*Re))));
Lambda = (-2.*log10(10.^(-0.4343.*A2)+RelEpsilon./3.71)).^-2;
```

MS Excel:

=LN(A1/(1.816*LN(1.1*A1/LN(1+1.1*A1)))) cell C1

=-2*LOG10((B1/3.71)+(POWER(10,-0.4343*C1))) cell D1

=POWER(1/D1,2)

After optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
B2 = log(Re./(1.895.*log(1.1.*Re./log(1+1.1.*Re))));
Lambda = (-2.013.*log10(10.^(-0.43.*B2)+RelEpsilon./3.71)).^-2;
```

MS Excel:

=LN(A1/(1.895*LN(1.1*A1/LN(1+1.1*A1)))) cell C1

=-2.013*LOG10((B1/3.71)+(POWER(10,-0.43*C1))) cell D1

=POWER(1/D1,2)

Table 2. Specific errors before and after genetic optimization; Brkić approximation [Appr. 2]

	Before	After
Maximal relative error δ_{max}	3.1560%	1.2868%
Average error δ_{avr}	0.8165%	0.8809%
Mean square error δ_{MSE}	$7.3959\cdot10^{-8}$	$1.3765\cdot10^{-7}$

Medium level of error before and after optimization; $10^{-8}<\delta_{MSE}<5\cdot10^{-6}$ according to:

-Winning, H.K., Coole, T., 2013. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow Turbulence Combust. 90(1), 1-27. https://dx.doi.org/10.1007/s10494-012-9419-7

Error estimated according to methodology from:

-Brkić, D., 2011. Review of explicit approximations to the Colebrook relation for flow friction. J. Petrol. Sci. Eng. 77(1), 34-48. https://dx.doi.org/10.1016/j.petrol.2011.02.006

3. Brkić approximation [Appr. 3]

Brkić, D. (2011). "New explicit correlations for turbulent flow friction factor." *Nucl. Eng. Des.*, 241(9), 4055-4059. http://dx.doi.org/10.1016/j.nucengdes.2011.07.042

Before optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (-2.*log10(150.39./Re.^0.98865-
152.66./Re+RelEpsilon./3.71)).^-2;
```

MS Excel:

=-2*LOG10(150.39/POWER(A1,0.98865)-152.66/A1+B1/3.71) cell C1

=POWER(1/C1,2)

After optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (-2.011.*log10(147.21./Re.^0.98865-
149.243./Re+RelEpsilon./3.71)).^-2;
```

MS Excel:

=-2.011*LOG10(147.21/POWER(A1,0.98865)-149.243/A1+B1/3.71) cell C1

=POWER(1/C1,2)

Table 3. Specific errors before and after genetic optimization; Brkić approximation [Appr. 3]

	Before	After
Maximal relative error δ_{max}	2.0715%	1.3326%
Average error δ_{avr}	0.3101%	0.8971%
Mean square error δ_{MSE}	$2.7622\cdot10^{-8}$	$1.0472\cdot10^{-7}$

Medium level of error before and after optimization; $10^{-8}<\delta_{MSE}<5\cdot10^{-6}$ according to:

-Winning, H.K., Coole, T., 2013. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow Turbulence Combust. 90(1), 1-27. https://dx.doi.org/10.1007/s10494-012-9419-7

Error estimated according to methodology from:

-Brkić, D., 2011. Review of explicit approximations to the Colebrook relation for flow friction. J. Petrol. Sci. Eng. 77(1), 34-48. https://dx.doi.org/10.1016/j.petrol.2011.02.006

4. Brkić approximation [Appr. 4]

Brkić, D. (2011). "New explicit correlations for turbulent flow friction factor." *Nucl. Eng. Des.*, 241(9), 4055-4059. http://dx.doi.org/10.1016/j.nucengdes.2011.07.042

Before optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (-2.*log10((1.25603./(Re./sqrt((-
0.0015702./log(Re))+(0.3942031./(log(Re))^2)+(2.5341533./(log(Re))^3))
))+RelEpsilon./3.71)).^-2;
```

MS Excel:

=-2*LOG10(1.25603/(A1*SQRT((-
0.0015702/LN(A1)+0.3942031/(POWER(LN(A1),2))+2.5341533/(POWER(LN(A1),3)))))+B1/3.71)
cell C1

=POWER(1/C1,2)

After optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (-2.013.*log10((1.216./(Re./sqrt((-
0.0013./log(Re))+(0.383./(log(Re))^2)+(2.997./(log(Re))^3))))+RelEpsil
on./3.71)).^-2;
```

MS Excel:

=2.013*LOG10(1.216/(A1*SQRT((-
0.013/LN(A1)+0.383/(POWER(LN(A1),2))+2.997/(POWER(LN(A1),3)))))+B1/3.71)
cell C1

=POWER(1/C1,2)

Table 4. Specific errors before and after genetic optimization; Brkić approximation [Appr. 4]

	Before	After
Maximal relative error δ_{max}	2.0111%	1.2866%
Average error δ_{avr}	0.3101%	0.7115%
Mean square error δ_{MSE}	$2.7565\cdot10^{-8}$	$1.2750\cdot10^{-7}$

Medium level of error before and after optimization; $10^{-8}<\delta_{MSE}<5\cdot10^{-6}$ according to:

-Winning, H.K., Coole, T., 2013. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow Turbulence Combust. 90(1), 1-27. https://dx.doi.org/10.1007/s10494-012-9419-7

Error estimated according to methodology from:

-Brkić, D., 2011. Review of explicit approximations to the Colebrook relation for flow friction. J. Petrol. Sci. Eng. 77(1), 34-48. https://dx.doi.org/10.1016/j.petrol.2011.02.006

5. Fang, Xu and Zhou approximation [Appr. 5]

Fang, X., Xu, Y., Zhou, Z., (2011). "New correlations of single-phase friction factor for turbulent pipe flow and evaluation of existing single-phase friction factor correlations." *Nucl. Eng. Des.*, 241(3), 897-902. http://dx.doi.org/10.1016/j.nucengdes.2010.12.019

Before optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = 1.613.*(log(0.234.*RelEpsilon.^1.1007-
60.525./Re.^1.1105+56.291./Re.^1.0712)).^-2;
```

MS Excel:

=1.613*POWER(LN(0.234*POWER(B1,1.1007)-60.525/POWER(A1,1.1105)+56.291/POWER(A1,1.0715)),-2)

After optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = 1.61.*(log(0.234.*RelEpsilon.^1.1007-
61.948./Re.^1.1105+57.449./Re.^1.0712)).^-2;
```

MS Excel:

=1.61*POWER(LN(0.234*POWER(B1,1.1007)-61.948/POWER(A1,1.1105)+57.449/POWER(A1,1.0715)),-2)

Table 5. Specific errors before and after genetic optimization; Fang et al. approximation [Appr. 5]

	Before	After
Maximal relative error δ_{max}	0.6167%	0.5669%
Average error δ_{avr}	0.3101%	0.1526%
Mean square error δ_{MSE}	$2.9324\cdot10^{-9}$	$2.8711\cdot10^{-9}$

Small level of error before and after optimization; $10^{-11}< \delta_{MSE}<10^{-8}$ according to:

-Winning, H.K., Coole, T., 2013. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow Turbulence Combust. 90(1), 1-27. https://dx.doi.org/10.1007/s10494-012-9419-7

Error estimated according to methodology from:

-Brkić, D., 2011. Review of explicit approximations to the Colebrook relation for flow friction. J. Petrol. Sci. Eng. 77(1), 34-48. https://dx.doi.org/10.1016/j.petrol.2011.02.006

6. Ghanbari, Farshad and Rieke approximation [Appr. 6]

Ghanbari, A., Farshad, F.F., Rieke, H.H. (2011). “Newly developed friction factor correlation for pipe flow and flow assurance.” *J. Chem. Eng. Mater. Sci.*, 2(6), 83-86.

Before optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (-
1.52.*log10((RelEpsilon./7.21).^1.042+(2.731./Re)^0.9152)).^-2.169;
```

MS Excel:

=POWER(-1.52*LOG10(POWER(B1/7.21,1.042)+POWER(2.731/A1,0.9152)),-2.169)

After optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (-
1.606.*log10((RelEpsilon./7.03).^0.967+(2.629./Re)^0.858)).^-2.195;
```

MS Excel:

=POWER(-1.606*LOG10(POWER(B1/7.03,0.967)+POWER(2.629/A1,0.858)),-2.195)

Table 6. Specific errors before and after genetic optimization; Ghanbari et al. approximation [Appr. 6]

	Before	After
Maximal relative error δ_{max}	2.8962%	2.5947%
Average error δ_{avr}	0.8028%	1.2359%
Mean square error δ_{MSE}	$9.1390 \cdot 10^{-7}$	$2.2629 \cdot 10^{-7}$

Medium level of error before and after optimization; $10^{-8}<\delta_{MSE}<5 \cdot 10^{-6}$ according to:

-Winning, H.K., Coole, T., 2013. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow Turbulence Combust. 90(1), 1-27. https://dx.doi.org/10.1007/s10494-012-9419-7

Error estimated according to methodology from:

-Brkić, D., 2011. Review of explicit approximations to the Colebrook relation for flow friction. J. Petrol. Sci. Eng. 77(1), 34-48. https://dx.doi.org/10.1016/j.petrol.2011.02.006

7. Papaevangelou, Evangelides and Tzimopoulos approximation [Appr. 7]

Papaevangelou, G., Evangelides, C., Tzimopoulos, C. (2010). “A new explicit relation for the friction factor coefficient in the Darcy–Weisbach equation.”, Protection and Restoration of the Environment, University of Ioannina Greece and Stevens Institute of Technology New Jersey, Corfu, Greece, 166–172.

Before optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (0.2479-0.0000947.*(7-
log10(Re)).^4)/(log10(RelEpsilon./3.615+7.366./Re.^0.9142)).^2;
```

MS Excel:

```
=(0.2479-0.0000947*POWER(7-
LOG10(A1),4))/POWER(LOG10(B1*(1/3.615)+7.366/POWER(A1,0.9142)),2)
```

After optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (0.249-0.0000974.*(abs(7.122-
log10(Re))).^3.769)/(log10(RelEpsilon./3.646+7.484./Re.^0.919)).^2;
```

MS Excel:

```
=(0.249-0.0000974*POWER(ABS(7.122-
LOG10(A1)),3.769))/POWER(LOG10(B1*(1/3.646)+7.484/POWER(A1,0.919)),2)
```

Table 7. Specific errors before and after genetic optimization; Papaevangelou et al. approximation [Appr. 7]

	Before	After
Maximal relative error δ_{max}	0.8248%	0.7312%
Average error δ_{avr}	0.2001%	0.2974%
Mean square error δ_{MSE}	$1.2984\cdot10^{-8}$	$1.5319\cdot10^{-8}$

Medium level of error before and after optimization; $10^{-8}<\delta_{MSE}<5\cdot10^{-6}$ according to:

-Winning, H.K., Coole, T., 2013. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow Turbulence Combust. 90(1), 1-27. https://dx.doi.org/10.1007/s10494-012-9419-7

Error estimated according to methodology from:

-Brkić, D., 2011. Review of explicit approximations to the Colebrook relation for flow friction. J. Petrol. Sci. Eng. 77(1), 34-48. https://dx.doi.org/10.1016/j.petrol.2011.02.006

8. Avci and Karagoz approximation [Appr. 8]

Avci, A., Karagoz, I. (2009). “A novel explicit equation for friction factor in smooth and rough pipes.” *J. Fluids Eng. ASME*, 131(6), 061203, 1-4. http://dx.doi.org/10.1115/1.3129132

Before optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = 6.4./(log(Re)-
log(1+0.01.*Re.*RelEpsilon.*(1+10.*sqrt(RelEpsilon)))).^2.4;
```

MS Excel:

=6.4/POWER((LN(A1)-LN(1+0.01*A1*B1*(1+10*SQRT(B1)))),2.4)

After optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = 6.264./(log(Re)-
log(1+0.009.*Re.*RelEpsilon.*(1+10.*sqrt(RelEpsilon)))).^2.383;
```

MS Excel:

=6.264/POWER((LN(A1)-LN(1+0.009*A1*B1*(1+10*SQRT(B1)))),2.383)

Table 8. Specific errors before and after genetic optimization; Avci and Karagoz approximation [Appr. 8]

	Before	After
Maximal relative error δ_{max}	4.7858%	3.1259%
Average error δ_{avr}	1.2521%	1.8650%
Mean square error δ_{MSE}	$1.1611 \cdot 10^{-6}$	$3.1516 \cdot 10^{-7}$

Medium level of error before and after optimization; $10^{-8}<\delta_{MSE}<5 \cdot 10^{-6}$ according to:
-Winning, H.K., Coole, T., 2013. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow Turbulence Combust. 90(1), 1-27. https://dx.doi.org/10.1007/s10494-012-9419-7
Error estimated according to methodology from:
-Brkić, D., 2011. Review of explicit approximations to the Colebrook relation for flow friction. J. Petrol. Sci. Eng. 77(1), 34-48. https://dx.doi.org/10.1016/j.petrol.2011.02.006

9. Buzzelli approximation [Appr. 9]

Buzzelli, D. (2008). "Calculating friction in one step." *Mach. Des.*, 80(12), 54–55.

Before optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
A3 = ((0.774.*log(Re))-1.41)./(1+1.32.*sqrt(RelEpsilon));
A4 = (RelEpsilon./3.7).*Re+2.51.*A3;
Lambda = (A3-((A3+2.*log10(A4./Re))/(1+2.18./A4))).^-2;
```

MS Excel:

=(((0.774*LN(A1))-1.41)/(1+1.32*SQRT(B1)))-((((((0.774*LN(A1))-1.41)/(1+1.32*SQRT(B1))))+2*LOG10(((1/3.7)*B1*A1+2.51*(((0.774*LN(A1))-1.41)/(1+1.32*SQRT(B1))))/A1))/(1+2.18/(B1*(1/3.7)*A1+2.51*((((0.774*LN(A1))-1.41)/(1+1.32*SQRT(B1)))))) cell C1

=POWER(1/C1,2)

After optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
B3 = ((0.7314.*log(Re))-1.3163)./(1.0025+1.2435.*sqrt(RelEpsilon));
B4 = (RelEpsilon./3.7165).*Re+2.5137.*B3;
Lambda = (B3-((B3+1.9999.*log10(B4./Re))/(0.9996+2.1018./B4))).^-2;
```

MS Excel:

=(((0.7314*LN(A1))-1.3163)/(1+1.32*SQRT(B1)))-((((((0.7314*LN(A1))-1.3163)/(1+1.32*SQRT(B1))))+1.9999*LOG10(((1/ 3.7165)*B1*A1+2.5137*(((0.7314*LN(A1))-1.3163)/(1+1.32*SQRT(B1))))/A1))/(1+2.1018/(B1*(1/3.7165)*A1+2.5137*((((0.7314*LN(A1))-1.3163)/(1+1.32*SQRT(B1)))))) cell C1

=POWER(1/C1,2)

Table 9. Specific errors before and after genetic optimization; Buzzelli approximation [Appr. 9]

	Before	After
Maximal relative error δ_{max}	0.1385%	0.0797%
Average error δ_{avr}	0.0526%	0.0265%
Mean square error δ_{MSE}	$1.4643 \cdot 10^{-9}$	$4.2014 \cdot 10^{-10}$

Small level of error before and after optimization; $10^{-11} < \delta_{MSE} < 10^{-8}$ according to:

-Winning, H.K., Coole, T., 2013. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow Turbulence Combust. 90(1), 1-27. https://dx.doi.org/10.1007/s10494-012-9419-7

Error estimated according to methodology from:

-Brkić, D., 2011. Review of explicit approximations to the Colebrook relation for flow friction. J. Petrol. Sci. Eng. 77(1), 34-48. https://dx.doi.org/10.1016/j.petrol.2011.02.006

10. Sonnad and Goudar approximation [Appr. 10]

Sonnad, J.R., and Goudar, C.T. (2006). “Turbulent flow friction factor calculation using a mathematically exact alternative to the Colebrook–White equation.” *J. Hydraul. Eng. ASCE*, 132(8), 863-867. http://dx.doi.org/10.1061/(ASCE)0733-9429(2006)132:8(863)

Vatankhah, A.R., Kouchakzadeh, S. (2008). “Discussion of ‘Turbulent flow friction factor calculation using a mathematically exact alternative to the Colebrook-White equation.’ by Jagadeesh R. Sonnad and Chetan T. Goudar.” *J. Hydraul. Eng. ASCE*, 134(8), 1187. http://dx.doi.org/10.1061/(ASCE)0733-9429(2008)134:8(1187)

Before optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
G = 0.124.*Re.*RelEpsilon+log(0.4587.*Re);
Lambda = (0.8686.*log(0.4587.*Re./G.^(G/(G+1)))).^-2;
```

MS Excel:

=0.8686*LN(0.4587*A1/POWER((0.124*A1*B1+LN(0.4587*A1))-0,(0.124*A1*B1+LN(0.4587*A1))/((0.124*A1*B1+LN(0.4587*A1))+1))) cell C1

=POWER(1/C1,2)

After optimization (Vatankhah and Kouchakzadeh):

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
G = 0.124.*Re.*RelEpsilon+log(0.4587.*Re);
Lambda = (0.8686.*log(0.4587.*Re./(G-0.31).^(G/(G+0.9633)))).^-2;
```

MS Excel:

=0.8686*LN(0.4587*A1/POWER((0.124*A1*B1+LN(0.4587*A1))-0.31,(0.124*A1*B1+LN(0.4587*A1))/((0.124*A1*B1+LN(0.4587*A1))+0.9633))) cell C1

=POWER(1/C1,2)

Table 10. Specific errors before and after genetic optimization; Sonnad and Goudar approximation [Appr. 10]

	Before	After
Maximal relative error δ_{max}	0.8007%	0.1473%
Average error δ_{avr}	0.2167%	0.0587%
Mean square error δ_{MSE}	$5.6447\cdot10^{-9}$	$1.5896\cdot10^{-9}$

Small level of error before and after optimization; $10^{-11}<\delta_{MSE}<10^{-8}$ according to:

-Winning, H.K., Coole, T., 2013. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow Turbulence Combust. 90(1), 1-27. https://dx.doi.org/10.1007/s10494-012-9419-7

Error estimated according to methodology from:

-Brkić, D., 2011. Review of explicit approximations to the Colebrook relation for flow friction. J. Petrol. Sci. Eng. 77(1), 34-48. https://dx.doi.org/10.1016/j.petrol.2011.02.006

11. Romeo, Royo and Monzón approximation [Appr. 11]

Romeo, E., Royo, C., Monzón, A. (2002). "Improved explicit equations for estimation of the friction factor in rough and smooth pipes." *Chem. Eng. J.*, 86(3), 369–374. http://dx.doi.org/10.1016/S1385-8947(01)00254-6

Ćojbašić, Ž, and Brkić, D. (2013). "Very accurate explicit approximations for calculation of the Colebrook friction factor." *Int. J. Mech. Sci.*, 67, 10-13. http://dx.doi.org/10.1016/j.ijmecsci.2012.11.017

Before optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (-2.*log10((RelEpsilon./3.7065)-
(5.0272./Re).*log10((RelEpsilon./3.827)-
(4.567./Re).*log10((RelEpsilon./7.7918).^0.9924+(5.3326./(208.815+Re))
.^0.9345)))).^-2;
```

MS Excel:

=-2*LOG10(B1*(1/3.7065)-5.0272/A1*LOG10(B1*(1/3.827)-(4.567/A1)*LOG10(POWER(B1*(1/7.7918),0.9924)+POWER(5.3326/(208.815+A1),0.9345))))

cell C1

=POWER(1/C1,2)

After optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (-2.*log10((RelEpsilon./3.7106)-
(5./Re).*log10((RelEpsilon./3.8597)-
(4.795./Re).*log10((RelEpsilon./7.646).^0.9685+(4.9755./(206.2795+Re))
.^0.8759)))).^-2;
```

MS Excel:

=-2*LOG10(B1*(1/3.7106)-5/A1*LOG10(B1*(1/3.8597)-(4.795/A1)*LOG10(POWER(B1*(1/7.646),0.9685)+POWER(4.9755/(206.2795+A1),0.8759))))

cell C1

=POWER(1/C1,2)

Table 11. Specific errors before and after genetic optimization; Romeo et al. approximation [Appr. 11]

	Before	After
Maximal relative error δ_{max}	0.1345%	0.0083%
Average error δ_{avr}	0.0544%	0.0037%
Mean square error δ_{MSE}	$3.4379 \cdot 10^{-10}$	$4.3087 \cdot 10^{-12}$

Small level of error before optimization; $10^{-11} < \delta_{MSE} < 10^{-8}$, and very small after, according to:

-Winning, H.K., Coole, T., 2013. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow Turbulence Combust. 90(1), 1-27. https://dx.doi.org/10.1007/s10494-012-9419-7

Error estimated according to methodology from:

-Brkić, D., 2011. Review of explicit approximations to the Colebrook relation for flow friction. J. Petrol. Sci. Eng. 77(1), 34-48. https://dx.doi.org/10.1016/j.petrol.2011.02.006

12. Manadilli approximation [Appr. 12]

Manadilli, G. (1997). "Replace implicit equations with signomial functions." *Chem. Eng. (New York)*, 104(8), 129-130.

Before optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (-2.*log10(RelEpsilon./3.7+95./Re.^0.983-96.82./Re)).^-2;
```

MS Excel:

=-2*LOG10(B1*(1/3.7)+95/POWER(A1,0.983)-96.82/A1) cell C1

=POWER(1/C1,2)

After optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (-1.98.*log10(RelEpsilon./3.949+95.974./Re.^0.986-
96.02./Re)).^-2;
```

MS Excel:

=-1.98*LOG10(B1*(1/3.949)+95.974/POWER(A1,0.986)-96.02/A1) cell C1

=POWER(1/C1,2)

Table 12. Specific errors before and after genetic optimization; Manadilli [Appr. 12]

	Before	After
Maximal relative error δ_{max}	2.0651%	1.5018%
Average error δ_{avr}	0.3716%	0.5956%
Mean square error δ_{MSE}	$3.4483 \cdot 10^{-8}$	$7.2942 \cdot 10^{-8}$

Medium level of error before and after optimization; $10^{-8}<\delta_{MSE}<5 \cdot 10^{-6}$ according to:
-Winning, H.K., Coole, T., 2013. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow Turbulence Combust. 90(1), 1-27. https://dx.doi.org/10.1007/s10494-012-9419-7
Error estimated according to methodology from:
-Brkić, D., 2011. Review of explicit approximations to the Colebrook relation for flow friction. J. Petrol. Sci. Eng. 77(1), 34-48. https://dx.doi.org/10.1016/j.petrol.2011.02.006

13. Chen J.J.J. approximation [Appr. 13]

Chen, J.J.J. (1984). "A simple explicit formula for the estimation of pipe friction factor." *Proc. Inst. Civ. Eng.*, 77, 49–55. http://dx.doi.org/10.1680/iicep.1984.1272

Before optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = 0.184.*(1./Re.^0.67+0.7.*RelEpsilon).^0.3;
```

MS Excel:

=0.184*POWER(0.7*B1+1/POWER(A1,0.67),0.3)

After optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = 0.208.*(0.327./Re.^0.541+0.697.*RelEpsilon).^0.315;
```

MS Excel:

=0.208*POWER(0.697*B1+0.321/POWER(A1,0.541),0.315)

Table 13. Specific errors before and after genetic optimization; Chen J.J.J. [Appr. 13]

	Before	After
Maximal relative error δ_{max}	27.5074%	18.4800%
Average error δ_{avr}	7.4537%	10.8465%
Mean square error δ_{MSE}	$1.0188\cdot10^{-5}$	$1.0171\cdot10^{-5}$

Large level of error before and after optimization; $>5\cdot10^{-6}$ according to:

-Winning, H.K., Coole, T., 2013. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow Turbulence Combust. 90(1), 1-27. https://dx.doi.org/10.1007/s10494-012-9419-7

Error estimated according to methodology from:

-Brkić, D., 2011. Review of explicit approximations to the Colebrook relation for flow friction. J. Petrol. Sci. Eng. 77(1), 34-48. https://dx.doi.org/10.1016/j.petrol.2011.02.006

14. Serghides approximation [Appr. 14]

Serghides, T.K. (1984). “Estimate friction factor accurately.” *Chem. Eng. (New York)*, 91(5), 63–64.
Ćojbašić, Ž, and Brkić, D. (2013). “Very accurate explicit approximations for calculation of the Colebrook friction factor.” *Int. J. Mech. Sci.*, 67, 10-13. http://dx.doi.org/10.1016/j.ijmecsci.2012.11.017

Before optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
A7 = -2*log10(RelEpsilon./3.7+12./Re);
A8 = -2*log10(RelEpsilon./3.7+2.51.*A7./Re);
A9 = -2*log10(RelEpsilon./3.7+2.51.*A8./Re);
Lambda = (A7-((A8-A7).^2)/(A9-2.*A8+A7)).^-2;
```

MS Excel:

```
=POWER(-2*LOG10(B1*(1/3.7)+12/A1)-POWER(-2*LOG10(B1*(1/3.7)+2.51*(-
2*LOG10(B1*(1/3.7)+12/A1))/A1)--2*LOG10(B1*(1/3.7)+12/A1),2)/((-
2*LOG(B1*(1/3.7)+(2.51*(-2*LOG10(B1*(1/3.7)+(2.51*(-
2*LOG10(B1*(1/3.7)+12/A1))/A1)))/A1)))-2*(-2*LOG10(B1*(1/3.7)+2.51*(-
2*LOG10(B1*(1/3.7)+12/A1))/A1))+(-2*LOG10(B1*(1/3.7)+12/A1))),-2)
```

After optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
B7 = -2*log10(RelEpsilon./3.71+12.585./Re);
B8 = -2*log10(RelEpsilon./3.71+2.51.*B7./Re);
B9 = -2*log10(RelEpsilon./3.71+2.51.*B8./Re);
Lambda = (B7-((B8-B7).^2)/(B9-2.*B8+B7)).^-2;
```

MS Excel:

```
=POWER(-2*LOG10(B1*(1/3.71)+12.585/A1)-POWER(-2*LOG10(B1*(1/3.71)+2.51*(-
2*LOG10(B1*(1/3.71)+12.585/A1))/A1)--2*LOG10(B1*(1/3.71)+12.585/A1),2)/((-
2*LOG(B1*(1/3.71)+(2.51*(-2*LOG10(B1*(1/3.71)+(2.51*(-
2*LOG10(B1*(1/3.71)+12.585/A1))/A1)))/A1)))-2*(-2*LOG10(B1*(1/3.71)+2.51*(-
2*LOG10(B1*(1/3.71)+12.585/A1))/A1))+(-2*LOG10(B1*(1/3.71)+12.585/A1))),-2)
```

Table 14. Specific errors before and after genetic optimization; Serghides [Appr. 14]

	Before	After
Maximal relative error δ_{max}	0.1385%	0.0026%
Average error δ_{avr}	0.508%	0.0004%
Mean square error δ_{MSE}	$1.4487\cdot10^{-9}$	$2.4495\cdot10^{-14}$

Small level of error before optimization; $10^{-11}<\delta_{MSE}<10^{-8}$ and very small after, according to:
-Winning, H.K., Coole, T., 2013. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow Turbulence Combust. 90(1), 1-27. https://dx.doi.org/10.1007/s10494-012-9419-7
Error estimated according to methodology from:
-Brkić, D., 2011. Review of explicit approximations to the Colebrook relation for flow friction. J. Petrol. Sci. Eng. 77(1), 34-48. https://dx.doi.org/10.1016/j.petrol.2011.02.006

15. Serghides approximation (simpler) [Appr. 15]

Serghides, T.K. (1984). "Estimate friction factor accurately." *Chem. Eng. (New York)*, 91(5), 63–64.

Before optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
A10 = -2*log10(RelEpsilon./3.7+12./Re);
A11 = -2*log10(RelEpsilon./3.7+2.51.*B10./Re);
Lambda = (4.781-((A10-4.781).^2)/(A11-2.*A10+4.781)).^-2;
```

MS Excel:

=POWER(4.781-POWER(-2*LOG10(B1*(1/3.7)+12/A1)-4.781,2)/((-2*LOG(B1*(1/3.7)+(2.51*(-2*LOG10(B1*(1/3.7)+(12/A1)))/A1))-2*(-2*LOG10(B1*(1/3.7)+12/A1))+4.781),-2)

After optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
B1 = -2*log10(RelEpsilon./3.71+12.585./Re);
B2 = -2*log10(RelEpsilon./3.71+2.51.*S1./Re);
Lambda = (4.83-((B1-4.83).^2)/(B2-2.*B1+4.83)).^-2;
```

MS Excel:

=POWER(4.83-POWER(-2*LOG10(B1*(1/3.71)+12.585/A1)-4.83,2)/((-2*LOG(B1*(1/3.71)+(2.51*(-2*LOG10(B1*(1/3.71)+(12.585/A1)))/A1))-2*(-2*LOG10(B1*(1/3.71)+12.585/A1))+4.83),-2)

Table 15. Specific errors before and after genetic optimization; Serghides - simpler [Appr. 15]

	Before	After
Maximal relative error δ_{max}	0.3543%	0.2739%
Average error δ_{avr}	0.1036%	0.0354%
Mean square error δ_{MSE}	$1.7284\cdot10^{-9}$	$9.9360\cdot10^{-11}$

Small level of error before and after optimization; $10^{-11}<\delta_{MSE}<10^{-8}$ according to:

-Winning, H.K., Coole, T., 2013. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow Turbulence Combust. 90(1), 1-27. https://dx.doi.org/10.1007/s10494-012-9419-7

Error estimated according to methodology from:

-Brkić, D., 2011. Review of explicit approximations to the Colebrook relation for flow friction. J. Petrol. Sci. Eng. 77(1), 34-48. https://dx.doi.org/10.1016/j.petrol.2011.02.006

16. Haaland approximation [Appr. 16]

Haaland, S.E. (1983). “Simple and explicit formulas for the friction factor in turbulent pipe flow.” *J. Fluids Eng.*, 105(1), 89-90. http://dx.doi.org/10.1115/1.3240948

Before optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (-1.8.*log10((RelEpsilon./3.7)^1.11+6.9./Re)).^-2;
```

MS Excel:

=-1.8*LOG10(POWER((1/3.7)*B1,1.11)+6.9/A1) cell C1

=POWER(1/C1,2)

After optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (-1.798.*log10((RelEpsilon./3.755)^1.106+6.891./Re)).^-2;
```

MS Excel:

=-1.798*LOG10(POWER((1/3.755)*B1,1.106)+6.891/A1) cell C1

=POWER(1/C1,2)

Table 16. Specific errors before and after genetic optimization; Haaland [Appr. 16]

	Before	After
Maximal relative error δ_{max}	1.4083%	1.1098%
Average error δ_{avr}	0.4657%	0.6167%
Mean square error δ_{MSE}	$2.2249 \cdot 10^{-8}$	$4.5480 \cdot 10^{-8}$

Small level of error before and after optimization; $10^{-11} < \delta_{MSE} < 10^{-8}$ according to:

-Winning, H.K., Coole, T., 2013. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow Turbulence Combust. 90(1), 1-27. https://dx.doi.org/10.1007/s10494-012-9419-7

Error estimated according to methodology from:

-Brkić, D., 2011. Review of explicit approximations to the Colebrook relation for flow friction. J. Petrol. Sci. Eng. 77(1), 34-48. https://dx.doi.org/10.1016/j.petrol.2011.02.006

17. Zigrang and Sylvester approximation [Appr. 17]

Zigrang, D.J., Sylvester, N.D. (1982). "Explicit approximations to the solution of Colebrook's friction factor equation." *AIChE J.*, 28(3), 514–515.

Before optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (-2.*log10((RelEpsilon./3.7)-
(5.02./Re)*log10((RelEpsilon./3.7)-
(5.02./Re)*log10(RelEpsilon./3.7+13./Re)))).^-2;
```

MS Excel:

=-2*LOG10((1/3.7)*B1-(5.02/A1)*LOG10((1/3.7)*B1-(5.02/A1)*LOG10((1/3.7)*B1+13/A1))) cell C1

=POWER(1/C1,2)

After optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (-2.0012.*log10((RelEpsilon./3.7027)-
(5.0605./Re)*log10((RelEpsilon./3.7027)-
(5.0605./Re)*log10(RelEpsilon./3.7027+12.513./Re)))).^-2;
```

MS Excel:

=-2.0012*LOG10((1/3.7027)*B1-(5.0605/A1)*LOG10((1/3.7027)*B1-(5.0605/A1)*LOG10((1/3.7027)*B1+12.513/A1))) cell C1

=POWER(1/C1,2)

Table 17. Specific errors before and after genetic optimization; Zigrang and Sylvester [Appr. 17]

	Before	After
Maximal relative error δ_{max}	0.1385%	0.0831%
Average error δ_{avr}	0.0696%	0.0521%
Mean square error δ_{MSE}	$1.5148 \cdot 10^{-10}$	$1.6359 \cdot 10^{-10}$

Small level of error before and after optimization; $10^{-11} < \delta_{MSE} < 10^{-8}$ according to:

-Winning, H.K., Coole, T., 2013. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow Turbulence Combust. 90(1), 1-27. https://dx.doi.org/10.1007/s10494-012-9419-7

Error estimated according to methodology from:

-Brkić, D., 2011. Review of explicit approximations to the Colebrook relation for flow friction. J. Petrol. Sci. Eng. 77(1), 34-48. https://dx.doi.org/10.1016/j.petrol.2011.02.006

18. Zigrang and Sylvester approximation (simpler) [Appr. 18]

Zigrang, D.J., Sylvester, N.D. (1982). “Explicit approximations to the solution of Colebrook’s friction factor equation.” *AIChE J.*, 28(3), 514–515.

Before optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (-2.*log10((RelEpsilon./3.7)-
(5.02./Re)*log10(RelEpsilon./3.7+13./Re))).^-2;
```

MS Excel:

=-2*LOG10((1/3.7)*B1-(5.02/A1)*LOG10((1/3.7)*B1+13/A1)) cell C1

=POWER(1/C1,2)

After optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (-2.0012.*log10((RelEpsilon./3.7027)-
(5.0605./Re)*log10(RelEpsilon./3.7027+15.202./Re))).^-2;
```

MS Excel:

=-2.0012*LOG10((1/3.7027)*B1-(5.0605/A1)*LOG10((1/3.7027)*B1+15.202/A1)) cell C1

=POWER(1/C1,2)

Table 18. Specific errors before and after genetic optimization; Zigrang and Sylvester – simpler [Appr. 18]

	Before	After
Maximal relative error δ_{max}	1.0075%	0.7496%
Average error δ_{avr}	0.2967%	0.1845%
Mean square error δ_{MSE}	$6.9576\cdot10^{-9}$	$2.0703\cdot10^{-9}$

Small level of error before and after optimization; $10^{-11}<\delta_{MSE}<10^{-8}$ according to:

-Winning, H.K., Coole, T., 2013. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow Turbulence Combust. 90(1), 1-27. https://dx.doi.org/10.1007/s10494-012-9419-7

Error estimated according to methodology from:

-Brkić, D., 2011. Review of explicit approximations to the Colebrook relation for flow friction. J. Petrol. Sci. Eng. 77(1), 34-48. https://dx.doi.org/10.1016/j.petrol.2011.02.006

19. Barr approximation [Appr. 19]

Barr, D.I.H. (1981). “Solutions of the Colebrook-White function for resistance to uniform turbulent flow.” *Proc. Inst. Civ. Eng.*, 71(2), 529–535. http://dx.doi.org/10.1680/iicep.1981.1895

Before optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (-
2.*log10((4.518.*log10(Re./7)/(Re.*(((1+Re.^0.52)./29).*RelEpsilon.^0.
7)))+(RelEpsilon./3.7))).^-2;
```

MS Excel:

=-2*LOG10(B1*(1/3.7)+(4.518*LOG10(A1/7))/(A1*(1+(POWER(A1,0.52)/29)*POWER(B1,0.7)))) cell C1

=POWER(1/C1,2)

After optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (-
1.998.*log10((4.509.*log10(Re./7.049)/(Re.*(((0.999+Re.^0.525)./28.102
).*RelEpsilon.^0.721)))+(RelEpsilon./3.737))).^-2;
```

MS Excel:

=-
1.998*LOG10(B1*(1/3.737)+(4.509*LOG10(A1/7.049))/(A1*(0.999+(POWER(A1,0.525)/28.102
)*POWER(B1,0.721)))) cell C1

=POWER(1/C1,2)

Table 19. Specific errors before and after genetic optimization; Barr [Appr. 19]

	Before	After
Maximal relative error δ_{max}	0.2774%	0.2644%
Average error δ_{avr}	0.0548%	0.1137%
Mean square error δ_{MSE}	$1.1399 \cdot 10^{-9}$	$2.9212 \cdot 10^{-9}$

Small level of error before and after optimization; $10^{-11} < \delta_{MSE} < 10^{-8}$ according to:

-Winning, H.K., Coole, T., 2013. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow Turbulence Combust. 90(1), 1-27. https://dx.doi.org/10.1007/s10494-012-9419-7

Error estimated according to methodology from:

-Brkić, D., 2011. Review of explicit approximations to the Colebrook relation for flow friction. J. Petrol. Sci. Eng. 77(1), 34-48. https://dx.doi.org/10.1016/j.petrol.2011.02.006

20. Round approximation [Appr. 20]

Round, G.F. (1980). “An explicit approximation for the friction factor-Reynolds number relation for rough and smooth pipes.” *Can. J. Chem. Eng.*, 58(1), 122-123.
http://dx.doi.org/10.1002/cjce.5450580119

Before optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (1.8.*log10(Re./(0.135.*Re.*RelEpsilon+6.5))).^-2;
```

MS Excel:

=1.8*LOG10(A1/(0.135*A1*B1+6.5)) cell C1

=POWER(1/C1,2)

After optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (1.898.*log10(Re./(0.202.*Re.*RelEpsilon+9.779))).^-2;
```

MS Excel:

=1.898*LOG10(A1/(0.202*A1*B1+9.779)) cell C1

=POWER(1/C1,2)

Table 20. Specific errors before and after genetic optimization; Round [Appr. 20]

	Before	After
Maximal relative error δ_{max}	10.9183%	5.5094%
Average error δ_{avr}	4.0149%	2.6418%
Mean square error δ_{MSE}	$6.8724\cdot10^{-6}$	$8.7303\cdot10^{-7}$

Large level of error before and after optimization; >$5\cdot10^{-6}$ according to:
-Winning, H.K., Coole, T., 2013. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow Turbulence Combust. 90(1), 1-27. https://dx.doi.org/10.1007/s10494-012-9419-7
Error estimated according to methodology from:
-Brkić, D., 2011. Review of explicit approximations to the Colebrook relation for flow friction. J. Petrol. Sci. Eng. 77(1), 34-48. https://dx.doi.org/10.1016/j.petrol.2011.02.006

21. Chen approximation [Appr. 21]

Chen, N.H. (1979). “An explicit equation for friction factor in pipes.” *Ind. Eng. Chem. Fund.*, 18(3), 296-297. http://dx.doi.org/10.1021/i160071a019

Before optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (-2.*log10((RelEpsilon./3.7065)-
(5.0452./Re)*log10((1./2.8257)*(RelEpsilon.^1.1098)+5.8506./Re.^0.8981
))).^-2;
```

MS Excel:

=-2*LOG10(B1*(1/3.7065)-
5.0452/A1*LOG10((1/2.8257)*POWER(B1,1.1098)+5.8056/POWER(A1,0.8981))) cell C1

=POWER(1/C1,2)

After optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (-2.003.*log10((RelEpsilon./3.689)-
(4.933./Re)*log10((1./2.762)*(RelEpsilon.^1.109)+5.89./Re.^0.923))).^-
2;
```

MS Excel:

=-2.003*LOG10(B1*(1/3.689)-
4.933/A1*LOG10((1/2.762)*POWER(B1,1.1098)+5.89/POWER(A1,0.923))) cell C1

=POWER(1/C1,2)

Table 21. Specific errors before and after genetic optimization; Chen [Appr. 21]

	Before	After
Maximal relative error δ_{max}	0.3649%	0.1851%
Average error δ_{avr}	0.1229%	0.0808%
Mean square error δ_{MSE}	$1.0862\cdot10^{-9}$	$5.2494\cdot10^{-10}$

Small level of error before and after optimization; $10^{-11}< \delta_{MSE}<10^{-8}$ according to:

-Winning, H.K., Coole, T., 2013. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow Turbulence Combust. 90(1), 1-27. https://dx.doi.org/10.1007/s10494-012-9419-7

Error estimated according to methodology from:

-Brkić, D., 2011. Review of explicit approximations to the Colebrook relation for flow friction. J. Petrol. Sci. Eng. 77(1), 34-48. https://dx.doi.org/10.1016/j.petrol.2011.02.006

22. Swamee and Jain approximation [Appr. 22]

Swamee, D.K., Jain, A.K. (1976). “Explicit equations for pipe flow problems.” *J. Hydraul. Div. ASCE*, 102(HY5), 657–664.

Before optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (-2.*log10(5.74./Re^0.9+RelEpsilon./3.7)).^-2;
```

MS Excel:

=-2*LOG10(B1*(1/3.7)+5.74/POWER(A1,0.9))   cell C1

=POWER(1/C1,2)

After optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (-1.972*log10(5.828./Re^0.916+RelEpsilon./4.04)).^-2;
```

MS Excel:

=-1.972*LOG10(B1*(1/4.04)+5.828/POWER(A1,0.916))   cell C1

=POWER(1/C1,2)

Table 22. Specific errors before and after genetic optimization; Swamee and Jain [Appr. 22]

	Before	After
Maximal relative error δ_{max}	2.1872%	1.7535%
Average error δ_{avr}	0.4314%	0.8932%
Mean square error δ_{MSE}	$3.3002 \cdot 10^{-8}$	$1.2769 \cdot 10^{-7}$

Medium level of error before and after optimization; $10^{-8}<\delta_{MSE}<5 \cdot 10^{-6}$ according to:

-Winning, H.K., Coole, T., 2013. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow Turbulence Combust. 90(1), 1-27. https://dx.doi.org/10.1007/s10494-012-9419-7

Error estimated according to methodology from:

-Brkić, D., 2011. Review of explicit approximations to the Colebrook relation for flow friction. J. Petrol. Sci. Eng. 77(1), 34-48. https://dx.doi.org/10.1016/j.petrol.2011.02.006

23. Eck approximation [Appr. 23]

Eck, B. (1973). Technische Stromungslehre. Springer, New York

Before optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (-2.*log10(15./Re+RelEpsilon./3.715)).^-2;
```

MS Excel:

=-2*LOG10(B1*(1/3.715)+15/A1)   cell C1

=POWER(1/C1,2)

After optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = (-1.963.*log10(14.064/Re+RelEpsilon./4.034)).^-2;
```

MS Excel:

=-1.963*LOG10(B1*(1/4.034)+14.064/A1)   cell C1

=POWER(1/C1,2)

Table 23. Specific errors before and after genetic optimization; Eck [Appr. 23]

	Before	After
Maximal relative error δ_{max}	8.1953%	5.6955%
Average error δ_{avr}	1.9256%	1.6722%
Mean square error δ_{MSE}	$1.18706\cdot10^{-7}$	$1.5222\cdot10^{-7}$

Medium level of error before and after optimization; $10^{-8}<\delta_{MSE}<5\cdot10^{-6}$ according to:

-Winning, H.K., Coole, T., 2013. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow Turbulence Combust. 90(1), 1-27. https://dx.doi.org/10.1007/s10494-012-9419-7

Error estimated according to methodology from:

-Brkić, D., 2011. Review of explicit approximations to the Colebrook relation for flow friction. J. Petrol. Sci. Eng. 77(1), 34-48. https://dx.doi.org/10.1016/j.petrol.2011.02.006

24. Wood approximation [Appr. 24]

Wood, D.J. (1966). "An explicit friction factor relationship." *Civil. Eng.*, 36 (12), 60–61.

Before optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda =
0.094.*RelEpsilon.^0.225+0.53.*RelEpsilon+88.*RelEpsilon.^0.44.*Re.^-
(1.62.*RelEpsilon.^0.134);
```

MS Excel:

=0.094*POWER(B1,0.225)+0.53*B1+88*POWER(B1,0.44)*POWER(A1,-
1.62*POWER(B1,0.134))

After optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda =
0.094.*RelEpsilon.^0.209+0.376.*RelEpsilon+85.005.*RelEpsilon.^0.33.*R
e.^-(1.501.*RelEpsilon.^0.101);
```

MS Excel:

=0.094*POWER(B1,0.209)+0.376*B1+85.005*POWER(B1,0.33)*POWER(A1,-
1.501*POWER(B1,0.101))

Table 24. Specific errors before and after genetic optimization; Wood [Appr. 24]

	Before	After
Maximal relative error δ_{max}	23.7204%	16.5910%
Average error δ_{avr}	3.7011%	7.2113%
Mean square error δ_{MSE}	$2.5046\cdot10^{-6}$	$3.8013\cdot10^{-6}$

Large level of error before and after optimization; $10^{-8}<\delta_{MSE}<5\cdot10^{-6}$ according to:

-Winning, H.K., Coole, T., 2013. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow Turbulence Combust. 90(1), 1-27. https://dx.doi.org/10.1007/s10494-012-9419-7

Error estimated according to methodology from:

-Brkić, D., 2011. Review of explicit approximations to the Colebrook relation for flow friction. J. Petrol. Sci. Eng. 77(1), 34-48. https://dx.doi.org/10.1016/j.petrol.2011.02.006

25. Moody approximation [Appr. 25]

Moody, L.F. (1947). “An approximate formula for pipe friction factors.” *Trans. ASME.*, 69(12), 1005–1006.

Before optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = 0.0055.*(1.+(2.*10^4.*RelEpsilon+10^6./Re).^1/3);
```

MS Excel:

=0.0055*(1+POWER(20000*B1+(1000000/A1),1/3))

After optimization:

MATLAB:

```
function Lambda=Lambda(Re,RelEpsilon)
Lambda = 0.006.*(0.775.+(2.443.*10^4.*RelEpsilon+10^6./Re).^0.343);
```

MS Excel:

=0.006*(0.775+POWER(24430*B1+(1000000/A1),0.343))

Table 25. Specific errors before and after genetic optimization; Moody [Appr. 25]

	Before	After
Maximal relative error δ_{max}	21.4855%	18.1024%
Average error δ_{avr}	4.5795%	8.3301%
Mean square error δ_{MSE}	$2.4454 \cdot 10^{-5}$	$9.9926 \cdot 10^{-6}$

Large level of error before and after optimization; >$5 \cdot 10^{-6}$ according to:

-Winning, H.K., Coole, T., 2013. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow Turbulence Combust. 90(1), 1-27. https://dx.doi.org/10.1007/s10494-012-9419-7

Error estimated according to methodology from:

-Brkić, D., 2011. Review of explicit approximations to the Colebrook relation for flow friction. J. Petrol. Sci. Eng. 77(1), 34-48. https://dx.doi.org/10.1016/j.petrol.2011.02.006