

Article

# Risk Assessment for Distribution Systems Using a Developed PEM-Based Method Considering Wind and Photovoltaic Power Distribution

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**Abstract:** the intermittency and variability of these permeated Distributed Generators (DGs) could critically cause many security and economy risks to distribution systems. This paper applied a certain mathematical distribution to imitate the output variability and uncertainty of DGs. And then, four risk indices EENS, PLC, EFLC and SI were established to reflect the system risk level of distribution system. For the certain mathematical distribution of the DGs' output power, a developed PEM (point estimate method)-based method was proposed to calculate these four system risk indices. In this developed PEM-based method, enumeration method was used to list the states of distribution systems, an improved PEM was presented to deal with the uncertainties of DGs and the value of load curtailment in distribution systems was calculated by an optimal power flow algorithm. Finally, the effectiveness and advantages of this proposed PEM-based method for distribution system assessment were verified by the tests of a modified IEEE 30-bus system. Simulation results have shown that this proposed PEM-based method has a high computational accuracy and highly reduced computational costs compared with other risk assessment methods and is very effective for risk assessments.

**Keywords:** distributed generators; risk assessment; distribution systems; developed PEM-based method; optimal power flow algorithm

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## 1. Introduction

The extensive penetration of renewable-type DGs (e.g., wind and PV) in distribution networks could bring many benefits to the grid as they are alternative to conventional generators [1-2]. However, the randomness of these DGs could critically cause some risks to distribution systems on security and economy aspects, such as power quality and stability, fault level and the value of load curtailment, which impose challenges when planning distribution systems [3-4]. Therefore, it is becoming increasingly important to assess these risks associated with the variability and uncertainty of DGs.

In the past decade, many researches concentrated on how to assess the impacts of random DGs on distribution systems. Many assessment risk indices have been established in [5-8]. In [5], risk indices of limit violation, load curtailments and socio-economic losses due to contingencies were used to assess power system operation planning risk with high penetration of renewable-type DGs. In [6-7], transmission line overload risk was established to assess the risk of wind-integrated power systems. In [8], voltage-limit violation risk was used to handle the two-fold uncertainty combining randomness and fuzziness in power system operations. These papers aimed to establish the assessment risk indices by the product of probability and severity but ignored many significant aspects such as the value of load curtailment.

Many other risk indices have also been presented. In [9], three risk indices including LOLP (loss of load probability), EENS (expected energy not supplied) and ECOST (expected customer interruption cost) were used for reliability and price risk assessment. In [10-11], the indices of SAIFI (system average interruption frequency index), SAIDI (system average interruption duration index)

and EENS were used to assess the reliability of active distribution network. In [12], EENS, ENLC (expected number of load curtailment) and ADLC (average duration of load curtailment) were used for power system reliability assessment. In [13-15], EENS, PLC (probability of load curtailment), EFLC (expected frequency of load curtailment) and SI (severity index) were presented as risk indices to assess the risk level of distribution system. In these papers, more considerations were concentrated on the operation risk level. In need to say, EENS, PLC, EFLC and SI are the most important risk indices to reflect the high penetration of DGs in distribution system.

The output uncertainties of DGs can be dealt with by many uncertainty modeling methods including probabilistic methods and other uncertainty methods [16]. Among the other uncertainty methods, IGDT (information gap decision theory), robust optimization and hybrid possibilistic-probabilistic approach were respectively introduced in [17-19]. In [17], IGDT was applied to choose the supplying resources for meeting the demand of customers. In [18], robust optimization was used to meet the challenges of supply and demand uncertainty. In [19], a hybrid possibilistic-probabilistic approach was introduced to deal with the uncertainty of random variability and imprecision. However, the power output of wind or photovoltaic generating units generally subjects to a certain mathematical distribution [20-21]. Namely, the randomness of wind and PV can be imitated by some mathematical formulas. Therefore, these other uncertainty methods as mentioned are not applicable for the risk assessment of distribution system with the penetration of probabilistic DGs. For probabilistic DGs, probabilistic methods including Monte Carlo method, scenario based decision making method and point estimate method can be suitably applied. But, Monte Carlo method which was introduced in [22-23] and the scenario based decision making method introduced in [24] are all computationally harder. Compared to the other probabilistic methods, point estimate method (PEM) in [25-27] has highly reduced computational costs and is extremely applicable for the probabilistic uncertainties of DGs. However, the accuracy of PEM is greatly low. Therefore, the traditional PEM should be improved for application.

In this paper, in order to reasonably assess the risks of distribution systems with the penetration of DGs, four risk indices EENS, PLC, EFLC and SI were applied to reflect the system risk level of distribution systems. And then, for the certain mathematical distribution of the DGs' output power, a developed PEM-based method was proposed to calculate these four system risk indices. In this developed PEM-based method, enumeration method was used to list the states of distribution system; an improved PEM was proposed to deal with the uncertainties of DGs in distribution system.

Finally, the effectiveness and advantage of this proposed PEM-based method for power system assessment were verified by the tests of a modified IEEE 30-bus system, which have shown that this proposed PEM-based method is very effective for risk assessments in distribution systems and has a high computational accuracy and largely reduced computational costs compared with other risk assessment methods. Simulation results also demonstrate that DGs' total generation capacity, DGs' type, DGs' location, DGs' dispersion, and DGs' capacity proportion have great influences on the system risk indices.

## 2. Distribution of Wind and photovoltaic DGs

In distribution systems, the output of DGs contributes to whether the load can be supplied when malfunction occurs. Therefore, the output randomness of DGs should be imitated by mathematical formulas. In this paper, wind generators and photovoltaic generators are only considered as DGs. It should be explained that other types of DGs could also be applied by this proposed PEM-based risk assessment method in this paper.

### 2.1. Output Uncertainty of Wind Generators

Large amounts of researches in the past decades have demonstrated that wind speed  $v$  is the main stochastic factor that determines the output power of wind generators. Generally, a Weibull distribution can be used to imitate the stochastic wind speed  $v$  [6]. The probability density function (PDF) for wind speed  $v$  is described as (1):

$$f_v(v) = \frac{k}{c} \left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^k\right] \quad (1)$$

Where,  $k$  and  $c$  are respectively the shape parameter and the scale parameter of the wind speed distribution. According to the historical data of wind speed  $v$ , these two indices can be approximately estimated.

Based on the PDF of wind speed  $v$ , the output of wind turbine generator can be acquired [28]:

$$P_w = \begin{cases} 0, & v < v_{in} \\ P_N \frac{v - v_{in}}{v_N - v_{in}}, & v_{in} \leq v < v_N \\ P_N, & v_N \leq v < v_{out} \\ 0, & v \geq v_{out} \end{cases} \quad (2)$$

Where  $P_w$  is the active output power,  $P_N$  is the rated output power of wind turbine generator.  $v_N$  is rated wind speed,  $v_{in}$  is cut-in wind speed and  $v_{out}$  is cut-out wind speed.

## 2.2. Output Uncertainty of photovoltaic Generators

In most occasions, illumination intensity is thought as the major factor that affects the active output of photovoltaic generators. Because of cloud cover and other insolation reducing phenomena, the illumination intensity  $I$  can also be represented by a random variable. In general, illumination intensity  $I$  approximately follows a Beta distribution with shape indexes  $\alpha$  and  $\beta$  [29]:

$$f(I) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \cdot \left(\frac{I}{I_{max}}\right)^{\alpha-1} \cdot \left(1 - \frac{I}{I_{max}}\right)^{\beta-1} \quad (3)$$

Where,  $I_{max}$  is the maximum intensity during a certain interval. The two shape indexes  $\alpha$  and  $\beta$  can be evaluated by the mean value and the variance of illumination intensity.

Many studies have showed that the active output of photovoltaic generators could be described as:

$$P_v = A \cdot \eta \cdot I \quad (4)$$

Where,  $A$  is photoelectric array area,  $\eta$  is the photoelectric transformation efficiency. Combine Equation (3) and (4), the PDF of photovoltaic generators' output power  $P_v$  can be acquired:

$$f(P_v) = \frac{\Gamma(\alpha + \beta)}{A \cdot \eta \cdot \Gamma(\alpha)\Gamma(\beta)} \cdot \left(\frac{P_v}{P_v^{max}}\right)^{\alpha-1} \cdot \left(1 - \frac{P_v}{P_v^{max}}\right)^{\beta-1} \quad (5)$$

Where,  $P_v^{max}$  is the maximum value of  $P_v$ , which can be calculated by  $P_v^{max} = A \cdot \eta \cdot I_{max}$ . According to (5),  $P_v$  also follows Beta distribution with shape indexes  $\alpha$  and  $\beta$ .

## 3. Risk Assessment Indices and method

### 3.1. Risk Assessment Indices

In order to do risk assessment for distribution systems, risk assessment indices should be primarily established. In [6], the overload risk index which is considered as the product of the probability and the severity  $Se(Z_l)$  of the transmission line overload was used to assessing line overload risk of wind-integrated power systems. In [10-11], three common indices SAIFI, SAIDI and EENS were used to assess the reliability of active distribution network. It should be noted that these indices can be suitably used as risk assessment indices in distribution systems, but other risk indices such as EENS, PLC, EFLC and SI in [13-15] are more considered with the penetration of DGs. Therefore, in this paper, these four risk indices were used to reflect the system risk level of

distribution systems. The calculation methods of these four risk indices are illustrated as the following:

1) EENS (unit: MWh/y) represents the energy that not supplied with the penetration of DGs in distribution system, which can be computed by (6):

$$EENS = \sum_{i=1}^{N_L} \left( \sum_{s \in Q_i} p_T(s) \cdot C_0(s) \right) \cdot T_i \quad (6)$$

Where,  $i$  is the load level,  $N_L$  is the set for load levels.  $T_i$  is the duration of  $i$ , and  $Q_i$  is the set for system state  $s$  at load level  $i$ .  $p_T(s)$  and  $C_0(s)$  are respectively the occurrence probability and total load curtailment of system state  $s$ .

2) PLC can be calculated by:

$$PLC = \sum_{i=1}^{N_L} \left( \sum_{s \in Q_i} p_T(s) \right) \cdot \frac{T_i}{T} \quad (7)$$

3) EFLC (unit: Times/y) can be calculated by:

$$EFLC = \sum_{i=1}^{N_L} \sum_{s \in Q_i} \left( p_T(s) \sum_{j=1}^{m(s)} \theta_j \right) \cdot \frac{T_i}{T} \quad (8)$$

Where,  $T$  is total duration of  $T_i$ ,  $m(s)$  is the set of component  $j$  at system states. For component  $j$ ,  $\theta_j$  represents repair or outage rate.

4) SI (unit: min/y) represents the equivalent duration under an entire system outage of peak conditions, which can be calculated:

$$SI = \frac{EENS \times 60}{L} \quad (9)$$

Where,  $L$  is the value of peak load in one year.

In these four risk indices, SI is much related to EENS. In order to calculate the index of EENS, the total load curtailment at state  $s$   $C_0(s)$  should be previously calculated, which can be calculated by optimal power flow (OPF) introduced in Part C in this Section. For all of these four risk indices, the occurrence probability of system state  $s$   $p_T(s)$  is needed. Therefore, enumeration method is applied for system state selection, which is introduced in 4.2.

### 3.2. Improved Point estimate methods

Point estimate methods (PEM), which was firstly proposed by Hong H. P. in 1998, focuses on the statistical information which is provided by the first few central moments of the input random variables. For each variable,  $K$  points are used and  $K$  is a parameter named concentrations, which depends on the Hong's method. These points and the function which relates input and output variables are used to obtain the information about the uncertainty associated with problem output random variables.

This paper carries on  $2 \times m$  and  $2 \times m + 1$  type scheme of PEM for computing the four risk indices in Part A in this Section, which gives a good tradeoff between the solution accuracy and the computational efforts.  $m$  denotes the number of random variables, which represent wind speed or illumination intensity  $I$ .  $2m$  type scheme of PEM uses the first two concentrations for each input random variable, i.e.,  $K=2$ . For  $2m+1$  type scheme of PEM,  $K=3$ .

Generally speaking, PEM is used to calculate the estimation value  $E(Z)$  of  $F(X)$  ( $Z=F(X)=F(X_1, X_2, \dots, X_m)$ ) when the stochastic numerical characteristics of variables  $X_1, X_2, \dots, X_m$  are known [26-27]. The point estimation principle can be depicted as:

$$E(Z^j) \approx \sum_{l=1}^m \sum_{k=1}^K \omega_{l,k} F^j(\mu_{X_1}, \dots, x_{l,k}, \dots, \mu_{X_m}) \quad (10)$$

Where,  $m$  is the number of DGs,  $\mu_{X_i}$ ,  $\xi_{i,k}$  are respectively the expectation and standard location of variable  $X_i$ , which represents illumination intensity  $I$  or wind speed  $v$  in this paper.  $\omega_{i,k}$  represents the weight of  $X_i$  at  $x_{i,k}$ , which can be calculated by (11):

$$x_{i,k} = \mu_{X_i} + \xi_{i,k} \sigma_{X_i}, i=1, 2, \dots, m; k=1, 2, \dots, 2K-1 \quad (11)$$

Where,  $\sigma_{X_i}$  represents the variance of  $X_i$ . Standard location  $\xi_{i,k}$  and weight  $\omega_{i,k}$  could be determined by (12):

$$\left\{ \begin{array}{l} \sum_{k=1}^K \omega_{i,k} \cdot \xi_{i,k}^j = \lambda_{i,j}, \quad j=1, \dots, 2K-1 \\ \lambda_{i,j} = E[(x_i - \mu_{X_i})^j] / \sigma_{X_i}^j \\ \sum_{k=1}^K \omega_{i,k} = \alpha_i \end{array} \right. \quad (12)$$

In traditional PEM,  $\sum_{k=1}^K \omega_{i,k}$  is set as  $\frac{1}{m}$  which means the impacts of  $X_i$  on  $Z$  is the same. This assumption can simplify the calculation but may cause big computational error as the different impacts of  $X_i$  on  $Z$ . In (12),  $\sum_{k=1}^K \omega_{i,k}$  is set as  $\alpha_i$  which can be determined by Analytic Hierarchy Process (AHP) or the contribution of  $X_i$  on  $Z$ . This equation appears to be more reasonable as it considered the different impacts of  $X_i$  on  $Z$ .

When  $K=2$  and  $\xi_{i,3}=0$ , Equations of (12) can be solved:

$$\left\{ \begin{array}{l} \xi_{i,1} = \frac{\lambda_{i,3}}{2} + \sqrt{\frac{1}{\alpha_i} + \left(\frac{\lambda_{i,3}}{2}\right)^2}, \xi_{i,2} = \frac{\lambda_{i,3}}{2} - \sqrt{\frac{1}{\alpha_i} + \left(\frac{\lambda_{i,3}}{2}\right)^2} \\ \omega_{i,1} = -\frac{\alpha_i \cdot \xi_{i,2}}{\xi_{i,1} - \xi_{i,2}}, \omega_{i,2} = \frac{\alpha_i \cdot \xi_{i,1}}{\xi_{i,1} - \xi_{i,2}} \end{array} \right. \quad (13)$$

The calculation results based on (13) is named 2m improved PEM. When  $K=3$  and  $\xi_{i,3}=0$ , 2m+1 improved PEM can be constructed and the solving results of Equations (12) is:

$$\left\{ \begin{array}{l} \xi_{i,1} = \frac{\lambda_{i,3}}{2} + \sqrt{\lambda_{i,4} - \frac{3}{4}\lambda_{i,3}^2}, \xi_{i,2} = \frac{\lambda_{i,3}}{2} - \sqrt{\lambda_{i,4} - \frac{3}{4}\lambda_{i,3}^2} \\ \omega_{i,1} = \frac{1}{\xi_{i,1}(\xi_{i,1} - \xi_{i,2})}, \omega_{i,2} = -\frac{1}{\xi_{i,2}(\xi_{i,1} - \xi_{i,2})} \\ \omega_{i,3} = \alpha_i - \frac{1}{\lambda_{i,4} - \lambda_{i,3}^2} \end{array} \right. \quad (14)$$

As Analytic Hierarchy Process (AHP) is a rough estimation of weights and may be complex in operation. Therefore, the contribution of  $X_i$  on  $Z$  which can be approximated by the capacity proportion of DG  $i$  was used to calculate the value of  $\alpha_i$ .

According to Equation (10)-(14), the  $j$ -th raw moment of the output random vector  $Z$  can be acquired. And then, the mean and the standard deviation of  $Z$  can be estimated by Equation (15):

$$\left\{ \begin{array}{l} \mu_Z = E(Z) \approx \sum_{i=1}^m \sum_{k=1}^K \omega_{i,k} F(\mu_{X_1}, \dots, x_{i,k}, \dots, \mu_{X_m}) \\ \sigma_Z = \sqrt{E(Z^2) - \mu_Z^2} \end{array} \right. \quad (15)$$

For each risk assessment index, the mean and the standard deviation can be respectively acquired by Equation (15). For a variable with a certain mathematical distribution, the mean is the

most probable value of the variable. Therefore, the mean of  $Z$   $\mu_z$  is used as the estimation value of each risk index in this paper.

### 3.3. Optimal power flow algorithm in Distribution Systems

In distribution systems, load curtailment and generation dispatch are extensively applied to enable the system from urgent state to normal state. In order to obtain the system security indices in distribution systems, an optimal power flow algorithm (OPF) was present in this paper to assess the total load curtailment  $C_0(s)$  in state  $s$ . The following Equations of (16) and (17) were used as this optimal power flow algorithm to simulate the curtailment and dispatch work and ensure the security of distribution system [30]:

$$\min f(\Delta P_{G1}, \dots, \Delta P_{Gm}, \Delta P_{L1}, \dots, \Delta P_{Ll}) = \sum_{k=1}^m C_{Gk} \Delta P_{Gk} + \sum_{h=1}^l C_{Lh} \Delta P_{Lh} \quad (16)$$

$$\begin{aligned} s.t. \quad & \sum_{k=1}^m \frac{\partial I_{ij}}{\partial P_{Gk}} \Delta P_{Gk} + \sum_{h=1}^l \frac{\partial I_{ij}}{\partial P_{Lh}} \Delta P_{Lh} \leq I_{ij}^{\max} - I_{ij}^0 \\ & \sum_{k=1}^m \frac{\partial P_s}{\partial P_{Gk}} \Delta P_{Gk} + \sum_{h=1}^l \frac{\partial P_s}{\partial P_{Lh}} \Delta P_{Lh} = 0 \\ & P_{Gk}^{\min} - P_{Gk}^0 \leq \Delta P_{Gk} \leq P_{Gk}^{\max} - P_{Gk}^0 \\ & 0 \leq \Delta P_{Lh} \leq P_{Lh}^0 \\ & 1 \leq k \leq m, 1 \leq h \leq l, 1 \leq i \leq N_b, 1 \leq j \leq N_b \end{aligned} \quad (17)$$

Where  $l$  is the number of load buses in distribution systems.  $\frac{\partial z}{\partial x}$  ( $z = I_{ij}, P_s; x = P_{Gk}, P_{Lh}$ ) are sensitivity coefficients.  $N_b$  is the number of buses,  $\Delta P_{Gk}$  and  $\Delta P_{Lh}$  are respectively the power variations of generator  $k$  and load  $h$ ;  $C_{Gk}$  and  $C_{Lh}$  are respectively the control cost of generator  $k$  and load  $h$ ;  $I_{ij}$  is the current through overload line  $ij$ .

According to the results of this optimal power flow algorithm in distribution systems, the total load curtailment in state  $s$ ,  $C_0(s)$ , could be calculated as Equation (18):

$$C_0(s) = \sum_{h=1}^l \Delta P_{Lh} + \Delta P_d \quad (18)$$

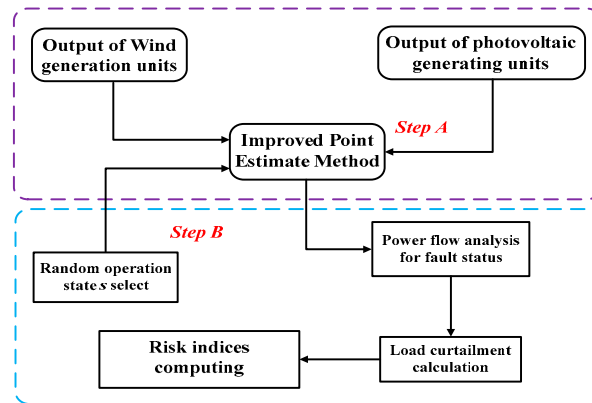
Where,  $\Delta P_d$  is the value of load curtailment caused by direct structure change of distribution system. In line with the value of  $C_0(s)$  and  $p_T(s)$ , the risk assessment indices in Equation (6)-(9) can be calculated.

## 4. Risk Assessment Procedure for Distribution systems

### 4.1. Structure for Risk assessment

To assess the four risk indices in distribution systems, PEM was used and the uncertainties of distribution generators are sufficiently considered. The flowchart of the developed PEM-based method for risk assessment of distribution systems is shown in Fig 1.

In step *A*, active output of photovoltaic generators and wind generators are calculated according to the stochastic wind speed  $v$  and illumination intensity  $I$ . In step *B*, random operation state  $s$  of distribution system was firstly selected. And then, optimal power flow algorithm in state  $s$  was applied to compute the value of load curtailment based on the improved point estimate method. And then, the four risk indices listed in Section 3.1 were computed until all the random operation state  $s$  is considered.

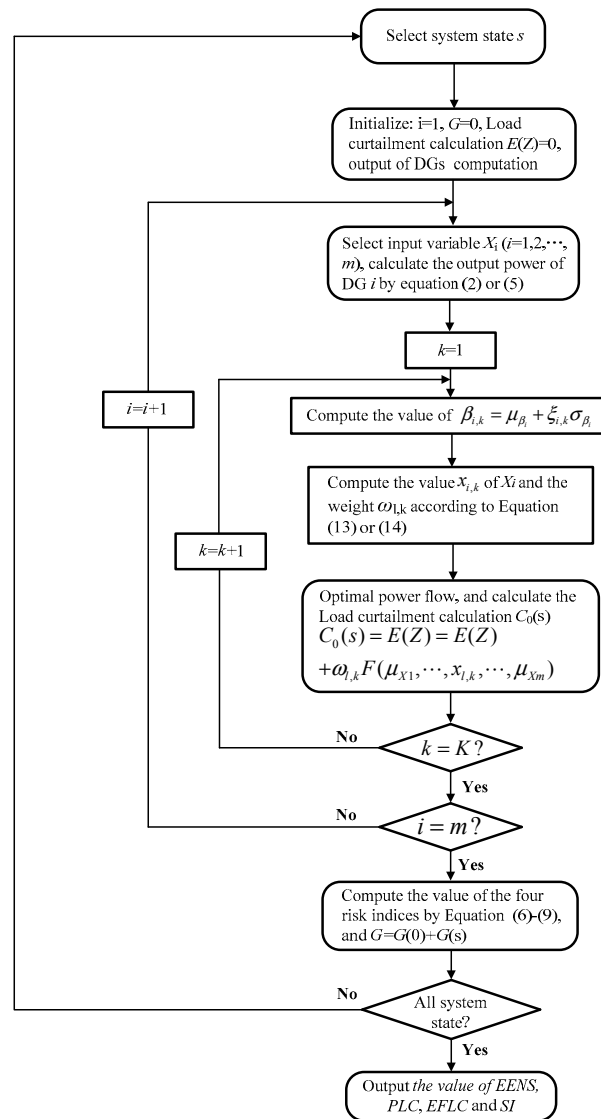


**Figure 1.** Diagram of the developed PEM-based method for risk assessment of Transmission line in distribution system

#### 4.2. Procedure for Risk indices Calculation

In [13], according to DGs' probability model, the indices of EENS, PLC, EFLC and SI were calculated by a proposed hierarchical risk assessment method which is the combination of Monte Carlo simulation and enumeration method. However, this proposed hierarchical method is relatively complex in application. In [6],  $2m+1$  PEM was utilized to assess the overload risk of transmission line. This  $2m+1$  PEM has lower accuracy compared to Monte Carlo simulation. However, the computational burden of  $2m+1$  PEM is greatly reduced. Consequently, the improved PEM is applied to compute the four risk indices in this paper. For comparison, the calculation results of the four risk indices based on the hierarchical risk assessment method in [13] was used as the exact value and the error in accuracy of the developed PEM-based method for risk assessment was analyzed.

For convenience, symbol  $G$  is used to represent any parameter of the four risk indices including EENS, PLC, EFLC and SI. Namely, to calculate the parameter  $G$  is to calculate these four risk indices. The designed procedure for computing the symbol  $G$  based on PEM is summarized as Fig. 2.



**Figure 2.** Flow chart of computing procedure for risk indices

In Fig. 2,  $2m$  and  $2m+1$  improved PEM were used respectively. The output power of DG  $i$  is calculated by equation (2) or (5). Further, OPF was applied to calculate the value of  $C_0(s)$ , which can be referred from Equation (16) and (17). The detail procedure of the developed PEM-based method for computing risk indices  $G$  is summarized as follows:

Step 1): Enumeration method was used to list the states of distribution system. The occurrence probability  $p_T(s)$  in system state  $s$  could be calculated as Equation (19):

$$p_T(s) = \prod_{i=1}^{N_f} \frac{\lambda_i}{\lambda_i + \mu_i} \prod_{j=1}^{N_n} \left(1 - \frac{\lambda_j}{\lambda_j + \mu_j}\right) \quad (19)$$

Where,  $N_f$  is the total number of failure components,  $N_n$  is the total number of normal components. For component  $i$ ,  $\mu_i$  is the repair rate,  $\lambda_i$  is the outage rate.

Step 2): Initialize and generate the constructed points of wind speed  $v$  as well as illumination intensity  $I$ . Initialize the initial value of load curtailment calculation  $E(Z)=0$  and the initial value of  $G=0$ .

Step 3): Constructing the parameters of improved point estimate method, it contains the following substeps:

1) Select the input variable  $X_i$  ( $i=1, 2, \dots, m$ ). For wind generating unit  $i$ ,  $X_i$  represents the stochastic wind speed  $v$ ; for photovoltaic generating units  $i$ ,  $X_i$  represents the stochastic illumination intensity  $I$ .  $m$  is the total number of wind generators and photovoltaic generators. And then, the



output power of DG  $i$  is calculated by equation (2) or (4).

2) With the first central moments, standard locations  $\xi_{i,k}$  and the weights  $\omega_{i,k}$  for random variables are computed by Equation (12) or (13). And then, the concentrations  $x_{i,k}$  can be calculated by Equation (10). Consequently, the point  $(\mu_{x1}, \mu_{x2}, \dots, x_{i,k}, \dots, \mu_{xm})$  is constructed.

3) Based on the output of DG  $i$ , optimal power flow in state  $s$  was applied to compute the value of  $C_0(s)$  according to equation (15) and (16).

4) Repeat substeps 2)–3) until variable  $k$  reaches  $K$ . For 2m improved PEM,  $K=2$  and for 2m+1 improved PEM,  $K=3$ .

5) Repeat substeps 2)–4) until variable  $i$  reaches the total number  $m$ , and the calculation of  $G$  in state  $s$  is completed.

Step 4): Repeat Steps 1)–3) until all the states in distribution system are considered.

Step 5): Output the values of the four risk indices EENS, PLC, EFLC and SI.

## 5. Case Studies

In order to verify the rationality of the proposed PEM-based method for risk assessment of distribution system, IEEE 30-bus system with DGs is applied, as shown in Fig. 3. For all nodes, upper voltage bound is 1.06 p.u. and lower voltage bound is 0.94 p.u. in this paper. All the cases are achieved by MATLAB in an Advanced Micro Devices 64 Dual Core 3.3 GHz PC. For photovoltaic farms, shape indexes  $\alpha=15.42$ ,  $\beta=4.3$ ,  $A=2.17\text{m}$ ,  $\eta=13.53\%$ . For wind farms, rated wind speed  $v_N=15\text{m/s}$ , cut-in wind speed  $v_{in}=4\text{m/s}$ , cut-out wind speed  $v_{out}=20\text{m/s}$ , shape index  $k=6.23$ , and scale index  $c=10.43$ .

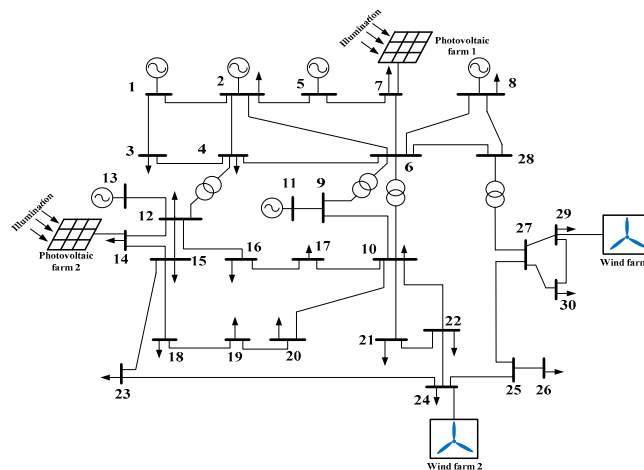


Figure 3. modified IEEE 30-bus system with DGs

### 5.1. Calculation of Risk indices based on PEM

In this modified IEEE 30-bus system, two wind farms and two photovoltaic farms are connected to node 7, 14, 24 and 29. The capacity proportion of these four farms is 20%:30%:20%:30%. But the total generation capacity of DGs  $K_d$  varies from 0% to 100% of the total load. With the known capacity proportion of DGs, the value of  $\alpha_i$  can be got:  $\alpha_1=0.2$ ,  $\alpha_2=0.3$ ,  $\alpha_3=0.2$ ,  $\alpha_4=0.2$ .

According to the designed procedure for computing risk indices in Fig. 2 and the parameters of DGs, the calculation results of EENS, PLC, EFLC and SI based on 2m and 2m+1 improved PEM were respectively listed in Table 1 and Table 2.

Table 1. Calculation results of Risk indices based on 2m improved PEM

$K_d$	2m PEM			
	EENS(MWh/y)	PLC	EFLC(Times/y)	SI(min/y)
0	3151.2	62247e-7	2.4899	66.716
0.2	2382.5	59981e-7	2.3992	50.441

0.4	1945.7	53176e-7	2.1285	41.193
0.6	1744.4	48635e-7	1.9454	36.932
0.8	1619.1	47516e-7	1.9006	34.279
1	1580.3	45243e-7	1.8097	33.457

**Table2.** Calculation results of Risk indices based on 2m+1 improved PEM

$K_d$	2m+1 PEM			
	EENS(MWh/y)	PLC	EFLC(Times/y)	SI(min/y)
0	3156.7	62293e-7	2.4917	66.832
0.2	2385.4	60033e-7	2.4013	50.502
0.4	1948.2	53212e-7	2.1285	41.246
0.6	1747.3	48687e-7	1.9475	36.993
0.8	1621.8	47551e-7	1.9020	34.336
1	1582.5	45285e-7	1.8114	33.504

As can be seen from Table 1 and Table 2, with the increase of total generation capacity  $K_d$ , all of these risk indices decrease gradually. When the generation capacity  $K_d=0$  which means that DGs are not permeated, the value of EENS is about 3156.7 MWh/year. But EENS decreases nearly a half and drops to about 1582.5 MWh/year when the generation capacity  $K_d$  rises to 1. Analogously, the value of PLC decreases from about 0.0062293 to 0.0045285 when generation capacity  $K_d$  rises from 0 to 1.

The variation tendency of SI is consistent with the variation tendency of EENS since the index of SI is calculated by EENS which is shown in Section 3.1. Similarly, the variation tendency of EFLC is consistent with the variation tendency of PLC. In addition, the calculation results of risk indices based on 2m improved PEM are very close with the calculation results of risk indices based on 2m+1 improved PEM.

From Table 1 and Table 2, EENS and SI decrease smoothly with the increment of generation capacity  $K_d$ . In addition, the slope of EENS and SI decreases with the growth of generation capacity  $K_d$  which means that the growing trend of EENS and SI is not as tangible as before increasingly when generation capacity  $K_d$  increases.

As shown in Table 1 and Table 2, the decrease of EENS, PLC, ELIC and SI becomes to be slow when generation capacity  $K_d$  increases to about 0.6. Namely, a large value of  $K_d$  to promote the decrease of risk indices of distribution systems is not significantly remarkable. However, the consumption of renewable energy becomes an increasing problem with the increase of generation capacity  $K_d$ . It is reluctance that too much wind and photovoltaic power is abandoned in distribution systems. Therefore, it is not necessary to increase the value of  $K_d$  to a certain large degree.

## 5.2. Deviation and computational cost comparison

In order to illustrate the effectiveness of this developed PEM-based method for risk assessment in distribution systems, the hierarchical risk assessment method in [13] was used as the exact value and the calculation results of EENS, PLC, EFLC and SI are shown in Table 3.

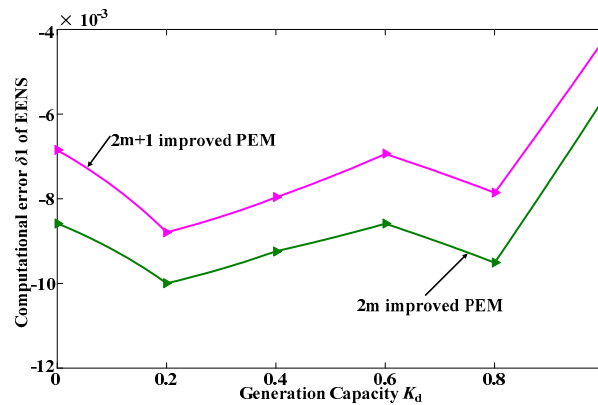
**Table3.** Calculation results of Risk indices based on hierarchical risk assessment method

$K_d$	Hierarchical Method			
	EENS(MWh/y)	PLC	EFLC(Times/y)	SI(min/y)
0	3178.5	62434e-7	2.4974	67.293
0.2	2406.6	60164e-7	2.4066	50.95
0.4	1963.8	53353e-7	2.1341	41.577
0.6	1759.5	48812e-7	1.9525	37.251
0.8	1634.6	47677e-7	1.9071	34.608
1	1589.2	45407e-7	1.8163	33.646

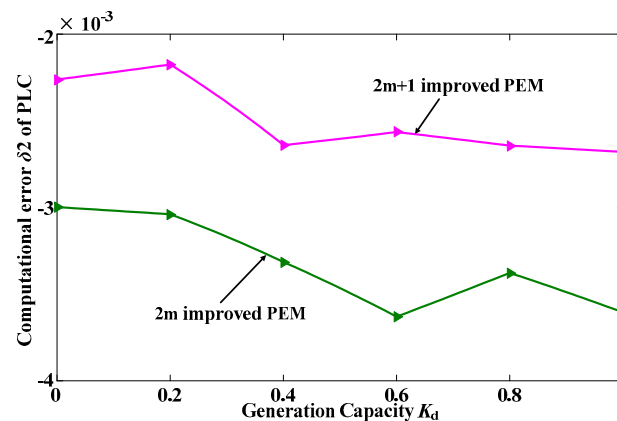
Compare the calculation results of risk indices in Table 1 and Table 2 with Table 3, these calculation results of each risk indices are very close in value. However, the calculation results of this developed PEM-based method are all less than that of hierarchical risk assessment method since PEM is an order approximation which ignores the higher order terms. Besides, the calculation

results of  $2m+1$  improved PEM are closer to the exact value than the calculation results of  $2m$  improved PEM as  $2m$  improved PEM is a third order approximation but  $2m+1$  improved PEM is a fourth order approximation in fact.

According to the analysis in Section 3.1, the variation tendency of SI is consistent with the variation tendency of EENS and the variation tendency of EFIC is consistent with the variation tendency of PLC. Therefore, the deviation comparison of EENS and PLC are only needed to be analyzed which can be shown in Fig. 4 and Fig. 5.



**Figure 4.** The computational error  $\delta_1$  of EENS in pace with  $K_d$



**Figure 5.** The computational error  $\delta_2$  of PLC in pace with  $K_d$

As can be seen from Fig. 4 and 5, the maximum value of computational error  $\delta_1$  is about 0.99% and the maximum value of computational error  $\delta_2$  is about 0.37%. The computational error  $\delta_1$  and  $\delta_2$  are all less than 1% which greatly verifies the effectiveness of this developed PEM-based method for risk assessment of transmission line in distribution system. Also, the computational error  $\delta_2$  of PLC is less than the computational error  $\delta_1$  of EENS. This is because optimal power flow algorithm based on Equation (15) and (16) is used in the calculation procedure of EENS.

Compared with hierarchical risk assessment method which is a combination of Monte Carlo method and enumeration method in fact, the computational burden of this PEM-based method is greatly reduced. When generation capacity  $K_d=0.5$ , the computational costs of risk indices are listed in Table 4.

**Table 4.** Calculation results of Risk indices based on hierarchical risk assessment method

Risk indices	The computational costs of Risk indices/(sec)		
	2m improved PEM	2M+1 improved PEM	Hierarchical method
EENS	25.845	27.742	67.293
PLC	1.985	2.031	5.950
EFLC	2.157	2.192	6.107
SI	25.845	27.742	67.293

As can be seen from Table 4, the computational costs of this developed PEM-based method for risk assessment in distribution systems are much less than the computational costs of hierarchical method. For EENS, the computational costs of 2m improved PEM and 2m+1 improved PEM are respectively 25.845s and 27.742s but the computational cost of hierarchical method is 67.293s. This is because that Monte Carlo method with much computational harder is used in the hierarchical method. On the contrary, only a bit of numerical operations are needed in the improved PEM. The calculation of risk indices has to be evaluated only  $K$  times for each input variable which is stochastic wind speed  $v$  or stochastic illumination intensity  $I$  at the  $K$  points.

In addition, the computational costs of 2m+1 improved PEM-based method are almost the same compared to 2m improved PEM-based method as only a bit of numerical operations differences are existed in these two methods. These simulations has demonstrated this developed PEM-based method has high accuracy and highly reduced computational costs, therefore it is extremely applicable for risk assessment of distribution systems.

### 5.3. Influence of DGs on System Risk

Many researches have shown that many other aspects such as DGs' type, DGs' location, DGs' dispersion, and DGs' capacity proportion have great influences on the system risk indices. In this Part, the influences of these aspects are analyzed.

In section 5.1, two wind farms and two photovoltaic farms are connected to node 7, 14, 24 and 29. Suppose that 4 wind generating units are respectively connected to node 7, 14, 24 and 29 and their capacity proportion is still 20%:30%:20%:30%. This case is named case 1 in this paper. According to the developed 2m+1 improved PEM-based method for risk assessment in fig. 2, the four system risk indices of EENS, PLC, EFLC and SI can be acquired when generation capacity  $K_d=0.2$  which can be shown in Table 5.

**Table5.** Calculation results of Risk indices in case 1

$K_d$	2m+1 improved PEM			
	EENS(MWh/y)	PLC	EFLC(Times/y)	SI(min/y)
0.2	2131.7	56823e-7	2.2729	45.131

As can be seen from Table 5, the index of EENS decreases from 2385.4MWh/y to 2131.7MWh/y and the values of other indices also cut down distinctly. For DGs' type, wind farms' power support is better than photovoltaic farms therefore. These results reflected that the randomness of the output power of photovoltaic farms is even bigger than the active output power of wind farms.

To illustrate the influences of DGs' location, suppose that two wind farms and two photovoltaic farms are respectively connected to node 3, 14, 19 and 30 in this Part and their capacity proportion is still 20%:30%:20%:30%. This case is named case 2 in this paper. Likewise, the calculation results of system risk indices based on 2m+1 improved PEM when generation capacity  $K_d=0.2$  are outlined in Table 6.

**Table6.** Calculation results of Risk indices in case 2

$K_d$	2m+1 improved PEM			
	EENS(MWh/y)	PLC	EFLC(Times/y)	SI(min/y)
0.2	2193.6	57232e-7	2.2893	46.442

Compare the calculation results of Table 6 with Table 2, the index of EENS decreases from 2385.4MWh/y to 2193.6MWh/y and the values of other indices also cut down distinctly. This comparison showed that the locations of node 3, 14, 19 and 30 are more suitable for optimal siting of DGs than node 7, 14, 24 and 29. But these locations may be not the most optimal siting for DGs and how to certain the optimal siting of DGs is worth of further study.

To illustrate the influences of DGs' dispersion, suppose that 3 wind farms and 3 photovoltaic farms are respectively connected to node 3, 7, 14, 24, 29 and 30 in this Part and their capacity

proportion is 15%:15%:20%:15%:15%:20%. This case is named case 3 in this paper and the calculation results of system risk indices are outlined in Table 7.

**Table 7.** Calculation results of Risk indices in case 3

$K_d$	2m+1 improved PEM			
	EENS(MWh/y)	PLC	EFLC(Times/y)	SI(min/y)
0.2	2058.4	55785e-7	2.2314	43.579

In case 3, the total proportion of wind farms and photovoltaic farms is still 50%:50% but the dispersion degree of DGs in case 3 is enhanced. The results in Table 7 and Table 2 showed that system risk indices decrease along with dispersion degree' increment. Therefore, DGs should be dispersed as much as possible for the optimal siting of DGs for application.

The influences DGs' capacity proportion is on the side of reflection the influences of DGs' type on system risk indices. As wind farms' power support is better than photovoltaic farms, the values of system risk indices cut down certainly when wind generating units' capacity proportion is increased.

These test results illustrated that appropriate DGs' siting and sizing have great influences on the system risk indices. For the sizing of DGs, system risk indices increase along with DGs' capacity but the corresponding investment costs will increase on the other hand. For the appropriate siting of DGs, different locations may suit for different types' DGs and DGs' dispersion can decrease the value of system risk indices.

## 6. Conclusions

DGs connected to distribution system could critically cause some risks to distribution systems on security and economy aspects. In order to reasonably assess the risks of distribution systems with the penetration of DGs, four risk indices EENS, PLC, EFLC and SI were used in this paper to reflect the system risk level in distribution systems. The output uncertainties of DGs were depicted by some certain mathematical distributions. And then, a developed PEM-based method was proposed to calculate these four system risk indices. In this developed PEM-based method, Enumeration method was used to list the states of distribution systems and an improved PEM was presented to deal with the uncertainties of DGs.

Test results of case studies have shown that this proposed PEM-based method is highly effective to assess the risk of distribution system with DGs as it has a high computational accuracy and highly reduced computational costs compared with other risk assessment methods. Simulation results also demonstrate that DGs' total generation capacity, DGs' type, DGs' location, DGs' dispersion, and DGs' capacity proportion have great influences on the system risk indices. The determining method of appropriate DGs' siting and sizing considering the operation risk of distribution system with DGs is worthy for further research.

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