Hysteresis Characteristic in Hump Region of a Pump-Turbine Model

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Abstract: The hump characteristic is one of the major instabilities in pump-turbines. When pump-turbines operate in the hump region, strong noise and serious fluctuations could be observed, which are harmful to the safe and stable operations and even destroy the whole unit as well as water conveyance system. In this paper, a low specific speed ($n_q = 36.1$ min$^{-1}$) pump-turbine model was experimentally investigated. Firstly, the hump characteristic was obtained under 19 mm guide vane opening. More interestingly, when the hump characteristic was measured in two directions (increasing and decreasing the discharge), the hysteresis characteristic was found during the hump region. The analysis of performance characteristics reveals that the hump instability is resultant of Euler momentum and hydraulic losses, and different Euler momentum and hydraulic losses in the two development processes lead to hysteresis phenomenon. Then, 12 pressure sensors were mounted in the different parts of the pump-turbine model to obtain the time and frequency characteristics. The analysis of fast Fourier transform confirms that the hump characteristic is related to the low-frequency (0.04–0.15 times rotational frequency) vortices. The occurrence and cease of vortices depend on the operating condition and measurement direction, which contribute to the hysteresis characteristic. Finally, the type of the low-frequency vortices was analyzed through the cross power spectrum.

Keywords: pump turbine; hump characteristic; hysteresis characteristic; experimental investigation

PACS: J0101

1. Introduction

Due to dramatic increase of renewable energy sources, pumped storage power plants obtain rapid development to balance the fluctuations of the electric grid. As a consequence, pump-turbines need rapid switch between the pump mode and turbine mode, which leads to frequent operation at off-design conditions.

Pump-turbines are usually designed prior to pump mode in consideration with turbine mode, due to which the flow in pump mode is much more sensitive to flow separation and cavitation. Practical results prove this method could make most of the operating conditions in the high efficiency region. However, two unstable features could be found at off-design operating conditions in pump-turbines. One occurs in the head-discharge curve in the high head and low discharge operating conditions in pump mode, which is well known as hump characteristic. Another appears in the turbine brake operating conditions called S-shaped characteristic.

With respect to the hump instability characteristic, although the engineers choose the normal operating range outside of the hump region, it is still not avoided when pump-turbines operate at the states of high level water of the reservoir or low frequency of the grid. Figure 1 presents a pump-
turbine undergoes the hump region during the start-up in pump mode. Pump-turbine starts from the 0 mm guide vane opening (GVO), and reaches point A at 11 mm GVO, and then point B at 13 mm GVO. When the GVO opens to 15 mm, operating point should reach at point C. However, due to the vibration of the water pressure caused by the hump instability characteristic, the operating point jumps to the point D. This process will delay the start-up of the pump-turbine in pump mode. Meanwhile, strong noise and serious fluctuations could be observed, which are significantly harmful to the stable and safe operations of pump-turbines.

In recent years, pump-turbines trend to higher head, higher specific speed and larger capacity. As a consequence, more and more researchers focus on the studies of the hump instability characteristic through the experimental and numerical methods. The hump characteristic is mainly caused by the prerotation, recirculation [1], and suction surface cavitation [2] at the runner inlet, backflow and vortex motion [3], rotating stall [4,5], and secondary flow pattern [6] in the diffuser channels. Lots of achievements have been obtained, which could give a deep understanding of the hump characteristic.

Interestingly, several researchers found the hysteresis phenomenon during the hump region. When the hump characteristic was measured in the decreasing and increasing discharge directions, a hysteresis loop will be formed. Hysteresis is the time-based dependence of a system’s output on present and past inputs, which is well known in physics.

However, it is merely presented in hydraulic machinery. It is firstly published in a high specific speed ($\omega_s = 1.7 \text{ s}^{-1}$) radial pump in 1997 by Kaupert and Staubli [7]. When measured in two directions (decreasing or increasing discharge), performance characteristic ($H-Q$) shows different during the hump region. Then, they carried out the simulation based on experimental validation and concluded that the hysteresis loop is related to the recirculation at the runner inlet. However, it is not necessary condition to form the hysteresis loop. Based on the above research, they [8,9] continued to perform the experiments of unsteady pressure fluctuations, and proposed that the hysteresis characteristic which pressure difference of the pump and impeller experiences is related to the occurrence and cease of the backflow at the impeller inlet and outlet.

In 2003, Iino et al. [10] presented experimental and numerical studies of a pump with/without fins in upstream side of the impeller. They found the pump with fins has a large hysteresis loop, while this phenomenon was not found in the pump without fins. Through the numerical analysis, they concluded that the difference in the hysteresis loop is caused by the existence of the fins, which suppress or promote the backflow at the impeller in the different directions.

In 2012, Ran et al. [11] confirmed this phenomenon also exists in a low specific speed pump-turbine. In the same year, they [12] conducted the simulations to predict this phenomenon. Numerical analysis shows this might come from the rotating stall in the vaneless space.
In 2015, during the Ješev’s [13] doctoral thesis, he also talked about the hysteresis phenomenon existing in pump-turbines. However, he did not carry out related studies on formation mechanism of the hysteresis characteristic.

To the author’s knowledge, there are no other papers or reports to present this interesting phenomenon in hydraulic machinery. Hence, the authors carried out related experiments to investigate why this phenomenon occurs. Usually, the most obvious hump characteristic happens in the relatively small GVO. For the pump-turbine investigated in this research, the hump region under 19 mm GVO is much more obvious, and the hump margin in real operations was chosen according to this GVO. Hence, 19 mm GVO was chosen to carry out a further investigation. Firstly, performance characteristics were obtained under 19 mm GVO in a low specific speed ($n_q = 36.1 \text{ min}^{-1}$) pump-turbine to validate the existing of the hysteresis phenomenon and find the causes of the hump characteristic. Then, the measurements of pressure fluctuations were carried out to find the reasons of the formation of the hysteresis loop. Finally, detailed analysis of time and frequency characteristics was performed to analyze the type of the unsteady sources.

2. Experimental Setup

2.1. Pump-Turbine Specification

The investigated pump-turbine model is scaled down (1:9.24) from a prototype in a real pumped storage power plant in China. It is a single stage, vertical shaft, low specific speed ($n_q = 36.1 \text{ min}^{-1}$), Francis pump-turbine. As shown in Figure 2, the pump-turbine consists of a runner with 9 blades, 20 guide vanes, 20 stay vanes including a special one, a draft tube and a spiral casing. The detailed information of main parameters is listed in Table 1.

![Figure 2. The pump-turbine model.](image)

Table 1. Main parameters of the pump-turbine model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
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<tbody>
<tr>
<td>Specific speed</td>
<td>$n_q$</td>
<td>36.1 min$^{-1}$</td>
<td>pump mode</td>
</tr>
<tr>
<td>Runner inlet diameter</td>
<td>$D_1$</td>
<td>0.250 m</td>
<td>pump mode</td>
</tr>
<tr>
<td>Runner outlet diameter</td>
<td>$D_2$</td>
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<td>pump mode</td>
</tr>
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<td>Guide vane height</td>
<td>$B_0$</td>
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<td></td>
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<tr>
<td>Guide vane distribution diameter</td>
<td>$D_0$</td>
<td>0.54117 m</td>
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<tr>
<td>Optimum GVO</td>
<td>$GVO_{Opt}$</td>
<td>25 mm</td>
<td>Pump mode</td>
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<tr>
<td>Investigated GVO</td>
<td>$GVO_{Invest}$</td>
<td>19 mm</td>
<td>Pump mode</td>
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2.2. Test Rig

The measurements in this research were carried out in Harbin Institute of Large Electrical Machinery (HILEM), in which six test rigs are available for turbine, pump-turbine and pump performance assessment in a closed loop (maximum head = 80 m, maximum discharge = 0.8 m$^3$s) within an accuracy of 0.2%. The sketch of the test rig is shown in Figure 3. In pump mode, the water from the basement was pumped using an addition water supply pump to inlet pipe connected with draft tube, and then passed through the runner, guide vanes, stay vanes, and the spiral casing, finally through the downstream tank back to the basement. Measurement system was programed by LabVIEW 2014, and NI PXI platform with a PXI-8336 controller and a PXI-1042Q chassis was chosen. Digital, current and pressure signals could be measured with a high sampling rate. Detailed information for the measurement system was described in the reference [14].

![Figure 3. Schematic diagram of the test rig.](image)

3. Performance Measurements and Analysis

3.1. Performance Measurements

Pump characteristics of the pump-turbine were carried out under 19 mm GVO without consideration cavitation, namely in the measurements, the cavitation coefficient was set higher than experimental facility coefficient to avoid the occurrence of the cavitation. The measurements firstly were started from the large discharge operating condition, and decreased the discharge step by step. Then, the measurements were repeated starting from the low discharge operating condition, and increased step by step until the initial state of the first process. All the measurements were processed strictly following the standard of International Electrotechnical Commission (IEC) [15], and all the uncertainties are below 0.2%. Performance parameters are normalized according to the following equations [15]. Figure 4 depicts the performance characteristics (energy, torque and efficiency).

$$E_{nD} = \frac{gH}{n_y^2 D_1^2}$$  \hspace{1cm} (1)

$$Q_{nD} = \frac{Q}{n_y D_1^2}$$  \hspace{1cm} (2)

$$T_{nD} = \frac{T}{\rho n_y^2 D_1^3}$$  \hspace{1cm} (3)
From the Figure 4, an obvious hump region is observed in the $E_{enD}^{-}\text{Q}_{enD}$ curve in the high energy (head) and low discharge region ($0.63\text{Q}_{\text{BEP}}-0.78\text{Q}_{\text{BEP}}$). More interestingly, obvious hysteresis phenomenon is found during the hump region. Meanwhile, another small hump region (the energy coefficient shows a downtrend) is found from $0.78\text{Q}_{\text{BEP}}$ to $0.94\text{Q}_{\text{BEP}}$, and a hysteresis loop could also be observed.

In the first hump region, when measured in the decreasing direction, the net energy coefficient begins to drop at point $0.69\text{Q}_{\text{BEP}}$, and the wave valley occurs at point $0.67\text{Q}_{\text{BEP}}$. The drop of the net energy coefficient in the decreasing direction is 0.281. When measured in the increasing direction, the net energy coefficient begins to drop at point $0.78\text{Q}_{\text{BEP}}$, and the wave valley happens at point $0.72\text{Q}_{\text{BEP}}$. The drop in the increasing direction is 0.187. Furthermore, the drop of net energy coefficient between the wave peak point and the wave valley point in the decreasing direction is much higher than that in the increasing direction. As for the torque-discharge curve, only small difference could be found in the obvious hump region, and no obvious hysteresis phenomenon could be observed. Hence, the influence of the torque is relatively little. More detailed analysis will be performed in the following section. The efficiency is the resultant of the torque and head, which also shows obvious hysteresis phenomenon. Moreover, the efficiency has a sharp drop from the wave peak point to the wave valley point, which is respectively 3.54% and 4.67% in the decreasing and increasing directions.

In the second hump region, the net energy coefficient just shows a decreasing tendency. In the torque-discharge and efficiency-discharge curves, just a small hysteresis loop could be observed. The cause of the hysteresis loop seems not the same with the first one. This will be discussed in the next section.

### 3.2. Analysis of Energy Drop

From the technical definition, there are two reasons for the energy drop. One is Euler momentum (the change of velocity momentum) of the fluid ($\Delta c_{u}\cdot u$), which is input for the hydraulic system and associated with the shape of the blades (inlet and outlet) giving rise to incidence and deviation effects. Another is hydraulic losses including frictional losses, collision losses and losses induced by flow separation and unstable vortex. The drop phenomenon is the resultant of Euler momentum and hydraulic losses.

Euler momentum ($\Delta c_{u}\cdot u$) is the function of incidence angle ($c_{u1}$) at inlet and deviation angle at the exit ($c_{u2}$), ($\Delta c_{u}\cdot u$) can be determined by Equations (4) and (5). For experimental method, Equation (4) gives an approximate value of Euler torque [16]. Energy coefficients for hydraulic losses and gross energy obtained from the rotating runner could be calculated through the Equations (6) and (7).

$$\Delta c_{u}\cdot u = c_{u2}\cdot u - c_{u1}\cdot u = T\cdot \omega / (\rho Q)$$  \hspace{1cm} (4)

$$\Delta c_{u}\cdot u / g = H_{\text{net}} + H_{\text{loss}}$$  \hspace{1cm} (5)
\[ E_{\text{D-gross}} = \Delta c_u \cdot u/(h^2D^2) = T\omega/(Qn^2D^2) \]  
\[ E_{\text{D-loss}} = E_{\text{D-gross}} - E_{\text{D-net}} \]

The curves for different energy coefficients are shown in Figure 5. The variation of gross energy coefficients obtained from the rotating runner is almost the same in the two directions. However, some small differences could still be observed in the hysteresis loops.

In the first hysteresis loop, gross energy coefficient in the increasing direction is a bit larger than the one in the decreasing direction. The largest drop is the energy coefficient of hydraulic losses, and clear hysteresis phenomenon could be found. In the two directions, the hydraulic losses increase sharply from the wave peak point to the wave valley point. It is found that the high hydraulic losses last to the point 0.72\(Q_{BEP}\) in the increasing direction, while it occurs from the point 0.67\(Q_{BEP}\) in the decreasing direction. During the first hump region, inlet recirculation might exist in the both directions due to operating conditions far away with BEP [17], so the Euler momentum in the two direction are almost the same due to the influence of the inlet recirculation. Furthermore, the efficiency show a sharp decrease. Hence, the first hysteresis loop only results from the sharp increase of the hydraulic losses in the increasing direction.

In the second hysteresis loop, gross energy coefficient in the increasing direction is a bit smaller than the one in the decreasing direction. In addition, the energy coefficient of hydraulic losses in the decreasing direction is just a little larger than the one in the increasing direction, unlike the variation in the obvious hump region. Hence, the second hysteresis loop might result from the decrease of the Euler momentum and the increase of the hydraulic losses. Namely, the incidence angle, deviation angle or both are different in the two directions. This might exist recirculation during this region in the increasing direction due to the initial unstable patterns, while not in the decreasing direction. Inlet recirculation will lower the gross energy coefficient because it changes the incidence angle.

Energy coefficients analysis reveals the hump characteristic and associated hysteresis phenomenon is result of Euler momentum (input parameter) and hydraulic losses (dissipated parameter). In the first hump region far away from BEP the hysteresis characteristic mainly results from the difference of the hydraulic losses, while the second one might come from the combined effect of Euler momentum and hydraulic losses. This needs a further validation through the simulation.

Figure 5. Energy coefficients for gross energy, net energy and hydraulic losses.

4. Experiments and Analysis of Pressure Fluctuations

Hydraulic losses mainly come from the unstable vortex, flow separation and collision losses, which usually couple with pressure fluctuations. In order to find if there are some relations between high hydraulic losses and pressure fluctuations, measurements of pressure fluctuations were carried out in this research.
During the measurements, 12 high resolution pressure sensors (PCB-112A22) were mounted in the whole passage. As shown in Figure 6, 1 is in the spiral casing outlet (SC1), 1 in the stay vanes (SV1), 2 in the guide vanes (GV1 and GV2), 2 in the vaneless space between the runner outlet and guide vane inlet (RG1 and RG2), 2 in the cone of the draft tube (CT1 and CT2), 2 in the elbow of the draft tube (ET1 and ET2), 1 in the top cover (TC1) and 1 in the bottom shroud (BS1). All the pressure sensors were well calibrated before the measurements, and the errors of the fitting are below 0.4%. All the measurements were performed at constant rotational speed (1000 rpm). Sampling rate was set 4000, and every operating point was measured for 10 s (40,000 sample points). In such case, rotational frequency is 16.67 Hz and the frequency resolution is 0.1 Hz.

![Diagram](a)

**Figure 6.** Monitoring points of pressure fluctuations in the measurements. (a) Monitoring points in front view; (b) Monitoring points in top view.

4.1. Analysis of Pressure Fluctuations

All the signals are unsteady and normalized according to Equation (8) as unsteady pressure coefficients. The intensity of pressure fluctuations is expressed as the standard deviation according to Equation (9). Figure 7 gives the standard deviation of pressure fluctuations for every monitoring point varying with the operating condition in the two directions under 19 mm GVO. In the Figure 7, the size of the circle indicates the magnitude of the standard deviation. Due to which the values in the vaneless space, guide vanes and stay vanes are relatively high compared with other positions. The legend of the circle was amplified for 4 times. It is found that the highest-level pressure fluctuations occur in the vaneless space, then the second is in the guide vanes. Along the flow direction from the draft tube inlet to the spiral casing outlet, the pressure fluctuation intensity increases slowly from the elbow to the cone of the draft tube, and jumps sharply to the vaneless space, and then decreases slowly to stay vanes, finally shows a rapid decline at the spiral casing outlet.
\[ C_p = \frac{p - \overline{p}}{\frac{1}{2} \rho U_i^2} \times 100\% \]  
\[ \overline{C}_r = \frac{1}{2} \rho U_i^2 \sqrt{\frac{1}{N} \sum_{i=1}^{N} (p_i - \overline{p})^2} \]
For every monitoring point, when the discharge is increased from the BEP, the pressure fluctuation intensity has a small increase, while it increases sharply when the discharge is decreased from the BEP. In the first hump region, from the wave peak point to the wave valley point in the two directions, the intensity has a different level increase, whereas the highest-level increase occurs in the vaneless space. Additionally, the level of the intensity in the two directions shows obviously different, and obvious hysteresis phenomenon could be found. Hence, the first hysteresis loop might be related to the pressure fluctuations in the vaneless space and guide vanes, which lead to high hydraulic losses.

In the second hump region, the pressure fluctuations in two directions also show obviously different, especially in the guide vanes (GV2). Hence, the increase of hydraulic losses in the increasing direction in the second hysteresis loop might come from high amplitude pressure fluctuations in the guide vanes.

To obtain the detailed frequencies, FFT analysis was performed in the decreasing direction. Figure 8 lists the frequencies varying with operating conditions for every monitoring point. In the draft tube, the amplitudes are relatively low for all the operating points, and some low frequency components (less than $3f_n$, BPF ($9f_n$) and its harmonic frequencies ($18f_n$, $27f_n$, ...)) could be observed. Only during the low load operating conditions, the amplitudes of the low frequencies increase sharply. In the vaneless space, guide vanes and stay vanes, the amplitudes of low frequency components (less than $0.8f_n$) show an amazing jump (maximum more than 20 times) around the hump region. Compared with low frequency components, BPF and its harmonic frequencies show not so important. In addition, for all the positions, a frequency $3.678f_n$ could be observed, whose amplitude gradually decreases from downstream spiral casing outlet to upstream draft tube inlet within range 0.4 and 0.1. The frequency keeps no change with the variation of the operating condition. This might come from an elastic fluctuation of the hydraulic system, or a pressure fluctuation generated by water supply pump, which is not dependent on the operating condition. Hence, this frequency has no direct relations with the hysteresis characteristic in this research.
Figure 9 depicts the detailed frequency spectrum at point 0.67$Q_{BEP}$. At the position of the top cover (TC1), two low frequency components (0.114$f_n$ and 1.002$f_n$) are found. The first one (0.114$f_n$) might come from the vaneless space. The second one might come from the runner, which may be rotating stall in the runner or rotational frequency. Due to the lack of pressure sensors in the runner, this cannot be analyzed in the measurements. This needs further studies through the simulation. Nevertheless, the amplitudes of the two frequency components show that pressure fluctuations in the vaneless space much larger than ones in the runner. In the draft tube, these two frequency components are also observed. The first one might be related to the frequency in the vaneless space, and the second one might come from the runner.

Based on the variation of the frequencies, the hump characteristic might be related with low frequency components in the vaneless space, guide vanes and stay vanes. Hence, the detailed low frequency components in the vaneless space, guide vanes and stay vanes from 0.03–0.8$f_n$ in the two directions are shown in Figure 10.

In the decreasing direction, from the 0.91$Q_{BEP}$ to 1.25$Q_{BEP}$, no high amplitude low frequency components are found. From 0.78$Q_{BEP}$ to 0.82$Q_{BEP}$, the highest amplitude low frequency components occur in the guide vanes, and the frequency increases with the decreasing of the discharge. As the discharge is reduced from 0.69$Q_{BEP}$, the highest amplitude low frequency components appear in the vaneless space. At the point 0.69$Q_{BEP}$ (wave peak), low frequency components 0.042$f_n$ and 0.09$f_n$ appear, but the amplitudes are relatively low. When reduced to the point 0.63$Q_{BEP}$ (wave valley), a higher amplitude low frequency component (0.114$f_n$) occurs. As the discharge continues to decrease, the amplitudes of the low frequency components increase and the frequency decreases. From point 0.60$Q_{BEP}$ to 0.49$Q_{BEP}$, the frequency and amplitudes of these low frequency components change with operating conditions.

In the increasing direction, the low frequency components from 1.00$Q_{BEP}$ to 1.25$Q_{BEP}$, and from 0.49$Q_{BEP}$ to 0.59$Q_{BEP}$ are totally the same with the ones in the decreasing direction. However, from 0.65$Q_{BEP}$ to 0.91$Q_{BEP}$, the frequency components are extremely different. The highest low frequency components in the guide vanes occur from the point 0.79$Q_{BEP}$ and last to the point 0.94$Q_{BEP}$. The amplitudes decrease slowly and the frequency increases. From the wave peak to the wave valley in this direction, the position of the highest amplitude low frequency components shifts from the guide
vanes to the vaneless space. Furthermore, from the 0.70\(Q_{\text{BEP}}\) to 0.94\(Q_{\text{BEP}}\), the amplitudes are obviously higher than the ones in the decreasing direction. This tendency is the same with the variation of the hydraulic losses, namely, the higher amplitudes of low frequency components are related with the higher hydraulic losses.

In this process, some other linear frequencies (2\(f_1\), 3\(f_1\)) and nonlinear frequencies (2\(f_1+ f_0\), 2\(f_1- f_0\), 3\(f_1+ f_0\) and 3\(f_1- f_0\)) could also be found, which are marked in the Figure 10. The first ones are harmonic frequencies from the highest frequency, while the second ones come from the interaction between the highest frequency and the basic frequency (minimum resolution frequency \(f_0 = 0.006f_n\)). Compared with the first dominant frequency (\(f_1\)), the amplitudes of the other frequencies are relatively low.

It can be concluded that the hysteresis characteristic is related to the hydraulic losses, which might be caused by low frequency pressure fluctuations. Moreover, the position and frequency of these low frequency components vary with the operating conditions.
4.2. Analysis of Low-Frequency Vortices

Low frequency components in the pressure fluctuations in pump-turbines usually come from stationary vortex, unsteady vortex and rotating stall [18]. To analyze the type of these low frequency components, low pass filter was used to obtain the time domain signals. In addition, cross power spectrum between the different signals was performed in order to obtain the correlation and phase difference of different frequency components. As shown in Figure 6, points RG1 and RG2 are mounted at the same circumference with 90° difference, and also the same for points GV1 and GV2.

Figure 11 gives low pass filter signals and corresponding cross power spectrum of several operating points in the two directions.
In the decreasing direction, for point 0.49$Q_{\text{BEP}}$, averaged phase difference between RG1 and RG2 ($\Delta \theta_1$) is $-98.8^\circ$ and averaged phase difference between GV1 and GV2 ($\Delta \theta_2$) is $-100.1^\circ$. Both are around 90° and cross power spectrum shows extremely high correlation for these four signals. Hence, this low frequency component was considered as coming from the rotating stall. Due to which the runner spins in the anticlockwise direction, and points RG1 and GV1 are at the upstream of the points RG2 and GV2, negative sign indicates that the vortices rotate in the opposite direction with the runner.

Using the same method to analyze the rest operating points, it is found that low frequency component for point 0.67$Q_{\text{BEP}}$ is just originated from unsteady vortex, due to the low correlation and no consistent phase difference.

In a sum, from the Figure 11, in the decreasing direction, above the point 0.91$Q_{\text{BEP}}$, there is no low frequency component (no vortices); from the point 0.63$Q_{\text{BEP}}$ to 0.91$Q_{\text{BEP}}$, low frequency component comes from unsteady vortex, below the point 0.63$Q_{\text{BEP}}$, unsteady vortices develop and become rotating stall.

In the increasing direction, below the point 0.82$Q_{\text{BEP}}$, all the low frequency components originate from the rotating stall. Then at the point 0.91$Q_{\text{BEP}}$, the rotating stall suddenly disappears, and only very low unsteady vortices in the guide vanes could be found.

In the decreasing direction, the flow patterns develop from initial good operating conditions as the discharge is reduced. Hence, the vortices develop from stationary vortex, and then unsteady vortex and finally become rotating stall. While in the increasing direction, the flow patterns begin from the bad operating condition (rotating stall). So the rotating stall slowly disappears and lasts to the point 0.91$Q_{\text{BEP}}$ as the discharge is increased. When the operating condition is not suitable, the rotating stall suddenly breaks down. As a consequence, the processes in the two directions show different, which leads to the different level hydraulic losses, and contributes to the hysteresis phenomenon in the first hump region.

5. Conclusions

Pump characteristics of a low specific speed ($n_q = 36.1 \text{ min}^{-1}$) pump-turbine were experimentally studied in this paper. The hump characteristic under 19 mm GVO was specially emphasized. When it was measured in the increasing and decreasing discharge directions, an interesting phenomenon (hysteresis characteristic) was found.

Firstly, the analysis of the performance characteristics reveals that the hump characteristic is resultant of Euler momentum and hydraulic losses. Moreover, the first hysteresis loop far away from BEP mainly originates from hydraulic losses, while the second hysteresis loop results from combined effect of Euler momentum (recirculation inlet) and hydraulic losses. Due to the different development
processes in the two directions, Euler momentum and hydraulic losses show different, which contribute to the hysteresis phenomenon.

Secondly, measurements of pressure fluctuations in the whole passage reveal the high amplitude low frequency components exist in the vaneless space or guide vanes. Moreover, the frequency and magnitudes of the low frequency components also show hysteresis characteristics, which indicate the high hydraulic losses might be related to the low frequency components.

Finally, the analysis of phase and cross power spectrum of low frequency signals obtains the development of the low frequency components. These low frequency components might come from the stationary vortices, or unsteady vortices, or rotating stall. Vortices will develop and become the rotating stall when the operating conditions are suitable.

Due to the existing of the hysteresis characteristic, the hump instability region is increased. Hence, in the hydraulic model acceptance test, the hump characteristic should be measured in two directions, and the selection of the hump margin should be considered the hysteresis characteristic. In this research, the hump characteristic and associated hysteresis phenomenon were investigated only through the experiments. However, the flow mechanism is still not clear. It should perform further studies through numerical method to obtain the detailed formation mechanism. To avoid the hump characteristic and associated hysteresis phenomenon, the root method is to optimize the design of pump-turbine.

Acknowledgments: This work was supported by Foundation for Innovative Research Groups of the National Natural Science Foundation of China (Grant No. 51121004) and China Scholarship Council (CSC) (Grant No. 201506120168).

Author Contributions: This paper is part work of Ph.D. work of Deyou Li. Hongjie Wang is supervisor of the Ph.D.-candidate. Torbjørn K. Nielsen and Daqing Qin are co-supervisors. Deyou Li programed the measurement system, carried out measurements, performed the analysis and prepared the manuscript. Jinxia Chen is an engineer in Harbin Institute of Large Electrical Machinery. She carried out the measurements together with Deyou Li. Hongjie Wang discussed with Deyou Li and proposed the structure of the manuscript and the method of the analysis. Torbjørn K. Nielsen, Daqing Qin and Xianzhu Wei gave a lot of suggestions and revised the manuscript.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

<table>
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<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tr>
<td>GVO</td>
<td>Guide vane opening</td>
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<tr>
<td>BEP</td>
<td>Best efficiency point</td>
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<td>BPF</td>
<td>Blade passing frequency</td>
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<td>FFT</td>
<td>Fast Fourier transform</td>
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<td>HILMEM</td>
<td>Harbin Institute of Large Electrical Machinery</td>
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<td>IEC</td>
<td>International Electrotechnical Commission</td>
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\[ Q_{\text{BEP}} = \text{Discharge at the best efficiency point under the } 19 \text{ mm GVO} \]

\[ n_q = \frac{n Q}{H^{3/4}}, \text{ min}^{-1} \]

\[ \omega_s = \frac{\omega Q}{(gH)^{3/4}}, \text{ s}^{-1} \]

<table>
<thead>
<tr>
<th>Symbol</th>
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<td>H</td>
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</tr>
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<td>Q</td>
<td>Pump volume discharge at the best efficiency point, m³·s⁻¹</td>
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<tr>
<td>T</td>
<td>Hydraulic torque, N·m</td>
</tr>
<tr>
<td>g</td>
<td>Gravitational acceleration, m·s⁻²</td>
</tr>
<tr>
<td>ρ</td>
<td>Density of the water, kg·m⁻³</td>
</tr>
<tr>
<td>D₁</td>
<td>Runner inlet diameter in pump mode, m</td>
</tr>
<tr>
<td>D₂</td>
<td>Runner outlet diameter in pump mode, m</td>
</tr>
<tr>
<td>B₀</td>
<td>Guide vane height, m</td>
</tr>
<tr>
<td>D₀</td>
<td>Guide vane distribution diameter, m</td>
</tr>
</tbody>
</table>
\[ n \] Rotational speed of the runner, rpm

\[ n_0 \] Rotational speed of the runner, \( \text{s}^{-1} \)

\[ \bar{p} \] Time averaged pressure of 10s for every position, pa

\[ U_2 \] Circumferential velocity at the tip of the blades at the runner outlet, \( \text{m} \cdot \text{s}^{-1} \)

\[ f_r \] Rotational frequency, Hz

\[ f_0 \] Minimum frequency resolution, 0.0006\( f_n \), Hz

\[ \Delta\theta_1 \] Phase difference between RG1 and RG2, \(^\circ\)

\[ \Delta\theta_2 \] Phase difference between GV1 and GV2, \(^\circ\)

\[ E_{\text{ed}} \] Energy coefficient, \( E_{\text{ed}} = \frac{gH}{n_0^2 D_1^2} \)

\[ Q_{\text{ed}} \] Discharge coefficient, \( Q_{\text{ed}} = \frac{Q}{n_0^2 D_1^3} \)

\[ T_{\text{ed}} \] Torque coefficient, \( T_{\text{ed}} = \frac{T}{\rho n_0^2 D_1^5} \)

\[ u \] Circumferential velocity, \( \text{m} \cdot \text{s}^{-1} \)

\[ C_u \] Component of the absolute velocity in the circumferential direction, \( \text{m} \cdot \text{s}^{-1} \)

\[ \Delta C_u \cdot u \] Euler momentum (the change of velocity momentum), \( \text{m}^2 \cdot \text{s}^{-2} \)

**Subscript**

1. Runner inlet
2. Runner outlet
3. Net
4. Gross
5. Loss

**References**


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