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THE MODIFIED KUDRYASHOV METHOD FOR THE CONFORMABLE TIME FRACTIONAL (3+1)- DIMENSIONAL KADOMTSEV-PETVIASHVILI AND THE MODIFIED KAWAHARA EQUATIONS

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Abstract. The three dimensional conformable time fractional Kadomtsev-Petviashvili and the conformable time fractional modified Kawahara equations are solved by implementing the Kudryashov's procedure. The corresponding wave transformation reduces both equations to some ODEs. Balancing the nonlinear and the highest order derivative terms gives the structure of the solutions in the finite series form. The useful symbolic tools are used to solve the resultant algebraic systems. The solutions are expressed in explicit forms.

1. Introduction

The Kadomtsev-Petviashvili (KP) equation (sometimes it is called as the two-dimensional Korteweg- de Vries equation) appears in the earlier 70s to study the stability of solitary waves in weakly dispersive media covering fluids or plasma[1]. It is an integrable equation owing to the fact that there exists a connection with a linear spectral problem to a particular Schrödinger equation[2, 3]. Modulation equations of the two-dimensional KP are developed by examining Riemann invariants problem[4]. The two- dimensional form of the KP equation passes the Painlevé(P-) test for the integrability and has soliton-type solutions[5]. The evolution of water wave packages traveling in one direction strongly despite the slowly modulated amplitudes in both directions are studied in details by Ablowitz and Segur[6].

The three- dimensional form of the KP equation is a model to describe rapidly propagating magnetosonic waves of small amplitudes in a low β-magnetized plasma of super fluid helium[7]. The properties of soliton dynamics are also examined deeply for slightly perturbations along z direction in the same study. Bouard and Saut[8] perform a classification of the existence of the localized solitary-type waves by the sign of the transverse dispersion term and by the nonlinearity. The solitary waves of the three- dimensional KP equation both are in cylindrical form for the transverse variables and decay with algebraically optimal rate[9]. Beside the existence of the solitary wave solutions, the three- dimensional KP equation has cnoidal wave solutions to be written as infinite sum of solitons[10] and traveling wave solutions to be expressed in the form of hyperbolic, rational and trigonometric functions[11].

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The modified Kawahara (mKawahara) equation is a cubic non-linear equation with a fifth-order derivative term. The quadratic nonlinear form of the equation is suggested by Kawahara [12] with its steady solutions. Such solutions may exist due to the sign of the dispersive term and these solutions can be in a oscillatory form because of the dominant fifth-ordered term. The mKawahara equation has implicit doubly periodic solutions and they can be determined by using the method of auxiliary equation[13, 14]. Some soliton and periodic solutions are derived by using a set of forecasting methods having some trigonometric and hyperbolic functions inside[15]. The mKawahara equation has also non-constant meromorphic solutions in explicit form[16]. The existence of traveling wave type solutions in hyperbolic or trigonometric forms are discussed by Al-Ali[17]. Tanh and exp-function methods are also capable to derive the exact solutions to the mKawahara equation[18].
So far, some methods belonging to different categories have been proposed for the exact solutions of both integer or fractional ordered PDEs [19, 20, 21, 22]. In ths study ,The main aim is to derive some exact solutions to the three-dimensional conformable time fractional KP and the conformable time fractional mKawahara equations. The solutions are obtained explicitly by using modified Kudryashov method. The existence of the chain rule and other required properties in the definition of the conformable derivative enable some wave transformations to generate the solutions.

2. Conformable Derivative and Some Significant Properties

Let α be $\in (0,1]$. Then, the conformable derivative of $f = f(\tau)$ defined in the positive half space $\tau > 0$ is given as

$$D_\tau^\alpha(f(\tau)) = \lim_{h\to 0} \frac{f(\tau + h\tau^{1-\alpha}) - f(\tau)}{h},\ \tau > 0,\ \alpha \in (0,1] \tag{2.1}$$

for $f : [0,\infty) \to \mathbb{R}$[23]. Even though it has just been defined, some significant properties covering derivative of multiplication or division of two functions. The following two theorems give a brief summary of those properties.

Theorem 1. *The conformable derivative of order $\alpha \in (0,1]$ for the α-differentiable functions $u = u(\tau)$ and $w = w(\tau)$ for all positive τ satisfies*

- $D_\tau^\alpha(c_1 u + c_2 w) = c_1 D_\tau^\alpha(u) + c_2 D_\tau^\alpha(w)$
- $D_\tau^\alpha(\tau^p) = p\tau^{p-\alpha}, \forall p \in \mathbb{R}$
- $D_\tau^\alpha(u(\tau)) = 0$, *when* $u(\tau) = c_3$ *is a constant function*
- $D_\tau^\alpha(uw) = uD_\tau^\alpha(w) + wD_\tau^\alpha(u)$
- $D_\tau^\alpha(\frac{u}{w}) = \dfrac{uD_\tau^\alpha(w) - wD_\tau^\alpha(u)}{w^2}$
- $D_\tau^\alpha(u)(\tau) = \tau^{1-\alpha}\frac{du}{d\tau}$

for all real c_1, c_2, c_3[24, 25].

Moreover, the definitions of many further properties of the conformable derivative (2.1) are discussed in details in [26]. The Gronwall's inequality, integration by parts, Laplace transform, the conformable derivative of the composite function and more are described in that study.

3. The Modified Kudryashov Method

Let P be

$$P(u, u_\tau^\alpha, u_x, u_y, u_z, u_{\tau\tau}^\alpha, u_{xx}, \ldots) = 0 \tag{3.1}$$

where $u = u(x, y, z, \ldots, \tau)$ and $\alpha \in (0, 1]$ be the fractional derivative order. The transformation

$$u(x, y, z, \ldots, \tau) = u(\xi), \xi = x + y + z + \ldots - \frac{c}{\alpha}\tau^{\alpha} \tag{3.2}$$

converts (3.1) to an ODE for new variable ξ

$$R(u, u', u'', \ldots) = 0 \tag{3.3}$$

where the prime ($'$) indicates the derivative operator $\frac{d}{d\xi}$ of u with respect to ξ[27]. Let

$$u(\xi) = a_0 + a_1 U(\xi) + a_2 U^2(\xi) + \ldots a_n U^n(\xi) \tag{3.4}$$

be predicted solution of the equation (3.3) for a finite n with all $a_i, 0 \leq i \leq n$ are constants satisfying $a_n \neq 0$. In fact, the n is determined by balancing nonlinear term and the maximal derivative order. Furthermore, this finite series of $U(\xi)$ satisfies the first-order ODE

$$U^{'}(\xi) = U(\xi)(U(\xi) - 1)\ln A \tag{3.5}$$

Accordingly, $U(\xi)$ is of the form

$$U(\xi) = \frac{1}{1 + dA^{\xi}}$$

where d and A are non-zero constants with the conditions $A > 0$ and $A \neq 1$. The balance between the non-linear term and the term having the maximal order derivative in (3.3) enables to determine the positive integer n, if exists. Ultimately, since a solution $u(\xi)$ is sought for (3.3), it must satisfy (3.3). Thus, it is substituted into (3.3) and the resultant equation is rearranged for the powers $U(\xi)$. All the coefficients of the powers of $U(\xi)$ including the remaining part including constants and other parameters are equated to zero. Finally, $a_0, a_1, a_2, \ldots a_n$ are obtained explicitly in terms of constants and coefficients used in the equation (3.1) or originated from the (3.2).

4. The Conformable Time Fractional (3+1) -dimensional KP equation

Consider the (3+1)- dimensional KP equation of the form

$$(D_{\tau}^{\alpha}(u) + \theta u u_x + \beta u_{xxx})_x - \varepsilon u_{yy} - \delta u_{zz} = 0 \tag{4.1}$$

where $u = u(x, y, z, \tau)$, θ, β, ε and δ are parameters. The transformation (3.2) reduces the KP equation (4.1) to

$$\left(-cu^{'} + \theta u u^{'} + \beta u^{'''}\right)^{'} - (\varepsilon + \delta)\, u^{''} = 0 \tag{4.2}$$

where $\left({}^{'}\right) = \frac{d}{d\xi}$ derivative operator. Integrating the last equation twice gives

$$-cu + \frac{\theta}{2}u^2 + \beta u^{''} - (\varepsilon + \delta)\, u = K_1 \xi + K_2 \tag{4.3}$$

where K_1 and K_2 are constants of integration. The balance between u^2 and $u^{''}$ occurs when $n = 2$. Assume that K_1 is zero. Substitution of the predicted solution

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$u(\xi) = a_0 + a_1 U(\xi) + a_2 U^2(\xi)$ into (4.3) and assuming $K_1 = 0$ gives

$$\begin{aligned}
&\left(\frac{1}{2}\,\theta\,{a_2}^2 + 6\,\beta\,a_2\,(\ln(A))^2\right) U^4(\xi) \\
&+ \left(2\,\beta\,a_1\,(\ln(A))^2 - 10\,\beta\,a_2\,(\ln(A))^2 + \theta\,a_1 a_2\right) U^3(\xi) \\
&+ \left(\frac{1}{2}\,\theta\,{a_1}^2 - \delta\,a_2 - c a_2 - \varepsilon\,a_2 + \theta\,a_0 a_2 - 3\,\beta\,a_1\,(\ln(A))^2 + 4\,\beta\,a_2\,(\ln(A))^2\right) U^2(\xi) \\
&+ \left(-c a_1 - \delta\,a_1 - \epsilon\,a_1 + \beta\,a_1\,(\ln(A))^2 + \theta\,a_0 a_1\right) U(\xi) \\
&- K_2 - \varepsilon\,a_0 - \delta\,a_0 - c a_0 + \frac{1}{2}\,\theta\,{a_0}^2 = 0
\end{aligned} \tag{4.4}$$

Thus, equating the coefficients of the powers of each $U(\xi)$ and the remaining constants to zero gives

$$\begin{aligned}
&U^4 : \frac{1}{2}\,\theta\,{a_2}^2 + 6\,\beta\,a_2\,(\ln(A))^2 = 0 \\
&U^3 : 2\,\beta\,a_1\,(\ln(A))^2 - 10\,\beta\,a_2\,(\ln(A))^2 + \theta\,a_1 a_2 = 0 \\
&U^2 : \frac{1}{2}\,\theta\,{a_1}^2 - \delta\,a_2 - c a_2 - \varepsilon\,a_2 + \theta\,a_0 a_2 - 3\,\beta\,a_1\,(\ln(A))^2 + 4\,\beta\,a_2\,(\ln(A))^2 = 0 \\
&U^1 : -c a_1 - \delta\,a_1 - \epsilon\,a_1 + \beta\,a_1\,(\ln(A))^2 + \theta\,a_0 a_1 = 0 \\
&U^0 : -K_2 - \varepsilon\,a_0 - \delta\,a_0 - c a_0 + \frac{1}{2}\,\theta\,{a_0}^2 = 0
\end{aligned} \tag{4.5}$$

The first solution set of the last system constituting five algebraic equations for $\{a_0, a_1, a_2, c\}$ gives

$$\begin{aligned}
a_0 &= \frac{-\beta\,(\ln(A))^2 + \sqrt{\beta^2\,(\ln(A))^4 - 2\,\theta\,K_2}}{\theta} \\
a_1 &= 12\,\frac{\beta\,(\ln(A))^2}{\theta} \\
a_2 &= -12\,\frac{\beta\,(\ln(A))^2}{\theta} \\
c &= -\delta - \varepsilon + \sqrt{\beta^2\,(\ln(A))^4 - 2\,\theta\,K_2}
\end{aligned} \tag{4.6}$$

where $\beta^2\,(\ln(A))^4 - 2\,\theta\,K_2 \geq 0$ and $\theta \neq 0$. The solution of the (4.3) takes the form

$$\begin{aligned}
U(\xi) = &\frac{-\beta\,(\ln(A))^2 + \sqrt{\beta^2\,(\ln(A))^4 - 2\,\theta\,K_2}}{\theta} + \frac{\beta\,(\ln(A))^2}{\theta}\,\frac{1}{1 + dA^{\xi}} \\
&- 12\,\frac{\beta\,(\ln(A))^2}{\theta}\,\frac{1}{(1 + dA^{\xi})^2}
\end{aligned} \tag{4.7}$$

Returning the original variables $\{x, y, z, \tau\}$ gives the solution for the conformable time fractional KP equation (4.1) as

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$$
\begin{aligned}
u_1(x,y,z,\tau) = {} & \frac{-\beta\,(\ln(A))^2+\sqrt{\beta^2\,(\ln(A))^4-2\,\theta\,K_2}}{\theta} \\
& + \frac{\beta\,(\ln(A))^2}{\theta}\,\frac{1}{1+dA^{x+y+z-\frac{(-\delta-\varepsilon+\sqrt{\beta^2\,(\ln(A))^4-2\,\theta\,K_2})\tau^\alpha}{\alpha}}} \\
& - 12\,\frac{\beta\,(\ln(A))^2}{\theta}\,\frac{1}{\left(1+dA^{x+y+z-\frac{(-\delta-\varepsilon+\sqrt{\beta^2\,(\ln(A))^4-2\,\theta\,K_2})\tau^\alpha}{\alpha}}\right)^2}
\end{aligned}
\tag{4.8}
$$

The second solution of the system (4.5) for $\{a_0, a_1, a_2, c\}$ gives

$$
\begin{aligned}
a_0 &= -\frac{\beta\,(\ln(A))^2+\sqrt{\beta^2\,(\ln(A))^4-2\,\theta\,K_2}}{\theta} \\
a_1 &= 12\,\frac{\beta\,(\ln(A))^2}{\theta} \\
a_2 &= -12\,\frac{\beta\,(\ln(A))^2}{\theta} \\
c &= -\delta-\varepsilon-\sqrt{\beta^2\,(\ln(A))^4-2\,\theta\,K_2}
\end{aligned}
\tag{4.9}
$$

where $\beta^2\,(\ln(A))^4-2\,\theta\,K_2 \geq 0$ and $\theta \neq 0$. Thus, the solution of the (4.3) is determined as

$$
\begin{aligned}
U(\xi) = {} & -\frac{\beta\,(\ln(A))^2+\sqrt{\beta^2\,(\ln(A))^4-2\,\theta\,K_2}}{\theta} + 12\,\frac{\beta\,(\ln(A))^2}{\theta}\,\frac{1}{1+dA^\xi} \\
& - 12\,\frac{\beta\,(\ln(A))^2}{\theta}\,\frac{1}{(1+dA^\xi)^2}
\end{aligned}
\tag{4.10}
$$

Ultimately, the solution of (4.1) is written in original variables as

$$
\begin{aligned}
u_2(x,y,z,\tau) = &-\frac{\beta\,(\ln(A))^2+\sqrt{\beta^2\,(\ln(A))^4-2\,\theta\,K_2}}{\theta} \\
&+12\,\frac{\beta\,(\ln(A))^2}{\theta}\frac{1}{1+dA^{x+y+z-\frac{(-\delta-\varepsilon-\sqrt{\beta^2\,(\ln(A))^4-2\,\theta\,K_2})\tau^\alpha}{\alpha}}} \\
&-12\,\frac{\beta\,(\ln(A))^2}{\theta}\frac{1}{\left(1+dA^{x+y+z-\frac{(-\delta-\varepsilon-\sqrt{\beta^2\,(\ln(A))^4-2\,\theta\,K_2})\tau^\alpha}{\alpha}}\right)^2}
\end{aligned}
\tag{4.11}
$$

5. The Conformable Time Fractional mKawahara Equation

The conformable time fractional mKawahara equation

$$D_\tau^\alpha(u)+pu^2u_x+qu_{xxx}+ru_{xxxxx}=0 \tag{5.1}$$

where $u=u(x,\tau)$ and p, q and r are constant parameters can be reduced to a nonlinear ODE of the form

$$-cu^{'}+pu^2u^{'}+qu^{'''}+ru^{'''''}=0 \tag{5.2}$$

by using the simplest form of the transformation (3.2) that is $u(\xi)=u(x,\tau)$, $\xi=x-c\frac{\tau^\alpha}{\alpha}$. Integrating the last equation once converts it to

$$-cu+\frac{p}{3}u^3+qu^{''}+ru^{''''}=K \tag{5.3}$$

where K denotes the constant of integration. The balance between u^3 and $u^{''''}$ determines the degree n of the solution as 2. Thus, the predicted solution is constructed in the form $u(\xi)=a_0+a_1U(\xi)+a_2U^2(\xi)$ with the condition $a_2\neq 0$. Substituting this solution into the equation (5.3) results

$$
\begin{aligned}
&\left(1/3\,p{a_2}^3+120\,ra_2\,(\ln(A))^4\right)U^6(\xi) \\
&+\left(pa_1{a_2}^2-336\,ra_2\,(\ln(A))^4+24\,ra_1\,(\ln(A))^4\right)U^5(\xi) \\
&+\left(p{a_1}^2a_2+6\,qa_2\,(\ln(A))^2+330\,ra_2\,(\ln(A))^4+pa_0{a_2}^2-60\,ra_1\,(\ln(A))^4\right)U^4(\xi) \\
&+\left(-130\,ra_2\,(\ln(A))^4+2\,pa_0a_1a_2+2\,qa_1\,(\ln(A))^2+1/3\,p{a_1}^3-10\,qa_2\,(\ln(A))^2+50\,ra_1\,(\ln(A))^4\right)U^3(\xi) \\
&+\left(-15\,ra_1\,(\ln(A))^4-ca_2+pa_0{a_1}^2+4\,qa_2\,(\ln(A))^2+16\,ra_2\,(\ln(A))^4+p{a_0}^2a_2-3\,qa_1\,(\ln(A))^2\right)U^2(\xi) \\
&+\left(ra_1\,(\ln(A))^4-ca_1+qa_1\,(\ln(A))^2+p{a_0}^2a_1\right)U(\xi) \\
&-ca_0+1/3\,p{a_0}^3-K=0
\end{aligned}
\tag{5.4}
$$

Equating the coefficients of each power $U(\xi)$ gives

$$
\begin{aligned}
&U^6 : 1/3\, p{a_2}^3 + 120\, r a_2 (\ln(A))^4 = 0\\
&U^5 : p a_1 {a_2}^2 - 336\, r a_2 (\ln(A))^4 + 24\, r a_1 (\ln(A))^4 = 0\\
&U^4 : p{a_1}^2 a_2 + 6\, q a_2 (\ln(A))^2 + 330\, r a_2 (\ln(A))^4 + p a_0 {a_2}^2 - 60\, r a_1 (\ln(A))^4 = 0\\
&U^3 : -130\, r a_2 (\ln(A))^4 + 2\, p a_0 a_1 a_2 + 2\, q a_1 (\ln(A))^2 + 1/3\, p{a_1}^3 - 10\, q a_2 (\ln(A))^2 + 50\, r a_1 (\ln(A))^4 = 0\\
&U^2 : -15\, r a_1 (\ln(A))^4 - c a_2 + p a_0 {a_1}^2 + 4\, q a_2 (\ln(A))^2 + 16\, r a_2 (\ln(A))^4 + p{a_0}^2 a_2 - 3\, q a_1 (\ln(A))^2 = 0\\
&U^1 : r a_1 (\ln(A))^4 - c a_1 + q a_1 (\ln(A))^2 + p{a_0}^2 a_1 = 0\\
&U^0 : -c a_0 + 1/3\, p{a_0}^3 - K = 0
\end{aligned}
\tag{5.5}
$$

The solution of this algebraic system for $\{a_0, a_1, a_2, c, K\}$ gives

$$
\begin{aligned}
a_0 &= \frac{1}{10r}\sqrt{-10\,\frac{r}{p}}\left(q + 5\,r(\ln(A))^2\right)\\
a_1 &= -6\sqrt{-10\,\frac{r}{p}}\,(\ln(A))^2\\
a_2 &= 6\sqrt{-10\,\frac{r}{p}}\,(\ln(A))^2\\
c &= -\frac{1}{10r}\left(15\,r^2(\ln(A))^4 + q^2\right)\\
K &= \frac{1}{150r^2}\left(-15\,qr^2(\ln(A))^4 + 50\,r^3(\ln(A))^6 + q^3\right)\sqrt{-10\,\frac{r}{p}}
\end{aligned}
\tag{5.6}
$$

with the condition $rp < 0$. Thus, the solution of (5.2) can be written as

$$
\begin{aligned}
U(\xi) = {} & \frac{1}{10r}\sqrt{-10\,\frac{r}{p}}\left(q + 5\,r(\ln(A))^2\right) - 6\sqrt{-10\,\frac{r}{p}}\,(\ln(A))^2\,\frac{1}{1+dA^{\xi}}\\
& + 6\sqrt{-10\,\frac{r}{p}}\,(\ln(A))^2\,\frac{1}{(1+dA^{\xi})^2}
\end{aligned}
\tag{5.7}
$$

Returning the original variables x and τ gives the solution of (5.1) as

$$
\begin{aligned}
u_3(x,\tau) = {} & \frac{1}{10r}\sqrt{-10\,\frac{r}{p}}\left(q + 5\,r(\ln(A))^2\right) - 6\sqrt{-10\,\frac{r}{p}}\,(\ln(A))^2\,\frac{1}{1+dA^{x+\frac{1}{10r}\left(15\,r^2(\ln(A))^4+q^2\right)\frac{t^{\alpha}}{\alpha}}}\\
& + 6\sqrt{-10\,\frac{r}{p}}\,(\ln(A))^2\,\frac{1}{\left(1+dA^{x+\frac{1}{10r}\left(15\,r^2(\ln(A))^4+q^2\right)\frac{t^{\alpha}}{\alpha}}\right)^2}
\end{aligned}
\tag{5.8}
$$

for $pr < 0$.

A second solution to (5.5) for $\{a_0, a_1, a_2, c, K\}$ is of the form

$$
\begin{aligned}
a_0 &= -\frac{1}{10r}\sqrt{-10\,\frac{r}{p}}\left(q+5\,r\,(\ln(A))^2\right)\\
a_1 &= 6\sqrt{-10\,\frac{r}{p}}\,(\ln(A))^2\\
a_2 &= -6\sqrt{-10\,\frac{r}{p}}\,(\ln(A))^2\\
c &= -\frac{1}{10r}\left(15\,r^2\,(\ln(A))^4+q^2\right)\\
K &= -\frac{1}{150r^2}\left(-15\,qr^2\,(\ln(A))^4+50\,r^3\,(\ln(A))^6+q^3\right)\sqrt{-10\,\frac{r}{p}}
\end{aligned}
\tag{5.9}
$$

and generates the solution to (5.2) as

$$
\begin{aligned}
U(\xi) = &-\frac{1}{10r}\sqrt{-10\,\frac{r}{p}}\left(q+5\,r\,(\ln(A))^2\right)+6\sqrt{-10\,\frac{r}{p}}\,(\ln(A))^2\,\frac{1}{1+dA^{\xi}}\\
&-6\sqrt{-10\,\frac{r}{p}}\,(\ln(A))^2\,\frac{1}{(1+dA^{\xi})^2}
\end{aligned}
\tag{5.10}
$$

Similarly, the solution to the conformable time fractional mKawahara equation (5.1) is obtained as

$$
\begin{aligned}
u_4(x,\tau) = &-\frac{1}{10r}\sqrt{-10\,\frac{r}{p}}\left(q+5\,r\,(\ln(A))^2\right)+6\sqrt{-10\,\frac{r}{p}}\,(\ln(A))^2\,\frac{1}{1+dA^{x+\left(\frac{1}{10r}\,15\,r^2(\ln(A))^4+q^2\right)\frac{\tau^{\alpha}}{\alpha}}}\\
&-6\sqrt{-10\,\frac{r}{p}}\,(\ln(A))^2\,\frac{1}{\left(1+dA^{x+\left(\frac{1}{10r}\,15\,r^2(\ln(A))^4+q^2\right)\frac{\tau^{\alpha}}{\alpha}}\right)^2}
\end{aligned}
\tag{5.11}
$$

where pr is negative.

6. Conclusion

The modified form of the Kudryashov method is implemented for the exact solutions of the conformable time fractional $(3+1)$- dimensional KP and the modified Kawahara equations. The corresponding wave transformations reduce the number of independent variables to one. Thus, both equations are converted some nonlinear ODEs. Balancing the highest ordered derivative term and the nonlinear term leads to determine the degree of the solution in the form of finite series. Substituting the predicted solution into the resultant ODE and equating all coefficients of the powers of the predicted solution gives an algebraic system of equations. Thus, the coefficients of the predicted solution are determined in explicit form by computer aided algebra.

In the study, a couple of explicit solutions to each conformable fractional (3+1) KP and the modified Kawahara equations are determined in the rational form containing exponential function by the modified Kudryashov method.

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