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Letter

Power Conservative Thevenin-Norton and Norton-Thevenin Transformations

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Abstract: A power conservative Thevenin-Norton and Norton-Thevenin transformations are proposed in this letter. The transformations introduce a voltage and a current generators for which parameters depend on the loading impedance value.

Keywords: Thevenin; Norton; voltage source; current source

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1. Introduction

Thevenin's theorem and its dual, Norton's theorem [1,2], also call Thevenin-Norton or Norton-Thevenin transformations are widely used and very useful in the domain of electrical and power signal processing; like for example in synthesis of power amplifiers, filters and antennas. However, considering the traditional transformations, we must always keep in mind that power dissipations of the Thevenin equivalent or the Norton equivalent circuits are not necessarily conservative, even if the power dissipated by the output terminals (loading impedance) is the same [1,2]. In this letter a new full conservative formulations of the Thevenin-Norton and Norton-Thevenin transformations are proposed in case of resistive impedances.

2. Traditional Thevenin-Norton and Norton- Thevenin transformations

Fig.1 illustrates the well-known traditional Thevenin and Norton generators and their corresponding transformations [1,2]. In Fig.1, The voltage source V_T and the current source I_N provide the energy. The Norton transformation (Fig.1b) from an original (or initial) Thevenin circuit (Fig.1a) follows the conditions [1]:

$$I_N=V_T/R_T \tag{1}$$

and

$$R_N=R_T \tag{2}$$

In a same way, parameters of the resulting Thevenin generator (Fig.1a) from an original (or initial) Norton circuit (Fig.1b) is obtained for [1]:

$$V_T=R_N.I_N \tag{3}$$

and

$$R_T=R_N \tag{4}$$

If we considered that the Thevenin circuit (Fig.1a) is the initial circuit, the power transferred to the overall initial circuit is provided by the voltage source V_T and its outputs current. The total dissipation (P_T) of this original Thevenin circuit is the sum of the energy consume by R_T and R_L [2]:

$$P_T = V_T^2/(R_T + R_L) = [R_N^2/(R_N + R_L)].I_N^2 \tag{5}$$

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Considering the power dissipation (P_N) of the equivalent (resulting) Norton (Fig.1b) circuit, we have [2]:

$$P_N = [R_N.R_L/(R_N + R_L)].I_N^2 \tag{6}$$

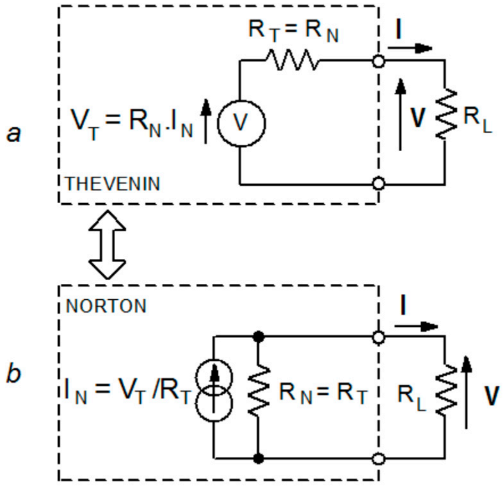

Fig.1. Voltage and current generators.

a Thévenin

b Norton

From equation (5) and equation (6) it follows that:

$$\frac{P_T}{P_N} = \frac{R_N}{R_L} \tag{7}$$

Equation (7) shows that the resulting circuit is strictly equivalent with the original one for $R_L=R_N=R_T$ only [2].

Starting from an original Norton circuit to obtain an equivalent Thevenin circuit will also leads to $P_N=P_T$ only in the case of matching impedances. Therefore, despite the case of $R_L=R_N=R_T$, the Thevenin-Norton or Norton-Thevenin transforms are only valid at the loading point of view [2].

3. Proposed conservative transformations

This section may be divided by subheadings. It should provide a concise and precise description of the experimental results, their interpretation as well as the experimental conclusions that can be drawn.

When the loading impedance doesn't match with the Norton or Thevenin impedance it is interesting to consider transformations which are conservative in term of power consumption of the entire resulting circuit. Similar idea regarding such formulations has been discussed in [2]. Based these last considerations, starting from the Thevenin circuit in Fig.1a, the equivalent conservative Norton circuit, shown in Fig.2a, is obtained from the following proposed transformation:

$$I_N = V_T / R_L \tag{8}$$

and

$$R_N = R_L^2 / R_T \tag{9}$$

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Stating from the Norton circuit in Fig.1b, the proposed conservative transformation are given by (Fig.2b):

$$V_T = R_L I_N \tag{10}$$

and

$$R_T = R_L^2 / R_N \tag{11}$$

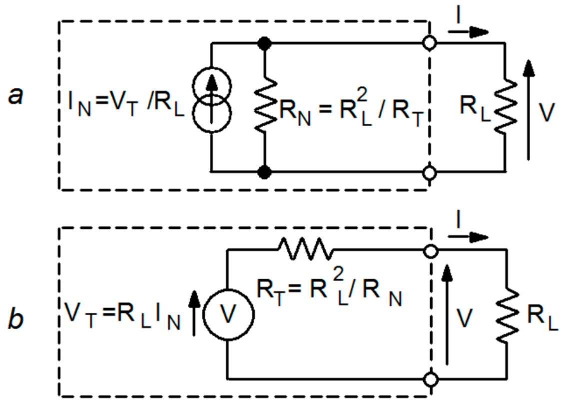

Fig.2. Conservative transformations.

a Resulting circuit of the Thévenin configuration in Fig.1a

b Resulting circuit of the Norton configuration in Fig.1b

Equations (8,9) provide a new transformation of a voltage source to a current source with power conservation. In the same way equations (10,11) provide a new conservative transformation from a current source to a voltage source. Equations (8) to (11) show that the resulting parameters are dependent of the value of the loading impedance. This is not the case of the traditional transforms.

4. Conclusions

The traditional Thévenin or Norton transformations remain very useful in loading impedance point of view. Unfortunately the traditional transforms lead to a power non-conservative topologies, except the case of impedance matching. This letter has proposed a new power-conservative formulation of the Thevenin-Norton or Norton-Thevenin transformation. The proposed formulations demonstrate that the loading impedance plays an essential role in the electrical description of the resulting sources. When power consumption is a key parameter, the formulations, proposed in this paper, must be useful for circuit synthesis based on Norton or Thevenin transforms.

References

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