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Communication

Some Topological Invariants of the Möbius Ladder

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Abstract: In this article we compute many topological indices for the family of Möbius ladder. At first we give general closed form of M-polynomial of this family and recover many degree based topological indices out of it. We also compute Zagreb indices and Zagreb polynomials of this family.

Keywords: Möbius ladder; topological indices

0. Introduction

A graph invariant is a number, a polynomial, or a matrix which uniquely represents the whole graph [2]. Topological index is a graph invariant which characterizes the topology of the graph and remains invariant under graph automorphism. Degree based topological indices are of great importance and play a vital role in chemical graph theory see[6,17,18]. Wiener [15], working on boiling point of paraffin, introduced the idea of topological index. The Wiener index is originally the first and most studied topological index and is defined as the sum of distances between all pairs of vertices in G, for more details see [5,14,16]. Zagreb indices, have been introduced 38 years ago by I. Gutman and N. Trinajstic [20]. First Zagreb index $M_1(G)$ is defined as sum of the squares of degrees of a graph G and second Zagreb $M_2(G)$ is sum of the product of all degrees corresponding to each edge in G see [20]. Second modified Zagreb index is defined by

$$^{m}M_2(G) = \Sigma_{uv\epsilon E(G)}\frac{1}{d(u)d(v)}$$

Where $d(u)$ and $d(v)$ are the degrees of vertices u and v respectively see [21]. General Randic index of G is defined as sum of $(d(u)d(v))^\alpha$ over all edges uv of G, where $d(u)$ denote the degree of vertex u of G,

$$R_\alpha(G) = \Sigma_{uv\epsilon E(G)}(d(u)d(v))^\alpha$$

Where α is an arbitrary real number see [22]. Symmetric division index is defined by

$$SDD(G) = \Sigma_{uv\epsilon E(G)}\left(\frac{min\{d_u, d_v\}}{max\{d_u, d_v\}} + \frac{min\{d_u, d_v\}}{max\{d_u, d_v\}}\right)$$

Where d_i is the degree of vertex i in Graph G. These indices can help to characterize the chemical and physical properties of molecules see [3,6,15,17–19,21–23]. Some standard degree based topological indices and the formulas how to compute them from M-polynomial.

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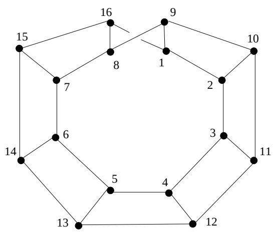
Topological Index	$f(x,y)$	Derivation from $M(G,x,y)$
First Zagreb	$x+y$	$(D_x+D_y)(M(G;x,y))\|_{x=y=1}$
Second Zagreb	xy	$(D_xD_y)(M(G;x,y))\|_{x=y=1}$
Second Modified Zagreb	$\frac{1}{xy}$	$(S_xD_y)(M(G;x,y))\|_{x=y=1}$
General Randić $\alpha\epsilon N$	$(xy)^\alpha$	$(D_x^\alpha D_y^\alpha)(M(G;x,y))\|_{x=y=1}$
General Randić $\alpha\epsilon N$	$\frac{1}{xy}^\alpha$	$(S_x^\alpha S_y^\alpha)(M(G;x,y))\|_{x=y=1}$
Symmetric Division Index	$\frac{x^2+y^2}{xy}$	$(D_xS_y+D_yS_x)(M(G;x,y))\|_{x=y=1}$,

where

$$D_x(f(x,y)) = x\frac{\partial f(x,y)}{\partial x}, D_y(f(x,y)) = y\frac{\partial f(x,y)}{\partial y}, S_x(f(x,y)) = \int_0^x \frac{f(t,y)}{t}dt, S_y(f(x,y)) = \int_0^y \frac{f(x,t)}{t}dt.$$

The Möbius ladder M_n which is a cubic circulant graph with an even number of vertices, formed from an $n-$cycle by adding edges (called "rungs") connecting opposite pair of vertices in the cycle. It is so-named because (with the exception of $M_6 = K_{3,3}$) M_n has exactly $\frac{n}{2}$ 4-cycles which link together by their shared edges to form a topological Möbius strip. Möbius ladders can also be viewed as a prism with one twisted edge. Two different views of Möbius ladders M_n have been shown in Fig.1. Möbius ladders have many applications in Chemistry, Chemical Stereography, Electronics and Computer Science.
For our convenience, we view the Möbius ladder M_n which is a cubic circulant graph with an even number of vertices, formed from an $n-$cycle by adding edges (called "rungs") connecting opposite pair of vertices in the cycle.



Möbius ladder M_{16}

Definition 1. Let G be a graph which is a simple molecular connected graph and $d_v(1 \leq d_v \leq n-1)$ be the degrees of vertices in G. We partition the set of vertex $V(G)$ and edge set $E(G)$ of G as follows $(\forall i, jandk : \delta \leq i,j,k \leq \triangle)$: $V_k = v\epsilon V(G)|d_v = k$ $E_{i,j} = \{e = uv\epsilon E(G)|d_u = jandd_v = i\}$ where δ and $\triangle$ are the minimum and maximum of degree of $d_v \forall v\epsilon V(G)$ and $\delta = Min\{d_v|v\epsilon V(G)\}$ and $\triangle = Max\{d_v|v\epsilon V(G)\}$, respectively. Now, let $G = (V,E)$ is a graph and let m_{ij} be the number of degrees $e = uv$ of G such that $\{d_v(G), d_u(G)\} = \{i,j\}$, then the M-polynomial of G define as follows:

$$M(G,x,y) = \Sigma_{\delta \leq i \leq j \triangle} m_{ij}x^iy^j,$$

where$d_u, d_v(1 \leq \delta \leq d_u, d_v \leq \triangle \leq |V(G)|-1)$ are the degrees of vertices $u, v\epsilon V(G)$.

Ghorbani and Azimi defined two new versions of Zagreb indices of a graph G in 2012 [9]. The first multiple Zagreb index $PM_1(G)$, second multiple Zagreb index $PM_2(G)$ and these indices are defined as:

$$PM_1(G) = \prod_{uv\epsilon E(G)} [deg(u) + deg(v)],$$

$$PM_2(G) = \prod_{uv\epsilon E(G)} [deg(u) \times deg(v)].$$

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In 2013, Shirdel *et al.* [26] introduced a new degree based Zagreb index named hyper-Zagreb index as:

$$HM(G) = \sum_{uv\epsilon E(G)} [deg(u) + deg(v)]^2.$$

The properties of $PM_1(G)$, $PM_2(G)$ indices for some chemical structures have been studied in [4,7,9, 11,22,25]. The first Zagreb polynomial $M_1(G,x)$ and second Zagreb polynomial $M_2(G,x)$ are defined as:

$$M_1(G,x) = \sum_{uv\epsilon E(M_n)} x^{[deg(u)+deg(v)]},$$

$$M_2(G,x) = \sum_{uv\epsilon E(M_n)} x^{[deg(u)\times deg(v)]}.$$

The properties of $M1(G,x), M2(G,x)$ polynomials for some chemical structures have been studied in [10]. Nowadays there is an extensive research activity on $PM_1(G), PM_2(G), HM(G)$, indices, $M_1(G,x), M_2(G,x)$ polynomials and their variants, see also [7–9,12,13,20,26].

1. Main Results

Theorem 2. *Let M_n be Möbius ladder. Then the M-polynomial of M_n is*

$$M(M_n, x, y) = 3nx^3y^3$$

Proof. Let M_n be Möbius ladder. From the structure of M_n Möbius ladder we can see that only one partition $V_3 = \{v\epsilon V(M_n)|d_v = 3\}$. By definition of M-polynomial, we can see edge set of M_n can be partition as follows: $E_{3,3} = \{e = uv\epsilon E(M_n)|d_u = d_v = 3\} \rightarrow |E_{\{3,3\}}| = 3n$
In figure the size of $E_{\{3,3\}}$ is equal to $3n$. Thus M-polynomial of M_n

$$\begin{aligned} M(M_n, x, y) &= \Sigma_{i\leq j} m_{ij}(M_n)x^iy^j, \\ &= \Sigma_{3\leq 3} m_{ij}(M_n)x^3y^3, \\ &= |E_{\{3,3\}}|x^3y^3, \\ &= 3nx^3y^3. \end{aligned}$$

□

Now we derive topological indices which are directly derivable from M-polynomial.

Theorem 3. *Let M_n be Möbius ladder. Then*

1. $M_1(M_n) = 18n$,
2. $M_2(M_n) = 81n^2$,
3. ${}^mM_2(M_n) = n^2$,
4. $R_\alpha(M_n) = (81n^2)^\alpha$,
5. $$SDD(M_n) = (n^2)^\alpha,$$
6. $SDD(M_n) = 18n^2$.

Proof. Let $f(x,y) = 3nx^3y^3$, then $D_x(f(x,y)) = 9nx^2y^3$, $D_y(f(x,y)) = 9nx^3y^2$, $S_x(f(x,y)) = S_y(f(x,y)) = nx^3y^3$, $D_x(f(x,y))|_{x=y=1} = D_y(f(x,y))|_{x=y=1} = 9n$, and $S_x(f(x,y))|_{x=y=1} = S_y(f(x,y))|_{x=y=1} = n$.

1. $M_1(M_n) : (D_x + D_y)f(x,y))(M(M_n;x,y))|_{x=y=1} = 18n$.
2. $M_2(M_n) : (D_xD_y)f(x,y))(M(M_n;x,y))|_{x=y=1} = 81n^2$.
3. ${}^mM_2(M_n) : (S_xS_y)f(x,y))(M(M_n;x,y))|_{x=y=1} = n^2$.

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4. $R_{\alpha}(M_n) : (D_xD_y)^{\alpha} f(x,y))(M(M_n;x,y))|_{x=y=1} = (81n^2)^{\alpha}$.
5. $R_{\alpha}(M_n) : (S_xS_y)^{\alpha} f(x,y))(M(M_n;x,y))|_{x=y=1} = (n^2)^{\alpha}$.
6. $SDD(M_n) : (D_xS_y + D_yS_x)(M(M_n;x,y))|_{x=y=1} = 18n^2$.

□

Theorem 4. *Let M_n be Möbius ladder. Then*

1. $PM_1(M_n) = 6^{3n}$,
2. $PM_2(M_n) = 9^{3n}$,
3. $HM(M_n) = 36(3n)$.

Proof. Let M_n be Möbius ladder. Edge set of M_n has one partition based on degree of vertices.The edge partition has $3n$ edges uv where $deg(u) = deg(v) = 3$ it can easy to see that $|E_1(M_n)| = d_{33}$. Now using equations (1)-(3)we have,

1.

$$\begin{aligned} PM_1(M_n) &:= \prod_{uv\epsilon E(M_n)} [deg(u) + deg(v)], \\ &= \prod_{uv\epsilon E_1(M_n)} [deg(u) + deg(v)], \\ &= 6^{|E_1(M_n)|}, \\ &= 6^{3n}. \end{aligned}$$

2.

$$\begin{aligned} PM_2(M_n) &:= \prod_{uv\epsilon E(M_n)} [deg(u) \times deg(v)], \\ &= \prod_{uv\epsilon E_1(M_n)} [deg(u) \times deg(v)], \\ &= 79^{|E_1(M_n)|}, \\ &= 9^{3n}. \end{aligned}$$

3.

$$\begin{aligned} HM(M_n) &:= \sum_{uv\epsilon E(M_n)} [deg(u) + deg(v)]^2, \\ &= \sum_{uv\epsilon E_1(M_n)} [deg(u) + deg(v)]^2, \\ &= 36|E_1(M_n)|, \\ &= 36(3n). \end{aligned}$$

□

Theorem 5. *Let M_n be Möbius ladder. Then*

1. $M_1(M_n, x) = 3nx^6$,
2. $M_2(M_n, x) = 3nx^9$.

Proof. Let M_n be Möbius ladder. Edge set of M_n has one partition based on degree of vertices.The edge partition has $3n$ edges uv where $deg(u) = deg(v) = 3$ it can easy to see that $|E_1(M_n)| = d_{33}$. Now using equations (4)-(5)we have,

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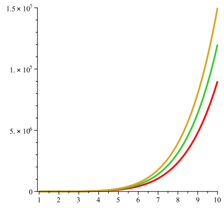
1.

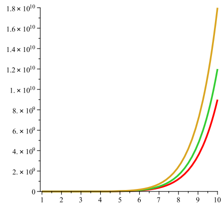
$$
\begin{aligned}
M_1(M_n &:= \sum_{uv\epsilon E(M_n)} x^{[deg(u)+deg(v)]}, \\
&= \sum_{uv\epsilon E_1(M_n)} x^{[deg(u)+deg(v)]}, \\
&= |E_1(M_n)|x^6, \\
&= 3nx^6.
\end{aligned}
$$



Zagreb 1st polynomial of M_n for n=3 Red,n=4 Green and n=5 Golden

2.

$$
\begin{aligned}
PM_2(M_n) &:= \sum_{uv\epsilon E(M_n)} x^{[deg(u)\times deg(v)]}, \\
&= \sum_{uv\epsilon E_1(M_n)} x^{[deg(u)\times deg(v)]}, \\
&= |E_1(M_n)|x^9, \\
&= 3nx^9.
\end{aligned}
$$



2nd polynomial of M_n n=3.Red n=4 Green and n=6 Golden

□

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Conflicts of Interest: "The authors declare no conflict of interest."

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