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*Article*

# The thermodynamics of collapsars

**Trevor W. Marshall**

Buckingham Centre for Astrobiology, The University of Buckingham, Buckingham MK18 1EG, UK; trevnat@talktalk.net; Tel.: +44-061-445-4872

**Abstract:** This article argues that there is a consistent description of gravitationally collapsed bodies, including neutron stars above the Tolman-Oppenheimer-Volkoff mass and also supermassive galactic centres, according to which collapse stops before the object reaches its gravitational radius, the density reaching a maximum close to the surface and then decreasing towards the centre. Models for such shell-like objects have been constructed using classic formulations found in the 1939 articles of Oppenheimer-Volkoff and Oppenheimer-Snyder. It was possible to modify the conclusions of the first article by changing the authors' boundary conditions at $r = 0$. In the second case we find that the authors' solution of the field equations needs no changes, but that the choice of their article's title led many of their successors to believe that it supports the black-hole hypothesis. However, it is easily demonstrated that their final density distribution accords with the shell models found in our articles. Because black holes, according to many formulations, "have no hair", their thermodynamics is rather simple. The kind of collapsar which our models describe are more like main-sequence stars; they have spatiotemporal distributions of pressure, density and temperature, that is they have hair. In this article we shall concentrate on the dynamics of the Oppenheimer-Snyder collapsar; both pressure and temperature are everywhere zero, so there can be no thermodynamics. Only in the time independent case of Oppenheimer-Volkoff type models is it currently feasible to consider some thermodynamic implications; here some valuable new insights are obtained through the incorporation of the Oppenheimer-Snyder dynamics.

**Keywords:** Gravitational collapse; Oppenheimer-Snyder; horizonless solutions; neutron stars.

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## 1. Introduction

This article extends the discussion I undertook in my article "Gravitational collapse without black holes"[1]. The latter showed that there is an approach to the field equations of General Relativity (GR) which excludes solutions with density singularities (black holes) and their paradoxical consequences[2]. I am therefore proposing that the theme of *Black Hole Thermodynamics* (BHT) should be placed within the wider one denoted by my article's title.

What my description of a collapsar has in common with BHT is that it is made of cooled down stellar material with a rather small luminosity of its own, but with a hotter accretion region, usually in the form of a disc, the latter being concentrated close to its gravitational radius. Where it differs from the BHT description is that it does not allow of the entire stellar mass collapsing inside the gravitational radius; on the contrary, as its radius approaches that limit, the stellar material is increasingly concentrated at the surface. Thus the accretion region is quite close to the surface, and observations of the light from the collapsar should be designed[3] to distinguish between it and the light from the hotter but less dense accretion region.

My colleagues' and my own studies of what BHT would describe as incomplete collapse, but which we consider to be its final phase, fall into the categories of either the time-independent case, for which the pioneering article is that of Oppenheimer and Volkoff (OV)[4], or its partner, the time-dependent case of Oppenheimer and Snyder (OS)[5]. Our modification[6] to the conventional understanding of OV has its origin in footnote 10 of the OV article, where the authors, having acknowledged that there exists a family of solutions of the Hilbert-Einstein equations for which the pressure is zero at $r = 0$, failed to follow up that possibility. We discovered that such solutions have a shell-like density profile, with most of the material density concentrated just inside the surface;

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inside the shell there is a region consisting almost exclusively of negative gravitational energy. This description has some features in common with that of *gravastars*[7][8][9].

There is, as yet, no calculation of the time evolution of a collapsar with a realistic equation of state like that of OV, so we have no rigorous means of deciding whether the shell model or the BHT model is correct for the final state. My study of the OS article[1][3][10], however, gives support to the shell model, because it shows that, as the radius of a collapsar approaches its asymptotic value, the density at the surface becomes infinite. In the present article I investigate the trajectory of a freefalling test particle as it travels from a surface point to the centre and then on to an antipodal point of the surface, thereby giving further illustration of the shell structure. However, the material of the OS collapsar, commonly referred to as "dust", is devoid of any self interaction; effectively it has dynamics but no thermodynamics. So, having used the OS collapsar to convince oneself that the shell version of OV is a plausible alternative to BHT, the programme one then follows can only be that of combining the normal thermodynamic variables of pressure, density and temperature with the OS gravitational field. Such a description will have much in common with that of normal stars; in particular there will be a smoothing of the infinite surface density of the OS collapsar.

## 2. The dynamics of Oppenheimer-Snyder

The OS collapsar[5] with gravitational mass $m$ has the vacuum Schwarzschild metric

$$ds^2 = \frac{r-2m}{r}dt^2 - \frac{r}{r-2m}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \quad , \tag{1}$$

in the exterior region $r > r_1(t) > 2m$, the surface separating it from the interior being defined by

$$t = -\frac{2}{3}\sqrt{\frac{r_1^3}{2m}} - \sqrt{2mr_1} + 2m\ln\frac{\sqrt{r_1}+\sqrt{2m}}{\sqrt{r_1}-\sqrt{2m}} \quad . \tag{2}$$

A particle projected radially inwards from $t = -\infty$ has the integrals of motion

$$\begin{aligned} \frac{r-2m}{r}\frac{dt}{ds} &= C \quad , \\ \left(\frac{dr}{ds}\right)^2 &= C^2 - 1 + \frac{2m}{r} \quad . \end{aligned} \tag{3}$$

If the particle is freefalling $C = 1$, and hence

$$\frac{dr}{dt} = -\frac{r-2m}{r}\sqrt{\frac{2m}{r}} \quad . \tag{4}$$

Then eq. (2) is the integral of this, indicating that points at the surface of the OS collapsar are in free fall. This means a freefalling particle takes an infinite time to arrive at the surface, a result which is summed up by the statement OS made in their Abstract

> .... an external observer sees the star asymptotically shrinking to its gravitational radius.

The OS interior metric replaces the coordinates $(t, r)$ by $(\tau, R)$, with $0 \leq R < 1$, in such a way that the surface point $(t, r_1)$ maps into $(\tau, 1)$ and the metric tensor is continuous there

$$ds^2 = d\tau^2 - \frac{r^2}{R^2}dR^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \quad , \tag{5}$$

where

$$r = 2mR\left(1 - \frac{3\tau}{4m}\right)^{2/3} \quad . \tag{6}$$

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A test particle in the interior falling freely from $\tau = -\infty$ has constant values for $(R, \theta, \phi)$, that is $R$ is a *comoving* ([11] section 11.9) coordinate. The OS time coordinate $t$ is then defined through the *cotime* $y$:

$$\frac{t}{2m} = -\frac{2}{3}y^{3/2} - 2\sqrt{y} + \ln\frac{\sqrt{y}+1}{\sqrt{y}-1} \quad , \tag{7}$$

with

$$y = \frac{r}{2mR} + \frac{R^2}{2} - \frac{1}{2} \quad , \tag{8}$$

So the cotime is equal to $r_1/(2m)$ at the surface, and it is always greater than 1.

The OS property regarding concentration of matter at $R = 1$ follows from consideration of the stress tensor there, that is the curvature tensor divided by $8\pi$. In particular the density $\rho$ has the expression ([11] section 11.9)

$$\rho = \frac{1}{16\pi}\left[2g^{RR}\frac{d^2g_{RR}}{d\tau^2} + 4g^{\theta\theta}\frac{d^2g_{\theta\theta}}{d\tau^2} - \left(g^{RR}\frac{dg_{RR}}{d\tau}\right)^2 - 2\left(g^{\theta\theta}\frac{dg_{\theta\theta}}{d\tau}\right)^2\right] \tag{9}$$

Substituting (5) and (6), this leads to

$$\rho\sqrt{-g} = \frac{3mR^2\sin\theta}{2\pi} \quad . \tag{10}$$

Integrated over the 3-sphere $R < 1$ for constant $\tau$, this gives the total mass $2m$; in the early stage of collapse, $\tau \to -\infty$, we do not need to distinguish $\tau$ from $t$, nor the gravitational from the material mass. Furthermore, in this early stage, the $R$-dependence shows that the density is uniform both in the $(R, \tau)$ and $(r, t)$ coordinates. From the comoving property of $R$ we infer that in the $(R, \tau)$ cordinates the density remains uniform inside $R = 1$, and in the $(r, t)$ coordinates its evolution is obtained by expressing $r$ as a function of $(R, t)$. This relation is obtained from (7) and (8), and in the limit $t \to +\infty$, that is $y \to 1$, it is

$$r = mR(3 - R^2) \quad (y = 1) \quad , \tag{11}$$

which means that, near $R = 1$,

$$\frac{dR}{dr} \sim \frac{1}{6m}\sqrt{\frac{3m}{2m - r}} \quad (R \to 1) \quad , \tag{12}$$

giving an infinite density as $r$ tends to the horizon at $2m$.

We may now investigate the dynamical effect of the shell structure by considering the behaviour of a test particle falling along a geodesic in the interior region. Geodesics along a radius ($d\theta = d\phi = 0$) may be found by defining $x = r/(2mR)$ and expressing $\tau$ as a function of $(x, R)$. The metric then becomes

$$\frac{ds^2}{4m^2} = xdx^2 - x^2dR^2 \quad , \tag{13}$$

and the cyclic variable $R$ satisfies the equation

$$2m\frac{dR}{ds} = -\frac{\alpha}{x^2} \quad , \tag{14}$$

where the constant $\alpha$ is positive, and $x$ takes the value $x_1 = r_1/(2m)$ at $R = 1$. Then the evolution of $x$ is given, for a particle going towards $r = 0$, by

$$2m\frac{dx}{ds} = -\sqrt{\frac{1}{x} + \frac{\alpha^2}{x^3}} \quad . \tag{15}$$

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If the particle was projected from $r = \infty$ with $C > 1$, as in (3), the constant $\alpha$ is related to $C$ by

$$\left(\frac{2m\alpha}{r_1} + \sqrt{\frac{2m}{r_1} + \frac{8m^3\alpha^2}{r_1^3}}\right)^2 = C^2 - 1 + \frac{2m}{r_1} \tag{16}$$

The freefall case $C = 1$ corresponds to $\alpha = 0$, for which eqs. (14) and (15) give, for all interior points,

$$R = const, \quad s - s_0 = \frac{2}{3}\left(x_0^{3/2} - x^{3/2}\right) \quad . \tag{17}$$

The constancy of $R$ in this case establishes that "dust" particles inside the OS collapsar, like those at the surface, are falling freely in the gravitational field of the other particles, and confirms that $R$ is a comoving coordinate.

For the general case $\alpha > 0$, we may eliminate $s$ between (14) and (15) to obtain

$$R(x) = 1 - \int_x^{x_1} \frac{dx'}{\sqrt{x' + x'^3/\alpha^2}} \quad . \tag{18}$$

In particular the particle reaches $R = 0$ with $x = x_0$ given by

$$\int_{x_0}^{x_1} \frac{dx}{\sqrt{x + x^3/\alpha^2}} = 1 \quad . \tag{19}$$

Furthermore, after passing through the origin both $dR/ds$ and $dr/ds$ change sign leaving the sign of $dx/ds$ unchanged, so that the particle exits at $R = 1$ with $x = x_2$ satisfying

$$\int_{x_2}^{x_1} \frac{dx}{\sqrt{x + x^3/\alpha^2}} = 2 \quad . \tag{20}$$

However, these values of $x_0$ and $x_2$ depend on the initial values $x_1$ and $\alpha$ allowing the time coordinate $t(x, R)$ to be real, that is $y > 1$, which implies, from (8), that $x_0 > 3/2$ and $x_2 > 1$. In particular the minimum value $x_{1m}(\alpha)$ allowing the particle to exit has $x_2 = 1$ and satisfies

$$\int_1^{x_{1m}(\alpha)} \frac{dx}{\sqrt{x + x^3/\alpha^2}} = 2 \quad . \tag{21}$$

We find, for example, that $x_{1m}(2) = 9.539$, $x_{1m}(4) = 4.878$, and $x_{1m}(\infty) = 4$; the latter value was obtained in [10] and represents the case where the initial value of $dR/ds$ is infinite, that is our particle has become a light ray.

I remark that, although my article[1] established that the OS choice (8) of the function $y(x, R)$ is not unique, the reality condition (21) is independent of that choice, and arises solely from the continuity of the metric tensor at $R = 1$. The wider family of solutions revealed in my article must also satisfy the reality condition, and in addition the condition that $y = 1$ when $R = 1$; this means the modification studied in my article[1] gives very similar dynamics to those of the original OS metric described above.

If the particle fails to satisfy the reality condition for $x_2$, that is if $x_1 < x_{1m}$, that means the trajectory reaches $y = 1$ before $R = 1$, and the final value of $r$ will then be less than the gravitational radius $2m$, that is it will lie inside the "event horizon". We encountered an extreme case of this in [10], where $x_1$ itself was taken so close to the horizon that the particle did not penetrate beyond the dense concentration of matter at the collapsar's surface. This enables us to understand that, in a supermassive collapsar like that at the centre of our galaxy[3], the greater part of the collapsar's matter lies close to its accretion disc. The geodesics we obtained above enable us now to investigate what happens when the surface shell is approached from the interior. Defining the cartesian coordinate $Z = R\cos\phi$, where $\theta = \pi/2$, and $\phi = 0$ as far as the centre and $\phi = \pi$ after that, we consider the

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trajectory for $-1 < Z < 1$ with the initial values $Z = 1$ and $dZ/ds = -\alpha/x_{1m}(\alpha)^2$; such a particle just reaches $Z = -1$ after an infinite $t$-interval. A typical trajectory has been plotted in Figure 1 with the values $\alpha = 2, r_1/(2m) = 9.538$, and in this case we see that the 4-velocity component $dr/ds$ changes sign at $R = 0.918$, which means the particle actually turns back towards the centre just before reaching the very high concentration of matter at the surface. We therefore have here further confirmation that the high-density shell produces a repulsive gravity field.

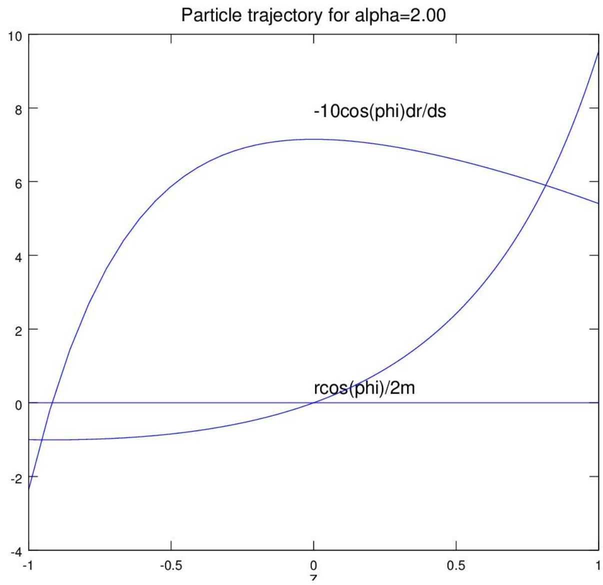


**Figure 1.** The trajectory of a particle projected inwards from the surface of an OS collapsar, with initial values $r_1/(2m) = 9.538, \alpha = 2$. The particle passes through $r = 0$ and then turns back towards the centre at $R = 0.918$, which corresponds to a value of $r$ just inside the "horizon".

In view of the importance now rightly accorded to the study of spinning collapsars, following the discovery of the Kerr[12] metric more than 50 years ago, my extensive discussion of the OS metric may seem hopelessly out of date. However, it seems to have been almost universally overlooked what Kerr stated in his final paragraph

> If we expand the metric in Eq. (5) as a power series in $m$ and $a$ .... and compare it with the third-order Einstein-Infeld- Hoffman approximation for a spinning particle, we find that $m$ is the Schwarzschild mass and $ma$ the angular momentum ... It has no higher order multipole moments in this approximation. Since there is no invariant definition of the moments in the exact theory, one cannot say what they are... It would be desirable to calculate an interior solution to get more insight into this.

The results obtained in the present article indicate that the time has come to resume the programme recommended by Kerr 50 odd years ago. Only then will we be able to extend the investigation of the interior geodesics from the spherosymmetric to the axisymmetric case.

## 3. The thermodynamic implications

The previous section considered the highly idealized OS dust model, whose properties are independent of the collapsar's gravitational mass $m$. With the addition of appropriate equations of state (EoS) it will be equally applicable to either neutron stars above the Tolman-Oppenheimer-Volkoff (TOV) limit[6] or the supermassive white giant[3], which is what I consider the object at the centre of our galaxy to be; in the former case the EoS may be taken as that of a Fermi liquid described by Cameron[13], and in the latter, a relativistic form of electron gas as described by Chandrasekhar[14].

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Such an extension is easy to describe, but the magnitude of its undertaking is truly awesome. I reproduce here the description given by Tolman[15], who pioneered not only the dust model of a collapsar, but also the application of thermodynamics to cosmology[16], the subject of this issue of *Entropy* :

> .... the fluid in the models was taken as dust exerting negligible pressure. Hence no allowance was made for effects such as thermal flow from one portion of matter to another, which in the actual universe might provide a non-gravitational kind of action which would tend to iron out inhomogeneities.

This was written over 80 years ago, and the author had in mind the "inhomogeneities" of the universe as a whole. What I have in mind is the application of the time-dependent OS model to the time-independent OV or the white-dwarf case, and requires, among other things, finding the effect of a nonzero pressure on the "inhomogeneity" presented by the OS surface $r_1(t)$, in particular the infinite density at that surface in the limit $t \to +\infty$. It seems modest by comparison with Tolman's programme, but it is a project which remains to be started.

I state with some confidence that both the upper mass limit on white dwarfs (the Chandrasekhar mass) and the similar limit on neutron stars (the TOV mass) should be disregarded. As for the former, it is now understood that, at around $1.2M_\odot$, that is before the Chandrasekhar mass is reached, a white dwarf collapses into a neutron star rather than a black hole[11]; this is because the density becomes so high that beta-capture of electrons by protons occurs. And as for the latter, neutron stars with mass somewhat in excess of the TOV mass have already been reported[17], and an unprejudiced assessment of the recent LIGO gravitational wave signal[18] would accept the possibility that it comes from the merger of a binary neutron-star system of total mass around $65M_\odot$. Furthermore, our preliminary investigation of super-TOV neutron stars indicates that, with the shell-like structure we propose, the density of nuclear matter decreases as the overall mass, and therefore the surface area, increases to the extent that, when the collapsar enters the supermassive range of a galactic nucleus, the beta-capture process has reversed, and these bodies are *supermassive white giants* (SWG).

The mass limits on white dwarfs[1] and neutron stars originated in the OV field equations[4]

$$\begin{aligned} \frac{du}{dr} &= 4\pi r^2 \rho \quad , \\ \frac{dp}{dr} &= -\frac{(p+\rho)(u+4\pi r^3 p)}{r(r-2u)} \quad . \end{aligned} \tag{22}$$

These may be numerically integrated; with $u(0) = 0$ and any initial value of $\rho$ at $r = 0$ one finds[11], for either a white-dwarf or neutron-star EoS of the form $p = p(\rho)$, a value $r = r_1$ at which $p(r_1) = 0$. The interior metric for $r < r_1$ is smoothly matched to an exterior Schwarzschild metric for $r > r_1$ corresponding to a gravitational mass $m$, where $m$ is equal to the integral of $\rho$ over the interior. The limiting masses, above which no solution of this type exists, are when the initial value $p(0)$ tends to infinity.

This conclusion is based on the assumption, essentially Newtonian, that the gravitational field must be attractive, and that consequently $p$ has its maximum at $r = 0$. However, if, instead of integrating outwards from $r = 0$, we fix $m$ and integrate inwards from an arbitrary value of $r_1$ greater than $2m$ we find[6] that we have a family of solutions of (22), all of which have $p$ increasing to a maximum and falling to $p = 0$ at a point $r_2$ between $r_1$ and zero.

A feature common to this family of solutions is that the variable $u$ takes a negative value at $r_2$ which is determined by our choice of $m$ and $r_1$. In Newtonian theory, which Chandrasekhar used for white dwarfs (see Weinberg's discussion in [11] Chap. 11), $u(r)$ is the mass contained inside a

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1 Chandrasekhar used the Newtonian form of these equations in the white-dwarf case.

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sphere of radius $r$, so its negative value may be a surprise. Our proposed interpretation[6] is that the quantity $m - u(r_2)$ is the total or proper mass, part of which is cancelled by a substantial negative gravitational energy, contained in $r < r_1$; the fact that this total mass is greater than the gravitational mass was acknowledged in Weinberg's presentation, where, however, the two parts of the energy do not have the same values as we are proposing.

A further novel feature of the new family is that it apparently gives zero pressure and density, though not zero gravitational field, for $r < r_2$. However, this is because the OV equation of state, although it gives a neat cutoff at both $r_1$ and $r_2$, is inappropriate for small $p$; it should be replaced by the EoS for an electron gas as beta decay is reestablished, and this in turn should ultimately be replaced by an atmospheric EoS appropriate for a population of atoms, probably largely $Fe^{56}$. It is remarkable that Oppenheimer and Volkoff[4] in footnote 10 of their article seem already to have been aware of the possibility that $u(0) < 0$ and that the EoS at $r = 0$ may be of a different character from the EoS in the high-density region. It was recognized long ago[13] that there is an outer atmosphere in $r > r_1$, so now we should reckon with a complementary inner one in $r < r_2$.

So far it has not been possible to make an estimate of how the surface radius $r_1$ may be related to the gravitational mass $m$. The values obtained experimentally indicate[17] that, for neutron stars in the solar-mass range, $r_1$ falls in the range $6 - 10m$, while the supermassive object at the centre of our galaxy is thought to be visible only on account of its accretion disc[19], whose image in our terrestrial telescope indicates[3] that its radius, which is greater than $r_1$, is no more than $3m$. We may infer that, over a mass range going from less than $M_\odot$ to more than $10^6 M_\odot$, the ratio $r_1/m$ decreases slowly from about 10 to less than 3. Estimates based on a theoretical model for an SWG suggest a lower limit closer to 2 than to 3.

I conjecture that galactic centres like Sagittarius A* will be the only category of collapsar to have truly thermodynamic properties of the type envisaged by Tolman[16]. Neutron stars in the TOV and super-TOV range have shell pressures which are too high for them to be affected by their temperature, and the atmospheres of neutron stars have been estimated[13] to have thicknesses of less than 1km. But when $m$ reaches the domain of the supermassive white giant, the collapsar's atmosphere becomes mixed up with the accretion region and, as in the above quotation from Tolman[15], thermal processes become paramount. Also the close proximity of the hot accretion region to the collapsar's surface allows an enormously greater energetic interaction of the collapsar with the galaxy as a whole; this makes more plausible the concept of the galactic centre as an energy bank for the galaxy. The key feature which restores what may be termed *normal thermodynamics* to such collapsed objects is the absence of a horizon, as also remarked by Spivey[2] and Chafin[20].

## Bibliography


1. T. W. Marshall, Gravitational collapse without black holes, *Astrophys. Space Sci.*, **2012** *342*, 329-332.
2. R. J. Spivey, Dispelling black hole pathologies through theory and observation *Progress in Physics*, **2015**, *11*, 321-328
3. T. W. Marshall, Optics of the event horizon telescope *Progress in Physics*, **2016** *12*, 236-240.
4. J. R. Oppenheimer and G. M. Volkoff, On massive neutron cores *Phys. Rev.* **55,** 374-381 (1939).
5. J. R. Oppenheimer and H. Snyder, On continued gravitational contraction *Phys. Rev.*, **1939**, *56*, 455-459.
6. M. K. Wallis and T. W.Marshall, Energy in General Relativity – the case of the neutron star. In *Physical Interpretations of Relativity Theory*; Proceedings of International Meeting, Bauman State Technical University, Moscow, 2015; pp 544-556. Also at http://tinyurl.com/PIRT-Moscow-15-MarshallWallis.
7. P. O. Mazur and E. Mottola, Gravitational condensate stars: an alternative to black holes *Proc. Natl. Acad. Sci. U.S.A.*, **2004**, *101*, 9545.
8. C. B. M. H. Chirenti and L. Rezzolla, Ergoregion Instability in Rotating Gravastars *Phys. Rev. D*, **2008**, *78*, 084011.
9. M. Visser and D.L. Wiltshire, Stable gravastar - an alternative to black holes? *Class. Quantum Gravity*, **2004**, *21*, 1-17.
10. T. W. Marshall, Repulsive gravity in the Oppenheimer-Snyder collapsar *Progress in Physics* **2016**, *12*, 219-221.

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Peer-reviewed version available at *Entropy* **2016**, *18*, 363; doi:10.3390/e18100363



11. S. Weinberg *Gravitation and Cosmology*; John Wiley, New York, 1972.
12. R. P. Kerr, Gravitational field of a spinning mass as an example of algebraically special metrics, *Phys. Rev. Lett.*, **1963**, *11*, 237-8.
13. A. G. W. Cameron, Neutron star models *Ap.J.*, **1959**,*130*, 884-894.
14. S. Chandrasekhar *Stellar Structure* ; Dover, New York, 1939; Chapter IV
15. R. C. Tolman, Effect of inhomogeneity on cosmological models *Proc. Nat. Acad. Sci.*, **1934**, *20*, 169-176.
16. R. C. Tolman, *Relativity, Thermodynamics and Cosmology*, Clarendon, Oxford, 1934.
17. J. M. Lattimer, The nuclear equation of state and neutron star masses, *Annual Review of Nuclear and Particle Science*, textbf2012, *62*, 485-515.
18. B. Abbott and 1003 others, Observation of gravitational waves from a binary black hole merger *Phys. Rev. Lett.*, **2016**, *116*, 061102.
19. S. S. Doeleman, J. W. Weintroub, A. E. E. Rogers, R. Plambeck, R. Freund, R. Tilanus, L. Ziurys, J. Moran, B. Corey, K. H. Young, D. L. Smythe, M. Titus, D. P. Marrone, R. J. Cappallo, D. C. J. Bock, G. C. Bower, R. Chamberlin, H. Maness, A. E. Niell, A. Roy, P. Strittmatter, D. Werhimmer, A. R. Whitney and D. Woody, Event-horizon-scale struct ure in the supermassive black hole candidate at the galactic centre, *Nature*, **2008**, *455*, 78.
20. C. E. Chafin, Globally causal solutions for gravitational collapse, **2014**, arXiv/1402.1524.