

EXACT SOLUTIONS TO (3+1) CONFORMABLE TIME FRACTIONAL JIMBO-MIWA, ZAKHAROV-KUZNETSOV AND MODIFIED ZAKHAROV-KUZNETSOV EQUATIONS

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ABSTRACT. Exact solutions to conformable time fractional (3+1)- dimensional equations are derived by using the modified form of the Kudryashov method. The compatible wave transformation reduces the equations to an ODE with integer orders. The predicted solution of the finite series of a rational exponential function is substituted into this ODE. The resultant polynomial equation is solved by using algebraic operations. The method works for the Jimbo-Miwa, the Zakharov-Kuznetsov and the modified Zakharov-Kuznetsov equations in conformable time fractional forms. All the solutions are expressed in explicit forms.

1. INTRODUCTION

The particular integer ordered form of the time fractional Jimbo-Miwa (JM) equation of the type

$$\eta_{xxxy} + p\eta_y\eta_{xx} + q\eta_x\eta_{xy} + rD_\tau^\alpha\eta_y - s\eta_{xz} = 0 \quad (1.1)$$

where D_τ^α is the α .th order fractional derivative operator with respect to τ is proposed in [1] as the second member of an integrable systems of the KP-hierarchy even though it is not successful to pass the integrability tests[2, 3]. Some multi-soliton solutions are determined by bilinear form with an homoclinic test method[3]. These multi-soliton solutions are different from the multi-solitons described by using homogeneous balance and a Bäcklund transformation[4]. Some exact solutions in the forms of solitary waves, periodic waves and variable separation solution to the JM equation are derived by the aid of improved mapping approach[5]. The generalized tanh method can be used to generate non traveling-soliton and moving solitary wave type exact solutions[6]. İc₂^{1/2}zİİ₂^{1/2} and Aslan also determine some exact solutions to the JM equation by using the method of exp-function[7]. The exact solutions to space-time fractional version of the JM equation are proposed in [8] by using generalized Bernoulli equation method.

The time fractional Zakharov-Kuznetsov (KZ) equation is of the form

$$D_\tau^\alpha\eta + p\eta\eta_x + q\eta_{zzz} + r\eta_{xxz} + s\eta_{yyz} = 0 \quad (1.2)$$

where p , q , r and s are real parameters. The original form of the equation has been proposed to define three dimensional ion-sound solitons in a magnetized plasma with a low pressure[9]. The three conservation laws describing mass, momentum and center mass are also formulated in the same study. The ZK equation also admits the ellipsoidal and plane type solitons[10]. Some solutions of the cnoidal, periodic, singular periodic, solitary wave and non topological soliton forms are constructed by using the extended hyperbolic tangent, G'/G and ansatz methods[11]. The traveling wave solitons in various forms are derived in Ebadi et al.'s study by implementing the exp-function, modified F-expansion and G'/G methods[12]. Zhang and Zhou [13] obtains kink, antikink, solitary wave and periodic solutions to the ZK equation in general form by employing the bifurcation theory.

When the nonlinearity of the ZK equation is increased to three, the equation is named as the modified ZK (mZK) equation and the conformable time fractional form of the equation becomes

$$D_\tau^\alpha\eta + p\eta^2\eta_x + q\eta_{zzz} + r\eta_{xxz} + s\eta_{yyz} = 0 \quad (1.3)$$

Liang[14] derived some exact solutions in forms of some trigonometric and hyperbolic functions to the mZK equation by using modified simple equation method. More than twenty exact solutions of the mZK equation are derived by using enhanced G'/G method in [15]. The fractional forms of both equations are solved exactly by using improved sub equation adapted for fractional cases[16].

In the present study, finite series of a rational exponential function types solutions are derived for the three dimensional fractional PDEs in conformable sense listed above. All the solutions are expressed explicitly. Before explaining the used procedure, some significant descriptions and calculus properties of the conformable derivative are summarized below.

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2. PRELIMINARIES

Consider a function $\zeta = \zeta(\tau)$ defined in the positive half space and α be number $\in (0, 1]$. Then, the conformable derivative of ζ for $\tau > 0$ is defined as

$$D_{\tau}^{\alpha}(\zeta(\tau)) = \lim_{h \rightarrow 0} \frac{\zeta(\tau + h\tau^{1-\alpha}) - \zeta(\tau)}{h}, \quad \tau > 0, \alpha \in (0, 1] \quad (2.1)$$

for $\zeta : [0, \infty) \rightarrow \mathbb{R}$ [17]. Even though this definition of the fractional derivative is pretty new, various important properties such as derivative of multiplication and division are defined clearly. The fundamental properties of the conformable derivative required to solve fractional PDEs are summarized below.

Theorem 1. Let $\zeta = \zeta(\tau)$ and $\eta = \eta(\tau)$ be two α -differentiable functions for $\alpha \in (0, 1]$. Then

- $D_{\tau}^{\alpha}(c_1\zeta + c_2\eta) = c_1D_{\tau}^{\alpha}(\zeta) + c_2D_{\tau}^{\alpha}(\eta)$
- $D_{\tau}^{\alpha}(\tau^p) = p\tau^{p-\alpha}, \forall p \in \mathbb{R}$
- $D_{\tau}^{\alpha}(\zeta(\tau)) = 0$, when $\zeta(\tau) = c_3$ is constant
- $D_{\tau}^{\alpha}(\zeta\eta) = \zeta D_{\tau}^{\alpha}(\eta) + \eta D_{\tau}^{\alpha}(\zeta)$
- $D_{\tau}^{\alpha}\left(\frac{\zeta}{\eta}\right) = \frac{\zeta D_{\tau}^{\alpha}(\eta) - \eta D_{\tau}^{\alpha}(\zeta)}{\eta^2}$
- $D_{\tau}^{\alpha}(\zeta)(\tau) = \tau^{1-\alpha} \frac{d\zeta}{d\tau}$

for all real c_1, c_2, c_3 [18, 19].

The significant properties covering Laplace transform, derivative of composite functions and Gronwall' inequality are defined in[20, 18].

Theorem 2. Let ζ be a differentiable and α -conformable differentiable function and η also be defined defined in the range of ζ and be differentiable. Then,

$$D_{\tau}^{\alpha}(\zeta \circ \eta) = \tau^{1-\alpha} \eta^{1-\alpha} \eta' D_{\tau}^{\alpha} \zeta(t) \Big|_{t=\eta(\tau)} \quad (2.2)$$

where ' denotes the derivative with respect to τ [18].

3. THE MODIFIED KUDRYASHOV METHOD

Let P be

$$P(\eta, \eta_{\tau}^{\alpha}, \eta_x, \eta_y, \eta_z, \eta_{\tau\tau}^{\alpha}, \eta_{xx}, \dots) = 0 \quad (3.1)$$

where $\eta = \eta(x, y, z, \dots, \tau)$ and $\alpha \in (0, 1]$ be the fractional derivative order. The transformation

$$\eta(x, y, z, \tau) = \eta(\xi), \quad \xi = ax + by + cz - \frac{\nu}{\alpha} \tau^{\alpha} \quad (3.2)$$

converts (3.1) to an ODE for new variable ξ

$$Q(\eta, \eta', \eta'', \dots) = 0 \quad (3.3)$$

where the prime (') indicates the derivative operator $\frac{d}{d\xi}$ of η with respect to ξ [21].

Assume that the equation (3.3) has a solution of the form

$$\eta(\xi) = a_0 + a_1 H(\xi) + a_2 H^2(\xi) + \dots + a_n H^n(\xi) \quad (3.4)$$

for a finite n with all $a_i, 0 \leq i \leq n$ and $a_n \neq 0$. The procedure start by determining the degree of the polynomial type series n by balancing the non linear term and the highest order derivative term. The function H is required to satisfy the first-order ODE

$$H'(\xi) = H(\xi)(H(\xi) - 1) \ln A \quad (3.5)$$

Thus, $H(\xi)$ is determined as

$$H(\xi) = \frac{1}{1 + dA\xi}$$

where d and A are non-zero constants with the conditions $A > 0$ and $A \neq 1$.

Substituting the predicted solution (3.4) and its derivatives into (3.3) gives a polynomial of $H(\xi)$. All the coefficients of the powers of $H(\xi)$ and the constant term are equated to zero. The resultant algebraic equation system is solved for $a_0, a_1, a_2, \dots, a_n$ and the other constants used in the wave transformation (3.2). This method is explained in details in [22].

4. SOLUTIONS TO (3+1)-DIMENSIONAL JM EQUATION

The JM equation given in the (1.1) is reduced to

$$a^3 b \eta^{(iv)} + p a^2 b \eta'' \eta' + q a^2 b \eta' \eta'' - \nu b r \eta'' - s a c \eta'' = 0 \quad (4.1)$$

by using the compatible form of the transformation (3.2). Rearrangement of the last equation by integrating once gives

$$a^3 b \eta''' + \frac{(p+q)}{2} a^2 b (\eta')^2 - (\nu b r + s a c) \eta' = K \quad (4.2)$$

where K stands for the constant of integration. The balance between the non linear and the highest ordered terms gives $n = 1$. Accordingly, the predicted solution (3.4) should be

$$\eta(\xi) = a_0 + a_1 H(\xi) \quad (4.3)$$

where a_0 and a_1 are the constants to be determined. Substituting this solution and its derivatives into (1.1) leads

$$\begin{aligned} & \left(6 a^3 b a_1 (\ln(A))^3 + 1/2 a^2 b a_1^2 (\ln(A))^2 p + 1/2 a^2 b a_1^2 (\ln(A))^2 q \right) H^4(\xi) \\ & + \left(-12 a^3 b a_1 (\ln(A))^3 - a^2 b a_1^2 (\ln(A))^2 p - a^2 b a_1^2 (\ln(A))^2 q \right) H^3(\xi) \\ & + \left(7 a^3 b a_1 (\ln(A))^3 + 1/2 a^2 b a_1^2 (\ln(A))^2 p + 1/2 a^2 b a_1^2 (\ln(A))^2 q - a_1 \ln(A) s a c - a_1 \ln(A) b r \nu \right) H^2(\xi) \\ & + \left(-a^3 b a_1 (\ln(A))^3 + a_1 \ln(A) s a c + a_1 \ln(A) b r \nu \right) H(\xi) - K = 0 \end{aligned} \quad (4.4)$$

It is clear that a $H(\xi)$ should be nonzero. Hence, the coefficients of all powers of $H(\xi)$ and K should be zero. Thus, the algebraic system of equations

$$\begin{aligned} & -a^3 b a_1 (\ln(A))^3 + a_1 \ln(A) s a c + a_1 \ln(A) b r \nu = 0 \\ & 7 a^3 b a_1 (\ln(A))^3 + 1/2 a^2 b a_1^2 (\ln(A))^2 p + 1/2 a^2 b a_1^2 (\ln(A))^2 q - a_1 \ln(A) s a c - a_1 \ln(A) b r \nu = 0 \\ & -12 a^3 b a_1 (\ln(A))^3 - a^2 b a_1^2 (\ln(A))^2 p - a^2 b a_1^2 (\ln(A))^2 q = 0 \\ & 6 a^3 b a_1 (\ln(A))^3 + 1/2 a^2 b a_1^2 (\ln(A))^2 p + 1/2 a^2 b a_1^2 (\ln(A))^2 q = 0 \end{aligned} \quad (4.5)$$

Solving (4.5) for a_0, a_1, a, b, c, ν gives

$$\begin{aligned} a_1 &= -12 \frac{a \ln(A)}{p+q} \\ c &= \frac{b \left(a^3 (\ln(A))^2 - \nu r \right)}{a s} \end{aligned} \quad (4.6)$$

for arbitrary choices of a_0, a, b, ν and $K = 0$. The formed solution

$$\eta(\xi) = a_0 - 12 \frac{a \ln(A)}{(p+q)(1+dA\xi)} \quad (4.7)$$

gives

$$\eta_1(x, y, z, \tau) = a_0 - 12 \frac{a \ln(A)}{(p+q) \left(1 + dA^{ax+by+\frac{b(a^3(\ln(A))^2-\nu r)}{as}} z - \nu \frac{\tau^\alpha}{\alpha} \right)} \quad (4.8)$$

for arbitrarily chosen a_0, a, b, ν . It must be noted that the system (4.5) has three more solutions for $a_1 \neq 0$ but at least one of a, b, c, ν are zero in those solution sets. That's why these solutions are not reported here.

5. SOLUTIONS TO (3+1)-DIMENSIONAL ZK EQUATION

The wave transformation (3.2) reduces the fractional ZK equation (1.2) to

$$-\nu \eta' + p a \eta \eta' + q c^3 \eta''' + r a^2 c \eta''' + s b^2 c \eta''' = 0 \quad (5.1)$$

where $'$ denotes $\frac{d}{d\xi}$. Integrating both sides of this equation converts it to

$$-\nu \eta + \frac{p a}{2} \eta^2 + (q c^3 + r a^2 c + s b^2 c) \eta'' = K \quad (5.2)$$

with integration constant K . The balance of η^2 and η'' gives the compatible n as 2. Hence, the predicted solution must be in the form $\eta(\xi) = a_0 + a_1 H(\xi) + a_2 H^2(\xi)$. Substituting this solution into (5.2) yields

$$\begin{aligned}
& \left(1/2 apa_2^2 + 6 b^2 csa_2 (\ln(A))^2 + 6 a^2 cra_2 (\ln(A))^2 + 6 c^3 qa_2 (\ln(A))^2 \right) H^4(\xi) \\
& + \left(2 a^2 cra_1 (\ln(A))^2 - 10 a^2 cra_2 (\ln(A))^2 + 2 b^2 csa_1 (\ln(A))^2 - 10 b^2 csa_2 (\ln(A))^2 + 2 c^3 qa_1 (\ln(A))^2 \right. \\
& \left. - 10 c^3 qa_2 (\ln(A))^2 + apa_1 a_2 \right) H^3(\xi) \\
& + \left(-\nu a_2 + 1/2 apa_1^2 + 4 a^2 cra_2 (\ln(A))^2 + 4 b^2 csa_2 (\ln(A))^2 - 3 a^2 cra_1 (\ln(A))^2 - 3 b^2 csa_1 (\ln(A))^2 \right. \\
& \left. + apa_0 a_2 + 4 c^3 qa_2 (\ln(A))^2 - 3 c^3 qa_1 (\ln(A))^2 \right) H^2(\xi) \\
& + \left(a^2 cra_1 (\ln(A))^2 + b^2 csa_1 (\ln(A))^2 + c^3 qa_1 (\ln(A))^2 + apa_0 a_1 - \nu a_1 \right) H(\xi) - \nu a_0 + 1/2 apa_0^2 - K = 0
\end{aligned} \tag{5.3}$$

The solution of this system for $a_0, a_1, a_2, a, b, c, \nu$ gives two different solutions as

$$\begin{aligned}
a_1 &= -6 \frac{apa_0^2 + 2K}{a_0 ap} \\
a_2 &= 6 \frac{apa_0^2 + 2K}{a_0 ap} \\
b &= \pm \frac{1}{\ln(A)} \sqrt{-1/2 \frac{2 (\ln(A))^2 a^2 cra_0 + 2 (\ln(A))^2 c^3 qa_0 + apa_0^2 + 2K}{csa_0}} \\
\nu &= -1/2 \frac{-apa_0^2 + 2K}{a_0}
\end{aligned} \tag{5.4}$$

for arbitrary constants a_0, a, c and K . Thus, the solution of (5.2) is determined as

$$\eta(\xi) = a_0 - 6 \frac{apa_0^2 + 2K}{a_0 ap} \frac{1}{1 + dA^\xi} + 6 \frac{apa_0^2 + 2K}{a_0 ap} \frac{1}{(1 + dA^\xi)^2} \tag{5.5}$$

arbitrary a_0 and K . The solution of the conformable time fractional ZK equation (1.2) is expressed as

$$\eta_{2,3}(x, y, z, \tau) = a_0 - 6 \frac{apa_0^2 + 2K}{a_0 ap} \frac{1}{1 + dA^{ax+by+cz-\nu \frac{\tau^\alpha}{\alpha}}} + 6 \frac{apa_0^2 + 2K}{a_0 ap} \frac{1}{\left(1 + dA^{ax+by+cz-\nu \frac{\tau^\alpha}{\alpha}} \right)^2} \tag{5.6}$$

where a_0, a, c arbitrary b, ν are as given in (5.4).

6. SOLUTIONS TO (3+1)-DIMENSIONAL MZK EQUATION

The modified form of the ZK equation in the time fractional form (1.3) is reduced to

$$-\nu \eta' + a \frac{p}{3} (\eta^3)' + (qc^3 + ra^2c + sb^2c) \eta''' = 0 \tag{6.1}$$

where $'$ denotes $\frac{d}{d\xi}$. Integrating this equation once converts it to

$$-\nu \eta + a \frac{p}{3} \eta^3 + (qc^3 + ra^2c + sb^2c) \eta'' = K \tag{6.2}$$

with the integration constant K . The balance between η^3 and η'' gives $n = 1$. Thus, the solution is formed as

$$\eta(\xi) = a_0 + a_1 H(\xi) \tag{6.3}$$

for a nonzero a_1 . Substituting this predicted solution and its derivative into (6.2), a polynomial equation of $H(\xi)$ of the form

$$\begin{aligned}
& \left(1/3 apa_1^3 + 2 a^2 cra_1 (\ln(A))^2 + 2 b^2 csa_1 (\ln(A))^2 + 2 c^3 qa_1 (\ln(A))^2 \right) H^3(\xi) \\
& + \left(-3 a^2 cra_1 (\ln(A))^2 - 3 b^2 csa_1 (\ln(A))^2 - 3 c^3 qa_1 (\ln(A))^2 + apa_0 a_1^2 \right) H^2(\xi) \\
& + \left(a^2 cra_1 (\ln(A))^2 + b^2 csa_1 (\ln(A))^2 + c^3 qa_1 (\ln(A))^2 + apa_0^2 a_1 - \nu a_1 \right) H(\xi) + 1/3 apa_0^3 - \nu a_0 - K = 0
\end{aligned} \tag{6.4}$$

is obtained. This algebraic system has two different solutions satisfying the condition $a_1 \neq 0$ and can be written in the form

$$\begin{aligned} a_0 &= \pm 3 \frac{c \ln(A) (a^2 r + b^2 s + c^2 q)}{ap} \frac{1}{\sqrt{-\frac{6ra^2c + 6sb^2c + 6qc^3}{ap}}} \\ a_1 &= \pm \sqrt{-\frac{6ra^2c + 6sb^2c + 6qc^3}{ap}} \ln(A) \\ \nu &= -\frac{1}{2} (\ln(A))^2 c (a^2 r + b^2 s + c^2 q) \end{aligned} \quad (6.5)$$

for arbitrarily chosen a, b, c and $K = 0$. Thus, the solution to (6.2) is constructed as

$$\eta(\xi) = \pm 3 \frac{c \ln(A) (a^2 r + b^2 s + c^2 q)}{ap} \frac{1}{\sqrt{-\frac{6ra^2c + 6sb^2c + 6qc^3}{ap}}} \pm \sqrt{-\frac{6ra^2c + 6sb^2c + 6qc^3}{ap}} \ln(A) \frac{1}{1 + dA\xi} \quad (6.6)$$

with the condition $\frac{(qc^2 + ra^2 + sb^2)c}{ap} < 0$, $ap \neq 0$. Hence, the solutions of the (1.3) are of the form

$$\begin{aligned} \eta_{4,5}(x, y, z, \tau) &= \pm 3 \frac{c \ln(A) (a^2 r + b^2 s + c^2 q)}{ap} \frac{1}{\sqrt{-\frac{6ra^2c + 6sb^2c + 6qc^3}{ap}}} \\ &\pm \sqrt{-\frac{6ra^2c + 6sb^2c + 6qc^3}{ap}} \ln(A) \frac{1}{1 + dA^{ax+by+cz + \frac{1}{2}(\ln(A))^2 c(a^2 r + b^2 s + c^2 q) \frac{\tau^\alpha}{\alpha}}} \end{aligned} \quad (6.7)$$

7. CONCLUSION

The method of Kudryashov in modified form is implemented to derive the exact solutions to (3+1)-dimensional conformable time fractional JM, ZK and mZK equations. The valid and compatible traveling wave transformation reduces these equations to integer ordered ODEs. The predicted solution of the finite series form of a rational exponential function is substituted into the resultant ODEs. The algebraic operations are used to determine the relations between the coefficients originated from both the equations and the transformation. Once these relations are determined, the traveling wave type solutions in three dimensions are developed explicitly.

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