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# A Hierarchical Risk Assessment Model using the Evidential Reasoning Rule

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**Abstract:** This paper aims to develop a hierarchical risk assessment model using the newly-developed evidential reasoning (ER) rule, which constitutes a generic conjunctive probabilistic reasoning process. In this paper, we first provide a brief introduction to the basics of the ER rule and emphasize the strengths for representing and aggregating uncertain information from multiple experts and sources. Further, we discuss the key steps of developing the hierarchical risk assessment framework systematically, including (1) formulation of risk assessment hierarchy, (2) representation of both qualitative and quantitative information, (3) elicitation of attribute weights and information reliabilities, (4) aggregation of assessment information using the ER rule and (5) quantification and ranking of risks using utility-based transformation. The proposed hierarchical risk assessment framework can potentially be implemented to various complex and uncertain systems. A case study on the fire/explosion risk assessment of marine vessels demonstrates the applicability of the proposed risk assessment model.

**Keywords:** Risk assessment; Evidential reasoning; Fire/explosion

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## 1. Introduction

Risk assessment plays a vital role in the whole risk management cycle from identifying, assessing, analysing, reporting and manipulating to monitoring risks. It helps decision makers to prioritise and manage risks in order to avoid potential threats and better utilise limited sources. Traditionally, risk is defined as a combination of likelihood and consequence. However, as many real-world systems are becoming increasingly complicated, along with the appearance of unexpected events and dramatic changes, the two high-level measurements of likelihood and consequence are unable to completely capture the entire characteristics of a risk (Curtis and Carey, 2012). In the past decades, a range of risk assessment methods, including preliminary hazard analysis, fault tree analysis, event tree analysis, and relevant quantitative techniques have been proposed to support risk management (Cox Jr, 2009; Rausand, 2013).

Compared with qualitative risk assessment techniques, which mainly focus on risk identification and evaluation, quantitative risk assessment techniques put more emphasis on the quantification as well as ranking of risks to support better decision making and perfect qualitative analysis. Under the umbrella of quantitative risk assessment methods, risk matrix has been widely accepted as a convenient and effective tool for risk assessment (Paul, 1998; Markowski and Mannan, 2008; Ni et al., 2010). Both qualitative and quantitative assessment can be incorporated into a risk matrix, where qualitative information, such as questionnaires and interviews, can be used to identify potential improvements, while quantitative information, such as historical data, can help to evaluate countable costs or benefits. In line with the definition of risks, the probability of occurrence and the severity of impact can be expressed as two input variables in the risk matrix. Their combination formulates an index to classify and discriminate different risks, and it can also be logically interpreted as "IF probability is  $p$  and severity is  $s$ , THEN risk is  $r$ " (Markowski and Mannan, 2008). Usually, both input

and output variables are described by qualitative scales in the risk matrix. For example, the probability of occurrence can be split into five levels, such as remote, unlikely, likely, high likely and almost certain, while the severity of impact can be categorised as negligible, minor, moderate, serious and critical.

As discussed above, risk matrix measures each risk mainly from two dimensions. However, many real-world systems are more complicated, and we may need to take into consideration more risk attributes and components in the risk evaluation process. It is difficult for the risk matrix to provide a holistic view from multiple aspects. Thus, multiple criteria decision analysis (MCDA) techniques can be employed to fill up the gap. MCDA provides a systematic process to formulate the hierarchical assessment model, aggregate assessment information and support better decision making (Roy, 1996). It can also be useful to select potential risks, categorise risks with similar characteristics and prioritise risks in terms of historical data and decision maker (DM)'s domain knowledge (Mayag et al., 2011). The evidential reasoning (ER) methodology among a series of MCDA techniques has attracted a lot of attention due to the capability of modelling qualitative and quantitative information in a unified way, aggregating probabilistic information rigorously and producing final distributed assessment results (Yang and Singh, 1994; Xu, 2012). Furthermore, the ER methodology was initially proposed in the context of MCDA, and it consists of three key components or features, specifically, belief structure for modelling various types of uncertainty (Yang and Singh, 1994), rule and utility based information transformation techniques (Yang 2001), and the ER algorithm for information aggregation (Yang and Xu 2002). The ER methodology has been widely applied to a wide range of decision and risk analysis problems (Wang et al., 2013; Liu et al., 2013; Dymova and Sevastjanov, 2014; Fu et al., 2014). The ER rule further improves the ER methodology, and it constitutes a generic conjunctive probabilistic reasoning process and combine multiple pieces of independent evidence conjunctively with taking into account both weights and reliabilities (Yang and Xu, 2013; 2014).

Through taking advantage of the key strengths of the ER rule in uncertainty modelling and aggregation, this paper aims to develop a hierarchical risk assessment framework with incorporating both qualitative and quantitative assessment information. The rest of the paper is organised as follows: in Section 2, the basics and key features of the ER rule are briefly introduced. In Section 3, the hierarchical risk assessment framework is presented with the key steps, (1) formulation of risk assessment hierarchy, (2) representation of both qualitative and quantitative information, (3) elicitation of attribute weights and information reliabilities, (4) aggregation of assessment information using the ER rule and (5) quantification and ranking of risks using utility-based transformation. A case study on the fire/explosion risk assessment of marine vessels is conducted to illustrate the applicability of the proposed hierarchical risk assessment model in Section 4. Some concluding remarks are presented in Section 5.

## 2. Basics and strengths of the ER rule in representing and aggregating uncertain information

In the ER rule, a piece of evidence or information in the context of risk and decision analysis,  $e_i$ , is profiled by the following belief distribution.

$$e_i = \left\{ (\theta, p_{\theta,i}), \forall \theta \subseteq \Theta, \sum_{\theta \in \Theta} p_{\theta,i} = 1 \right\} \quad (1)$$

where  $\Theta$  denotes a frame of discernment consisting of a set of mutually exclusive and collectively exhaustive hypotheses, mathematically,  $\Theta = \{\theta_1, \dots, \theta_N\}$  with  $\theta_n \cap \theta_m = \emptyset$  for any  $n, m \in \{1, \dots, N\}$  and  $n \neq m$ .  $P(\Theta)$  or  $2^\Theta$  can be used to represent the power set of  $\Theta$  with  $2^N$  subsets of  $\Theta$ , i.e.,  $P(\Theta) = 2^\Theta = \{\emptyset, \theta_1, \dots, \theta_N, \{\theta_1, \theta_2\}, \dots, \{\theta_1, \theta_N\}, \dots, \{\theta_1, \theta_{N-1}\}, \Theta\}$ . Thus,  $p_{\theta,i}$  represents the belief degree to which the evidence  $e_i$  supports proposition  $\theta$  being any element of  $P(\Theta)$  except for the empty set (Yang and Xu, 2013).

From the perspective of modelling uncertainty, the belief degree assigned exactly to the frame of discernment  $\Theta$  reflects global ignorance, to a smaller subset of  $\Theta$  except for any singleton proposition measures local ignorance, and to any singleton can be regarded as probability (Yang and Xu, 2013). Each piece of evidence  $e_i$  can also be associated with a weight  $w_i$  and a reliability  $r_i$

respectively. In the ER rule, the weight  $w_i$  is used to reflect the relative importance of evidence  $e_i$ , while the reliability  $r_i$  is regarded as the inherent property of the evidence (Yang and Xu, 2013; Chen et al., 2015).

Once all the three components, namely, belief distribution, weight and reliability are given, each piece of evidence  $e_i$  can be further transformed to a weighted belief distribution with reliability (Yang and Xu, 2013).

$$m_i = \{(\theta, \tilde{m}_{\theta,i}), \forall \theta \subseteq \mathcal{O}, (P(\mathcal{O}), \tilde{m}_{P(\mathcal{O}),i})\} \quad (2)$$

where  $\tilde{m}_{\theta,i}$  is calculated below to measure the degree of support for  $\theta$  from  $e_i$ .

$$\tilde{m}_{\theta,i} = \begin{cases} 0, & \theta = \emptyset \\ \tilde{w}_i p_{\theta,i}, & \theta \subseteq \mathcal{O}, \theta \neq \emptyset \\ 1 - \tilde{w}_i, & \theta = P(\mathcal{O}) \end{cases} \quad (3)$$

The new hybrid weight is defined as  $\tilde{w}_i = w_i / (1 + w_i - r_i)$ . The residual support  $\tilde{m}_{P(\mathcal{O}),i} = 1 - \tilde{w}_i = 0$ , when the piece of evidence  $e_i$  is fully reliable, i.e.,  $r_i = 1$ .

Further, the ER rule can be used to combine multiple pieces of evidence in a recursive way. For illustration purpose, two pieces of independent evidence  $e_i$  and  $e_j$  can be combined as follows

$$p_{\theta,e(2)} = \begin{cases} 0, & \theta = \emptyset \\ \frac{\hat{m}_{\theta,e(2)}}{\sum_{D \subseteq \mathcal{O}} \hat{m}_{D,e(2)}}, & \theta \subseteq \mathcal{O}, \theta \neq \emptyset \end{cases} \quad (4)$$

$$\hat{m}_{\theta,e(2)} = [(1 - r_j)m_{\theta,i} + (1 - r_i)m_{\theta,j}] + \sum_{B \cap C = \theta} m_{B,i} m_{C,j}, \quad \forall \theta \subseteq \mathcal{O} \quad (5)$$

where  $p_{\theta,e(2)}$  denotes the combined belief degrees to which the proposition  $\theta$  is jointly supported. The first square bracket term in Eq. (5) is regarded as the bounded sum of individual support on proposition  $\theta$ . Specifically, the unreliability of evidence  $e_i$ , i.e.,  $(1 - r_i)$  sets a bounded role which  $e_j$  can play. While the second term is the orthogonal sum of collective support on proposition  $\theta$ .

The ER rule generalises the Bayesian inference, the seminal Dempster-Shafer (D-S) theory of evidence (Dempster, 1968; Shafer, 1976) and the ER algorithm (Yang and Singh, 1994; Xu, 2012). Each piece of evidence in the Bayesian inference is formulated by a probability distribution, which can be regarded as a belief distribution without local or global ignorance (Yang and Xu, 2013; Chen et al., 2015). The ER rule can rigorously combine two pieces of highly or completely conflicting evidence, where Dempster's rule combination was found to generate counter-intuitive results (Zadeh, 1984; Yager 1987). The original ER algorithm considers a special case where the reliability of evidence is equal to its normalised weight (Yang and Xu, 2013). As discussed above, the ER rule can deal with both qualitative and quantitative attributes, which provides DMs with the flexibility of providing their preferences using either numeric values or linguistic variables. In addition, a variety of other uncertainties, such as fuzziness, interval beliefs, can also be formulated under the unified structure of belief distributions. In the context of risk analysis, these features are extremely useful in representing and aggregating uncertain risk assessment information in different formats and from multiple experts and sources.

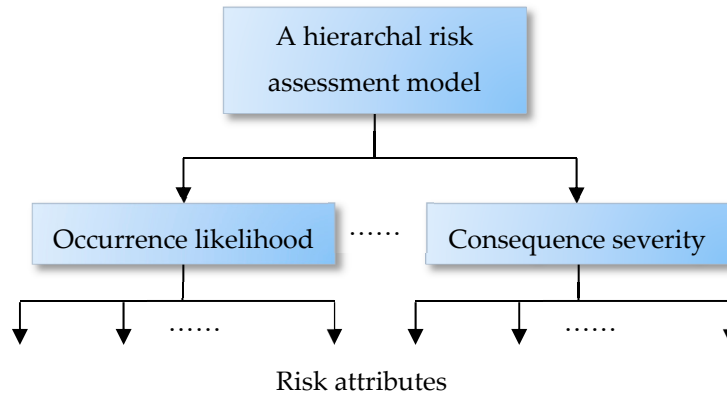
### 3. A hierarchal risk assessment model using the ER rule

It is worth noting that we mainly focus on the step of risk assessment, instead of the whole process of risk management in this section.

### 3.1. Formulation of risk assessment hierarchy

In risk assessment, different stakeholders, including experts and decision makers are usually involved, and they play respective roles in identifying risk attributes or factors and providing assessment information in terms of their domain knowledge and risk perception.

In this paper, the scheme of MCDA is employed to build up the risk assessment hierarchy. As illustrated in the Figure 1, an identified risk can be assessed by two or more dimensional parameters, such as occurrence likelihood and consequence severity used in a risk matrix. Under each of the top-level parameters, relevant risk attributes or factors should be identified to support reliable risk assessment. Even lower-level risk attributes or factors can be split into a number of sub-attributes.



**Figure 1.** Illustration of a hierarchal risk assessment model

From the perspective of collecting assessment information, the bottom level risk attributes or factors can be further classified as quantitative evaluation and subjective judgment. It is rather straightforward to collect quantitative data, while subjective judgment may involve stakeholders and experts' perception as well as various risk guidelines.

### 3.2. Representation of both qualitative and quantitative assessment information

As discussed previously, the top-level risk parameters can be assessed by linguistic variables. Correspondingly, a set of linguistic variables can also be defined for both lower-level qualitative and quantitative risk attributes. The set of linguistic grades can be regarded as a frame of discernment, for example,  $\Theta = \{\theta_1: \text{remote}, \theta_2: \text{unlikely}, \theta_3: \text{likely}, \theta_4: \text{high likely}, \theta_5: \text{almost certain}\}$ . Under the frame of discernment, accurate and exclusive definition of each linguistic term should be provided in order to facilitate rigorous risk assessment. In addition, it worth mentioning that the so-called local ignorance of assigning belief degrees to smaller subsets of  $\Theta$  except for any singleton proposition is usually disregarded in the context of risk analysis in order to reduce the difficulty of eliciting assessment information.

With regard to a qualitative risk attribute, subjective assessment information of assigning belief degrees to each assessment grade can be collected from decision makers and experts directly. However, the bias and variations from qualitative assessment is likely to result in a lack of objectivity and consistency for risk management.

For a quantitative risk attribute  $e_i$ , a set of referential values  $A_i = \{A_{n,i}; n = 1, \dots, N\}$  should be defined to cover its value interval. Then the following information transformation technique can be used to generate the corresponding belief distribution.

$$S(e_i) = \{(A_{n,i}, p_{n,i}); n = 1, \dots, N\} \quad (6)$$

where

$$p_{n,i} = \frac{A_{n+1,i} - e_i}{A_{n+1,i} - A_{n,i}} \quad \text{and} \quad p_{n+1,i} = 1 - p_{n,i}, \quad \text{if} \quad A_{n,i} \leq e_i \leq A_{n+1,i} \quad (7)$$

$$p_{n',i} = 0, \text{ for } n' = 1, \dots, N \text{ and } n' \neq n, n + 1 \quad (8)$$

Here,  $p_{n,i}$  represents the belief degree to which the risk attribute is assessed as the referential value  $A_{n,i}$ .

### 3.3. Representation of both qualitative and quantitative assessment information

The weight  $w_i$  can be assigned as a measurement of the degree of importance, where  $0 \leq w_i \leq 1$  and  $\sum_{i=1}^I w_i = 1$  to ensure the completeness of the total  $I$  risk attributes under a parameter or a risk category.

Apart from direct assignment techniques in weight elicitation, the analytic hierarchy process (AHP) method can be used to generate weights for attributes (Saaty, 1990). DMs provide multiple pairwise comparisons of one attribute against another by assigning subjective degree of importance, ranging from 1~9, and we can exhaust  $I(I - 1)/2$  pairwise comparisons completely. DMs also have the flexibility to use verbal intuitive expressions, such as 'strongly' or 'moderately', to elicit relative importance of attributes (Belton and Stewart, 2002). Specifically, in a pairwise comparison matrix  $C$ , an entry  $c_{ij}$  represents how much more important attribute  $i$  is over attribute  $j$  ( $i, j = 1, \dots, I$ ). In principle,  $c_{ij} = w_i/w_j$  where the diagonal entries are equal to 1. Thus the pairwise comparison matrix  $C$  can be represented as

$$C = \begin{bmatrix} 1 & \dots & c_{1I} \\ \vdots & \ddots & \vdots \\ c_{I1} & \dots & 1 \end{bmatrix} = \begin{bmatrix} \frac{w_1}{w_1} & \dots & \frac{w_1}{w_I} \\ \vdots & \ddots & \vdots \\ \frac{w_I}{w_1} & \dots & \frac{w_I}{w_I} \end{bmatrix} \quad (9)$$

The normalized weights can then be estimated by the eigenvector of  $C$ . A consistency index can further be calculated to check whether the weights generated are sufficiently consistent, and it is allowed to have a small degree of inconsistency resulting from biased judgement. However, re-examination of the pairwise comparisons should be conducted, when the consistency index approximated from maximum eigenvalue and eigenvector is smaller than 10% (Triantaphyllou, 2013). Given a sufficiently acceptable level of the consistency index, the AHP weights are regarded as more reliable than the directly assigned weights.

Furthermore, a group of DMs or experts are usually involved in the risk assessment process, and it is difficult for them to reach consensus on both assessing risk attributes and assigning their weights. For a cooperative risk assessment situation, where DMs or experts share accountability, the weight assignment process can be elicited from negotiation, brainstorming, voting schemes, etc. Alternatively, a supra DM in the collective decision environment can potentially lead the whole process (Bell et al., 1988; Kenney and Gregory, 2005).

In the ER rule, weight and reliability need to be considered simultaneously in order to obtain the hybrid weight  $\tilde{w}_i = w_i/(1 + w_i - r_i)$  as discussed above. However, there are lack of theoretical research and practical solutions with regard to eliciting information reliabilities, and it has often been linked with evidence discounting (Rogova and Nimier, 2004; Elouedi et al., 2004; Yang et al., 2013; Xu et al., 2016). In the context of risk analysis, the reliabilities of information sources can be obtained in terms of contextual information, experts' domain knowledge, and historical data.

When both weights  $w_i$  and reliability  $r_i$  are available, the implementation of the formula  $\tilde{w}_i = w_i/(1 + w_i - r_i)$  produces the hybrid weight  $\tilde{w}_i$ , for which  $\tilde{w}_i < w_i$  if  $r_i < w_i$ ,  $\tilde{w}_i = w_i$  if  $r_i = w_i$ , and  $\tilde{w}_i > w_i$  if  $r_i > w_i$ . Specifically, when a piece of evidence is fully reliable, i.e.,  $r_i = 1$ , there will be  $\tilde{w}_i = 1$ . The hybrid weight  $\tilde{w}_i$  will be discounted  $w_i$ , when the piece of evidence is fully unreliable i.e.,  $r_i = 0$ .

### 3.5. Aggregation of assessment information using the ER rule

With the collection of the three types of risk assessment information, specifically, belief distributions on bottom-level risk attributes, reliabilities of assessment information on bottom-level risk attributes, and weights for all risk parameters and attributes, the ER rule is then applied to

aggregate assessment information from bottom to up. As illustrated in Figure 2, the risk level on a middle-level risk attribute or parameter is usually aggregated from the assessment information collected from the bottom-level risk attributes. The aggregation process is performed in a recursive way. Finally, the overall risk level  $S(e)$  can be obtained in the form of a belief distribution.

$$S(e) = \{(\theta_n, p_n), n = 1, \dots, N; (\theta, p_\theta)\} \quad (10)$$

where  $p_n$  denotes the belief degree to which the overall risk level is assessed as the linguistic grade  $\theta_n$ , though taking into account all relevant risk assessment information. The remaining belief degree  $p_\theta > 0$ , when there are incomplete assessment information from the bottom-level risk attributes. The belief distribution provides a more informative risk profile.

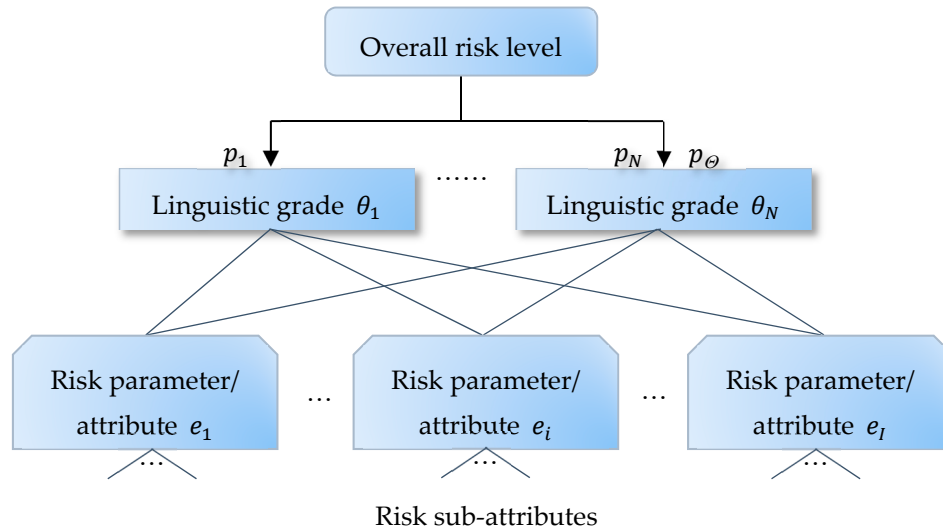


Figure 2. Aggregation of risk assessment information

It is assumed that the same number of linguistic grades is used to assess all the risk parameters and attributes. Otherwise, rule and utility based information transformation approaches should be implemented to match a common set of linguistic variables (Yang, 2001).

### 3.4. Quantification and ranking of risks using utility-based transformation

The overall belied distribution can be quantified as a risk using utility-based transformation. Assume that the utility of the linguistic grade  $\theta_n$  is  $u(\theta_n)$ . The risk score  $R(e)$  can be given as

$$R(e) = \sum_{n=1}^N u(\theta_n) p_n \quad (11)$$

As discussed above, incomplete assessment information from the bottom-level risk attributes will lead to  $p_\theta > 0$ . Correspondingly, the overall risk level can be characterised by a set of minimum, maximum and average risk scores.

$$R_{\min}(e) = u(\theta_1)(p_1 + p_\theta) + \sum_{n=2}^N u(\theta_n) p_n \quad (12)$$

$$R_{\max}(e) = \sum_{n=1}^{N-1} u(\theta_n) p_n + u(\theta_N)(p_N + p_\theta) \quad (13)$$

$$R_{\text{avg}}(e) = \frac{R_{\min}(e) + R_{\max}(e)}{2} \quad (14)$$

The interval  $[R_{\min}(e), R_{\max}(e)]$  can capture the range of potential risk levels. If there doesn't exist incomplete assessment information, we can easily rank a series of risks in terms of their risk scores. Otherwise, the above risk intervals should be considered for ranking.

#### 4. A case study on the fire/explosion risk assessment of marine vessels

Fire/explosion is one of the major accidents or risks having the potential to cause disastrous consequences for marine vessels (Wang, 2002; Chen et al., 2013). According to the UK marine accident investigation branch (MAIB) statistics (MAIB, 2012), around 11% of the total 1639 accidents of UK merchant vessels ( $\geq 100$  gross tonnage) from 2001 to 2012 were resulted from fire/explosion as shown in Figure 3.

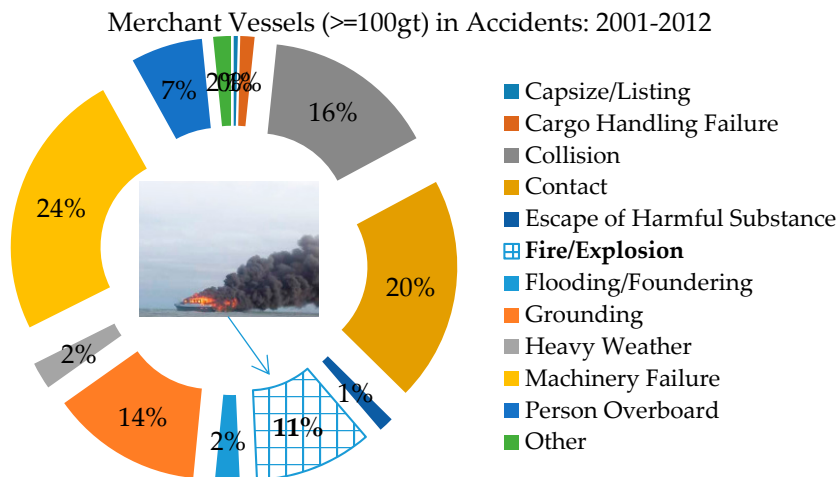


Figure 3. MAIB statistics on fire/explosion risks of UK merchant vessels

Thus, a set of regulations, codes and guidelines have been issued to support the safety management over fire/explosion and relevant risks (Chen et al., 2013). The focus of the section is to demonstrate how to apply the proposed hierarchical risk assessment model, rather than to formulate a complete assessment criteria hierarchy. From this perspective, this section only picks up a small part of the assessment criteria hierarchy, specifically, the assessment of measures to prevent the occurrence of fire/explosion in the engine room of marine vessels as shown in Figure 4. Next, we consider the above discussed three types of risk assessment information, which are weights, reliabilities, belief distributions on the lower-level risk attributes categorised as managerial measures, operative measures and technical measures respectively.

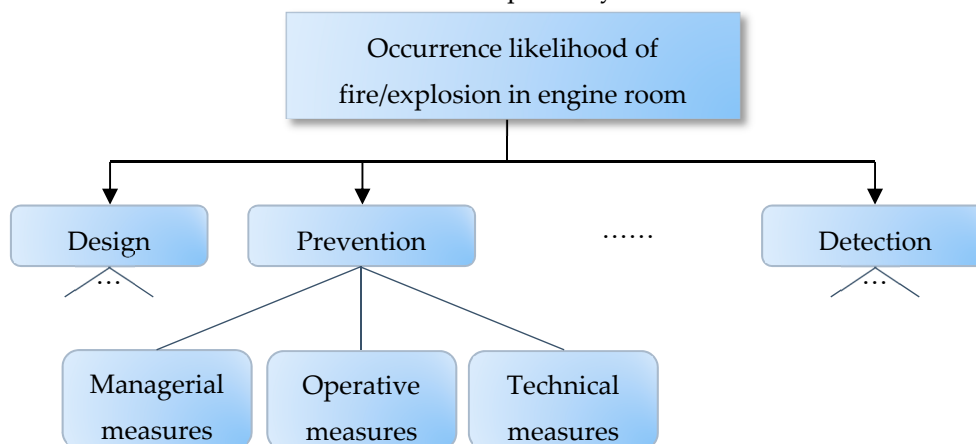


Figure 4. Assessment of the occurrence likelihood of fire/explosion in engine room

The relative importance of these prevention measures can be captured by pairwise comparisons. As shown in Table 1, the upper diagonal entries denote how many times the row attribute is more

important than the column attribute, the diagonal entries are equal to 1 for self-comparisons, and the lower diagonal entries are the reciprocals of the corresponding upper diagonal entries. Further, the AHP method is applied to obtain the last column of the calculated weights.

Table 1. An example of pairwise comparison matrix

Pairwise comparisons	Managerial	Operative	Technical	Calculated AHP weights
Managerial	1	2	3	0.55
Operative	$\frac{1}{2}$	1	1	0.24
Technical	$\frac{1}{3}$	$\frac{1}{2}$	1	0.21

It can be assumed that reliabilities are equal to weights, if both pieces of information are collected from the same group of experts. In practice, it is always very demanding to assign weights and reliabilities separately based on subjective judgement. However, there are potentially historical data available to calibrate reliabilities in some applications.

All the three attributes related to prevention measures can be regarded as qualitative, and a set of questions can be designed to collect judgement from experts. For example, the questions “Is there any fire/explosion prevention plan?”, “Are there any rules in place for processing flammable and/or explosive materials?” and “Whether the fire prevention and protection system is installed and tested appropriately?” can be asked for the assessment of managerial measures; “Is there any assessment for planned operations for fire/explosion risks?” and “Is there any regular assessment on the functionality of fire/explosion prevention systems?” for operative measures; “What is the capability of fire/gas detection system?” and “What is the capability of static electricity protection system” for technical measures. Assume that the following belief distributions are obtained under the unified frame of discernment  $\Theta = \{\theta_1: \text{remote}, \theta_2: \text{unlikely}, \theta_3: \text{likely}, \theta_4: \text{high likely}, \theta_5: \text{almost certain}\}$ .

Managerial:  $S(e_1) = \{(\theta_1, 0.5), (\theta_2, 0.5), (\theta_3, 0), (\theta_4, 0), (\theta_5, 0)\}$

Operative:  $S(e_2) = \{(\theta_1, 0.2), (\theta_2, 0.5), (\theta_3, 0.3), (\theta_4, 0), (\theta_5, 0)\}$

Technical:  $S(e_3) = \{(\theta_1, 0.3), (\theta_2, 0.5), (\theta_3, 0), (\theta_4, 0), (\theta_5, 0)\}$

Here it worth noting that there is incomplete assessment information for technical measures, and the remaining belief degree  $p_\emptyset = 1 - \sum_{n=1}^5 p_{n,3} = 1 - (0.3 + 0.5) = 0.2$ .

Then, the ER rule can be applied to combined three pieces of belief distributions recursively, which produces the overall assessment of prevention measures in the format of a belief distribution as shown in Figure 5.

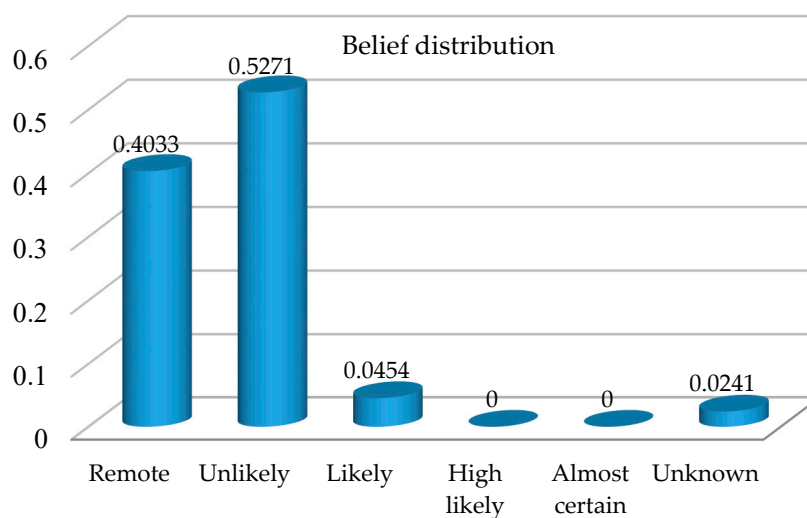


Figure 5. Belief distribution on the overall assessment of prevention measures

The belief distribution provides more informative fire/explosion risk assessment profile with regard to the attribute of prevention measures, and it also covers unknown part due to incomplete knowledge or limited information. Sensitivity analysis can further be conducted to identify lower-



level risk attributes causing high belief degrees on certain linguistic grades to be avoided. In addition, the quantified risk cores discussed in Section 3.5 can be used to rank and prioritise the fire/explosion risk among a set of potential risks.

## 5. Concluding remarks

In this paper, we proposed a hierarchical risk assessment model using the evidential reasoning (ER) rule. The key steps from the formulation of risk assessment hierarchy to the quantification and ranking of risks have been discussed in a systematic way. The applicability of the proposed risk assessment model was demonstrated by a case study on the fire/explosion risk assessment of marine vessels. In addition, the paper further takes into account the reliabilities of information sources and expert's judgement for risk assessment, in comparison to the previous application of the ER approach to risk analysis. In order to reduce the complexity of collecting information, local ignorance isn't considered in the belief distributions on the bottom-level attributes. It can be useful to support more rigorous risk assessment in our future research. In addition, the mutual independence of risk attributes also needs to be investigated in specific applications.

**Acknowledgments:** This work is partially supported by the Natural Science Foundation of China under Grant No. 71671186, 71331008.

**Conflicts of Interest:** The authors declare no conflict of interest. The founding sponsors had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, and in the decision to publish the results.

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