Dark Matter and Its Relationship with Gravity, Electromagnetism and the Two Nuclear Forces

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Summary
In a search for a solution of the galaxy rotation problem, a novel view on cosmological gravity is developed. In this view, the origin of gravity is traced back to the basic nuclear field of energy, spread by quarks as elementary pointlike sources. This gives a common basis for gravity, electromagnetism, the strong nuclear force and the weak nuclear force. After a review on gravimagnetism and the causality problem of gravity, a hypothetical concept for the gravity field is proposed, which solves the rotation problem without the need to accept dark matter as the deus ex machina.

Keywords: dark matter; cosmological gravity; galaxy rotation problem; gravimagnetism; antigravity; cosmological constant

I. INTRODUCTION

Dark matter is a hypothetical physical massive substance, the existence of which is proposed to explain particular phenomena observed in cosmology that cannot be understood from common physical laws. An intriguing example of such a phenomenon is the experimental evidence that stars rotating at the edge of a galaxy show an orbital velocity that remains more or less constant as a function of their spacing from the galaxy center. Although this phenomenon can be explained for particular mass density distributions, the orbital velocity appears to be larger than allowed by the Newtonian law of gravity, which limits this velocity by the amount of enclosed mass. Therefore, "something else" should be responsible for the phenomenon of excessive orbital velocity.

Several mechanisms have been proposed to explain the unexpected. The hypothetical existence of mass within the galaxy that escapes from visible observation, is one of these. Such dark matter would explain an increase of the gravitational force, thereby making the orbital velocity higher than expected from the amount of visible mass. Another mechanism is Milgrom's MOND hypothesis [1]. MOND stands for MOdified Newtonian Dynamics. In this approach, it is hypothesized that Newton's gravitational law is incomplete and that it can be adapted in a way that its influence becomes manifest at a cosmological scale and remains hidden in our daily world. This adaption, however, is not clearly physically justified, but is made on the basis of a curve fit to empirical results from observations. A third mechanism is proposed by Verlinde [2], who regards gravitation as an entropy phenomenon, emergent from other physical laws, rather than as a fundamental force of nature. He adopts the holographic principle of the string theory. According to this principle, the physical laws of our three-dimensional world, allows a (two-dimensional) holographic mapping on a shell around space. Verlinde hypothesizes fictitious molecules on this shell, which show the entropy as prescribed by the second law of thermodynamics. Physical mass objects in this fictitious world appear to show a motion behavior that can be described mathematically in a format that equals Newton's law of gravity. He claims that, similarly as with MOND, this mechanism gives a clue to understand the behavior of mass objects at a cosmological scale without the need to accept the existence of dark matter.

In this article, I wish to show that there might be fundamental physical reasons why mass objects in deep space behave differently from the behavior of mass objects in non-cosmological space. These reasons have to do with the relationship between mass, gravity, electromagnetism and the two basic nuclear forces. It all starts with the concept of quarks as described in previous work [3,4]. In this concept, a bare quark is a massless pointlike source of an energetic flux. This flux produces two energetic fields, namely a vector type repelling "far field" and a scalar type attracting "near field". This enables the origin of stable bonds between two quarks, known as mesons, and between three quarks, known as baryons. These configurations behave as quantum mechanical oscillators. The energetic state of these oscillators becomes manifest as mass. A quark on its own, has no relevant mass attribute. Mass is the consequence of the quantum mechanical junction of quarks (confinement). Electric charge is a consequence of this junction as well. It is the manifestation of the (iso)spin condition of the quark junction. As a result, electric charge is quantized in integer units. Mass is quantized as well, albeit as a quantization level of the quantum mechanical oscillator state. This shows an irregular energy spacing. It is due to the characteristics of this oscillator, which is not purely harmonic as it would be in a quadratic field of energy. Instead, it is slightly anharmonic as a consequence of the Proca type of the quark's nuclear far field and the Yukawa type of the nuclear near field. There is, however, a clear parallel. Electromagnetism, which originates from the phenomenon of electric charge, and gravity, which originates from the phenomenon of mass, are indirect forces that follow from the existence of the energetic field of the two basic nuclear forces.

We know that the electromagnetic force has a vector potential next to a scalar potential. Taking into account the parallel between electromagnetism and gravity as just described, it would be reasonable to suppose that such would hold for gravity as well. So, where is the vector potential of gravity? It might well be that this potential escapes from our observation because of its weakness. Would it be possible that this vector potential may show its existence at the cosmological scale? It is the aim of this article, to explore if such could be true. If gravity would be incomplete, in the sense that next to its scalar field a hidden vectorial field would exist, it could well be possible that currents of mass particles would show similar properties as currents of electrically charged particles. In that case, mass currents would create the equivalent of a magnetic field and so could execute the equivalent of the Lorentz force on other mass currents. Such an effect may manifest itself at a cosmological scale as the force that enhances the Newtonian force, thereby suggesting that some dark matter increases the strength of the scalar gravity force.

The idea of gravimagnetism is not new. In fact, it traces back to Heaviside [5]. The present approach to it is based upon a relativistic formalism, derived from Einstein's Field Equation. The suggestion that the (equivalent) Lorentz force might be related with dark matter is not new either. Attempts to explain the dark matter phenomenon in terms of a gravimagnetic Lorentz force have remained unsuccessfully, because its strength is considered as being too weak. In this article, I wish to review the dark matter problem once more as well as the shortcoming of gravimagnetism to explain outstanding cosmological problems. After that, I'll propose a novel concept of Cosmological Gravity in an attempt to find a way out.

II. THE GALAXY ROTATION CURVE PROBLEM: DARK MATTER OR DARK FORCE?

Newtonian laws prescribe that the *transverse* velocity $v_{\varphi}(r)$ of a cosmic object revolving in a circular orbit with radius r in a gravity field is determined by

$$v_{\varphi}^2(r) = \frac{M(r)G}{r}. \tag{1}$$

where $M(r)$ is the amount of enclosed mass and where G is the gravitational constant. This relationship is often denoted as Kepler's third law. As illustrated in figure 1, the velocity curve of cosmic objects in a galaxy, such as, for instance, the Milky Way, appears to be almost flat. It is tempting to believe that this can be due to a particular spectral distribution of the spectral density to compose $M(r)$. This, however, cannot be true, because $M(r)$ builds up to a constant value of the overall mass. And Kepler's law states in fact that a flat mass curve $M(r)$ is not compatible with a flat velocity curve.

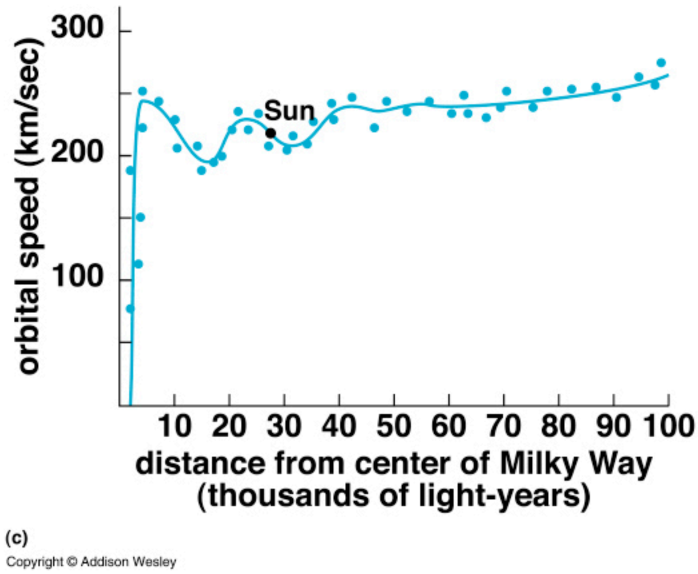


Figure 1: Rotation curves of solar objects in the Milky Way (From ircamera.as.arizona.edu)

Figure 2 illustrates the problem. It is one of the two: either some additional force on top of the Newtonian law is responsible for the phenomenon, or dark matter, affecting the mass distribution is responsible. Apart from this, it can be readily concluded from Table I that the mass data of the Milky Way are inadequate to explain the high value of $v_{\varphi}(r)$ at the edge of the galaxy. Figure 1 shows that the revolution speed of the sun (at $R = 8$ kpc) in the Milky Way has about the same value of some 220 km/s as the one of a star at the edge of the Milky Way ($R = 20$ kpc). For a stellar object at the edge, all Milky Way mass can be considered as if it were central. Using the values shown in Table I, it would result into a revolution speed of 132 km/s. The amount of missing mass can thus be estimated from

$$\frac{M_G + M_{dark}}{M_G} = \frac{1}{1-\alpha} = (\frac{220}{132})^2 \rightarrow \alpha = 0.64. \tag{2}$$

It means that some 64% of the Milky Way would consist of dark matter.

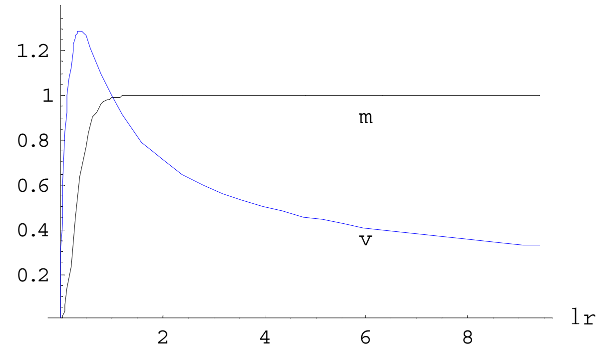


Figure 2. Incompatibility of a flat enclosed mass curve with a flat rotation curve.

Table I. Characteristics of the Milky Way

distance unit	1 kpc	$3.06\ 10^{19}$ [m]
grav. constant	G	6.67×10^{-11} [$m^3kg^{-1}s^{-2}$]
solar mass	M_{sol}	$1.99\ 10^{30}$ [kg]
mass of the bulge	M_{bulge}	$2 \times 10^{10}\ M_{sol}$
mass of the disc	M_{disc}	$6 \times 10^{10}\ M_{sol}$
Mass Milky Way	M_R	$8 \times 10^{10}\ M_{sol}$
Radius Milky Way	R	20 kpc

III. GRAVIMAGNETISM AS DARK FORCE?

As already stated, another possibility for explaining the anomalous rotation curve is the hypothetical existence of some dark force. Let us consider if the concept of gravimagnetism may reveal such. Many galaxies, such as, for instance, the Milky Way show a typical topological structure of the type as shown in figure 3.

Figure 3: Milky Way (from http://www.atlasoftheuniverse.com/milkyway2.jpg)

This structure is a collection of spiral arms in which solar objects show an orbital motion around the center of the galaxy. It will be clear that neighboring arms can be conceived as parallel currents of massive particles. In terms of the analogy between electromagnetism and gravity, such as summarized in Table II, one might hypothesize that these currents execute an attracting Lorentz force on each other of the type as shown in the last row of Table I. This would mean that, apart from the Newtonian gravity force that attracts massive objects to the center of the galaxy, an additional central force influences the equation of motion of such masses. This effect is formally expressed in the energy relationship

$$\frac{1}{2}mv^2+\frac{1}{2}J\omega^2+V_G(r)+V_L(r)=E\,, \tag{3}$$

where m is the mass of the particle, v its *radial* velocity, J its moment of inertia with respect to the galaxy center, ω(= $\mathrm{d}\varphi/\mathrm{d}t$) its angular frequency, V_G its potential energy due to gravity, V_L its potential energy due to the Lorentz force and E the total particle's energy. Next to conservation of energy as expressed by (3), the angular momentum is a conserved quantity, so

$$mr^2\frac{\mathrm{d}\varphi}{\mathrm{d}t}=b\,, \tag{4}$$

where b is a constant. It is well known that, by evaluating of this expression for the case that V_G is the potential energy associated with Newton's gravity law and that no Lorentz force is involved, under consideration of these conservation laws , elliptic orbits appear for the motion of the mass particle m. Among the variety of elliptic orbits possible, the circular motion, where the radial velocity $v=0$, is a valid special case. As shown in figure 3, orbits in the Milky Way are more or less circular.

Table II: Analogy of (naive) gravimagnetism with electromagnetism.

	electric	gravitational	(gravi)dimension
current	$I=\Delta q/\Delta t$	$I=\Delta m/\Delta t$	[kg s^{-1}]
Coulomb/Newton	$F=q_1q_2/4\pi\varepsilon_0r^2$	$F=m_1m_2G/r^2$	[kg m s^{-2}]
Lorentz force	$qv\times B$	$mv\times B$	[kg m s^{-2}]
(gravi) magn. field	$B=\mu_0I/2r$	$B=\mu_GI/2r$	[s^{-1}]
constant of nature	ε_0	$\varepsilon_G=1/4\pi G$	[(m^3 kg^{-1} $s^{-2})^{-1}$]
propagation speed	$c=(\varepsilon_0\mu_0)^{-1/2}$	c	[m s^{-1}]
gravimagnetic constant		$\mu_G=(c_G^2\varepsilon_G)^{-1}$	[kg^{-1} m]
force between two l long current wires	$F=\mu_0I_1I_2l/2\pi r$	$F=\mu_GI_1I_2l/2\pi r$	[kg m s^{-2}]

This picture shows a core (the bulge) and a number of spiral arms. Both the bulge and the massive objects (stellar) in the spiral arms rotate around the center. Experimental evidence shows that the tangential velocity of the stellars is different from the Keplian law as expressed by (1). As discussed before and shown in figure 1, this value appears to approach a value independent of r, at remote distances from the center. A few number of spiral arms can be distinguished. These spiral arms have a certain width and a certain spacing between them. To keep the model simple, it will be supposed that the width Δ of the spiral arms is about the same as the spacing between them. The particle streams in the arms will be considered as gravitational currents I_i in parallel wires. In accordance with the hypothetical equivalence of electromagnetism and gravity, the currents in the arms execute a gravitational Lorentz force that can be calculated according to the formula as shown in the last row of Table I. This builds a certain potential energy $U_L(r)$ of the outer spiral arm with respect to the galaxy center, such that

$$U_L(r) = -\int_0^r F(r)\mathrm{d}r. = -\{\int_0^\Delta F(r)\mathrm{d}r + \int_\Delta^{2\Delta} F(r)\mathrm{d}r + ...\int_{r-\Delta}^r F(r)\mathrm{d}r\} \quad \text{, where} \tag{5}$$

$$\int_{i\Delta}^{(i+1)\Delta} F(r)\mathrm{d}r = \mu_G \int_{i\Delta}^{(i+1)\Delta} \frac{I_i I_{i+1}}{2\pi\Delta} 2\pi r_i \mathrm{d}r \,, \tag{6}$$

with $I_i = v_t \rho_i \Delta$ [kg s^{-1}],

where v_t is the *tangential* velocity of the massive objects and where $\rho(r)$ is the mass density (in kg/m^2) in the spiral arms. The observer's position here and elsewhere in this analysis, is assumed being in the center of the galaxy. In the case that the orbit of the massive object under consideration is circular, the tangential velocity v_t is the same as the transverse velocity v as defined in (1). In non-circular orbits, the value of the latter one decreases with r as a consequence of the conserved angular momentum as defined by (4). It has to be noted, though, that this conservation law only holds within an orbit under consideration. It does not apply when comparing massive objects in different orbits. Experimental observation reveals that v_t is more or less independent of r. So, let us continue under this assumption and let us investigate if a consistent theory can be built that is not in conflict with it.

Further analysis reveals

$$\int_{i\Delta}^{(i+1)\Delta} F(r)dr = \mu_G \int_{i\Delta}^{(i+1)\Delta} \frac{I_i I_{i+1}}{2\pi\Delta} 2\pi r_i \mathrm{d}r = \mu_G \int_{i\Delta}^{(i+1)\Delta} \frac{(v_t \rho_i \Delta)^2}{\Delta} r_i \mathrm{d}r = \mu_G v_t^2 r_i \rho_i^2 \Delta^2 \tag{7}$$

Under use of the definitions $\varepsilon_G = 1/4\pi G$ and $c_G = (\sqrt{\varepsilon_G \mu_G})^{-1}$, it follows from (5) and (7),

$$U_L(r) = -\mu_G v_t^2 \Delta^2 \sum_{i=1}^{N} r_i \rho_i^2 = -\frac{4\pi G}{c_G^2} v_t^2 \Delta^2 \sum_{i=1}^{N} r_i \rho_i^2 \ . \tag{8}$$

as the total potential energy of the N^{th} spiral arm due to the gravitational Lorentz force.

Per unit of mass, the contribution is

$$U'_L(r) = \frac{1}{2\pi r \rho_N \Delta} U_L(r) = -\frac{2G v_t^2 \Delta}{c_G^2 r} \sum_{i=1}^{N} \frac{r_i \rho_i^2}{\rho_N} = \frac{G v_t^2}{\pi c_G^2 r} M_{rs} , \tag{9}$$

with $$M_{rs} = 2\pi\Delta \sum_{i=1}^{N} \frac{r_i \rho_i^2}{\rho_N} . \tag{10}$$

It will be clear that M_{rs} has about the same value as the sum M_r of mass in the N spiral arms within the radius r under consideration.

From (3) and (8), we have, without Lorentz force

$$\frac{1}{2} m v_t^2 = \frac{mGM_r}{r} , \tag{11}$$

and with Lorentz Force,

$$\frac{1}{2} m(1-\alpha) v_t^2 = \frac{mGM_r}{r} , \text{ where} \tag{12}$$

$$\alpha = \frac{2}{\pi c_G^2} \frac{GM_{rs}}{r} . \tag{13}$$

It will be clear that if α would have a substantial value, it would have a substantial influence on the tangential velocity v_t of the star in the spiral arms rotating around the center of the galaxy. Note that GM_r / r has the dimension of a squared velocity. As long as $\alpha < 1$, equation (12) is consistent with a circular orbit equation. The circular motion would change into a spiral one if α exceeds 1. If α is about 0.9, it would seem as if the central mass M_r is raised to a tenfold. It would seem as if 90% of its mass is "dark". Depending on the radial dependency of M_r and on the value $(1-\alpha)$, it might well be that v_t will not only show a flat behavior as a function of r, but that it will be substantially larger than expected from the amount of visible mass as well.

So, the crucial question is whether α may have a substantial value. It will be clear that if c_G would equate the light velocity in empty space, gravimagnetism cannot be the substitute for dark matter. So, the crucial question is if there is a reason to suppose that c_G can be different from the light velocity. This issue will be further addressed in the next paragraph. Before doing so, I wish to point to a seeming anomaly. In this analysis it has been taken for granted that the velocity curve of moving objects in the galaxy is flat, implying that the tangential velocity of orbiting objects is independent of the distance to the center. This corresponds with experimental evidence. It means, though, that the objects are revolving in a circle. The structure of the galaxy, however, shows spirals. The implication is that stellars cannot maintain their position in the spiral arms. If they did, their velocity should increase with the radius, because it would seem as if they were part of a rigid body. This seems being a paradox. In fact, it is not, because the spirals have to be considered as a pattern of mass density, which for some reason, is created in the dark initial phase of the galaxy. So, where the spiral arms with their high mass density preserve their position in the galaxy, individual stars don't. The spiral arms have to be considered as traffic jams, where stellars move in and move out. If this were not the case, the flatness of the velocity curve would demolish the spiral characteristic.

IV. GENERAL RELATIVITY AND GRAVIMAGNETISM

The resemblance between the electromagnetic field and the gravitational field does not necessarily mean that the one is a Chinese copy of the other. One thing is, for instance, the fact that where in electromagnetism charges are bipolar, charges in gravity are unipolar. Where the vectorial character of the electromagnetic field's potential function is without any doubt, it might still be that the potential function of the gravitational field is scalar. The observation that the weak field limit of the gravitational field, as expressed by Einstein's field equation, gives rise to a wave equation, does not necessarily imply that the gravity field's potential function is vectorial. To provide more insight in this, it might be useful to compare the origin of Maxwell's wave equation with the gravity wave equation that results from Einstein's Field Equation. Maxwell's wave equation is straightforwardly derived from the four well known Maxwell equations, written in SI notation as,

$$\nabla \cdot \mathbf{E} = \rho / \varepsilon_0 ;\ \nabla \cdot \mathbf{B} = 0 ;\ \nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t \text{ and } \nabla \times \mathbf{B} = \mu_0 (\mathbf{j} + \varepsilon_0 \partial \mathbf{E} / \partial t) \tag{14}$$

Combining the time derivatives of these equations followed by elementary algebraic manipulation, results in the well known Maxwellian wave equations for the electric field strength $\mathbf{E}$ and the magnetic field strength $\mathbf{B}$, which are spatially and temporally of second order. These fields are created as a consequence of a spatial charge ρ (which may reduce to a Dirac type distribution) and a current density $\mathbf{j}$.The Maxwell equations as such are not sufficient for describing mechanical forces on charged particles. As soon as a force F is assigned on a particle with electrical charge q as $F = qE$, Einstein's description of Maxwell's laws in terms of special relativity prescribes the generalization of the force by including the Lorentz force as a consequence of a moving charge q with velocity v as

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B} . \tag{15}$$

The Faraday induction $\partial \mathbf{B} / \partial t$ and the displacement current $\varepsilon_0 \partial \mathbf{E} / \partial t$, which mutually couple a magnetic field with an electric field, are the essential ingredients to generate the wave equation. This evokes the question in how far the bipolarity of electric charges in the source terms ρ / ε_0 and $\mathbf{j}$ is responsible for these essential ingredients. Anyhow, where the displacement current in electromagnetism can be readily understood from the interaction between positive and negative charges in the interrupt of a charged conductor by a condenser, there is not such equivalence available with unipolar charges in gravity. Whether a one-to-one mapping of the electromagnetic Maxwellian equations to gravity is allowed is an open question still. This holds in particular for the dynamic terms $\varepsilon_0 \partial \mathbf{E}/\partial t$ (displacement current) and $\partial \mathbf{B}/\partial t$ (Faraday induction). It might well be that these ingredients are absent in gravimagnetism.

Let us now consider the gravitational wave equation as a consequence of the weak field limit of the Einsteinean Field Equation. The equation reads as,

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad \text{with} \quad G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} . \tag{16}$$

where $T_{\mu\nu}$ is the stress-energy tensor, which describes the energy and the momenta of the source(s) and where $R_{\mu\nu}$ and R are respectively the so-called Ricci tensor and the Ricci scalar, which can be calculated if the metric tensor components $g_{\mu\nu}$ are known [6,7]. In the case that a particle under consideration is subject to a central force only, the time-space condition shows a rotational symmetric isotropy. This allows to read the metric elements g_{ij} from a simple line element that can be written as

$$\mathrm{d}s^2 = g_{00}(r,t)\mathrm{d}q_0^2 + g_{rr}(r,t)\mathrm{d}r^2 + r^2 \sin^2 \vartheta \mathrm{d}\varphi^2 + r^2 \mathrm{d}\vartheta^2 , \tag{17}$$

where $q_0 = \mathrm{i}ct$ and $\mathrm{i} = \sqrt{-1}$.

It means that the number of metric elements g_{ij} reduce to a few, and only two of them are time and radial dependent. Schwarzschild's solution of Einstein's equation for empty space and $\Lambda = 0$ relates the metric components as,

$$g_{rr} g_{00} = 1 . \tag{18}$$

Solving Einstein's equation under the weak field limit

$g_{00}(r,t) = 1 + h_\varphi(r,t)$, where $\left| h_\varphi(r,t) \right| << 1$,

under adoption of a massive source with pointlike distribution $T_{00} = Mc^2 \delta^3(r)$, results in a wave equation with the format

$$-\frac{1}{r}\frac{\partial^2}{\partial t^2}(r\Phi) + \frac{1}{r}\frac{\partial^2}{\partial r^2}(r\Phi) = \frac{8\pi GM}{c^2}\delta^3(r) , \tag{19}$$

where $h_\varphi = \dfrac{2\Phi}{c^2}$.

Its stationary solution is the well-known Newtonian potential,

$$\Phi = -\frac{MG}{r}. \tag{20}$$

Obviously, a central force from a sudden source in the center of empty space is enough to create a gravitational wave function. This wave function propagates with the vacuum light velocity, thereby proving the causality of the gravitational force.

So far in these considerations, possible rotation energy of the source in the center, has not been considered. Inclusion of it will, of course, affect the metric tensor. It makes it more complex, because one might expect that the nice symmetric isotropy will be lost. Generically, the line element has to be expressed in terms of the Kerr metric [8], which in the canonical gravimagnetic formalism is simplified to [9]

$$\mathrm{d}s^2 = -c^2(1-2\frac{\Phi}{c^2})\mathrm{d}t^2 - \frac{4}{c}(\mathbf{A}\cdot.\mathrm{d}\mathbf{x})\mathrm{d}t + (1+2\frac{\Phi}{c^2})\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j, \tag{21}$$

where δ_{ij} is the Kronecker delta and where, far from the source,

$\Phi = \frac{GM}{r}$, $\mathbf{A} = \frac{G}{c}\frac{\mathbf{J}\times\mathbf{x}}{r^3}$, $r = |\mathbf{x}|$ and where $\mathbf{J}$ is the angular momentum of the source. As will be shown later in this text, this metric is the basis for a canonic formulation of the gravimagnetic equation set, which will appear somewhat different from a naïve transposition of the Maxwellian set that we shall pursue first.

In the special case that a particle subject to this metric is revolving in the equatorial plane of the source, the influence of the source's angular momentum becomes manifest as a quasi Lorentz force, which can be modeled as an add-on to the central force. Under this condition the nice symmetric isotropy is restored. It will be clear that the value of the proportionality constant μ_G has a large impact on the strength of the quasi Lorentz force. It is quite common to establish this value from the following observation. First of all, Poisson's law for gravity is invoked, which states

$$\nabla^2\Phi = 4\pi G\rho. \tag{22}$$

In analogy with Poisson's law for electromagnetism a gravity equivalent ε_G for electric permeability is defined, such that

$$\varepsilon_G = 1/4\pi G. \tag{23}$$

The next step is relating

$$c^2 = \frac{1}{\varepsilon_0\mu_0} \Leftrightarrow c_G^2 = \frac{1}{\varepsilon_G\mu_G}? \tag{24}$$

The justification for equating c^2 with c_G^2, as is common practice, is the statement that the presence of a vector potential associated with the angular momentum brings the causality of the gravity field such that it propagates with the vacuum light velocity. The presence of the angular momentum is not required for bringing the causality. As proven in the Appendix, the causality is implicitly provided by Einstein's Field equation even in the absence of an angular momentum of the source. Therefore, causality of the gravity field is no reason to equate c^2 with c_G^2.

Gravimagnetism might be essentially different from electromagnetism in the sense that the mutual coupling between a gravitational equivalent of an electric field generated by a scalar potential and a gravitational equivalent of a magnetic field generated by a vector potential might be absent. The two types of fields might exist, however, maybe without mutual coupling. Without the coupling, the quantity c_G is just a constant that has nothing to do with radiation or propagation of gravitational energy by a Poynting vector. Arguments for believe in the absence of this coupling is given above, where the role of the difference in polarity of charges has been discussed. Let us continue by estimating the value of μ_G from known experiments. This will be done in the next paragraph.

V. IMPLICATIONS FROM THE GRAVITY B PROBE EXPERIMENT

Probably the most relevant experiment for establishing the value of the gravimagnetic constant μ_G has been offered by the Gravity B Probe project [10,11]. This project has been set up to compare theoretical values of two gravitational effects with experimental evidence. These effects are, respectively, the *geodetic effect* and *frame dragging* (also known as the Lense-Thirring effect [12,13]). The geodetic effect is related with the phenomenon of the perihelium shift of planets in the solar system. This shift is predicted by Einstein's General Relativity. The perihelium of a planet in an almost circular loop with radius r_0 shifts each revolution by an amount of [7,14]

$$\Delta\varphi' = \frac{3\pi R_S}{r_0}, \text{ where } R_S = \frac{2M_{sol}G}{c^2}. \tag{25}$$

The quantity R_S is known as the Schwartzschild radius. A satellite orbiting around the earth will show the same effect. It can be measured by a gyroscope on board of a satellite with its axis pointing to a cosmological object in deep space. As a consequence of the curving of space-time due to the energetic gravitational field of the earth, the gyroscope on board of the satellite will show a precession motion. One might think that, per revolution of the satellite at a radius r_0' around the earth enter, the angle of the gyroscope axis will shift by an amount that equals the value of the perihelium shift. Actually, it is more complicated than that. The theoretical value of the perihelium shift is calculated from the line element of an isotropic non-rotating metric of the type as given by (17). More particularly,

$$\mathrm{d}s^2 = -(1-\frac{R_s}{r})\mathrm{d}t^2 + (1-\frac{R_s}{r})^{-1}\mathrm{d}r^2 + r^2\sin^2\vartheta\mathrm{d}\varphi^2 + r^2\mathrm{d}\vartheta^2 . \quad (26)$$

The calculated value of the perihelium shift nicely fits with experimental evidence from orbiting planets indeed. However, as noted before, this line element is incomplete, because the influence of the rotation energy of the central mass (of the sun for planets or the earth for orbiting satellites) is not included. Nevertheless, the perihelium shift behaves as predicted by (25). This can be understood as follows. The influence of the rotation energy can be accounted for by two different mechanisms. These mechanisms are physically equivalent, but are different in modeling. The first one is changing the line element of field metric followed by an analysis of the angular momentum of a mass object moving in this field. The second one is leaving the field metric unchanged and modeling the influence of the rotation energy as the add-on of a gravimagnetic field. In the latter model, a gravitational Lorentz force is associated with this field and as long as the revolving of a massive object takes place in the equatorial plane of a rotating central mass, the gravitational Lorentz force is central. This force is just a slight add-on on the Newtonian force, thereby hardly affecting the amount of perihelium shift of a revolving massive object in the equatorial plane. The angle of the axis of a gyroscope placed in a satellite orbiting in the equatorial plane of the earth will show per orbit a slight additional amount of phase shift in the equatorial plane. Later in this article, I'll show that this additional amount is virtually indistinguishable from the amount as predicted by the perihelium shift of a non-rotating metric. For that reason the perihelium shift of planets is not different from the one as calculated from the Schwarzschild metric (26). In fact, the only thing to be done is to relate the radius of Mercury's orbit and the solar mass with the radius of the Gravity B satellite ($\approx$7000 km) and the earth mass. This gives 13188 macs/yr (milliarcsec per year). Curiously, this is about twice the value as reported from the Gravity B project [10,11]. The reason of the difference will be discussed later in this text.

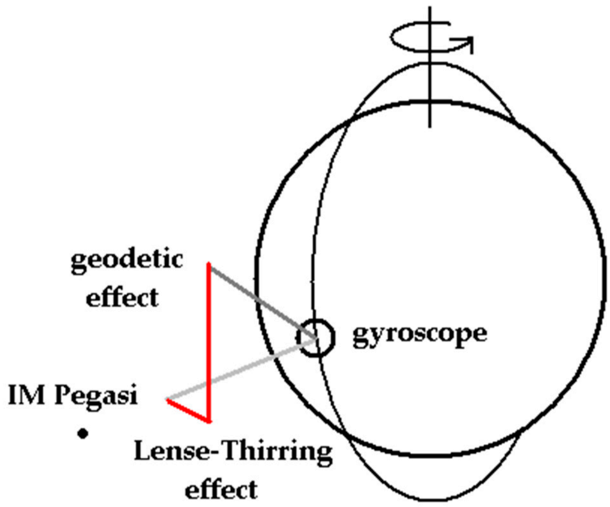


Figure 4. The two orbital precession influences (perihelium shift/geodetic effect and Lorentz force/Lense-Thirring effect) are coherent (i.e. in the same direction) for massive objects in an equatorial plane and non-coherent for massive objects in a polar plane. IM Pegasi is the guide star for the gyroscope. From: large.stanford.edu/courses/2007/ph210

This picture will change in the case that the satellite with gyroscope(s) orbits in the polar plane. A gyroscope is a device that always keeps its orientation, irrespective of the bottom on which it is placed. If, for some reason whatever, the bottom tilts or turns, the orientation the gyroscope's axis is preserved. For an observer fixed to the bottom, however, it seems as if the gyroscope's axis shifts (precession). A particular example is Foucault's pendulum. The earth rotates under the plane of swinging. For the observer on earth it seems as if the pendulum plane rotates. The Lorentz force on a massive object orbiting in a polar plane is no longer oriented in the same direction as the gravitational force. The force is maximum and orthogonal to the gravitational force when the satellite passes the poles and the force is zero when the satellite passes the equator. As a consequence the orientation of the plane of orbit will be subject to a spatial phase shift orthogonal to the polar plane. This will be experienced by an observer in the satellite as a precession of the gyroscope next to the orbital perihelium phase shift. Figure 4 illustrates the process. Let us first consider the gravimagnetic view.

A. The gravimagnetic view

In line with the reflections on gravimagnetism earlier in this article, I wish to develop the gravimagnetic view by conceiving mass particles as charged particles with charge m. I wish to do so on the basis of the naive analogy as summarized in Table II. The correspondence/difference with the canonical view will be discussed later. A large massive volume will be modeled as a massive body with some internal mass density ρ. As a consequence of this mass density, a gravimagnetic field is generated, similar to the magnetic field of a volume with some electrical space charge. According to this picture, the earth will be modeled as a sphere with a uniform mass density ρ. The calculation of the generated (gravi)magnetic field $\mathbf{B_G}$ is not trivial. The field for a sphere with uniform surface charge density σ can be found in textbooks [15]. It appears that,

$$\mathbf{B_G} = \frac{\mu_G \omega\sigma}{3}\frac{R^4}{r^3}\{2\cos\vartheta\hat{\mathbf{r}} + \sin\vartheta\hat{\vartheta}\}\text{, for } r \geq R\text{,} \tag{27}$$

($\cos\vartheta\hat{\mathbf{r}} = \cos\vartheta\sin\vartheta\mathbf{x} + \cos\vartheta\cos\vartheta\mathbf{z}$; $\sin\vartheta\hat{\vartheta} = -\sin\vartheta\cos\vartheta\mathbf{x} - \sin\vartheta\sin\vartheta\mathbf{z}$)

where ω is the angular velocity of the earth rotation and where $\hat{\mathbf{r}}$ and $\hat{\vartheta}$, respectively, are unit vectors in $r-$direction and in $\vartheta-$direction. This can be converted into the field of a sphere with uniform volume charge density ρ by integrating over shells $\rho\mathrm{d}r = \sigma$, so that

$$\mathbf{B_G} = \frac{\mu_G \omega\rho}{3r^3}\frac{R^5}{5}\{2\cos\vartheta\hat{\mathbf{r}} + \sin\vartheta\hat{\vartheta}\}\text{, for } r \geq R\text{.} \tag{28}$$

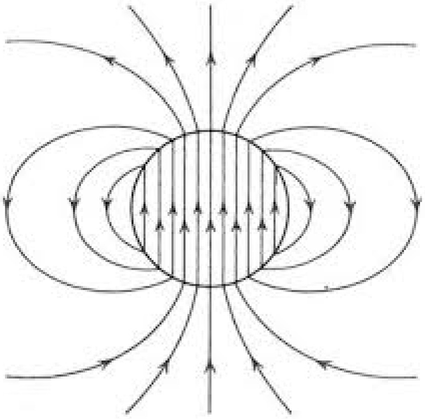


Figure 5. The rotational symmetry of the gravimagnetic field strength B.

In terms of the peripheral equatorial speed $v_e = \omega R_e$ and the mass $M_e = 4\pi\rho R_e^3/3$, the magnitude of the equatorial magnetic field ($\vartheta = \pi/2$) can be expressed as

$$B_G = \frac{\mu_G}{20\pi}\frac{R^3}{r^3}\frac{M_e}{R_e^2}v_e \ . \tag{29}$$

As a consequence of this gravimagnetic field, a massive object with mass m, orbiting in the equatorial plane with tangential velocity v, is subject to a Lorentz force, the magnitude F_L of which is determined from $m\mathbf{v}\times\mathbf{B_G}$ as

$$F_L = m\frac{\mu_G}{20\pi}\frac{R_e^3}{r^3}\frac{M_e}{R_e^2}vv_e = m\frac{M_e G}{5R_e^2}\frac{R_e^3}{r^3}\frac{v_e v}{c_G^2} \tag{30}$$

In the equatorial plane, this force is balanced by a centripetal force, so that

$$\frac{mv^2}{r} = m\frac{M_e G}{5R_e^2}\frac{R_e^3}{r^3}\frac{v_e v}{c_G^2} \rightarrow v = r\frac{M_e G}{5R_e^2}\frac{R_e^3}{r^3}\frac{v_e}{c_G^2} \rightarrow$$
$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{M_e G}{5R_e^2}\frac{R_e^3}{r^3}\frac{v_e}{c_G^2} \tag{31}$$

This gives per orbit an additional shift on top of the perihelium shift to the amount of

$$\Delta\varphi = \frac{2\pi}{5}\frac{M_e G}{c_G^2 R_e}\frac{R_e^2}{r^2}\frac{v_e}{v} = \frac{2\pi}{5}\frac{M_e G}{c_G^2}\frac{R_e}{r^2}\frac{v_e}{v} = \frac{\pi}{5}\frac{R_{SG}}{r}(\frac{R_e}{r}\frac{v_e}{v}), \tag{32}$$

with $R_{SG} = \dfrac{2M_e G}{c_G^2}$.

If c_G is equated with the vacuum light velocity c, R_{SG} is the same as the Schwarzschild radius.

It will be clear that if this is the case, this additional shift as a consequence of the rotation of the central mass is negligible for planets in orbits around the sun, because of the large mismatch between the radius of the sun and the average radius of the orbit. This is different for a satellite orbiting in the periphery of the earth.

In the Gravity Probe B project the distance of the satellite to the earth center amounts to $r = 7000$ km. The peripheral speed of the earth is $v_e = 2\pi$ x 6400/(24 x 60 x 60) km/s. From these expressions is straightforwardly calculated that, under assumption of $c_G = c$, this additional precession over a year amounts to 50 macs, while, as noted before, the perihelium shift amounts to 13188 macs/yr (milliarcsec per year). Note that the perihelium shift is not dependent on c_G, but on c instead.

Let us now suppose that the satellite orbits in the polar plane. Now the satellite experiences a Lorentz force orthogonal to the orbit plane. From (28) it is obvious that the Lorentz force while passing the poles is twice as large as in the equatorial case. It is zero while passing the equator. In other positions, the strength of the Lorentz force is composed as

$F_L = B_z v_x + B_x v_z$, where (33)

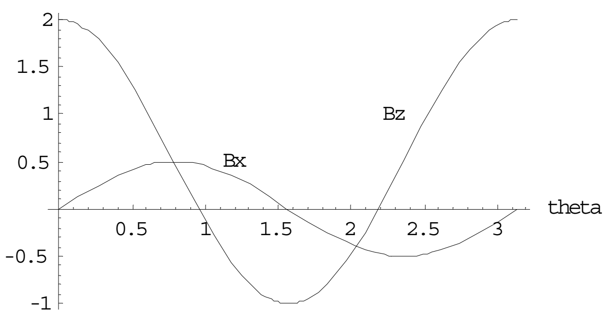
the two components of the gravimagnetic field strength are given by,

$B_z = B_G(2\cos\vartheta\cos\vartheta - \sin\vartheta\sin\vartheta)$ and $B_x = B_G(2\cos\vartheta\sin\vartheta - \sin\vartheta\cos\vartheta)$,

and where the tangential velocity v is decomposed as

$v_z = -v\sin\vartheta$ and $v_x = v\cos\vartheta$.

Figure 6 shows these quantities as a function of ϑ.



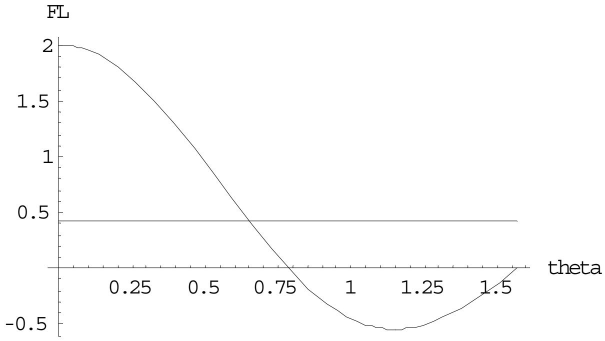


Figure 6. Upper part: The z and x component of the gravimagnetic fieldstrength B as a function of ϑ (the y component is zero). Lower part: The Lorentz force as a function of ϑ and its average value (0.42).

The Lorentz force is orthogonal to the plane of motion. Averaged over the orbit, the effective Lorentz force is 0.42 x in magnitude as in the equatorial case (thereby resulting in a frame dragging effect of 21 macs/yr). Now, however, the phase shift is orthogonal to the perihelium phase shift. The labor executed by the Lorentz force is converted into the energy of a precession motion orthogonal on the plane of motion. In the equatorial case the two precessions are coherent, i.e. pointing in the same direction, while they are incoherent in the polar case. In the latter case observation of the gyroscope allows to measure the two effects independently.

The reported measurement data from the Gravity B probe project are, respectively, for the geodetic effect, 6606.1 macs/yr (milliarcsec per year), and for the frame dragging effect 37.2 macs/yr. Curiously, where, as already noted, the figure for the geodetic effect is almost exactly a factor 2 smaller than results from the analysis above of the perihelium shift, it is about a factor 2 larger for the gravimagnetic effect. The reasons for these discrepancies will be discussed below.

B. Schiff's view

The gravity B probe project has been set-up for testing the gravity theory as documented by L.I. Schiff in his classic article [16] . This theory describes the influence of a rotatic metric, such as produced by the earth, on the spin vector (= angular momentum vector) $\mathbf{S}$ of a gyroscope placed in an orbiting satellite. The conclusion of the theory is that the time behavior of the spin vector from the perspective of an observer in the satellite and corrected for the earth time t, can be expressed as

$$\frac{\mathrm{d}\mathbf{S}}{\mathrm{d}t} = \mathbf{\Omega} \times \mathbf{S}, \tag{34}$$

where

$$\mathbf{\Omega} = \mathbf{\Omega}_{\mathbf{G}} + \mathbf{\Omega}_{\mathbf{FD}} = \frac{3GM}{2c^2 r^3}(\mathbf{r} \times \mathbf{v}) + \frac{GI}{c^2 r^3}\{\frac{3}{r^2}(\boldsymbol{\omega} \cdot \mathbf{r})\mathbf{r} - \boldsymbol{\omega}\}.$$

where I is the earth's moment of inertia. The first term is independent of the earth's rotation, while the second term determines its impact. In terms of the earth's angular momentum $\mathbf{J} = I\boldsymbol{\omega}$, this can be rewritten as,

$$\boldsymbol{\Omega} = \boldsymbol{\Omega}_{\mathbf{G}} + \boldsymbol{\Omega}_{\mathbf{FD}} = \frac{3GM}{2c^2r^3}(\mathbf{r}\times\mathbf{v}) + \frac{G}{c^2r^3}\{\frac{3}{r^2}(\mathbf{J}\cdot \mathrm{r})\mathbf{r} - \mathbf{J}\}\,. \tag{35}$$

The energy E_G associated with the first term amounts to

$$E_G = \mathbf{S}\cdot\boldsymbol{\Omega}_{\mathbf{G}} = mr^2\frac{\mathrm{d}\varphi}{\mathrm{d}t}\frac{3GM}{2c^2r^2}v\,. \tag{36}$$

This energy is converted into a rotational precession energy E_p , so that

$$E_p = E_G \rightarrow \frac{1}{2}J\omega^2 = \frac{1}{2}r^2m(\frac{\mathrm{d}\varphi}{\mathrm{d}t})^2 = mr^2\frac{\mathrm{d}\varphi}{\mathrm{d}t}\frac{3GM}{2c^2r^2}v = mr^2\frac{\mathrm{d}\varphi}{\mathrm{d}t}\frac{3}{4}R_s\frac{1}{r^2}v\,, \tag{37}$$

and so

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{3R_s}{2r^2}v = \frac{3R_s}{2r^2}(\frac{2\pi r}{\Delta T})\,. \tag{38}$$

Per orbit of the satellite, it results in a phase shift $\Delta\varphi_G$ to the amount of

$$\Delta\varphi_G = \frac{3\pi R_s}{r}\,. \tag{39}$$

This is the same as the value of the perihelium shift defined in the previous paragraph. For the second term precession energy of a gyroscope in the equatorial plane, we have from (36)

$$E_G = \mathbf{S}\cdot\boldsymbol{\Omega}_{\mathbf{FD}} = mr\frac{\mathrm{d}\varphi}{\mathrm{d}t}\frac{G}{c^2r^3}J\,. \tag{40}$$

For the angular momentum of the earth, interpreted as a spherical object, we have

$$J = M_e\frac{4\pi R_e^2}{5T_e} = M_e\frac{4\pi R_e^2}{5T_e} = \frac{2}{5}M_eR_ev_e\,, \tag{41}$$

where T_e is the rotation period of the earth.

With these expressions the second term of the gyroscope precession in the equatorial plane results from

$$E_p = E_G \rightarrow \frac{1}{2} r^2 m (\frac{\mathrm{d}\varphi}{\mathrm{d}t})^2 = mr^2 \frac{\mathrm{d}\varphi}{\mathrm{d}t} \frac{G}{c^2 r^3} \frac{2}{5} M_e R_e v_e = \frac{1}{5} mr^2 \frac{\mathrm{d}\varphi}{\mathrm{d}t} R_s \frac{R_e v_e}{r^3}, \tag{42}$$

so that

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{2}{5} R_s \frac{R_e v_e}{r^3}, \tag{43}$$

and therefore

$$\Delta\varphi_G = \frac{2}{5} R_s \frac{R_e v_e}{r^3} (\frac{2\pi r}{v}) = \frac{4\pi R_s}{5r} \frac{R_e v_e}{rv} \text{ per orbit of the satellite.} \tag{44}$$

This gives 200 macs/yr. It is 4x larger than in the gravimagnetic view for $c_G = c$. A gyroscope in a polar plane shifts (200 x 0.42 =) 84 macs/yr. Similar as the perihelium shift, it is 2x larger than quoted from the Gravity B probe project. The reason for this discrepancy has to do with the semantics of $\mathbf{\Omega_{FD}}$. Although the dimensionality of $\mathbf{\Omega_{FD}}$ is [s^{-1}], it should not be interpreted as $\mathrm{d}\varphi/\mathrm{d}t$. In fact $\mathbf{\Omega_{FD}}$ represents the gravimagnetic field, which has a dimensionality of [s^{-1}] as well (see Table II). This implies that the product $\mathbf{S} \cdot \mathbf{\Omega}$ is an energy. Equating this energy with the rotational energy of the gyroscope, we get,

$$\frac{1}{2} r^2 m (\frac{\mathrm{d}\varphi}{\mathrm{d}t})^2 = r^2 m \frac{\mathrm{d}\varphi}{\mathrm{d}t} \Omega_{FD} \rightarrow \frac{\mathrm{d}\varphi}{\mathrm{d}t} = 2\Omega_{FD}. \tag{45}$$

This makes the $\mathrm{d}\varphi/\mathrm{d}t$ twice the value of $\mathbf{\Omega_{FD}}$. It might be instructive here to note that other authors, e.g. Iorio [17], have expressed the Lense-Thirring formulae as $\dot{\Omega}_{LT}$, i.e, with a "dot". These formulae represent $\mathrm{d}\varphi/\mathrm{d}t$ and have twice the value as Schiff's $\mathbf{\Omega_{FD}}$ indeed. It is therefore quite probable that the measurement data reported from the Gravity B project apply to $\mathbf{\Omega}$ instead to $\mathrm{d}\varphi/\mathrm{d}t$.

C. Comparing Schiff's theory with gravimagnetism

The result of the Gravity B probe experiment confirms Schiff's conclusion about the weak limit of Einstein's Field Equation. Compared with the gravimagnetic view, there is no difference in the geodetic part, which, in a proper interpretation of the reported data, is the same as the well known perihelium shift of planetary objects in the solar system. The frame dragging result in the weak limit is exactly 4x larger than follows from a one-to-one transposition of Maxwellian laws to gravity. Repairing this discrepancy requires a modification of the Lorentz force by a factor 4. This can be done in the naive view on gravimagnetism by accepting $\mu_G = 16\pi G/c^2$, implying that $c^2 \neq c_G^2$. The alternative approach is to derive a canonical formulation of the gravimagnetism that does not show the discrepancy, while sticking to $c^2 = c_G^2$ [9]. Starting point is the metric shown in (21). This allows a derivation for the field's Lagrangian density, which executes a (Lorentz) force on a test particle with mass m to the amount of

$$\mathbf{F} = m\mathbf{E} + (2m)\mathbf{v} \times \mathbf{B}, \tag{46}$$

where, under stationary conditions, i.e. $\partial\mathbf{A}/\partial t = 0$, the gravimagnetic quantities $\mathbf{E}$ and $\mathbf{B}$ are related with the quantities in the metric tensor (21) as

$$\mathbf{E} = -\nabla\Phi \text{ and } \mathbf{B} = \nabla \times \mathbf{A}. \tag{47}$$

Comparing (15) with (46), it is obvious that the factor 4 is not yet fully accommodated. To do so, Mashhoon has proposed to modify the naïve Maxwellian transposition to

$$\nabla \cdot \mathbf{E} = \rho/\varepsilon_G;\ \nabla \cdot (\mathbf{B}/2) = 0;\ \nabla \times \mathbf{E} = -\partial(\mathbf{B}/2)/\partial t \text{ and } \nabla \times (\mathbf{B}/2) = \mu_G(\mathbf{j} + \varepsilon_G \partial\mathbf{E}/\partial t). \tag{48}$$

It will be obvious from the last member of this set that this doubles the gravimagnetic component $\mathbf{B}$. This doubling together with the term $2m$ in (46) makes the gravimagnetic equation set (46-48) compatible with experimental evidence. It has to be noted, however, that where the increase of the Lorentz force by a factor 4 straightforwardly follows from the metric, the gravitational equivalents for the Faraday induction and the displacement current are heuristically introduced to obtain symmetry between gravimagnetism and electromagnetism. It might well be that the gravitational field is not radiating and that a spin-1 graviton, such as suggested by the equation set, is non-existing.

VI. COSMOLOGICAL GRAVITY

The experimental results of the Gravity B project have shown that gravimagnetism exists, but that it is inadequate to explain the origin of black matter, because of the weakness of the frame dragging effect. So, something else is required to explain the phenomenon of flat rotation curves in a galaxy. Unusual problems ask for unusual explanations. In this paragraph, I wish to propose an unusual explanation, albeit that use will be made of common concepts in field theory. To this end, the Lagrangian density of the cosmological gravity field will be derived from the generic expression,

$$\text{☹} = -\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + U(\Phi) + \rho\Phi, \tag{49}$$

where $U(\Phi)$ is the potential energy of the field and where $\rho\Phi$ is the source term. Let us compare three different options for $U(\Phi)$, respectively $U(\Phi) = 0, U(\Phi) = \lambda^2\Phi/2$ and $U(\Phi) = -\lambda^2\Phi/2$, where $\lambda > 0$ and real. Application of the Principle of Action as embodied in the Lagrange-Euler equation yields a differential equation for the spatial behavior of the field's potential energy. The homogenous equations are respectively

$$\text{(a) } \frac{1}{r}\frac{\mathrm{d}^2}{\mathrm{d}r^2}(r\Phi) = 0;\ \text{(b) } \frac{1}{r}\frac{\mathrm{d}^2}{\mathrm{d}r^2}(r\Phi) - \lambda^2\Phi = 0 \text{ and (c) } \frac{1}{r}\frac{\mathrm{d}^2}{\mathrm{d}r^2}(r\Phi) + \lambda^2\Phi = 0. \tag{50}$$

The non-trivial solutions for the first and second case are

$$\Phi = \frac{\Phi_0}{\lambda r} \text{ and } \Phi = \Phi_0 \frac{\exp(-\lambda r)}{\lambda r}. \tag{51}$$

The first case applies to electromagnetism (for $\Phi_0 = Q\lambda / 4\pi\varepsilon_0$) and Newtonian gravity (for $\Phi_0 = MG\lambda$). The second case applies to Proca's generalization of the Maxwellian field. It reduces to the first case if $\lambda \to 0$, while keeping Φ_0 / λ constant. Generically, it represents a field with a format that corresponds with the potential as proposed by Yukawa to explain the short range of the nuclear force. It has been used by the author of this article for the purpose to express the Gravitational Constant in quantum mechanical quantities[3].

Let us now consider the third case. It can be readily verified from (50c) that a non-trivial solution for this case is,

$$\Phi = \Phi_0 \frac{\cos \lambda r + \sin \lambda r}{\lambda r}. \tag{52}$$

In accordance with the concepts of classical field theory, the field strength can be established as the spatial derivative of the potential Φ. Identifying Φ_0 / λ as MG and λ as a range parameter, we may identify this field strength as a cosmological gravity force F_{CG}. Let us compare this force with the Newtonian force F_N. To do so more explicitly, we compare $F_N r^2$ with $F_{CG} r^2$. The comparison is shown in figure 7.

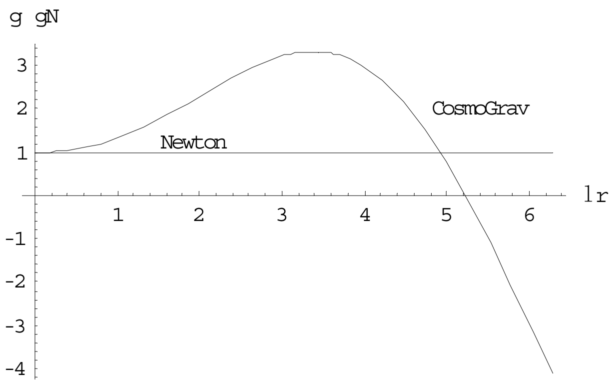


Figure 7: The cosmological gravity force compared with the Newtionian force

This figure shows that, for relative small values of r, the cosmological gravity force behaves similarly as the Newtonian force, but that its relative strength over the Newtonian force increases significantly for large values of r. This is a similar behavior as heuristically implemented in MOND. The effective range is determined by the parameter λ. It might therefore well be that the cosmological gravity force manifests itself only at cosmological scale. Figure 8 shows that under influence of this force, the rotation curves in the galaxy may assume a flat behavior. This hypothetical cosmological gravity shows an intriguing phenomenon. At very far cosmological distance, the attraction of gravity is inverted into repulsion. There is some

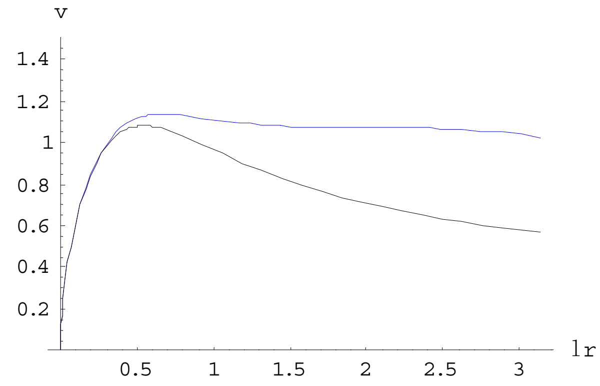


Figure 8: boost of the rotation curve under influence of cosmological gravity.

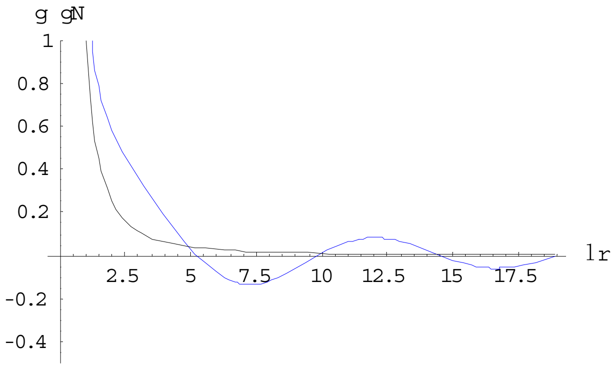


Figure 9: Inversion of the gravity force to antigravity at large cosmological distances. Black: Newtonian. Blue: Cosmological Gravity.

speculation reported in literature that such antigravity is required to explain the phenomenon of dark energy, responsible for the accelerated expansion of the universe [18]. Exploration of this phenomenon is a subject outside the scope of this article. It has to be noted that the solution (49) is not unique. There are more solutions possible by modifying the magnitude of $\sin \lambda r$ over $\cos \lambda r$. I have simply chosen here for the symmetrical solution. Cosmological observations would be required to obtain more insight in this. Such observations are required as well for establishing meaningful values for λ. In fact, combining a variety of mass density distributions and values for λ offer a wealth of galaxy rotation curves that are possible within the validity of the hypothesis.

This view on gravity has an exciting beauty. It unifies the four forces of nature in a single expression for the scalar part of their Lagrangian density, because of

☹ $= -\frac{1}{2}\partial_{\mu}\Phi\partial^{\mu}\Phi + U(\Phi) + \rho\Phi$, where

$U(\Phi) = -\lambda^2\Phi/2$ for gravity,

$U(\Phi) = \lambda^2\Phi/2$ for the nuclear forces [3,4],

$U(\Phi) = 0$ for electromagnetism. (53)

A remaining challenge now is, to harmonize the symmetry that shows up in this classical field view, with Einstein's Field Equation, in which the potential function concept is absent. It might well be that the cosmological parameter λ, which I have introduced here, can be related with the cosmological constant Λ that has been added as a degree of freedom by Einstein in his field equation. Let us see if such could be true. Most generically, Einstein's Field Equation reads as

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \,. \tag{54}$$

Because no experimental evidence could be found that this constant Λ would be different from zero, Einstein regretted its introduction by saying "*The introduction of the cosmological constant was the biggest blunder in my life*". But I believe, it wasn't. As shown in the Appendix, we may relate Einstein's cosmological constant Λ with the cosmological parameter λ indeed, as

$$\Lambda = -\lambda^2 \frac{\Phi}{2c^2} = \lambda^2 \frac{MG}{2rc^2} \,, \tag{55}$$

where M is the enclosed mass in the galaxy. It is probably fair to say that, apart from the explanation given for dark matter as proposed in this paragraph, this interpretation of the Einstein's cosmological constant, gives a possible support for the cosmological gravity as developed in this article. From textbooks [19], it can be readily concluded that Friedmann's equations predict an accelerated expansion of the universe under a positive value for the cosmological constant. That means that the hypothesis developed in this article does not only give a possible explanation for dark matter, but for dark energy as well.

A. Comparison with MOND

It might be instructive to compare this view on cosmological gravity with MOND. MOND is a heuristic approach based on a modification of the gravitational acceleration g such that

$$g = \frac{g_N}{\mu(x)} \text{, with } x = g / a_0 \tag{56}$$

where $\mu(x)$ is an interpolation function, $g_N (= MG / r^2)$ the Newtonian gravitational acceleration and where a_0 is an empirical constant acceleration. The format of the interpolation function is not known, but the objectives of MOND are met by a simple function like [20]

$$\mu(x) = \frac{x}{\sqrt{1+x^2}} \,. \tag{57}$$

If $g / a_0 << 1$, such as happens for large r, (56) reduces to

$$g = \sqrt{a_0 g_N} \ . \tag{58}$$

Under this condition, the gravitational acceleration decreases as r^{-1} instead of r^{-2}. As a result the orbital velocity curves as a function of r show up as flat curves.

Algebraic evaluation of (57) and (58) results into,

$$\frac{g}{g_N} = \sqrt{\frac{1+\sqrt{1+4k^2(\lambda r)^4}}{2}} \quad \text{with } k = \frac{a_0}{MG\lambda^2} \ . \tag{59}$$

This expression allows a comparison with the hypothesis as developed in this article. From (52), under consideration of $\Phi_0 = MG\lambda$,

$$\Phi = \Phi_0 \frac{\cos\lambda r + \sin\lambda r}{\lambda r} \rightarrow g = -\nabla\Phi = \frac{MG}{r^2}\{(1-\lambda r)\cos\lambda r + (1+\lambda r)\sin\lambda r\} \ , \tag{60}$$

hence

$$\frac{g}{g_N} = (1-\lambda r)\cos\lambda r + (1+\lambda r)\sin\lambda r \ . \tag{61}$$

As illustrated in figure 10, a pretty good fit between (59) and (61) is obtained if

$$k = \frac{a_0}{MG\lambda^2} = 2.5 \rightarrow a_0 = 2.5MG\lambda^2 \ . \tag{62}$$

Observations on various galaxies have shown that a_0 can be regarded as a galaxy-independent constant with a value about $a_0 \approx$ 1x10^{-10} m/s^2. The implication of (62) is, that $a_0 \approx$ 1x10^{-10} m/s^2 is a second gravitational constant next to G. The two constants determine the range λ of the gravitational force in solar systems and galaxy systems as $\lambda^2 \approx 2a_0/5MG$, where M is the enclosed mass in those systems.

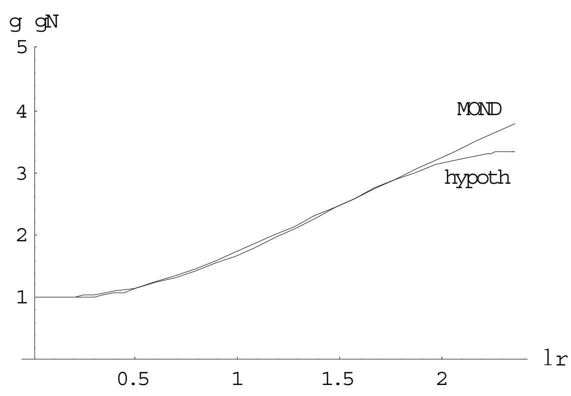


Figure 10: MOND's interpolation function compared with the theory as developed.

There is no reason why these observations, made on solar and galaxian systems, would not apply to the universe as a whole. It has to be noted, though, that modeling enclosed mass as central mass in empty space as we did, is a simplification with limitations. This is particularly true for applying the model to the universe, which in accordance with Friedman's view, is considered as an equi-temporal plane without a center. Having stated this, let us continue by elaborating the model as developed so far, a bit further. Let us do so by relating electromagnetism and gravitation in terms of the field's energy density w created by the mass M as

$$w = \frac{1}{2}\varepsilon_0 \left|\nabla\Phi_e\right|^2 \Leftrightarrow \frac{1}{8\pi G}\left|\nabla\Phi\right|^2 . \tag{63}$$

I have invoked here $\varepsilon_G = (4\pi G)^{-1}$, in correspondence with the view on gravimagnetism, as outlined before. Relationship (63) allows relating the massive energy Mc^2 and the energetic field density w of the mass M, under consideration of (60) as

$$Mc^2 = \int_V w\mathrm{d}V = \frac{1}{8\pi G}(MG)^2 \int_V f(r)\mathrm{d}V \text{ with } f(r) = \left|\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{\cos\lambda r + \sin\lambda r}{r}\right)\right|^2 . \tag{64}$$

To cope with the renormalization problem of classical field theory, let us split the volume in a part covered by $r < r_0$ and in a part covered by $r \geq r_0$, with energy contributions, respectively, $(1-\gamma)Mc^2$ and γMc^2, where γ is an unknown dimensionless fraction. This, in fact, implies that below the boundary $r < r_0$ the mass is no longer considered as pointlike. It enables evaluating (64) as

$$Mc^2 = \frac{(MG)^2}{8\pi G}\int_r f(r) 4\pi r^2 \mathrm{d}r \rightarrow Mc^2 = \frac{(MG)^2}{8\pi G}\int_{r_0}^{r_\infty} f(r) 4\pi r^2 \mathrm{d}r . \tag{65}$$

In the case that $\lambda = 0$, we would have the Newtonian condition. In that case,

$$\int_{r_0}^{r_\infty} f(r) r^2 \mathrm{d}r = \frac{1}{r_0} \quad \text{and} \quad M = \frac{2c^2 r_0}{G} . \tag{66}$$

This result is dimension-wise correct. Like in electromagnetism, apart from an empirical assessment from experimental evidence, there are no means to establish a value for the classical radius r_0 of the "pointlike" mass M. Under the condition $\gamma = 1$, there is no field left for $r_0 < MG/2c^2$, i.e. no field below a quarter of the Schwarzschild radius.

More interestingly is the case $\lambda > 0$. Evaluation of the integral under the condition $\lambda << 1$ and $r_\infty >> 1$ gives

$$\int_{r_0}^{r_\infty} f(r) r^2 \mathrm{d}r = \frac{1}{r_0} + \lambda^2 r_\infty .$$
(67)

The radius r_∞ is the ultimate horizon, which in the Newtonian limit ($\lambda = 0$) does not play any role, while it does in the non-Newtonian case. If the contribution $\lambda^2 r_\infty$ would dominate the contribution $1/r_0$, it follows readily from (64-67),

$$\lambda^2 = \gamma \frac{2c^2}{r_\infty MG} . \tag{68}$$

This raises a similar problem as in the Newtonian limit. Under the condition $\gamma = 1$, the quantity $(\lambda^2 r_\infty)^{-1}$ has the same value as the lower boundary r_0, plays a similar role in the sense that the field density has to be truncated to zero for $r > r_\infty$. Mond's acceleration constant (62) can then be written as

$$a_0 = 5\gamma \frac{c^2}{r_\infty} .$$

Considering that 1 light year = 9. 46x10^{15} m and that $a_0 \approx$ 1x10^{-10} m/s^2, it would make the ultimate horizon $r_\infty \approx 5\gamma$ x 10^{11} light years. This is considerably larger than the cosmological horizon, estimated from red shift experiments as $r_c \approx$ 40 x 10^9 light years [21]. A way to explain this is by stating that, as long as the cosmological horizon is smaller than the ultimate horizon, the stationary condition of the energy field spread by the pointlike mass M is not yet reached and that a considerable amount of the energy still is contained in the transient part of the solution of the field equation. Once the stationary condition is reached, the massive field energy, which equates the energy equivalent of the mass M, is spread over space. Under the constraint $\gamma = 1$, it is truncated between two boundaries. The parameter γ allows playing with the hard boundary truncation. This touches the subjects of the physics of black holes and the cosmological horizon [22].

It is probably fair to conclude that, where the cosmological gravity model as developed in this article, applies rather well to solar systems and galaxian systems, it is inadequate still for the universe as a whole, albeit that we may conclude as well that there is no conflict. Crucial here is the conclusion that Einstein's Λ should be regarded as a cosmological parameter instead of a cosmological constant. MOND's acceleration constant (62) a_0 is the more basic true invariant. The two are related as $\Lambda = 2a_0 / 5rc^2$ [m^{-2}].

VII. SYNOPSIS AND CONCLUSIONS

The search for relationships between the four basic forces of nature is an on-going challenge. In this respect, the gravitational force is the most problematic one. Even to the extent that the question if gravity is a basic force indeed, or just emergent from the other basic forces, is still subject to debate. Where electromagnetism and the two nuclear forces are unified in the context of quantum physics, gravity is still on its own. This, in spite of the beauty of Einstein's General Relativity, which has revealed and explained so many gravity related cosmological phenomena. In this article, I have described gravity from a point of view inherited from my earlier work on quantum physics, in which I have positioned quarks as pointlike sources spreading a composite field of energy, identified by me as the Higgs field. It gives rise to bonds of two quarks (mesons) and three quarks (baryons), to which attributes can be assigned that we know as electric charge and mass. In this view, the fields of the strong nuclear force and the weak nuclear force are basic and the fields of electromagnetism and gravity are sourced by the attributes of the quark bonds. In earlier work, this view has enabled me to express the Gravitational Constant in terms of quantum mechanical quantities [4. It appeared being possible by combining views of General Relativity with the views of classical field theory. One of the instruments used, was applying Proca's generalization of Maxwell's laws to the nuclear fields of energy. In the context of this article, I have done something similar for gravity. Just by changing the polarity of the mass term in Proca's generalization. In this article, gravity has been modeled as a scalar field, i.e., as a field with a scalar potential. Whether this field can be generalized to a vectorial one (that therefore radiates) depends on the question if the gravitational equivalents of the Faraday induction and the displacement current exist in the gravimagnetic view on gravity. It is one of the reasons why I have included an appendix, in which I have shown that the causality of the gravity field does not need a vectorial bosonic description. Therefore, the existence of a spin-1 gravimagnetic analogon of a photon that would follow from a Maxwellian description of the gravitational field, is unlikely. The more, because it would need an artificial adaption of the Lorentz force to explain a factor 4 discrepancy. Instead, spin-2 (i.e. non-radiating) gravimagnetism appears being a useful equivalent model for the weak limit view of Einstein's Field Equation for a rotational metric.

In this article, the four forces of nature have been unified in a single expression for the scalar part of their underlying Lagrangian density. This Lagrangian density is Proca's generalization of the Maxwellian one. For electromagnetism, Proca's "mass term" is zero, for the nuclear forces the "mass term" is positive, for gravity the "mass term" is negative. As a consequence, the electromagnetic field potential decays as $1/r$, the nuclear potential decays more aggressively as $\exp(-\lambda r)/r$ and the gravity potential decays less aggressively as $(\cos\lambda r + \sin\lambda r)/r$. Effectively, the gravity potential remains the Newton one in our common world, but is different at cosmological scale. This property explains the cosmological phenomenon that is usually assigned to dark matter. Because of the match in results, the developed model can be regarded as an underlying theory for the heuristic MOND approach, albeit that the prognosis that, at very large cosmological distances, gravity periodically turns on-and-off into antigravity marks a decisive difference. It is shown in this article that the range determining parameter λ is related with a second gravitational constant $a_0 \approx 1\text{x}10^{-10}$ m/s^2 next to G. The two constants determine the range λ of the gravitational force in solar systems and galaxy systems as $\lambda^2 \approx 2a_0/5MG$, where M is the enclosed mass in those systems. Where this second gravitational quantity a_0 is a constant, this is not true for the Einsteinean parameter Λ, which appears being radial related as $\Lambda = 2a_0/5rc^2$ [m^{-2}]. The theory as developed in this article gives an adequate explanation for the galaxian phenomenon of flat rotation curves and for the cosmological phenomenon that our universe is expanding in acceleration, such as predicted by Friedmann's law, under influence of a positive value of Einstein's cosmological parameter.

APPENDIX : THE GRAVITATIONAL WAVE EQUATION

The objective in this appendix is to derive the weak field limit of the gravitational wave equation with inclusion of the Cosmological Constant. This objective implies that we have to solve Einstein's Field Equation for a spherically symmetric space-time metric that is given by the line element (2),

$$\mathrm{d}s^2 = g_{tt}(r,t)\mathrm{d}q_0^2 + g_{rr}(r,t)\mathrm{d}r^2 + r^2\sin^2\vartheta\mathrm{d}\varphi^2 + r^2\mathrm{d}\vartheta^2\,, \tag{A-1}$$

where $q_0 = \mathrm{i}ct$.

Note: The space-time (i*ct*, *r*, ϑ, φ) is described on the basis of the "Hawking" metric (+,+,+,+). The components $g_{\mu\mu}$ are part of the metric tensor $g_{\mu\nu}$, which determine the Ricci tensor $R_{\mu\nu}$ and the Ricci scalar R. These quantities play a decisive role in Einstein's Field Equation, which reads as

$$G_{\mu\nu} + \Lambda_{\mu\nu} g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad \text{with} \quad G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}\,. \tag{A-2}$$

At this point of the description, $\Lambda_{\mu\nu}$ is a tensor with covariant derivative zero. Under spherically symmetric conditions, the tensors $g_{\mu\nu}, R_{\mu\nu}, \Lambda_{\mu\nu}$ and $T_{\mu\nu}$ have diagonal elements only. On top of that, the metric tensors $\Lambda_{\mu\nu}$ and $T_{\mu\nu}$ are particular. Apart from T_{00}, the other metric elements in $T_{\mu\nu}$ are zero. This is imposed by the assumption that the curving of space-time is caused by a pointlike massive object with mass M. The heuristic tensor $\Lambda_{\mu\nu}$ is particular in the sense that all diagonal elements are equal, i.e. $\Lambda_{\mu\mu} = \Lambda_{00}$ for $\mu = 0,1,2,3$. This is a relaxation of Einstein's Cosmological Constant from just being a constant scalar. More details will follow later.

In empty space, the Einstein Field Equation (1) under this symmetric spherical isotropy, reduces to a simple set of equations for the elements $R_{\mu\mu}$ of the Ricci tensor,

$$R_{tt} - \frac{1}{2} R g_{tt} + \Lambda_{00} g_{tt} = 0\,; \quad R_{rr} - \frac{1}{2} R g_{rr} + \Lambda_{00} g_{rr} = 0\,; \tag{A-3a,b,c,d}$$
$$R_{\vartheta\vartheta} - \frac{1}{2} R g_{\vartheta\vartheta} + \Lambda_{00} g_{\vartheta\vartheta} = 0 \quad R_{\varphi\varphi} - \frac{1}{2} R g_{\varphi\varphi} + \Lambda_{00} g_{\varphi\varphi} = 0.$$

The results of the calculation of the Ricci tensor as shown later in this Appendix is summarized in Table A-1. Note: g' and g'' means differentiation, respectively double differentiation of g into r ; $\dot{g}$ and $\ddot{g}$ means differentiation, respectively double differentiation of g into t. Furthermore, the subscripts and superscripts 00, 11 ,22, and 33 are, respectively, identical to $tt, rr, \vartheta\vartheta$ and $\varphi\varphi$.

Subtraction after multiplying (A-3a) by $1/g_{tt}$ and (A-3b) by $1/g_{rr}$ under consideration of the expressions from Table A-1, results into,

$$\frac{g'_{rr}}{g_{rr}} + \frac{g'_{tt}}{g_{tt}} = 0\,. \tag{A-4}$$

Condition (A-4) is met if

$$g_{rr} g_{tt} = 1\,. \tag{A-5}$$

Imposing this condition on the Ricci tensor component R_{00} in Table A-1, gives

$$R_{00} = -\frac{1}{2} g''_{tt} + \frac{1}{r} g'_{tt} + \frac{\ddot{g}_{rr}}{2c^2} = -\frac{1}{2r} \frac{\partial^2 (r g_{tt})}{\partial t^2} + \frac{1}{2c^2} \frac{\partial^2 g_{tt}}{\partial t^2}\,. \tag{A-6}$$

Let us proceed by considering the Ricci scalar. It is defined generically as

$$R = \sum_{\mu=0}^{3} \sum_{\nu=0}^{3} g^{\mu\nu} R_{\mu\nu}\,. \tag{A-7}$$

In spherical symmetry the matrices contain diagonal elements only, so that (A-7) reduces to

$$R = \sum_{\mu=0}^{3} g^{\mu\mu} R_{\mu\mu}\,. \tag{A-8}$$

This result can be applied to Einstein's Field Equation, which actually is a set of four simultaneous equations. Multiplying the first one with g^{00}, the second one with g^{11}, etc., and subsequent addition results of the terms $\mu = 1,2,3$ gives (A-9),

$$\sum_{\mu=1}^{3} g^{\mu\mu} R_{\mu\mu} - \frac{3}{2} R + 3\Lambda_{00} = -g^{00} R_{00} + \sum_{\mu=0}^{3} g^{\mu\mu} R_{\mu\mu} - \frac{3}{2} R + 3\Lambda_{00} = -g^{00} R_{00} - \frac{1}{2} R + 3\Lambda_{00} = 0\,,$$

so that $g^{00} R_{00} \approx -\frac{1}{2} R + 3\Lambda_{00}$. (A-10)

Applying this result to the non-homogeneous member of Einstein's equation set gives,

$$2g^{00} R_{00} - 2\Lambda_{00} = \frac{8\pi G T_{00} g^{00}}{c^4}\,. \tag{A-11}$$

In the weak field limit, such that

$g_{tt}(r,t) = 1 + h_\varphi(r,t)$, where $|h_\varphi(r,t)| << 1$, (A-12)

and under consideration of (A-6), (A-11) can be written as

$$\frac{1}{r}\frac{\partial^2 (rh_\varphi)}{\partial r^2} - \frac{1}{c^2}\frac{\partial^2 h_\varphi}{\partial t^2} - 2\Lambda_{00} = \frac{8\pi G T_{00}}{c^4}\,. \tag{A-13}$$

Identifying the mass density as pointlike, such that $T_{00} = Mc^2\delta^3(r)$, we have

$$\frac{1}{r}\frac{\partial^2 (rh_\varphi)}{\partial r^2} - \frac{1}{c^2}\frac{\partial^2 rh_\varphi}{\partial t^2} - 2r\Lambda_{00} = r\frac{8\pi GM}{c^2}\delta^3(r)\,. \tag{A-14}$$

If $\Lambda_{00} = 0$, the solution assumes the format of the Newtonian gravity law. However, if Λ_{00} is a scalar constant, (A-14) is not a proper wave equation for a pointlike massive source. So, what is wrong here? The escape comes from the following observation. As long as Λ_{00} is a uniform quantity within all four members of the equation set (A-2) and as long its covariant derivative with respect to any of the four coordinates is zero, other semantics, different from just being a constant scalar, might be adhered to $\Lambda_{00} \equiv \Lambda$. A possibility that results into a proper gravitational wave equation is offered by a format given by,

$$\Lambda = \lambda^2(1 - g_{tt})/2\,. \tag{A -15}$$

The format is uniform for $\mu =$ 0,1,2,3 indeed and, because of the zero value of the covariant derivative of the metric component g_{tt}, the covariant derivative of (A-15) is zero as well. Insertion this format into equation (A-14) results into the wave equation

$$\frac{\partial^2 (rh_\varphi)}{\partial r^2} - \frac{1}{c^2}\frac{\partial^2 rh_\varphi}{\partial t^2} + \lambda^2 rh_\varphi = \frac{8\pi GM}{c^2} r\delta^3(r)\,. \tag{A-16}$$

Let us consider the static solution for various options of λ.

If $\lambda = 0$, we have under static conditions,

$$\frac{1}{r}\frac{\partial^2 (rh_\varphi)}{\partial r^2} = \frac{8\pi GM}{c^2}\delta^3(r)\,. \tag{A-17}$$

In accordance with Einstein's weak field limit hypothesis, this is similar to Poisson's equation,

$$\nabla^2\Phi = \frac{1}{r}\frac{\partial^2 (r\Phi)}{\partial r^2} = 4\pi G\rho = 4\pi GM\delta^3(r)\,, \tag{A-18}$$

the solution of which is the Newtonian potential,

$$\Phi = -\frac{GM}{r}\ [\mathrm{m^2 s^{-2}}]. \tag{A-19}$$

Comparing (A-17) with (A-18) gives the equivalence

$$h_\varphi = \frac{2\Phi}{c^2}. \tag{A-20}$$

If $\lambda < 0$, we have under static conditions, a similarity with Helmholtz' equation with the screened Poisson's equation, the solution of which is Yukawa's potential,

$$\Phi = \frac{GM}{r}\exp(-\lambda r), \tag{A-21}$$

which reduces to Poisson's one for $\lambda \to 0$.

If $\lambda > 0$, we have under static conditions, a similarity with Helmhotz' equation [19] with a characteristic solution,

$$\Phi = \frac{GM}{r}\{\cos\lambda r + \sin\lambda r\}. \tag{A-22}$$

This solution reduces to Poisson's one for $\lambda \to 0$ as well.

This would have been the weak field limit solution of Einstein's Equation if he had not taken the validity of Poisson's equation of gravity for granted, but had adopted Helmholtz equation instead under an appropriate choice of the Cosmological Constant.

Table A1: metric tensor and Ricci tensor

metric tensor	Ricci tensor
$g_{tt} \equiv g_{00}$	$R_{tt} = -\frac{1}{2}\frac{g''_{tt}}{g_{rr}} - \frac{\ddot{g}_{rr}}{2c^2 g_{rr}} + \frac{g'_{tt}}{4g_{rr}}(\frac{g'_{rr}}{g_{rr}} + \frac{g'_{tt}}{g_{tt}}) - \frac{\dot{g}_{rr}}{4c^2 g_{rr}}(\frac{\dot{g}_{rr}}{g_{rr}} + \frac{\dot{g}_{tt}}{g_{tt}}) - \frac{1}{r}\frac{g'_{tt}}{g_{rr}}$
$g_{rr} \equiv g_{11}$	$R_{rr} = -\frac{1}{2}\frac{g''_{tt}}{g_{tt}} - \frac{\ddot{g}_{rr}}{2c^2 g_{tt}} + \frac{g'_{tt}}{4g_{tt}}(\frac{g'_{rr}}{g_{rr}} + \frac{g'_{tt}}{g_{tt}}) - \frac{\dot{g}_{rr}}{4c^2 g_{tt}}(\frac{\dot{g}_{rr}}{g_{rr}} + \frac{\dot{g}_{tt}}{g_{tt}}) + \frac{1}{r}\frac{g'_{rr}}{g_{rr}}$
$g_{\vartheta\vartheta} \equiv g_{22} = r^2$	$R_{\vartheta\vartheta} = 1 + \frac{r}{2g_{rr}}(\frac{g'_{rr}}{g_{rr}} - \frac{g'_{tt}}{g_{tt}}) - \frac{1}{g_{rr}}$
$g_{\varphi\varphi} \equiv g_{33} = r^2\sin^2(\vartheta)$	$R_{\kappa\varphi} = \sin^2(\vartheta) R_{\vartheta\vartheta}$

Calculation of the Ricci tensor
The Ricci tensor is described in expanded form by

$$R_{ij} = \sum_{k=0}^{3} \left(\frac{\partial \Gamma_{ij}^{k}}{\partial q_k} - \frac{\partial \Gamma_{ik}^{k}}{\partial q_j} \right) + \sum_{i=0}^{3} \sum_{k=0}^{3} \left(\Gamma_{ij}^{i} \Gamma_{ki}^{k} - \Gamma_{ik}^{i} \Gamma_{ji}^{k} \right) . \qquad \text{(A-23)}$$

The Christoffel symbols Γ_{ij}^{k} represent functions of the metric elements, such that

$$\Gamma_{ij}^{k} = \frac{1}{2} \sum_{m=0}^{3} g^{km} \left\{ \frac{\partial g_{jm}}{\partial q_i} + \frac{\partial g_{im}}{\partial q_j} - \frac{\partial g_{ji}}{\partial q_m} \right\} . \qquad \text{(A-24)}$$

Under symmetric spherical isotropy, only diagonal terms remain, so that the expression reduces to

$$R_{ii} = \sum_{k=0}^{3} \left(\frac{\partial \Gamma_{ii}^{k}}{\partial q_k} - \frac{\partial \Gamma_{ik}^{k}}{\partial q_i} \right) + \sum_{i=0}^{3} \sum_{k=0}^{3} \left(\Gamma_{ii}^{i} \Gamma_{ki}^{k} - \Gamma_{ik}^{i} \Gamma_{ii}^{k} \right) , \qquad \text{(A-25)}$$

and the Christoffel symbols reduce to

$$\Gamma_{ij}^{k} = \frac{1}{2 g_{kk}} \left\{ \frac{\partial g_{kj}}{\partial q_i} + \frac{\partial g_{ki}}{\partial q_j} - \frac{\partial g_{ji}}{\partial q_k} \right\} , \qquad \text{(A-26)}$$

such that only three different forms remain,

$$\Gamma_{kk}^{k} = \frac{1}{2 g_{kk}} \frac{\partial g_{kk}}{\partial q_k} ; \quad \Gamma_{ii}^{k} = -\frac{1}{2 g_{kk}} \frac{\partial g_{ii}}{\partial q_k} \quad (k \neq i) \text{ and } \Gamma_{ik}^{k} = \Gamma_{ki}^{k} = \frac{1}{2 g_{kk}} \frac{\partial g_{kk}}{\partial q_i} . \qquad \text{(A-27)}$$

Table A2: Christoffel elements and affine connections of the isotropic non-rotating metric

Γ_{tt}^{t}	Γ_{tr}^{r}		
Γ_{rt}^{t}	Γ_{rr}^{t}		

Γ_{tt}^{r}	Γ_{tr}^{r}		
Γ_{rt}^{r}	Γ_{rr}^{r}		
		$\Gamma_{\vartheta\vartheta}^{r}$	
			$\Gamma_{\varphi\varphi}^{r}$

		$\Gamma_{r\vartheta}^{\vartheta}$	
	$\Gamma_{\vartheta r}^{\vartheta}$		
			$\Gamma_{\varphi\varphi}^{\vartheta}$

			$\Gamma_{r\varphi}^{\varphi}$
			$\Gamma_{\vartheta\varphi}^{\varphi}$
	$\Gamma_{\varphi r}^{\varphi}$	$\Gamma_{\varphi\vartheta}^{\varphi}$	

Table A2 shows the Christoffel elements different from zero, where

$$\Gamma^t_{tt} = \frac{1}{2ic}\frac{\dot{g}_{tt}}{g_{tt}} \quad \Gamma^t_{rr} = -\frac{1}{2ic}\frac{\dot{g}_{rr}}{g_{tt}} \quad \Gamma^t_{tr} = \Gamma^t_{rt} = \frac{1}{2}\frac{g'_{tt}}{g_{tt}}$$

$$\Gamma^r_{rr} = \frac{1}{2}\frac{g'_{rr}}{g_{rr}} \quad \Gamma^r_{tt} = -\frac{1}{2}\frac{g'_{tt}}{g_{rr}} \quad \Gamma^r_{rt} = \Gamma^r_{tr} = \frac{1}{2ic}\frac{\dot{g}_{rr}}{g_{rr}}$$

$$\Gamma^r_{\vartheta\vartheta} = -\frac{r}{g_{rr}} \quad \Gamma^r_{\varphi\varphi} = -\frac{r\sin^2\vartheta}{g_{rr}}$$

$$\Gamma^\vartheta_{r\vartheta} = \Gamma^\vartheta_{\vartheta r} = \frac{1}{r} \quad \Gamma^\vartheta_{\varphi\varphi} = -\sin\vartheta\cos\vartheta$$

$$\Gamma^\varphi_{r\varphi} = \Gamma^\varphi_{\varphi r} = \frac{1}{r} \quad \Gamma^\varphi_{\vartheta\varphi} = \Gamma^\varphi_{\varphi\vartheta} = \cot\vartheta \qquad \text{(A-28)}$$

Application of (A-28) on (A-23) gives the Ricci tensor as listed in Table A1.

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