

## Article

# Chiral Optical Tamm States: Temporal Coupled-Mode Theory

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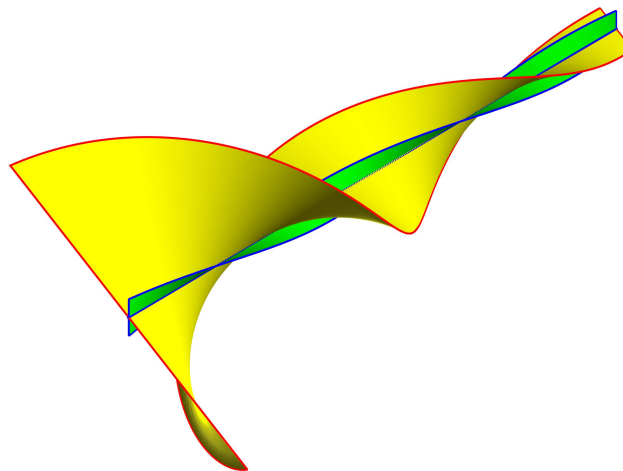
**Abstract:** The chiral optical Tamm state (COTS) is a special localized state at the interface of a handedness-preserving mirror and a structurally chiral medium such as a cholesteric liquid crystal or a chiral sculptured thin film. The spectral behavior of COTS, observed as reflection resonances, is described by the temporal coupled-mode theory. Mode coupling is different for two circular light polarizations because COTS has a helix structure replicating that of the cholesteric. The mode coupling for co-handed circularly polarized light exponentially attenuates with the cholesteric layer thickness since the COTS frequency falls into the stop band. Cross-handed circularly polarized light freely goes through the cholesteric layer and can excite COTS when reflected from the handedness-preserving mirror. The coupling in this case is proportional to anisotropy of the cholesteric and theoretically it is only anisotropy of magnetic permittivity that can ultimately cancel this coupling. These two couplings being equal results in a polarization crossover (the Kopp–Genack effect) for which a linear polarization is optimal to excite COTS. The corresponding cholesteric thickness and scattering matrix for COTS are generally described by simple expressions.

**Keywords:** optical Tamm state; cholesteric liquid crystal; handedness-preserving mirror

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## Introduction

Matter tends to order thus forming crystals. Orientation alignment is the preferred order in liquid crystals. Due to its cyclic nature, it can generate an echo of a translational order in chiral superlattices of cholesteric liquid crystal. Under diffraction, in cholesteric crystals the helical structure of the matter is rendered to the field because the co-handed circularly polarized wave diffracts (Fig. 1) while the cross-handed circularly polarized wave travels virtually unaffected. This phenomenon is referred to as selective reflection [1] or, alternatively, Bragg circular diffraction for electromagnetic and acoustic waves [2].



**Figure 1.** Circular Bragg diffraction forming COTS. The cholesteric director is shown in blue and green, the electric field is in red and yellow. The angle between them does not change with depth

The results obtained are readily generalized for any material with a helix-like response, including widely tunable heliconical structures [3]. The selective reflection obstructs observation of localized states when the order at the interface or at the structural defect is disturbed. This is paid off by a simpler description of the states, which is an advantage offered by smooth helix symmetry [1,4] as opposed to discrete translational symmetry of crystals. The structural defect is conventionally represented as a cavity confined by mirrors, where the role of mirrors is played by the Bragg grating. This counterpart of a Fabry–Perot resonator possesses a number of defect modes. The latter are localized optical states normally corresponding to the whole number of halfwaves accommodated in the cavity. There are a number of distinctive polarization features [5–11] associated with the chiral defect applications [2,12–15], among which the twist defect, lacking an intermediate layer and having zero thickness, is the most prominent [16,17]. Strict theoretical [4,18–20] and experimental [16,21–23] studies of the phenomenon gave rise to a discussion on polarization and relaxation time of the localized state [24,25]. Theoretically, infinite relaxation time is only possible if there is anisotropy of magnetic permittivity [4,26]. Otherwise it appears impossible to simultaneously match electric and magnetic field strengths at the interface. Note that infinitely increasing the cholesteric thickness does not provide infinite improvement of the quality factor. The quality factor saturates with increasing cholesteric thickness and circular polarization of the transmitted light changes from co-handed to cross-handed. This polarization crossover [17] has been called the Kopp–Genack effect [27]. Unlike the Fabry–Perot resonator, the twist defect generates a single localized state. The spatial field distribution curve in this case has no flat top and consists of two waves exponentially descending in opposite directions. This resembles a surface wave with the only difference that for the twist defect there is no limitation on the angle of incidence of the excitation wave, and surface waves at the cholesteric-isotropic dielectric interface are only observed at the angles ensuring complete internal reflection [28].

There exists a surface wave beyond the restriction of complete internal reflection. It is known as an optical Tamm state (OTS) [29–35], which is similar to the Tamm state of electrons at the superlattice interface. The dispersion of OTS lies outside the light cone given by  $k = \omega/c$  [36]. Such a state can be excited even perpendicular to the surface without energy transfer along the surface, which is an advantage for various applications. A question naturally arises if there is an OTS at the cholesteric-metal interface when the light is incident perpendicularly. When dealing with this problem, one should bear in mind two things: first, semitransparency of the cholesteric due to the circular Bragg diffraction and, second, polarization change caused by alternating circular polarizations, namely: reflection from cholesteric does not change the handedness of circular

polarization whereas reflection from metal does [37]. This alternation acts like traffic lights: a co-handed circular polarized wave is not allowed to go through the cholesteric until after it has been twice reflected from the metal [38]. The polarization match at the interface between chiral and nonchiral mirrors can be achieved by adding an extra anisotropic layer. This layer produces a set of localized states with nontrivial polarization properties [39–41]. Various combinations involving two mirrors have been extensively studied and offered for practical applications [37,42], including less-than-one-pitch chiral layer [43–48]. The closer to the mirror, the higher the energy density of the states may become, but still the states are not localized at surface but within the bulk of the extra layer. It is possible to do without an extra layer provided a handedness-preserving mirror (HPM) is used [25,49–51]. HPM maintains not only the handedness but also the ellipticity magnitude upon reflection, therefore, such a mirror is also referred to as a polarization-preserving anisotropic mirror [51]. Also HPM can be defined as a reflector with the effect of half-wave phase plate [52]. A localized state at the HPM/cholesteric interface was described in the low anisotropy approximation of cholesteric crystal and was called a chiral optical Tamm state (COTS) [46].

In this paper we seek to answer the question whether this state is possible in principle with an ideal HPM and a semi-infinite non-absorbing cholesteric layer having finite anisotropy. A detailed description is given of the simplest case when the electric and magnetic anisotropies are identical. Two types of deviation are considered: the lack of magnetic anisotropy and the finite thickness of the cholesteric layer. The ideal state here becomes a resonance with a finite quality factor and relaxation time.

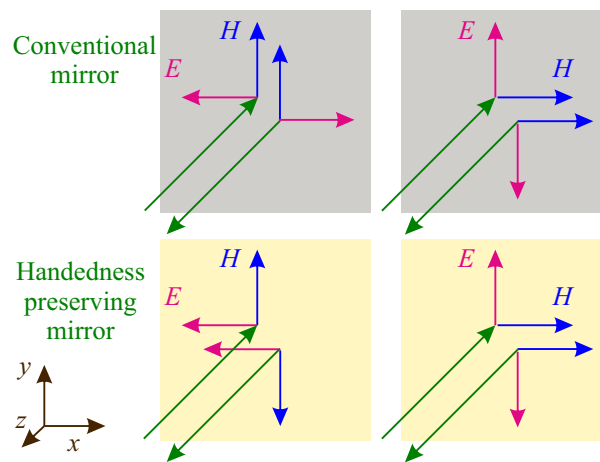
#### *A method to describe spectral peaks*

The spectra of interest and the field distribution are conveniently described by the Berreman formalism [53,54]. For a normal incidence on cholesteric there is an uncomplicated exact solution [55–58]. By matching the tangential strengths at the cholesteric interface we can write down general closed-form equations [4,18–20]. For the sake of simplicity and clarity, we additionally use an approximate analytical method – the temporal coupled-mode theory (TCMT), or the theory of coupled modes in the time domain [59–61]. TCMT provides an instrument to describe the field in coupled resonators where there is coupling between the resonator and the waveguide. The spatial structure of the localized mode here is not involved. It is the complex amplitude of this mode and its time derivative that matter for this theory. TCMT is a popular approach for dealing with stationary processes where the time derivative is zero. The first word, ‘temporal’, in the term [62] can be treated as historically coined. Essentially the same method is employed to describe open resonators [63,64] and it goes back to the Lippmann and Schwinger’s solution of the scattering problem in quantum mechanics [65]. This method is not to be confused with the theory of coupled waves [66,67], or the spatial theory of coupled modes widely used in the optics of cholesterics [28,68–72]. While both approaches rely on the concept of coupled modes [73], the latter theory deals with coupled amplitudes of propagating waves, leaving the amplitude of the resonator mode outside the scope of consideration.

#### **Model**

A sketch of the cholesteric interface is shown in Fig. 1. The cholesteric helical axis is perpendicular to the mirror surface. In other words, the cholesteric director, i.e. the unit vector of the preferred orientation of molecules, is constant in the interface-parallel cross-sections and it uniformly rotates with increasing distance from the interface. Near the interface, a chiral optical Tamm state (COTS) is possible, described in [46] in the limit of low cholesteric anisotropy. This state can be represented as a superposition of two circularly polarized counter-propagating co-handed waves with their strengths rotating in time in opposite directions. The resultant polarization is linear at each point of space, and the plane of polarization uniformly rotates together with the cholesteric director as the distance from the interface increases. The amplitude exponentially drops without

standing-wave nodes and antinodes. The field structure resembles a twisted onion dome of Saint Basil's Cathedral in Moscow or Dutch Renaissance style in Copenhagen Stock Exchange dome.



**Figure 2.** HPM and a conventional mirror. The  $x$ -polarized light reflection is different. For HPM the reflected electric strength  $E_x$  preserves its phase and the phase of the magnetic strength  $H_y$  alters instead.

Fig. 2 compares HPM to a conventional mirror. In a conventional metallic mirror, the electric field, when reflected, reverses its phase whereas the magnetic field does not. A reverse situation is possible, in which case the mirror is called a magnetic mirror. HPM combines the electric and magnetic types of reflection [50]. Let  $y$  be such an axis in the HPM plane that an electric field linearly polarized along that axis changes its phase after reflection. The orthogonal axis  $x$  corresponds to magnetic reflection. It is important that the magnetic field is orthogonal to the electric one, and hence to the  $x$  axis, and is directed along  $y$ . In other words, the electric and magnetic field components directed along  $y$  are subject to a phase jump and yield a node, i.e. zero intensity. It is only  $x$ -components of the field that remain non-zero.

*Maxwell equations in the basis associated with the cholesteric director*

Here we limit ourselves to the case of normal light incidence. For  $\mu = 1$ , the Maxwell equation for a wave propagating along the helix axis  $z$  can be written as

$$\frac{\partial^2 \vec{E}}{\partial z^2} = \frac{\hat{\epsilon}}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}. \quad (1)$$

The wave is described by a vector  $\vec{E}$  of complex amplitudes for the electric field components in the orthogonal directions  $x$  and  $y$ . Projection of the dielectric permittivity tensor  $\hat{\epsilon}$  on the plane  $x, y$  at the depth of cholesteric  $z$  is given by

$$\hat{\epsilon} = \begin{bmatrix} \epsilon_e \cos^2 \tilde{\phi} + \epsilon_o \sin^2 \tilde{\phi} & \sin 2\tilde{\phi} (\epsilon_e - \epsilon_o)/2 \\ \sin 2\tilde{\phi} (\epsilon_e - \epsilon_o)/2 & \epsilon_e \sin^2 \tilde{\phi} + \epsilon_o \cos^2 \tilde{\phi} \end{bmatrix}. \quad (2)$$

Here the optical axis coinciding with the cholesteric director is given by the twist angle  $\tilde{\phi}(z) = 2\pi z/p + \phi$ , which is measured from the  $x$  axis toward the  $y$  axis;  $p$  is the helix pitch. The positive pitch refers to a right-handed helix, the negative pitch refers to a left-handed one. Taking into account magnetic permittivity  $\mu_{xy}$  makes an explicit form of the magnetic strength  $\vec{H}$  expression more preferred and increases the field vector dimensionality from 2 to 4.

$$\vec{J} = [E_x, H_y, E_y, -H_x]^T.$$

Consequently, the order of the differential equation goes down from second to first. Let us assume the principal axes of the magnetic and electric permittivity tensors to coincide. On this assumption, we can use the orthonormal basis  $u, v, z$  uniformly rotating together with the cholesteric director so that the  $u$  axis always goes along the director:

$$\vec{J}_R = [E_u, H_v, E_v, -H_u]^T.$$

By the Berreman method [53], Maxwell equations in a stationary case have the form:

$$\frac{\partial \vec{J}_R}{\partial \tilde{z}} = i\hat{T}\vec{J}_R. \quad (3)$$

A differential transfer matrix for rotating basis is formulated in [4,18] and it can be reduced as follows:

$$\hat{T} = \begin{bmatrix} 0 & \mu_o & \tilde{\lambda} & 0 \\ \epsilon_e & 0 & 0 & \tilde{\lambda} \\ \tilde{\lambda} & 0 & 0 & \mu_e \\ 0 & \tilde{\lambda} & \epsilon_o & 0 \end{bmatrix}, \quad (4)$$

where  $\tilde{\lambda} = \lambda_0/p = 2\pi c/\omega p$  is the nondimensional wavelength,  $\tilde{z} = z\omega/c = 2\pi z/\lambda_0$  is the nondimensional coordinate. There are different units for electric and magnetic strengths in the SI system, therefore they have to be normalized via the vacuum impedance  $Z_0 = E/H = \sqrt{\mu_0/\epsilon_0}$ .

#### *Solution without low anisotropy approximation*

Four normal waves correspond to Eq.(3). They are determined by the eigenvalues of the  $\hat{T}$  matrix. These eigenvalues have the sense of refractive indices  $n$  and the respective eigenvectors of the  $\hat{T}$  matrix have the sense of polarizations  $\vec{J}_0$ . Based on the  $z$ -axis reversal symmetry  $z \rightarrow -z$ , these four normal waves can be classified as two pairs of counter-directed waves. In each pair, the wave with a larger refractive index has a lower phase velocity. We refer to this wave as a slow wave. The other wave is called a fast wave:

$$\vec{J}_{R0} = \vec{J}_{s,f}^{\pm} \exp(\pm i n_{s,f} \tilde{z}). \quad (5)$$

Substituting the solution for  $\vec{J}_{s,f}^+$  into Eq. (3) yields:

$$i[\hat{T} - n_{s,f}\hat{I}]\vec{J}_{s,f} = 0, \quad (6)$$

where  $\hat{I}$  is the unit matrix, the index  $+$  in  $\vec{J}_{s,f}^+$  has been omitted. The refractive indices are as follows

$$n_{s,f}^2 = \tilde{\lambda}^2 + \overline{\epsilon\mu} \pm \sqrt{4\tilde{\epsilon}\tilde{\mu}\tilde{\lambda}^2 + d_{\epsilon\mu}^2}, \quad (7)$$

where  $d_{\epsilon\mu} = (\epsilon_e\mu_o - \epsilon_o\mu_e)/2$  is the antisymmetry coefficient of permittivities and the overbar means the arithmetic mean over ordinary and extraordinary permittivities:

$$\tilde{\epsilon} = \frac{\epsilon_e + \epsilon_o}{2}, \tilde{\mu} = \frac{\mu_o + \mu_e}{2}, \overline{\epsilon\mu} = \overline{\epsilon_o\mu_o} = \frac{\epsilon_e\mu_o + \epsilon_o\mu_e}{2}. \quad (8)$$

Scale invariance of Maxwell equations (3) and normalization of material parameters (see Supplement in [4]) reduces the structure to two crucial parameters: electric and magnetic anisotropies

$$\delta_\epsilon = \frac{\epsilon_e - \epsilon_o}{\epsilon_e + \epsilon_o}, \delta_\mu = \frac{\mu_e - \mu_o}{\mu_e + \mu_o}. \quad (9)$$

The case of equal anisotropies,  $\delta_\epsilon = \delta_\mu$

Consider the case of equal anisotropies  $\delta = \delta_\epsilon = \delta_\mu$ . The clarity and fineness of this single-parametric set of structures should compensate the difficulty of their physical realization in the optical range for the reader [26]. The wavelength parameterizes the set of differential transfer matrixes (4) the way eccentricity does to a set of conical cross-sections. The dispersion equation simplifies due to the symmetry of permittivities  $d_{\epsilon\mu} = 0$

$$n_{s,f}^2 = (\tilde{\lambda} \pm \sqrt{\bar{\epsilon}\bar{\mu}})^2 - (\bar{\epsilon}\bar{\mu} - \bar{\epsilon}\bar{\mu}), \quad (10)$$

Without further prejudice to the generality, we assume the normalization  $\bar{\mu} = \bar{\epsilon}$ ,  $\bar{\epsilon}\bar{\mu} = 1$ . Then  $\sqrt{\bar{\epsilon}\bar{\mu}} = \bar{\epsilon} \geq 1$ . In other words, the permittivity is normalized to the geometric mean of  $\epsilon_o$  and  $\epsilon_e$ :  $\bar{\epsilon}_g = \sqrt{\epsilon_o\epsilon_e} = 1$ , the arithmetic mean being not less than unity  $\bar{\epsilon} \geq 1$ . The second term in the right-hand part of the dispersion equation becomes squared anisotropy  $\bar{\epsilon}\bar{\mu} - \bar{\epsilon}\bar{\mu} = \delta^2$ . Anisotropy here acquires the meaning of the standard deviation of permittivities:

$$n_{s,f}^2 = (\tilde{\lambda} \pm \bar{\epsilon})^2 - \delta^2. \quad (11)$$

Unlike the parabolic approximation typical of periodic media, the dispersion curves have a hyperbola shape, except for the stop band where they have the shape of a circle. Dispersion equation (11) can be written for the refractive index as well as for the wave vector

$$\begin{aligned} n_{s,f}^2 &= \tilde{\lambda}^2 \pm 2\bar{\epsilon}\tilde{\lambda} + 1, \\ \tilde{k}_{s,f}^2 &= n_{s,f}^2 / \tilde{\lambda}^2 = \tilde{\omega}^2 \pm 2\bar{\epsilon}\tilde{\omega} + 1. \end{aligned} \quad (12)$$

This symmetry of  $\tilde{\lambda}(n)$  and  $\tilde{\omega}(\tilde{k}) = 1/\tilde{\lambda}$  dispersions indicates the symmetry of the longwave and shortwave limits. In the longwave limit, the medium is homogenized and becomes isotropic. The negative optical activity ceases as the situation comes close to the static field case. In the shortwave limit, positive optical activity is supported by the Mauguin waveguide regime (Fig. 3). This symmetry is destroyed when the anisotropies become unequal  $\delta_\epsilon \neq \delta_\mu$ . The ordinary and extraordinary waves can then be distinguished in the high-frequency limit equivalent to helix untwisting. Next we focus on the circular Bragg diffraction, when  $(\tilde{\lambda} - \sqrt{\bar{\epsilon}\bar{\mu}})^2 \leq \delta^2$  and the refractive index for the fast wave  $n_f$  acquires purely imaginary values. This is the case when the phase velocity becomes infinite and the group velocity – meaningless. It would be reasonable here to write down dispersion equation (11) in the form of a trigonometric identity where some angle  $\chi \in [0, \pi/2]$  acts instead of the wavelength, on the physical meaning of which we will dwell later.

$$-\delta^2 \sin^2 2\chi = \delta^2 \cos^2 2\chi - \delta^2. \quad (13)$$

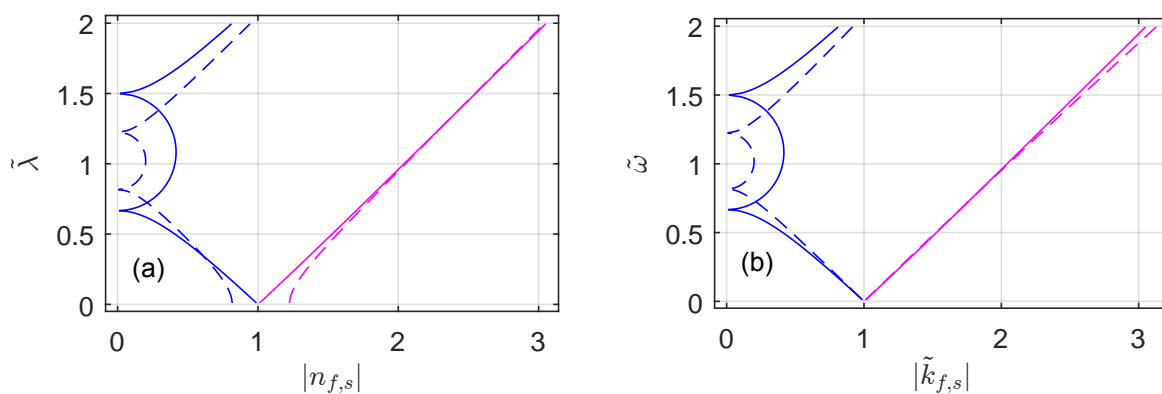
This means that at wavelength

$$\tilde{\lambda}_0 = \bar{\epsilon} + \delta \cos 2\chi, \quad (14)$$

the refractive index of the fast wave  $n_f = i\delta \sin 2\chi$  describes full reflection in the cholesteric bulk. This wave is conventionally called a diffracting wave [1] and its polarization is derived from nontrivial solvability of Eq. (11). Subtracting the refractive index from transfer matrix (6) yields the following identity:

$$[\hat{T} - n_f \hat{I}] \vec{f} = \delta \begin{bmatrix} -i \sin 2\chi & \Delta - \bar{\epsilon} & -i\Delta - i \cos 2\chi & 0 \\ \Delta + \bar{\epsilon} & -i \sin 2\chi & 0 & -i\Delta - i \cos 2\chi \\ i\Delta + i \cos 2\chi & 0 & -i \sin 2\chi & \Delta + \bar{\epsilon} \\ 0 & i\Delta + i \cos 2\chi & \Delta - \bar{\epsilon} & -i \sin 2\chi \end{bmatrix} \begin{bmatrix} \cos \chi \\ -i \sin \chi \\ -\sin \chi \\ -i \cos \chi \end{bmatrix} \equiv 0,$$

where  $\Delta = \bar{\epsilon}/\delta$ . We deal here with a manifest symmetry with respect to exchange between electric and magnetic fields and permittivities,  $E_u/E_v = (-H_v/H_u)^*$ . The electric and magnetic permittivities being equal is equivalent to the Umov-Poynting vector going to zero the same way as for conventional standing-wave nodes and antinodes. The electric and magnetic fields are linearly polarized at the same angle  $-\chi$  to the cholesteric director. This direction coincides with the  $x$  axis for  $\chi = \phi = \tilde{\phi}(z = 0)$ . Surface conditions for an ideal HPM are exactly the same, i.e. both the electric field and the magnetic field are linearly polarized along the magnetic axis of the mirror (Fig. 2). Hence the derived expression provides an exact description of COTS for a finite anisotropy.



**Figure 3.** Symmetry of dispersion curves. (a) Wavelength  $\tilde{\lambda}$  as a function of the refractive index  $|n_{f,s}|$ . (b) Frequency  $\tilde{\omega} = 1/\tilde{\lambda}$  as a function of the wave number  $|\tilde{k}_{f,s}|$ . The blue curve is the fast wave, the purple curve – slow wave. The solid curve is drawn for  $\epsilon_e = \mu_e = 3/2, \epsilon_o = \mu_o = 2/3, \delta_e = \delta_\mu$ . The semicircle refers to a diffracting wave for which the refractive index acquires purely imaginary values  $|n_f| = \text{Im}(n_f)$ . The Mauguin regime,  $\tilde{\lambda} \ll 1$ , is equivalent to homogenization  $\omega \ll 2\pi$  according to dispersion equations (4). The dashed line is  $\epsilon_o = 2/3, \epsilon_e = 3/2, \mu_e = \mu_o = 1$ . The symmetry is violated

*The case of unequal anisotropies,  $\delta_e \neq \delta_\mu$*

In a general case, we denote directions of electric and magnetic polarizations by the angles  $\chi_E$  and  $\chi_H$ . Generalized nontrivial solution (12) provides eigenvectors of the transfer matrix (6) and exact solutions for the angles [19]:

$$\begin{aligned} \tan \chi_E &= \frac{E_u}{E_v} = -\tilde{\lambda}_0 n_f \frac{\mu_e + \mu_o}{\epsilon_e \mu_e \mu_o - n_f^2 \mu_o - \tilde{\lambda}_0^2 \mu_e}, \\ -\cot \chi_H &= \frac{H_v}{-H_u} = -\tilde{\lambda}_0 n_f \frac{\epsilon_e + \epsilon_o}{\mu_e \epsilon_e \epsilon_o - n_f^2 \epsilon_o - \tilde{\lambda}_0^2 \epsilon_e}, \\ \psi &= \chi_E - \chi_H \neq 0, \\ \tilde{\chi} &= (\chi_E + \chi_H)/2, \end{aligned} \quad (15)$$

where the refractive index  $n_f$  should be borrowed from dispersion equation (4). It is evident that the polarizations are linear for the purely imaginary refractive index  $n_f$  whereas for the real refractive index  $n_s$  they are almost circular in the stop band. Dispersion equation (7) can be generalized as:

$$\tilde{\lambda}_0 \approx \sqrt{\bar{\epsilon}\bar{\mu}} + \sqrt{\bar{\epsilon}\bar{\mu} - \bar{\epsilon}\bar{\mu}} \cos 2\tilde{\chi}. \quad (16)$$



For a cholesteric without magnetic anisotropy,  $\mu = 1$ , we have:

$$\tilde{\lambda}_0 \approx \sqrt{\bar{\epsilon}} + \sqrt{\bar{\epsilon} - 1} \cos 2\tilde{\chi}. \quad (17)$$

The eigenfrequency is:

$$\tilde{\omega}_0 \approx \frac{1}{\sqrt{\bar{\epsilon}} + \sqrt{\bar{\epsilon} - 1} \cos 2\tilde{\chi}}. \quad (18)$$

This expression is more exact compared to the earlier obtained approximation (Eq.(14) in [46]) and still there remains approximation. We have to use a unified angle  $\tilde{\chi}$  between the cholesteric optical axes and HPM hence the electric and magnetic polarizations are directed differently in a general case. This does not meet the condition on the HPM surface. The non-zero angle of polarization mismatch  $\psi$  introduces a new non-local COTS component [19]. This results in the state becoming a leaking mode (or resonance) and acquiring finite relaxation time  $\tau_\psi$ .

#### COTS relaxation time

By definition, the time of relaxation of a vibrational state is the ratio between the stored energy,  $\mathcal{E}$ , and the lost power,  $P$ , taken with a positive sign:

$$\frac{\tau}{2} = \frac{\mathcal{E}}{P} = \frac{\text{Energy stored}}{\text{Power of leakage}}. \quad (19)$$

Here  $\tau$  is the amplitude relaxation time, which is twice the energy relaxation time. Since field does not penetrate into depth of an ideal HPM, the whole stored energy is the energy of diffracting waves inside the cholesteric layer. Let us find this energy by integrating its density over space  $W = [\vec{E}\vec{D} + \vec{H}\vec{B}]/8\pi$ , in Gaussian units. The period-averaged densities of electric and magnetic energy components are equal because  $\vec{E}\vec{D} = \vec{H}\vec{B}$ . Moreover  $|\vec{E}^2| = |E_0^2|/2$ , where  $E_0$  is the wave amplitude. Therefore,  $\bar{W} = \bar{\epsilon}|E_0^2|/8\pi$ . A standing wave in the layer is formed by two waves travelling in opposite directions. Their constructive interference is compensated by a destructive one, and their energy densities add up. In the defect layer,  $\bar{W}_0 = \bar{\epsilon}|E_0^2|/4\pi$ . Integration yields

$$\mathcal{E} = \int_0^\infty \bar{W}_0 \exp(2in_f \cdot 2\pi z/\lambda_0) dz = \frac{\bar{W}_0 \lambda_0}{4\pi |n_f|}. \quad (20)$$

With account of Eq. (7,17) for  $\delta_\mu = 0$  and  $\bar{\epsilon} \approx 1$ ,

$$|n_f| = (\delta_\epsilon/2) \sin 2\tilde{\chi}, \quad \lambda_f = \lambda_0/|n_f| \quad (21)$$

Here  $\lambda_0 = \tilde{\lambda}_0 p$  is given by (16). The power of leakage is proportional to the flow velocity  $c/\sqrt{\bar{\epsilon}}$  and the energy density  $\bar{W}_\psi = \bar{\epsilon}|E_\psi^2|/8\pi$  carried away by the travelling wave. The strength is governed by the boundary conditions:

$$E_\psi = 2E_0 \sin(\psi/2). \quad (22)$$

We finally have [19]

$$\tau_\psi = \frac{\lambda_f}{4\pi c} \frac{1}{\sin^2(\psi/2)}. \quad (23)$$

Consider a cholesteric layer of finite thickness  $L$  embedded in a medium with permittivity  $\bar{\epsilon}_g = \sqrt{\epsilon_o \epsilon_\epsilon}$ . Instead of (22), the power of leakage at the edge of the cholesteric is given by:

$$E_L = 2E_0 \exp\left(-\frac{2\pi L}{\lambda_f}\right). \quad (24)$$



Note that because of cholesteric boundary condition  $E_L$  is twice higher than just exponentially decreasing by Eq. (5). Then the corresponding relaxation time is

$$\tau_L = \frac{\lambda_f}{4\pi c} \exp\left(\frac{4\pi L}{\lambda_f}\right), \quad (25)$$

which agrees with the expressions obtained in [10,19]. The last formula is also directly applicable when  $\bar{\epsilon}_g \neq 1$ , assuming  $L$  being optical density  $L = L_0 \sqrt{\bar{\epsilon}_g}$

#### Temporal coupled-mode theory

By the temporal coupled-mode theory [61,74] the resonance is described by the cyclic eigenfrequency  $\omega_0$  and complex amplitude  $A$ . This theory should not be confused with the spatial coupled-wave theory where the amplitudes of propagating waves are involved [66,73]. The resonance manifests itself through the amplitudes  $s_{\ell\pm}$  of incoming and outgoing energy fluxes:

$$\begin{aligned} \frac{dA}{dt} &= -i\omega_0 A - \sum_{\ell=1}^N \frac{A}{\tau_\ell} + \sum_{\ell=1}^N \sqrt{\frac{2}{\tau_\ell}} s_{\ell+}, \\ s_{\ell-} &= -s_{\ell+} + \sqrt{\frac{2}{\tau_\ell}} A. \end{aligned} \quad (26)$$

Excitation via one of the ports  $s_{\ell+} = s_0 \exp(-i\omega t)$  yields the amplitude

$$A_\ell(\omega) = \frac{\sqrt{\frac{2}{\tau_\ell}}}{i(\omega - \omega_0) + \sum_{\ell=1}^N \frac{1}{\tau_\ell}} s_{\ell+}. \quad (27)$$

Amplitudes of reflection from port  $\ell$  into port  $\ell'$  form a scattering matrix

$$r_{\ell\ell'} = \frac{s_{\ell'-}}{s_{\ell+}} = -\hat{\delta}_{\ell\ell'} + \frac{\sqrt{\frac{2}{\tau_\ell}} \sqrt{\frac{2}{\tau_{\ell'}}}}{i(\omega - \omega_0) + \sum_{\ell''=1}^N \frac{1}{\tau_{\ell''}}}, \quad (28)$$

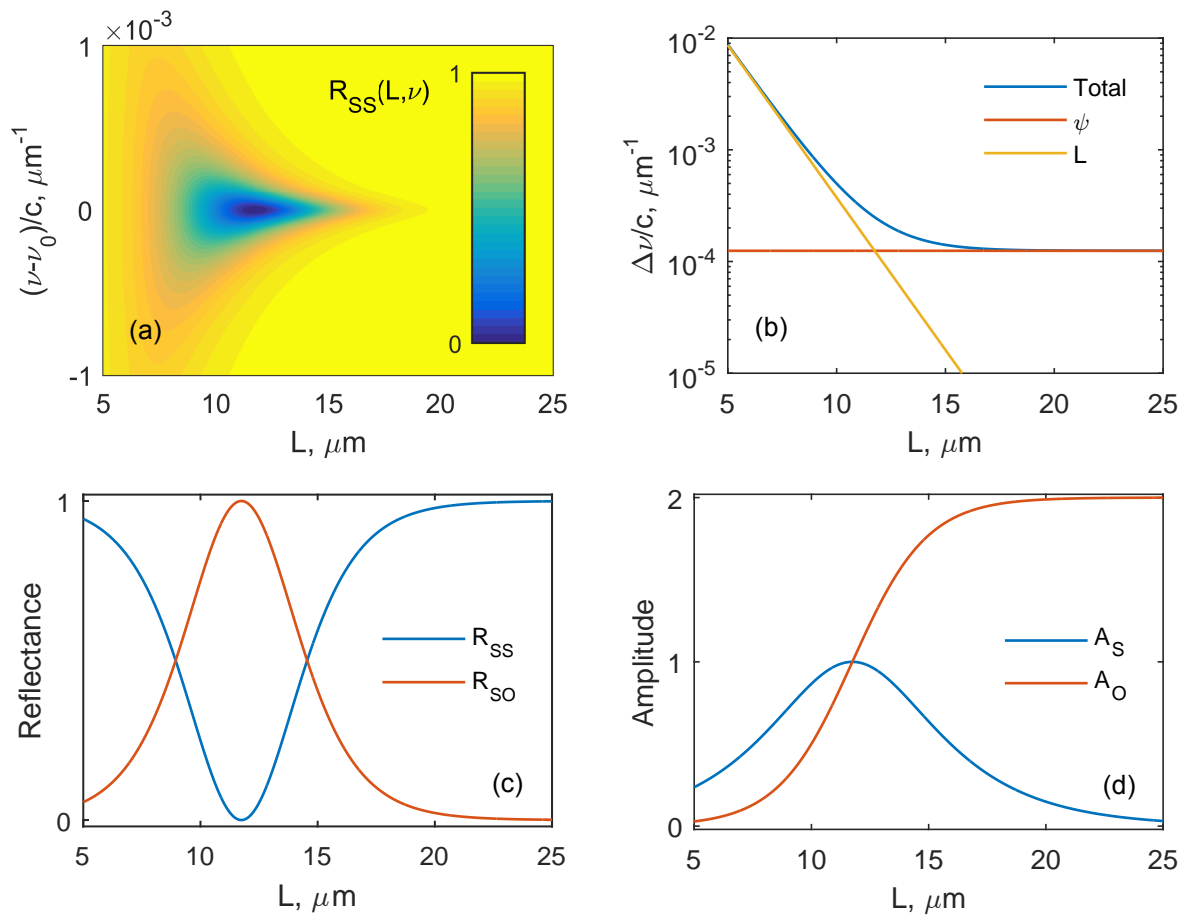
where  $\hat{\delta}_{\ell\ell'}$  is the Kronecker symbol. Reflections are observed as spectral peaks in the shape of Lorentz profiles with full width at half maximum <FWHM>

$$2\gamma = 2 \sum_{\ell=1}^N \frac{1}{\tau_\ell}. \quad (29)$$

Scaling from the cyclic frequency  $\omega = 2\pi\nu$  to the frequency  $\nu$  yields a  $2\pi$  times narrower width of the peak (29):

$$\Delta\nu = \frac{\gamma}{\pi} = \frac{1}{\pi} \sum_{\ell=1}^N \frac{1}{\tau_\ell}. \quad (30)$$

This is sufficient to allow the spectral behavior of the state to be described in terms of temporal coupled-mode theory.



**Figure 4.** The Kopp–Genack effect. The linewidth saturates with increasing cholesteric layer thickness. Polarization reversal of the optimal exciting light. (a) Reflection spectrum, Eq. (28). (b) Spectral dip width, Eq. (30). The cholesteric helix pitch is  $p = 1\mu\text{m}$ , electric anisotropy is  $\delta_\epsilon = 0.1$ ; there is no magnetic anisotropy,  $\delta_\mu = 0$ . (c) Reflection at the resonance frequency  $\omega = \omega_0$ , Eq. (31). (d) State amplitude at the resonance frequency  $\omega = \omega_0$ , Eq. (27).

In the previous section we dwelt in detail on ideal COTS with its eigenfrequency given by (16). Relaxation times (23) and (25) were used to express deviations from the ideal state associated, respectively, with the angle  $\psi$  and the layer thickness  $L$ . This appears to be sufficient to fully describe COTS in terms of TCMT. The port of coupling via angle  $\psi$  has the cross-handed circular polarization with respect to the cholesteric twist while coupling via the cholesteric thickness  $L$  has the co-handed circular polarization. Equation (28) at the resonance frequency  $\omega = \omega_0$  generates the following reflection matrix:

$$\hat{R} = \begin{bmatrix} r_{SS}^2 & r_{SO}^2 \\ r_{OS}^2 & r_{OO}^2 \end{bmatrix} = \begin{bmatrix} \cos^2 2\theta & \sin^2 2\theta \\ \sin^2 2\theta & \cos^2 2\theta \end{bmatrix}, \quad (31)$$

where  $\tan \theta = \tau_\psi / \tau_L$ , indices  $S$  and  $O$  stand for the same and the opposite circular polarizations, respectively. This matrix satisfies the energy conservation law. Maximum nondiagonal reflections occur at crossover when the relaxation times become equal  $\theta = \pi/4$ .

An analytical expression for the length of crossover  $L_c$  is obtained by equalizing relaxation times (23) and (25), which yields:

$$\exp\left(\frac{4\pi L_c}{\lambda_f}\right) = \frac{1}{\sin^2(\psi/2)}, \quad (32)$$

$$L_c = \frac{\lambda_f}{4\pi} |\log \sin^2(\psi/2)|. \quad (33)$$

This length, as is fairly noted in [25], is difficult to measure in the twist defect of a cholesteric because of the high  $\tau_\psi$  and the high requirements to the experiment accuracy. In the case of COTS, the  $\tau_\psi$  magnitude appears to be substantially larger because of the HPM imperfectness and the crossover length reduces, which should simplify its experimental measuring.

### TCMT applicability limits

When using TCMT to tackle the problem of coupling between a localized state and waveguides, there are certain limits imposed such as linearity, time-invariant structural parameters, energy conservation, time-reversible energy flux and weak coupling [61]. The model chosen a priori meets the first four requirements. The fifth one incorporates, in fact, two requirements. First, the solution derived from (26) ignores corrections with respect to a small parameter that is defined as a vibration period-to-relaxation time ratio [59]. Second, the waveguide dispersion should not be large over the frequency range of the resonance Lorentz profile. For a fairly large relaxation time the Lorentz profile width tends to zero and this automatically takes care of the second requirement [61]. In this case, the cholesteric acts as a waveguide for waves with cross-handed circular polarization. Dispersion in the middle of the stop band is moderate. So, we believe the set of linear differential equations (26) provides an adequate description of the model when  $L \geq 5p$  and  $\delta_\epsilon = 0.1$ .

### Results

Fig. 4 shows that there is a Kopp–Genack effect observed for COTS. For definiteness, we took the helix pitch to be  $p = 1\mu\text{m}$ . For the parameters specified in the figure, the eigenfrequency  $\nu_0/c = 1\mu\text{m}^{-1}$  is achieved at the angle  $\phi = \tilde{\chi} \approx 45.7^\circ$ , the mismatch angle being  $\psi \approx 2.9^\circ$ . The crossover length  $L_c \approx 11.75\mu\text{m}$  agrees with Eq. (33). For substantially thicker cholesteric layers,  $L \gg L_c$ , the line width is  $\Delta\nu_\infty/c \approx 1.25 \cdot 10^{-4}\mu\text{m}^{-1}$  and the quality factor saturates  $Q_\infty = \nu_0/\Delta\nu_\infty \approx 8000$ . Note that for crossover,  $L = L_c$ , the quality is half that magnitude. Fig. 4a illustrates that for crossover  $L = L_c$ , the structure reflects light as a conventional mirror, i.e. the circular polarization handedness changes for the opposite one. The growing thickness of the cholesteric layer  $L \gg L_c$  takes its polarization properties back to the HPM properties when  $L = 0$ , i.e. the circular polarization handedness no longer changes under reflection. Fig. 4b proves that the co-handed and cross-handed (right and left) circular polarizations excite COTS equally efficiently when  $L = L_c$  (crossover). Hence, the most efficient excitation occurs when both polarizations with equal amplitudes and a certain phase difference are superimposed to produce linearly polarized light. No COTS is excited by orthogonally polarized light. For thicker cholesterics,  $L \gg L_c$ , excitation is only possible with cross-handed circularly polarized light as co-handed circularly polarized light is totally reflected from the cholesteric.

### Conclusion

The chiral optical Tamm state can be considered strictly localized only provided the electric and magnetic permittivities are equal and the respective tensor axes coincide. Otherwise COTS is observed as a polarized reflection resonance with two relaxation constants determined by the permittivity difference and the cholesteric layer thickness. COTS is closely connected with the twist defect of cholesteric and renders the Kopp–Genack crossover effect. A scattering matrix, where spectral peaks are described by Lorentz profiles, has been found in terms of the temporal coupled-mode theory. It is for the first time that a formula for the cholesteric crossover thickness has been suggested, which is equally suitable for COTS and for the twist defect. The analytical result agrees with the direct numerical one.

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## Abbreviations

The following abbreviations are used in this manuscript:

COTS    chiral optical Tamm state  
TCMT    temporal coupled-mode theory  
HPM    handedness-preserving mirror

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