

## ON FENG QI-TYPE INTEGRAL INEQUALITIES FOR CONFORMABLE FRACTIONAL INTEGRALS

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ABSTRACT. In this paper, we establish the generalized Qi-type inequality involving conformable fractional integrals. The results presented here would provide extensions of those given in earlier works.

### 1. INTRODUCTION

In the last few decades, much significant development of integral inequalities had been established. Integral inequalities have been frequently employed in the theory of applied sciences, differential equations, and functional analysis. In the last two decades, they have been the focus of attention in [4]-[7]. Recently, especially Qi inequality, one of the integral inequalities, has been studied by many authors. Recall the famous integral inequality of Feng Qi type:

**Theorem 1.** (Proposition 1.1, [7]) *Let  $f(x)$  be differentiable on  $(a, b)$  and  $f(a) = 0$ . If  $f'(x) \geq 1$  for  $x \in (a, b)$ , then*

$$(1.1) \quad \left( \int_a^b [f(t)]^3 dt \right) \geq \left( \int_a^b f(t) dt \right)^2.$$

*If  $0 \leq f'(x) \leq 1$ , then the inequality (1.1) reverses.*

**Theorem 2.** (Proposition 1.3, [7]) *Let  $n$  be a positive integer. Suppose  $f(x)$  has continuous derivative of the  $n$ -th order on the interval  $[a, b]$  such that  $f^{(i)}(a) \geq 0$ , where  $0 \leq i \leq n-1$ ,  $f^{(n)}(x) \geq n!$ , then*

$$(1.2) \quad \left( \int_a^b [f(t)]^{n+2} dt \right) \geq \left( \int_a^b f(t) dt \right)^{n+1}.$$

*In [6], Ngô et al. gave the following inequality which is one of the open problem's solution.*

**Theorem 3.** *If  $f$  is a nonnegative, continuous function on  $[0, 1]$  satisfying,*

$$(1.3) \quad \int_x^1 f(t) dt \geq \frac{1-x^2}{2},$$

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then, for every positive number  $n$ ,

$$(1.4) \quad \int_0^1 f^{n+1}(t) dt \geq \int_0^1 t^n f(t) dt,$$

holds.

**Theorem 4.** ([2]) Let  $f(x) \geq 0$  be a continuous function on  $[a, b]$ , then

$$(1.5) \quad \int_a^b f^{\alpha+\beta}(x)x^\rho dx \geq \int_a^b (x^{\rho+1} - a^{\rho+1})^\alpha f^\beta(x)x^\rho dx,$$

holds for every positive real number  $\alpha > 0$ ,  $\beta > 0$  and  $\rho \geq 0$ .

**Theorem 5.** ([2]) Let  $f(x) \geq 0$  be a continuous function on  $[a, b]$ , then

$$(1.6) \quad \int_a^b (x^{\rho+1} - a^{\rho+1})^\alpha f^\beta(x)x^\rho dx \geq \frac{(b^{\rho+1} - a^{\rho+1})^{\alpha+\beta+1}}{(\rho+1)(\alpha+\beta+1)},$$

holds for every positive real number  $\alpha > 0$ ,  $\beta > 0$  and  $\rho \geq 0$ .

**Lemma 1.** (General Cauchy inequality). Let  $\alpha$  and  $\beta$  be positive real numbers satisfying  $\alpha + \beta = 1$ . Then for all positive real numbers  $x$  and  $y$ , we have

$$\alpha x + \beta y \geq x^\alpha y^\beta.$$

## 2. DEFINITIONS AND PROPERTIES OF CONFORMABLE FRACTIONAL DERIVATIVE AND INTEGRAL

The following definitions and theorems with respect to conformable fractional derivative and integral were referred in [1], [9]-[13].

**Definition 1.** (*Conformable fractional derivative*) Given a function  $f : [0, \infty) \rightarrow \mathbb{R}$ . Then the “conformable fractional derivative” of  $f$  of order  $\alpha$  is defined by

$$(2.1) \quad D_\alpha(f)(t) = \lim_{\epsilon \rightarrow 0} \frac{f(t + \epsilon t^{1-\alpha}) - f(t)}{\epsilon}$$

for all  $t > 0$ ,  $\alpha \in (0, 1)$ . If  $f$  is  $\alpha$ -differentiable in some  $(0, a)$ ,  $\alpha > 0$ ,  $\lim_{t \rightarrow 0^+} f^{(\alpha)}(t)$  exist, then define

$$(2.2) \quad f^{(\alpha)}(0) = \lim_{t \rightarrow 0^+} f^{(\alpha)}(t).$$

We can write  $f^{(\alpha)}(t)$  for  $D_\alpha(f)(t)$  to denote the conformable fractional derivatives of  $f$  of order  $\alpha$ . In addition, if the conformable fractional derivative of  $f$  of order  $\alpha$  exists, then we simply say  $f$  is  $\alpha$ -differentiable.

The idea of derivative of non-integer order was motivated by the question, “What does it mean by  $\frac{d^n f}{dx^n}$ , if  $n = \frac{1}{2}$ ?”, asked by L’Hospital in 1695 in his letters to Leibniz ([15]-[17]). Afterwards, many mathematicians tried to answer this question for centuries in several points of view. Various types of fractional derivatives were introduced by many authors, most of them are defined via fractional integrals, but many of those fractional derivatives have some non-local behaviors. Among the inconsistencies of the existing fractional derivatives are:

(1) Most of the fractional derivatives do not satisfy  $D_\alpha^\alpha(1) = 0$ , if  $\alpha$  is not a natural number.

(2) All fractional derivatives do not obey the familiar Product Rule and Quotient Rule for two functions.

(3) All fractional derivatives do not obey the Chain Rule.

(4) Fractional derivatives do not have a corresponding Rolle's Theorem, Mean Value Theorem.

(5) All fractional derivatives do not obey:  $D_\alpha^\alpha D_\beta^\alpha f = D_{\alpha+\beta}^\alpha f$ , in general.

To solve some of these and other difficulties, Khalil et al. [10], introduced the following.

**Theorem 6.** Let  $\alpha \in (0, 1]$  and  $f, g$  be  $\alpha$ -differentiable at a point  $t > 0$ . Then

i.  $D_\alpha (af + bg) = aD_\alpha (f) + bD_\alpha (g)$ , for all  $a, b \in \mathbb{R}$  (linearity),

ii.  $D_\alpha (\lambda) = 0$ , for all constant functions  $f(t) = \lambda$ ,

iii.  $D_\alpha (fg) = fD_\alpha (g) + gD_\alpha (f)$  (Product Rule),

iv.  $D_\alpha \left( \frac{f}{g} \right) = \frac{fD_\alpha (g) - gD_\alpha (f)}{g^2}$  (Quotient Rule),

v.  $D_\alpha (f \circ g)(t) = f'(g(t))D_\alpha (g)(t)$  (Chain rule).

If  $f$  is differentiable, then  $D_\alpha (f)(t) = t^{1-\alpha} \frac{df}{dt}(t)$ .

Also:

1.  $D_\alpha (1) = 0$ ,

2.  $D_\alpha (e^{ax}) = ax^{1-\alpha} e^{ax}$ ,  $a \in \mathbb{R}$ ,

3.  $D_\alpha (\sin(ax)) = ax^{1-\alpha} \cos(ax)$ ,  $a \in \mathbb{R}$ ,

4.  $D_\alpha (\cos(ax)) = -ax^{1-\alpha} \sin(ax)$ ,  $a \in \mathbb{R}$ ,

5.  $D_\alpha \left( \frac{1}{\alpha} t^\alpha \right) = 1$ ,

6.  $D_\alpha \left( \sin\left(\frac{t^\alpha}{\alpha}\right) \right) = \cos\left(\frac{t^\alpha}{\alpha}\right)$ ,

7.  $D_\alpha \left( \cos\left(\frac{t^\alpha}{\alpha}\right) \right) = -\sin\left(\frac{t^\alpha}{\alpha}\right)$ ,

8.  $D_\alpha \left( e^{\left(\frac{t^\alpha}{\alpha}\right)} \right) = e^{\left(\frac{t^\alpha}{\alpha}\right)}$ .

**Theorem 7** (Mean value theorem for conformable fractional differentiable functions). Let  $\alpha \in (0, 1]$  and  $f : [a, b] \rightarrow \mathbb{R}$  be a continuous on  $[a, b]$  and an  $\alpha$ -fractional differentiable mapping on  $(a, b)$  with  $0 \leq a < b$ . Then, there exists  $c \in (a, b)$ , such that

$$D_\alpha (f)(c) = \frac{f(b) - f(a)}{\frac{b^\alpha}{\alpha} - \frac{a^\alpha}{\alpha}}.$$

**Definition 2** (Conformable fractional integral). Let  $\alpha \in (0, 1]$  and  $0 \leq a < b$ . A function  $f : [a, b] \rightarrow \mathbb{R}$  is  $\alpha$ -fractional integrable on  $[a, b]$  if the integral

$$(2.3) \quad \int_a^b f(x) d_\alpha x := \int_a^b f(x) x^{\alpha-1} dx$$

exists and is finite.

**Remark 1.**

$$I_{\alpha}^{\alpha}(f)(t) = I_1^{\alpha}(t^{\alpha-1}f) = \int_a^t \frac{f(x)}{x^{1-\alpha}} dx,$$

where the integral is the usual Riemann improper integral, and  $\alpha \in (0, 1]$ .

**Theorem 8.** Let  $f : (a, b) \rightarrow \mathbb{R}$  be differentiable and  $0 < \alpha \leq 1$ . Then, for all  $t > a$  we have

$$(2.4) \quad I_{\alpha}^{\alpha} D_{\alpha}^{\alpha} f(t) = f(t) - f(a).$$

**Theorem 9. (Integration by parts)** Let  $f, g : [a, b] \rightarrow \mathbb{R}$  be two functions such that  $fg$  is differentiable. Then

$$(2.5) \quad \int_a^b f(x) D_{\alpha}^{\alpha}(g)(x) d_{\alpha}x = fg|_a^b - \int_a^b g(x) D_{\alpha}^{\alpha}(f)(x) d_{\alpha}x.$$

**Theorem 10.** Assume that  $f : [a, \infty) \rightarrow \mathbb{R}$  such that  $f^{(n)}(t)$  is continuous and  $\alpha \in (n, n+1]$ . Then, for all  $t > a$  we have

$$D_{\alpha}^{\alpha} f(t) I_{\alpha}^{\alpha} = f(t).$$

**Theorem 11.** Let  $\alpha \in (0, 1]$  and  $f : [a, b] \rightarrow \mathbb{R}$  be a continuous on  $[a, b]$  with  $0 \leq a < b$ . Then,

$$|I_{\alpha}^{\alpha}(f)(x)| \leq I_{\alpha}^{\alpha}|f|(x).$$

In this paper, we propose and prove some new results on the recently proposed conformable fractional derivatives and fractional integral, [Khalil, R., et al., A new definition of fractional derivative, J. Comput. Appl. Math. 264, (2014)]. The simple nature of this definition allows for many extensions of some classical theorems in calculus for which the applications are indispensable in the fractional differential models that the existing definitions do not permit.

### 3. MAIN RESULTS

We start the following important inequality for conformable fractional integrals:

**Theorem 12.** Let  $f(x) \in C_{\alpha}[a, b]$  and  $f(a) = 0$ . If  $0 \leq f'(x) \leq 1$ , then

$$(3.1) \quad \int_a^t [f(x)]^3 d_{\alpha}x \leq \left( \int_a^t f(x) d_{\alpha}x \right)^2.$$

*Proof.* Let

$$F(t) = \left( \int_a^t f(x) d_{\alpha}x \right)^2 - \int_a^t [f(x)]^3 d_{\alpha}x.$$

Simple computation yields

$$\begin{aligned} F'(t) &= 2 \left( \int_a^t f(x) dx \right) f(t) t^{\alpha-1} - [f(t)]^3 t^{\alpha-1} \\ &= \left[ 2 \int_a^t f(x) dx - [f(t)]^2 \right] [f(t)] t^{\alpha-1} \\ &= G(t) [f(t)] t^{\alpha-1}, \end{aligned}$$

and

$$\begin{aligned} G'(t) &= 2f(t)t^{\alpha-1} - 2f(t)f'(t) \\ &= 2f(t) [t^{\alpha-1} - f'(t)]. \end{aligned}$$

Since  $f'(t) \geq 0$  and  $f(a) = 0$ , thus  $f(t)$  is increasing and  $f(t) \geq 0$ .

When  $0 \leq f'(x) \leq 1$ , we have  $G'(t) \geq 0$ ,  $G(t)$  increases and  $G(t) \geq 0$  because of  $G(a) = 0$ , hence  $F'(t) = G(t)[f(t)]t^{\alpha-1}$ ,  $F'(t)$  is increasing. Since  $F(a) = 0$ , we have  $F(t) > 0$ , and  $F(b) > 0$ . Therefore, the inequality (3.1) holds.  $\square$

**Remark 2.** If we choose  $\alpha = 1$  in (3.1), then we have Proposition 1.1 in [7].

**Theorem 13.** Let  $f(x) \in C_\alpha[0, 1]$  satisfying

$$(3.2) \quad \int_t^1 f^\alpha(x) d_\alpha x \geq \frac{1-t^{2\alpha}}{2\alpha}, \quad \forall t \in [0, 1],$$

then

$$(3.3) \quad \int_0^1 t^\alpha f(t) d_\alpha x \geq \frac{1}{3\alpha}.$$

*Proof.* Let

$$\int_0^1 \left( \int_t^1 f^\alpha(x) d_\alpha x \right) d_\alpha t.$$

By using our assumption we have

$$\begin{aligned} \int_0^1 \left( \int_t^1 f^\alpha(x) d_\alpha x \right) d_\alpha t &\geq \int_0^1 \frac{1-t^{2\alpha}}{2\alpha} d_\alpha t \\ &= \frac{1}{3\alpha^2}. \end{aligned}$$

On the other hand, integrating by parts, we also get

$$\begin{aligned} \int_0^1 \left( \int_t^1 f^\alpha(x) d_\alpha x \right) d_\alpha t &= \frac{t^\alpha}{\alpha} \left( \int_t^1 f^\alpha(x) d_\alpha x \right) \Big|_0^1 + \frac{1}{\alpha} \int_0^1 t^\alpha f^\alpha(t) d_\alpha t \\ &= \frac{1}{\alpha} \int_0^1 t^\alpha f^\alpha(t) d_\alpha t \end{aligned}$$

Thus,

$$\int_0^1 t^\alpha f^\alpha(t) d_\alpha t \geq \frac{1}{3\alpha}.$$

The proof is completed.  $\square$

**Theorem 14.** *If (3.2) holds then we have,*

$$(3.4) \quad \int_0^1 t^{(n+1)\alpha} f^\alpha(t) d_\alpha t \geq \frac{1}{n\alpha + 3\alpha}.$$

*Proof.* Let

$$\int_0^1 t^{n\alpha} \left( \int_t^1 f^\alpha(x) d_\alpha x \right) d_\alpha t.$$

Integrating by parts, we have

$$\begin{aligned} \int_0^1 t^{n\alpha} \left( \int_t^1 f^\alpha(x) d_\alpha x \right) d_\alpha t &= \left. \frac{t^{(n+1)\alpha}}{(n+1)\alpha} \left( \int_t^1 f^\alpha(x) d_\alpha x \right) \right|_0^1 \\ &\quad + \frac{1}{(n+1)\alpha} \int_0^1 t^{(n+1)\alpha} f^\alpha(t) d_\alpha t \\ &= \frac{1}{(n+1)\alpha} \int_0^1 t^{(n+1)\alpha} f^\alpha(t) d_\alpha t \end{aligned}$$

On the other hand, by using our assumption we have

$$\begin{aligned} \int_0^1 t^{n\alpha} \left( \int_t^1 f^\alpha(x) d_\alpha x \right) d_\alpha t &\geq \int_0^1 t^{n\alpha} \left( \frac{1-t^{2\alpha}}{2\alpha} \right) d_\alpha t \\ &= \frac{1}{\alpha(n+1)(n\alpha+3\alpha)} \end{aligned}$$

Thus,

$$\int_0^1 t^{(n+1)\alpha} f^\alpha(t) d_\alpha t \geq \frac{1}{n\alpha + 3\alpha}.$$

The proof is completed.  $\square$

**Remark 3.** *If we choose  $\alpha = 1$  in (3.4), then we have Lemma 1.3 in [6].*

**Theorem 15.** *Let  $f(x) \geq 0$  be a continuous function on  $[0, 1]$  satisfying*

$$\int_t^1 f^m(x) d_\alpha x \geq \int_t^1 x^m d_\alpha x, \quad \forall t \in [0, 1],$$

*then the inequality*

$$(3.5) \quad \int_0^1 x^{n+\alpha-1} f^m(x) d_\alpha x \geq \frac{1}{n+m+2\alpha-1}$$

*holds for every positive real number  $n > 0$ ,  $m > 0$  and  $\alpha > 0$ .*

*Proof.* Integrating by parts, we have

$$\begin{aligned} \int_0^1 t^{n-1} \left( \int_t^1 f^m(x) d_\alpha x \right) d_\alpha t &= \frac{t^{n+\alpha-1}}{n+\alpha-1} \left( \int_t^1 f^m(x) d_\alpha x \right) \Big|_0^1 \\ &\quad + \frac{1}{n+\alpha-1} \int_0^1 t^{n+\alpha-1} f^m(t) d_\alpha t \\ &= \frac{1}{n+\alpha-1} \int_0^1 t^{n+\alpha-1} f^m(t) d_\alpha t \end{aligned}$$

On the other hand, by using our assumption we have

$$\begin{aligned} \int_0^1 t^{n-1} \left( \int_t^1 f^m(x) d_\alpha x \right) d_\alpha t &\geq \int_0^1 t^{n\alpha} \left( \int_t^1 x^m d_\alpha x \right) d_\alpha t \\ &= \frac{1}{(n+\alpha-1)(n+m+2\alpha-1)} \end{aligned}$$

Thus,

$$\int_0^1 t^{n+\alpha-1} f^m(t) d_\alpha t \geq \frac{1}{n+m+2\alpha-1}.$$

The proof is completed.  $\square$

**Remark 4.** If we choose  $\alpha = 1$  in (3.5), then we have Lemma 2.3 in [14].

**Theorem 16.** Let  $f(x) \geq 0$  be a continuous function on  $[0, 1]$  satisfying

$$\int_t^1 f^m(x) d_\alpha x \geq \int_t^1 x^m d_\alpha x, \quad \forall t \in [0, 1],$$

then the inequality

$$(3.6) \quad \int_0^1 f^{n+m+\alpha-1}(x) d_\alpha x \geq \int_0^1 x^{n+\alpha-1} f^m(x) d_\alpha x$$

holds for every positive real number  $n > 0$ ,  $m > 0$  and  $\alpha > 0$ .

*Proof.* By using the Cauchy inequality, we obtain

$$\frac{m}{n+m+\alpha-1} f^{n+m+\alpha-1}(x) + \frac{n+\alpha-1}{n+m+\alpha-1} x^{n+m+\alpha-1} \geq x^{n+\alpha-1} f^m(x).$$

Thus

$$\begin{aligned} & m \int_0^1 f^{n+m+\alpha-1}(x) d_\alpha x + (n+\alpha-1) \int_0^1 x^{n+m+\alpha-1} d_\alpha x \\ & \geq (n+m+\alpha-1) \int_0^1 x^{n+\alpha-1} f^m(x) d_\alpha x. \end{aligned}$$

Moreover, by using Theorem 15, we get

$$\begin{aligned} & (n+m+\alpha-1) \int_0^1 x^{n+\alpha-1} f^m(x) d_\alpha x \\ &= (n+\alpha-1) \int_0^1 x^{n+\alpha-1} f^m(x) d_\alpha x + m \int_0^1 x^{n+\alpha-1} f^m(x) d_\alpha x \\ &\geq \frac{n+\alpha-1}{n+m+2\alpha-1} + m \int_0^1 x^{n+\alpha-1} f^m(x) d_\alpha x \end{aligned}$$

that is

$$m \int_0^1 f^{n+m+\alpha-1}(x) d_\alpha x + \frac{n+\alpha-1}{n+m+2\alpha-1} \geq \frac{n+\alpha-1}{n+m+2\alpha-1} + m \int_0^1 x^{n+\alpha-1} f^m(x) d_\alpha x$$

which completes this proof.  $\square$

**Remark 5.** If we choose  $\alpha = 1$  in (3.6), then we have Theorem 2.1 in [14].

**Theorem 17.** If (3.2) holds then we have

$$(3.7) \quad \int_0^1 f^{(n+1)\alpha}(x) d_\alpha x \geq \int_0^1 x^{n\alpha} f^\alpha(x) d_\alpha x$$

*Proof.* By using the Cauchy inequality, we obtain

$$f^{(n+1)\alpha}(x) + nx^{(n+1)\alpha} \geq (n+1)x^{n\alpha} f^\alpha(x).$$

Thus

$$\int_0^1 f^{(n+1)\alpha}(x) d_\alpha x + n \int_0^1 x^{(n+1)\alpha} d_\alpha x \geq (n+1) \int_0^1 x^{n\alpha} f^\alpha(x) d_\alpha x.$$

Moreover, by using Theorem 14, we get

$$\begin{aligned} (n+1) \int_0^1 x^{n\alpha} f^\alpha(x) d_\alpha x &= n \int_0^1 x^{n\alpha} f^\alpha(x) d_\alpha x + \int_0^1 x^{n\alpha} f^\alpha(x) d_\alpha x \\ &\geq \frac{n}{n\alpha+2\alpha} + \int_0^1 x^{n\alpha} f^\alpha(x) d_\alpha x \end{aligned}$$

that is

$$\int_0^1 f^{(n+1)\alpha}(x) d_\alpha x + \frac{n}{n\alpha+2\alpha} \geq \frac{n}{n\alpha+2\alpha} + \int_0^1 x^{n\alpha} f^\alpha(x) d_\alpha x$$

which completes this proof.  $\square$

**Remark 6.** If we choose  $\alpha = 1$  in (3.7), then we have Theorem 2.1 in [6].

**Conclusion 1.** In the present paper, we establish the generalized Qi-type inequalities involving conformable fractional integrals. Some special cases are also discussed.



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## REFERENCES

- [1] A. Akkurt, M.E. Yildirim and H. Yildirim, On Some Integral Inequalities for Conformable Fractional Integrals, *Asian Journal of Mathematics and Computer Research*, (2016), (In Press).
- [2] A. Akkurt, and H. Yildirim, Genelleştirilmiş Fractional Integral için Feng Qi Tipli İntegral Eşitsizlikleri Üzerine, *IAAOJ, Scientific Science*, 1(2), 22-25, 2013.
- [3] L. Bougoffa, Note on an open problem, *J. Inequal. Pure Appl. Math.*, 8(2), Art. 58, (2007).
- [4] K. Boukernoua, A. Guezane-Lakoud, (2007), On an open question regarding an integral inequality, *J. Inequal. Pure Appl. Math.*, 8(3), Art. 77.
- [5] Z. Dahmani and S. Belarabi, Some inequalities of Qi type using fractional integration, *I.J.N.S* (2011).
- [6] Q.A. Ngô, D.D. Thang, T.T. Dat, and D.A. Tuan, Notes on an integral inequality, *J. Inequal. Pure Appl. Math.*, 7(4), Art. 120 (2006). [ONLINE: <http://jipam.vu.edu.au/article.php?sid=737>].
- [7] F. Qi, Several integral inequalities, *J. Inequal. Pure and Appl. Math.*, 1(2), Art. 19 (2000). [ONLINE: <http://jipam.vu.edu.au/article.php?sid=113>].
- [8] M.E. Yildirim, A. Akkurt and H. Yildirim, Generalized Qi's Integral Inequality, *Cumhuriyet Science Journal*, vol: 37 (1) pp: 12, (2016), DOI:10.17776/csj.10671
- [9] T. Abdeljawad, On conformable fractional calculus, *Journal of Computational and Applied Mathematics* 279 (2015) 57–66.
- [10] R. Khalil, M. Al horani, A. Yousef, M. Sababheh, A new definition of fractional derivative, *Journal of Computational Applied Mathematics*, 264 (2014), 65-70.
- [11] O.S. Iyiola and E.R. Nwaeze, Some new results on the new conformable fractional calculus with application using D'Alambert approach, *Progr. Fract. Differ. Appl.*, 2(2), 115-122, 2016.
- [12] M. Abu Hammad, R. Khalil, Conformable fractional heat differential equations, *International Journal of Differential Equations and Applications* 13(3), 2014, 177-183.
- [13] M. Abu Hammad, R. Khalil, Abel's formula and wronskian for conformable fractional differential equations, *International Journal of Differential Equations and Applications* 13(3), 2014, 177-183.
- [14] W-J. Liu, C-C. Li, J-W. Dong, On an Open Problem Concerning an Integral Inequality, *J. Inequal. Pure and Appl. Math.*, 8(3), Art. 73.
- [15] G. W. Leibniz, "Letter from Hanover, Germany to G.F.A. L'Hospital, September 30, 1695", *Leibniz Mathematische Schriften*, Olms-Verlag, Hildesheim, Germany, 1962, p.301-302, First published in 1849.
- [16] G. W. Leibniz, "Letter from Hanover, Germany to Johann Bernoulli, December 28, 1695", *Leibniz Mathematische Schriften*, Olms-Verlag, Hildesheim, Germany, 1962, p.226, First published in 1849.
- [17] G. W. Leibniz, "Letter from Hanover, Germany to John Wallis, May 28, 1697", *Leibniz Mathematische Schriften*, Olms-Verlag, Hildesheim, Germany, 1962, p.25, First published in 1849.



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