 doi:10.20944/preprints201612.0049.v1

On Equivalence of Quantum Liouville Equation And Metric Compatibility Condition, A Ricci Flow Approach

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In this paper after introducing a model of binary data matrix for physical measurements of an evolving system (of particles), we develop a Hilbert space as an ambient space to derive induced metric tensor on embedded parametric manifold identified by associated joint probabilities of particles observables (parameters). Parameter manifold assumed as space-like hypersurface evolving along time axis, an approach that resembles 3+1 formalism of ADM and numerical relativity. We show the relation of endowed metric with related density matrix. Identification of system density matrix by this metric tensor, leads to the equivalence of quantum Liouville equation and metric compatibility condition $\nabla_0 g_{ij} = 0$ while covariant derivative of metric tensor has been calculated respect to Wick rotated time coordinate. After deriving a formula for expected energy of the particles and imposing the normalized Ricci flow as governing dynamics, we prove the equality of this expected energy with local scalar curvature of related manifold. Consistency of these results with Einstein tensor, field equations and Einstein-Hilbert action has been verified. Given examples clarify the compatibility of the results with well-known principles. This model provides a background for geometrization of quantum mechanics compatible with curved manifolds and information geometry.

Keywords: Quantum Liouville equation; metric compatibility condition; Joint probability; Binary Data Matrix; Ricci flow

1. Introduction

Liouville theorem in statistical physics was first introduced by Joseph Liouville. Theorem states that the density of particles in a system with Hamiltonian regime through time evolution, remains constant in phase space, i.e. $\frac{d\rho}{dt} = 0$ [1] .The quantum version of this theorem, namely Liouville -Von Neumann theorem presented in density matrix formalism [2]. Density matrix evolution in Liouville -Von Neumann theorem could be derived directly from Schrödinger equation and acts on the same Hilbert space where the wave function and related operators are defined. This equation is in analogy with the evolution of classical phase space distribution by replacing the density matrix with phase space distribution and commutator with Poisson bracket. One of the major differences between classical and quantum measurement is the limitations induced by Heisenberg uncertainty law and its consequences that constrains the accuracy of joint (simultaneous) measurements of incompatible observables and divides the observables to compatible and incompatible category. Compatible observable refers to those that their operators are commutative and hence could be measured simultaneously while incompatibles are non-commutative and their precise simultaneous measurements are impossible. In spite of this restriction, recent advents reveal some solution for this constraints by imposing some approximations on joint measurements of incompatible observables at the price of introducing some errors with respect to the ideal measurement [3-5]. Then one may consider experiments with acceptable simultaneous measurements of incompatibles with definite concept of *joint probability*. However joint probability in quantum mechanics remains as an old and challenging area of research. One of the main approaches for quantum correction to classical statistical mechanics and

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 9 December 2016 doi:10.20944/preprints201612.0049.v1

consequently joint probability in quantum mechanics are brought by Wigner distribution (function) to formulate the quantum physics in a phase space through introduction of *Quasi probabilities*. The Quasi prefix is considered because of emerging some negative probabilities in the context of Wigner distribution. It has been proved that these negative probabilities often refer to small parts of phase space and could be ignored in most problems [6]. Actually whenever the Wigner function takes just the non-negative values it represents a true joint probability distribution of position and momentum [6]. At the time being joint measurements of incompatible observable with some error becomes feasible [5]. Therefore implementing joint measurements to record the magnitude of observables with a possible range of errors is achievable. This means that one may define *joint probabilities* in quantum approach especially in density matrix formalism[5]. We will present in sec (3) a density matrix which fitted for the present measurement model with entries proportional to joint probabilities of observables. In sec (2,3,4) we set a model of recording measurements by introducing a binary data matrix $\boldsymbol{D}$ as well as Hilbert spaces and related metric tensors. We will show in these sections the equivalence of density matrix model with $\boldsymbol{DD^T}$.

Metric compatibility condition exhibited as a pure mathematical inference in differential geometry and tensor analysis [7].This theorem states that for any chosen local coordinates the covariant derivative of metric tensor g_{ij} vanishes i.e. $\nabla_k g_{ij} = \nabla_k g^{ij} = 0$ [8]. When we apply the covariant time derivative of metric tensor, these two apparently far concepts may appear as two sides of a common reality when the deep connection of *metric tensor* and *joint probability* has been shown to be based on an abstract background of measurement process. In some attempts toward the geometrization of statistical inferences and probability distributions, Amari and Fisher contributed to develop the metric tensor concept of manifolds constructed by points correspond to probability distributions in order to geometrize the information theory. In some approach to general relativity like numerical relativity and ADM formalism [9], the concept of foliation of space-time manifolds into space like hyper-surfaces has been introduced and used to solve some related problems. These hyper-surfaces embedded in space-time manifold with time-like unit normal vectors. We generalize this method to $n + 1$ dimensional parametric manifolds with hyper-surfaces of n space-like dimensions. Accordingly in this approaches the hyper-surfaces and their induced metrics could be evolved through time and consequently one may be able to impose some kind of Ricci flow dynamics to develop a new approach to dynamical systems. Ricci flow is a well-known geometric flow was first introduced by R.Hamilton and used for solution of the Poincare conjecture. As an evolution equation of metric tensor, Hamilton (1982) showed the existence of unique solution of Ricci flow equation on a closed manifold over a sufficiently short time. We would have shown in sec (3) the equivalence of metric tensor and joint probabilities which appear in $\boldsymbol{DD^T}$. Thereafter we demonstrate equivalence between quantum Liouville equation and metric compatibility condition which explores some kind of geometrization of related problems. This connection could be possible by assuming the metric compatibility condition as an evolution equation when be applied for covariant derivative of metric tensor respect to time dimension after a Wick rotation to get a Riemannian manifold fitted for Ricci flow evolution. Mainstay of general relativity has been based on the relation of space-time manifold structure and stress energy tensor in the presence of gravitational field by presumption of equivalence principle [10]. Einstein field equation represents this equivalence by equating a pure geometrical term (left side) well known as Einstein tensor with a pure physical term (i.e. stress energy tensor) [10]. This great assumption leads to geometrization of all gravitational and non-gravitational field theories through introduction of Einstein-Hilbert action integral in such a way that metric tensor of space-time appears in all actions of field theories. In recent years some attempts devoted to introduce the gravity as an emerging force i.e. entropic force [11]. By this scenario the distribution of mass-energy dictates the gravitational potential [11, 12].These theories support the relation of geometry and probability. Intuitively an immediate result is a probable deep connection between the geometry (of space-time) and physical probability concept. As an interesting example we have shown in sec (6) the more basic interconnection between joint probability density (as an induced metric tensor) and the Einstein tensor under Ricci flow dynamic. Geometrization of probability distribution and information has been achieved by some authors. Fisher information and covariance based metric in phase space and

 doi:10.20944/preprints201612.0049.v1

information geometry are among the original works in this field [13, 14]. However these approaches limited to phase space with definitions of metric tensor as the expectation values of probability distribution moments and likelihoods. Moreover there has not revealed a clear connection to physical applications. Therefor local approaches have not been yet developed properly in order to be used in Riemannian curved spaces and general relativity. Some authors also indicated the relation of thermodynamic rules with Einstein field equations [15, 16]. These theories describe the gravitational forces with entropic force assuming entropy as a function of matter distribution [12]. Although the pure geometrical part of Einstein field equations could be served in arbitrary dimensional space, however its physical side can be realized in four dimensional space-time continuum, accordingly it seems to be a special case of a more general form of basic laws. In this article we generalize the physical concept of geometrical part of Einstein field equation in $n+1$ dimensional manifolds defined through exploring a deep connection between the concepts of *metric* and *joint probability density*. To approach this aim, as described before, we set up a thought experiment to record the measurement result in a *binary data matrix* whereby a tangent space and its base vectors will be defined, and then the metric tensor appears as joint probabilities where some similarities reveals with Hessian manifolds. While all the metric and Ricci tensors and related curvatures will be defined on this manifold, the *entropy* concept enters under the dynamic of (backward) Ricci flow and related solitons.

2. Binary Data Matrices and Hilbert space

In this section, I have used the methods which described previously in my other work [28].

Definition: Parametric space $\mathcal{M}$ specified by x^{ν} coordinates with $\boldsymbol{\nu}$ which varies from 1 to the dimension of parametric space μ:

$$1 \leq \nu \leq \mu \tag{1}$$

Let construct a binary data matrix on the basis of sequential measurements take place in a time interval $\Delta\mathbb{T}$ on N particles in a system with conserved total number. One may label each particle by a number so that the first measurement implemented on first particle and second measurement on second particle and so on. $\Delta\mathbb{T}$ represents the least time required to achieve measurements of all particles and is assumed to be a small time interval. We label these set of measurements by $\{\alpha |\ 1 \leq \alpha \leq N\}$ with time ordering. If our measurements include μ independent parameters (μ = degree of freedom) are being denoted by x^{ν} (ν denotes the ν th degree of freedom) then we can divide the possible range of these parameters to a large number of intervals Δx^{ν} in order to obtain such small intervals that satisfy the order of predicted error of measurement setting and the accuracy of measurements. If the number of these intervals for each parameter x^{ν} denoted by m_{ν}, the total number of interval reads as:

$$m = \sum_{\nu} m_{\nu} \tag{2}$$

Accordingly any measurement outcome of a particle to determine the value of specific parameter x^{ν} falls just in one interval labeled by '$\boldsymbol{i}$' denoting the $\boldsymbol{i}$ th interval meanwhile stands for a specific value of x^{ν}. Let show this interval by $\Delta x^{\nu}(i)$ and attribute the binary value $\mathbf{1}$ for this interval while the other intervals take the value $\mathbf{0}$. Consequently the result of x^{ν} measurement for a particle will be represented by some column binary matrix with non-zero ($\mathbf{1}$) element only at row specified by $x^{\nu}(i)$. Iteration of measurement on other parameters turn out other column binary matrices. The outcome of all parameters could be represented by μ column binary matrix with μ non-zero entries. Conjunction of these column binary matrices as a single column binary matrix result in a matrix $\boldsymbol{\xi}^{\alpha}_{\boldsymbol{m}\times\mathbf{1}}$. Each of these $\boldsymbol{\xi}^{\alpha}_{\boldsymbol{m}\times\mathbf{1}}$ gives the parameter values of the α th particle. Union of $\boldsymbol{\xi}^{\alpha}_{\boldsymbol{m}\times\mathbf{1}}$constructs a data matrix $\boldsymbol{D}_{\boldsymbol{m}\times\boldsymbol{N}}$. Rows of these binary data matrices i.e. each interval $\Delta x^{\nu}(i)$ can be denoted by a vector $e^{*\nu}(i)$:

$$e^{*\nu}(i) = (0\,,1\,,0\,,0\,,1\,,1\,,0\,,0\,,1\,,\dots\ ) \tag{3}$$

Let call these base vectors as *data basis vectors*. Each vector $e^{*\nu}(i)$ could be regarded as a *base vector* spanned in an abstract N dimensional space with binary components. We will define this N dimensional space as *particle oriented coordinates* in next paragraph. $\boldsymbol{D_{m\times N}}$ could be partitioned to $D_{m_\nu\times N}$ matrices for each parameter x^ν. Thus matrix product $\boldsymbol{DD^T}$ contains block matrices for each parameter as diagonal entries and block matrices produced by different parameters as non-diagonal entries.

$$\boldsymbol{DD^T} = \begin{bmatrix} D_{m_1\times m_1} & D_{m_1\times m_2} & \cdots & \\ D_{m_2\times m_1} & \ddots & & \vdots \\ \vdots & \cdots & & D_{m_\mu\times m_\mu} \end{bmatrix}$$

As we will prove in Lemma 2, the entries of this matrix carry the set of joint probabilities of parameters. For space coordinate of particles the involved block matrices yield the spatial distribution of particles.

Postulate At the limit $\Delta x^\nu(i) \to dx^\nu(i)$, the vectors $e^{*\nu}(i)$ approaches the basis of cotangent bundle (space) as *1-form* i.e. $e^{*\nu}(i) \equiv dx^\nu(i) \equiv \omega^\nu(i)$ (4)

Definition: Here any particle specifies an independent coordinate with two possible values **0** and **1.** These coordinates are *orthogonal*, because at the initial setting the parameter values of each particle (such as position and momentum etc.) considered to be independent of all other particles. We call these set of coordinate as *particle oriented coordinate* that as a coordinate chart is homeomorphic to a subset of Euclidean flat space $\mathbb{R}^N$ which span a manifold $\boldsymbol{M}$. Moreover we define a parametric space $\mathcal{M}$ of considered system including all coordinates x^ν and their dual basis $e^{*\nu}$ where the latter span a dual tangential (cotangent) vector space $T_P^*\mathcal{M}$ at a point $\boldsymbol{p}$ in parametric space $\mathcal{M}$ i.e.

$$Span\{\, e^{*\nu}\} = T_P^*\mathcal{M} \subset \boldsymbol{M} \tag{5}$$

Lemma 1. It is straight forward to deduce the orthogonality of $e^{*\nu}(i)$ in each parametric range of x^ν by scalar products:

$$\langle e^{*\nu}(i), e^{*\nu}(j)\rangle = 0 \quad i \neq j \tag{6}$$

Proof: components of $e^{*\nu}(i)$ defined in an orthogonal *particle oriented coordinates*. Let n-th component be denoted by: $[\, e^{*\nu}(i)]_n$

Then the scaler product in an orthogonal coordinate for a fixed ν reads as:

$$\langle e^{*\nu}(i), e^{*\nu}(j)\rangle = \textstyle\sum_n [\, e^{*\nu}(i)]_n [\, e^{*\nu}(j)]^T{}_n \tag{7}$$

If a specific component $[\, e^{*\nu}(i)]_p$ takes the value **1**, this means that the value of parameter x^ν for p -th particle falls in i-th interval and other intervals as $[\, e^{*\nu}(j)]_p$ could not take the same value, and vice versa, therefore we have: $[\, e^{*\nu}(j)]_p = 0 \qquad i \neq j$ (8)

Consequently in equation (7) $[\, e^{*\nu}(i)]_p$ and $[\, e^{*\nu}(j)]_p$ could not take the value **1** simultaneously and this sum as the inner (scaler) product vanishes.

In order to derive a matrix containing the relative and simultaneous abundance of positive interval population (i.e. total number of particles of different parameters) we need to extract all scalar products $e^{*\mu}(i).[e^{*\nu}(j)]^{T} = \langle e^{*\mu}(i),\ e^{*\nu}(j)\rangle$ obtained by means of the matrix product $\boldsymbol{DD^T}$.

Lemma 2. Diagonal entries of the matrix $\boldsymbol{DD^T}$ are equivalent to the separate probability of each interval and non-diagonal entries return the joint probabilities of different intervals after necessary normalization.

Proof: Elements of $\boldsymbol{DD^T}$ could be represented as:

$$\langle e^{*\nu}(i), e^{*\mu}(j)\rangle = \sum_n [e^{*\nu}(i)]_n [e^{*\mu}(j)]^T{}_n \tag{9}$$

Obviously this sum enumerate the total number of particles that have common parameter value of $\boldsymbol{i}$-th interval of x^{ν} and $\boldsymbol{j}$ th interval of x^{μ}. Hence the joint probability of $e^{*\nu}(i)$ and $e^{*\mu}(j)$ events reads as:

$$f^{\mu\nu} = \frac{1}{N}\langle e^{*\nu}(i), e^{*\mu}(j)\rangle \tag{10}$$

Lemma 3. Paired joint probabilities $f^{\mu\nu}$ indicate the local metric tensor of $\mathcal{M}$.
The general definition of metric tensor for a manifold with local base vectors $e^{*\nu}(i)$ is compatible with equation (10): $g^{\mu\nu} = \langle e^{*\nu}(i), e^{*\mu}(j)\rangle$

Therefore $$\frac{1}{N} g^{\mu\nu} = f^{\mu\nu} \tag{11}$$

The total information bits collected during measurement on such a system with μ as the degree of freedom will be read as: $$\mathbb{N} = \mu N \tag{12}$$

In this model we define a Hilbert space $\mathcal{H}$ with all basis of the form $\langle j_1, j_2, \dots j_n|$ with $j_m \in \{\mathbf{0},\mathbf{1}\}$. In quantum computation however, these basis well known as *(quantum) computational basis vectors* (states) of the Hilbert space $\mathcal{H}_{2^n}$ [17, 18]. 2^n refers to the total number of elements of this Hilbert space. $\mathcal{H}_{2^n}$ contains all $e^{*\nu}(i)$ and related $\mathcal{N}_\nu$ spaces. The $\mathcal{N}_\nu$ spaces are sub-spaces of $\mathcal{H}_{2^n}$ and could be described as Hilbert spaces $\mathcal{H}_\nu$ for each parameter x^{ν}.This approach, is in close relation to qubit basis definition in quantum computation theory[17]. Indeed for construction of tangent spaces compatible with our model we need to choose a sub-space of base vectors of $\mathcal{H}_\nu$ in such a way that inner product of any pair of them vanishes:

$$\langle e^{*\nu}(i), e^{*\nu}(j)\rangle = 0 \qquad i \neq j \tag{13}$$

Obviously these sub-spaces may be regarded as $\mathcal{H}_\nu$. Sub-spaces $\mathcal{H}_\nu$ are spanned by m_ν base vectors $e^{*\nu}(i)$. Then the whole space could be represented as the sum (not direct sum) of sub-spaces $\mathcal{H}_\nu$:

$$T_P^*\mathcal{M} = \mathcal{H}_1 + \mathcal{H}_2 \dots + \mathcal{H}_\mu = \mathcal{H} \tag{14}$$

The collected information of system of (particles) along time interval $\Delta\mathbb{T}$, leads to a binary data matrix $\boldsymbol{D}_{\boldsymbol{m}\times\boldsymbol{N}}$.

Each sub-space $\mathcal{H}_\nu$ considered as a *tangent sub-manifold* $\mathcal{N}_\nu$ at a point p. The union of these tangent spaces results in the total space of a tangent bundle $T_P^*\mathcal{M}$. The state of the system could be represented by such matrix and the evolution of this quantum system obeys the equation of quantum Liouville theorem as well as Hamiltonian operator. The inner product property of this Hilbert space leads to definition of metric tensor and related curvatures induced on manifold $\mathcal{M}$.

Definition: We have shown Hilbert space $\mathcal{H}$ spanned by $e^{*\nu}(i)$ as base vectors of related vector space. The "bra" notation determines these bases in the sense of quantum mechanics. If one shows the "bra" with $\langle e^{*\nu}(i)|$ then the related dual base vector will be denoted by "ket" i.e.$|e^{*}_{\hat{\mu}}(j)\rangle$ and lives in dual vector space $\mathcal{H}^*$. In matrix form, $\langle e^{*\nu}(i)|$ presented by a row matrix as depicted in equation (3) and $|e^{*\nu}(i)\rangle$ by a column matrix that is transpose of $\langle e^{*\nu}(i)|$. For compatibility with tensor representation we use reasonably the lower index for "ket" vector and therefore we have $|e^{*}_{\hat{\mu}}(j)\rangle$ instead of.$|e^{*\hat{\mu}}(j)\rangle$ and the scaler of the "bra" and "ket" in this notation reads as:

$$\langle e^{*\nu}(i), e^{*}_{\hat{\mu}}(j)\rangle \tag{15}$$

We sued $\hat{\mu}$ instead of μ to emphasize that this index refers to the double dual of parametric space while we know the *isomorphism* of double dual with original vector space [29]. The joint probability as proved in lemma 3 is a tensor because is proportional to metric tensor. In the notation of (15) this joint probability should be shown by a mixed tensor defined in dual and double dual vector space:

$$f^{\nu}_{\hat{\mu}} = \frac{1}{N}\langle e^{*\nu}(i), e^{*}_{\hat{\mu}}(j)\rangle \tag{16}$$

The value of scaler product $\langle e^{*\nu}(i), e^{*}_{\hat{\mu}}(j)\rangle$ equals $\langle e^{*\nu}(i), e^{*\mu}(j)\rangle$. The upper index of $f^{\nu}_{\hat{\mu}}$ related to dual space while the lower index to double space. Because of isomorphism between original and double dual space [29] this tensor could be considered as a mixed rank 2 tensor in parametric (original) vector space. Thus for compatibility with bra and ket notation we apply this tensor as metric tensor evolving by time.

$$\frac{1}{N}g^{\nu}_{\hat{\mu}} = f^{\nu}_{\hat{\mu}} = \frac{1}{N}\langle e^{*\nu}(i), e^{*}_{\hat{\mu}}(j)\rangle \tag{17}$$

3. Equivalence of metric compatibility condition and quantum Liouville equation

Density matrix formalism is the quantum version of phase space probability measure of classical statistical mechanics. Accordingly it deals with ensembles of *mixed and pure states.* The general definition of density matrix could be read as:

$$\rho = \sum_{ij} |i\rangle\rho_{ij}\langle j| \tag{18}$$

$|i\rangle$ denotes the basis vector labelled by "i" and $|i\rangle\langle j|$ denotes the projection matrix with non-zero element at row "i" and column "j". The corresponding element presented by ρ_{ij} . Diagonal entries ρ_{ii} of density matrix represents the population (probability) of a specific basis (state) therefore the trace of density matrix is unit. Off diagonal entries would provide information about the degree of coherence (or polarization) between two states, in other words it represents the *correlation* of basis states. Although off-diagonal elements have no simple physical interpretation it always gives information on quantum correlation between particles and fields [19, 20]. we consider these off-diagonal elements as the usual correlations between parameters (random variables) x_i and x_j which could be encoded by their *joint probabilities* [20], whereby we assume in our definition the equivalent notion of off-diagonal entries of density matrix ($\rho_{\mu\nu}$) and joint probability density function:

$$\rho_{\mu\nu} = f^{\nu}_{\hat{\mu}} = \frac{1}{N}g^{\nu}_{\hat{\mu}}$$

Because the factor $\frac{1}{N}$ is a scaler constant of system, it could be absorbed by $g_{\hat{\mu}}^{\nu}$ and from now on we use te term $g_{\hat{\mu}}^{\nu}$ instead of $\frac{1}{N} g_{\hat{\mu}}^{\nu}$ without any change in dynamic and topology of $\mathcal{M}$.

$$\rho_{\mu\nu} = f_{\hat{\mu}}^{\nu} = g_{\hat{\mu}}^{\nu} \tag{19}$$

In the sense of quantum computation $\langle j|$ vectors are computational basis vector in the form $\langle j_1, j_2, \ldots j_n|$ with $j_m \in \{\mathbf{0}, \mathbf{1}\}$. In present model these vectors substituted by data basis vector $e^{*\nu}(i)$ which corresponds the $\boldsymbol{i}$ th row of $\boldsymbol{D}_{m_\nu \times N}$ matrix. With the identification of bra $\langle \nu|$ by $e^{*\nu}(i)$ and ket $|\mu\rangle$ by $e_{\hat{\mu}}^{*}(j)$ the density matrix entries $\rho_{\mu\nu}$ respect to (19) could be represented by:

$$\rho_{\mu\nu}(i,j) = \frac{1}{\mathrm{N}} \langle e^{*\nu}(i), e_{\hat{\mu}}^{*}(j), \rangle = f_{\hat{\mu}}^{\nu}(i,j) \tag{20}$$

$\boldsymbol{i}, \boldsymbol{j}$ determine the corresponding intervals (values) of x^{ν} and x^{μ} respectively. The off diagonal entries give the classical joint probabilities f_{μ}^{ν}.

Recalling the equation (16) also reveals the equivalence of $\rho_{\mu\nu}$ and $g_{\mu\nu}$ and their symmetric and positive definite properties. One may compare these correspondence with similarities of covariance matrix and metric of thermodynamic state manifold [19]. One may use f_{μ}^{ν} as a mixed tensor defined by inner product of a base $e^{*\nu}(i)$ with a dual base $e_{\mu}^{*}(j)$, by the same components of $g_{\mu\nu}$ and consequently with vanishing covariant derivative as could be seen for $g_{\mu\nu}$ due to metric compatibility. Respect to binary data matrix mentioned in previous section we can imply a new relation between metric compatibility in differential geometry and Liouville equation in quantum density matrix notion. It should be reminded that the trace of defined ρ_{ij} equals the constant μ (the degree of freedom). Evidently, this fact does not interfere the validity of what will be followed.

Definition: Let $(\mathcal{M}, g)$ stands for a μ dimensional space-like Riemannian manifold described in sections (1),(2) with $g_{\mu\nu}$ as metric and $f_{\mu\nu}$ as joint probabilities described in section (2). Evolution of particles system evolves this manifold through time axis. The overall manifold M comprises space-like manifolds $\mathcal{M}$ and time coordinate generally constructs a Lorentzian manifold where a Wick rotation (i.e.$t = ic\tau = i\tau$) convert it to a Riemannian manifold of dimension $\nu + 1$. Therefore M foliated by hypersurfaces $\mathcal{M}$ through time axis. This approach is close to ADM formalism and numerical relativity [9]. From now on we use alphabetic indices instead of Greek letters. Metric compatibility known as vanishing of covariant derivative of metric tensor i.e. $\nabla_k g_{ij} = g_{ij;k} = 0$. Here we use the covariant derivative respect to Wick rotated time axis: $g_{ij;0} = 0$ because the evolution of these systems occur along the time axis and this reveals the rational for exclusive role of time covariant derivative of metric tensor in comparison with the spatial derivatives.

Theorem: For a system of particles and associated manifold $\mathcal{M}$ endowed by the metrics g_{ij} defined in section (2) vanishing covariant derivative of metric tensor (respect to Wick rotated time) is equivalent to quantum Liouville equation.

Proof: Density matrix evolution in quantum setting and its Liouville-von Neumann equation for time evolution with H_{mj} as matrix form of Hamiltonian operator could be read as [2, 21]:

$$\frac{\partial \rho_{mn}}{\partial t} = -\frac{i}{\hbar} \sum_j (H_{mj}\rho_{jn} - H_{jn}\rho_{mj}) \tag{21}$$

In the Planck units $\hbar = c = 1$ by taking into account the Euclidean coordinate after a Wick rotation i.e. $t = ic\tau = i\tau$ and substituting it in above equation we have:

$$\frac{\partial \rho_{mn}}{\partial \tau} = \sum_j (H_{mj}\rho_{jn} - H_{jn}\rho_{mj}) \tag{22}$$

Regarding metric compatibility in differential geometry [4] i.e.$\nabla_k g_{ij} = \nabla_k g^{ij} = 0$ and equation (19) i.e. $\frac{1}{N} g^{mn} = f_n^m = \rho_{mn}$, taking into consideration the temporal component (covariant derivative of metric tensor respect to Wick rotated time τ) of tensor compatibility, by definition of covariant derivative we obtain:

$$\nabla_0 f_m^n = 0 \quad \Rightarrow \frac{\partial f_m^n}{\partial \tau} = \Gamma_{0m}^j f_j^n - \Gamma_{0j}^n f_m^j \tag{23}$$

Then we get (by Einstein summation convention on j index and $g_{mn} = f_n^m = f_m^n$):

$$\frac{\partial f_m^n}{\partial \tau} = \Gamma_{0m}^j f_j^n - \Gamma_{0j}^n f_m^j \tag{24}$$

Where Γ_{0m}^j terms denote the Christoffel symbols. Comparing equations (22) and (24) reveals a new relation between Christoffel symbol and Hamiltonian matrix of the considered state:

$$\sum_l H_{ml}\rho_{ln} = \sum_l H_{ml} f_l^n \sim \Gamma_{0m}^l f_l^n \tag{25}$$

Accordingly we achieve a correspondence: $\boxed{H_{mj} \sim \Gamma_{0m}^j}$ (26)

For general strict equation instead (26), one needs a constant additional term to Γ_{0m}^j which does not depend on metric tensor, namely:

$$\boxed{H_{mj} = \Gamma_{0m}^j + C_m^j} \tag{27}$$

Then the equation (22) and (24) remain compatible. In next sections taking $C_m^j = 0$, leads Hamiltonian operator to be reduced to $H_{mj} = \Gamma_{0m}^j$. The term C_m^j stands for a constant trace mixed tensor which independent of indices remains with constant trace i.e.

$$Tr(C_m^j) = K \tag{28}$$

4. Derivation of Mean energy

Taking into account the relation of energy expectation value $\langle E \rangle$ of a system with Hamiltonian $\widehat{H}$ and density matrix ρ_{mj} :

$$\langle E \rangle = Tr(\rho\widehat{H}) = \sum_{mj} H_{mj}\,\rho_{mj} \tag{29}$$

With substitution of H_{mj} and ρ_{mj} from (19) and (27) and identity $\Gamma_{0m}^j = g^{jk} g_{km,0}$ and using Einstein summation convention we have:

$$Tr(\rho\widehat{H}) = g_j^m(\Gamma_{0m}^j + C_m^j) = \left(g_j^m g^{jk} g_{km,0} + g_j^m C_m^j\right) = (g^{mk} g_{km,0} + C_m^m) \tag{30}$$

Using the formula for trace of Christoffel symbol ($\Gamma_{0m}^m = g^{mk} g_{km,0} = \frac{1}{2}\frac{\partial}{\partial \tau}\log g$) [10] we get a relation between energy expectation value as trace of $\rho\widehat{H}$ and the trace of Γ_{0m}^j as follows:

$$\langle E\rangle = Tr\left(\rho\widehat{H}\right) = \left(\Gamma_{0m}^{m} + C_{m}^{m}\right) = \frac{1}{2}\frac{\partial}{\partial\tau}\log g + K \tag{31}$$

Where $\langle E\rangle$ denotes the energy per particle (constituent) [2] and g stands for determinant of metric tensor i.e. g_{mn} matrix. The trace of C_m^j substituted by K and appears as a constant related to the energy of system through the evolution toward the thermal equilibrium. By approaching the thermal equilibrium the term $\frac{\partial}{\partial\tau}\log g$ vanishes because the change rate of determinant g getting closer to zero and equation (31) reduces to the ensemble average of energy at thermal equilibrium $\mathbb{U}_0$:

$$\langle E\rangle = \mathbb{U}_0 = K \tag{32}$$

As described above, $\langle E\rangle$ stands for mean energy per constituent (particle) at an exact interval of parameters (one point of the related manifold) and since at *equilibrium state*, each particle contains μ *bit of information*, therefore in our model $\langle E\rangle$ is equivalent to energy of μ *bit*. We will show in sec (7) the consequences of this result. Accordingly the whole expected energy of N particle system at thermal equilibrium, $\mathbb{E}$ can be read as:

$$\mathbb{E} = N\langle E\rangle = \frac{N}{2}\frac{\partial}{\partial\tau}\log g \tag{33}$$

The main result of this equation, regarding the energy conservation of system, is a continuous evolution and matric change. Metric of considered system and its determinant g should change by a rate determined by the total energy content of system. If we denote $\boldsymbol{g}_{ij}$ as the corresponding matrices of spatial coordinates in $\boldsymbol{DD^T}$ this metric is also involves in time evolution. In a system with equilibrium state respect to parameters other than space parameters, we expect the change rate of determinant g is determined by the time evolution of $\boldsymbol{g}_{ij}$:

$$\mathbb{E} \sim \frac{\partial}{\partial\tau}\log(det\boldsymbol{g}_{ij}) \tag{34}$$

The default positive sign of $\mathbb{E}$ yields: $$\frac{\partial}{\partial\tau}\log(det\boldsymbol{g}_{ij}) > 0 \tag{35}$$

This reveals that in any open system there is a tendency toward the expansion of spatial coordinate. We realize this result in section 7. Of course, $N\langle E\rangle$ stands for the mean energy of total system consisting of particles or a hierarchy of information bits or the energy density. We will present $\mathcal{M}$ in next sections as a *non-compact manifold* specified for a class of ensembles with certain energy and particle number.

5. Evolution of manifold $\mathcal{M}$ under Ricci flow Dynamic

One of the most famous recent breakthrough in differential geometry is the notion of Ricci flow[22, 23] which Introduced by Hamilton (1984) as an evolution equation of metric tensor:

$$g_{ij,0} = -2R_{ij} \tag{36}$$

Hamilton showed the unique solution of Ricci flow equation on a closed manifold for sufficiently short time. It is noteworthy that Ricci flow is an evolution equation comparable to Heat diffusion and is not a tensor equation because the derivative of metric tensor is not generally a tensor. The equation (36) has similarities with evolution model in ADM and Numerical relativity [9].We show that this flow and its solution is in agreement with our notion of Ricci tensor and mean energy based on binary data matrix. These equations reveal straight- forward similarity between Einstein gravity emerging from curvature of space-time and curvature in *data space*, perhaps includes leading reasons for emerging gravity as an entropic force. Taking into account expression for Γ^{j}_{0m}:

$$\Gamma^{j}_{0m} = \frac{1}{2} g^{jk}\left(g_{k0,m} + g_{km,0} - g_{0m,k}\right) \tag{37}$$

And orthogonality of *time* base vector (Killing vector) relative to the other bases, by $g^{m0} = g_{k0} = 0$ for $k, m \neq 0$ results in:

$$H_{mj} = \Gamma^{j}_{0m} + C^{j}_{m} = \frac{1}{2} g^{jk} g_{km,0} + C^{j}_{m} \tag{38}$$

Assuming a dynamic of *Ricci flow* i.e. $-2R_{km} = g_{km,0}$; equation (37) and $g_{k0} = 0$ gives:

$$\Gamma^{j}_{0m} = -g^{jk} R_{km} \tag{39}$$

By equations (31), (32) and (39) equation (31) converts to:

$$\langle E \rangle = \Gamma^{j}_{0j} + \mathbb{U}_0 = -g^{jk} R_{kj} + \mathbb{U}_0 = -R + \mathbb{U}_0 = \frac{1}{2}\frac{\partial \varphi}{\partial \tau} + \mathbb{U}_0 \tag{40}$$

Thus gives rise to:

$$\langle E \rangle = -R + \mathbb{U}_0 \tag{41}$$

Here we use the expression for trace of Γ^{j}_{0m} , and definition for Ricci scalar curvature. As before φ stands for the logarithm of determinant of metric tensor; $\varphi = \log g$, now with $\mathbb{U}_0 = 0$ (41) reduces to:

$$\langle E \rangle = -R \tag{42}$$

Obviously at thermal equilibrium where the time derivative in equation (40) vanishes, we have $\langle E \rangle = \mathbb{U}_0$. Regarding the positive sign of energy, the negative sign of right side implies the negative curvature of the manifold $\mathcal{M}$. This conveys the *Hyperboloid* nature of this manifold.

To preserve the volume of manifold, one should use the normalized flows. Regarding the *normalized Ricci flow* [23]:

$$g_{ij,0} = -\,2R_{ij} + \frac{2}{n} g_{ij}\bar{r} \tag{43}$$

Where $\bar{r}$ denotes mean curvature of $\mathcal{M}$ defined by:

$$\bar{r} = \frac{\int_{\mathrm{M}} R d\mu}{\int_{\mathrm{M}} d\mu} \tag{44}$$

This normalization preserves the volume of related manifold. With (38) and substitution $g_{km,0}$ from this normalized Ricci flow, (40) converts to:

$$\langle E \rangle = -g^{jk} R_{kj} + \frac{1}{2} g^{jk}\left(\frac{2}{n} g_{kj}\bar{r}\right) + \mathbb{U}_0 \tag{45}$$

Then:

$$\langle E \rangle = -R + \mathbb{U}_0 + \bar{r} \tag{46}$$

Thus $\bar{r}$ denotes the ultimate curvature of manifold at equilibrium and consequently we reach the identity $\mathbb{U}_0 = \bar{r}$. Then (46) reads as:

$$\boxed{\langle E \rangle = -\,R + 2\bar{r}} \tag{47}$$

This and equation (42) implies that local scalar curvature of $\mathcal{M}$ corresponds to the mean constituent energy of particles confined in a volume element of parametric space of related system. Therefore if we consider a set of states (ensembles) with finite mean energy $\mathbb{E}_1 \leq \langle E \rangle \leq \mathbb{E}_2$ consisted of N particle evolving toward thermal equilibrium with a heat bath with mean energy $\mathbb{E}_1 \leq \bar{r} \leq \mathbb{E}_2$ then any member of this set corresponds to a point on a manifold $\mathcal{M}$ with local negative curvature R being determined by $\langle E \rangle = -R$. Through an evolution of metric tensor by normalized Ricci flow, while preserving the volume, manifold

$\mathcal{M}$ deforms in such a way that all local curvatures tend to $\bar{r}$ as final curvature at equilibrium. Accordingly any certain infinite set of states of a system which evolves to an equilibrium state with mean ensemble energy $\mathbb{U}_0 = \bar{r}$ could be mapped to a non-compact negative curvature manifold $\mathcal{M}$, where any point stands for an element of manifold with energy density $\langle E \rangle = -R$. By time evolution, scalar curvature (i.e. $\langle E \rangle$) diffuses through the manifold and any point (system) undergoes the change of its $\langle E \rangle$ while approaches the final equilibrium energy $\mathbb{U}_0 = \bar{r}$.

Corollary If metric tensor g_{ij} for a system is defined by equation (11) then the related partition function could be derived by equation:

$$Z = g^{-\frac{1}{2}} \tag{48}$$

Proof: The relation for mean energy of a mixed system with density partition function $\mathbb{Z}$ in thermal equilibrium at temperature T, given by [24]:

$$\langle E \rangle = U = k_B T^2 \frac{\partial}{\partial T} \log \mathbb{Z} \tag{49}$$

Where k_B stands for Boltzmann constant. By replacing T by $(k_B \tau)^{-1}$ as in thermal field theory [25] this equation transforms to:

$$\langle E \rangle = k_B T^2 \left(\frac{-1}{k_B T^2} \frac{\partial}{\partial \tau}\right) \log \mathbb{Z} = -\frac{\partial}{\partial \tau} \log \mathbb{Z}$$

Comparing this equation with (42) results in:

$$-\frac{\partial}{\partial \tau} \log \mathbb{Z} = \frac{1}{2} \frac{\partial}{\partial \tau} \log g$$

Then we have:

$$Z = C g^{-\frac{1}{2}}$$

Where C is a constant respect to time.

6. Least action principle

In section (5) we showed the manifold $\mathcal{M}$ with a local curvature R representing the mean energy per particle at any point on this manifold. The particle density ρ_n (the number of particles per unit spatial volume at each point) evidently is not a strict function of metric $g_{\mu\nu}$ and consequently its variation respect to $g_{\mu\nu}$ vanishes. Then energy density on manifold will be denoted by $\rho_n \langle E \rangle$. By this energy density the variation of action integral in our model could be read as:

$$\delta S = \delta \int_{\mathrm{M}} \rho_n \langle E \rangle \sqrt{g} d^n \boldsymbol{\omega} = \rho_n \delta \int_{\mathrm{M}} \frac{1}{2} \frac{\partial \varphi}{\partial \tau} \sqrt{g} d^n \boldsymbol{\omega} = 0 \tag{50}$$

It is straight forward to impose the relation $\delta S = 0$ which guarantees the role of $\rho_n \langle E \rangle$ as a Hamiltonian. Subsequently the action S will remain invariant under coordinate transformation. This means that the covariant divergence of $H_{\mu\nu} = \left(\frac{\delta S}{\delta g^{\mu\nu}}\right)$ should be vanished:

$$H_{\mu\nu;\nu} = \nabla_\nu \left(\frac{\delta S}{\delta g^{\mu\nu}}\right) = 0 \tag{51}$$

This can be easily verified by taking the covariant derivative of $\frac{\delta S}{\delta g^{\mu\nu}}$ as follows. By replacing $\frac{\partial \varphi}{\partial \tau}$ by term $\left(\frac{\partial}{\partial \tau} \log g\right)$ and some variation calculus, equation (50) gives rise to:

$$\delta S = \rho_n \delta \int_{\mathrm{M}} \frac{1}{2} \frac{\partial \varphi}{\partial \tau} \sqrt{g} d^n \boldsymbol{\omega} = \rho_n \frac{1}{2} \int_{\mathrm{M}} \left(g_{\mu\nu,0} - \frac{1}{2} g^{jk} g_{jk,0} g_{\mu\nu}\right) \delta g^{\mu\nu} \sqrt{g} d^n \boldsymbol{\omega} = 0 \tag{52}$$

Let $H_{\mu\nu} = g_{\mu\nu,0} - \frac{1}{2} g^{jk} g_{jk,0} g_{\mu\nu}$, and recall the variation of Einstein-Hilbert action namely:

$$\delta S^{E-H} = \delta \int_{\mathrm{M}} R \sqrt{g} d^n \boldsymbol{x} = \int_{\mathrm{M}} \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}\right) \delta g^{\mu\nu} \sqrt{g} d^n \boldsymbol{x} = 0 \tag{53}$$

Where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ is Einstein tensor. Comparing $H_{\mu\nu}$ and $G_{\mu\nu}$ reveals that substitution of $g_{\mu\nu,0}$ by $\alpha R_{\mu\nu}$ (with α as a constant) in (52), results in the equation $\alpha\rho_n \delta S^{E-H} = 0$ with the same solutions of equation $\delta S^{E-H} = 0$. Thus under the condition of Ricci flow like dynamics:

$$g_{\mu\nu,0} = \alpha R_{\mu\nu} \tag{54}$$

Two integrand remain proportional i.e. $H_{\mu\nu} = \alpha\rho_n G_{\mu\nu}$ and solution of two action integral will be identical. This reveals that imposing a Ricci flow on evolution of manifold gives the structure of space-time in General relativity. In section (5) by the exclusive role of Ricci flow as governing dynamic we reached the equation (47) i.e.

$$\langle E \rangle = -R + 2\bar{r}$$

This equation emphasizes $\langle E \rangle$ role as a local energy density comparable with *local temperature* and its equivalence with curvature in the sense of evolving related manifold by Ricci flow:

$$\langle E \rangle \sim T \tag{55}$$

Recalling the Lagrangian density in the notion of AdS_n(n-dimensional anti-desitter space) with negative cosmological constant $\Lambda < 0$:

$$\mathcal{L} = \frac{1}{16\pi G}(R - 2\Lambda) \tag{56}$$

Ignoring the sign, there reveals a strict resemblance of this Lagrangian with equation (47). Interestingly the cosmological constant Λ , showing the vacuum curvature, could be considered as the far ultimate curvature of an inflating universe and therefor plays the role of $\bar{r}$ in equation (47).

7. Results:

In this section we bring some examples for compatibility of the results of previous sections with some well-known results of astronomy, information theory, thermodynamics etc. Main result of the theorem is the equation (42):

$$-R = \langle E \rangle = \frac{1}{2}\frac{\partial\varphi}{\partial\tau}$$

In subsequent examples we verify this equation in various situations.

1. Relation to average energy in canonical ensemble statistics

In this section we prove an interesting relation between energy averages $\langle E \rangle$ and partition function in the path integral notion of quantum field statistics. First we note the relation of imaginary time periods $\tau = -it$ in thermal field theories which coincides the Wick rotation we used in section.3 and $\beta = \frac{1}{k_B T}$ (with k_B as Boltzmann constant) in statistical mechanics:

$$\tau = \beta = \frac{1}{k_B T} \tag{57}$$

Now recall the well-known derivation of average energy from canonical partition function [24].

$$\langle E \rangle = U = -\frac{\partial \log \mathbb{Z}}{\partial \beta} \tag{58}$$

According to equations (40) and (41) and assuming $\mathbb{U}_0 = 0$:

$$\langle E\rangle = \frac{1}{2}\frac{\partial}{\partial\tau}\log g = \frac{1}{2}\frac{\partial\varphi}{\partial\tau} \tag{59}$$

Considering the relation between partition function $\mathbb{Z}$ and determinant of a non-negative self adjoint (symmetric) operator A in the context of field theory [25] gives:

$$\mathbb{Z} = \int_{\mathrm{M}} e^{-<\varphi,A\varphi>} D\varphi = (\det A)^{-\frac{1}{2}} \tag{60}$$

Or in a brief notation [26]: $\mathbb{Z} = \int_{\mathrm{M}} e^{-\beta<E>} D\varphi$

Here $\langle E\rangle$ should be introduced as Dirichlet energy [26]. We see if one assumes g_{ij} as an operator A in above equations, then equations (59), (60) and corollary (i.e. $Z = g^{-\frac{1}{2}}$) yields the identity:

$$\langle E\rangle = \frac{\partial}{\partial\tau}\log\sqrt{g} = -\frac{\partial}{\partial\tau}\log\mathbb{Z} = -\frac{\partial}{\partial\tau}\log\int_{\mathrm{M}} e^{-\tau<E>} D\varphi = \langle E\rangle \tag{61}$$

This reveals the compatibility of expected energy formula (59) of the model with field theory formalism.

2. Stress-Energy tensor

Energy momentum tensor of model can be derived by variation of action S respect to $g_{\mu\nu}$:

$$\bar{T}_{\mu\nu} = \frac{\delta S}{\delta g^{\mu\nu}} = H_{\mu\nu} \sim \frac{1}{2}(g_{\mu\nu,0} - \frac{1}{2}g^{jk}g_{jk,0}g_{\mu\nu}) \tag{62}$$

Recall the Einstein field equation: $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}$ (63)

With assumption of Ricci flow as the governing dynamics and replacing $g_{kj,0} = -2R_{kj}$in (39), we obtain the energy momentum tensor of model: $\bar{T}_{\mu\nu} = H_{\mu\nu} \sim R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}$ (64)

This is also consistent with the result of previous section and implies a linear relation between $\langle E\rangle$ and temperature.

3. Entropy and information

As we showed in previous sections, the state of system could be characterized by a data matrix containing a set of information bits. Any column of this matrix (i.e. $\boldsymbol{\xi}^{\alpha}_{\boldsymbol{m}\times\boldsymbol{1}}$) contains the set of information of one particle in which the values of parameters given by μ bit of information, each bit for a positive interval in each degree of freedom (μ denotes the degree of freedom). If $\bar{\epsilon}$ stands for the mean energy of a bit of information, then the mean energy per particle $\langle E\rangle$ is given by:

$$\langle E\rangle = \mu\bar{\epsilon} \tag{65}$$

Regarding the Landauer's principle which states that for erasing a bit of information the minimum required energy is: $\bar{\epsilon} = k_B T \log 2$ (66)

Substitution of $\bar{\epsilon}$ in equation (65) gives:

$$\langle E\rangle = \mu\bar{\epsilon} = \mu\, k_B T \log 2 \tag{67}$$

This is in analogy with the *equipartition theorem of energy* which states that the mean energy of each particle is proportional to the degree of freedom, Boltzmann constant and temperature, and shows the compatibility of the model with these two basic principles.

4. Ricci flow soliton

By definition, the combination $(\mathcal{M}^n, g, f)$ is a gradient Ricci soliton if it satisfies the equation[22, 23]:

$$R_{ij} + \partial_i \partial_j f = \lambda g_{ij} \quad (68)$$

This coincides our assumption provided that tangent manifold is Ricci flat ($R_{ij} = 0$) and scalar f stands for N defined in equation (7) with $\lambda = 1$:

$$\partial_i \partial_j f = g_{ij} \quad (69)$$

This is consistent with geometric structure of our tangential spaces $T_P\, \mathcal{M}$ which consisted of $\mathcal{N}_K$ and their union $\bigcup_K \mathcal{N}_K$ provided that the manifold assumed to be locally flat. Because if f stands for the cumulative probability function, $\partial_i \partial_j f$ in the classical probability notion presents the joint probability of x_i and x_j parameters and consequently if we replace f with N as total number of particles $\partial_i \partial_j N$ is proportional to joint probability and (69) supports the result of our model that shows the equality of metric tensor with joint probability density.

5. Universe Inflation and extrinsic curvature

If except for spatial dimensions all other parameters confine in a relatively equilibrium range, the large part of g_{ij} will be constant with a fairly good approximation while the 3D-space metric $\boldsymbol{g}_{ij}$ increases as was proved in (35). In this situation which could be realized by our universe, the change rate of the whole metric g_{ij} equals the change rate of spatial part of the metric. Therefor we can substitute the 3-dimensional spatial metric $\boldsymbol{g}_{ij}$ with $j = 1,2,3$ into FRW equation which reveals a relation between spatial metric tensor and scale factor $a(t)$[27]:

$$\boldsymbol{g}_{ij} = a^2(t)\delta_{ij} \quad (70)$$

In n dimensional manifold the determinant of above metric reads as:

$$\boldsymbol{g} = a^{2n}(t)$$

Accordingly we have:

$$\mathbb{E} \sim \frac{1}{2}\frac{\partial}{\partial \tau}\log \boldsymbol{g} = n\frac{\partial}{\partial \tau}\log a(t) = n\frac{\dot{a}}{a} \quad (71)$$

With $\dot{a}$ as time derivative of scale factor a. Hubble parameter has been defined as:

$$H = \frac{\dot{a}}{a}$$

Substitution in (71) gives:

$$\mathbb{E} \sim R = nH \quad (72)$$

Usually Hubble parameter H considered to be equal to $\mathcal{R}^{-1}$ with $\mathcal{R}$ as the observable Universe radius. Respect to the equations (72) we achieve the equation:

$$\mathbb{E} = n\,\mathcal{R}^{-1} \quad (73)$$

On the other hand, in ADM notion of general relativity, 3+1 dimensional splitting of space-time reveals a relation between K (the trace of extrinsic curvature K_{ij}) and determinant of metric tensor i.e. g [9]:

$$\frac{1}{2}\frac{\partial}{\partial \tau}\log \boldsymbol{g} = -\alpha K + D_i \beta^i \quad (74)$$

Where β^i stands for shift vector and assumed to be vanished. Therefore by equation (71) we have:

$$\mathbb{E} = R = -\alpha K$$

This relation supports the curvature concept of $\langle E \rangle$ as predicted in our model. In 3+1 decomposition of general relativity, one of the important concepts related to 3 spatial submanifold is external curvature K_{ij} with a well-known equation as follows[9]:

$$\partial_0 \boldsymbol{g}_{ij} = -2NK_{ij} \quad (75)$$

Where N considered as a constant respect to time. On the other hand as mentioned in previous sections by applying the Ricci flow, dynamic for metric tensor reads as:

$$\mathcal{g}_{ij,0} = -\,2R_{ij} \tag{76}$$

This reveals that the Ricci flow can be traced out as the main dynamic in time evolution of spatial hypersurfaces in ADM formalism and Numerical relativity and supports the main idea of our model in applying this flow as a universal dynamics.

Interpretation of system evolution

Eventually, the evolution of our system represented by deformation of manifold $\mathcal{M}$ and related tangential space $T_p\,\mathcal{M}$. Obviously because of the bounded nature of system parameters this manifold is not a compact (closed) manifold but could be considered as a connected region on a closed manifold that evolves by time through a diffeomorphism transformation under *normalized Ricci flow* dynamics and while the volume of manifold $\mathcal{M}$ will be preserved, the local scalar curvature changes to approach the equilibrium curvature $\bar{r}$. This final curvature stands for the final temperature that the system takes while $t \to \infty$.

Conclusion

Binary data matrix and constructed Hilbert space fitted for physical measurements recording, represents a set of base vectors with associated metric tensor and entries that interpreted as observed joint density probabilities of related system parameters. Both metric tensor and joint probabilities are symmetric and positive semi-definite. Definition of density matrix in the sense of quantum statistics conveys the full analogy between these matrices and metric tensor. We define a manifold with the dimension of the whole parameters intervals number, and its submanifolds expanded by basis vector subsets identified on each independent parameter intervals. We prove that this geometry and induced metric not only reveals the properties of a Riemannian manifold, but also proves the equivalence of metric compatibility and Liouville-Von Neumann equation. This formalism also explores the relation of the manifold curvature and ensemble average energy of the under measurement system, and uncovers the rational for equating the pure geometrical side and stress energy tensor of Einstein field equation. We have shown that this mean energy is proportional to energy per bit of information that recovered from measurements. We assert the compatibility of normalized Ricci flow dynamics with our Hamiltonian action integral and equivalence of this integral with Einstein-Cartan integral. Other successful interpretations included in this model, consist of equipartition theorem of energy, Landauer's and minimum energy principles. Compatibility with Universe inflation and FRW equation is mentioned in last section.

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