Article

A Fuzzy Approach to Design Motion Profiles of Automatic Machines

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Abstract: The design of machine motion profiles may be a complex and time-consuming process. The application of a structured method permits to reduce the design cycle time and optimize the problem solutions. This paper presents a fuzzy approach to design indexing systems composed by a rotary globoidal cam coupled with a rotary table. The fuzzy theory provides a systematic framework to model vague variables and supports the designer in selecting effective solutions. This study proposes a mix of design problems, generating a single fuzzy multiobjective problem. The global solution is represented by a fuzzy mix of each single solution related to each single minor problem. The final result may be used to support a designer in the definition of a proper motion profile and to generate an initial solution for optimization algorithms. This paper also wants to compare this fuzzy logic with the traditional approach in order to understand advantages and constraints during the design stage. For this reason, a group of designers has been involved. Half of them (experimental group) has adopted the experimental fuzzy approach, and the other half (control group) has applied a conventional approach. The experimental group exhibited a higher and more repeatable speed than the control group in the realization of this design task, guaranteeing the quality of the resulting motion profiles.

Keywords: fuzzy approach; motion profile; cam; automatic machine

1. Introduction

The demand of automatic production processes is growing world-wide, driving the design of cam mechanisms for high speed machining and assembling [1-3]. These automatic systems need to satisfy a set of key requirements, such as high positioning accuracy, high-speed operations, intermittent motion control and high-load application [4,6]. In addition, they are usually a complex frame, composed by different sub-machines, that realize single actions on the objects, and by a moving sub-system, that moves the objects to the sub-machines in a predefined route [8,9].

A common layout is realized with a rotating table with different stations, that corresponds to the active sub-machines [10-12]. The layout is very specialized to the realization of a limited type of parts, with minor variations. When the life of the product is commercially finished, the whole system may be decomposed and its parts refurbished and reconfigured to construct a new system.

Many technologies may be used to design the machine layout [13,14], nevertheless full mechanical approaches resist for their ability to guarantee very fast and optimized movements using low cost solutions [15-17]. In particular, cam transmissions are commonly adopted to realize indexing systems that move the rotary table [18]. This paper focuses on this application, that needs an effective design of the cam shape in order to provide an optimized motion profile of the rotary table.

The most elementary scheme of a machine can be represented by a sequence of three elements:
- the motor that produces the mechanical power;
- the transmission that adapts and transfers the mechanical power;
- the user that realizes a functional work for an external object or for the environment through the mechanical power (Fig. 1) [19].
During the first stage of the machine design, the designer must define the motion profile of the user. Many characteristics of this motion are constrained to the function that must be performed by the user; nevertheless a part of them may be freely chosen. This choice can affect the qualitative realization of the function and, thus, the overall quality of the machine [20,21]. In particular, further evaluations need to be performed in defining these qualitative characteristics, focusing on the presence of inertia actions, compliances in the transmission, external loads on the user and friction. This activity is associated to a preliminary selection of the machine member and allows an estimation of the qualitative behavior of the system.

Figure 1. Machine scheme: M is the motor; T is the transmission; U is the user; T_m and T_u are, respectively, the motor and the user torques; \( \omega_m \) and \( \omega_u \) are, respectively, the motor and user angular speeds; \( \tau \) is the transmission ratio and \( \eta \) is the transmission efficiency.

The adoption of appropriate criteria in designing the machine mechanisms plays a fundamental role to achieve shorter design cycles and decreasing costs. In particular, an improvement of a single machine characteristic may degrade the other required features. A good sense solution cannot be easily automatized. For this reason a fuzzy approach represents an effective solution to answer to this complexity, realizing an adequate mix of these requirements.

The fuzzy logic provides a systematic framework designed to model vague variables. The goal is to determine some automation functions for which fuzzy logic offers attractive possibilities and advantages. Fuzzy set theory and fuzzy logic have been applied in a great variety of applications [22-24]. The state of art offers a broad range of examples and studies, highlighting the main benefits and limits [25,26].

The design of an automatic machine is based on multiple technical choices and selections. This process may be complex and time consuming for a designer, the application of a structured method permits to reduce design cycle time and optimize the problem solutions. This paper aims to develop a fuzzy logic in designing the motion profiles of automatic machines. By using a fuzzy approach, this study proposes a mix of design problems, generating a single fuzzy multiobjective problem. The solution of this multiobjective problem may be decomposed in sub-solutions of each single problem. Then, the global solution is represented by a fuzzy mix of each single solution related to each single minor problem. The result may be used to support a designer in the definition of a proper motion profile and to generate an initial solution for optimization algorithms. The proposed approach has been implemented in a software program applied to the design indexing systems with a visual interface. In particular, this study wants also to compare the fuzzy logic with the traditional approach in order to understand advantages and constraints during the design stage. For this reason, a group of designers has been involved in designing an indexing system composed by a rotary globoïdal cam coupled with a rotary table. Half of them (experimental group) has adopted the experimental fuzzy approach, and the other part of designers (control group) has applied a conventional approach. The experimental group exhibited a higher and more repeatable speed than the control group in the realization of this design task, guaranteeing the quality of the resulting motion profiles.

2. Problem definition

Consider an industrial indexing system realized with a globoïdal cam [27], as shown in Fig. 2. The cam rotates around its axis with a quasi-constant rotation produced by an asynchronous AC
electric motor. Two rollers of the rotary table are constrained to a rib on the cam; in this way the
motion is transmitted to the rotary table that can rotate around its axis. The rib on the globoidal cam
is shaped to permit sequential movements alternated with the rotary table.

![Figure 2. Indexing system realized with a globoidal cam.](image)

Considering a single movement phase, it is assumed that all dynamic phenomena associated to
the movement phase and to other imprecisions become negligible during a single phase after a
settling period. In this way, the behavior of all movement phases and their sequential rests is similar
(eventually after an initial transient state).

The moving phase is constrained to be performed in a moving period $T$ to realize a moving
angle $h$, while the rest phase is constrained to be performed in a remaining period $TR$. Generally,
one or more objects are transported by the rotary table and, during the remaining phase, they are
subjected to one or more operations by some other machines. For this reason, after a settling time,
the rotary table is requested to be motionless during the rest phase. The moving phase of the rotary
table is considered an inactive phase for the whole mechanical system as the objects moved by the
rotary table are not subjected to operations during this phase. Thus, a reduced moving period $T$ is
usually requested.

This reduction of the moving period $T$ produces different undesired dynamical phenomena
that may degrade the quality of the movement and the reliability of the mechanical system. Once the
parts of the mechanical system and the realization technologies are identified, there are other
possibilities to optimize the quality of the movement, focusing on the rib shape, that may improve
the motion profile of the rotary table during the motion phase.

The undesired mechanical phenomena are associated to the inertial actions that interact with
compliances and clearances. In particular, dissipations may also be noted due to viscous effects.

As proposed in literature [28], these dynamical characteristics of the system are concentrated in
a single virtual joint between the rollers and the rotary table (Fig. 3). In this way, the geometrical
constraint between the cam rib and the roller can represent an ideal commanded motion profile. The
selection of the commanded motion profile is devoted to optimize the motion of the rotary table.
Figure 3. An overview of the system dynamical model.

The Lumped parameters model the system with a virtual compliant joint, where:

- $\gamma$, $\alpha$, $\beta$ and $\varphi$ are the angular positions, respectively, of the motor, of the cam input, of the cam output and of the rotary table;
- $J_m$, $J_c$, $J_f$ and $J_r$ are the inertias, respectively, of the motor, of the cam, of the compliant joint and of the rotary table;
- $T_m$ and $T_r$ are the torque of the motor and of the rotary table;
- $\tau$ is the transmission ratio, $k$ is the lumped stiffness, $r$ is the lumped friction coefficient and $c$ is the lumped clearance.

This model leads to the power equilibrium equation (1) applied to the subsystem from the motor to the rollers of the rotary table, where:

- $T_m$ is the motor torque and $T_v e$ is the viscous-elastic torque associated to the virtual joint;
- $\gamma$ is the angular position of the motor;
- $\beta$ is the angular position of the rotary table;
- $\alpha$ is the angular position of the cam;
- $\beta$ is the angular position of the rollers;
- $J_m$ is the inertia of the parts rotating around the motor shaft;
- $J_c$ is the inertia of the cam and $J_f$ is the inertia immediately after the cam constraint.

As mentioned, the compliances are concentrated in the virtual joint, thus the cam constraint is ideal and the expression (2) can be derived to express the first and second temporal derivatives as function of the geometric derivatives $\beta'$ and $\beta''$ and of the rotation of the cam $\alpha$. Then, it is noted that the rotation of the cam $\alpha$ is linearly dependent to the rotation of the motor $\gamma$ through the transmission ratio $\tau$ of the gearbox (3).

\[
\beta = \beta'(\alpha) \cdot \alpha, \quad \dot{\beta} = \beta''(\alpha) \cdot \dot{\alpha}^2 + \beta'(\alpha) \cdot \dot{\alpha}, \quad \alpha = \tau \cdot \gamma,
\]

Incorporating expression (2) and (3) in (1), the equation (4) represents the first part of the non-linear differential system associated to the model in Fig. 3, where $J_{eq}$ is shown in (5).
\[
\dot{\gamma} = \frac{T_m(\dot{\gamma}) - \tau \beta'(\tau \cdot \gamma) T_{eq} - \tau^3 J_f \beta'(\tau \cdot \gamma) \beta''(\tau \cdot \gamma) \dot{\gamma}^2}{J_{eq} + \tau^2 J_f \beta'(\tau \cdot \gamma)^2},
\]

(4)

\[
\alpha = \tau \cdot \gamma,
\]

(5)

The torque at the ends of the virtual joint is shown in (6), where \(k\) is the stiffness, \(r\) is the viscous friction coefficient, \(c\) is the clearance.

\[
T_{eq} = \begin{cases} 
0, & \text{if } |\beta - \varphi| \leq \frac{c}{2} \\
 k\left(\beta - \varphi - \frac{c}{2}\right) + r(\beta - \varphi), & \text{if } (\beta - \varphi) > \frac{c}{2} \\
 k\left(\beta - \varphi + \frac{c}{2}\right) + r(\beta - \varphi), & \text{if } (\beta - \varphi) < -\frac{c}{2}
\end{cases}
\]

(6)

The power equilibrium equation from the virtual joint to the object on the rotary table is presented in (7).

\[
J_r \cdot \dot{\varphi} + T_r - T_{eq} = 0,
\]

(7)

In this way, the whole non-linear differential problem can be described by expressions (8), (9) and (10), according to the value of \((\beta - \varphi)\).

\[
\begin{align*}
|\beta - \varphi| & \leq \frac{c}{2} \\
\dot{\gamma} &= \frac{T_m(\dot{\gamma}) - \tau \beta'(\tau \cdot \gamma) T_{eq} - \tau^3 J_f \beta'(\tau \cdot \gamma) \beta''(\tau \cdot \gamma) \dot{\gamma}^2}{J_{eq} + \tau^2 J_f \beta'(\tau \cdot \gamma)^2}, \\
\phi &= -\frac{T_r}{J_r}
\end{align*}
\]

(8)

\[
\begin{align*}
(\beta - \varphi) & > \frac{c}{2} \\
\dot{\gamma} &= \frac{T_m(\dot{\gamma}) - \tau \beta'(\tau \cdot \gamma) \left[k\left(\beta - \varphi - \frac{c}{2}\right) + r(\beta - \varphi)\right] - \tau^3 J_f \beta'(\tau \cdot \gamma) \beta''(\tau \cdot \gamma) \dot{\gamma}^2}{J_{eq} + \tau^2 J_f \beta'(\tau \cdot \gamma)^2}, \\
\phi &= \frac{k\left(\beta - \varphi - \frac{c}{2}\right) + r[\tau \beta'(\tau \cdot \gamma) \dot{\gamma} - \varphi] - T_r}{J_r}
\end{align*}
\]

(9)

\[
\begin{align*}
(\beta - \varphi) & < \frac{c}{2} \\
\dot{\gamma} &= \frac{T_m(\dot{\gamma}) - \tau \beta'(\tau \cdot \gamma) \left[k\left(\beta - \varphi + \frac{c}{2}\right) + r(\beta - \varphi)\right] - \tau^3 J_f \beta'(\tau \cdot \gamma) \beta''(\tau \cdot \gamma) \dot{\gamma}^2}{J_{eq} + \tau^2 J_f \beta'(\tau \cdot \gamma)^2}, \\
\phi &= \frac{k\left(\beta - \varphi + \frac{c}{2}\right) + r[\tau \beta'(\tau \cdot \gamma) \dot{\gamma} - \varphi] - T_r}{J_r}
\end{align*}
\]

(10)
When the rotary table and the cam are disconnected, the two differential equations of the system are decoupled due to the clearance (8). In the other two conditions (9) and (10), the equations are coupled, thus the fourth order Runge-Kutta algorithm may be applied to solve numerically the system.

As mentioned, the dynamical problems affecting the cam indexing systems arise from the interaction of inertial actions with the compliances of the system. For this reason, it seems physically adequate to design the profile of the cam \( \beta(\alpha) \) in the geometric acceleration diagram, as presented in many works of the state of art [29-31].

3. The proposed method

The function that describes the expression \( \beta(\alpha) \) can be presented in different ways. As observed in other works [32], a special class of trigonometric splines is composed by seven pieces that is completely evaluated in the appendix of this work. This special class of trigonometric splines (hereafter TSplines) is described by seven parameters in accordance to important constraints for cam systems, e.g. constant sign of the geometric speed, null speed at the start and at the end of the moving phase, exact angular extension of the moving phase, and some important constraints for high speed cams, e.g. continuity of the geometric acceleration, null geometric acceleration at the end of the motion phase.

As confirmed in other works [33], the TSplines may be use to optimize algorithms, in fact their parameter changes produce always an acceptable motion profile for a cam system. In this way, the TSplines are considered to describe the profile \( \beta(\alpha) \) of the cam.

The TSplines are also suitable to solve other simple optimization problems, that can improve the quality of the movement of the system. One of these problems is the reduction of the maximal positive acceleration. The reduction of the maximal positive acceleration can be performed incrementing the parameter \( \delta_2 \) of the TSpline. The analogous problem is the reduction of the maximal negative acceleration, in fact its inertial effects may be observed also during the reaming phase. It can be solved, analogously, incrementing the parameter \( \delta_6 \). An equal distribution of the inertial actions during the acceleration and deceleration phase, reducing their maximal values, is especially important in high speed cams and it can be obtained reducing the parameter \( \delta_4 \) and maintaining \( \delta_2 = \delta_6 \).

The opposite objective is to reduce the maximal speed, that may be interesting in different applications to minimize the cam dimensions. In this case, the solution is obtained incrementing the parameter \( \delta_4 \). A further interesting problem is the reduction of the motion torque, that is generally obtained reducing the product between the speed and the acceleration in speed cams, incrementing the parameters \( \delta_3 \) and \( \delta_5 \).

Finally, a demanding problem is the minimization of vibrations that emerge during the motion phase and can also remain during the rest phase. Many approaches can be considered to compensate this undesired effect, one of them consists of incrementing the continuity level of the acceleration with the increment of the parameters \( \delta_\tau, \delta_5, \delta_6 \) and \( \delta_8 \).

In real application, a mix of these problems must be solved together in order to obtain a useful motion profile. This multiobjective problem is complex and not always convergent to an optimal solution.

The multiobjective problem is a mix of single objective problems. For this reason, this study proposes a fuzzy approach to mix these problems and to generate a single fuzzy multiobjective problem. The solution of this fuzzy multiobjective problem can be decomposed in sub-solutions of each single problem. Then, the global solution will be represented by a fuzzy mix of each single solution to each single minor problem. The result can be used to help a designer in the definition of a proper motion profile and to generate an initial solution for optimization algorithms.

To describe the multiobjective problem \( P \), consider that it is exactly equal to one of the single problems \( P_i \); thus the generic solution \( S \) of the multiobjective problem \( P \) is exactly equal to the solution \( S_i \) of the single problem \( P_i \) (11).
If a fuzzy logic is introduced, it is possible to associate a degree of trueness for the equivalence between Pi and P that can be described by a variable \( \mu_i \). This variable can assume a value between 0 and 1. If the set of problems Pi is finite and its size is equal to n, the general assumption (12) may be considered valid also in case of fuzzy logic. Thus the relation (13) represents the proposed solution S for the problem P.

\[
( P = P_1 ) \land ( P = P_2 ) \land \ldots ( P = P_n ) = 1 \Rightarrow \\
( S = S_1 ) \land ( S = S_2 ) \land \ldots ( S = S_n ) = 1
\]

\[P = \frac{\sum_{i=1}^{n} \mu_i P_i}{\sum_{i=1}^{n} \mu_i} \Rightarrow S = \frac{\sum_{i=1}^{n} \mu_i S_i}{\sum_{i=1}^{n} \mu_i},
\]

Precisely, the following problems need to be solved:

- P1: Minimize the absolute value of the acceleration;
- P2: Minimize the negative acceleration peak;
- P3: Minimize the speed peak;
- P4: Minimize the product between acceleration and speed;
- P5: Minimize the overshooting.

The proposed solutions of these problems are described adopting the TSpline formulation, as defined in appendix. These solutions are expressed in terms of values of the parameters \( \delta_i \), as shown in Tab. 1.

The problems Pi are not exactly linearly superimposed to generate the problem P, thus the variable \( \mu_i \), also called membership function in a fuzzy logic nomenclature, is defined as a function of the assumption variable \( \xi_i \) through the expression (14), that depends on the shape parameters \( s_{1,i} \) and \( s_{2,i} \).

<table>
<thead>
<tr>
<th>Problem</th>
<th>Associated solution (( \delta_i ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>0.1250-0.2500-0.1250-0.0000-0.1250-0.2500-0.1250</td>
</tr>
<tr>
<td>P2</td>
<td>0.1250-0.1250-0.1250-0.0000-0.1250-0.3750-0.1250</td>
</tr>
<tr>
<td>P3</td>
<td>0.0625-0.1250-0.0625-0.5000-0.0625-0.1250-0.0625</td>
</tr>
<tr>
<td>P4</td>
<td>0.0125-0.0000-0.3750-0.0000-0.3750-0.0000-0.1250</td>
</tr>
<tr>
<td>P5</td>
<td>0.2500-0.0000-0.2500-0.0000-0.2500-0.0000-0.2500</td>
</tr>
</tbody>
</table>

The domain of the assumption variable \( \xi_i \) is between -10 and 10, while the domains of the shape parameters \( s_{1,i} \) and \( s_{2,i} \), respectively, [-5;5] and [-10;10].

\[
\mu_i(\xi_i) = \frac{1}{1 + e^{-s_{1,i}(\xi_i-s_{2,i})}},
\]

\[
\mu_i(\xi_i) : [-10;10] \rightarrow \{0,1\},
\]

Thus the problem P can be defined in terms of standard fuzzy set nomenclature, as in (16), where the global problem P is the fuzzy set, the problem Pi is an element of the fuzzy set, the couple \( \mu_i/P_i \) describes the degree of membership of Pi to P.
\[ P = \sum_{i=1}^{5} \mu_i / P_i, \quad P = \{ P_1, P_2, P_3, P_4 \} \]

The proposed approach has been implemented in a software program applied to the design of an indexing system with a visual interface, as shown in Fig. 4.

4. Results

To test the proposed algorithm, a case study related to the globoidal cam indexing system has been considered. The mechanical parameters are described in Tab. 2. It is assumed that dynamical effects before the virtual joint are negligible and their residual dynamic effects are incorporated into the virtual joint during the experimental calibration of the model.

The constraints for the shape of the cam are listed in Tab. 3, for an indexing system with: 20 stations, double profile, a moving angle \( 3\pi/4 \) and a rest angle of \( \pi/4 \).

The experimental test has been performed on 40 designers, assigned randomly to two groups. Half of them (experimental group) were asked to design an optimal cam profile for the proposed problem and they adopted the multiobjective fuzzy algorithm implemented in a software able to simulate the behaviour of the indexing system. A control group of the other 20 designers was asked to realize the same task with the support of a software able to simulate the cam system and changing only the \( \delta_i \) parameters of the Tspline.

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<tbody>
<tr>
<td>0.005</td>
<td>0.01</td>
<td>2.0</td>
<td>0.015</td>
<td>0.0</td>
<td>32000</td>
<td>30</td>
<td>0.00025</td>
</tr>
</tbody>
</table>

Table 2. Mechanical parameters of the system.

<table>
<thead>
<tr>
<th>( \alpha_M ) [rad]</th>
<th>( \beta_M ) [rad]</th>
<th>( \omega ) [rad/s]</th>
<th>( \tau ) [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 3\pi/4 )</td>
<td>( \pi/10 )</td>
<td>50( \pi )</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 3. The constraints of the cam shape.

The designers were free to perform the task in the desired time to obtain an optimal solution. Then, the time consumed to perform this task for each designer was measured. In particular, a commission of other 4 designers assigned a vote from 0 to 10 to each designed motion profile.
without knowing the designers that proposed the motion profile and the group of designers (blind vote).

An independent samples Mann Whitney U Test was performed to compare the time of the test between the two populations (experimental and control group) with significance level equal to 0.05 and confidence interval equal to 95% (without missing values). As shown in Fig. 5, there is a significant difference between the two populations with respect to the time to perform the task.

![Figure 5](image)

**Figure 5.** Statistical comparison of the time to realize the task in seconds.

Specifically, the experimental and control group were able to realize the task with a mean time of 212 s and 835 s, respectively, and with a standard deviation of 67 s and 344 s.

The same test was performed on the vote expressed by the commission. As shown in Fig. 6, there is not a significant difference between the votes received by the two populations.

In particular, the experimental and control group received a vote equal to 36 and 35, respectively, with a standard deviation of 3.4 and 3.0 votes.

5. Discussion

Focusing the attention on the task speed of the two groups, it was noted that the experimental group exhibited a mean time to perform the task that was significantly smaller ($p<0.001$) than the mean time of the control group. Furthermore, the standard deviation associated to this variable is equal to 67 s for the experimental group while it is close to 345 s for the control group. Thus, the task time is more predictable using the fuzzy approach.

This result can be associated to the linguistic interface of the fuzzy approach that is more adapt to the human reasoning than the pure numeric method. In addition, the experimental group adopted an algorithm that is based on solved problems (high level), while the control group used an algorithm based on the direct setting of the parameters of the motion profiles (low level) that needed to solve problems through the knowledge of the parameter effects on the dynamics of the system. This characteristic has played a crucial role in speed reduction between the two groups of designers.
At the end of the task, the quality of the proposed solution was evaluated with a blind peer review process and the experimental group obtained a mean score equal to 36, whereas the control group obtained a mean score equal to 35. This difference is not statistically significant (p=0.362). In the same way, the standard deviations are comparable: 3.4 s for the experimental group and 3.0 s for the control group. However, the sample size is small to detect a significant difference, in fact, performing a sample analysis based on the mean values and on the standard deviations of the obtained votes, it is possible to forecast a significant difference when the sample size is greater than 406 subjects. From a practical viewpoint, the vote difference is not excessive and the designers are able to obtain similar results, eventually incrementing the task time (for the control group), in fact they have an strong ability to evaluate the result quality and compensate the lack of design instruments incrementing their amount of work. Probably, the use of the control approach is more intellectually demanding than the use of the experimental algorithm and it needs more professional skills. This aspect can be studied in future works performing a statistical analysis on a large group of designers stratified by year of experience.

6. Conclusion

This work presents a fuzzy approach to design motion profiles of automatic machines, applied to cam indexing systems. The fuzzy theory provides a mathematical theory designed to model the vague variables and helps the designer in selecting effective solutions. This study proposes a mix of design problems, generating a single fuzzy multiobjective problem. The global solution is represented by a fuzzy mix of each single solution related to each single minor problem. The final result may be used to support a designer in the definition of a proper motion profile and to generate an initial solution for optimization algorithms. The proposed approach has been implemented in a software program applied to the design indexing systems with a visual interface. In particular, this study also wants to compare the fuzzy logic with the traditional approach in order to understand advantages and constraints during the design stage. The fuzzy logic has been tested on a group of designers divided in two groups. The experimental group applied the fuzzy approach while the
control group used a conventional method in performing the design task. The research results highlighted that the experimental group exhibited a higher and more repeatable speed than the control. In particular, the time to realize the task was unlimited, so the control group designers were able to compensate the complexity of their design tool simply incrementing the working time. In the light of these results, the next steps of the research will be based on the increment of the designer sample, emphasizing the grade of problem complexity and limiting the task time. In this way, the fuzzy logic may highlight all potential benefits due to a structured method in process criteria decision.

Author Contributions: All authors contributed extensively to the study presented in this manuscript.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

The model of a special adimensional trigonometric spline is presented. It is defined on a domain \([0;1]\) and divided in seven subintervals \(\delta_i\) \((a1)\). The codomain of the function is \([0;1]\).

\[
\hat{\alpha}_i = \sum_{j=1}^{i} \delta_j, \quad \hat{\alpha}_7 = 1, \quad (a1)
\]

The trigonometric spline is defined in the geometric acceleration diagram, as shown in \((a2)\), where the values of the constant terms \(A\) and \(B\) can be obtained imposing the constraints \((a3)\) with the linear system \((a4)\), where the \(a_{ij}\) coefficients are shown in \((a5)\). Then, the expression of the geometric speed \((a6)\) and of the shape of the cam \((a7)\) can be obtained through integration of the expression \((a2)\), constraining the continuity of these expression with a proper selection of the constants of integration. Finally the dimensional expressions of the shape \(\beta(\alpha)\) of the cam and of its geometrical derivatives can be obtained from equations \((a8)\), where \(\alpha_M\) and \(\beta_M\) are the maximum values of \(\alpha\) and \(\beta\) during the moving phase, i.e., the angular extensions of the moving phase.

\[
\tilde{\beta}^s(\hat{\alpha}) = \begin{cases} 
A \cdot \sin \left(\frac{\hat{\alpha} \cdot \pi}{2 \cdot \delta_1}\right) & \text{if } 0 \leq \hat{\alpha} < \hat{\alpha}_1 \\
A & \text{if } \hat{\alpha}_1 \leq \hat{\alpha} < \hat{\alpha}_2 \\
A \cdot \cos \left(\frac{\hat{\alpha} - \hat{\alpha}_2 \cdot \pi}{2 \cdot \delta_2}\right) & \text{if } \hat{\alpha}_2 \leq \hat{\alpha} < \hat{\alpha}_3 \\
0 & \text{if } \hat{\alpha}_3 \leq \hat{\alpha} < \hat{\alpha}_4' \\
-B \cdot \sin \left(\frac{\hat{\alpha} - \hat{\alpha}_4 \cdot \pi}{2 \cdot \delta_3}\right) & \text{if } \hat{\alpha}_4 \leq \hat{\alpha} < \hat{\alpha}_5 \\
-B & \text{if } \hat{\alpha}_5 \leq \hat{\alpha} < \hat{\alpha}_6 \\
-B \cdot \cos \left(\frac{\hat{\alpha} - \hat{\alpha}_6 \cdot \pi}{2 \cdot \delta_7}\right) & \text{if } \hat{\alpha}_6 \leq \hat{\alpha} < \hat{\alpha}_7 \\
\end{cases} \quad (a2)
\]

\[
\hat{\beta}(\hat{\alpha})_{|_{k=1}} = 1, \quad \hat{\beta}'(\hat{\alpha})_{|_{k=1}} = 0, \quad (a3)
\]

\[
\begin{align*}
A &= \frac{-a_{12}}{a_{11} \cdot a_{22} - a_{12} \cdot a_{21}} \\
B &= \frac{a_{11}}{a_{11} \cdot a_{22} - a_{12} \cdot a_{21}} \\
\end{align*} \quad (a4)
\]
\[ a_{11} = \frac{2 \cdot \delta_i + \delta_j}{\pi} + \frac{2 \cdot \delta_i}{\pi} \]
\[ a_{12} = -\frac{2 \cdot \delta_i - \delta_j}{\pi} - \frac{2 \cdot \delta_i}{\pi} \]
\[ a_{21} = \frac{2 \cdot \delta_i}{\pi} \cdot \left( \frac{\pi - 2 \cdot \delta_i + \frac{\delta_j}{2}}{\pi} \right) + \frac{2 \cdot \delta_i}{\pi} \cdot \left( \frac{\delta_j}{2} + \frac{\pi - 2 \cdot \delta_i}{\pi} \right) + \left( \frac{\frac{\delta_j}{2} + \frac{\pi - 2 \cdot \delta_i}{\pi}}{\pi} \right) \]
\[ a_{22} = \left( \frac{\pi - 2 \cdot \delta_i + \frac{\delta_j}{2}}{\pi} \right) + \frac{2 \cdot \delta_i}{\pi} \cdot \left( \frac{\delta_j}{2} + \frac{\pi - 2 \cdot \delta_i}{\pi} \right) \]

\[
\beta' (\hat{\alpha}) = \begin{cases} 
\frac{A \cdot 2 \cdot \delta_i}{\pi} \cdot \left[ 1 - \cos \left( \frac{\hat{\alpha} \cdot \pi}{2 \cdot \delta_i} \right) \right] & \text{if } 0 \leq \hat{\alpha} < \hat{\alpha}_1 \\
\hat{\beta}^* (\hat{\alpha}_1) + A \cdot (\hat{\alpha} - \hat{\alpha}_1) & \text{if } \hat{\alpha}_1 \leq \hat{\alpha} < \hat{\alpha}_2 \\
\hat{\beta}^* (\hat{\alpha}_2) + A \cdot 2 \cdot \delta_i \cdot \left[ \sin \left( \frac{(\hat{\alpha} - \hat{\alpha}_2) \cdot \pi}{2 \cdot \delta_i} \right) \right] & \text{if } \hat{\alpha}_2 \leq \hat{\alpha} < \hat{\alpha}_3 \\
\hat{\beta}^* (\hat{\alpha}_3) - B \cdot 2 \cdot \delta_i \cdot \left[ 1 - \cos \left( \frac{(\hat{\alpha} - \hat{\alpha}_3) \cdot \pi}{2 \cdot \delta_i} \right) \right] & \text{if } \hat{\alpha}_3 \leq \hat{\alpha} < \hat{\alpha}_4 \\
\hat{\beta}^* (\hat{\alpha}_4) - B \cdot 2 \cdot \delta_i \cdot \left[ \sin \left( \frac{(\hat{\alpha} - \hat{\alpha}_4) \cdot \pi}{2 \cdot \delta_i} \right) \right] & \text{if } \hat{\alpha}_4 \leq \hat{\alpha} \leq \hat{\alpha}_5 \\
\end{cases}
\]

\[
\beta (\hat{\alpha}) = \begin{cases} 
\frac{A \cdot 2 \cdot \delta_i}{\pi} \cdot \left[ \hat{\alpha} - 2 \cdot \delta_i \cdot \sin \left( \frac{\hat{\alpha} \cdot \pi}{2 \cdot \delta_i} \right) \right] & \text{if } 0 \leq \hat{\alpha} < \hat{\alpha}_1 \\
\hat{\beta}' (\hat{\alpha}_1) + \hat{\beta}^* (\hat{\alpha}_1) \cdot (\hat{\alpha} - \hat{\alpha}_1) + A \cdot \left( \frac{\hat{\alpha} - \hat{\alpha}_1}{2} \right)^2 & \text{if } \hat{\alpha}_1 \leq \hat{\alpha} < \hat{\alpha}_2 \\
\hat{\beta}' (\hat{\alpha}_2) + \hat{\beta}^* (\hat{\alpha}_2) \cdot (\hat{\alpha} - \hat{\alpha}_2) + A \cdot \left( \frac{2 \cdot \delta_i}{\pi} \right)^2 \cdot \left[ 1 - \cos \left( \frac{\hat{\alpha} - \hat{\alpha}_2}{2 \cdot \delta_i} \right) \right] & \text{if } \hat{\alpha}_2 \leq \hat{\alpha} < \hat{\alpha}_3 \\
\hat{\beta}' (\hat{\alpha}_3) + \hat{\beta}^* (\hat{\alpha}_3) \cdot (\hat{\alpha} - \hat{\alpha}_3) & \text{if } \hat{\alpha}_3 \leq \hat{\alpha} < \hat{\alpha}_4 \\
\hat{\beta}' (\hat{\alpha}_4) + \hat{\beta}^* (\hat{\alpha}_4) \cdot (\hat{\alpha} - \hat{\alpha}_4) - B \cdot \left( \frac{\hat{\alpha} - \hat{\alpha}_4}{2} \right)^2 & \text{if } \hat{\alpha}_4 \leq \hat{\alpha} < \hat{\alpha}_5 \\
\hat{\beta}' (\hat{\alpha}_5) + \hat{\beta}^* (\hat{\alpha}_5) \cdot (\hat{\alpha} - \hat{\alpha}_5) & \text{if } \hat{\alpha}_5 \leq \hat{\alpha} < \hat{\alpha}_6 \\
\hat{\beta}' (\hat{\alpha}_6) + \hat{\beta}^* (\hat{\alpha}_6) \cdot (\hat{\alpha} - \hat{\alpha}_6) & \text{if } \hat{\alpha}_6 \leq \hat{\alpha} \leq \hat{\alpha}_7 \\
\end{cases}
\]
\[ \alpha = \hat{\alpha} \cdot \alpha_M \]

\[ \beta(\alpha) = \hat{\beta} \left( \frac{\alpha}{\alpha_M} \right) \cdot \beta_M, \quad \beta'(\alpha) = \hat{\beta}' \left( \frac{\alpha}{\alpha_M} \right) \cdot \beta_M, \quad \beta''(\alpha) = \hat{\beta}'' \left( \frac{\alpha}{\alpha_M} \right) \cdot \beta_M \]  

(a8)

References


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