A Graphical Survey of the Various Trajectories of Thermal Electrons in Fu & Fu's Heat-electric Conversion

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Key words

Maxwell's demon   magnetic demon

entropy decrease   entropy elimination

Abstract

This is a graphical survey of the various thermal electron trajectories and their contribution to the output current in Fu and Fu's experiment of heat-electric conversion. The thermal electrons were emitted at room temperature from two symmetric Ag-O-Cs surfaces in a vacuum tube, with various exiting angles, speeds, and exiting spots. Due to the magnetic field, a certain part of the thermal electrons were transferred from A to B or from B to A, and the survey shows that the former exceeds the latter a little bit more, resulting in an electric charge distribution, with A positively charged and B negatively (or vice versa). A potential difference between A and B emerges, enabling an output current and a power to a load (a resistor, e.g.)

Introduction

In our experiment of electron tube FX12-51, two identical and parallel Ag-O-Cs surfaces (work function 0.8eV) ceaselessly emit thermal electrons at room temperature. The speeds of the thermal electrons, as discovered by Richardson in his famous retarding potential experiments in 1907~1909, are governed by Maxwell's gas molecule speed distribution law,

$$f(v)dv = 4\pi n\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m}{2kT}v^2} v^2 dv \qquad (1)$$

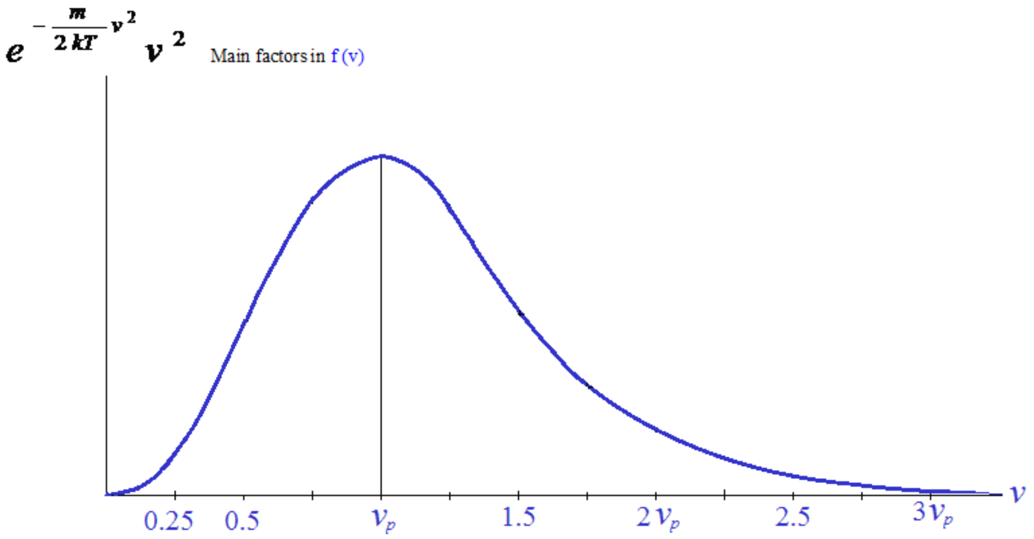


Fig.1 Maxwell's speed distribution law

volume distribution, three-dimension speed

Equation (1) and Fig.1 gives the numbers of gas molecules or thermal electrons **in unit space volume** (n = N/V) and in the speed interval from v to v + dv for an equilibrium state at temperature T.

It is a **volume distribution.**

As is well known, we may easily derive the three characteristic speeds and the mean kinetic energy of the gas molecules or thermal electrons by equation (1).

$$v_p = \sqrt{\frac{2kT}{m}} \quad \bar{v} = \sqrt{\frac{8kT}{\pi m}} \quad \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}} \quad \bar{\varepsilon} = \frac{3}{2}kT \qquad (2)$$

Maxwell's speed distribution concentrates in the speed range of $0.125v_p$ ~ $2.75v_p$, here v_p is **the most probable speed** of the gas molecules or thermal electrons.

$$0.125v_p < v < 2.75v_p, \qquad \Delta N/N = 99.70\%$$

When a static uniform magnetic field is applied to the electron tube in the direction of the axis of the tube, along axis OZ, as shown in Fig.2 (b), the electrons will fly clockwise in the XOY plane along circles of different radii according to the component of their speed in the XOY plane, u

$$R = \frac{m}{eB}u \qquad u = \sqrt{v_x^2 + v_y^2} \qquad (3)$$

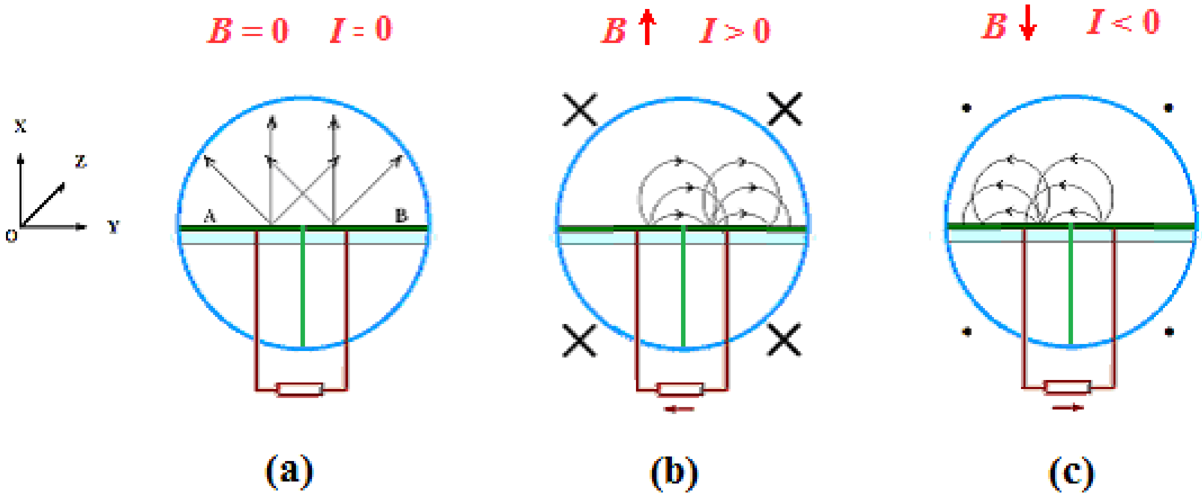


Fig.2 Thermal electrons eject from A or B, with or without a magnetic field.

Hence, for the graphical survey of the trajectories of the thermal electrons in the XOY plane, we need a Maxwell's speed distribution of two dimensions.

The Maxwell's velocity distribution of three dimensions may also take the following form,

$$f(v_x, v_y, v_z)dv_x dv_y dv_z = n(\frac{m}{2\pi kT})^{3/2} e^{-\frac{m}{2kT}(v_x^2+v_y^2+v_z^2)} dv_x dv_y dv_z . \quad (4)$$

By integration over all the v_z, equation (4) reduces to two dimensions

$$f(v_x, v_y)dv_x dv_y = n(\frac{m}{2\pi kT}) e^{-\frac{m}{2kT}(v_x^2+v_y^2)} dv_x dv_y$$

With a cylindrical coordinate system, we may have

$$f(u)du = n(\frac{m}{2\pi kT}) e^{-\frac{m}{2kT}u^2} 2\pi u du$$

$$f(u)du = n\frac{m}{kT} e^{-\frac{m}{2\pi kT}u^2} u du \quad (5)$$

It is the number of gas molecules or thermal electrons **in unit volume** and in the speed interval from u to $u+d\,u$. This is also a **volume distribution**, and speed u is of two dimensions.

Moreover, in the case of our experiment, for all the trajectories, the thermal electrons are just ejected from the two emitters and will soon return back to them. The speed distribution of the ejected electrons is actually not a volume distribution, but a **beam distribution**, or a **wall emission distribution**,.

What is a beam distribution?

It is a gas molecule hole-passing or wall colliding speed distribution, or, a thermal electron wall emission speed distribution. Conventionally, all these are called the **beam distribution.**

Beam speed distribution of two dimensions

(For beam speed distribution of gas molecules or thermal electrons of three dimensions, see the appendix.)

As shown in Fig 3, at time *t*, all the gas molecules or thermal electrons in a cylinder $udtdA\cos\theta$ with a speed from *u to u* + *du* and in the

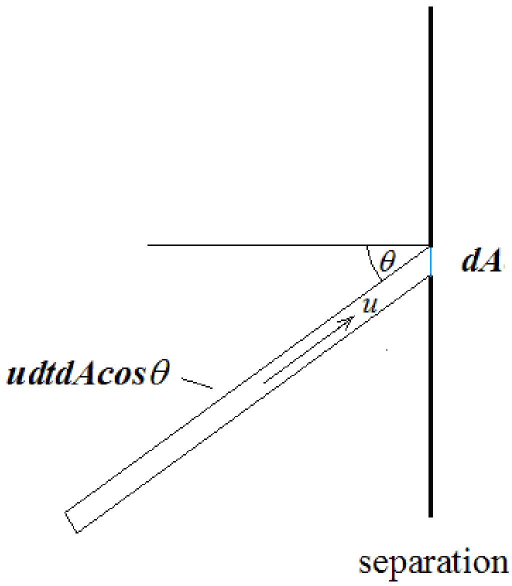


Fig 3 the number of molecules of $u \sim u + du$ and $\theta \sim \theta + d\theta$ pass *dA* in *dt.*

direction from θ *to* $\theta + d\theta$, will pass through the hole *dA* on the separation in a duration *dt*, and their number is

$$d\Gamma(u,\theta)dAdtd\theta = f(u)du \times (udtdA\cos\theta)\frac{d\theta}{2\pi}$$

$$= n\frac{m}{2\pi kT}e^{-\frac{m}{2kT}u^2}udu \times udtdA\cos\theta d\theta$$

$$= n\frac{m}{2\pi kT}e^{-\frac{m}{2kT}u^2}u^2du\mathrm{dtdA}\cos\theta d\theta$$

The factor $cos\theta$ repersents Lambert law.

For dA = 1 and dt = 1, and integrate over θ from −π/2 to −π/2, we derive

$$d\Gamma(u) = n\frac{m}{\pi kT}e^{-\frac{m}{2kT}u^2}u^2du \qquad (6)$$

The general collision (or emission) number of thermal electrons in unit time and on unit area is the so called **wall collision number (or wall emission number)**, which may be derived by integrate (6) over u from 0 to ∞

$$\Gamma = n\frac{m}{\pi kT}\int e^{-\frac{m}{2kT}u^2}u^2du$$ Briefly

$$= n\frac{m}{\pi kT}\bullet\frac{1}{4}\sqrt{\pi}(\frac{2kT}{m})^{\frac{3}{2}}$$

$$= \frac{1}{4}n\sqrt{\frac{8kT}{\pi m}}$$

Briefly $$\Gamma = \frac{1}{4}n\bar{u} \qquad (7)$$

The beam distribution, i.e., the ratio of the number of molecules or thermal electrons of speed interval u ~ $u+du$ in the beam to the number of the molecules or thermal electrons of all speeds in the same beam, is

$$g(u)du = \frac{d\Gamma}{\Gamma} = \frac{n\frac{m}{\pi kT}e^{-\frac{m}{2kT}u^2}u^2du}{\frac{1}{4}n\bar{u}}$$

$$g(u)du == 4\pi(\frac{m}{2\pi kT})^{\frac{3}{2}}e^{-\frac{m}{2kT}u^2}u^2du \qquad (8)$$

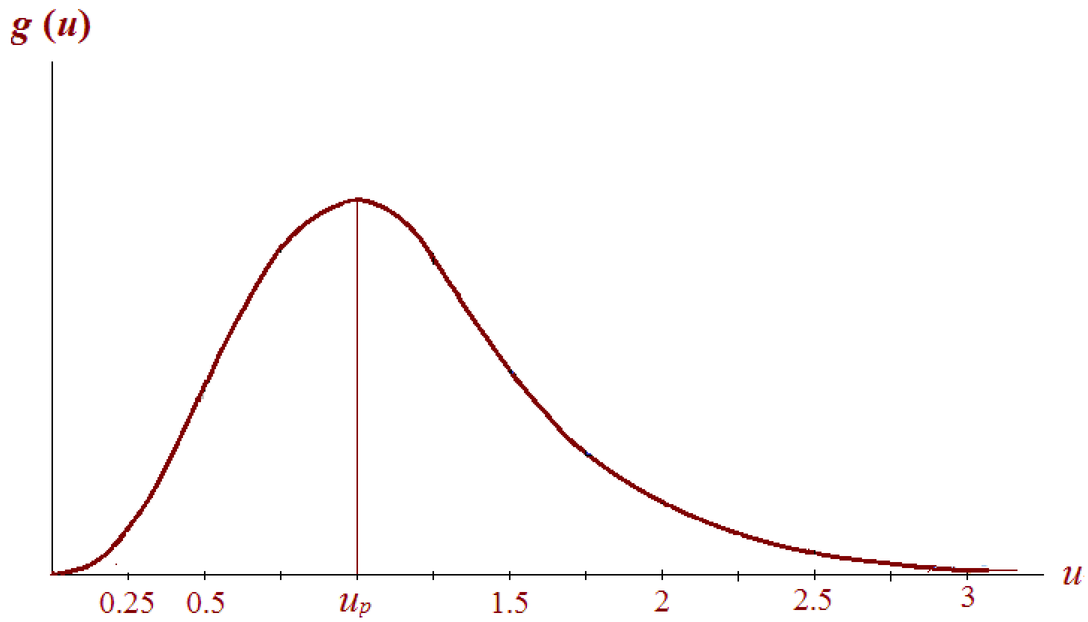


Fig 4 Beam distribution, two-dimensional speed

The two different distribution functions, equation (1) and (8), one is the volume distribution of three dimensions and the other is the beam distribution of two dimensions, are alike exactly!

We may also easily derive the three characteristic speeds and the mean kinetic energy of the gas molecules or thermal electrons for the beam distribution of two dimensional speeds,

$$u_p = \sqrt{\frac{2kT}{m}} \quad \bar{u} = \sqrt{\frac{8kT}{\pi m}} \quad \sqrt{\overline{u^2}} = \sqrt{\frac{3kT}{m}} \quad \bar{\varepsilon} = \frac{3}{2}kT \quad (9)$$

Equation (2) and (9) are also alike exactly.

The beam distribution of two dimensions also concentrates in the speed range of $0.125u_p$~$2.75u_p$, here u_p is **the most probable speed** of u, $u_p = v_p$.

Divide the area under the beam distribution graph of Fig 4 into nine parts, A_0, A_1, A_2, A_3, A_4, A_5 A_6, A_7, A_∞, as shown in Fig 5: for A_0, $\Delta u = 0.25u_p$, for each of A_1 to A_7, $\Delta u = 0.5u_p$, and for A_∞, $\Delta u = \infty$ (from $3.75u_p \sim \infty$).

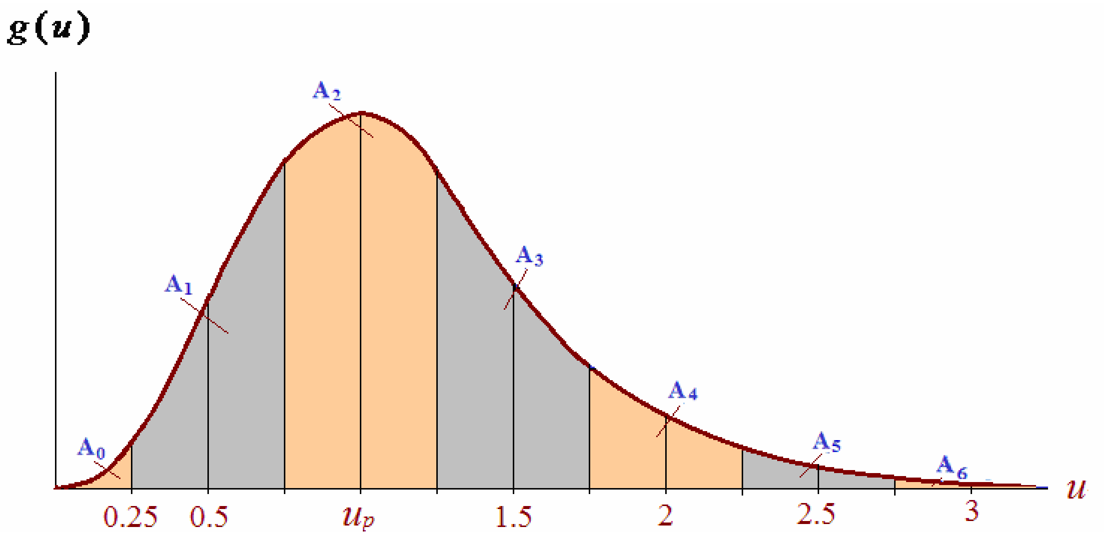


Fig 5 The nine areas under Maxwell's beam distribution, two dimensional speeds.

Range of speed Δu	$A_i = \dfrac{\Delta N_{(u_i \sim u_{i+1})}}{N} = \dfrac{\Delta N_{0\sim u_{i+1}} - \Delta N_{0\sim u_i}}{N}$
0.0 ~ 0.25u_p	A_0 =（0.276326 - 0.265004） = 0.011358 = 1.13%
0.0 ~ 0.125u_p	A_{01} =（0.140222-138860） = 0.001362 = 0.14 %
0.125 ~ 0.25u_p	A_{02} =（0.011358-0.001362） = 0.009996 = 0.99%
0.25 ~ 0.75u_p	A_1 = 0.228958 - 0.011358 = 0.217599 = 21.76%
0.75 ~ 1.25u_p	A_2 = 0.627286 - 0.228958 = 0.398328 = 39.83%
1.25 ~ 1.75 u_p	A_3 = 0.894316 - 0.627249 = 0.267067 = 26.71%
1.75 ~ 2.25u_p	A_4 = 0.982467 - 0.894316 = 0.088151 = 8.82%
2.25 ~ 2.75u_p	A_5 = 0.998287 - 0.982467 = 0.015820 = 1.58%
2.75 ~ 3.25u_p	A_6 = 0.999900 - 0.998287 = 0.001613 = 0.16%
3.25 ~ 3.75u_p	A_7 = 0.999996 - 0.999901 = 0.000095≈0.01%
3.75u_p ~ ∞	A_∞= 1.000000 - 0.999996 = 0.000004 = 0.0004%

Table 1 The nine areas under Maxwell's distribution graph, $\sum A_i$=100%
(beam distribution, two dimensions)

Calculate with the help of error functions the relative ratios of numbers of gas molecules or thermal electrons in the nine ranges, and the results are listed in Table 1. The error functions are

$$N_{0\sim u}=\int_0^u N4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}e^{-\frac{m}{2kT}u^2}u^2d\,u$$

$$=\int_0^u N4\pi\left(\frac{1}{\pi u_p^2}\right)^{\frac{3}{2}}e^{-\frac{u^2}{u_p^2}}u^2d\,u=\int_0^v N\frac{4}{\sqrt{\pi}}e^{-x^2}x^2d\,x$$

$$\frac{N_{0\sim v}}{N}=\frac{4}{\sqrt{\pi}}\int_0^x e^{-x^2}x^2dx=\frac{4}{\sqrt{\pi}}\left[\frac{1}{2}\int_0^x e^{-x^2}dx-\frac{1}{2}\int_0^x d(xe^{-x^2})\right]$$

$$=\left[erf(x)-\frac{2}{\sqrt{\pi}}xe^{-x^2}\right]$$

Now, taking equation (8), Fig 5 and table 2 together as a new platform, we begin our graphical survey of the various trajectories of thermal electrons in such an experiment. What we deal with hereafter is the wall emission distribution of thermal electrons (beam distribution), and the speeds are of two dimensions.

When a magnetic field is applied to the electron tube, thermal electrons of the nine different ranges of speed *u* contribute to the output current very differently. There are two causes.

The first cause, of course, is Maxwell's speed distribution itself (beam distribution, two dimensions). The numbers of electrons in the nine different speed ranges, as shown in Fig.5 and table 2, are explicitly different, hence their contributions to the output current are explicitly different, too.

The second cause is that the trajectories of the thermal electrons of different speeds also related to the exiting angles and exiting spots, are tremendously different. Tremendously different trajectories result in tremendously different contributions to the output current.

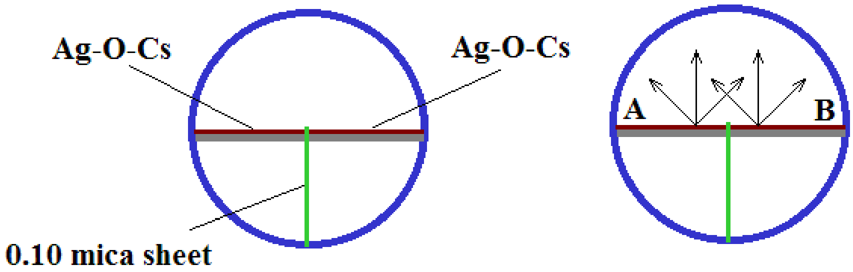


Fig 6 The cross section of an ideal simple and symmetric electron tube, (1 : 1)

For simplicity of discussion, we analyze here the electron migration between A and B with an ideal simple and symmetric tube as shown in Fig.6, which is alike to FX12-51, the actual tube in our experiment, meanwhile the two tubes have some differences from each other, as shown in Fig.7.

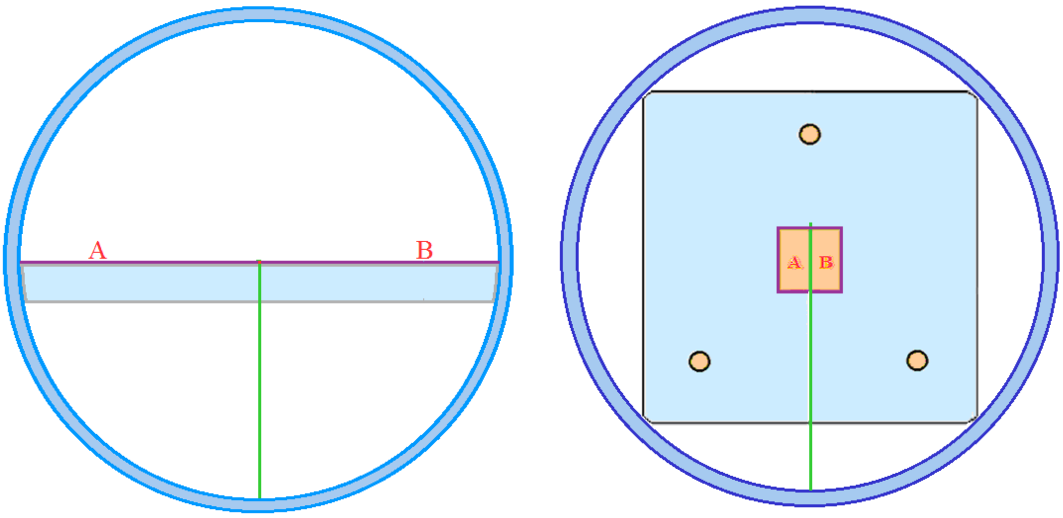


Fig.7 The cross section of the ideal simple and highly symmetric tube (left), and the cross section of the actual experimental tube FX12-5 (right) (2.5:1)

At 20°C, the mean speed of the thermal electrons is $\bar{u} = 106 km/s$, compare to the mean velocity of common gas molecules (say, the remnant gas molecules in the same tube), 460 m/s, **they are extremely fast!**

The mean free path of thermal electrons in a vacuum of 10^{-6} mmHg is determined by their collisions with the remnant gas molecules, about 50m, which is much greater than the dimensions of our tube, ϕ28mm$\times$$L$60 mm. Hence, these collisions may be neglected and the actual free paths of the thermal electrons in the tube may be regarded to be just determined by their ejection from the emitters, their collisions with the tube's glass wall, and their absorption by the emitters.

Numerous thermal electrons frequently collide with the glass wall. These collisions are due to the kinetic energy of their quick motion. At t = 20°C, the average kinetic energy of the thermal electrons and also of the remnant molecules is,

$$\bar{\varepsilon} = \frac{3}{2}kT = 0.038eV = 6.07 \times 10^{-21} J \tag{10}$$

So, the collisions are extremely weak. The glass wall is sufficiently hard and smooth for such collisions, and may be regarded here as a perfect elastic solid.

Now, we start the graphic method to survey the numerous various trajectories of the thermal electrons emitted from A or B in a magnetic field with different exiting angles, different speeds, and different exiting spots, and estimate their contributions to the output current.

All the electron trajectories in the tube may be classified into four groups: A-A, B-B, A-B, B-A. Here *directly* means no collision with the glass wall.

A-A = A-directly-A + A-glass-A B-B = B-directly-B + B-glass-B

A-B = A-directly-B + A-glass-B B-A = B-glass-A

The whole graphical survey has about 150 pages, 250 trajectory figures. The following representative exciting angles and speeds are selected for drawing the trajectories:

θ = 0^{o}, -15^{o}, 15^{o}, -30^{o}, 30^{o}, -45^{o}, 45^{o}, -60^{o}, 60^{o}, -75^{o}, 75^{o}

$u = 0.5u_p$, u_p, $1.5u_p$, $2u_p$, $2.5u_p$, $3u_p$, $3.5u_p$, $4.5u_p$.

We divided the survey into two parts, the first part and the second part. If the readers have enough time, we suggest them read the whole survey, which is rather long, but very interesting, comprehensive and beneficial. However, many readers may have not so much time to read the whole survey. We suggest them to read just the first part, which deals with only the thermal electrons emitted vertically from A or B ($\theta = 0^{o}$) with different speeds. It has only 16 pages, 33 trajectory figures, providing a primary understanding of the whole survey.

The First Part of the survey ($\theta = 0^o$)

In this first part, we survey all the trajectories of electrons that emitted vertically ($\theta = 0^o$) from all the points of the two Ag-O-Cs surfaces of the ideal electron tube shown in Fig.7 (left) with the following representative exiting speeds,

$u = 0.125u_p$, $0.25u_p$, $0.5u_p$, u_p, $1.5u_p$, $2u_p$, $2.5u_p$, $3u_p$, $4.5u_p$.

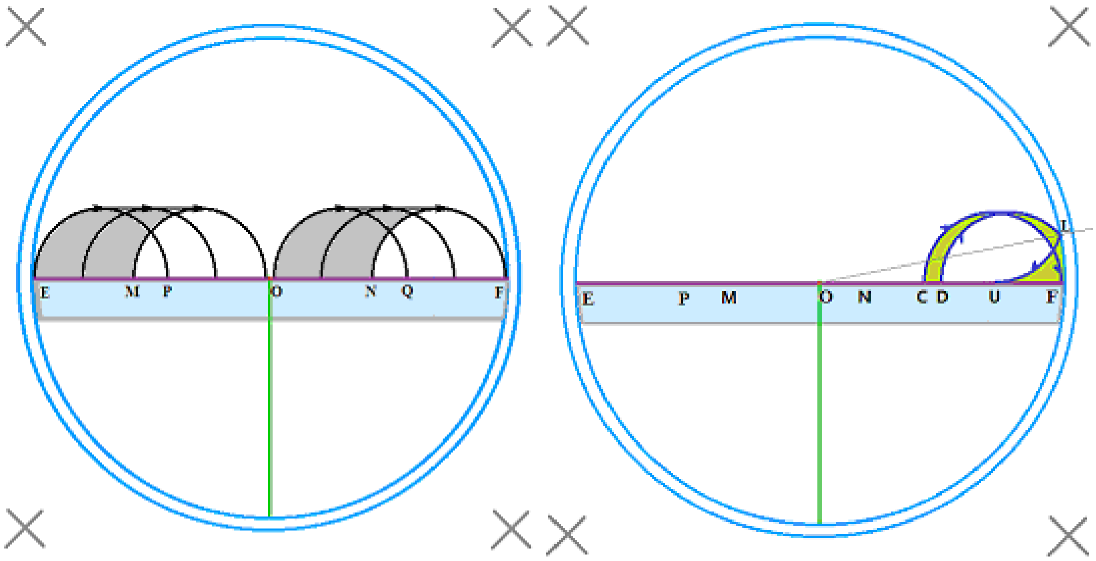
According to Lambert cosine law, normal is the direction of the strongest emission, $j \propto \cos\theta$.

For convenience of drawing and discussion, as we have mentioned previously, we choose a static uniform magnetic field of a special magnetic induction intensity, $B = 1.34 \times 10^{-4}$tesla = 1.34 gausses, to be applied to the ideal electron tube in the direction parallel to the tube axis. In such a magnetic field, for $t = 20^o$C, the thermal electrons of speed of $u = u_p = 94.3$km/s rotate with a radius of $R = 4$mm (more precisely, 4.002mm). Thus, for the electrons of speed $u = 0.5u_p$, the corresponding radius is $R = 2$mm. For the electrons of speed $u = 2u_p$, $R = 8$mm. The rest may de deduced by analogy.

The inner diameter of the electron tube is 28mm.

1. Trajectories of electrons of $\theta = 0^o$ and different speeds u

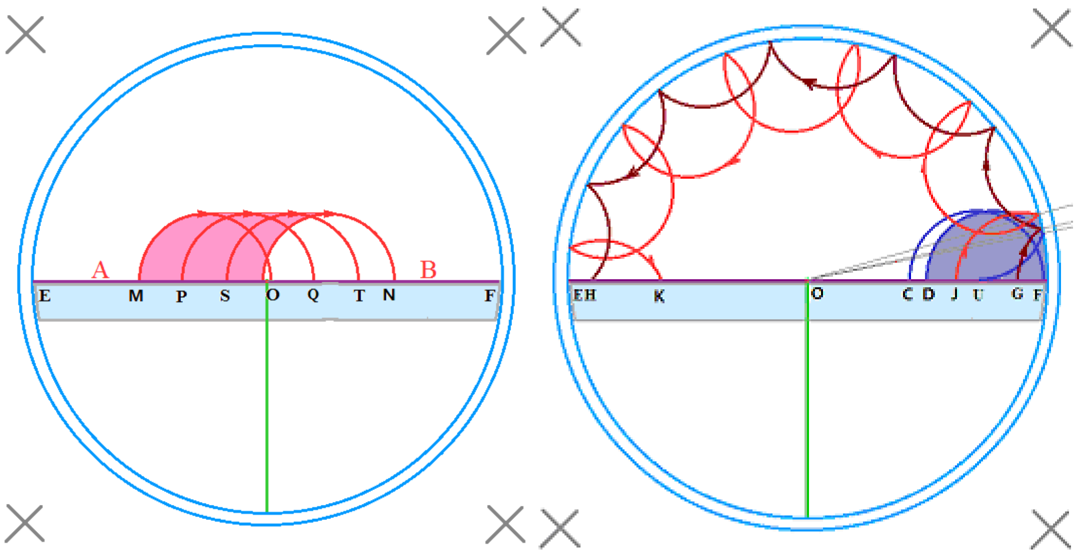
(1) Fig 1-1 $\theta = 0^o$ $u = u_p$ $R = 4$mm (B =1.34 gauss)



(a) A-directly-A & B-directly-B
(from EM to PO, ON to QF, grey)

(b) B-glass-B
(from C to F, D to U, etc., green)

No electron migration between A and B due to these trajectories.



(c) A-directly-B
(from MO to ON, red)
(MO=40, EO=70)
MO/EO = 40/70 = 0.57

(d) B-glass-A
(from J to K, G to H, etc, blue)
(from DF to B, DF=35, OF=70)
DF/OF = 35/70 = 0.50

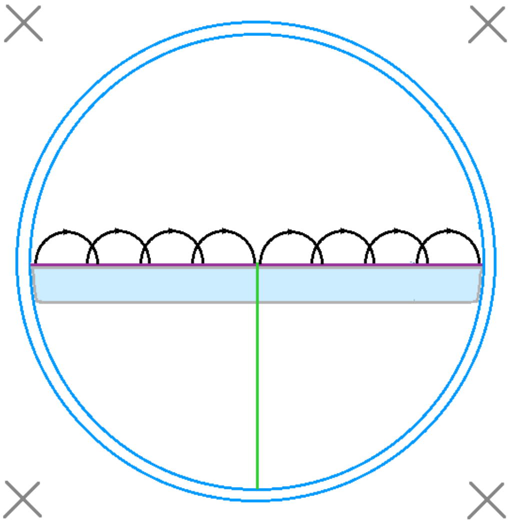
A-B 57% of the electrons of (0^o, u_p) emitted from A migrate to B,

B-A 50% of the electrons of (0^o, u_p) emitted from B migrate to A.

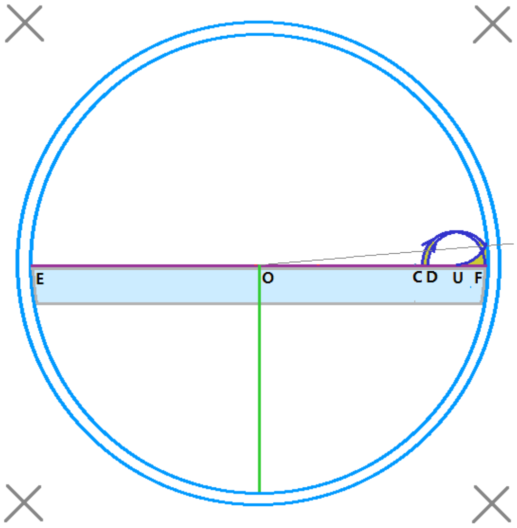
For all the electrons of (0^o, u_p), migration A-B exceeds B-A, and their difference (equals CD) is the corresponding contribution to the output current

$$D_{0^o}(u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{0^o, u_p} = 0.57 - 0.50 = 0.07.$$

(2) Fig 1-2 $\theta = 0^{o}$ $u = 0.5u_p$ R = 2mm ($cos0^{o} = 1$)

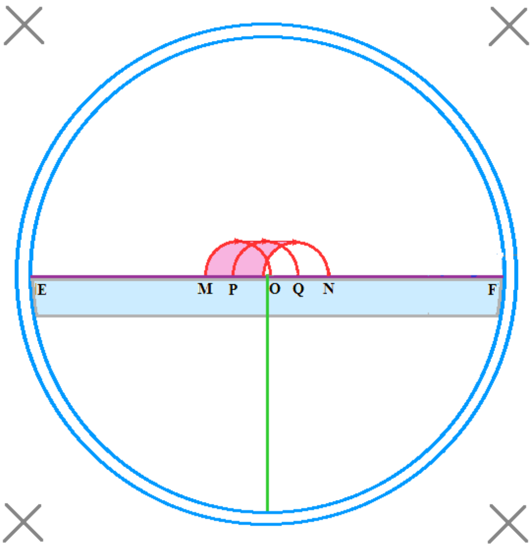


(a) A-directly-A & B-directly-B

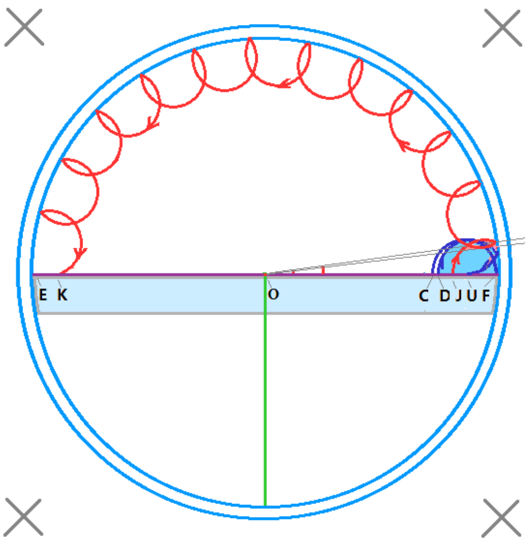


(b) B-glass-B

No electron migration between A and B due to these trajectories.



(c) A-directly-B
(from M to O, P to Q, O to ON, etc., red)
(from MO to ON, MO=20, EO=70)
MO/EO=20/70=0.29



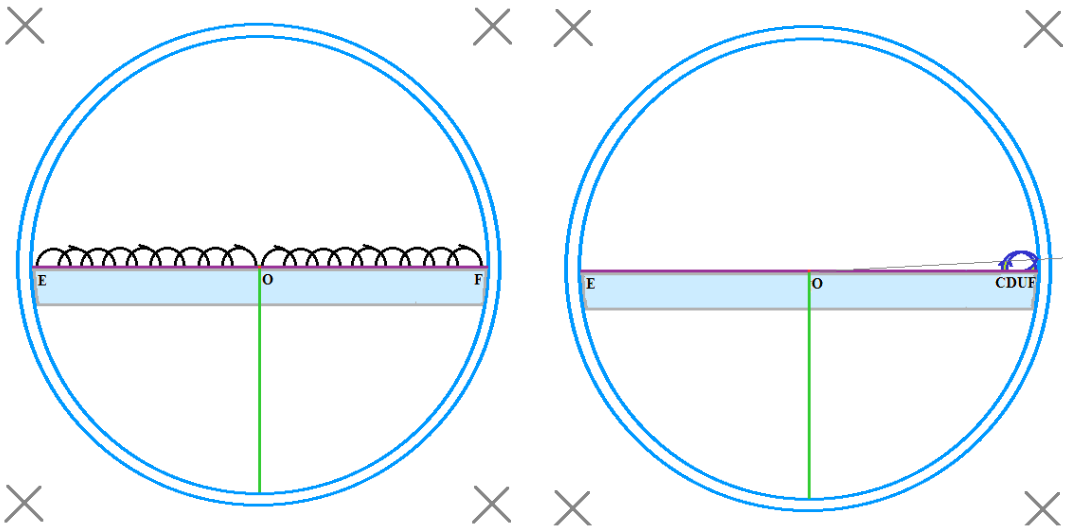
(d) B-glass-A
(from J to K, etc., blue)
(from DF to B, DF =17.5, OF = 70)
DF/OF=17.5/70=0.25

A-B 29% of the electrons of (0^{o}, $0.5u_p$) emitted from A migrate to B.
B-A 25% of the electrons of (0^{o}, $0.5u_p$) emitted from B migrate to A.

For all the electrons of (0^{o}, $0.5u_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current

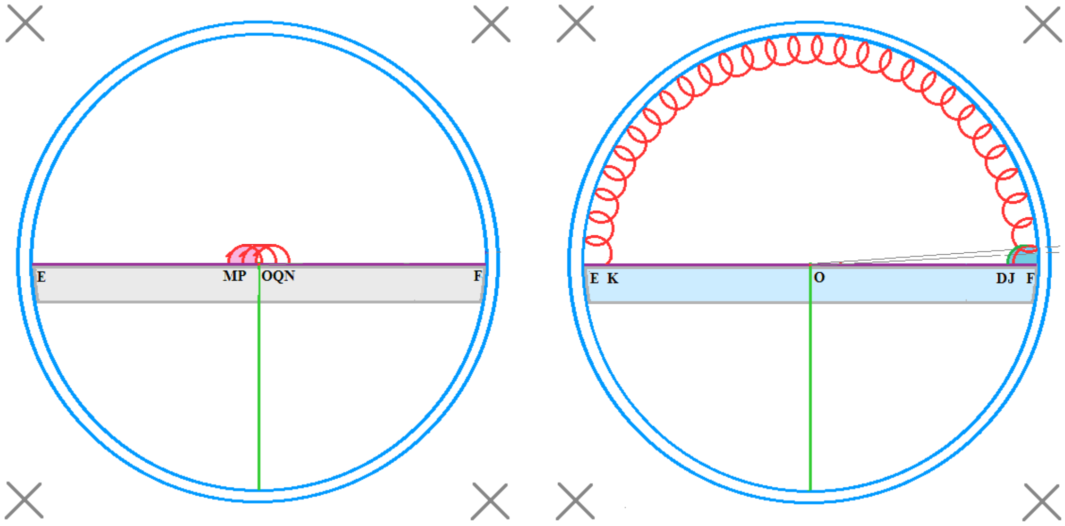
$$D_{0^{o}}(0.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{0^{o},\,0.5u_p} = 0.29 - 0.25 = 0.04$$

(3) Fig 1-3 $\theta = 0^{o}$ $u = 0.25u_p$ R = 1mm



(a) A-directly-A & B-directly-B (b) B-glass-B

No electron migration between A and B due to these trajectories.



(c) A-directly-B
(from MO to ON, red)
(MO=10, EO=70)
MO/EO=10/70=0.14

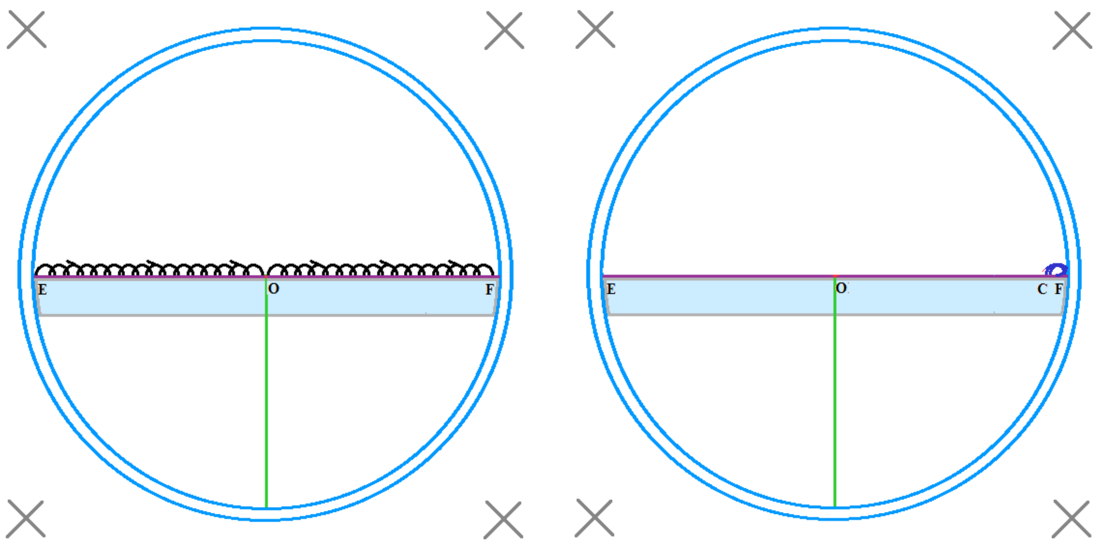
(d) B-glass-A
(from J to K, etc., blue)
(DF =8.5, OF = 70)
DF/OF=8.5/70=0.12

A-B 14% of the electrons of (0^{o}, $0.25u_p$) emitted from A migrate to B.
B-A 12% of the electrons of (0^{o}, $0.25u_p$) emitted from B migrate to A.

For all the electrons of (0^{o}, $0.25u_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

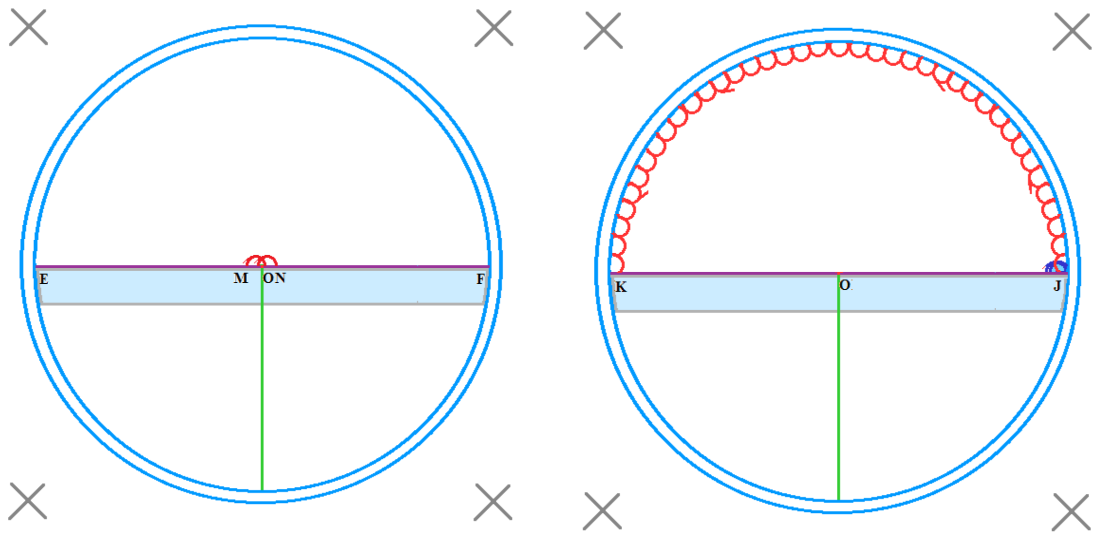
$$D_{0^{o}}(0.25u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{0^{o}\ 0.25u_p} = 0.14 - 0.12 = 0.02$$

(4) Fig 1-4 $\theta = 0^{o}$ $u = 0.125u_p$ $R = 0.5\text{mm}$



(a) A-directly-A & B-directly-B (b) B-glass-B

No electron migration between A and B due to these trajectories.



(c) A-directly-B
(from MO to ON)
(MO=5, EO=70)
MO/EO=5/70=0.071

(d) B-glass-A
from J to K, etc.)
(DF = 4.5, OF = 70)
DF/OF=4.5/70=0.064

A-B 7.1% of the electrons of (0^{o}, $0.125u_p$) emitted from A migrate to B.

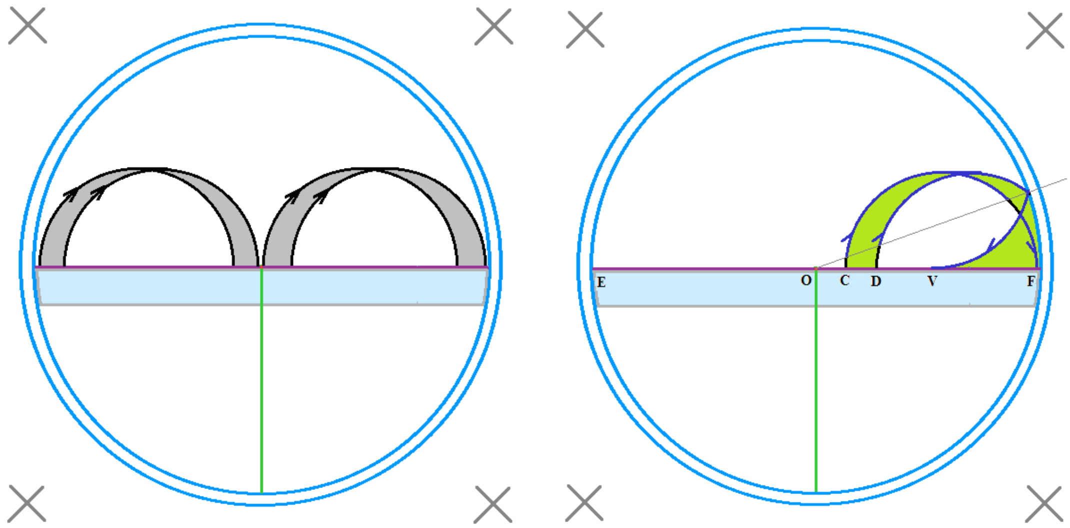
B-A 6.4% of the electrons of (0^{o}, $0.125u_p$) emitted from B migrate to A.

For all the electrons of (0^{o}, $0.12u_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current

$$D_{0^{o}}(0.125u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{0^{o}\,0.125u_p} = 0.071 - 0.064 = 0.007 \approx 0.01$$

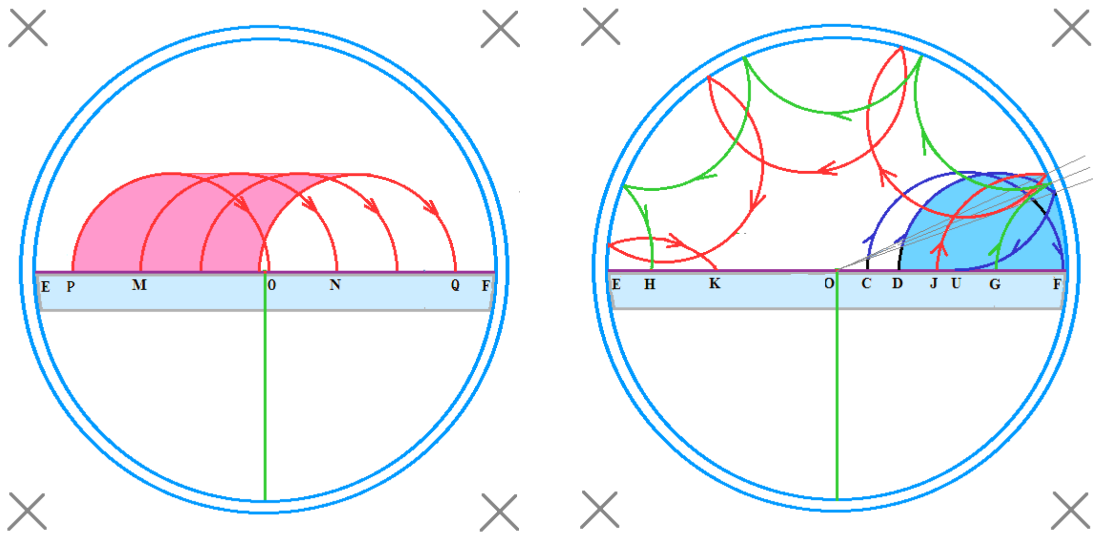
Now, let us see the trajectories of the faster electrons.

(5) Fig 1-5 $\theta = 0^o$ $u = 1.5u_p$ R = 6mm



(a) A-directly-A & B-directly-B (b) B-glass-B

No electron migration between A and B due to these trajectories.



(c) A-directly-B
(from PO to OQ, red)
(PO=60, EO=70)
PO/EO=60/70=0.857

(d) B-glass-A
(from J to K, G to H, etc., blue)
(DF =50, OF = 70)
DF/OF=50/70=0.714

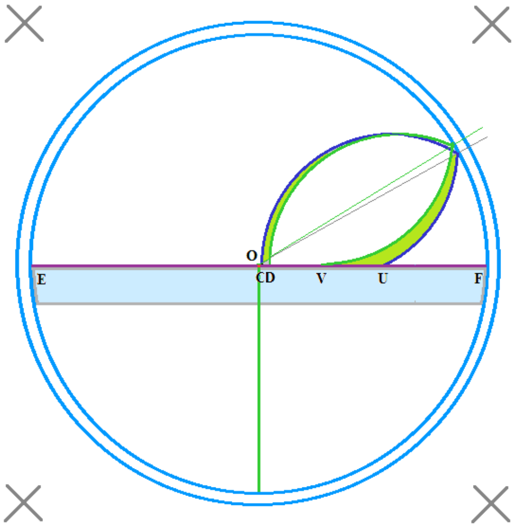
A-B 85.7% of the electrons of (0^o, $1.5u_p$) emitted from A migrate to B.

B-A 71.4% of the electrons of (0^o, $1.5u_p$) emitted from B migrate to A.

For all the electrons of (0^o, $1.5u_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current

$$D_{0^o}(1.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{0^o\ 1..5up} = 0.857 - 0.714 = 0.14 .$$

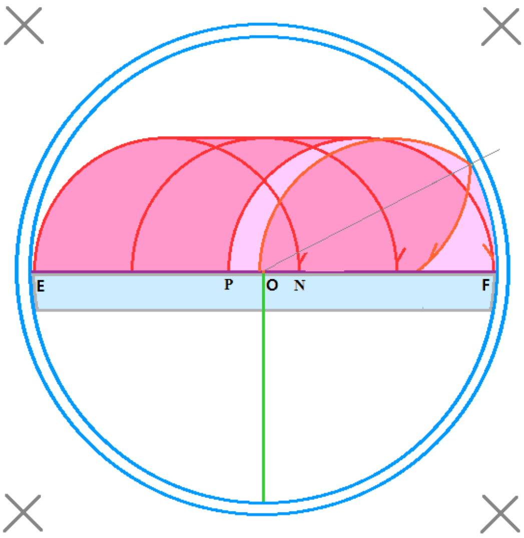
(6) Fig 1-6 $\theta = 0^o$ $u = 2\,u_p$ R = 8mm



(a) B-glass-B

(from C to E, D to V, CD = 3, green)

No electron migration between A and B due to these trajectories.

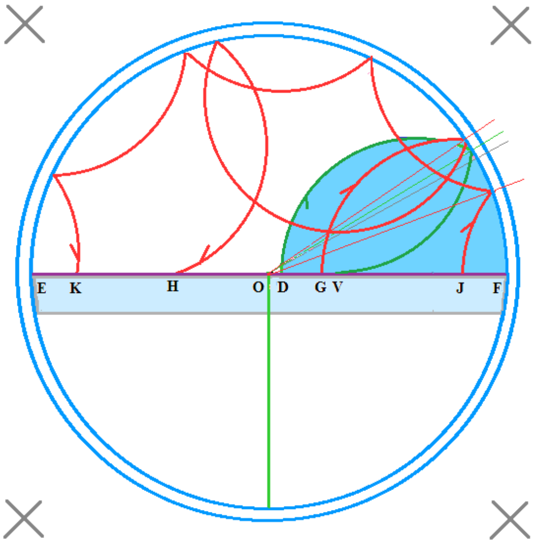


(b) A-directly-B & A-glass-B

(red, violet)

(EP+PO=EO =70, EO=70)

EO/EO=70/70=1.00



(c) B-glass-A

(from J to K, G to H, etc., blue)

(DF =67, OF = 70)

DF/OF=67/70=0.96

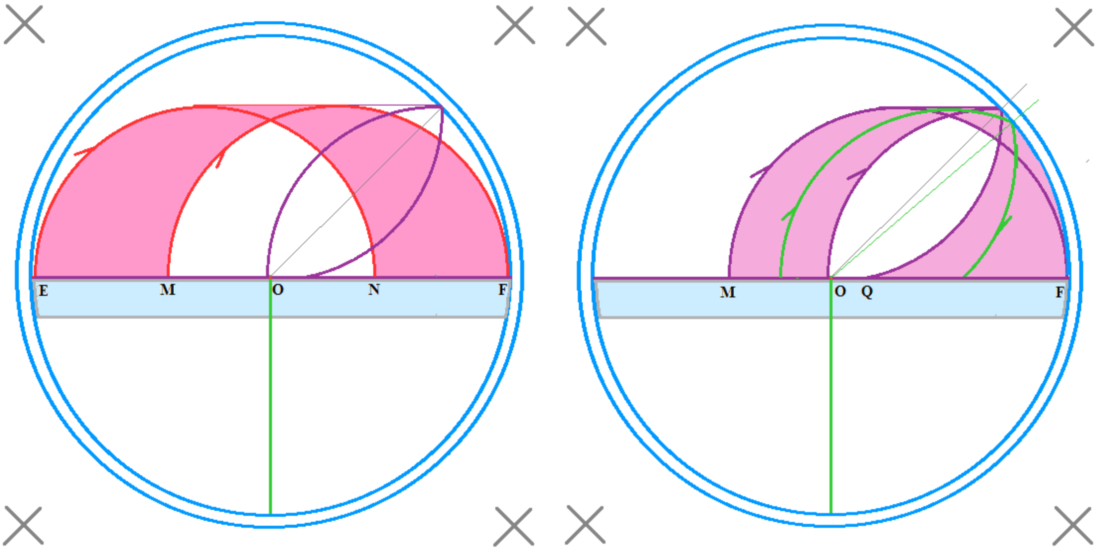
A-B 100% of the electrons of (0^o, $2u_p$) emitted from A migrate to B.

B-A 96% of the electrons of (0^o, $2\,u_p$) emitted from B migrate to A.

For all the electrons of (0^o, $2u_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$D_{0^o}(2u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{0^o\ 2up} = 1.00 - 0.96 = 0.04.$$

(7) Fig 1-7 $\theta = 0^{o}$ $u = 2.5\, u_p$ R = 10mm

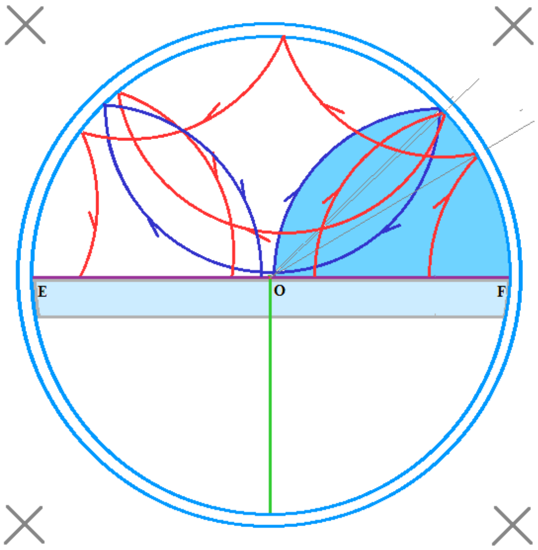


(a) A-directly-B (red) (b) A-glass-B (violet)

A-B = A-directly-B + A-glass-B EM + MO = 70

(EM +MO)/EO =70/70 = 1.00

A-B 100% of the electrons of (0°, 2.5u_p) emitted from A migrate to B.



(c) B-glass-A (blue)

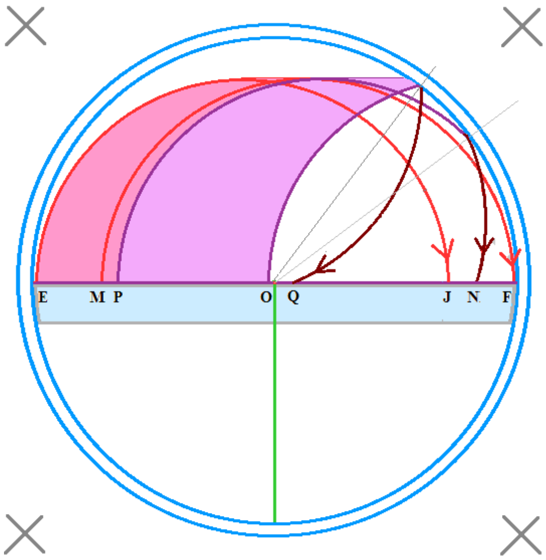
OF/OF = 1.00

B-A 100% of the electrons of (0°, 2.5u_p) emitted from B migrate to A.

For all the electrons of (0°, 2.5u_p), A-B and B-A cancel each other, there is no net contribution to the output current.

$D_{0^o}(2.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{0^o\ 2..5up} = 1.00 - 1.00 = 0.$

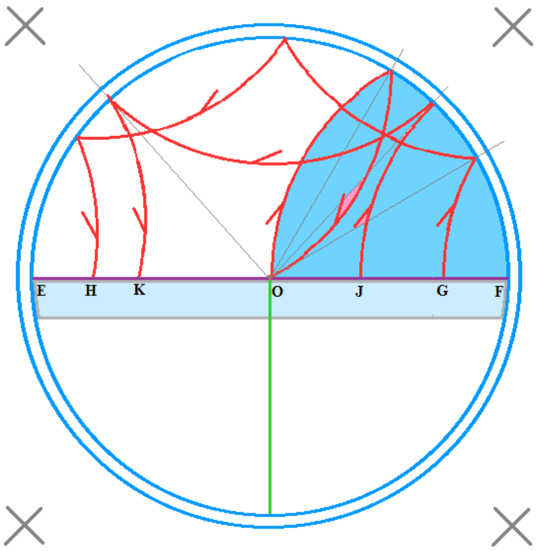
(8) Fig 1-8 $\theta = 0^o$ $u = 3u_p$ R = 12mm



(a) A-B = A-directly-B (red) + A-glass-B (violet)

70/70=1.00

A-B 100% of the electrons of (0^o, $3u_p$) emitted from A migrate to B.



(b) B-glass-A

(from G to H, J to K, etc. blue)

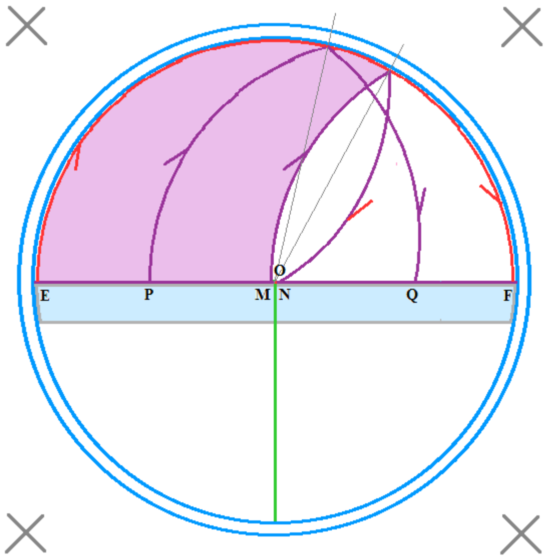
70/70 = 1.00

B-A 100% of the electrons of (0^o, $3u_p$) emitted from B migrate to A.

For all the electrons of (0^o, $3u_p$), A-B and B-A cancel each other, there is no net contribution to the output current..

$D_{0^o}(3u_p)$ = {(A-B) – (B-A)} $_{0^o, 3u_p}$ = 0.

(9) Fig 1-9 $\theta = 0^o$ $u = 3.5u_p$ $R = 14$mm

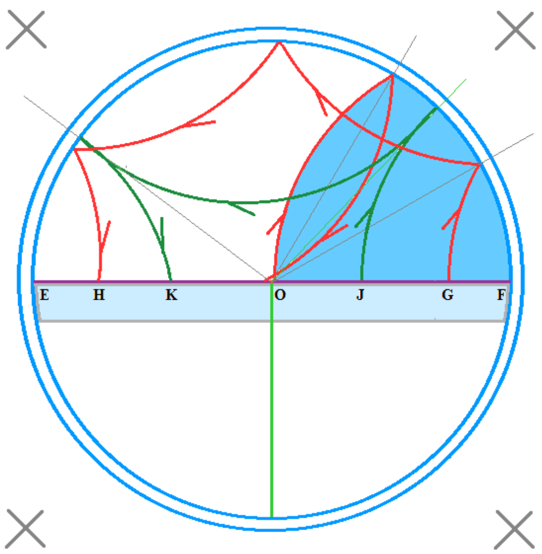


(a) A-directly-B (red), A-glass-B (violet)

A-B = A-directly-B +A-glass-B (red + violet)

70/70 = 1.00

A-B 100% of the electrons of (0^o, $3.5u_p$) emitted from A migrate to B.



(b) B-glass-A

(from G to H, J to K, etc.) (OF = 70, blue)

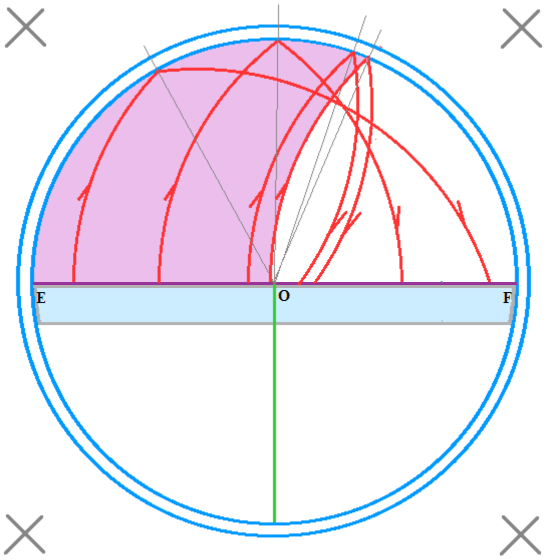
70/70 = 1.00

B-A 100% of the electrons of (0^o, $3.5u_p$) emitted from B migrate to A.

For all the electrons of (0^o, $3.5u_p$), migrations A-B and B-A cancel each other, there is no net contribution to the output current.

$$D_{0^o}(3.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{0^o\,3.5up} = 1.00 - 1.00 = 0.$$

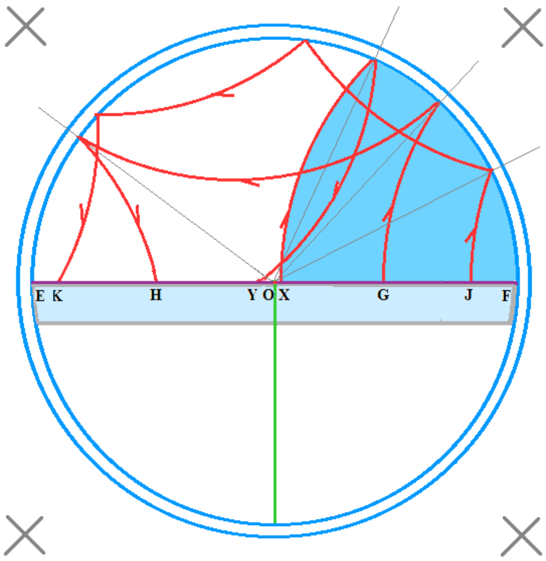
(10) Fig 1-10 $\theta = 0^o$ $u = 4.5\, u_p$ R = 18mm



(a) A-glass-B

EO/EO = 70/70 = 1.00 (violet)

A-B 100% of the electrons of (0°, 4.5u_p) emitted from A migrate to B.



(b) B-glass-A

OF/OF = 70/70 =1.00 (blue)

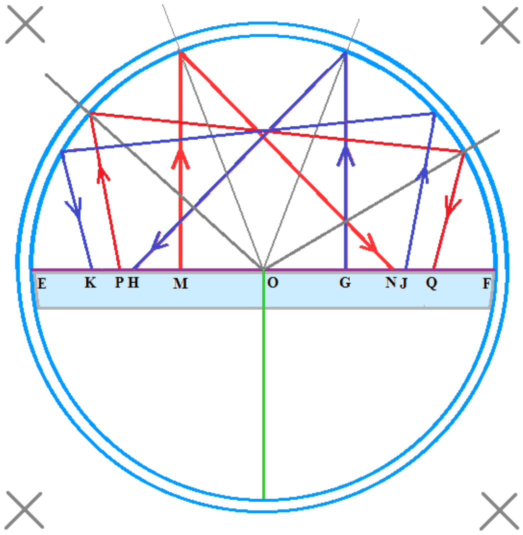
A-B 100% of the electrons of (0°, 4.5u_p) emitted from B migrate to A.

For all the electrons of (0°, 4.5u_p), migrations A-B and B-A cancel each other, there is no net contribution to the output current.

$$D_{0^o}(4.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{0^o,\, 4.5u_p} = 1.00 - 1.00 = 0.$$

(11) Fig 1-11 $\theta = 0^o$ $u >> 4.5\ u_p$ $R >> 18$mm

(i.e., $\theta = 0^o$ \ $u = \infty$ $R = \infty$)



(a) A-glass-B & B-glass-A

(Red and blue)

(from M to N, G to H; from P to Q, J to K; etc.)

$D_{0^o}(u_\infty) = \{(A\text{-}B) - (B\text{-}A)\}_{0^o\ \infty} = 1.00 - 1.00 = 0.$

For thermal electrons of extremely high speed, their trajectories are approximately straight lines although a magnetic field is applied.

Statistically, the electrons are now in symmetric pairs (mirror reflect symmetry, left-right symmetry), one electron exits from A and the other from B, with the same speeds, symmetric exiting angles and exiting spots, and they have symmetric trajectories. Their contributions to the output current cancel each other.

Every 100 + 100 electrons (1.00 = 100% from A, 1.00 = 100% from B) of exiting angle $\theta = 0^o$ and of different speeds contribute to the electron migration between A and B differently, as list in Tab 1 (1) ($cos0^o = 1$).

$$D_{0^o}(u) = \{(A\text{-}B)-(B\text{-}A)\}_{0^o}\ cos\theta\ (u)$$

speed *u*	$\{(A\text{-}B) - (B\text{-}A)\}_{0^o}$	$D_{0^o}(u) = \{(A\text{-}B)-(B\text{-}A)\}_{0^o} cos\theta\ (u)$
Fig 1-4 $u = 0.125u_p$	0.07 - 0.06 = 0.01	0.07 - 0.06 = 0.01
Fig 1-3 $u = 0.25u_p$	0.14 - 0.12 = 0.02	0.14 - 0.12 = 0.02
Fig 1-2 $u = 0.5u_p$	0.29 - 0.25 = 0.04	0.29 - 0.25 = 0.04
Fig 1-1 $u = u_p$	0.57 - 0.50 = 0.07	0.57 - 0.50 = 0.07
Fig 1-5 $u = 1.5u_p$	0.857 - 0.714 = 0.14	0.857 - 0.714 = 0.14
Fig 1-6 $u = 2u_p$	1.00 - 0.96 = 0.04	1.00 - 0.96 = 0.04
Fig 1-7 $u = 2.5u_p$	1.00 - 1.00 = 0	1.00 - 1.00 = 0
Fig 1-8 $u = 3u_p$	1.00 - 1.00 = 0	1.00 - 1.00 = 0
Fig 1-9 $u = 3.5u_p$	1.00 - 1.00 = 0	1.00 - 1.00 = 0
Fig 1-10 $u = 4.5u_p$	1.00 - 1.00 = 0	1.00 - 1.00 = 0
Fig 1-11 $u = \infty$	1.00 - 1.00 = 0	1.00 - 1.00 = 0

Tab. 1 (1) the contributions of the electrons of $\theta = 0^o$ and of different speed, $D_{0^o}(u) = \{(A\text{-}B)-(B\text{-}A)\}_{0^o} cos\theta\ (u)$.

Fig 1 (1) is the corresponding graph,

$D_{0^o}(u) = \{(A\text{-}B)–(B\text{-}A)\}_{0^o} \sim u$

\

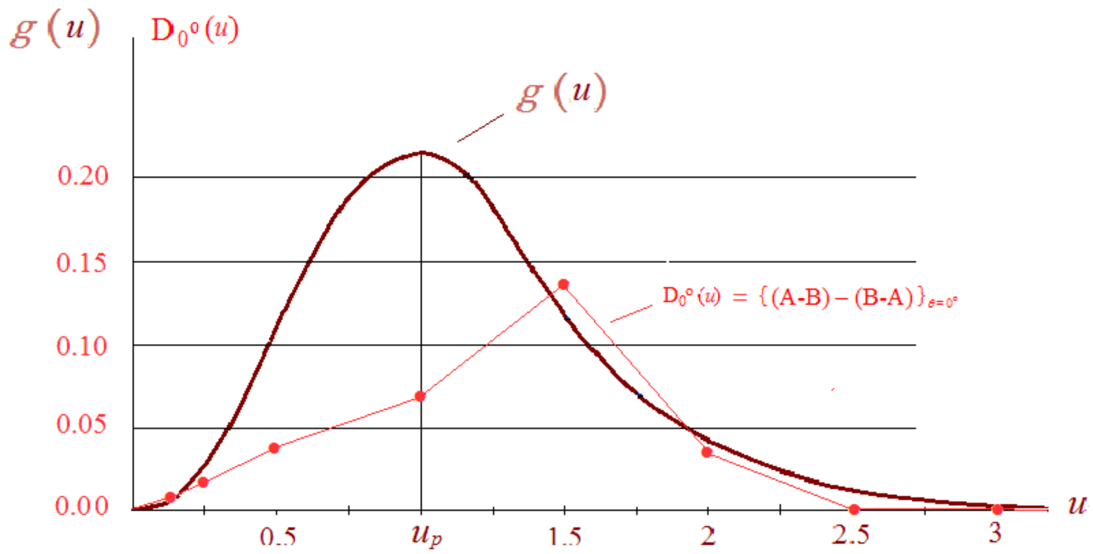


Fig 1(1) Graph of the contributions of electrons of $\theta = 0^o$ and of different speeds, $D_{0^o}(u) = \{(A\text{-}B)–(B\text{-}A)\}_{0^o} \sim u$.

Take Maxwell's speed distribution $g(u)$ into account to derive the actual contributions of the thermal electrons of θ = 0° and of different speed ranges $\varDelta u$, i.e., $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{0^o}(u) \varDelta u \sim u$,

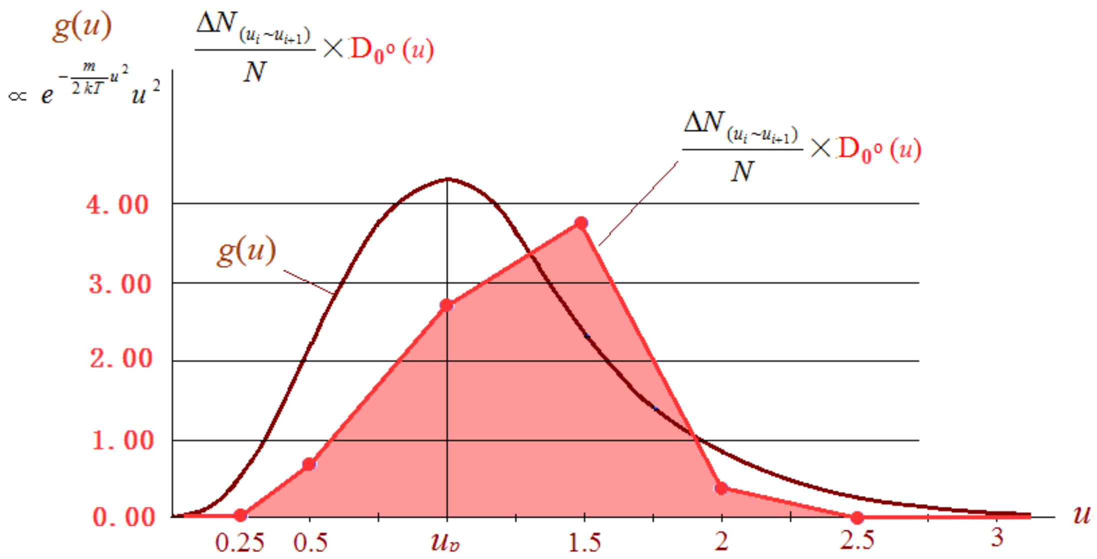
Speed range $\varDelta u$ ($u_i \sim u_{i+1}$)	$\frac{\Delta N_{(u_i \sim u_{i+1})}}{N}$	$D_{0^o}(u)$ {(A-B)–(B-A)} $_{0^o}$ $\cos\theta(u)$	$\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{0^o}(u) \varDelta u$
0.00~0.125u_p	A_{01} = 0. 15%	0.07 - 0.06 = 0.01	0. 15×0.01×0.125 (triangle) = 0.0001875 ≈ 0. 0002
0.125~0.25u_p	A_{02} = 4. 00%	0.14 - 0.12 = 0.02	4 × (0. 02+0. 01) × 0. 5 × 0. 125 = 0. 0075 (trapezoid)
0.25~0.75u_p	A_1 = 18. 74%	0.29 - 0.25 = 0.04	5.4346 – 4.685 = 0.7496
0.75~1.25u_p	A_2 = 39. 83%	0.57 - 0.50 = 0.07	22.7031 -19,915 = 2.7881
1.25~1.75u_p	A_3 = 26. 71%	0.857 - .714 = 0.14	22.8905–19.0709= 3.8196
1.75 ~ 2.25u_p	A_4 = 8. 82%	1.00 - 0.96 = 0.04	8.82 – 8.4672 = 0.3528
2.25 ~ 2.75u_p	A_5= 1. 58%	1.00 - 1.00 = 0	1.58 – 1.58 = 0
2.75 ~ 3.25u_p	A_6 = 0. 16%	1.00 - 1.00 = 0	0.16 - 0.16 = 0
3.25 ~ 3.75u_p	A_7 = .0096%	1.00 - 1.00 = 0	0.0096 -0.0096 = 0
3.75u_p ~ ∞	A_8 = .0003%	1.00 - 1.00 = 0	0.0003 -0.0003 = 0
$\sum_u \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{0^o}(u) \varDelta u$ = **61.5981 – 53.888 +0.0002 + 0.0075 = 7.7178**			

Tab. 1 (2) The actual contributions of electrons θ = 0° with different speeds u.

In the above table, we see, the contributions of the thermal electrons of 0.000~0.125~0.25u_p are very few compared to the contributions of other thermal electrons (compared to the others,

0.0002 and 0.0075 are not of significant figures). For simplicity, we will neglect these very few contributions (0.000~0.125~0.25u_p) in our discussion hereunder.

Fig.1 (1) is the graph of $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{0^o}(u) \triangle u \sim u$, the real contributions of thermal electrons of θ = 0°, and of different speed ranges $\triangle u$.



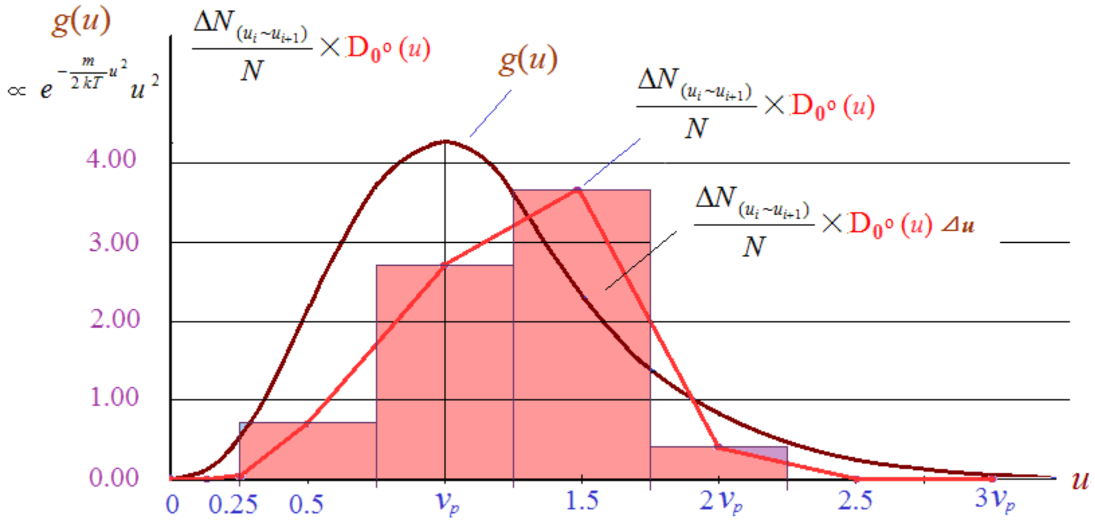


Fig. 1(2) Graph of the actual contributions of electrons of θ = 0° and of different speeds, $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{0^o}(u) \sim u$ and $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{0^o}(u) \triangle u \sim u$.

From the above discussion, we see, **the general net contribution to**

the output current of all the vertically emitted electrons from A and B is not zero. It is positive. This is sharply in contradiction to Boltzmann's principle of detailed balance.

The Second Part of the survey

Trajectories of thermal electrons of other exiting angles ($\theta \neq 0^o$) and of different speeds

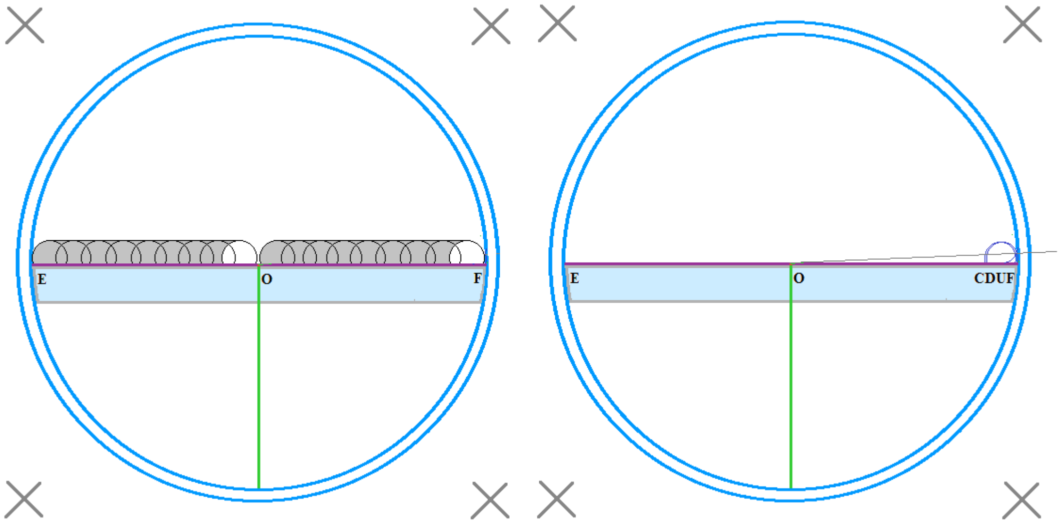
The following exiting angles and speeds are adopted

θ = -15º, 15º, -30º, 30º, -45º, 45º, -60º, 60º, -75º, 75º

$u = 0.5u_p$, u_p, $1.5u_p$, $2u_p$, $2.5u_p$, $3u_p$, $3.5u_p$, $4.5u_p$.

2. Trajectories of electrons of θ = -15° and different speeds

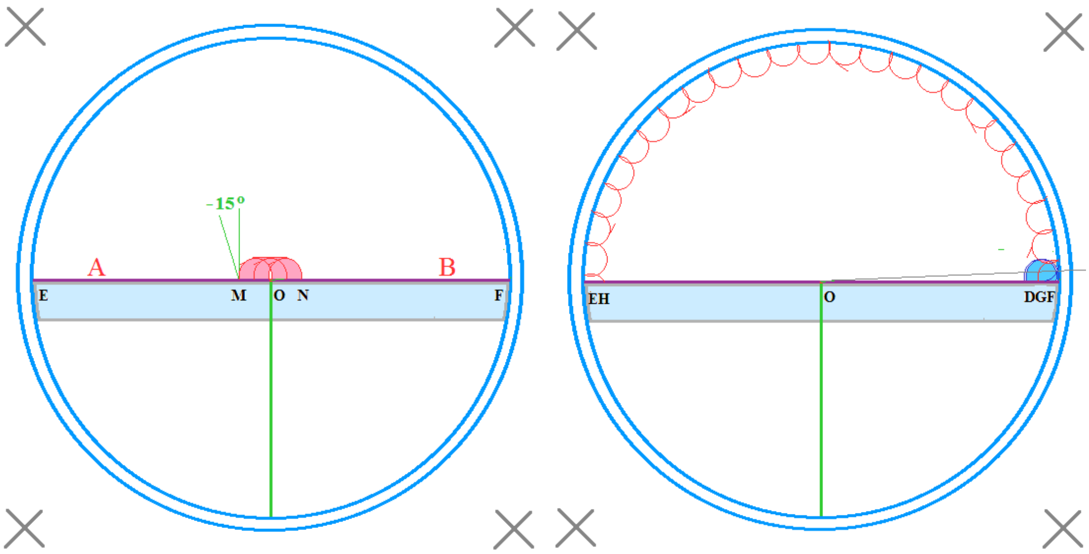
(1) Fig 2-1 θ = -15° $u = 0.25u_p$ R = 1mm (B =1.34 gauss)



A-directly-A B-directly-B (grey)

B-glass-B
CD/OF= 0.5/70 = 0.007≈0. 01

No electron migration between A and B due to these trajectories.



A-directly-B
(from MO to ON, MO = 9.5, red)
MO/EO = 9.5/70 = 0.136

B-Glass-A
(from G to H, etc., DF = 9, blue)
DF/OF= 9/70 = 0.129

A-B 13.6% of the electrons of (-15°, $0.25u_p$) emitted from A migrate to B.

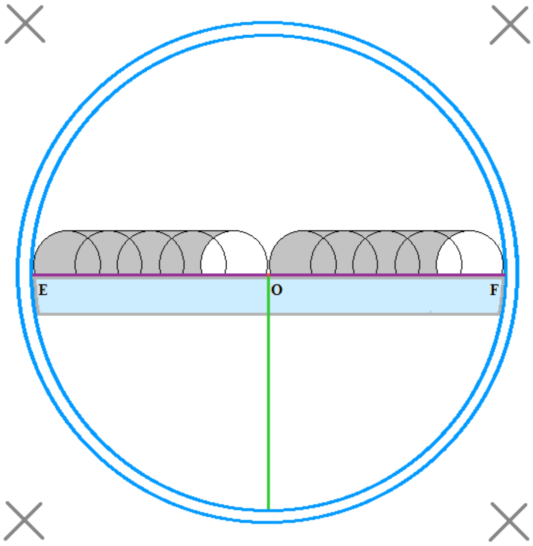
B-A 12.9% of the electrons of (-15°, $0.25u_p$) emitted from B migrate to A.

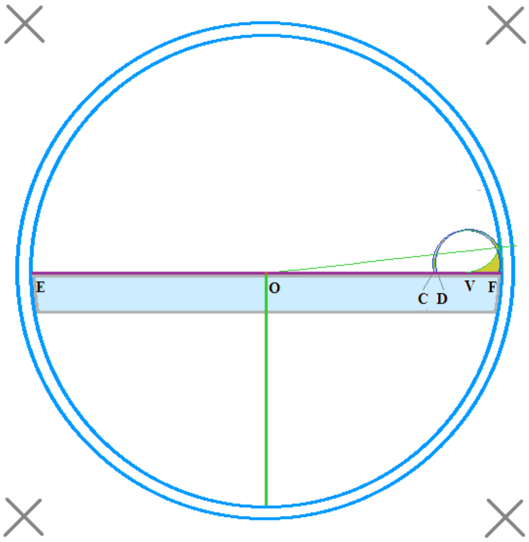
For all the electrons of (-15°, $0.25u_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current

$$\{(A\text{-}B) - (B\text{-}A)\}_{-15^\circ\ 0.25u_p} = 0.136 - 0.129 = 0.007$$

$$D_{-15^\circ}(0.25u_p) = 0.007 \times \cos 15^\circ = 0.007 \times 0.9659 \approx 0.007$$

(2) Fig 2-2 θ = -15° $u = 0.5u_p$ R = 2mm (cos-15° = 0.9659)

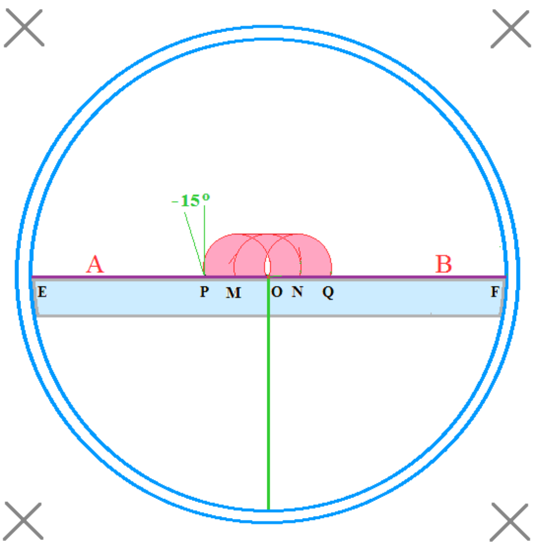


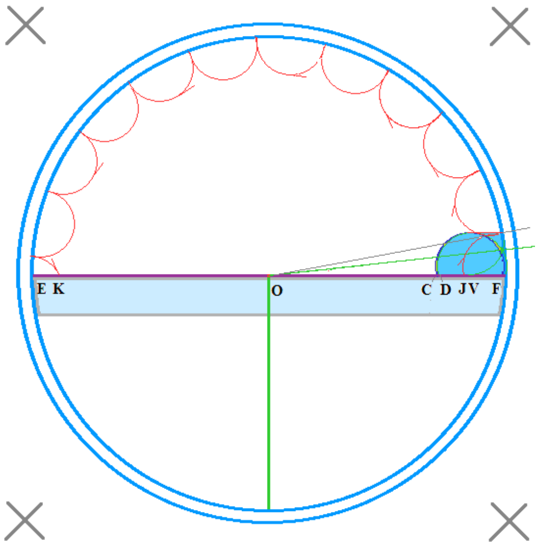


A-directly-A B-directly-B (grey)

B-Glass-B CD/OF=1/70 = 0.014 (CD = 1, OF =70) (green)

No electron migration between A and B due to these trajectories.





A-directly-B
(from PO to OQ, red)
(PO = 19, EO = 70)
PO/EO = 19/70 = 0.27

B-Glass-A
(from J to K, etc., blue)
(DF = 18, OF = 70) 0
DF/OF=18/70 = 0.26

A-B 27% of the electrons of (-15°, $0.5u_p$) emitted from A migrate to B.

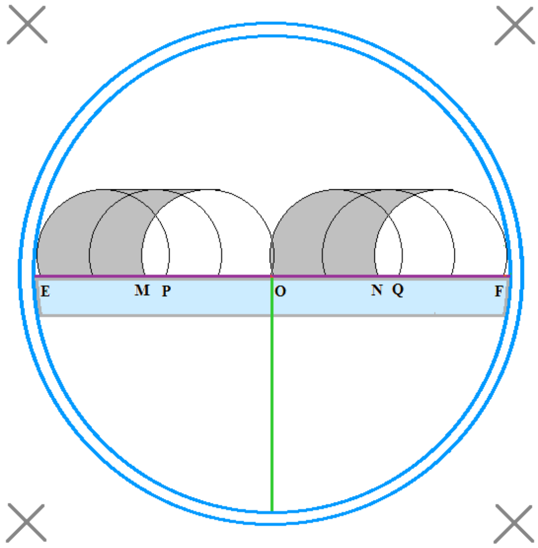
B-A 26% of the electrons of (-15°, $0.5u_p$) emitted from B migrate to A.

For all the electrons of (-15°, $0.5u_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current

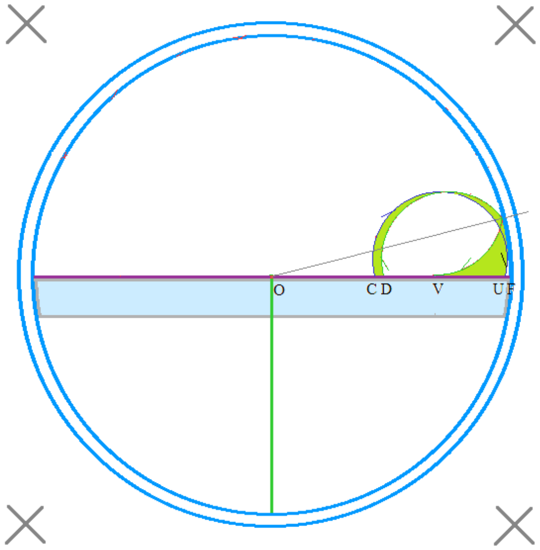
$$\{(A\text{-}B) - (B\text{-}A)\}_{-15^\circ\, 0.5u_p} = 0.27 - 0.26 = 0.01$$

$$D_{-15^\circ}(0.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-15^\circ\, 0.5u_p} \times cos15^\circ = 0.01 \times 0.9659 \approx 0.01$$

(3) Fig 2-3 θ = -15° $u = u_p$ R = 4mm (cos-15° = 0.9659)

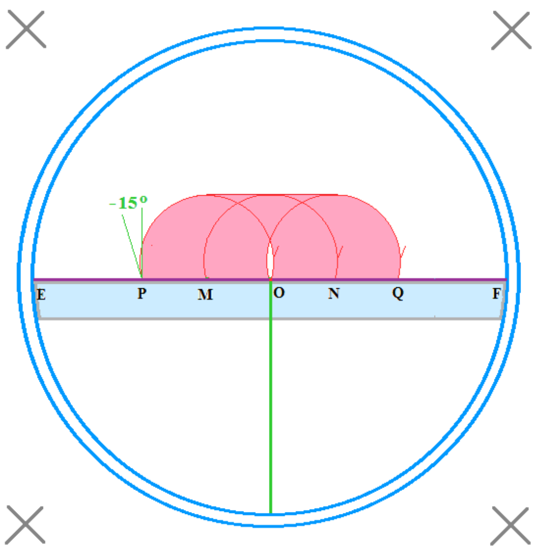


A-directly-A B-directly-B
(grey)

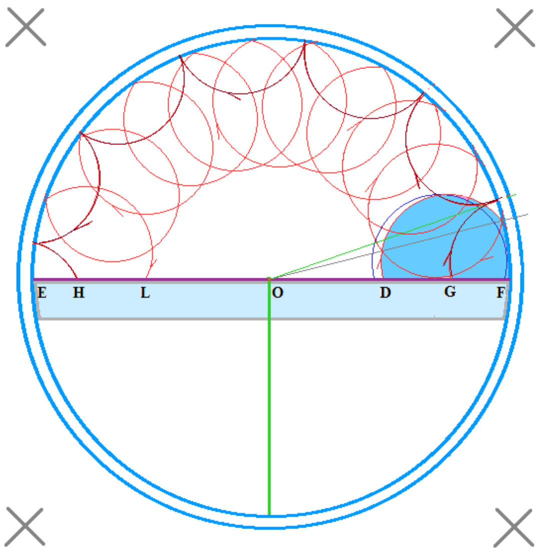


B-Glass-B
CD/OF =2.5/70≈0.04 (green)
(CD=2.5, OF =70)

No electron migration between A and B due to these trajectories.



A-directly-B
(from P to O, M to N, O to Q, etc., red)
(PO = 40, EO = 70)
PO/EO=40/70 = 0.57



B-Glass-A
(from G to H, D to L, etc., blue)
(DF = 37.5, OF = 70)
DF/OF=37.5/70 = 0.54

A-B 57% of the electrons of (-15°, u_p) emitted from A migrate to B.

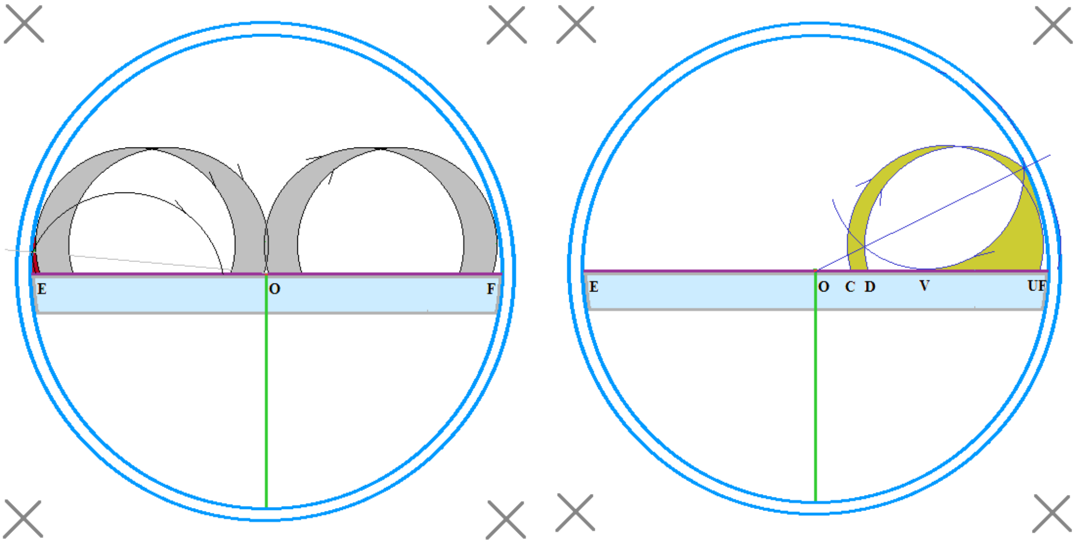
B-A 54% of the electrons of (-15°, u_p) emitted from B migrate to A.

For all the electrons of (-15°, u_p), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{-15^\circ\, u_p} = 0.57 - 0.54 = 0.03$$

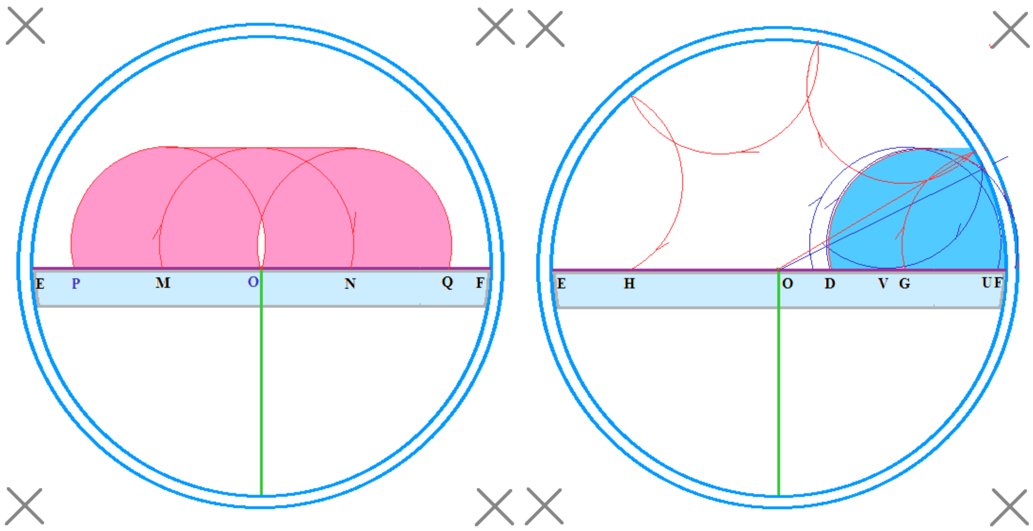
$$D_{-15^\circ}(u_p) = 0.03 \times cos15^\circ = 0.03 \times 0.9659 \approx 0.03$$

(3) Fig 2-4 $\theta = -15^{o}$ $u = 1.5u_p$ $R = 6mm$



A-glass-A (brown) A-directly-A (grey) B-directly-B (grey) B-Glass-B CD/OF = 5/70 =0.07 (green)

No electron migration between A and B due to these trajectories.



A-directly-B (from PMO to ONQ, PO = 58, red)
PO/EO=58/70 = 0.83

B-glass-A (from G to H, etc., blue, DF = 54)
DF/OF=54/70 = 0.77

A-B 83% of the electrons of $(-15^{o}, 1.5u_p)$ emitted from A migrate to B.

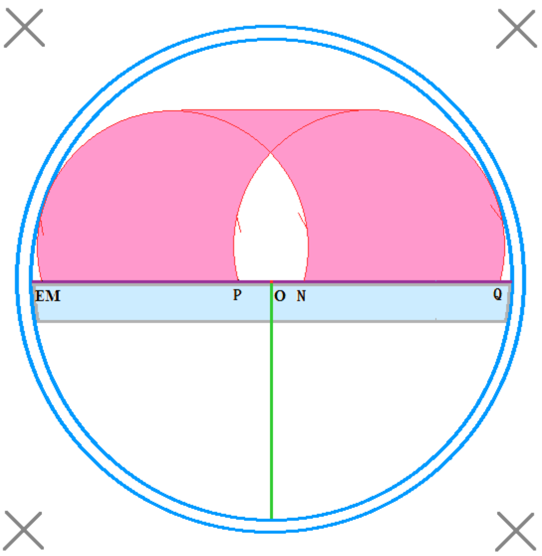
B-A 77% of the electrons of $(-15^{o}, 1.5u_p)$ emitted from B migrate to A.

For all the electrons of $(-15^{o}, 1.5u_p)$, migration A-B exceeds B-A, and their difference is the contribution to the output current,

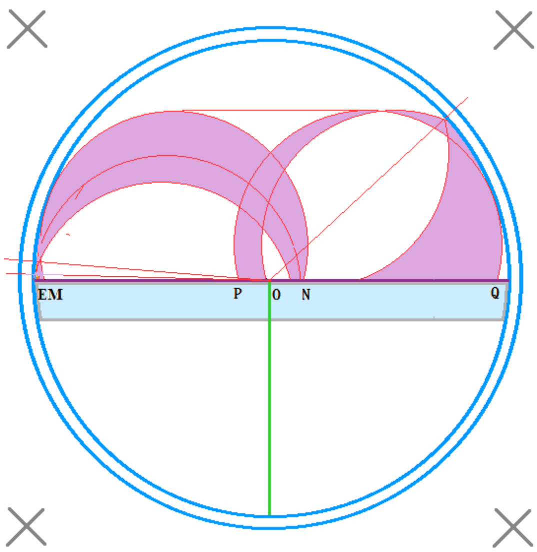
$$\{(A\text{-}B) - (B\text{-}A)\}_{-15^{o}\,1.5up} = 0.83 - 0.77 = 0.06$$

$$D_{-15^{o}}(1.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-15^{o}\,1.5up} \times \cos 15^{\circ} = 0.06 \times 0.9659 = 0.058 \approx 0.06$$

(4) Fig 2-5 $\theta = -15^{o}$ $u = 2u_p$ R = 8mm



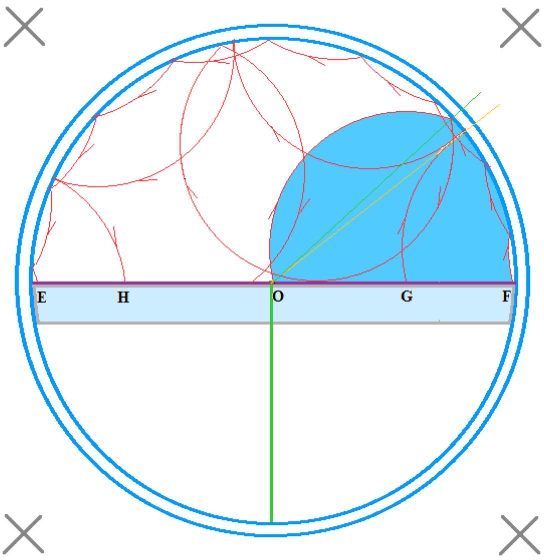
A-directly-B
(from MP to NQ, red. MP=58, EO=70)
58/70 = 0.83



A-glass-B
(EM=2.5, PO=9.5, violet)
12/70 = 0.17

EM + MP + PO = 70
70/70 =1.00

A-B 100% of the electrons of (-15^{o}, $2u_p$) emitted from A migrate to B.



B-glass-A
(from G to H, etc., blue, OE = 70)
70/70 = 1.00

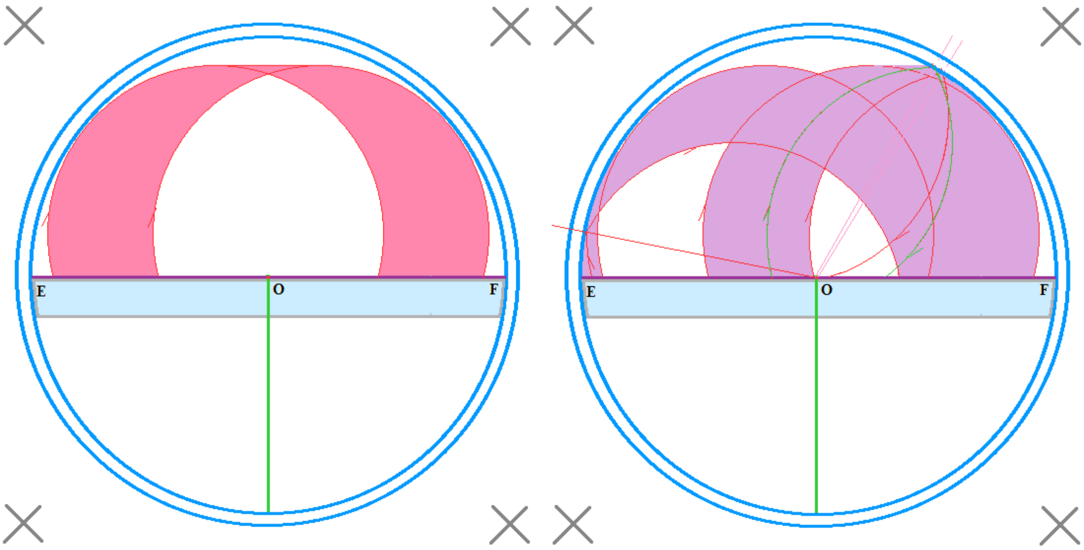
B-A 100% of the electrons of (-15^{o}, $2u_p$) emitted from B migrate to A.

For all the electrons of (-15^{o}, $2u_p$), migration A-B equals B-A, no net contribution to the output current,

$$\{(A\text{-}B) - (B\text{-}A)\}_{-15^{o}\ 2up} = 1.00 - 1.00 = 0$$

$$D_{-15^{o}}(2u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-15^{o}\ 2up} \times \cos 15^{\circ} = 0$$

(6) Fig 2-6 $\theta = -15^{o}$ $u = 2.5u_p$ $R = 10\text{mm}$

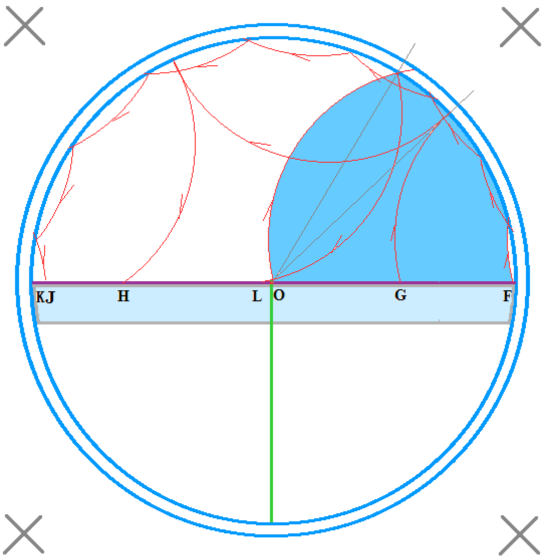


A-directly-B (red) A-glass-B (violet + volet)

A-B = A-glass-B + A-directly-B + A-glass-B (violet + red + violet = 70)

70/70 = 1.00

A-B 100% of the electrons of (-15^{o}, $2.5u_p$) emitted from A migrate to B.



B-Glass-A

(from G to H, F to J, O to L, etc., blue)

70/70 = 1.00

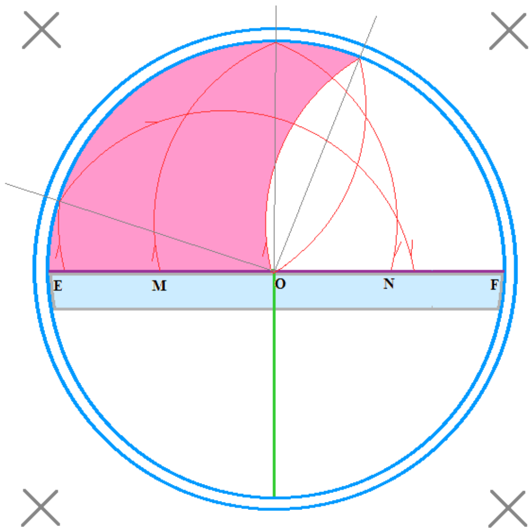
B-A 100% of the electrons of (-15^{o}, $2.5u_p$) emitted from B migrate to A.

For all the electrons of (-15^{o}, $2.5u_p$), migration A-B equals B-A, and their contribution to the output current cancel each other.

$$(\text{A-B}) - (\text{B-A})\}_{-15^{o}\,2.5up} = 1.00 - 1.00 = 0$$

$$D_{-15^{o}}(2.5u_p) = \{(\text{A-B}) - (\text{B-A})\}_{-15^{o}\,2.5up} \times \cos 15^{\circ} = 0$$

(7) Fig 2-7 $\theta = -15^{o}$ $u = 3u_p$ $R = 10\text{mm}$

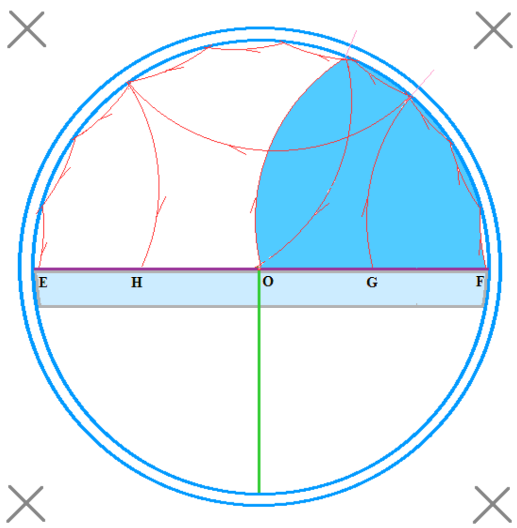


A-glass-B

(from M to N, etc., red)

70/70 = 1.00

A-B 100% of the electrons of (-15^{o}, $3u_p$) emitted from A migrate to B.



B-Glass-A

(from G to H, etc., blue)

70/70 = 1.00

B-A 100% of the electrons of (-15^{o}, $3u_p$) emitted from B migrate to A.

For all the electrons of (-15^{o}, $3u_p$), migrations A-B and B-A cancel each other, no net contribution to the output current,

$$\{(A\text{-}B) - (B\text{-}A)\}_{-15^{o}\,3u_p} = 1.00 - 1.00 = 0$$

$$D_{-15^{o}}(3u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-15^{o}\,3u_p} \times cos15^{\circ} = 0$$

Every 100 + 100 electrons (1.00 = 100% from A, 1.00 = 100% from B) of exiting angle θ = -15º and of different speeds contribute to the electron migration between A and B differently, as list in Tab 2 (1) (*cos*-15º = 0.9659),

speed u	{ (A-B) - (B-A) } $_{-15^o}$	$D_{-15^o}(u)$ { (A-B) - (B-A) } $_{-15^o}$ $\cos\theta$
Fig 2-1 u =0.25u_p	0.136 – 0.129= 0.007	0.13136-0.12460=0.00676
Fig 2-2 u = 0.5u_p	0.27 – 0.26 = 0.01	0.26079-0.25113 =0.0096
Fig 2-3 $u = u_p$	0.57 – 0.54 = 0.03	0.55056-0.52159 =0.029
Fig 2-4 u = 1.5u_p	0.83 – 0.77 = 0.06	0.8017-0.7437=0.058
Fig 2-5 u = 2u_p	1.00 – 1.00 = 0	0.9659-0.9659 = 0
Fig 2-6 u = 2.5u_p	1.00 – 1.00 = 0	0.9659-0.9659 = 0
Fig 2-7 u = 3u_p	1.00 – 1.00 = 0	0.9659-0.9659 = 0
Fig 2-8 u =4.5u_p	1.00 - 1.00 = 0	0.9659-0.9659 = 0

Tab 2 (1) Contributions of electrons of θ = -15º with different speeds.

And Fig 2(1) is the corresponding graph.

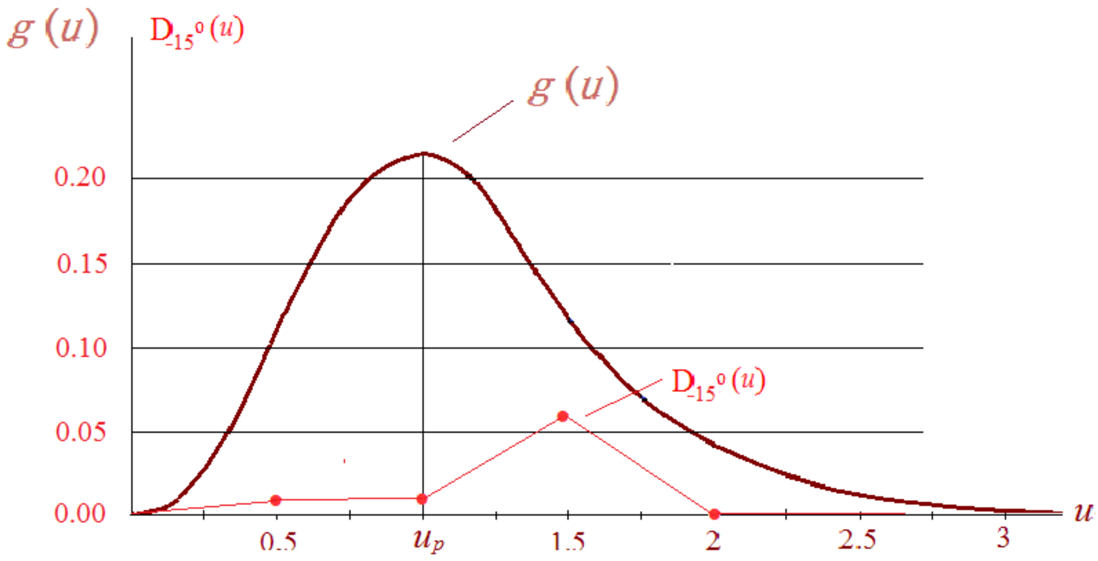


Fig 2 (1) Graph of the contributions of θ = -15° with different speeds

Take Maxwell speed distribution g(u) into account to derive the actual contributions of the thermal electrons of θ = -15º and of different speed ranges, as list in Tab 2 (2),

$$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-15^o}(u) \varDelta u \sim u.$$

Speed range $\varDelta u$	$\frac{\Delta N_{(u_i \sim u_{i+1})}}{N}$	$D_{-15^o}(u)$ = { (A-B)-(B-A) } $_{-15^o}$ $\cos\theta$	$\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{-15^o}(u) \varDelta u$
0.00~0.25u_p	$A_0 \approx$ 0. 004	0.1314-0.1246=0.0068	. 00052−0. 000498=0. 00002
0.25~0.75u_p	A_1= 18. 74%	0.26079-0.25113=0.0096	4. 8874−4. 7056 = 0. 1818
0.75~1.25u_p	A_2= 39. 83%	0.55056-0.52159=0.029	21.9304-20.7753 = 1.1551
1.25~1.75u_p	A_3= 26. 71%	0.8017-0.7437=0.058	21.5576-19.8642 = 1.6934
1.75~2.25u_p	A_4= 8. 82%	0.9659-0.9659 = 0	8.5192 - 8.9152 = 0
2.25~2.75u_p	A_5= 1. 58%	0.9659-0.9659 = 0	1.5261 – 1.5261 = 0
2.75~3.25u_p	A_6= 0. 16%	0.9659-0.9659 = 0	0.1545 – 0.1545 = 0
3.25~3.75u_p	A_7=0.0096%	0.9659-0.9659 = 0	0.00927 -0.00927 = 0
3.75u_p~∞	A_8=0.0003%	0.9659-0.9659 = 0	0.00029-0.00029 = 0
$\Sigma_u \frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{-15^o}(u) \varDelta u$ = 58.5852 – 55.9509 = 2.6343			

Tab 2 (2) Actual contributions of electrons of θ = -15º with different speed ranges.

The corresponding graph of $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{-15^{o}}(u) \sim u$ is shown in Fig 2 (2).

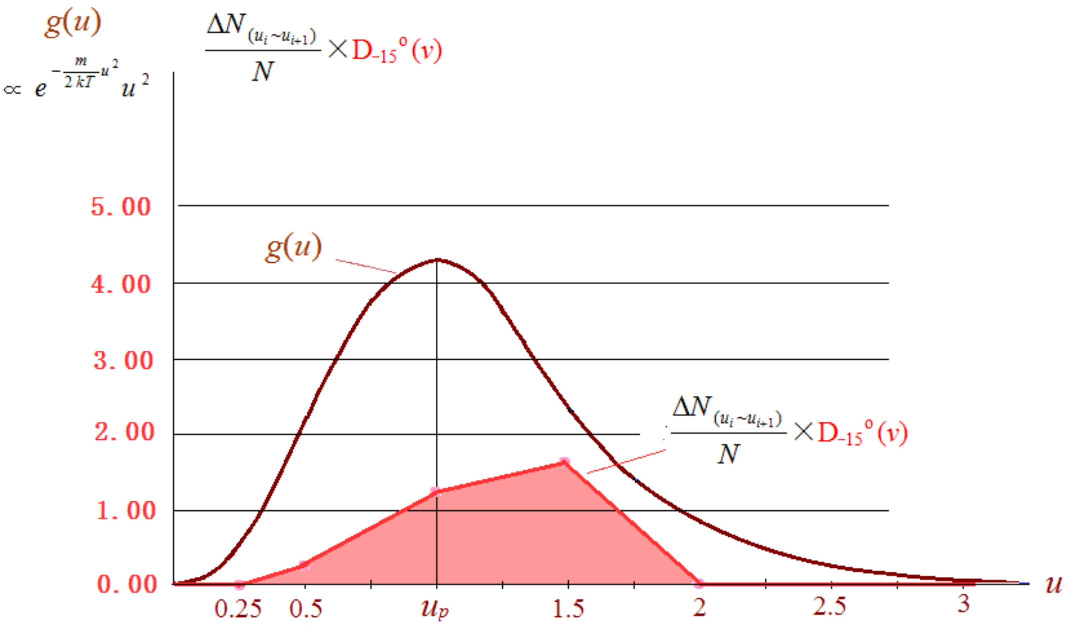
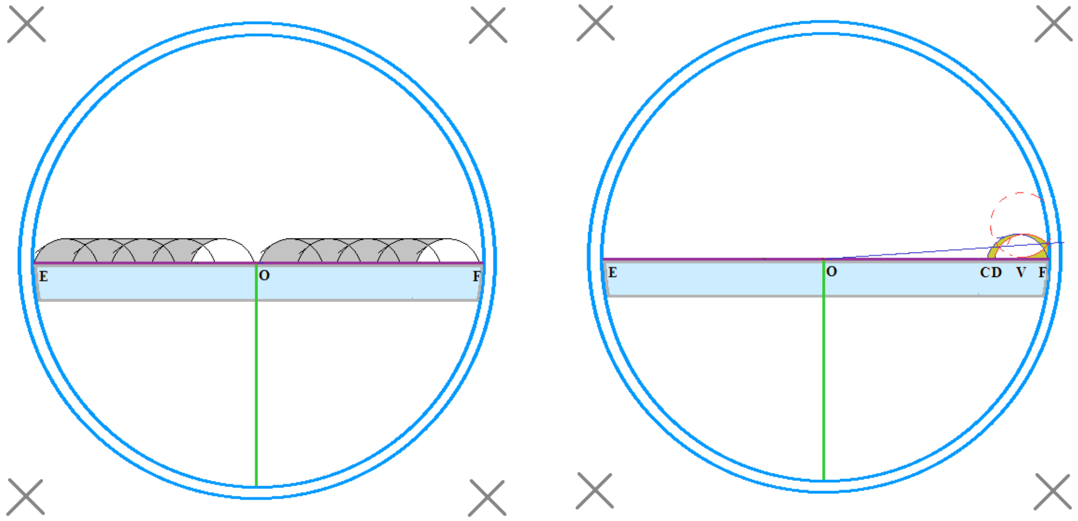


Fig 2 (2) Graph of $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N}$ $D_{-15^{o}}(u) \sim u$.

3. Trajectories of electrons of θ = 15° and different speeds

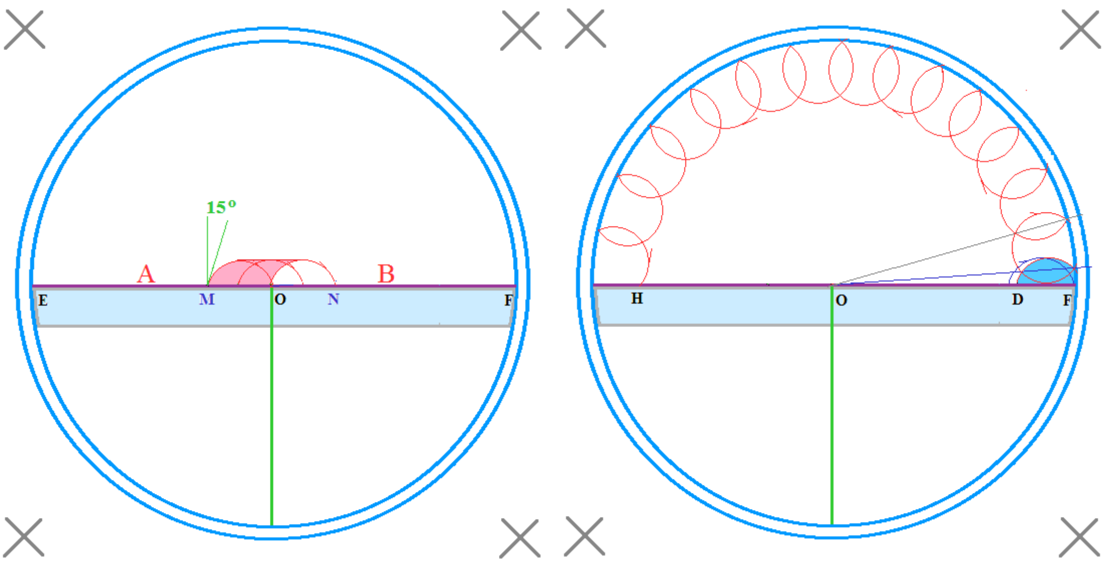
(1) Fig 3-1 $\theta = 15^{\circ}$ $u = 0.5u_p$ R = 2mm (B =1.34 gauss)



A-directly-A & B-directly-B (grey)

B-glass-B 2.5/70=0.0357 (from CD to FV, CD=2.5, OF=70)

No electron migration between A and B due to these trajectories.



A-directly-B (from MO to ON, red, MO=19, EO=70) 19/70 = 0.271

B-glass-A (from D to H, etc. blue) (DF=16.5, OF=70) 16.5/70 = 0.235

A-B 27% of the electrons of (15°, $0.5u_p$) emitted from A migrate to B.

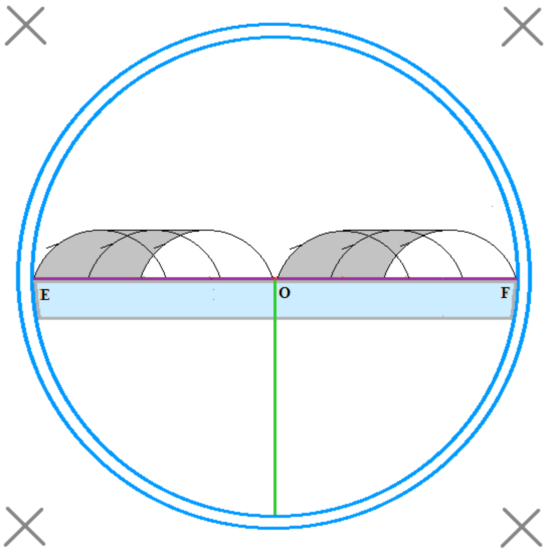
B-A 23.5% of the electrons of (15°, $0.5u_p$) emitted from B migrate to A.

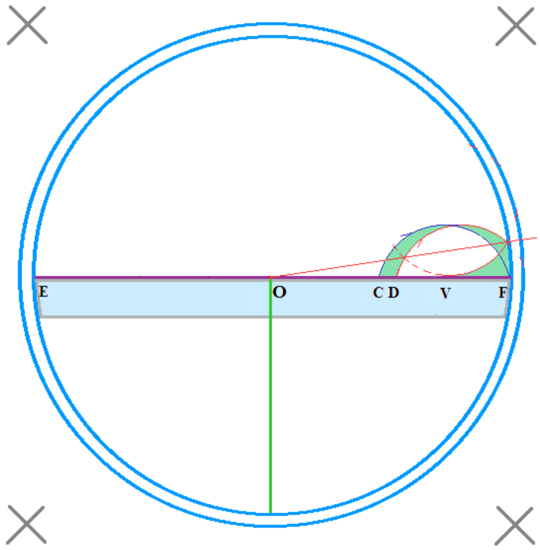
For all the electrons of (15°, $0.5u_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current

$$\{ (A\text{-}B) - (B\text{-}A) \}_{15^{\circ}\, 0.5u_p} = 0.271 - 0.235 = 0.036 \approx 0.04$$

$$D_{15^{\circ}}(0.5u_p) = \{ (A\text{-}B) - (B\text{-}A) \}_{15^{\circ}\, 0.5u_p} \cos 15^{\circ} = 0.04 \times 0.9659 \approx 0.04$$

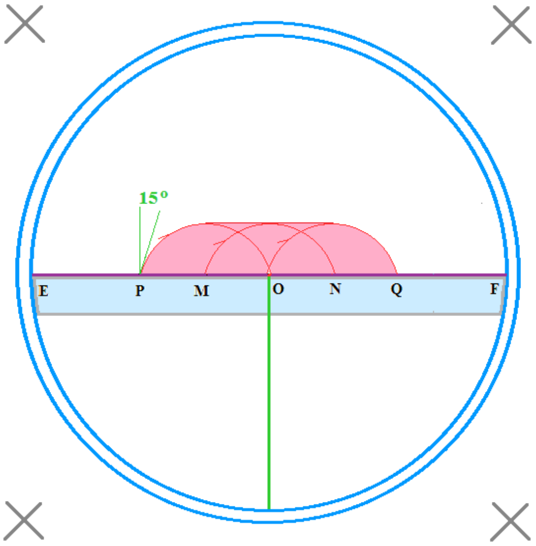
(2) Fig 3-2 $\theta = 15^{o}$ $u = u_p$ $R = 4\text{mm}$ $(cos15^{o} = 0.9659)$

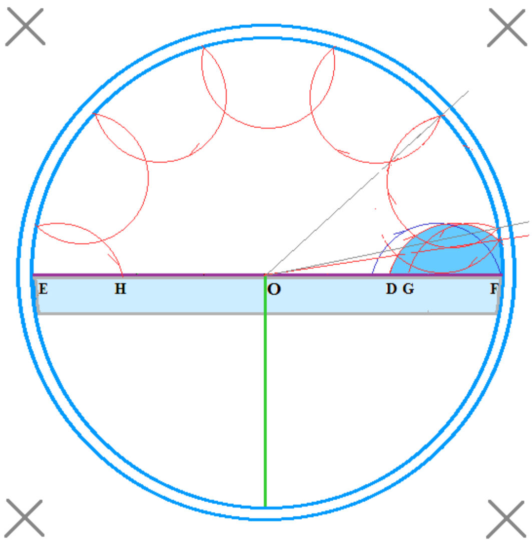




A-directly-A & B-directly-B B-glass-B

No electron migration between A and B due to these trajectories.





A-directly-B
(from PO to OQ, etc., red)
PO/EO = 39/70 = 0.56 (PO=39)

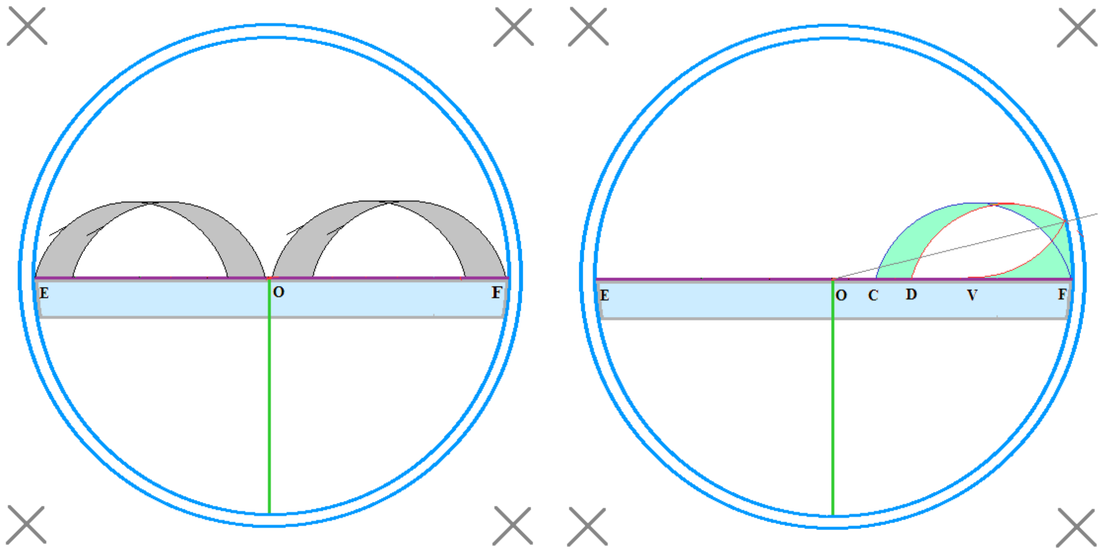
B-glass-A
(from G to H, etc., blue)
DF/OF = 33.7/70 = 0.48 (DF=33.7)

A-B 56% of the electrons of (15^{o}, u_p) emitted from A migrate to B.
B-A 48% of the electrons of (15^{o}, u_p) emitted from B migrate to A.

For all the electrons of (15^{o}, u_p), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current

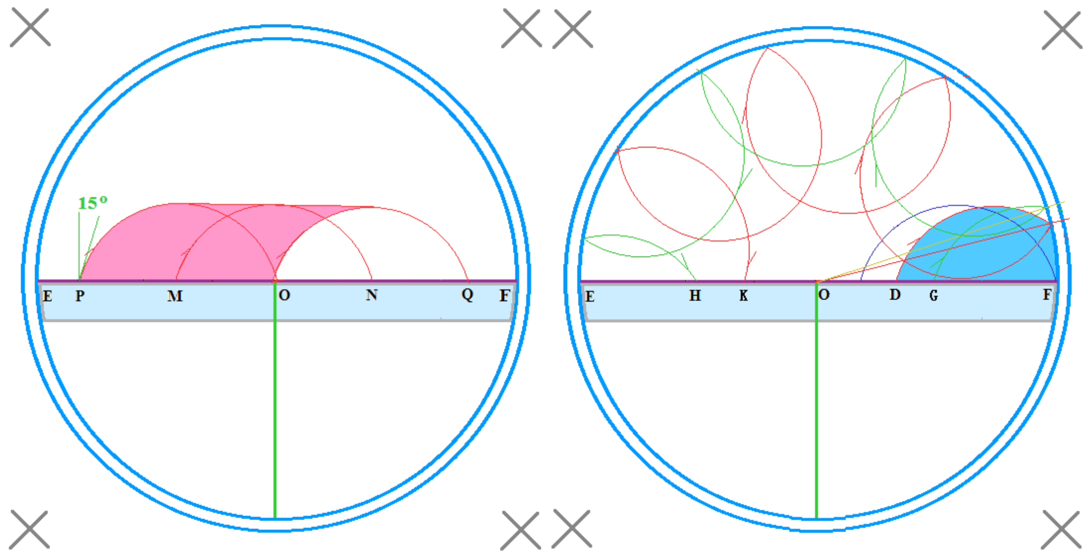
$$\{ (A\text{-}B) - (B\text{-}A) \}_{15^{o}u_p} = 0.56 - 0.48 = 0.08$$
$$D_{15^{o}}(u_p) = \{ (A\text{-}B) - (B\text{-}A) \}_{15^{o}u_p} \cos 15^{o} = 0.08 \times 0.9659 \approx 0.08$$

(3) Fig 3-3 $\theta = 15^{o}$ $u = 1.5u_p$ $R = 6\text{mm}$



A-directly-A & B-directly-B B-Glass-B

No electron migration between A and B due to these trajectories.



A-directly-B
(from PO to OQ, red)
PO/EO = 58/70 = 0.83 (PO=58)

B-glass-A
(from G to H, D to K, etc., blue)
DF/OF = 47/70 = 0.67 (DF = 47)

A-B 83% of the electrons of (15^{o}, $1.5u_p$) emitted from A migrate to B.

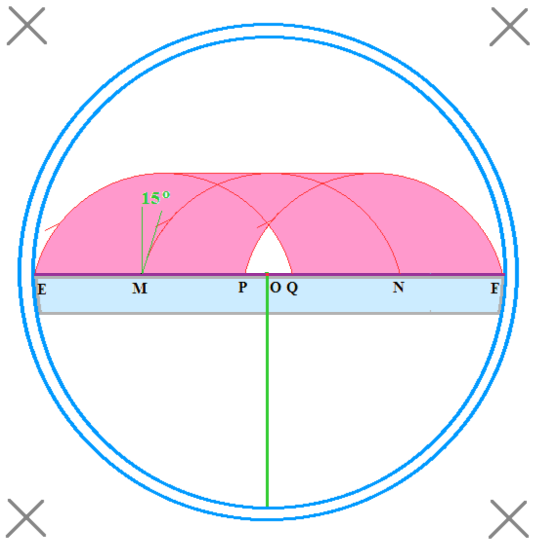
B-A 67% of the electrons of (15^{o}, $1.5u_p$) emitted from B migrate to A.

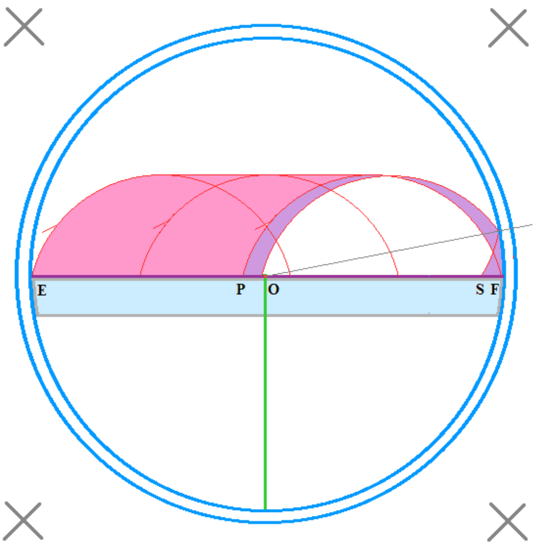
For all the electrons of (15^{o}, u_p), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current

$$\{ (A\text{-}B) - (B\text{-}A) \}_{15^{o} 1.5u_p} = 0.83 - 0.67 = 0.16$$

$$D_{15^{o}}(1.5u_p) = \{ (A\text{-}B) - (B\text{-}A) \}_{15^{o} 1.5u_p} \cos 15^{o} = 0.16 \times 0.9659 \approx 0.16$$

(4) Fig 3-4 $\theta = 15^{o}$ $u = 2u_p$ R = 8mm



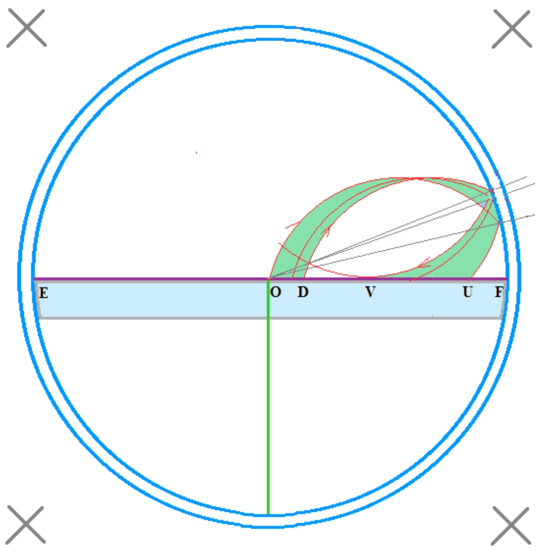


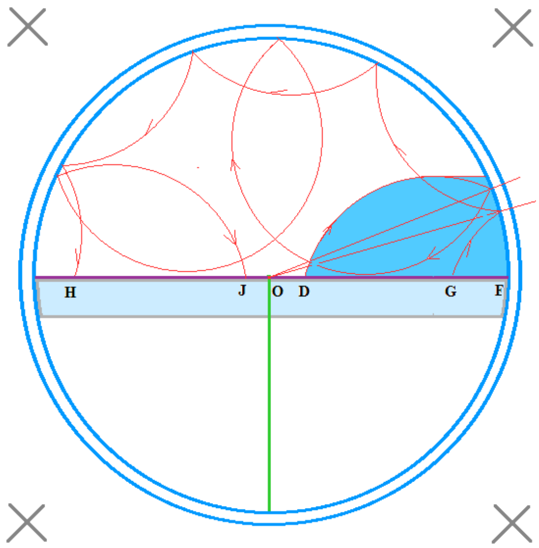
A-directly-B 64/70 = 0.91
(from E to Q, M to N, P to F, red)

A-glass-B 6/70 = 0.09
(from PO to FS, violet)

A-B = A-directly-B + A-glass-B (64 + 6 =70, red + violet) 70/70 = 1.00

A-B 100% of the electrons of (15^{o}, $2u_p$) emitted from A migrate to B.





B-glass-B
(from O to U, D to V, etc., green)
No electron migration

B-glass-A
(from D to J, G to H, etc., blue)
DF/OF=59.5/70 = 0.85

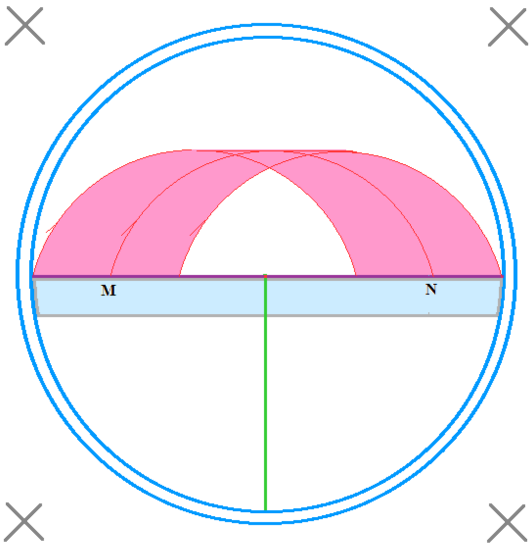
B-A 85% of the electrons of (15^{o}, $2u_p$) emitted from B migrate to A.

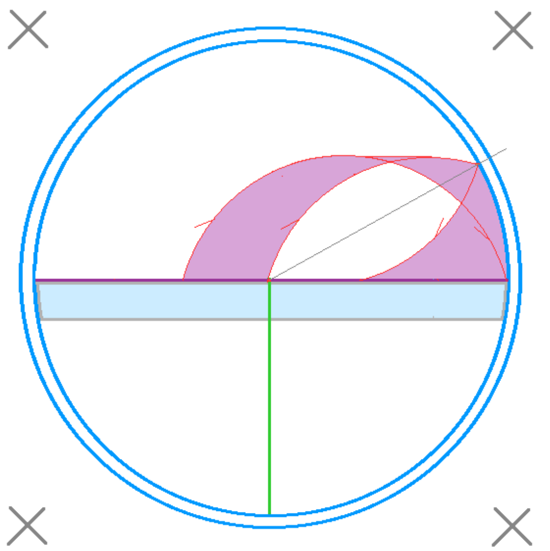
For all the electrons of (15^{o}, $\mathbf{2}u_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current

$$\{ (A\text{-}B) - (B\text{-}A) \}_{15^{o}\, 2u_p} = 1.00 - 0.85 = 0.15$$

$$D_{15^{o}}(2u_p) = \{ (A\text{-}B) - (B\text{-}A) \}_{15^{o}\, 2u_p} \cos 15^{o} = 0.15 \times 0.9659 \approx 0.15$$

(5) Fig 3-5 $\theta = 15^{\circ}$ $u = 2.5u_p$ R = 10mm

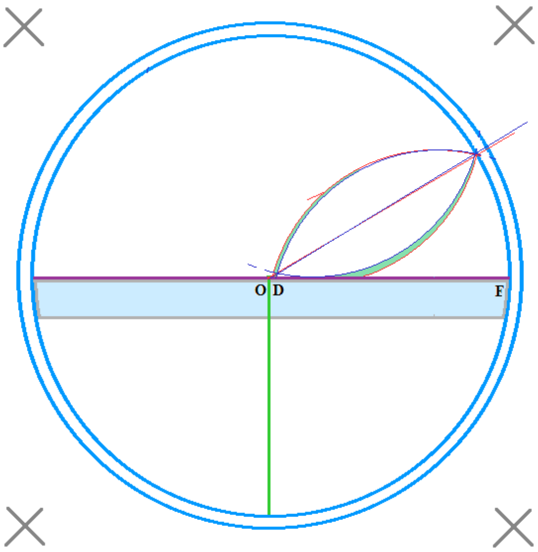


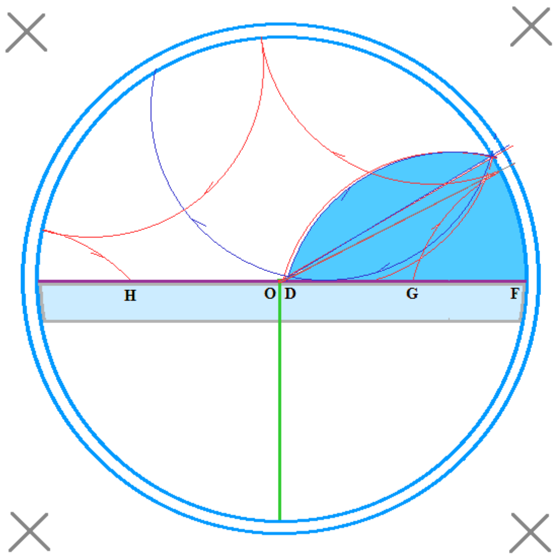


A-directly-B (45/70) (red) A-glass-B (25/70) (violet)

A-B (total) 70/70 = 1.00 (red +violet, 45 + 25 = 70)

A-B 100% of the electrons of (15°, $2.5u_p$) emitted from A migrate to B.





B-glass-B 1/70 = 0.014
(OD = 1, OF = 70)
No electron migration between A and B due to these trajectories.

B-glass-A
(DF = 69, OF =70)
(from G to H, etc., blue)
DF/OF = 69/70 = 0.986

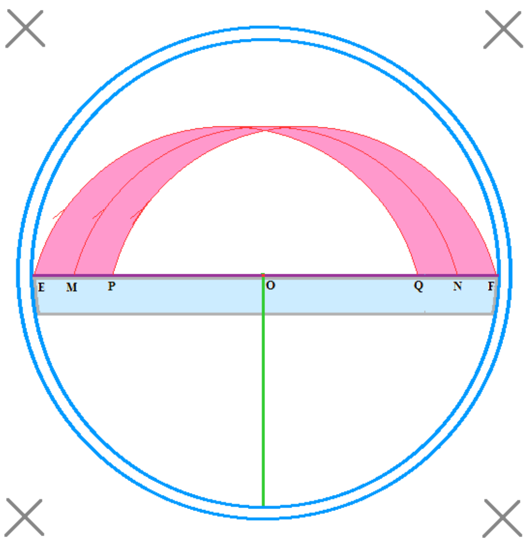
B-A 98.6% of the electrons of (15°, $2.5u_p$) emitted from B migrate to A.

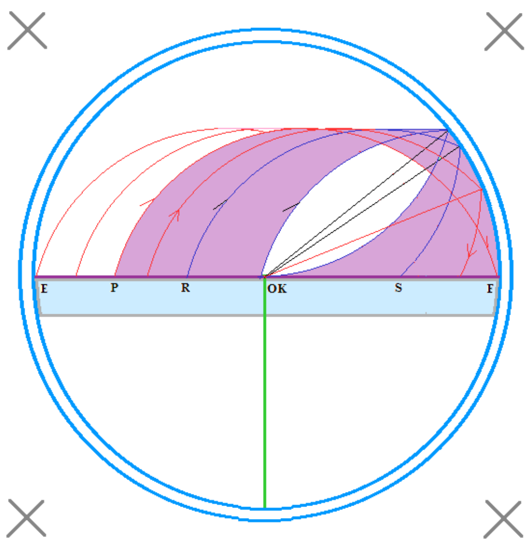
For all the electrons of (15°, $2.5u_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current

$$\{ (A\text{-}B) - (B\text{-}A) \}_{15^{\circ}\ 2.5u_p} = 1.00 - 0.986 = 0.014$$

$$D_{15^{\circ}}(2.5u_p) = \{ (A\text{-}B) - (B\text{-}A) \}_{15^{\circ}\ 2.5u_p} \cos 15^{\circ} \approx 0.01$$

(6) Fig 3-6 θ = 15^{o} $u = 3u_p$ R = 12mm





A-directly-B

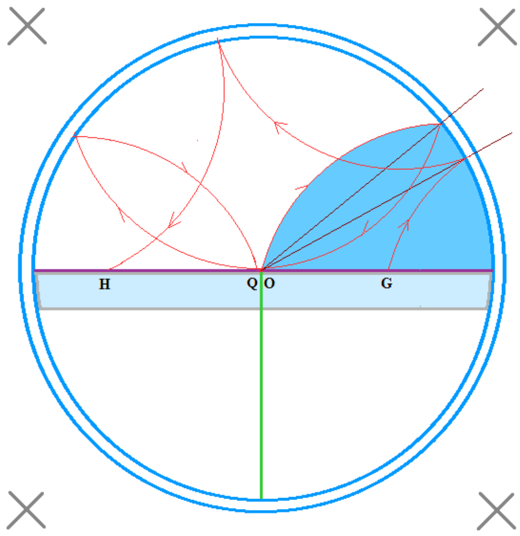
(from E to Q, M to N, P to F, etc., red)

A-glass-B

(from P to F, R to S, O to K, etc., violet)

A-B (total) 70/70 = 1.00 (EP + PO = 20 + 50 = 70)

A-B 100% of the electrons of (15^{o}, $3u_p$) emitted from A migrate to B.



B-glass-A

(from G to H, O to Q, etc., blue)

70/70 = 1.00

B-A 100% of the electrons of (15^{o}, $3u_p$) emitted from B migrate to A.

For all the electrons of (15^{o}, $3u_p$), migration A-B equals B-A, and their contribution to the output current cancel each other.

$$\{ \text{(A-B) - (B-A)} \}_{15^{o}3u_p} = 1.00 - 1.00 = 0$$

$$D_{15^{o}}(3u_p) = \{ \text{(A-B) - (B-A)} \}_{15^{o}3u_p} \times \cos 15^{\circ} = 0$$

Every 100 + 100 electrons (1.00 = 100% from A, 1.00 = 100% from B) of exiting angle $\theta = 15^o$ and of different speeds contribute to the electron migration between A and B differently, as list in Tab 3 (1) ($cos15^o$ = 0.9659),

speed *u*	$\{(A\text{-}B) - (B\text{-}A)\}_{15^o}$	$D_{15^o}(u) = \{(A\text{-}B) - (B\text{-}A)\}_{15^o} \cos\theta$
Fig 3-1 $u = 0.5u_p$	0.271 – 0.235 =0. 04	0.2618 - 0.2270 = 0.0348
Fig 3-2 $u = u_p$	0.56 – 0.48 = 0. 08	0.5409 – 0.4636 = 0.0773
Fig 3-3 $u = 1.5u_p$	0.83 – 0.69 = 0.16	0.8017 – 0.6665 = 0.1352
Fig 3-4 $u = 2u_p$	1.00 – 0.85 = 0.15	0.9659 – 0.8210 = 0.1449
Fig 3-5 $u = 2.5u_p$	1.00 – 0.986=0.014	0.9659 – 0.9524 = 0.0135
Fig 3-6 $u = 3u_p$	1.00 – 1.00 = 0	0.9659 – 0.9659 = 0
$u = 3.5u_p$	1.00 - 1.00 = 0	0.9659 – 0.9659 = 0
$u = 4.5u_p$	1.00 - 1.00 = 0	0.9659 – 0.9659 = 0

Tab 3 (1) Contributions of electrons ($\theta = 15^o$) of different speeds

Fig 3 (1) is the corresponding graph.

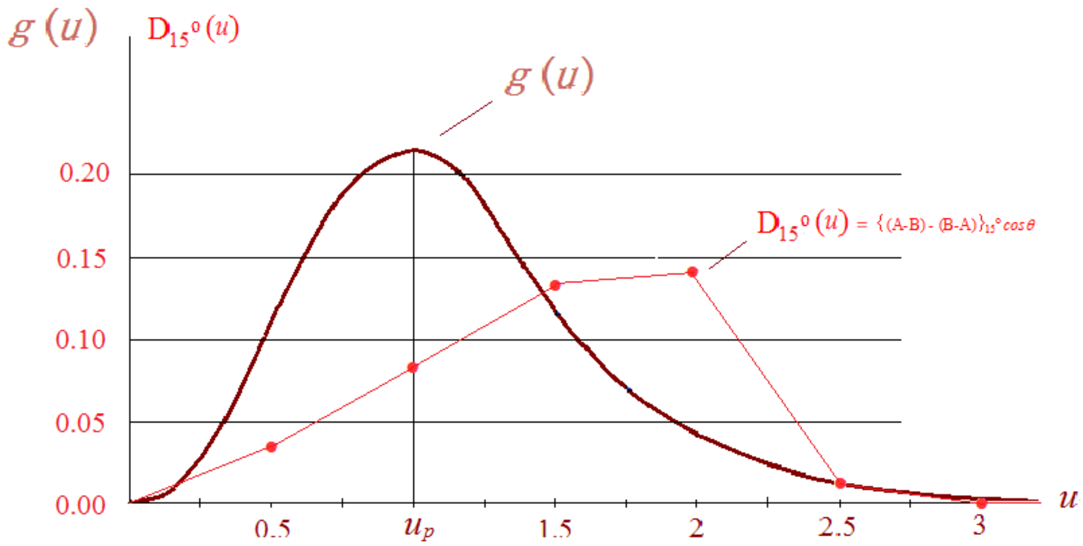


Fig 3 (1) Graph of the contributions of $\theta = 15^o$ with different speeds

Take Maxwell's speed distribution $g(u)$ into account to derive the actual contributions of the thermal electrons of $\theta = 15^{o}$ and of different speed ranges, i.e., $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{15^{o}}(u) \varDelta u \sim u$, as shown in table 3(2).

Speed range $\varDelta u$	$\frac{\Delta N_{(u_i \sim u_{i+1})}}{N}$	$\{(A\text{-}B)\text{-}(B\text{-}A)\}_{15^{o}} \cos\theta$	$\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{15^{o}}(u) \varDelta u$
$0.00 \sim 0.25u_p$	$A_0 \approx 0.004$	0.2618-0.2270= 0.0348	0. 0011−0. 0009=0. 0002
$0.25 \sim 0.75u_p$	$A_1 =$ 18. 74%	0.5409–0.4636=0.0773	10.1365-8.6879=1.4486
$0.75 \sim 1.25u_p$	$A_2 =$ 39. 83%	0.8017–0.6665=0.1352	31.9317-26.5466=5.3851
$1.25 \sim 1.75u_p$	$A_3 =$ 26. 71%	0.9659–0.8210=0.1449	25.7992-21.9289=3.8703
$1.75 \sim 2.25u_p$	$A_4 =$ 8. 82%	0.9659–0.9524=0.0135	8.5192-8.4002=0.1190
$2.25 \sim 2.75u_p$	$A_5 =$ 1. 58%	0.9659 – 0.9659 = 0	1.5261-1.5261= 0
$2.75 \sim 3.25u_p$	$A_6 =$ 0. 16%	0.9659 – 0.9659 = 0	0.1545-0.1545= 0
$3.25 \sim 3.75u_p$	A_7=0.0096%	0.9659 – 0.9659 = 0	0.0093-0.0093=0
$3.75u_p \sim \infty$	A_8=0.0003%	0.9659 – 0.9659 = 0	0.00029-0.00029=0
$\sum_u \frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{15^{o}}(u) \varDelta u$ = 78.0799-67.2541=10.8232			

Tab 3 (2) Actual contributions of electrons of $\theta = 15^{o}$ and different speeds

Fig 3 (2) is the corresponding graph

$$\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{15^o}(u) .$$

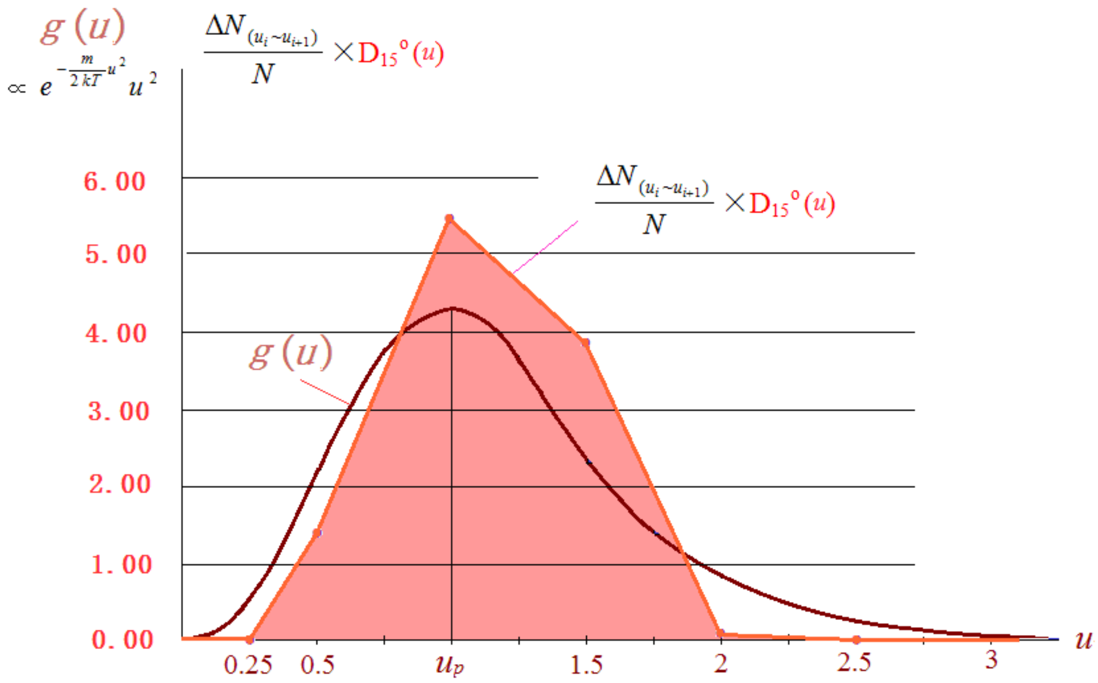
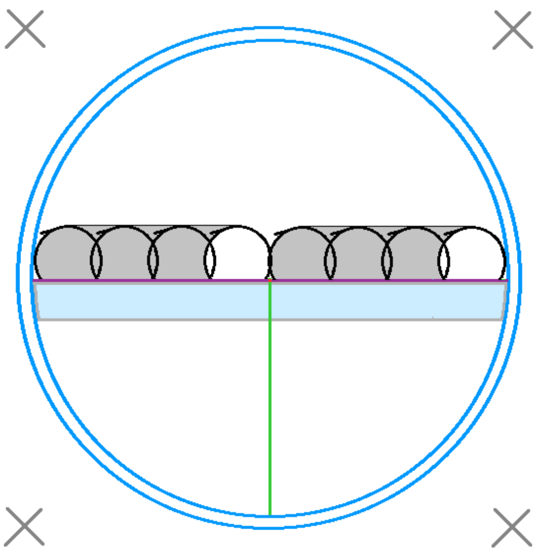


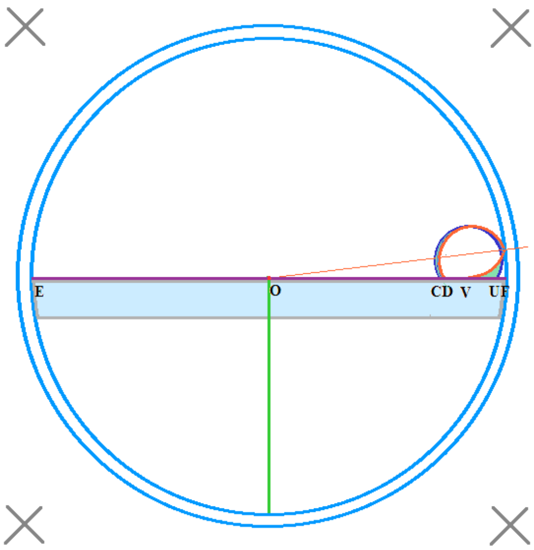
Fig 3 (2) Graph of $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{15^o} \sim u$

\

4. Trajectories of electrons of θ = - 30° and different speeds

(1) Fig 4-1 θ = - 30° $u = 0.5u_p$ R = 2mm (B = 1.34 gauss)

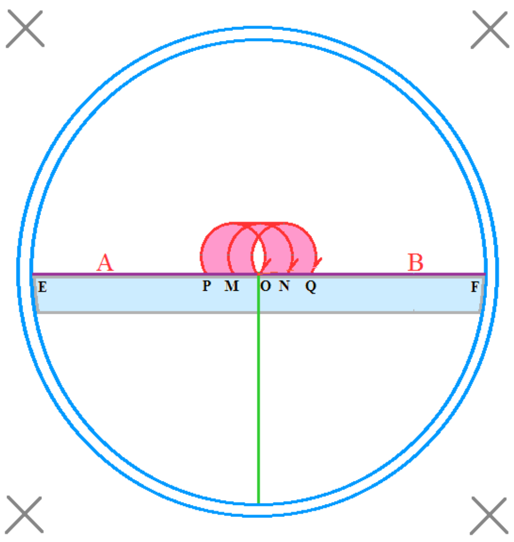


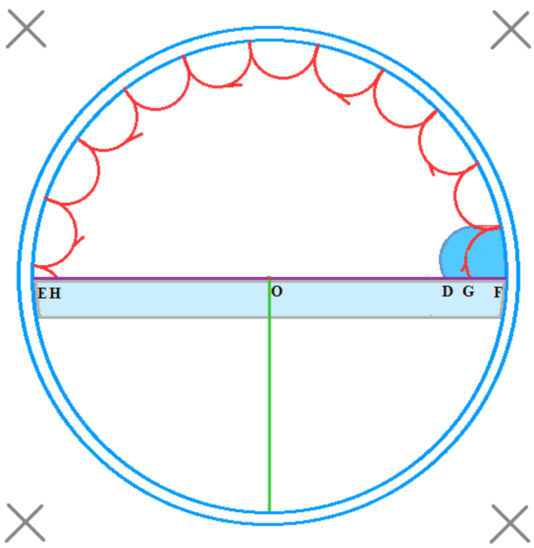


A-directly-A B-directly-B (grey)

B-glass-B 1/70 = 0.014 (green) (CD = 1, OF =70) (from CD to UV)

No electron migration between A and B due to these trajectories.





A-directly-B
(from PO to OQ, red)
(PO = 16.5, EO = 70)
16.5/70 = 0.236

B-Glass-A
(from G to H, etc., blue)
(DF =18, OF = 70)
18/70 = 0.257

A-B 24% of the electrons of (- 30°, $0.5u_p$) emitted from A migrate to B.

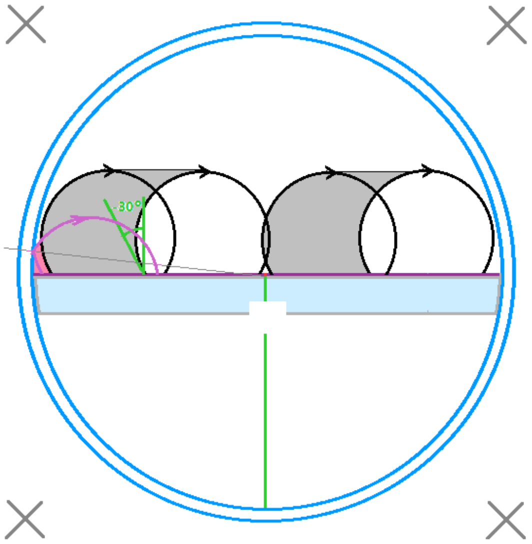
B-A 26% of the electrons of (- 30°, $0.5u_p$) emitted from B migrate to A.

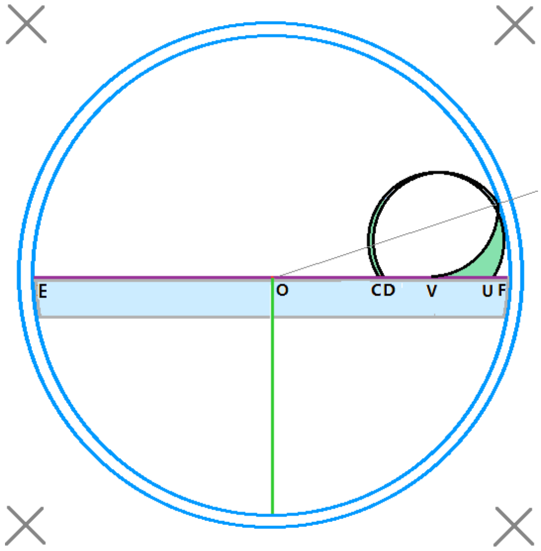
For all the electrons of (-30°, $0.5u_p$), migration A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

$$\{(A\text{-}B) - (B\text{-}A)\}_{-30^\circ\, 0.5u_p} = 0.236 - 0.257 = -0.021$$

$$D_{-30^\circ}(0.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-30^\circ\, 0.5u_p} \cos 30^\circ = -0.021 \times 0.8660 \approx -0.02$$

(2) Fig 4-2 θ = - 30° $u = u_p$ R = 4mm ($\cos$-30° = 0.8660)

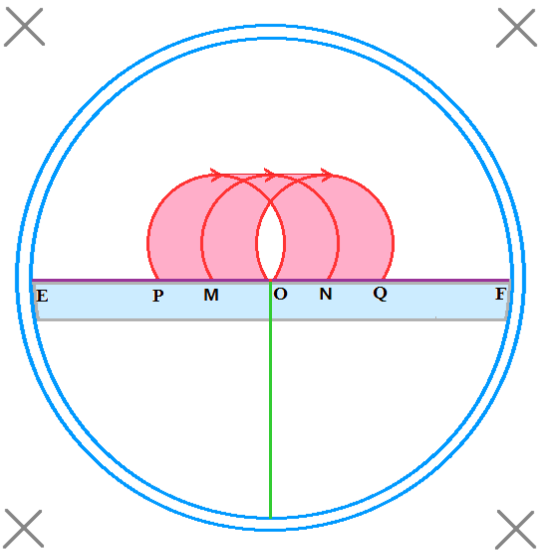


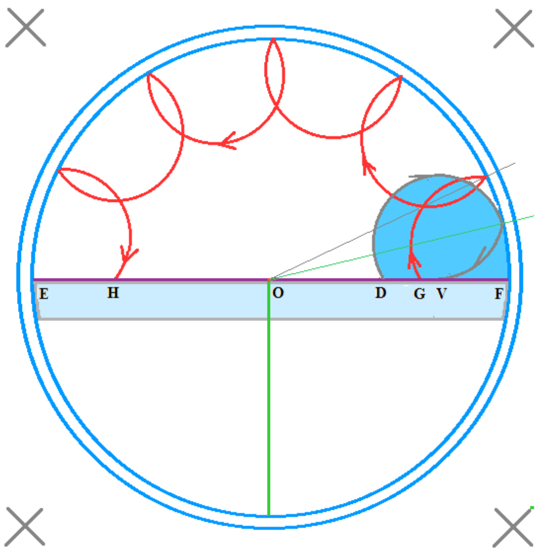


A-glass-A (violet) A-directly-A (grey) B-directly-B (grey)

B-glass-B
CD/OF = 1.5/70 = 0.021 (green)

No electron migration between A and B due to these trajectories.





A-directly-B
(PO =34) (from PO to OQ)
PO/EO = 34/70 = 0.49

B-Glass-A
(DF = 37) (from G to H, etc., blue)
DF/OF = 37/70 = 0.53

A-B 49% of the electrons of (- 30°, u_p) emitted from A migrate to B.

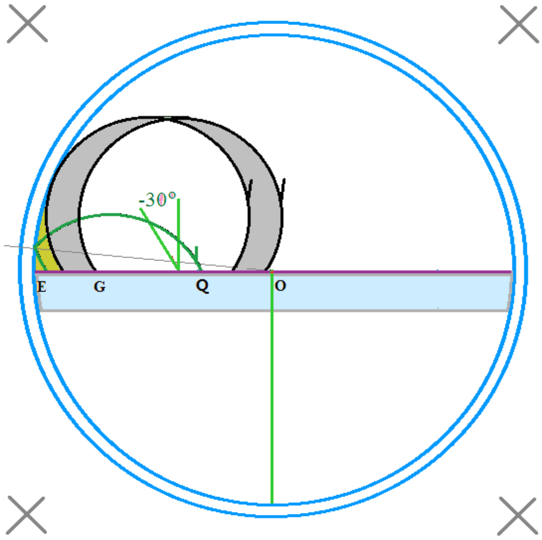
B-A 53% of the electrons of (-30°, u_p) emitted from B migrate to A.

For all the electrons of (-30°, u_p), migration A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

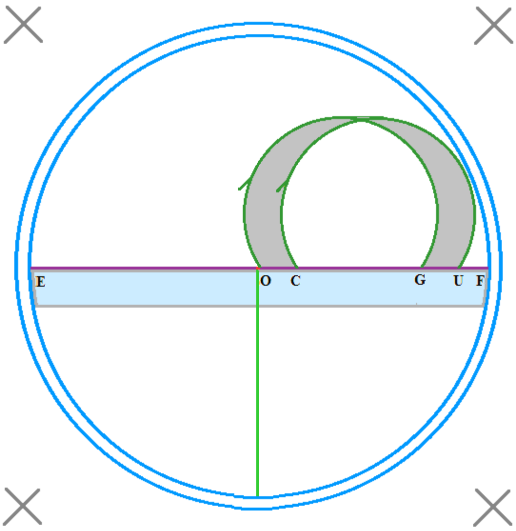
$$\{(A\text{-}B) - (B\text{-}A)\}_{-30^\circ u_p} = 0.49 - 0.53 = -0.04$$

$$D_{-30^\circ}(u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-30^\circ u_p} \cos 30^\circ = -0.04 \times 0.8660 \approx -0.03$$

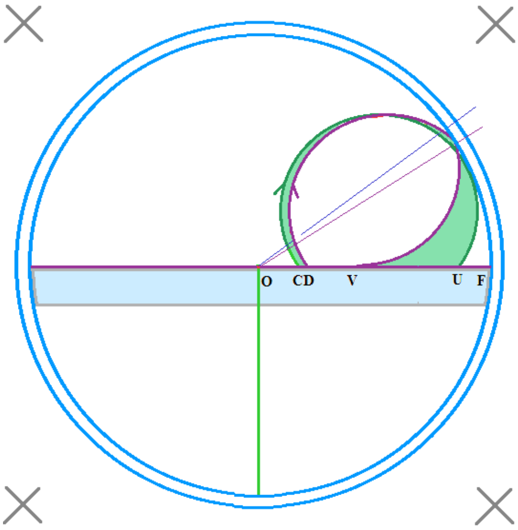
(3) Fig 4-3 θ = - 30° $u = 1.5u_p$ R = 6mm



(a) A-glass-A (green) + A-directly-A (grey)

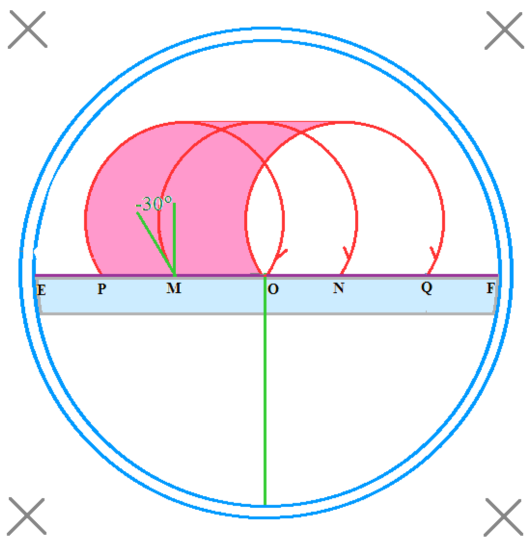


(b) B-directly-B
(grey)



(c) B-glass-B
(green)

No electron migration between A and B due to these trajectories.

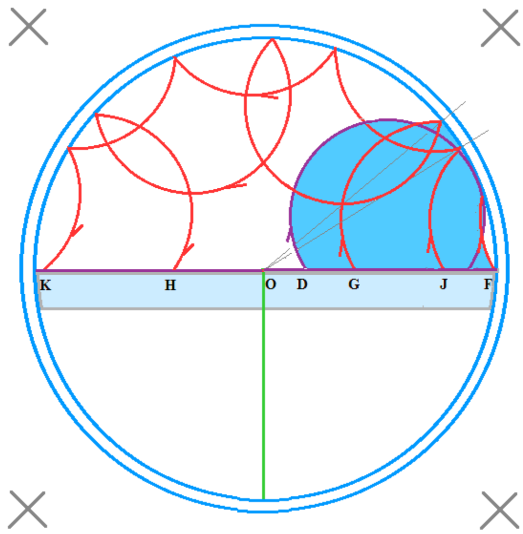


(d) A-directly-B

(PO = 50, EO =70) (from PO to OQ, red)

50/70 = 0.71

A-B 71% of the electrons of (- 30°, 1.5u_p) emitted from A migrate to B.



(e) B-glass-A

(from G to H, J to K, etc., blue) (DF = 55, OF = 70)

55/70 = 0.79

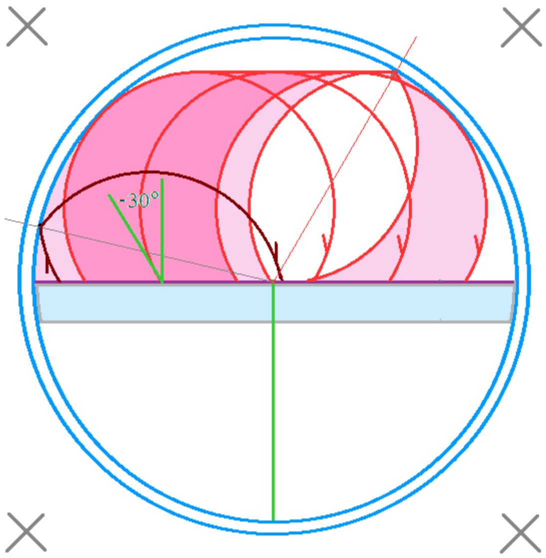
B-A 79% of the electrons of (-30°, 1.5u_p) emitted from B migrate to A.

For all the electrons of (-30°, 1.5u_p), migration A-B is less than B-A, their difference is the corresponding contribution to the output current (negative).

$$\{(A\text{-}B) - (B\text{-}A)\}_{-30^o\ 1.5u_p} = 0.71 - 0.79 = -0.08$$

$$D_{-30^o}(1.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-30^o\ 1.5u_p} \times \cos 30^o = -0.08 \times 0.866 = -0.07$$

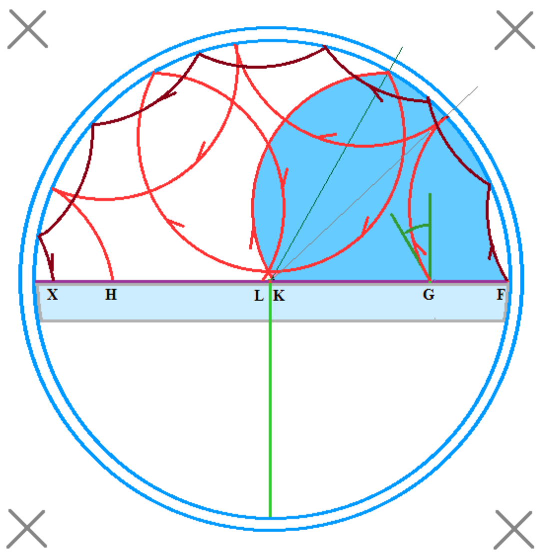
(4) Fig 4-4 θ = - 30° $u = 2\ u_p$ R = 8mm



A-glass-B + A-directly-B + A-glass-B
violet red violet

70/70 =1.00

A-B 100% of the electrons of (-30°, $2u_p$) emitted from A migrate to B.



B-glass-A
(from G to H, K to L, F to X, etc. blue)

70/70 = 1.00

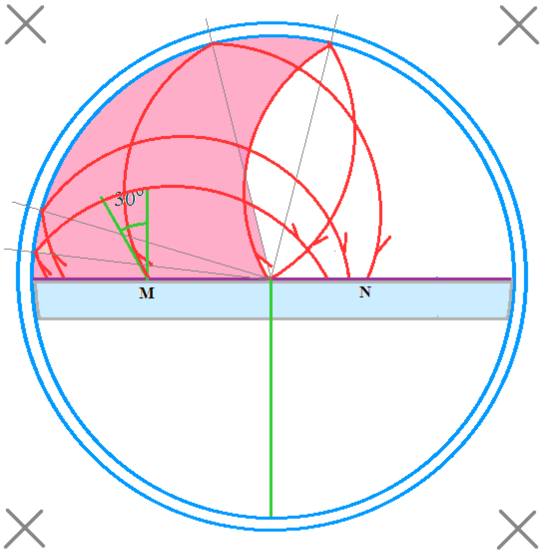
B-A 100% of the electrons (-30°, $2u_p$) emitted from B migrate to A.

For all the electrons of (-30°, $2u_p$), migration A-B and B-A cancel each other, no contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{-30^{o}\ 2u_p} = 1.00 - 1.00 = 0$$

$$D_{-30^{o}}(2u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-30^{o}\ 2u_p} \times \cos 30^{o} = 0$$

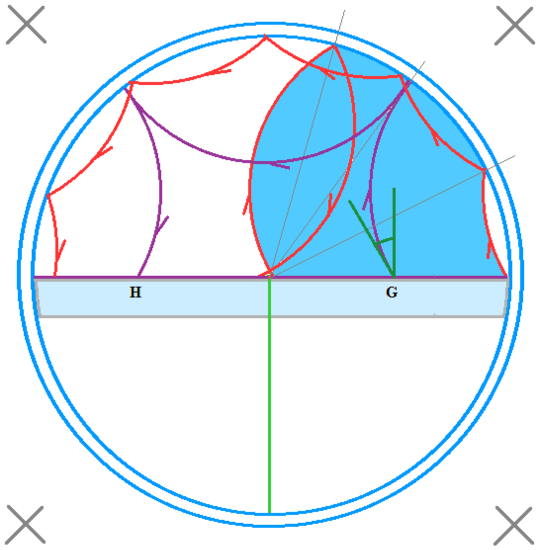
(5) Fig 4-5 θ = - 30° $u = 2.5u_p$ R = 10mm



A-glass-B (red)

70/70 = 1.00

A-B 100% of the electrons of (-30°, $2.5u_p$) emitted from A migrate to B.



B-glass-A (blue)

70/70 = 1.00

B-A 100% of the electrons of (-30°, $2.5u_p$) emitted from B migrate to A.

For all the electrons of (-30°, $2.5u_p$), migration A-B and B-A cancel each other, no contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{\theta = -30^\circ\, u = 2.5u_p} = 1.00 - 1.00 = 0$$

$$D_{-30^\circ}(2.5u_p) = 0 \times \cos 30^\circ = 0$$

Every 100 + 100 electrons (from A 1.00 = 100% and from B 1.00 = 100%) of exiting angle $\theta = -30^{o}$ and of different speeds contribute to the electron migration between A and B differently, as list in Tab 4 (1). Fig 4 (1) is the corresponding graph ($cos\text{-}30^{o} = 0.8660$).

$$D_{-30^{o}}(u) = \{(A\text{-}B) - (B\text{-}A)\}_{\theta = -30^{o}} \cos\theta \sim u .$$

speed *u*	$\{(A\text{-}B) - (B\text{-}A)\}_{-30^{o}}$	$D_{-30^{o}}(u)$
Fig 4-1 $u = 0.5u_p$	0.235 - 0.257 = - 0.022	0.2035 – 0.2226 = - 0.0191
Fig 4-2 $u = u_p$	0.49 – 0.54 = - 0.04	0.4243 – 0.4676 = - 0.0433
Fig 4-3 $u = 1.5u_p$	0.71 - 0.79 = - 0.08	0.6149 – 0.6814 = - 0.0665
Fig 4-4 $u = 2u_p$	1.00 – 1.00 = 0	0866 – 0.866 = 0
Fig 4-5 $u = 2.5u_p$	1.00 – 1.00 = 0	0866 – 0.866 = 0
Fig 4-6 $u = 3u_p$	1.00 – 1.00 = 0	0866 – 0.866 = 0

Tab 4 (1) $D_{-30^{o}}(u) = \{(A\text{-}B) - (B\text{-}A)\}_{\theta = -30^{o}} \cos\theta \sim u$

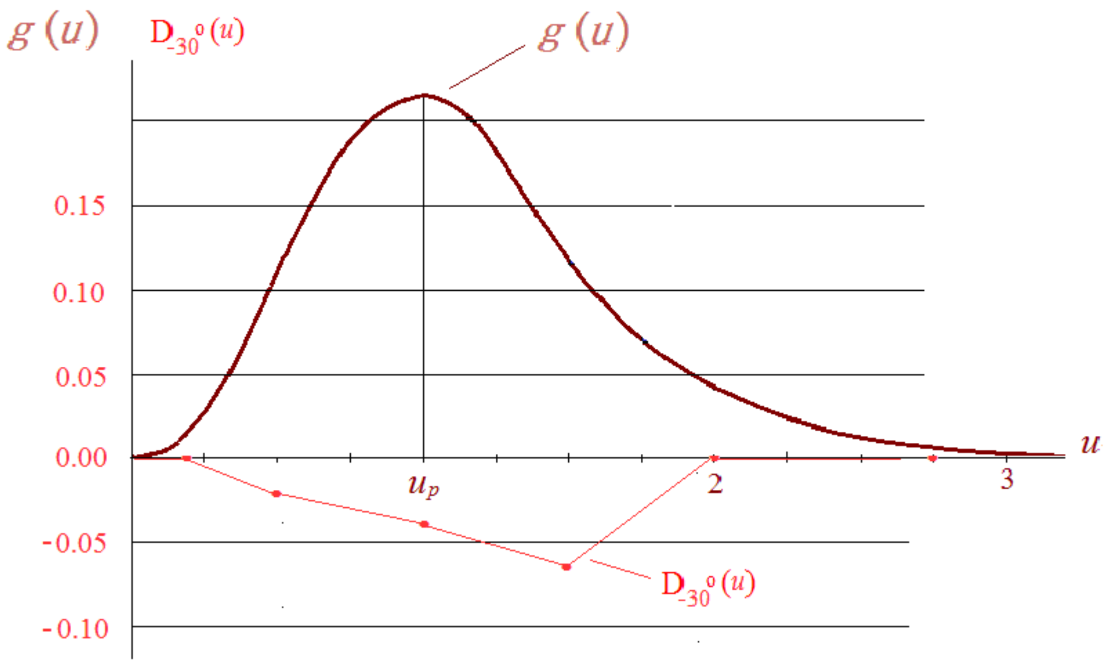


Fig 4 (1) Graph of contributions of electrons of (-30°, *u*)

Take Maxwell's speed distribution $g(u)$ into account to derive the actual contributions to the output current of electrons of $\theta = -30^{o}$ and of different speed ranges, i.e., $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{-30^{o}}(u) \varDelta u \sim u$, as shown in Tab 4(2).

Speed range $\varDelta u$	$\frac{\Delta N_{(u_i \sim u_{i+1})}}{N}$	$D_{-30^{o}}(u)$	$\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{-30^{o}}(u) \varDelta u$
$0.00 \sim 0.25 u_p$	$A_0 \approx$ 0. 004	$\approx$ 0	$\approx$ 0
$0.25 \sim 0.75 u_p$	$A_1 =$ 18. 74%	0.2035–0.2226=-0.0191	3.8136–4.1715=-0.3579
$0.75 \sim 1.25 u_p$	$A_2 =$ 39. 83%	0.4243–0.4676=- 0.0433	16.8999-18.6245=-1.7246
$1.25 \sim 1.75 u_p$	$A_3 =$ 26. 71%	0.6149–0.6814=-0.0665	16.4240-18.2002=-1.7762
$1.75 \sim 2.25 u_p$	$A_4 =$ 8. 82%	0866 – 0.866 = 0	7.6381 – 7.6381 = 0
$2.25 \sim 2.75 u_p$	$A_5 =$ 1. 58%	0866 – 0.866 = 0	1.3683 – 1.3683 = 0
$2.75 \sim 3.25 u_p$	$A_6 =$ 0. 16%	0866 – 0.866 = 0	0.1386 - 0.1386 = 0
$3.25 \sim 3.75 u_p$	$A_7 =$ 0.0096%	0866 – 0.866 = 0	0.0083 - 0.0083 = 0
$3.75 u_p \sim \infty$	$A_8 =$ 0.0003%	0866 – 0.866 = 0	0.00026 - 0.00026 = 0
$\sum_u \frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{-30^{o}}(u) \varDelta u$ = 46.291-50.1498=-3.8587			

Tab 4 (2) Actual contributions of electrons of $\theta = -30^{o}$ with different speed ranges

Fig 4(2) is the corresponding graph, the contributions of thermal electrons of θ = -30º to the output current with respect to the different speed u.

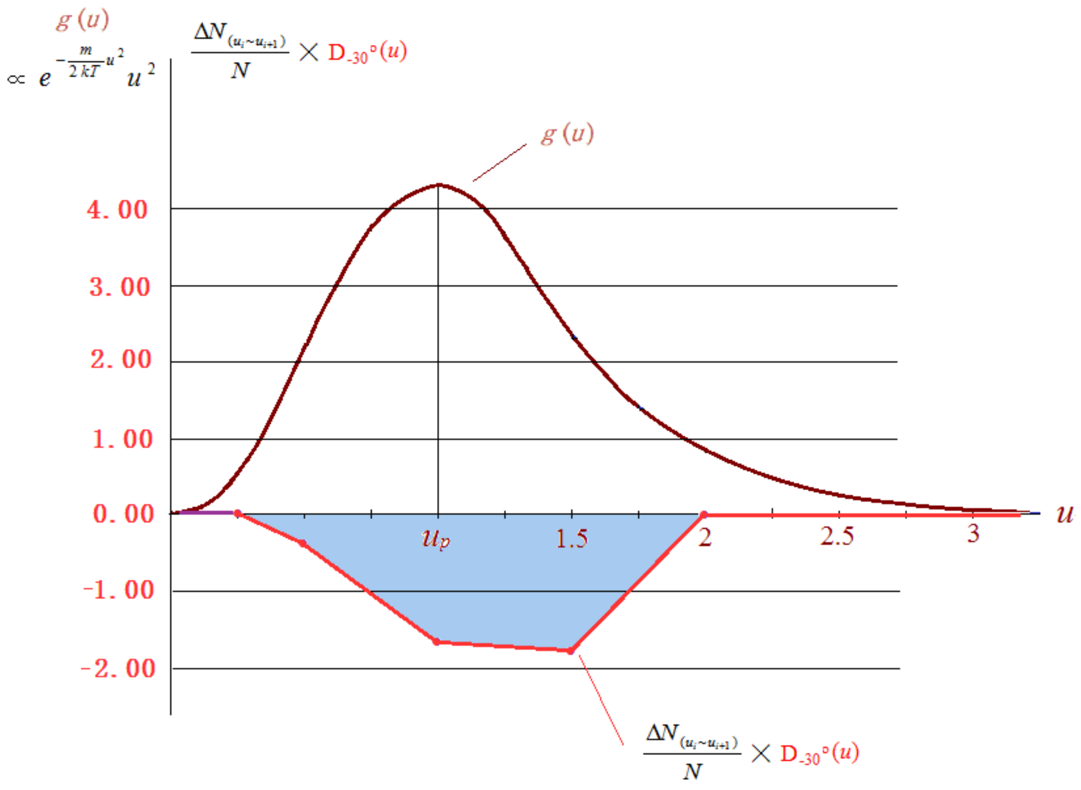
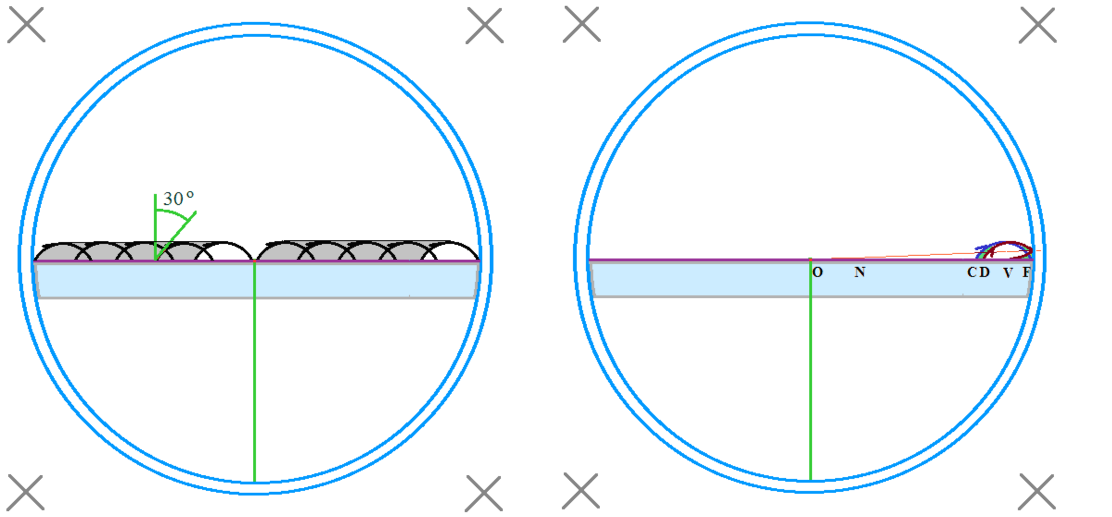


Fig 4 (2) Graph of $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-30^o} \sim u$.

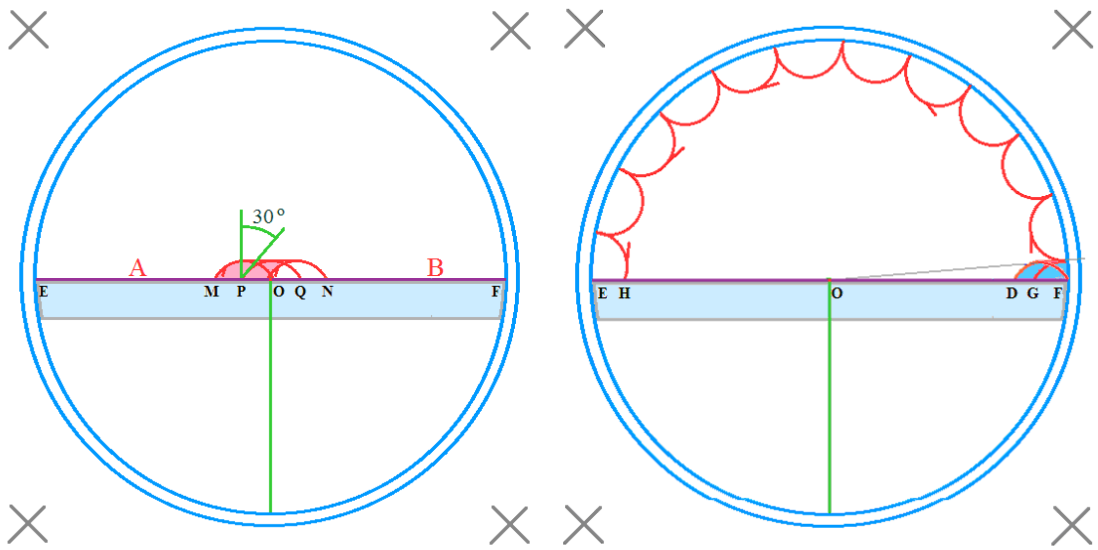
5. Trajectories of electrons of θ = 30° and different speeds

(1) Fig 5-1 $\theta = 30^o$ $u = 0.5u_p$ R = 2mm (B = 1.34 gauss)



A-directly-A & B-directly-B (grey) B-glass-B 2/70 =0.029 (CD = 2) (green)

No electron migration between A and B due to these trajectories.



A-directly-B (from MO to ON, red) B-glass-A (from G to H, etc., blue)

17.5/70 = 0.25 (MO = 17.5, EO = 70) 15.5/70 =0.22 (DF=15.5, OF=70)

A-B 25% of the electrons of (30°, $0.5u_p$) emitted from A migrate to B.

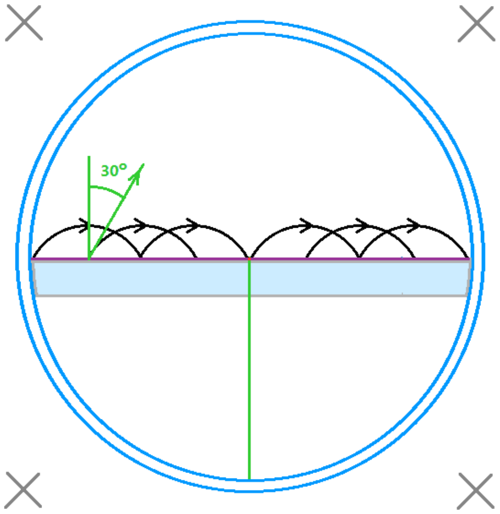
B-A 22% of the electrons of (30°, $0.5u_p$) emitted from B migrate to A.

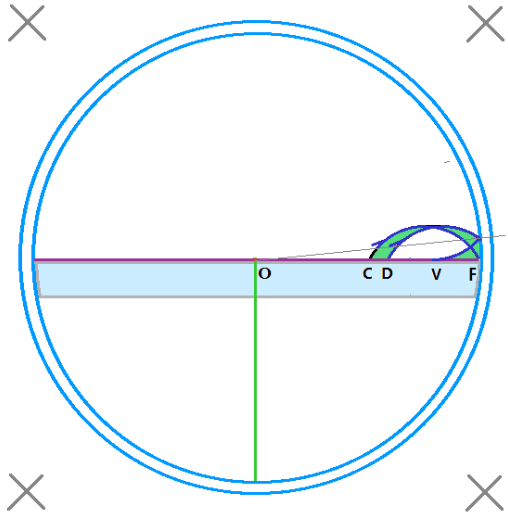
For all the electrons of (30°, $0.5u_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{30^o\,0.5u_p} = 0.25 - 0.22 = 0.03$$

$$D_{30^o}(0.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{30^o\,0.5u_p} \times cos30^o = 0.03 \times 0.8660 \approx 0.03$$

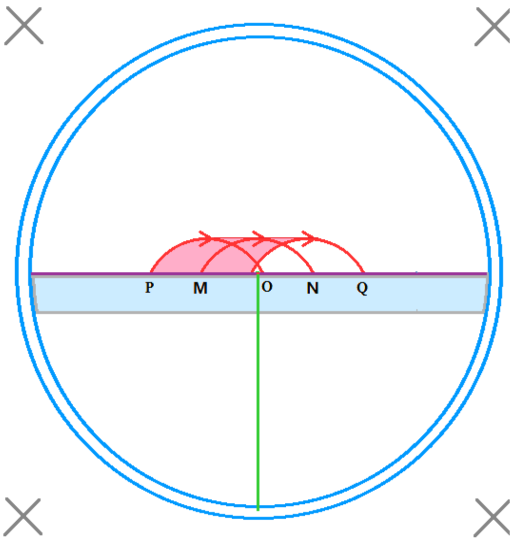
(2) Fig 5-2 $\theta = 30^{\circ}$ $u = u_p$ $R = 4\text{mm}$ ($\cos 30^{\circ} = 0.8660$)





A-directly-A & B- directly-B B-glass-B 5.5/70 = 0.08 (CD = 5.5)

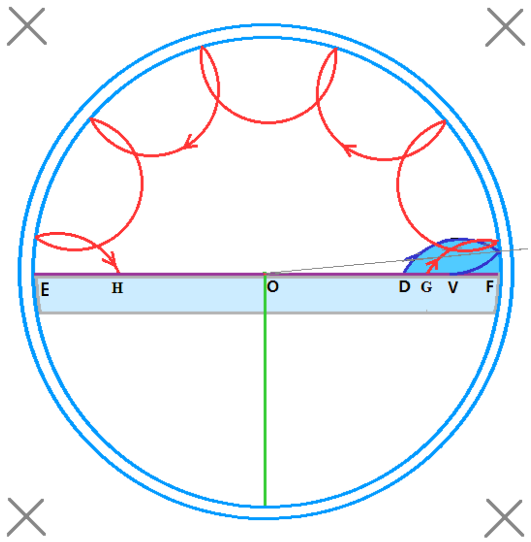
No electron migration between A and B due to these trajectories.



A-directly-B

(from PO to OQ，PO = 34.5, OE =70)

34.5/70 = 0.49 (red)



B-glass-A

(DF = 29, OF = 70，from G to H, etc.)

29/70 = 0.41 (blue)

A-B 49% of the electrons of (30°, u_p) emitted from A migrate to B.

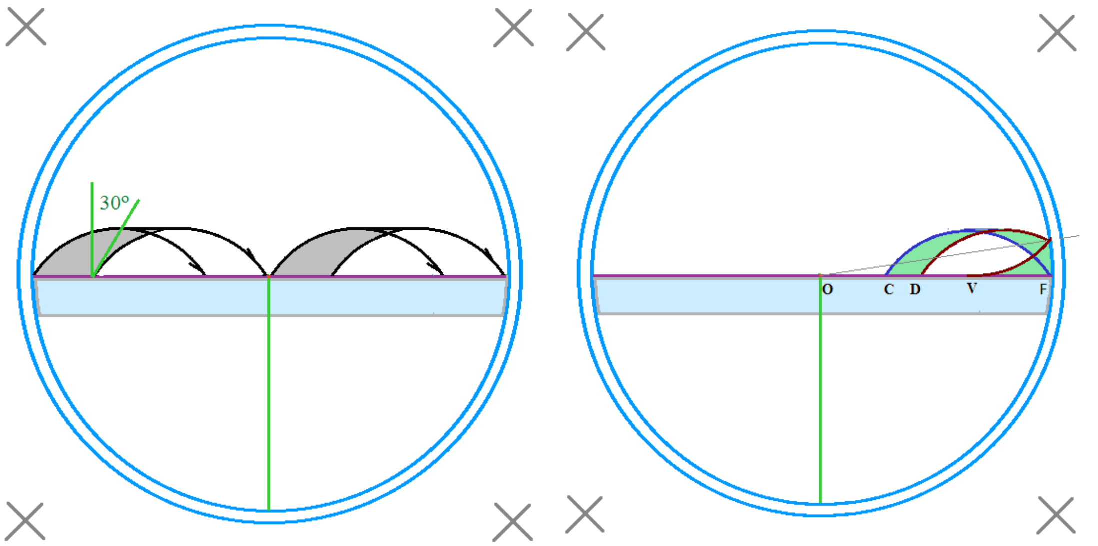
B-A 41% of the electrons of (30°, u_p) emitted from B migrate to A.

For all the electrons of (30°, u_p), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{30^{\circ}\, u_p} = 0.49 - 0.41 = 0.08$$

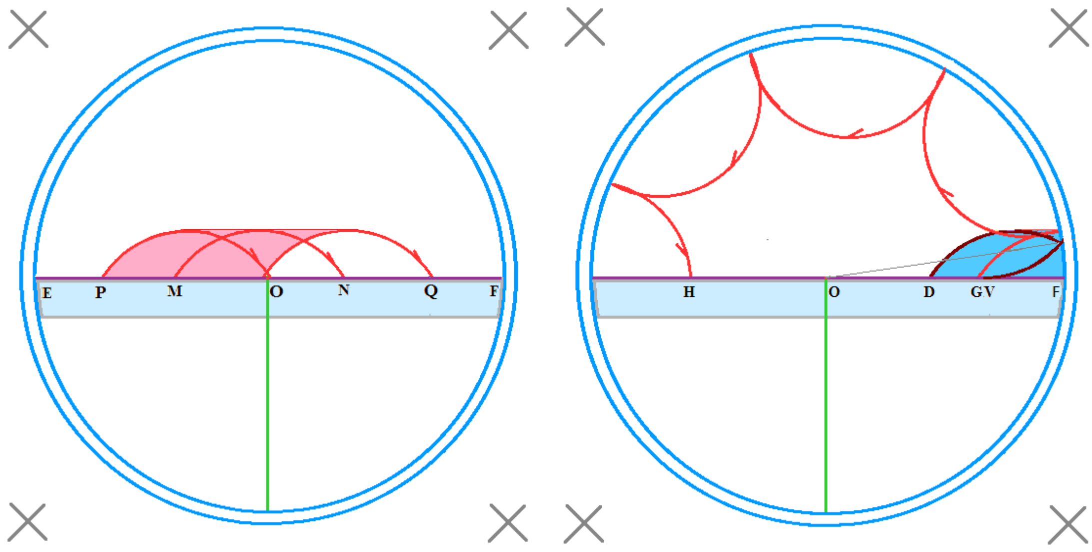
$$D_{30^{\circ}}(u_p) = 0.08 \times \cos\theta = 0.08 \times 0.866 = 0.069 = 0.07$$

(3) Fig 5-3 $\theta = 30^{o}$ $u = 1.5u_p$ $R = 6\text{mm}$



A-directly-A & B-directly-B B-glass-B (from CD to FV, CD = 11)

No electron migration between A and B due to these trajectories.



A-directly-B (from PO to OQ, red) (PO=51, EO=70) 51/70 = 0.729

B-glass-A (from G to H, etc., blue) (DF=40, OF=70) 40/70 = 0.57

A-B 73% of the electrons of (30°, 1.5u_p) emitted from A migrate to B.

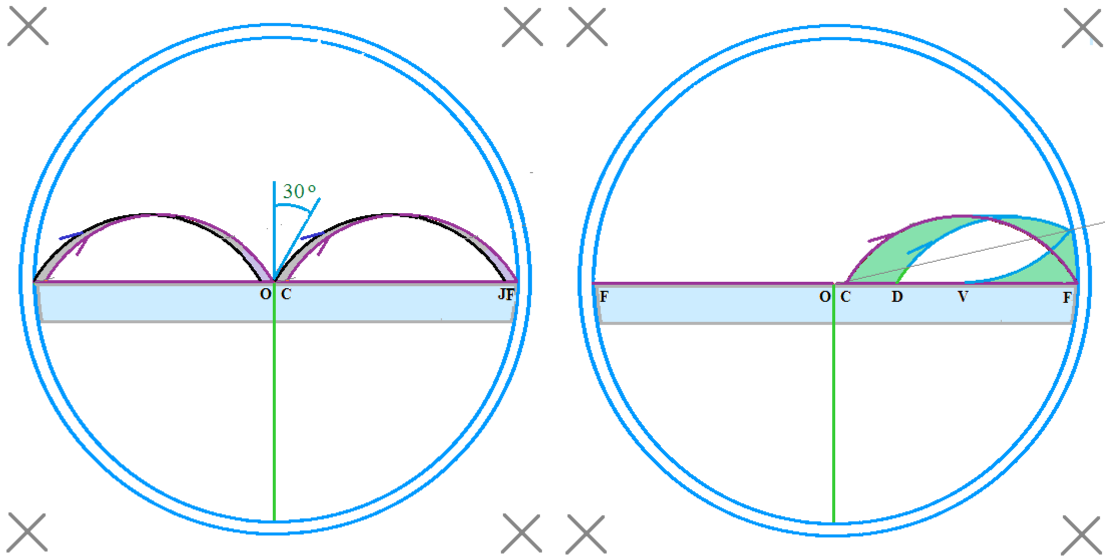
B-A 57% of the electrons of (30°, 1.5u_p) emitted from B migrate to A.

For all the electrons of (30°, 1.5u_p), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{30^{o}\,1.5u_p} = 0.73 - 0.57 = 0.16$$

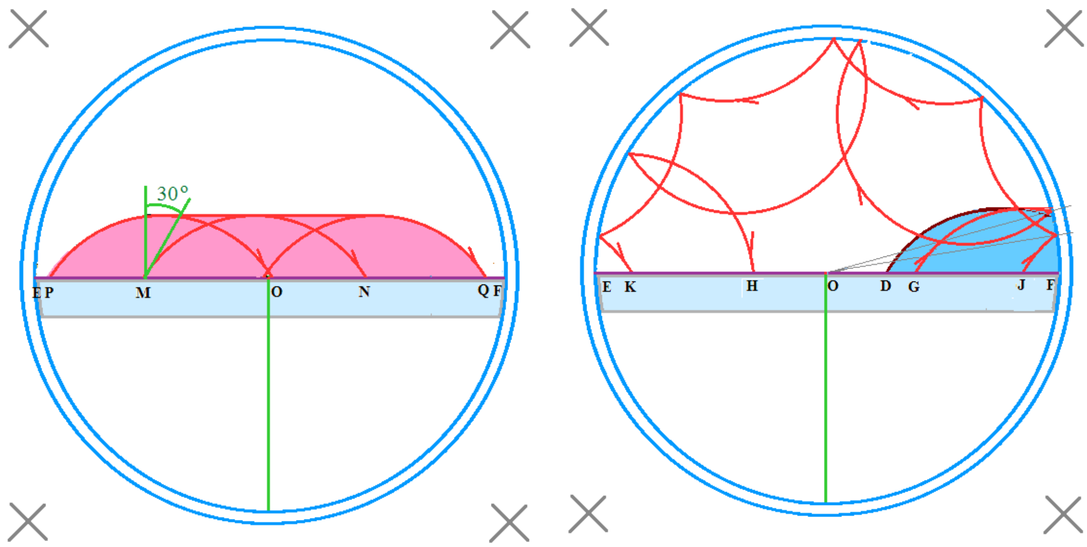
$$D_{30^{o}}(1.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{30^{o}\,1.5u_p} \times \cos\theta = 0.16 \times 0.8660 = 0.14$$

(4) Fig 5-4 $\theta = 30^o$ $u = 2u_p$ $R = 8mm$



A-directly-A & B-directly-B B-glass-B (CD = 15)

No electron migration between A and B due to these trajectories.



A-directly-B
(PO = 67.5, EO = 70)
(from PO to OQ, red)
PO/EO = 67.5/70 = 0.96

B-glass-A
(DF = 52.5, OF = 70)
(from G to H, J to K, etc., blue)
DF/OF = 52.5/70 =0.75

A-B 96% of the electrons of (30°, $2u_p$) emitted from A migrate to B.

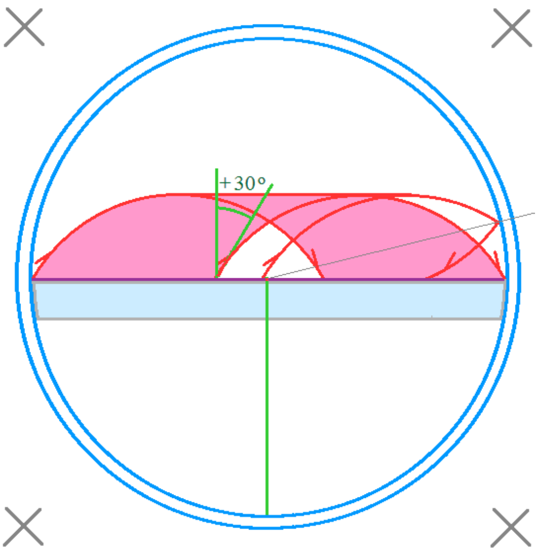
B-A 75% of the electrons of (30°, $2u_p$) emitted from B migrate to A.

For all the electrons of (30°, $2u_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

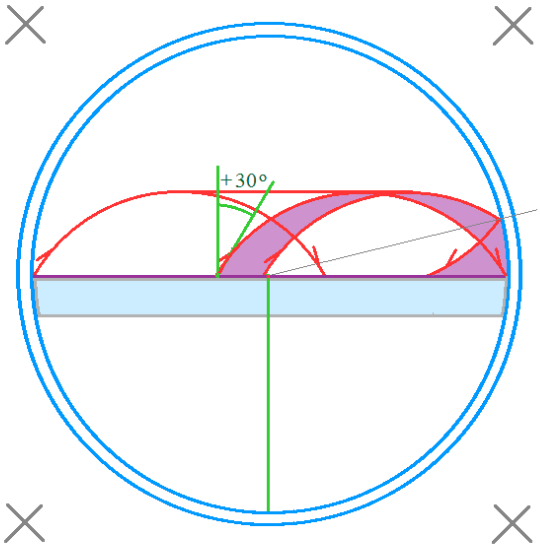
$$\{(A\text{-}B) - (B\text{-}A)\}_{30^o\,2up} = 0.96 - 0.75 = 0.21$$

$$D_{30^o}(2u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{30^o\,2up} \times \cos\theta = 0.21 \times 0.8660 = 0.18$$

(5) Fig 5-5 $\theta = 30^{o}$ $u = 2.5u_p$ R = 10mm



A-directly-B (red)

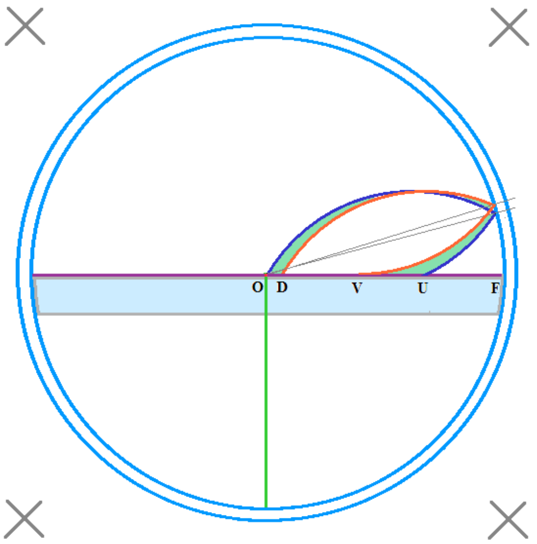


A-glass-B (violet)

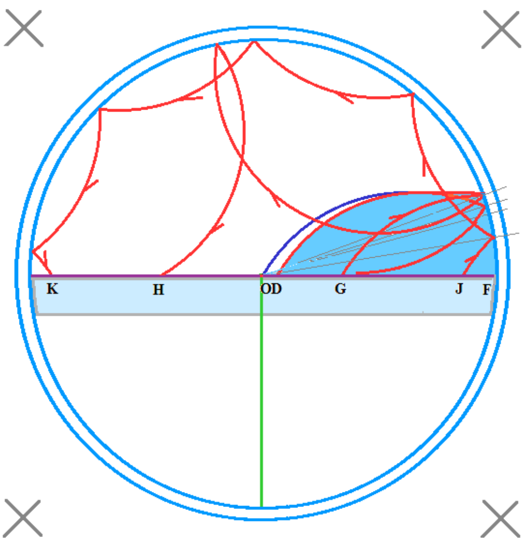
A-B = A-directly-B + A-glass-B EO = (red + violet) = 70

70/70 = 1.00

A-B 100% of the electrons of (30^{o}, $2.5u_p$) emitted from A migrate to B.



B-glass-B OD/OF = 4.5/70 = 0.06
No electron migration between A and B due to these trajectories.



B-glass-A (from G to H. J to K, etc., blue.)
(DF=65.5, OF =70)
DF/OF = 65.5/70 = 0.94

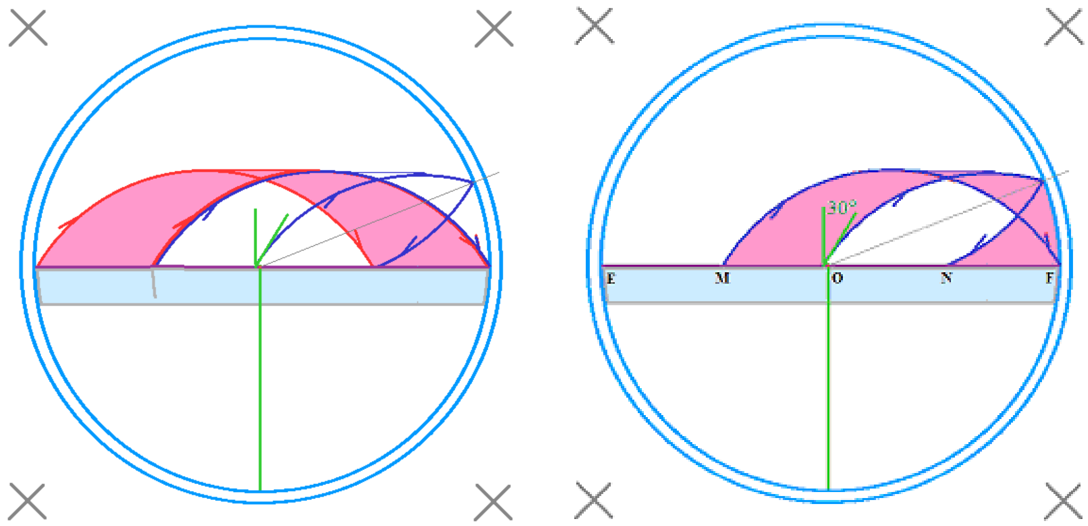
B-A 94% of the electrons of (30^{o}, $2.5u_p$) emitted from B migrate to A.

For all the electrons of (30^{o}, $2.5u_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{30^{o}\,2.5up} = 1.00 - 0.94 = 0.06$$

$$D_{30^{o}}(2.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{30^{o}\,2.5up} \times \cos\theta = 0.06 \times 0.866 = 0.05$$

(6) Fig 5-6 $\theta = 30^{o}$ $u = 3u_p$ R = 12mm

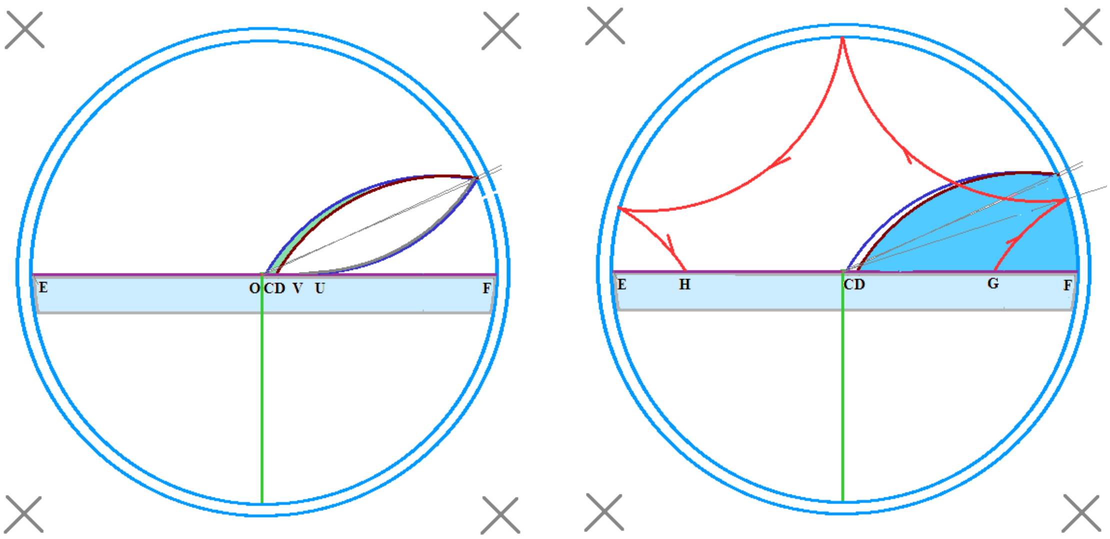


(a) A-directly-B (red) (b) A-glass-B (violet)

A-B = A-directly-B + A-glass-B (red + violet) = 70

70/70 = 1.00

A-B 100% of the electrons of (30^{o}, $3u_p$) emitted from A migrate to B.



(c) B-glass-B 3/70 = 0.04 (CD = 3)
No electron migration between A and B due to these trajectories.

(d) B-glass-A
(from G to H, etc., blue) (DF = 67)
67/70 = 0.96

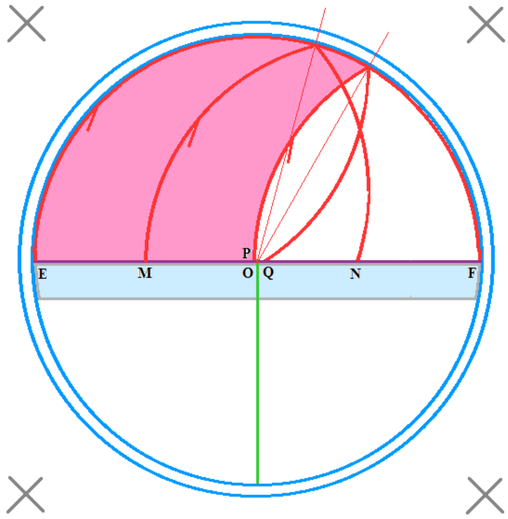
B-A 96% of the electrons of (30^{o}, $3u_p$) emitted from B migrate to A.

For all the electrons of (30^{o}, $3u_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{30^{o}\ 3u_p} = 1.00 - 0.96 = 0.04$$

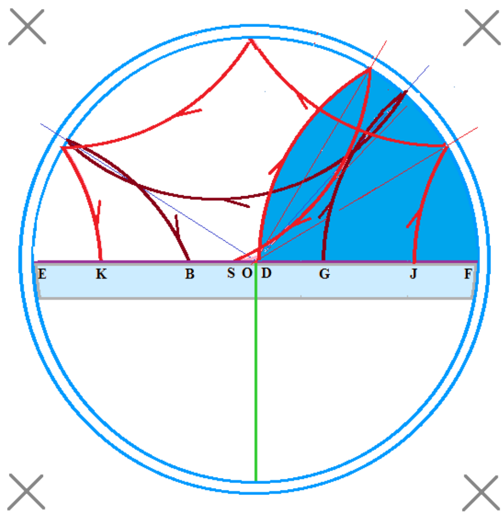
$$D_{30^{o}}(3u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{30^{o}\ 3u_p} \times \cos\theta = 0.04 \times 0.8660 = 0.03$$

(7) Fig 5-7 $\theta = 30^{o}$ $u = 3.5u_p$ $R = 18\text{mm}$



(a) A-glass-B (red)

A-B 100% of the electrons of (30^{o}, $3.5u_p$) emitted from A migrate to B.



(b) B-glass-A (blue)

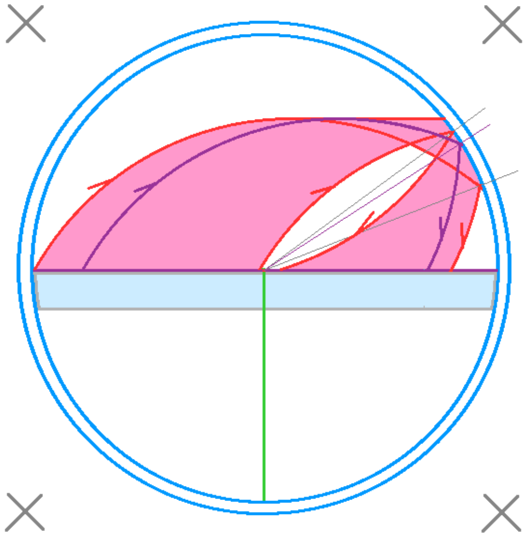
B-A 100% of the electrons of (30^{o}, $3.5u_p$) emitted from B migrate to A.

For all the electrons of (30^{o}, $3.5u_p$), migration A-B and B-A cancel each other, the corresponding contribution to the output current is zero.

$$\{(A\text{-}B) - (B\text{-}A)\}_{30^{o}\,3u_p} = 1.00 - 1.00 = 0$$

$$D_{30^{o}}(3u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{30^{o}\,3u_p} \times \cos\theta = 0.0 \times 0.8660 = 0.0$$

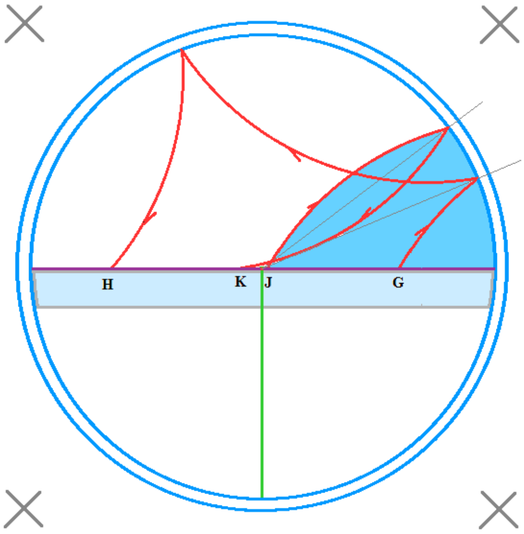
(8) Fig 5-8 $\theta = 30^o$ $u = 4.5u_p$ R = 18mm



A-glass-B

70/70 = 1.00 (red)

A-B 100% of the electrons of (30^o, $4.5u_p$) emitted from A migrate to B.



B-glass-A

70/70 = 1.00

(from G to H, J to K, etc., blue.)

B-A 100% of the electrons of (30^o, $4.5u_p$) emitted from B migrate to A.

For all the electrons of (30^o, $4.5u_p$), migration A-B and migration B-A cancel each other, there is no net contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)_{30^o\ 4.5u_p} = 1.00 - 1.00 = 0$$
$$D_{30^o}(4.5u_p) = 0$$

Every 100 + 100 electrons (1.00 = 100% from A, 1.00 = 100% from B) of exiting angle $\theta = 30^o$ and of different speeds contribute to the electron migration between A and B differently, as list in Tab 5 (1), ($cos30^o$ = 0.8660). Fig.5 (1) is the corresponding graph

$D_{30^o}(u)$ = {(A-B) – (B-A)} $_{30^o} \cos\theta \sim u$.

speed u		{(A-B) – (B-A)} $_{30^o}$	{(A-B) – (B-A)} $_{30^o} \cos\theta$
Fig 5-1	$u = 0.5u_p$	0.25 – 0.22 = 0.03	0.2165–0.1905=0.0260
Fig 5-2	$u = u_p$	0.49 - 0.41 = 0.08	0.4243-0.3551=0.0692
Fig 5-3	$u = 1.5u_p$	0.73 - 0.57 = 0.16	0.6322-0.4936=0.1386
Fig 5-4	$u = 2u_p$	0.96 - 0.75 = 0.21	0.8314-0.6495=0.1819
Fig 5-5	$u = 2.5u_p$	1.00 – 0.94 = 0.06	0.866-0.8140=0.0520
Fig 5-6	$u = 3u_p$	1.00 – 0.96 = 0.04	0.866-0.8314= 0.0346
Fig 5-7	$u = 3.5u_p$	1.00 - 1.00 = 0	0.8660-0.8660=0.0000
Fig 5-8	$u = 4.5u_p$	1.00 - 1.00 = 0	0.8660-0.8660=0.0000

Tab 5 (1) Contributions of electrons of $\theta = 30^o$ with different speeds.

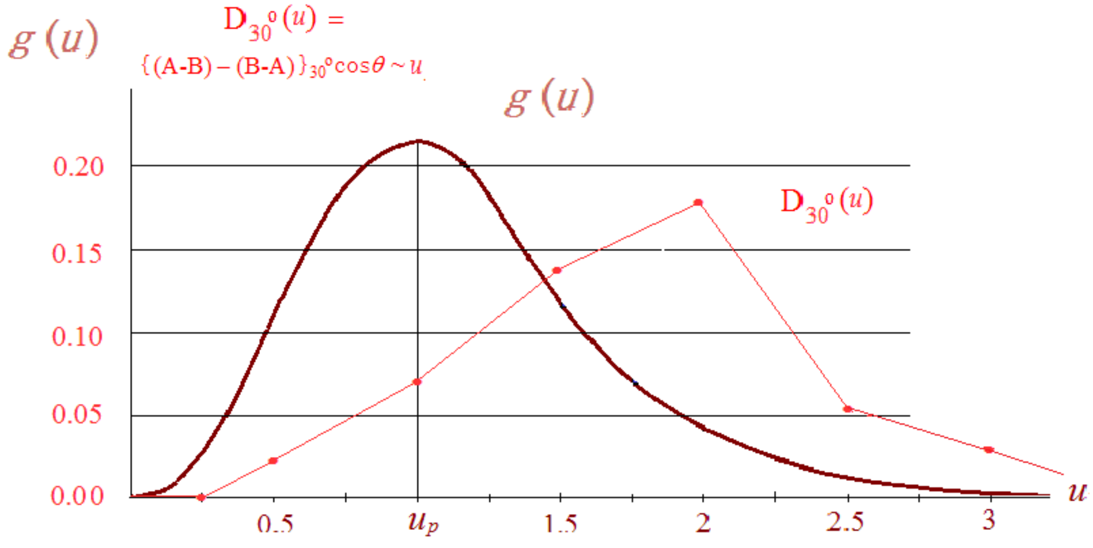


Fig. 5 (1) Graph of $D_{30^o}(u)$ = {(A-B) – (B-A)} $_{30^o} \cos\theta \sim u$

Take Maxwell's speed distribution $g(u)$ into account to derive the actual contributions of the thermal electrons of $\theta = 30^o$ with respect to different speed ranges, i.e., $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{30^o}(u) \varDelta u \sim u$, as shown in Tab 5 (2).

speed range $\varDelta u$	$\frac{\Delta N_{(u_i \sim u_{i+1})}}{N}$	$D_{30^o}(u)$ {(A-B) – (B-A)} $_{30^o}\cos\theta$	$\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{30^o}(u) \varDelta u$
$0.25 \sim 0.75u_p$	A_1 =18. 74%	0.4243-0.3551=0.0692	7.9514-6.6546=1.2968
$0.75 \sim 1.25u_p$	A_2 =39. 83%	0.6322-0.4936=0.1386	25.1805-19.6601=5.5204
$1.25 \sim 1.75u_p$	A_3 =26. 71%	0.8314-0.6495=0.1819	22.2067-17.3481=4.8586
1.75 ~ 2.25	A_4 =8. 82%	0.866-0.8140=0.0520	7.6381-7.1795=0.4586
2.25 ~ 2.75	A_5=1. 58%	0.866-0.8314= 0.0346	1.3683-1.3136=0.0547
2.75 ~ 3.25	A_6 =0. 16%	0.8660-0.8660=0.0000	0.1386-0.1386= 0
3.25~3.75	A_7=.0096%	0.8660-0.8660=0.0000	0.0083-0.0083= 0
$3.75u_p \sim \infty$	$A_8 \approx$.0003%	0.8660-0.8660=0.0000	0.00026-0.00026=0
$\sum_u \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{30^o}(u) \varDelta u$ = 64.3536 – 52.1645 = 12.1891			

Tab 5 (2) $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{30^o} \varDelta u \sim u$.

Fig 5 (2) is the corresponding graph.

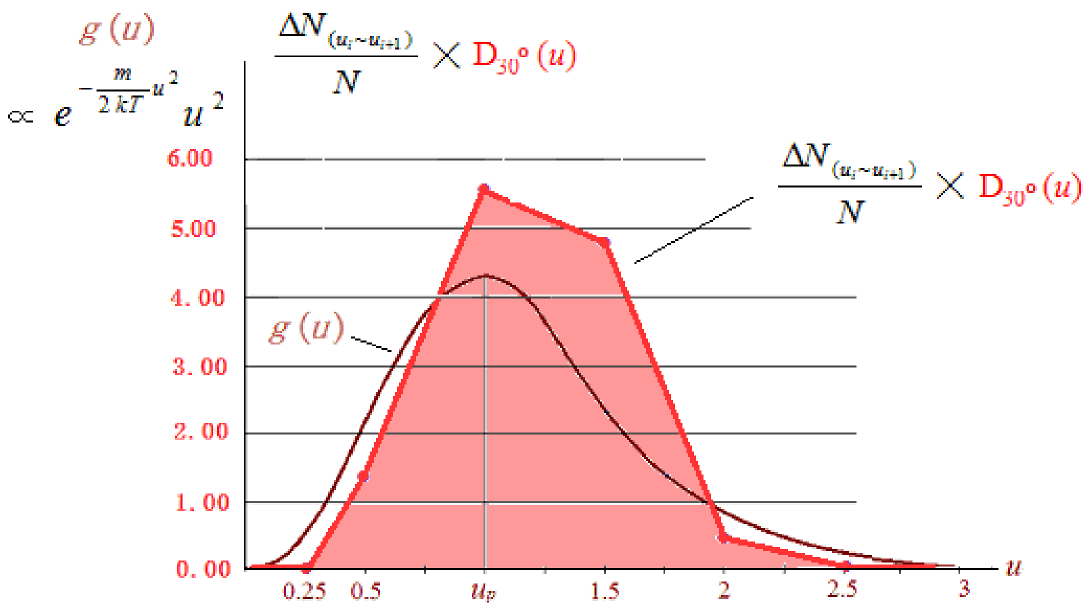
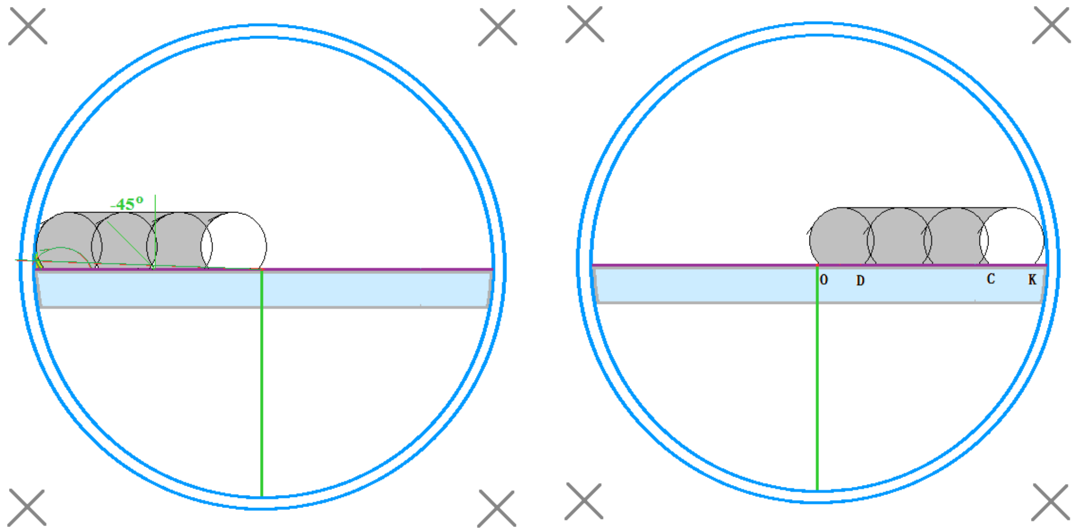


Fig 5 (2) Graph of $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{30^o} \sim u$.

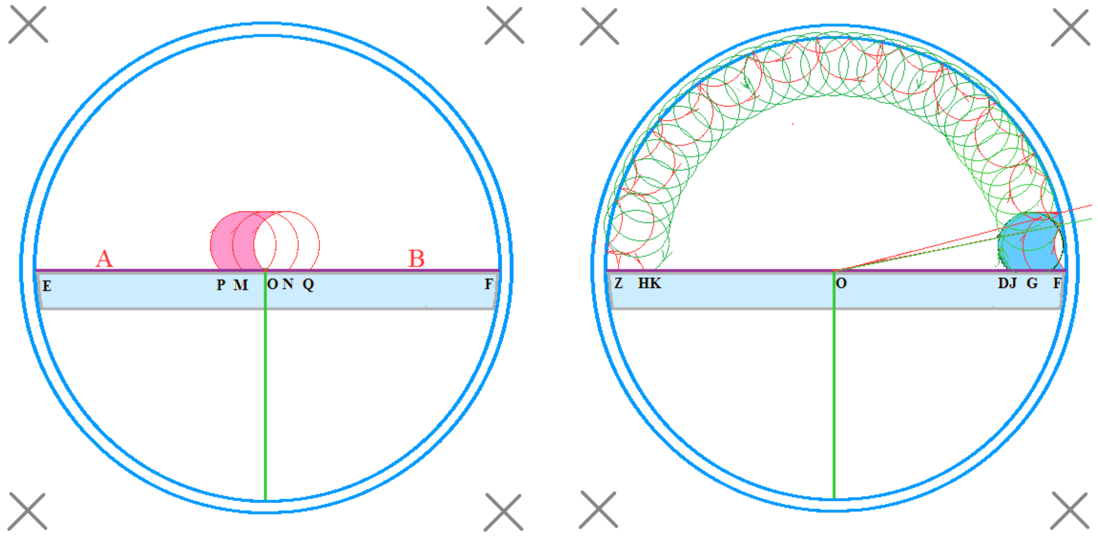
6. Trajectories of electrons of θ = - 45º and different speeds

(1) Fig 6-1 θ = - 45º $u = 0.5u_p$ R = 2mm (B = 1.34 gauss)



A-glass-A (green) & A-directly-A (grey) B-directly-B (grey)

No electron migration between A and B due to these trajectories.



A-directly-B
(from PO to OQ, red)
(PO = 15, EO =70)
15/70 = 0.21

B-glass-A
(from G to H, F to L, J to K,etc., blue.)
(DF=18, OF=70)
18/70 = 0.26

A-B 21% of the electrons of (- 45º, $0.5u_p$) emitted from A migrate to B.

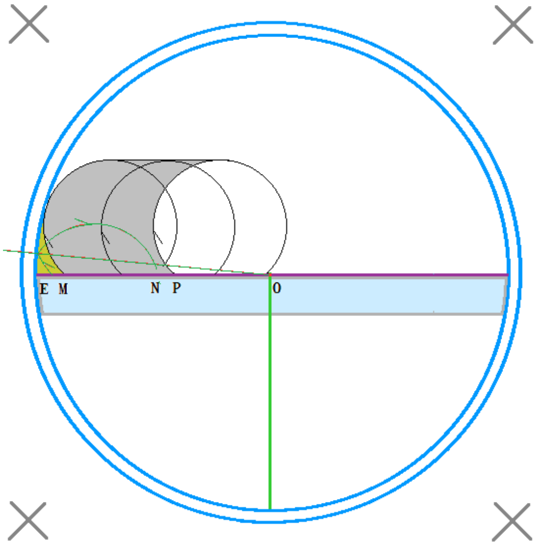
B-A 26% of the electrons of (- 45º, $0.5u_p$) emitted from B migrate to A.

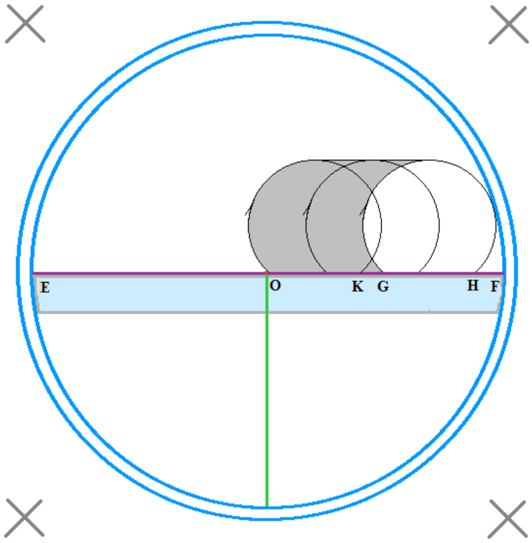
For all the electrons of (θ =- 45º, $u = 0.5u_p$), migration A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

$$\{(A\text{-}B) - (B\text{-}A)\}_{-45^o\ 0.5up} = 0.21 - 0.26 = -0.05$$

$$D_{-45^o}(0.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-45^o\ 0.5up} \times \cos 45^o = -0.05 \times 0.7071 = -0.04$$

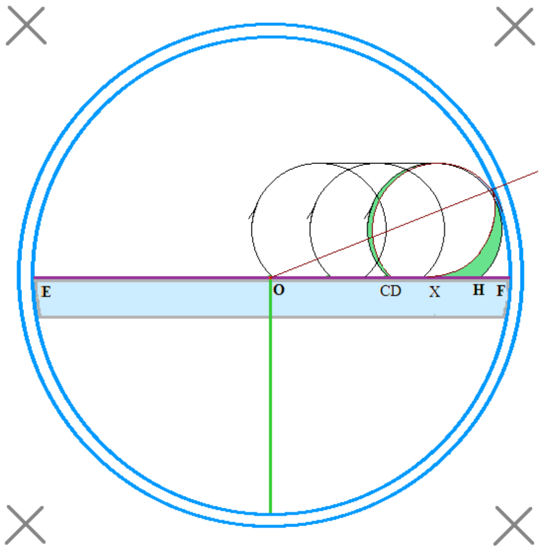
(2) Fig 6-2 θ = - 45º $u = u_p$ R = 4mm (*cos*-45º = 0.7071)



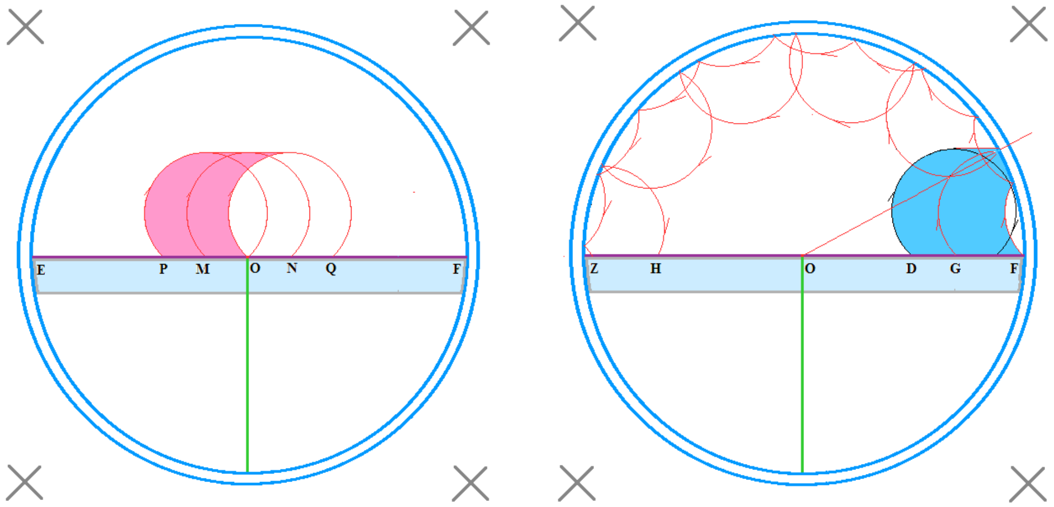


A-glass-A & A-directly-A B-directly-B (from O to K, G to H, etc.)
(green) (grey) (grey)

No electron migration between A and B due to these trajectories.



B-glass-B
(from CD to HX)
CD = 0.8 (green) 0.8/70 = 0.01
No electron migration between A and B due to these trajectories.



A-directly-B (from PO to OQ) (PO = 28, EO =70) (red) 28/70 = 0.40

B-glass-A (from G to H, etc.) (DF=36, OF=70) (blue) 35/70 = 0.50

A-B   40% of the electrons of (- 45°, u_p) emitted from A migrate to B.

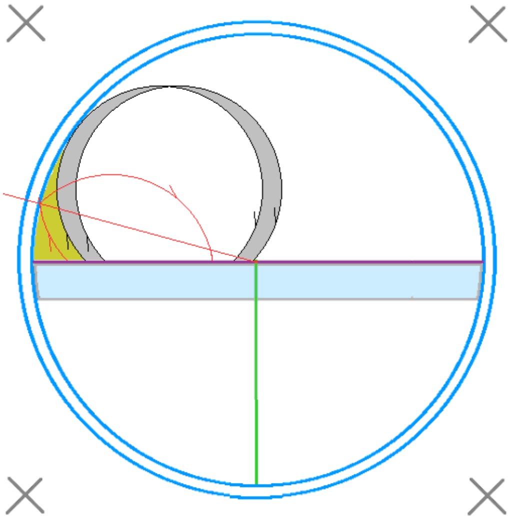
B-A   51% of the electrons of (- 45°, u_p) emitted from B migrate to A.

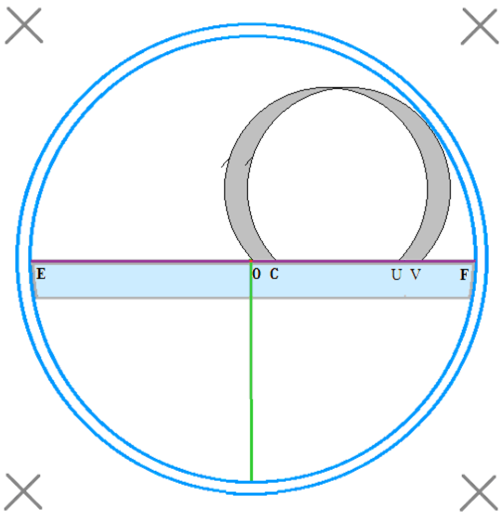
For all the electrons of (- 45°, u_p), migration A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

$$\{(A\text{-}B) - (B\text{-}A)\}_{-45^o\, up} = 0.40 - 0.50 = -0.10$$

$$D_{-45^o}(u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-45^o\, up} \cos 45^o = -0.10 \times 0.7071 = -0.07$$

(3) Fig 6-3 θ = - 45° $u = 1.5u_p$ R = 6mm





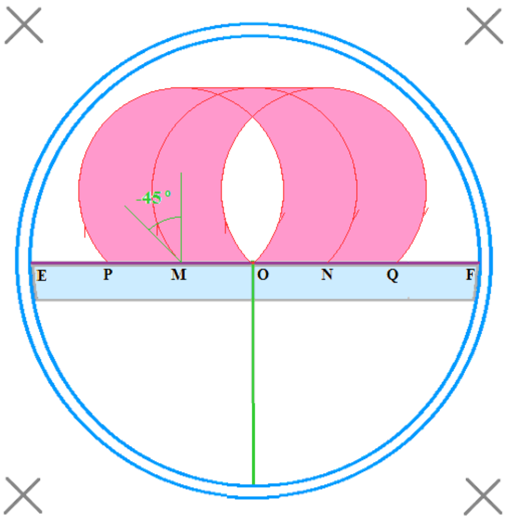
A-glass-A (17/70) (green)

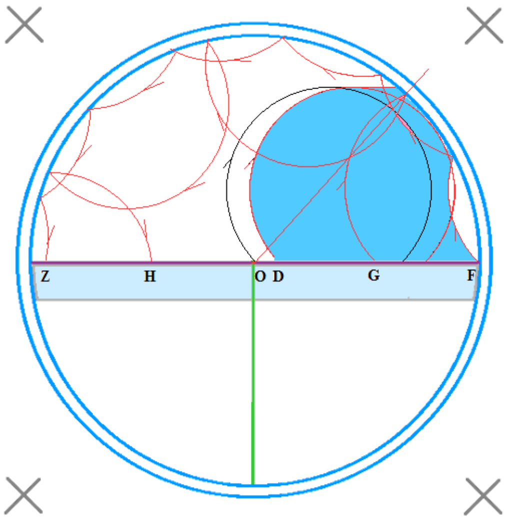
A-directly-A (7/70) (grey)

B-directly-B (from OC to UV)

(8/70) (OC = 8, grey)

No electron migration between A and B due to these trajectories.





A-directly-B

(from P to O, M to N, O to Q, etc., red)

(PO = 46, EO =70)

46/70 = 0.66

B-glass-A

(from G to H, F to Z, etc., blue)

(DF = 62, OF =70)

62/70 = 0.86

A-B 66% of the electrons of (- 45°, $1.5u_p$) emitted from A migrate to B.

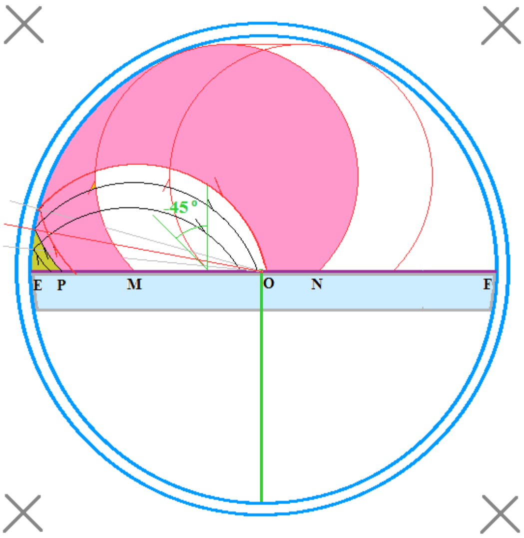
B-A 86% of the electrons of (- 45°, $1.5u_p$) emitted from B migrate to A.

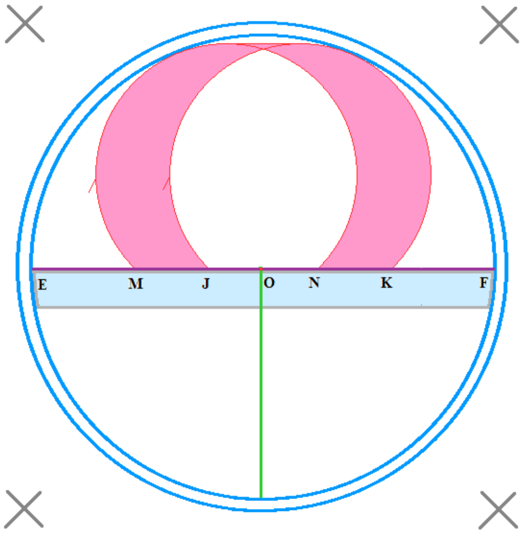
For all the electrons of (- 45°, $1.5u_p$), migration A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

$$\{(A\text{-}B) - (B\text{-}A)\}_{-45^\circ\ 1.5u_p} = 0.66 - 0.86 = -0.20$$

$$D_{-45^\circ}(1.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-45^\circ\ 1.5u_p} \cos 45^\circ = -0.20 \times 0.7071 = -0.14$$

(4) Fig 6-4 θ = - 45° $u = 2u_p$ R = 8mm



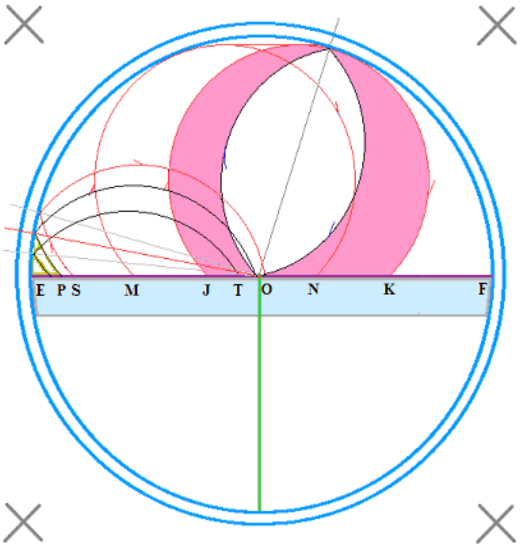


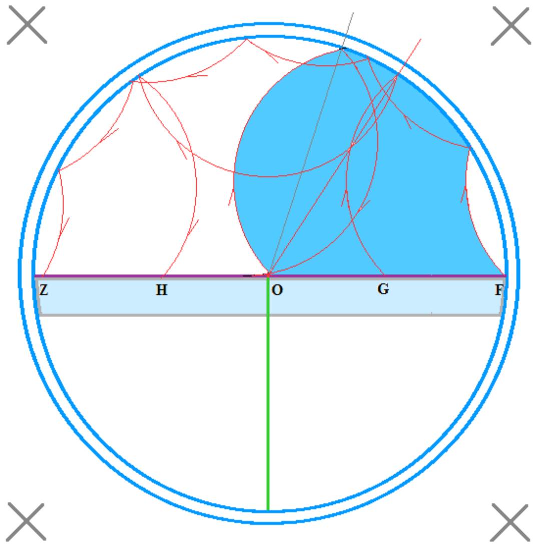
A-glass-A 9/70 = 0.13 (green, EP = 9)

A-glass-B 22/70 = 0.31 (red, PM =22)

A-directly-B (from MJ to NK)

23/70 = 0.32 (red, MJ = 23, OE =70)





A-glass-B (from JO to KO)

16/70 = 0.23 (JO =16, red)

B-glass-A (from G to H, etc.)

(OF=70, blue)

70/70 = 1.00

A-B = A-directly-B + A-glass-B

(PO = PM + MJ + JO = 22 + 23 + 16 = 61)

61/70 = 0.87

A-B 87% of the electrons of (- 45°, $2u_p$) emitted from A migrate to B.

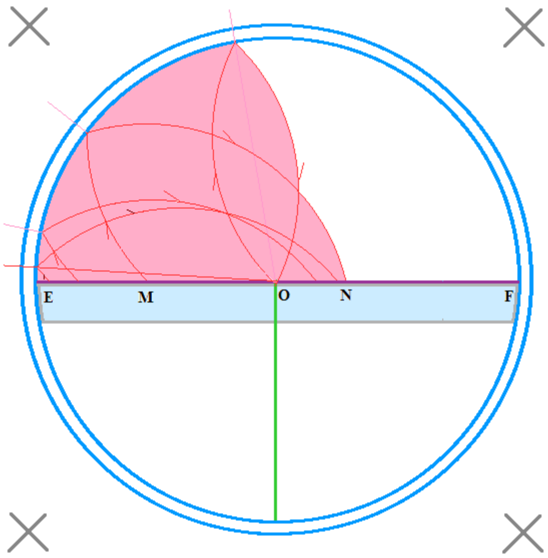
B-A 100% of the electrons of (- 45°, $2u_p$) emitted from B migrate to A.

For all the electrons of (- 45°, $2u_p$), A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

$$\{(\text{A-B}) - (\text{B-A})\}_{-45^\circ\, 2up} = 0.87 - 1.00 = -0.13$$

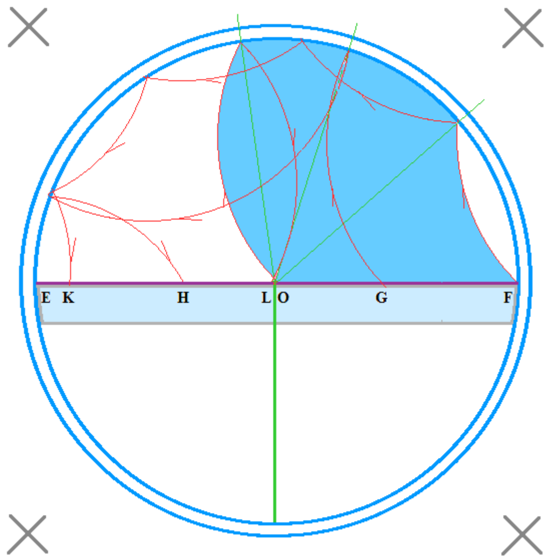
$$D_{-45^\circ}(2u_p) = \{(\text{A-B}) - (\text{B-A})\}_{-45^\circ 2up}\, cos45^\circ = -0.13 \times 0.7071 = -0.09$$

(5) Fig 6-5 θ = -45° $u = 2.5u_p$ R = 10mm



A-glass-B
(from M to N, etc., red)
70/70 = 1.00

A-B 100% of the electrons of (- 45°, 2.5u_p) emitted from A migrate to B.



B-glass-A
(from G to H, etc., blue)
70/70 = 1.00

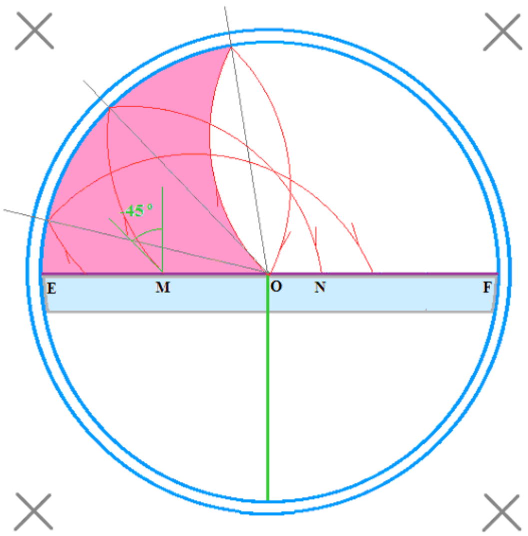
B-A 100% of the electrons of (- 45°, 2.5u_p) emitted from B migrate to A.

For all the electrons of (- 45°, 2.5u_p), migration A-B and B-A cancel each other, no contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{-45^\circ\ 2.5u_p} = 1.00 - 1.00 = 0$$

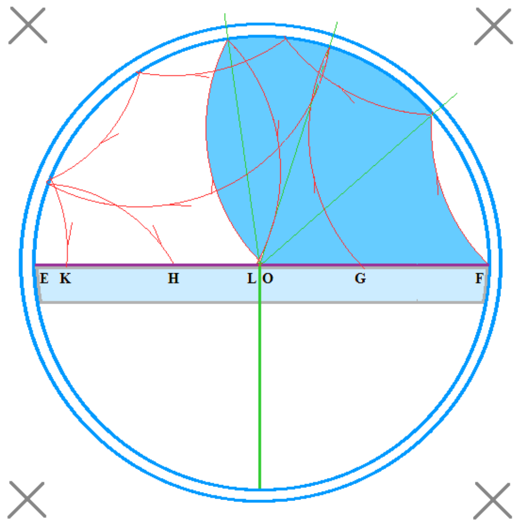
$$D_{-45^\circ}(2.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-45^\circ\ 2.5u_p} \times \cos 45^\circ = 0$$

(6) Fig 6-6 θ = -45° $u = 3u_p$ R = 12mm



A-glass-B
(from M to N, etc., red)
70/70 = 1.00

A-B All the electrons of (- 45°, $3u_p$) emitted from A migrate to B.



B-glass-A
(from G to H, F to K, O to L, etc., blue)
70/70 = 1.00

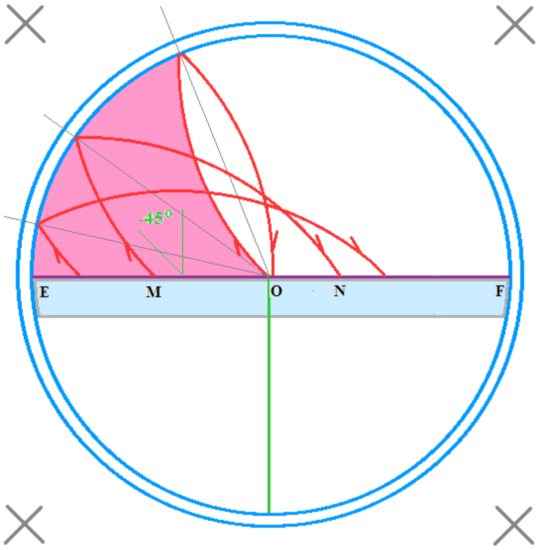
B-A All the electrons of (- 45°, $3u_p$) emitted from B migrate to A.

For all the electrons of (- 45°, $3u_p$), migration A-B and B-A cancel each other, no contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{-45^o\, 3u_p} = 1.00 - 1.00 = 0$$

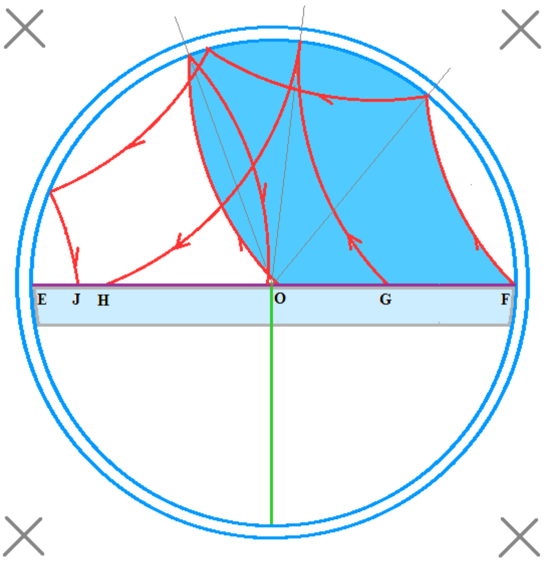
$$D_{-45^o}(3u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-45^o\, 3u_p} \cos 45^o = 0$$

(7) Fig 6-7 θ = -45° $u = 4.5u_p$ R = 18mm



A-glass-B
(from M to N, etc., red) \
70/70 = 1.00

A-B All the electrons of (- 45°, $4.5u_p$) emitted from A migrate to B.



B-glass-A
(from G to H, F to J, etc., blue)
70/70 = 1.00

B-A All the electrons of (- 45°, $4.5u_p$) emitted from B migrate to A.

For all the electrons of (- 45°, $4.5u_p$), migration A-B and B-A cancel each other, no contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{\theta = -45^\circ\ u = 4.5u_p} = 1.00 - 1.00 = 0$$

$$D_{-45^\circ}(4.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-45^\circ\ 4.5u_p} \cos 45^\circ = 0$$

Every 100 + 100 electrons (1.00 = 100% from A, 1.00 = 100% from B) of exiting angle θ = - 45° and of different speeds u contribute to the electron migration between A and B differently, as list in Tab 6 (1) (cos-45° = 0.7071)

$$D_{-45^o}(u) = \{(A\text{-}B) - (B\text{-}A)\}_{-45^o} \cos\theta \sim u.$$

speed u	$\{(A\text{-}B) - (B\text{-}A)\}_{-45^o}$	$\{(A\text{-}B) - (B\text{-}A)\}_{-45^o} \cos\theta$
Fig 6-1 $u = 0.5u_p$	0.21 – 0.26 = - 0.05	0.1485 – 0.183 = 0.025
Fig 6-2 $u = u_p$	0.40 - 0.51 = - 0.11	0.2828 - 0. 3606 = 0. 0778
Fig 6-3 $u = 1.5u_p$	0.66 – 0.89 = - 0.23	0.4667 – 0.6293 = 0.1626
Fig 6-4 \$u = 2u_p$	0.87 – 1.00 = - 0.13	0.6152 – 0.7071 = - 0.0919
Fig 6-5 $u = 2.5u_p$	1.00 – 1.00 = 0	0.7071 -0.7071 = 0
Fig 6-6 $u = 3u_p$	1.00 – 1.00 = 0	0.7071 -0.7071 = 0
Fig 6-7 $u = 4.5u_p$	1.00 - 1.00 = 0	0.7071 -0.7071 = 0

Tab 6(1) $D_{-45^o}(u) = (A\text{-}B) - (B\text{-}A)\}_{\theta=-45^o} \cos\theta \sim u$.

Fig 6 (1) is the corresponding graph.

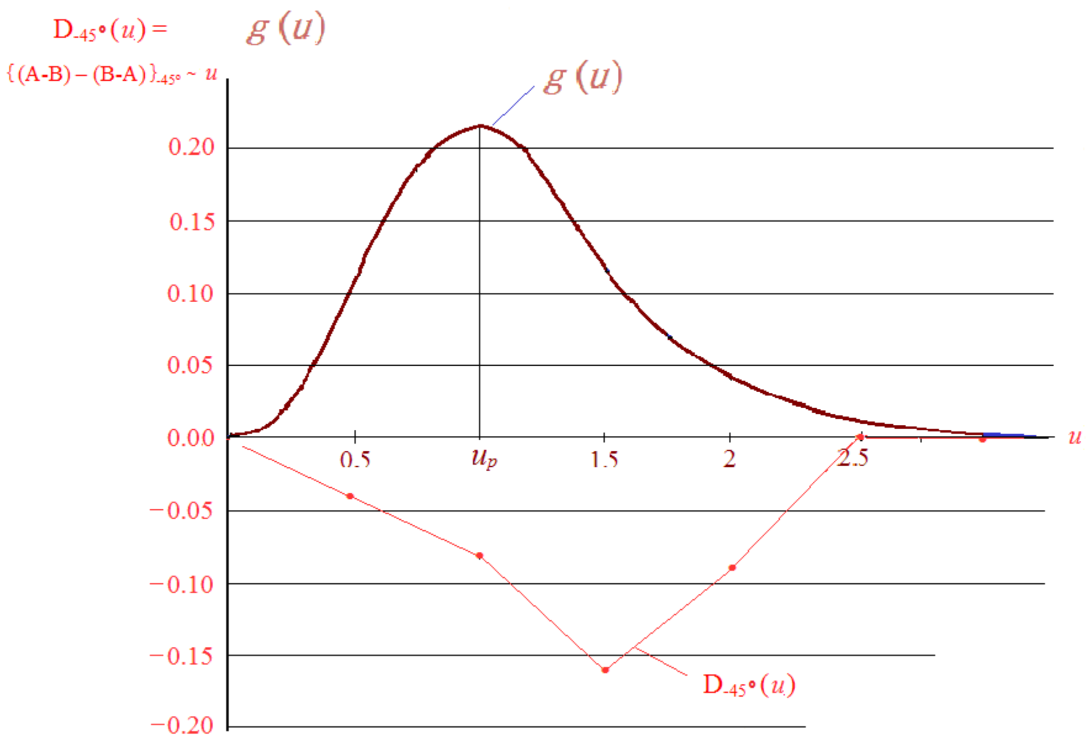


Fig 6 (1) graph of $D_{-45^{\circ}}(u)$ = { (A-B) - (B-A) } $_{\theta=-45^{\circ}}\cos\theta \sim u$.

Take Maxwell's speed distribution $g(u)$ into account to derive the actual contributions of the thermal electrons of $\theta = -45^{o}$ and of different speed ranges, i.e., $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{-45^o}(u) \varDelta u \sim u$., as shown in Tab 6 (2).

speed range $\varDelta u$	$A_i = \frac{\Delta N_{(u_i \sim u_{i+1})}}{N}$	$D_{-45^o}(u)$ $\{(A\text{-}B) - (B\text{-}A)\}_{-45^o} \cos$	$\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{-45^o}(u) \varDelta u$
$0.25 \sim 0.75u_p$	$A_1 =$ 18. 74%	0.1485 - 0.183 =-0.0345	2.7829–3.4292=-0.6465
$0.75 \sim 1.25u_p$	$A_2 =$ 39. 83%	0.2828- 0.3606=-0.0778	11.2639-14.3627=-3.0988
$1.25 \sim 1.75u_p$	$A_3 =$ 26. 71%	0.4667- 0.6293=-0.1626	12.4656-16.8086=-4.3430
$1.75 \sim 2.25u_p$	$A_4 =$ 8. 82%	0.6152-0.7071=-0.0919	5.4261-6.2366=-0.8106
$2.25 \sim 2.75u_p$	$A_5 =$ 1. 58%	0.7071 - 0.7071 = 0	1.1172-1.1172 = 0
$2.75 \sim 3.25u_p$	$A_6 =$ 0. 16%	0.7071 - 0.7071 = 0	0.1131-0.1131 = 0
$3.25 \sim 3.75\ u_p$	$A_7 =$. 0096%	0.7071 - 0.7071 = 0	0.0068-0.0068 = 0
$3.75u_p \sim \infty$	$A_8 \approx$ .0003%	0.7071 - 0.7071 = 0	0.0002-0.0002 = 0
$\sum_u \frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{-45^o}(u) \varDelta u$ = 33.1758 – 42.3881= - 9.2123			

Tab. 6 (2) $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{-45^o} \varDelta u \sim u$

Fig 6 (2) is the corresponding graph.

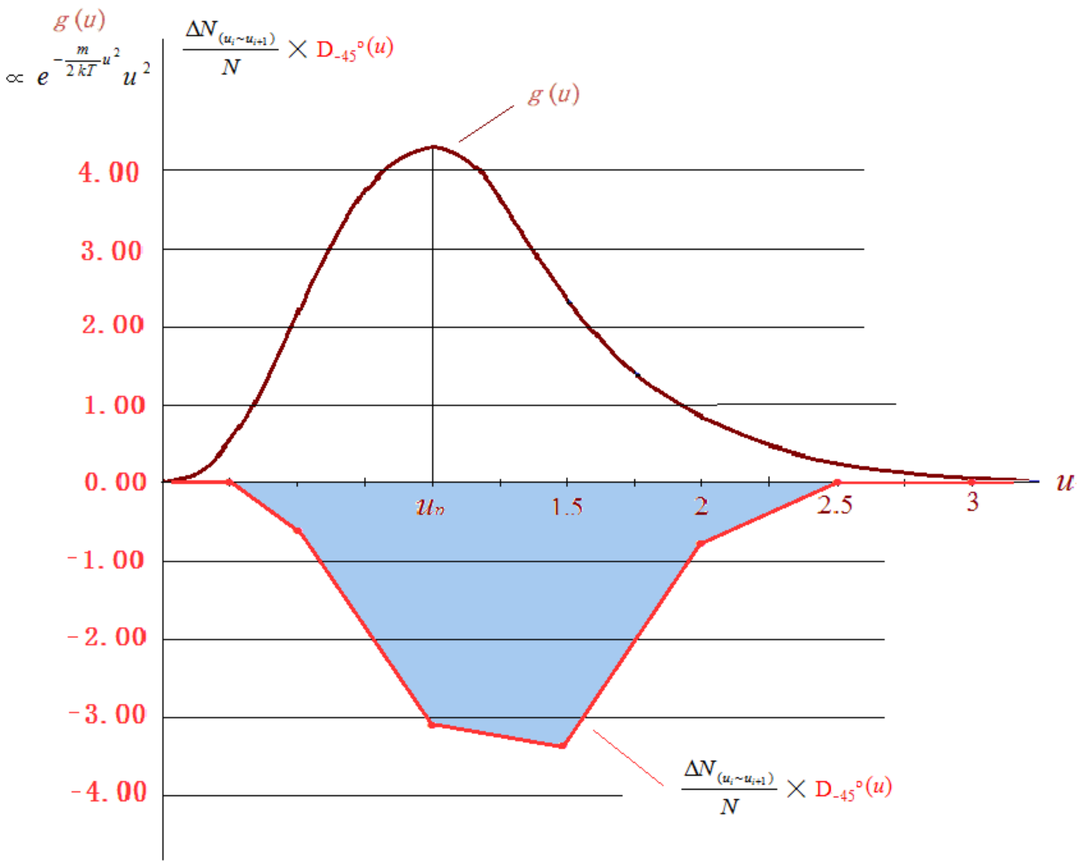
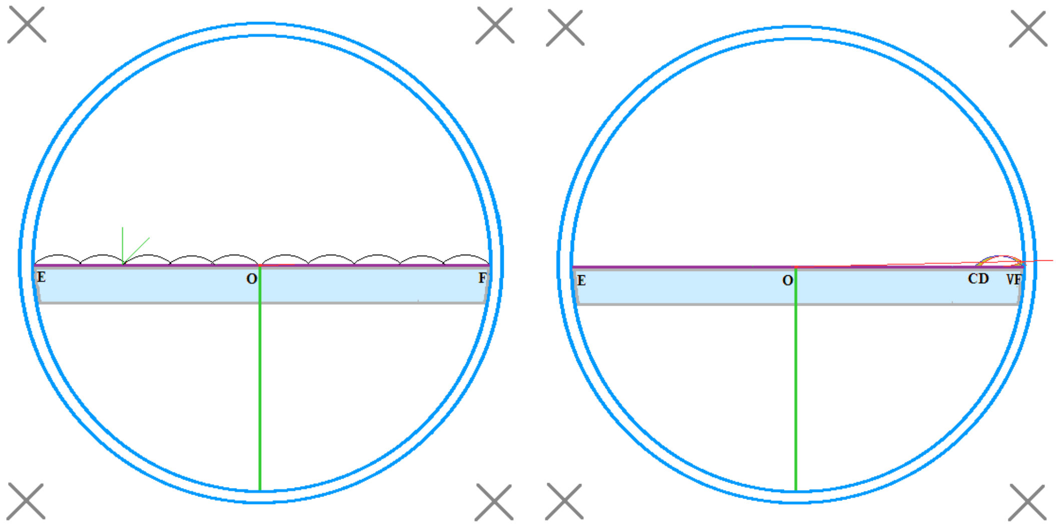


Fig 6 (2) Graph of $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{-45^o} \sim u$

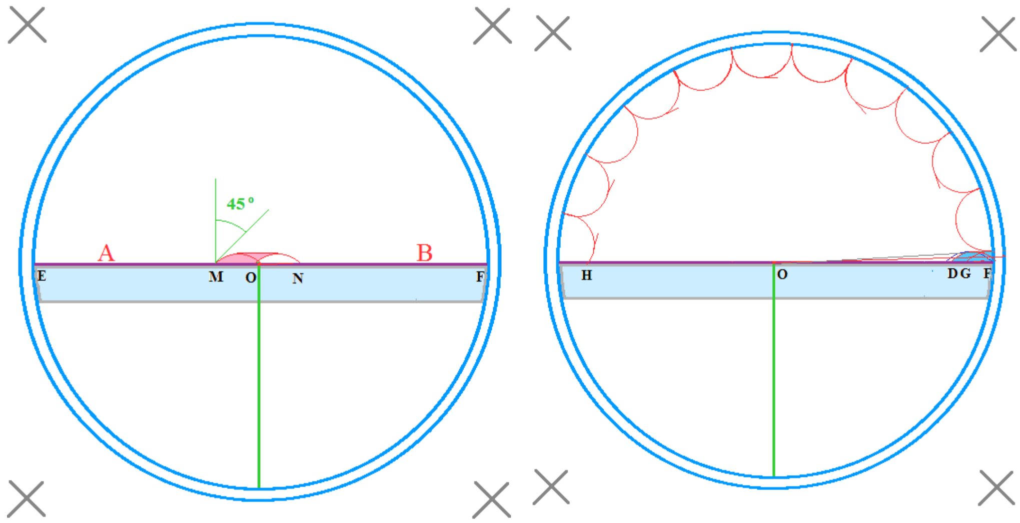
7. Trajectories of electrons of θ = 45° and different speeds

(1) Fig 7-1 $\theta = 45^o$ $u = 0.5u_p$ R = 2mm (B = 1.34 gauss)



A-directly-A B-directly-B B-glass-B 1.5/70≈0.02 (CD = 1.5)

No electron migration between A and B due to these trajectories.



A-directly-B (from MO to ON, MO = 12.5, red)

MO/EO = 14/70 = 0.20

B-glass-A (from G to H, etc., DF=12.5, blue)

DF/OF=12.5/70=0.18

A-B 20% of the electrons of (45°, $0.5u_p$) emitted from A migrate to B.

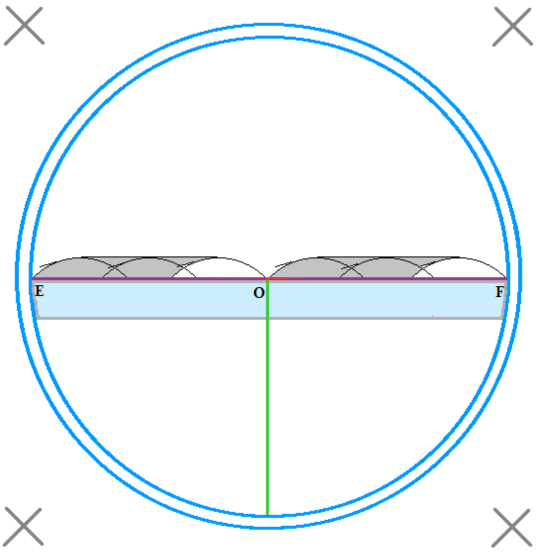
B-A 18% of the electrons of (45°, $0.5u_p$) emitted from B migrate to A.

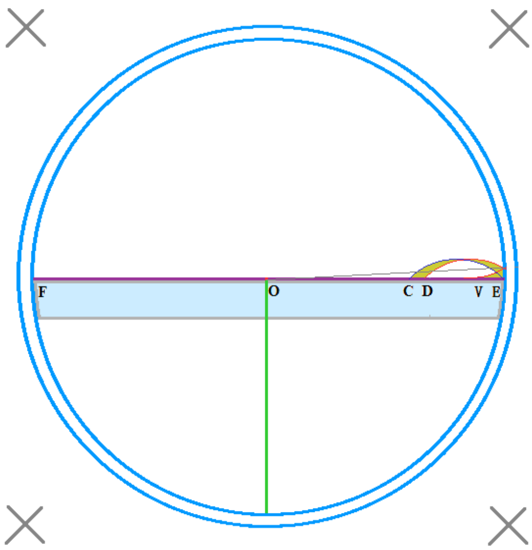
For all the electrons of (45°, $0.5u_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{45^o\,0.5u_p} = 0.20 - 0.18 = 0.02$$

$$D_{45^o}(0.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{45^o\,0.5u_p} \cos 45^o = 0.02 \times \cos 45^o = 0.014$$

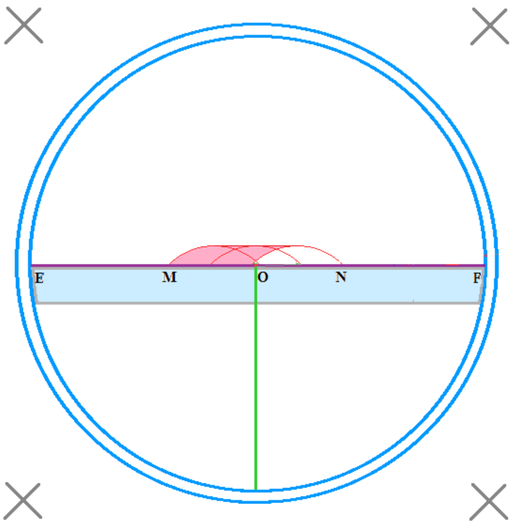
(2) Fig 7-2 $\theta = 45^{o}$ $u = u_p$ R = 4mm ($\cos 45^{o} = 0.7071$)





A-directly-A B-directly-B B-glass-B

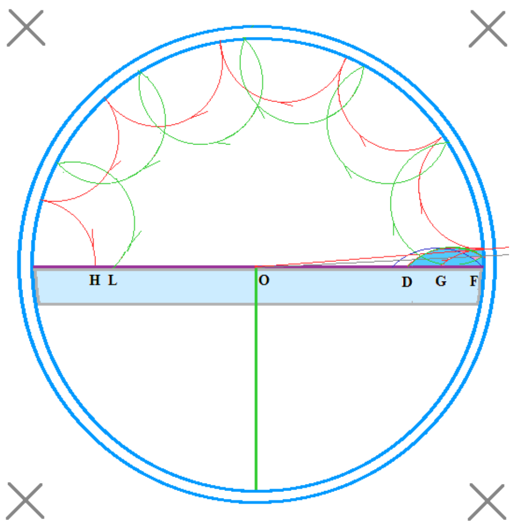
No electron migration between A and B due to these trajectories.



A-directly-B

(from MO to ON, MO = 28, red)

MO/EO=28/70=0.40



B-glass-A

(from G to H, D to L, etc., DF = 23.5, blue)

DF/OF=23.5/70 = 0.34

A-B 40% of the electrons of (45^{o}, u_p) emitted from A migrate to B.

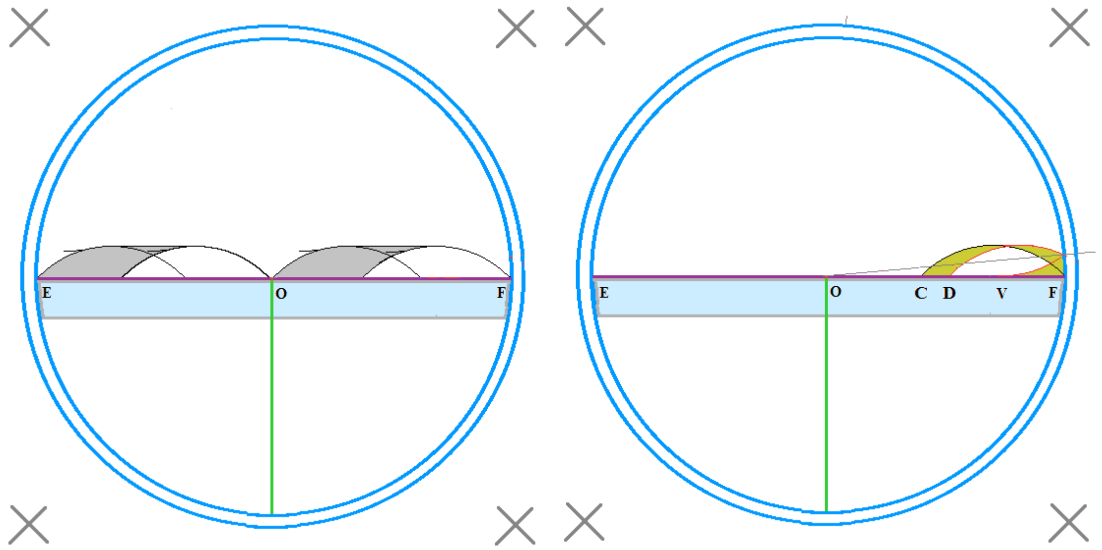
B-A 34% of the electrons of (45^{o}, u_p) emitted from B migrate to A.

For all the electrons of (45^{o}, u_p), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{45^{o}\, u_p} = 0.40 - 0.34 = 0.06$$

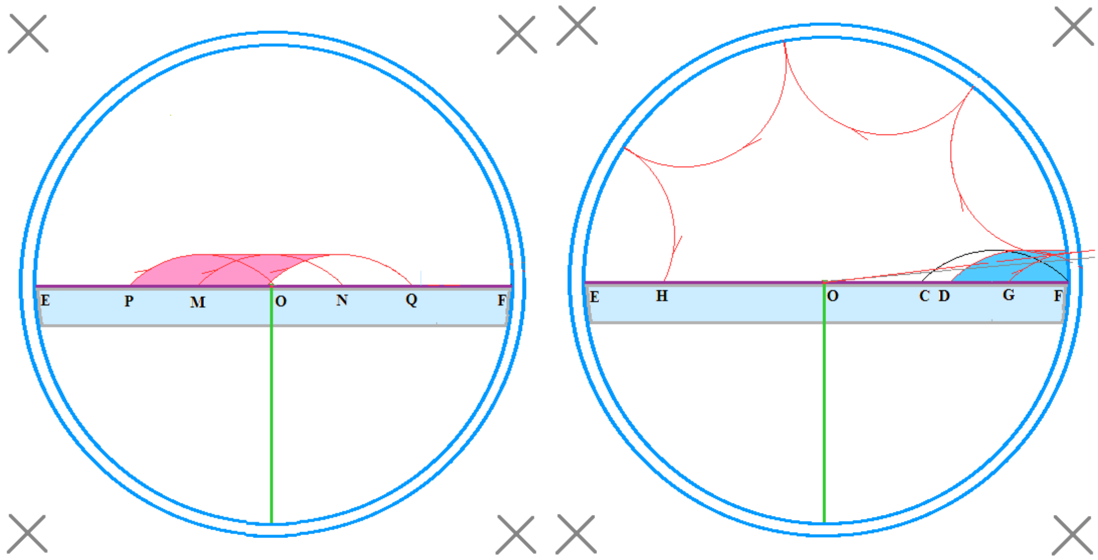
$$D_{45^{o}}(0.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{45^{o}\, u_p} \cos 45^{o} = 0.06 \times 0.7071 = 0.04$$

(3) Fig 7-3 $\theta = 45^{o}$ $u = 1.5u_p$ R = 6mm



A-directly-A B-directly-B B-glass-B

No electron migration between A and B due to these trajectories.



A-directly-B
(from PO to OQ, PO = 42.5, red)
PO/EO=42.5/70 = 0.61

B-glass-A
(from G to H, etc., DF = 34, blue)
DF/OF=34/70 = 0.49

A-B 61% of the electrons of (45^{o}, $1.5u_p$) emitted from A migrate to B.

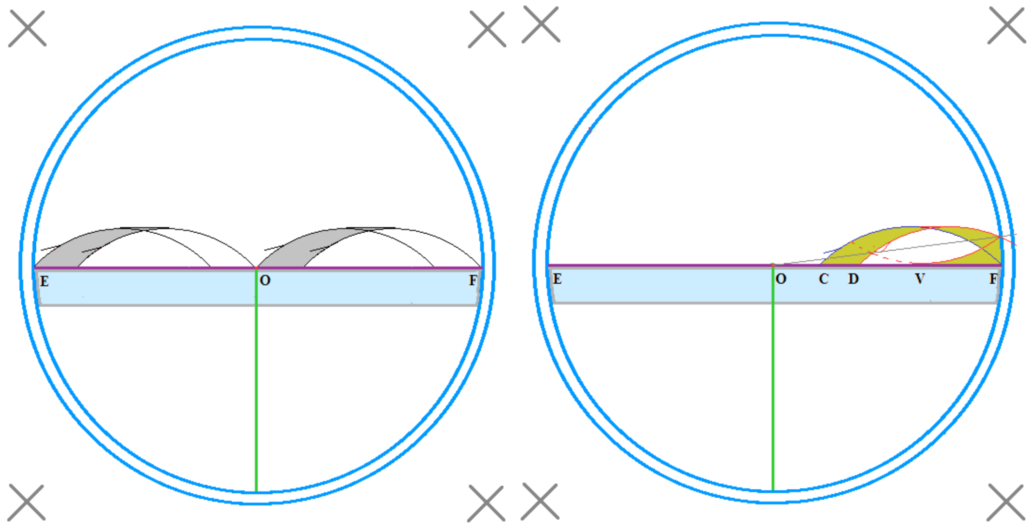
B-A 49% of the electrons of (45^{o}, $1.5u_p$) emitted from B migrate to A.

For all the electrons of (45^{o}, $1.5u_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{45^{o}\ 1.5u_p} = 0.61 - 0.49 = 0.12$$

$$D_{45^{o}}(1.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{45^{o}\ 1.5u_p} \cos 45^{o} = 0.12 \times \cos 45^{o} = 0.0849$$

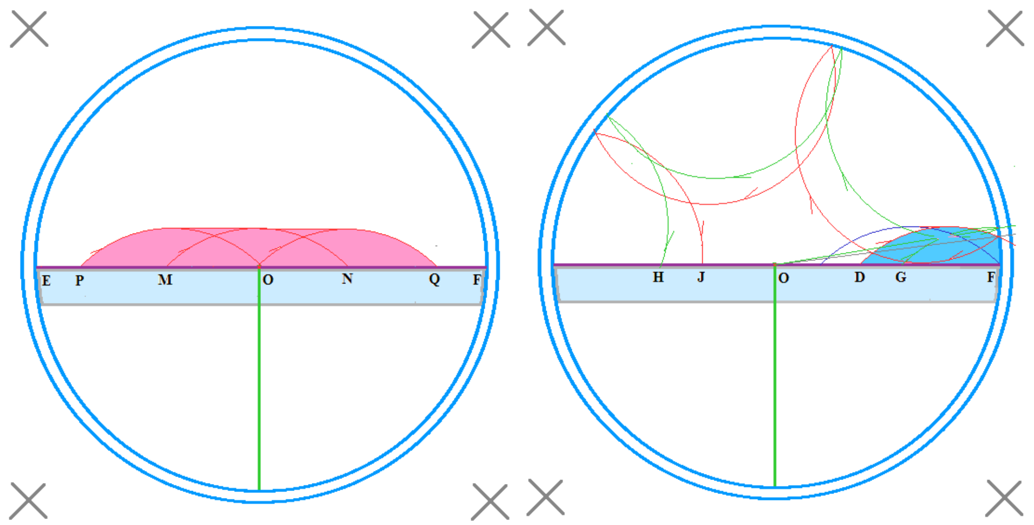
(4) Fig 7-4 $\theta = 45^o$ $u = 2u_p$ R = 8mm



A-directly-A B-directly-B B-glass-B

(grey) (green)

No electron migration between A and B due to these trajectories.



A-directly-B
(from PO to OQ, red)
PO/EO = 56.5/70 = 0.81

B-glass-A
(from G to H, D to J, etc., blue)
DF/OF=44/70 = 0.63

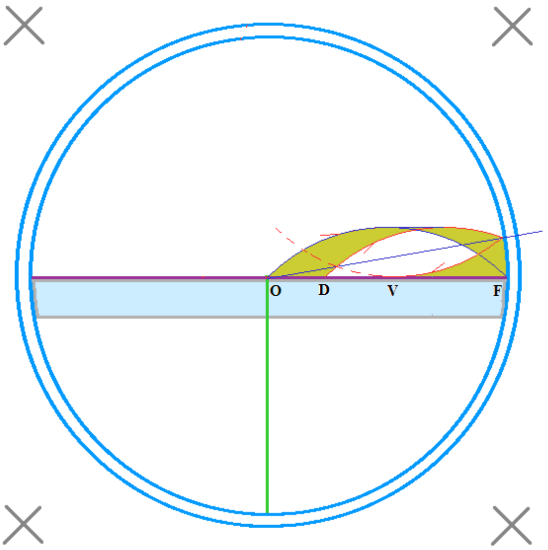
A-B 81% of the electrons of (45^o, $2u_p$) emitted from A migrate to B.
B-A 63% of the electrons of (45^o, $2u_p$) emitted from B migrate to A.

For all the electrons of (45^o, $2u_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

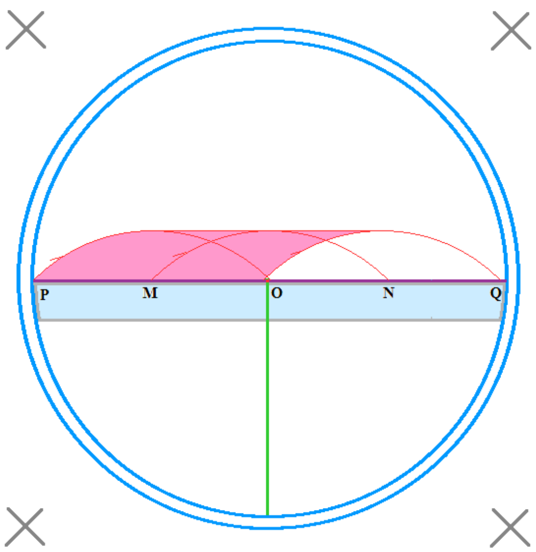
$$\{(A\text{-}B) - (B\text{-}A)\}_{45^o\ 2up} = 0.81 - 0.63 = 0.18$$

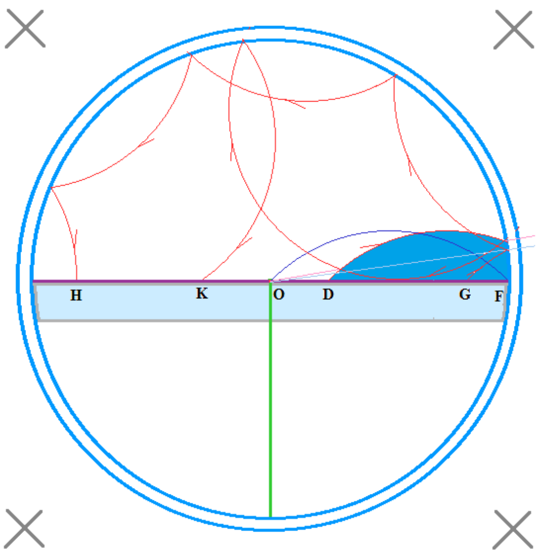
$$D_{45^o}(2u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{45^o\ 2up} \times \cos 45^o = 0.18 \times 0.7071 = 0.13$$

(5) Fig 7-5 θ = 45° $u = 2.5u_p$ R = 10mm



B-glass-B OD/OF= 16/70 = 0.23 (from OD to FV, green)
No electron migration between A and B due to these trajectories.





A-directly-B
(from PO to OQ, red)
PO/EO=70/70 = 1.00

B-glass-A
(from G to H, D to K, etc., DF = 54, blue)
DF/OF=54/70 = 0.77

A-B 100% of the electrons of (45°, $2.5u_p$) emitted from A migrate to B.

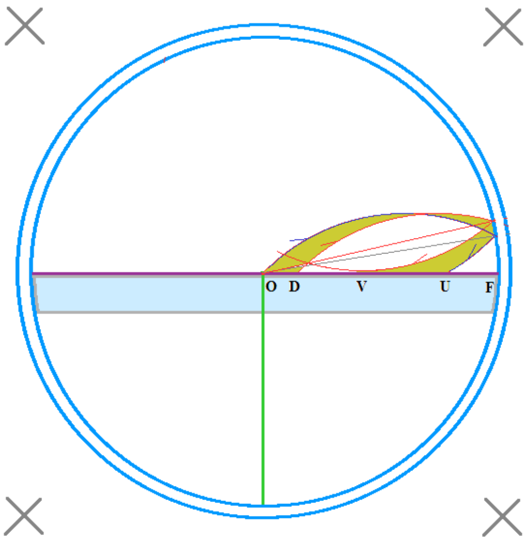
B-A 77% of the electrons of (45°, $2.5u_p$) emitted from B migrate to A.

For all the electrons of (45°, $2.5u_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{45^\circ\,2.5u_p} = 1.00 - 0.77 = 0.23$$

$$D_{45^\circ}(2.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{45^\circ\,2.5u_p}\cos 45^\circ = 0.23 \times 0.7071 = 0.16$$

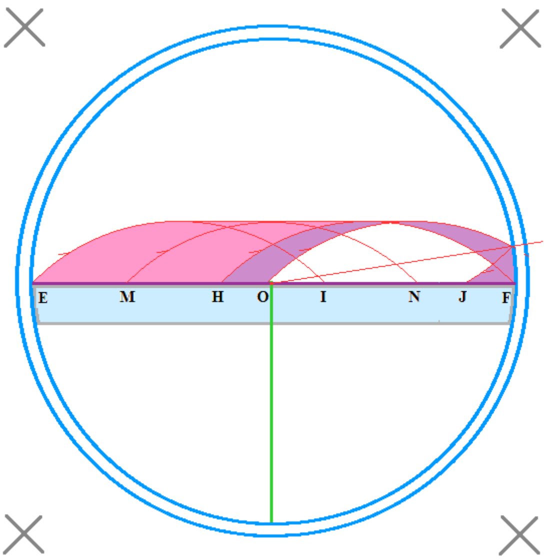
(6) Fig 7-6 θ = 45° $u = 3u_p$ R = 12mm

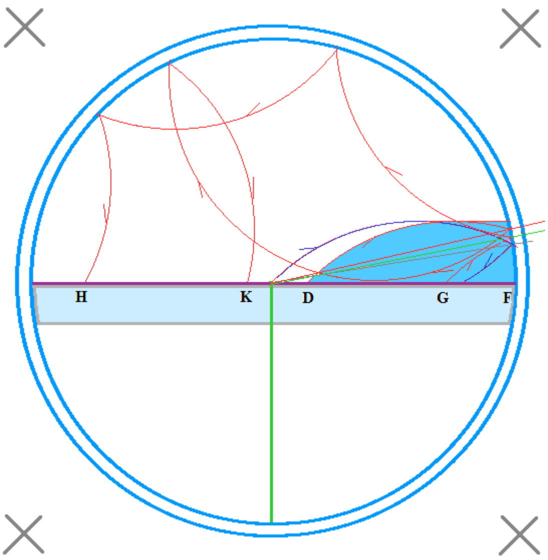


B-glass-B

10/70 = 0.14 (from OD to UV, green, OD = 10, OF =70)

No electron migration between A and B due to these trajectories.





A-directly-B (from EMH to INF, red)
A-glass-B (from HO to FJ, violet)
A-B = A-directly-B + A-glass-B
(FH + HO = 70, red + violet)
70/70 = 1.00

B-glass-A
(from G to H, D to K, etc., blue)
(DF = 60, OF =70)
60/70 = 0.86

A-B 100% of the electrons of (45°, $3u_p$) emitted from A migrate to B.

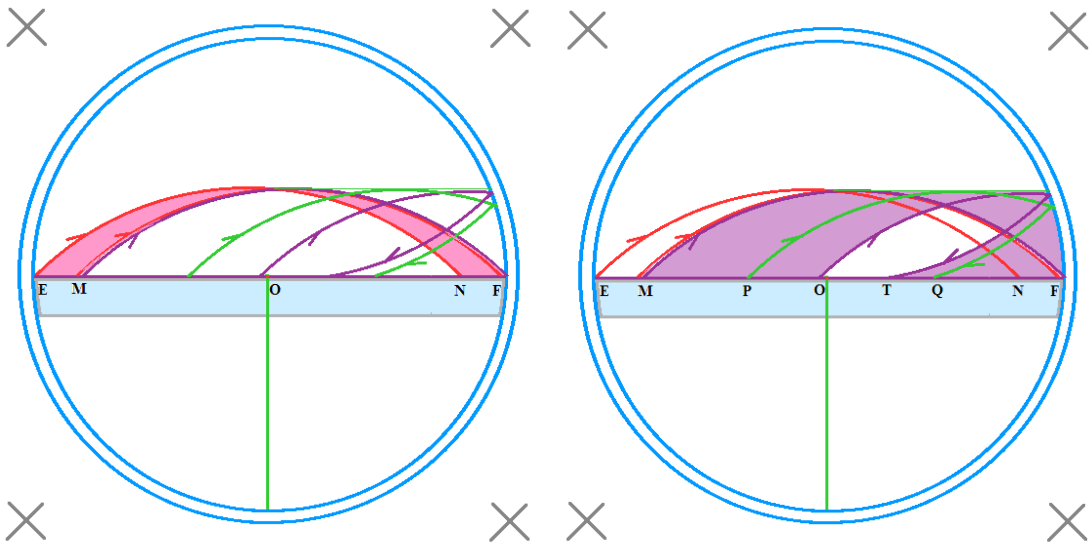
B-A 86% of the electrons of (45°, $3u_p$) emitted from B migrate to A.

For all the electrons of (45°, $3u_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{45^\circ\,3up} = 1.00 - 0.86 = 0.14$$

$$D_{45^\circ}(3u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{45^\circ\,3up} \times \cos 45^\circ = 0.14 \times 0.7071 = 0.10$$

(7) Fig 7-7 θ = 45° $u = 4.5u_p$ R = 18mm

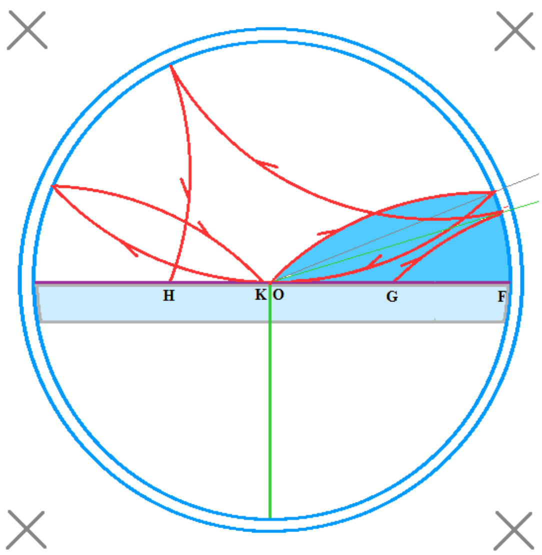


A-directly-B (red) A-glass-B (violet)

A-B = A-directly-B + A-glass-B (red +violet)

70/70 = 1.00

A-B 100% of the electrons of (45°, $4.5u_p$) emitted from A migrate to B.



B-glass-A

(from G to H, O to K, etc., blue)

70/70 = 1.00

B-A 100% of the electrons of (45°, $4.5u_p$) emitted from B migrate to A.

For all the electrons of (45°, $4.5u_p$), A-B equals B-A, and their contributions to the output current cancel each other.

$$\{(A\text{-}B) - (B\text{-}A)\}_{45^\circ\ 4.5up} = 1.00 - 1.00 = 0$$

$$D_{45^\circ}(4.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{45^\circ\ 4.5up} \cos 45^\circ = 0$$

Every 100 + 100 electrons (1.00 = 100% from A, 1.00 = 100% from B) of exiting angle θ = 45° and of different speeds u contribute to the electron migration between A and B differently, as list in Tab 7 (1) (cos45° = 0.7071),

$$D_{45^o}(u) = \{ (A\text{-}B) - (B\text{-}A) \}_{\theta=45^o} \cos\theta \sim u.$$

speed u	$\{(A\text{-}B) - (B\text{-}A)\}_{45^o}$	$(A\text{-}B) - (B\text{-}A)\}_{45^o} \cos\theta$
Fig 7-1 $u = 0.5u_p$	0.20 – 0.18 = 0.02	0.1414 – 0.1273 = 0.0141
Fig 7-2 $u = u_p$	0.40 – 0.34 = 0.06	0.2828 – 0.2404 = 0.0424
Fig 7-3 $u = 1.5u_p$	0.61 – 0.49 = 0.12	0.4313 – 0.3465 = 0.0878
Fig 7-4 $u = 2u_p$	0.81 – 0.63 = 0.18	0.5728 – 0.4455 = 0.1273
Fig 7-5 $u = 2.5u_p$	1.00 – 0.77 = 0.23	0.7071 – 0.5445 = 0.1626
Fig 7-6 $u = 3u_p$	1.00 – 0.86 = 0.14	0.7071 – 0.6081 = 0.099
Fig 7-7 $u = 4.5u_p$	1.00 - 1.00 = 0	0.7071 – 0.7071 = 0

Tab 7 (1) Contributions of trajectories of electron of θ = 45° with different speeds, $D_{45^o}(u) = \{ (A\text{-}B) - (B\text{-}A) \}_{\theta=45^o} \cos\theta \sim u$.

Fig 7 (1) is the corresponding graph.

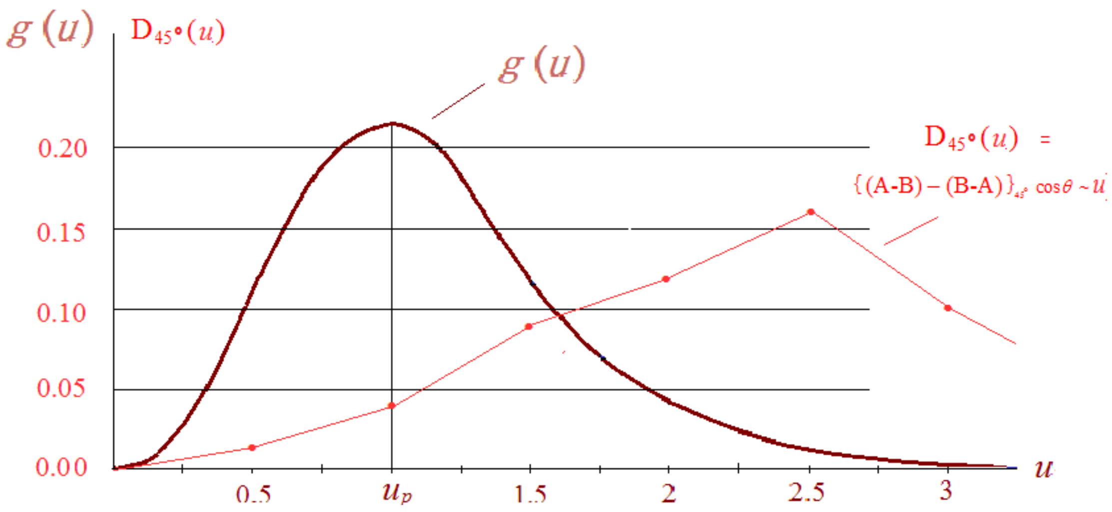


Fig 7 (1) Graph of $D_{45^o}(u) = \{ (A\text{-}B) - (B\text{-}A) \}_{\theta=45^o} \cos\theta \sim u$.

Take Maxwell's speed distribution $g(u)$ into account to derive the actual contributions of the thermal electrons of $\theta = 45^o$ with different speed ranges Δu, i.e., $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{45^o}(u) \Delta u \sim u$, as shown in Tab 7 (2).

Speed range Δu	$g(u)\Delta u = \frac{\Delta N_{(u_i \sim u_{i+1})}}{N}$	$D_{45^o}(u)$ { (A-B)-(B-A) } $_{45^o}$ $\cos\theta$	$\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{45^o}(u)\Delta u$
$0.25 \sim 0.75 u_p$	$A_1 =$ 18. 74%	0.1414–0.1273=0.0141	2.6498-2.3856=0.2642
$0.75 \sim 1.25 u_p$	$A_2 =$ 39. 83%	0.2828-0.2404=0.0424	11.2639-9.5751=1.6889
$1.25 \sim 1.75 u_p$	$A_3 =$ 26. 71%	0.4313–0.3465=0.0878	11.5200-9.2550=2.265
$1.75 \sim 2.25 u_p$	$A_4 =$ 8. 82%	0.5728–0.4455=0.1273	5.0521-3.9293=1.1228
$2.25 \sim 2.75 u_p$	$A_5 =$ 1. 58%	0.7071–0.5445=0.1626	1.1172-0.8603=0.2569
$2.75 \sim 3.25 u_p$	$A_6 =$ 0. 16%	0.7071–0.6081=0.099	0.1131-0.0973=0.0158
$3.25 \sim 3.75 u_p$	$A_7 =$0.0096%	0.7071–0.7071 = 0	0.0068-0.0068= 0
$3.75\ u_p \sim \infty$	$A_8 \approx$.0003%	0.7071–0.7071 = 0	0.00021-0.00021=0
$\sum_u \frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{45^o}(u)\Delta u =$ 31.7231 – 26.1096 = 5.6135			

Tab 7 (2) Data of $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{45^o} \Delta u \sim u$

And Fig 7 (2) is the corresponding graph.

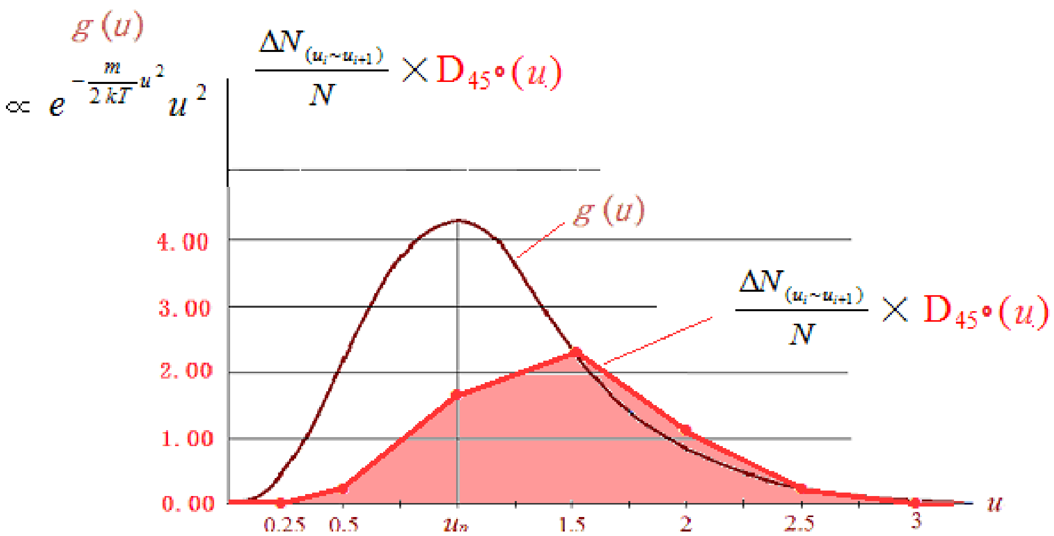
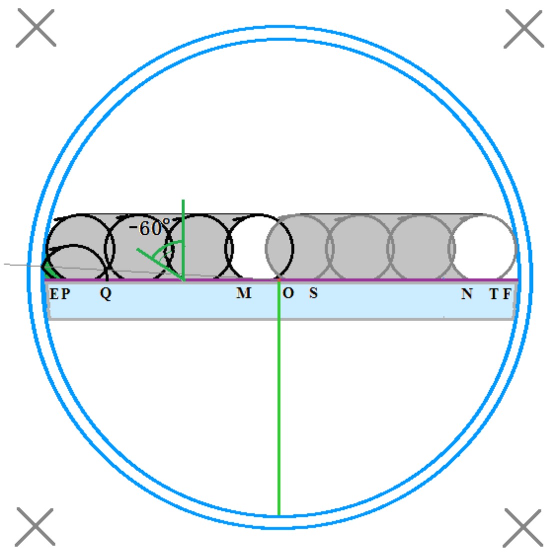


Fig 7 (2) Graph of $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{45^o}$ ~ u.

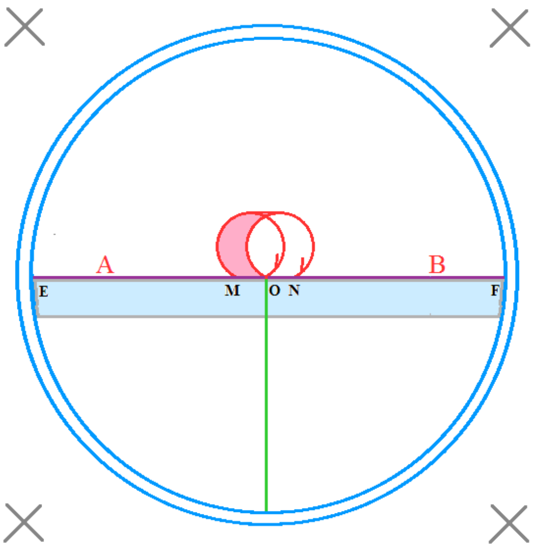
8. Trajectories of electrons of θ = - 60° and different speeds

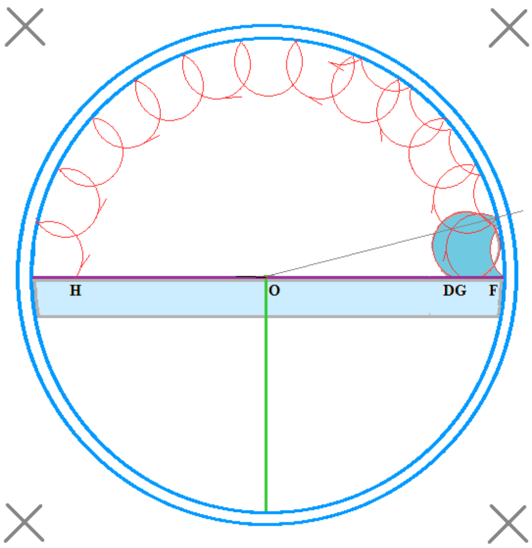
(1) Fig 8-1 θ = - 60° $u = 0.5u_p$ R = 2mm (B = 1.34 gauss)



A-glass-A & A-directly-A B-B (from ON to ST , ON = 57)
(EP green) (PM grey) (ON grey)

No electron migration between A and B due to these trajectories.





A-directly-B
(from MO to ON, red)
10/70 = 0.14 (MO = 10, EO = 70)

B-glass-A
(from G to H, etc., blue)
16/70 = 0.23 (DF =16, OF =70)

A-B 14% of the electrons of (- 60°, $0.5u_p$) emitted from A migrate to B.

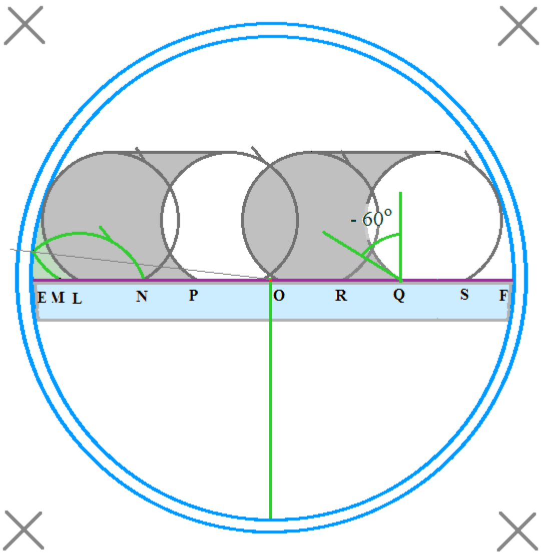
B-A 23% of the electrons of (- 60°, $0.5u_p$) emitted from B migrate to A.

For all the electrons of (-60°, $0.5u_p$), migration A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

$$\{(A\text{-}B) - (B\text{-}A)\}_{-60^\circ 0.5u_p} = 0.14 - 0.23 = -0.09$$

$$D_{-60^\circ}(0.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-60^\circ\, 0.5u_p} \cos 60^\circ = -0.09 \times 0.50 \approx -0.05$$

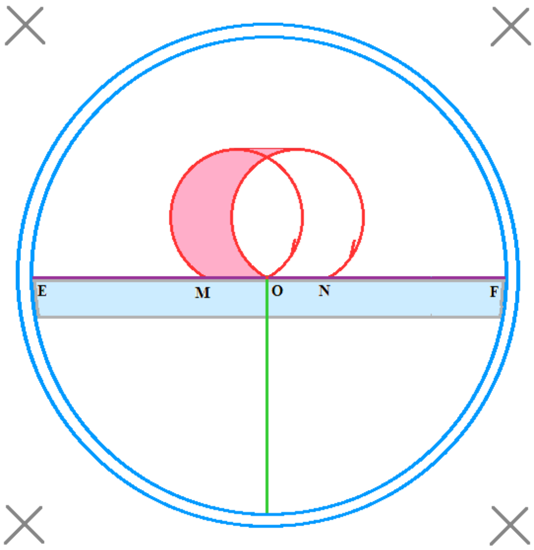
(2) Fig 8-2 θ = - 60° $u = u_p$ R = 4mm (cos-60° = 0.5000)



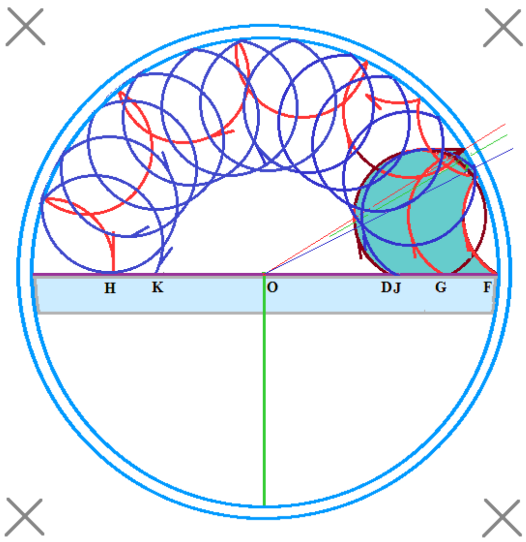
A-glass-A (from M to N, EL, green)
A-directly-A (from LP to NO, grey)

B-directly-B
(from OQ to RS, grey)

No electron migration between A and B due to these trajectories.



A-directly-B
(from MO to ON, MO = 20, red)
20/70 = 0.29



B-glass-A
(from G to H, etc., DF = 32, blue)
32/70 = 0.46

A-B 29% of the electrons of (- 60°, u_p) emitted from A migrate to B.

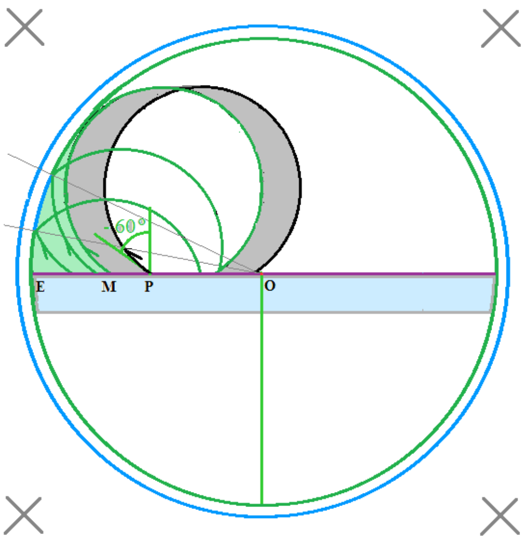
B-A 46% of the electrons of (- 60°, u_p) emitted from B migrate to A.

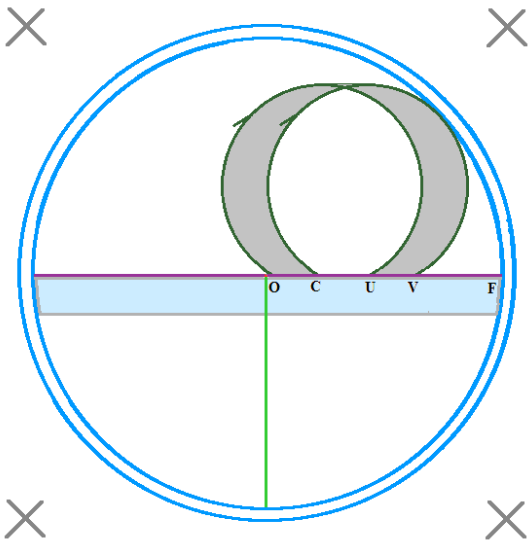
For all the electrons of (-60°, u_p), migration A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

$$\{(A\text{-}B) - (B\text{-}A)\}_{-60^\circ\, u_p} = 0.29 - 0.46 = -0.17$$

$$D_{-60^\circ}(u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-60^\circ\, u_p} \times cos\ 60^\circ = -0.17 \times 0.50 = -0.09$$

(3) Fig 8-3 θ = - 60° u = 1.5 u_p R = 6mm

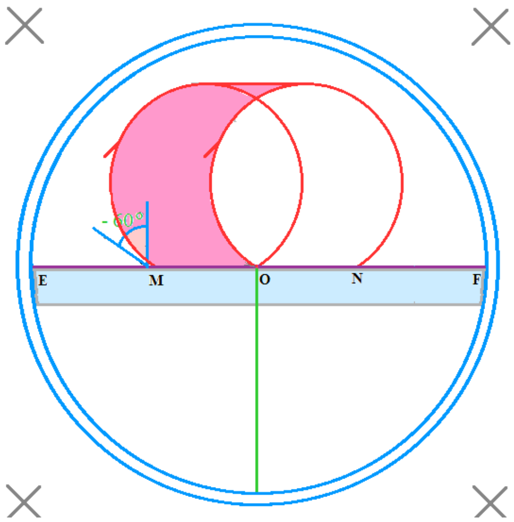


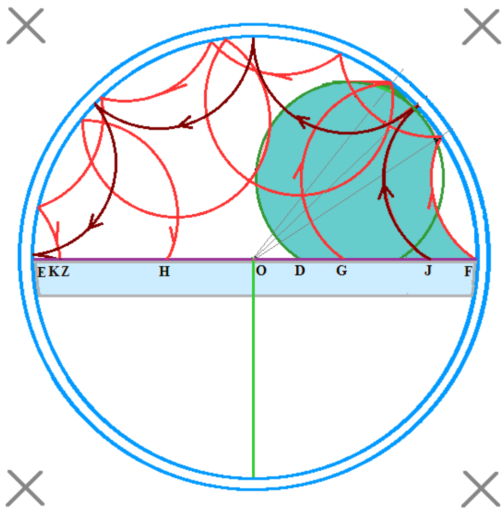


A-glass-A (EM, green)
A-directly-A (MP, grey)

B-directly-B
(from OC to UV, grey)

No electron migration between A and B due to these trajectories.





A-directly-B
(from MO to ON, red)
MO/EO = 33/70 = 0.47

B-glass-A
(from G to H, J to K, F to Z, etc., blue)
DF/OF = 56/70 = 0.80

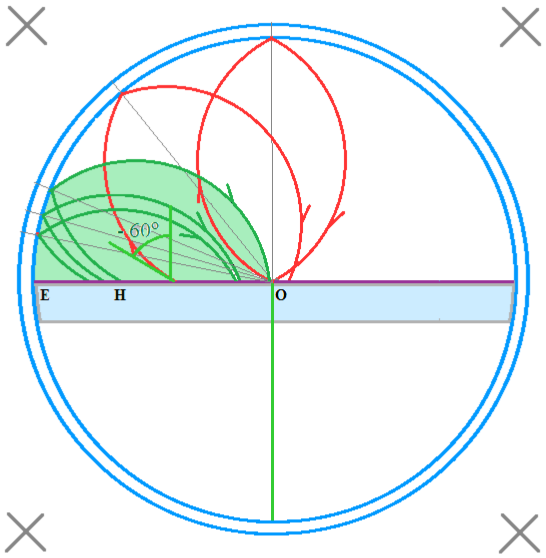
A-B 47% of the electrons of (- 60°, 1.5u_p) emitted from A migrate to B.
B-A 80% of the electrons of (- 60°, 1.5u_p) emitted from B migrate to A.

For all the electrons of (-60°, 1.5u_p), migration A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

$$\{(A\text{-}B) - (B\text{-}A)\}_{-60^\circ\ 1.5u_p} = 0.47 - 0.80 = -0.33$$

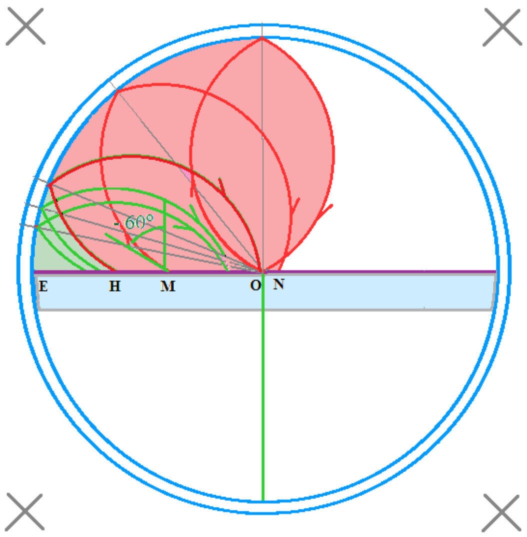
$$D_{-60^\circ}(1.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-60^\circ\ 1.5u_p} \cos 60^\circ = -0.33 \times 0.50 = -0.17$$

(4) Fig 8-4 θ = - 60° u = 2 u_p R = 8mm



A-glass-A 26/70 = 0.37 (EH = 26, EO = 70) (green)

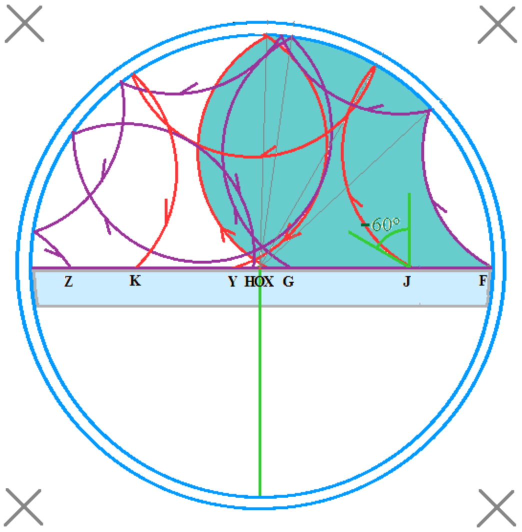
No electron migration between A and B due to these trajectories.



A-glass-B

(from H to O, M to N, O to O, etc., red)

HO/EO =44/70 = 0.63



B-glass-A

(from G to H, J to K, X to Y, F to Z, blue)

70/70 = 1.00

A-B 63% of the electrons of (- 60°, 2u_p) emitted from A migrate to B.

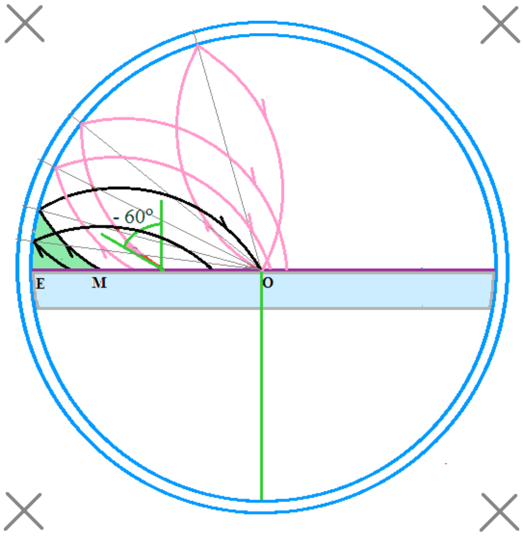
B-A 100% of the electrons of (- 60°, 2u_p) emitted from B migrate to A.

For all the electrons of (- 60°, 2u_p), migration A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

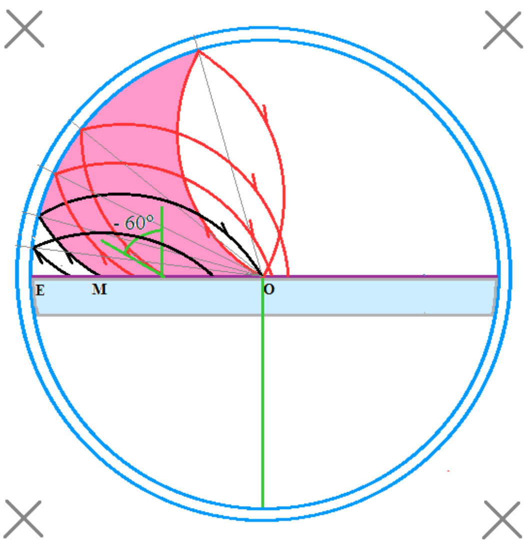
$$\{(A\text{-}B) - (B\text{-}A)\}_{-60^\circ\ 2u_p} = 0.63 - 1.00 = -0.37$$

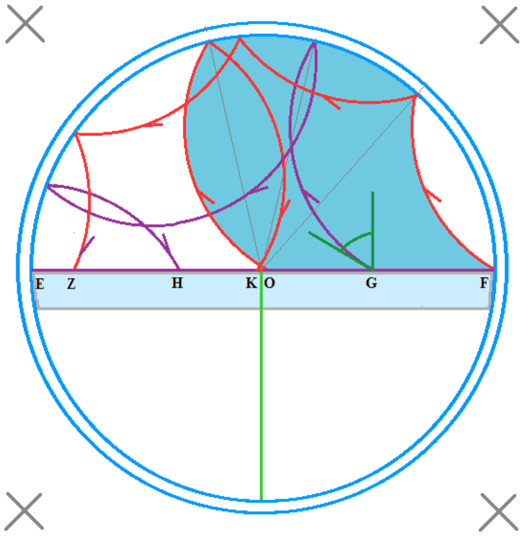
$$D_{-60^\circ}(2u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-60^\circ\ 2u_p} \cos 60^\circ = -0.37 \times 0.50 = -0.19$$

(5) Fig 8-5 θ = - 60° $u = 2.5\ u_p$ R = 10mm



A-glass-A (20/70 = 0.286)
(EM = 20, FO =70) (green)
No electron migration between A and B due to these trajectories.





A-glass-B 50/70 = 0.714
(MO = 50, EO = 70) (red)
50/70 = 0.714

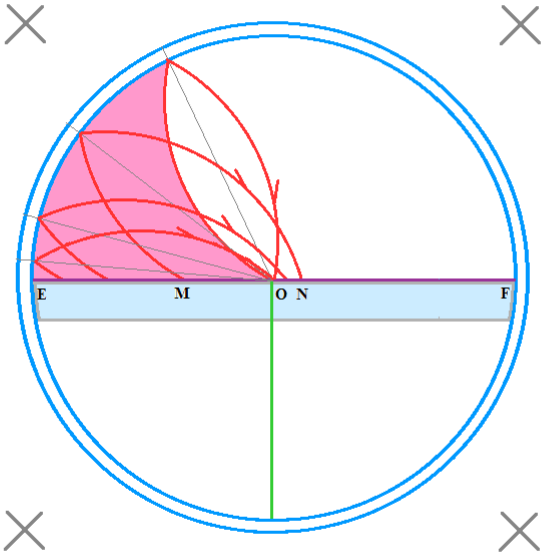
B-glass-A 70/70 = 1.00 (DF = 70)
(from G to H, O to K, F to Z, blue)
70/70 = 1.00

A-B 71% of the electrons of (- 60°, 2.5u_p) emitted from A migrate to B.
B-A 100% of the electrons of (- 60°, 2.5u_p) emitted from B migrate to A.

For all the electrons of (-60°, 2.5u_p), migration A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

$$\{(A\text{-}B) - (B\text{-}A)\}_{-60^\circ\ 2.5u_p} = 0.71 - 1.00 = -0.29$$
$$D_{-60^\circ}(2.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-60^\circ\ 2.5u_p} \cos 60^\circ = -\ 0.29 \times 0.50 = -0.15$$

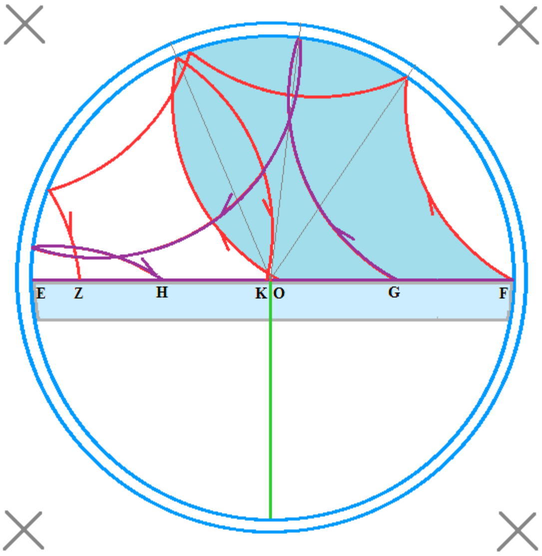
(6) Fig 8-6 θ = - 60° $u = 3u_p$ R = 12mm



A-glass-B

(from M to N, etc., red) (EO = 70)

70/70 = 1.00



A-glass-A

(from G to H, O to K, F to Z, etc., blue) (OF = 70)

70/70 = 1.00

A-B 100% of the electrons of (- 60°, $3u_p$) emitted from A migrate to B.

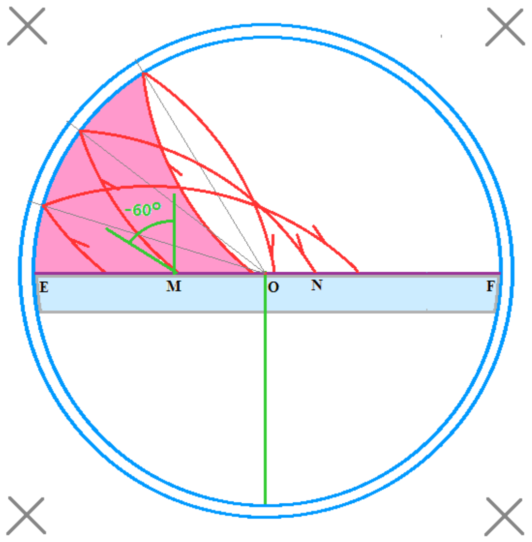
B-A 100% of the electrons of (- 60°, $3u_p$) emitted from B migrate to A.

For all the electrons of (-60°, $3u_p$), migration A-B equals B-A and their contributions to the output current cancel each other.

$$\{(\text{A-B}) - (\text{B-A})\}_{-60^{\circ}\,3up} = 1.00 - 1.00 = 0$$

$$D_{-60^{\circ}}(3u_p) = \{(\text{A-B}) - (\text{B-A})\}_{-60^{\circ}\,3up}\, \cos 60^{\circ} = 0$$

(7) Fig 8-7 θ = - 60º $u = 4.5\ u_p$ R = 18mm

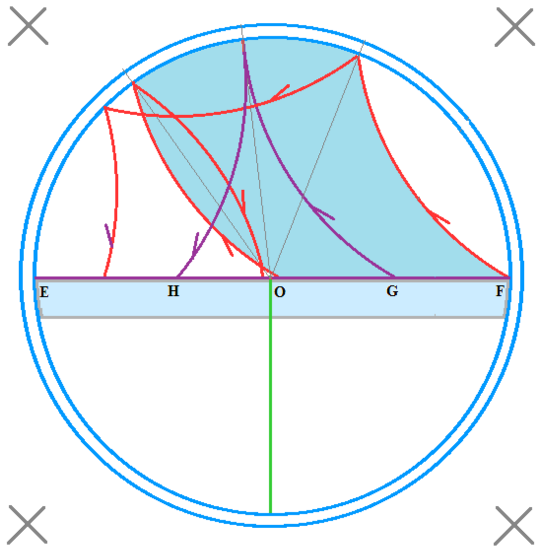


A-glass-B

(from M to N, etc., red)

EO/EO = 70/70 = 1.00

A-B 100% of the electrons of (θ = - 60º, $u = 4.5u_p$) emitted from A migrate to B.



B-glass-A

(from G to H, etc., blue)

OF/OF = 70/70 = 1.00

B-A 100% of the electrons of (θ = - 60º, $u = 4.5u_p$) emitted from B migrate to A.

For all the electrons of (-60º, $4.5u_p$), migration A-B equals B-A, and their contributions to the output current cancel each other.

$\{(A\text{-}B) - (B\text{-}A)\}_{-60^o\ 4.5up} = 1.00 - 1.00 = 0$

$D_{-60^o}(4.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-60^o\ 4.5up}\ cos\ 60^o = 0$

Every 100 + 100 electrons (1.00 = 100% from A, 1.00 = 100% from B) of exiting angle $\theta = -60^{o}$ and of different speeds contribute to the electron migration between A and B differently, as list in Tab 8 (1) ($cos\text{-}60^{o} = 0.5000$),

$$D_{-60^{o}}(u) = \{(A\text{-}B) - (B\text{-}A)\}_{-60^{o}} \cos\theta \sim u,$$

speed u	$\{(A\text{-}B) - (B\text{-}A)\}_{-60^{o}}$	$(A\text{-}B) - (B\text{-}A)\}_{-60^{o}} \cos\theta$
Fig 8-1 $u = 0.5u_p$	0.14 - 0.23 = - 0.09	0.07 – 0.115 = -0.045
Fig 8-2 $u = u_p$	0.29 - 0.46 = - 0.17	0.145 – 0.23= -0.085
Fig 8-3 $u = 1.5u_p$	0.47 - 0.80 = - 0.33	0.235 – 0.40 = -0.165
Fig 8-4 $u = 2u_p$	0.63 - 1.00 = - 0.37	0.315 – 0.500 = -0.165
Fig 8-5 $u = 2.5u_p$	0.71 - 1.00 = - 0.29	0.355 – 0.500 = -0.145
Fig 8-6 $u = 3u_p$	1.00 - 1.00 = 0	0.500 - 0.500 = 0.00
Fig 8-7 $u = 4.5u_p$	1.00 - 1.00 = 0	0.500 - 0.500 = 0.00

Tab. 8 (1) $D_{-60^{o}}(u) = \{(A\text{-}B) - (B\text{-}A)\}_{-60^{o}} \cos\theta \sim u$

And Fig 8(1) is the corresponding graph

$$D_{-60^o}(u) = \{(A\text{-}B) - (B\text{-}A)\}_{-60^o} \cos\theta \sim u .$$

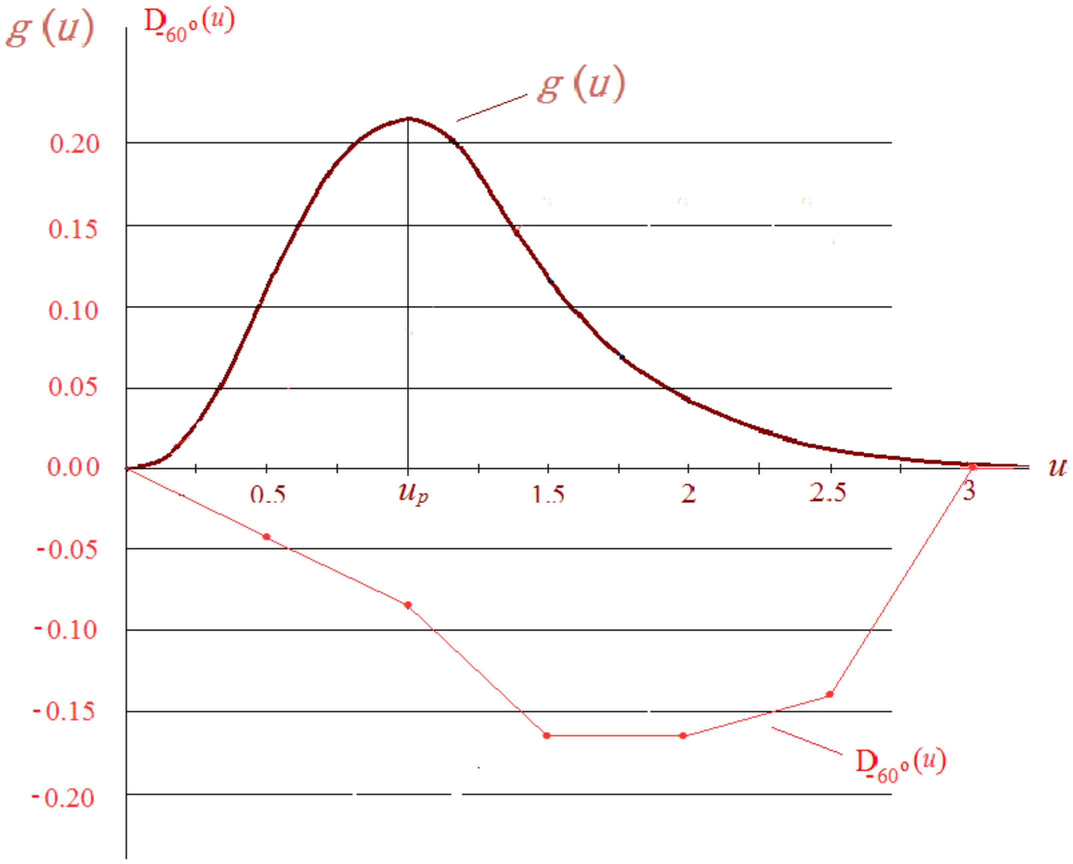


Fig 8 (1) Graph of $D_{-60^o}(u) = (A\text{-}B) - (B\text{-}A)\}_{-60^o} \cos\theta \sim u$

Take Maxwell's speed distribution $g(u)$ into account to derive the actual contributions of the thermal electrons of θ = -60° with respect to different speed ranges, i.e., $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N}$ ×D-60° $(u)\varDelta u$ ~ u, as shown in Tab 8 (2).

Speed range $\varDelta u$	$\frac{\Delta N_{(u_i \sim u_{i+1})}}{N}$	D-60° (u) (A-B) – (B-A)} -60° cosθ	$\frac{\Delta N_{(u_i \sim u_{i+1})}}{N}$ ×D-60° $(u)\varDelta u$
0.25~0.75u_p	A_1 = 18. 74%	0.07 – 0.115 = -0.045	1.3118-2.1551= -0.8433
0.75~1.25u_p	A_2 = 39. 83%	0.145 – 0.23= -0.085	5.7754-9.1609 = -3.3855
1.25~1.75u_p	A_3 = 26. 71%	0.235 – 0.40 = -0.165	6.2769-10.6840=-4.4071
1.75 ~ 2.25u_p	A_4 = 8. 82%	0.315 – 0.500 = -0.185	2.7783- 4.41= -1.6317
2.25 ~ 2.75u_p	A_5= 1. 58%	0.355 – 0.500 = -0.145	0.5609-0.79 = -0.2291
2.75 ~ 3.25u_p	A_6 = 0. 16%	0.500 - 0.500 = 0.00	0.08 -0.08 = 0
3.25~ 3.75u_p	A_7=0.0096%	0.500 - 0.500 = 0.00	0. 0048−0. 0048 = 0
3.75u_p ~ ∞	A_8≈.0003%	0.500 - 0.500 = 0.00	0.00015-0.00015 = 0
$\Sigma_u \frac{\Delta N_{(u_i \sim u_{i+1})}}{N}$ ×D-60°$(u)\varDelta u$ = 16.7883 – 27.285 = -10.4967			

Tab 8 (2) The actual contributions of electrons of θ = - 60° with different speeds, $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N}$ ×D-60° $\varDelta u$ ~ u .

Fig 8 (2) is the corresponding graph.

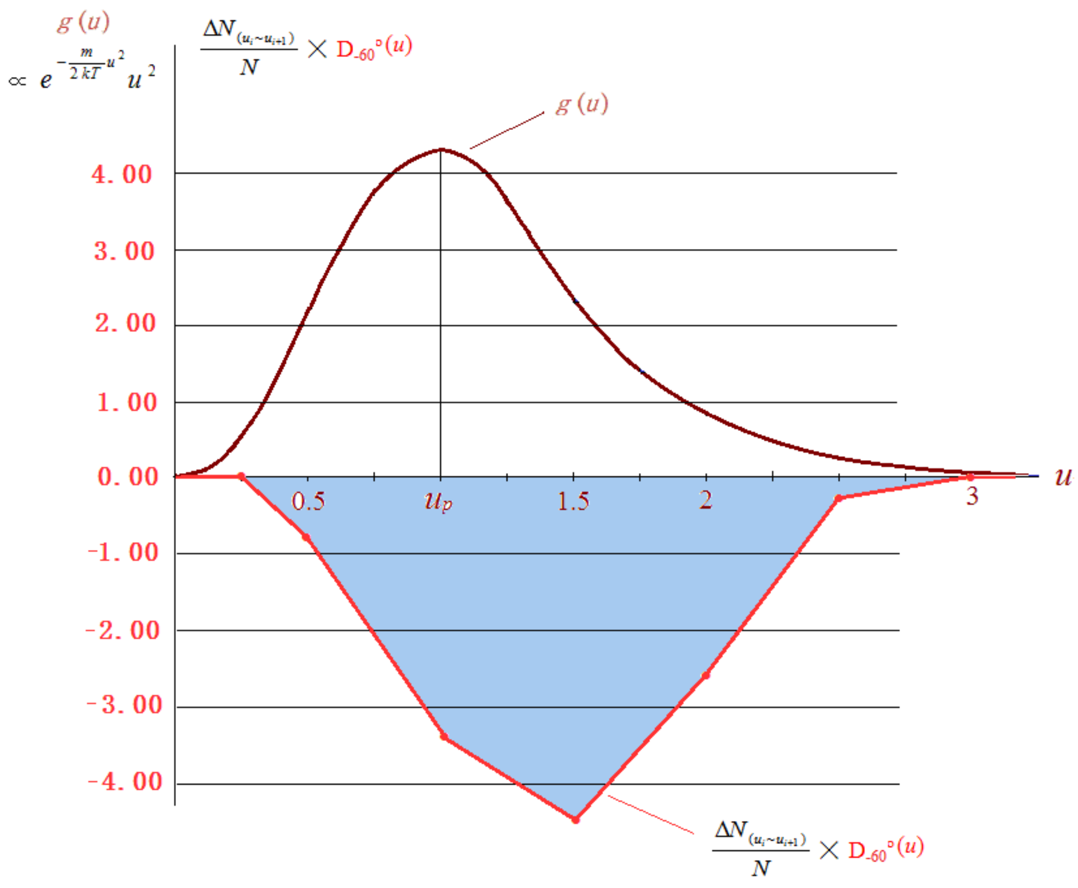
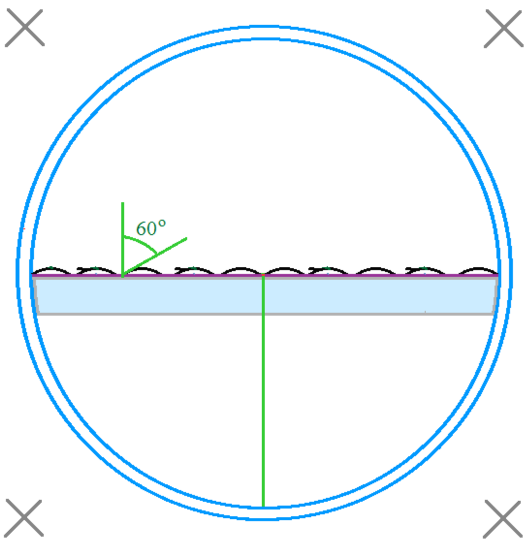
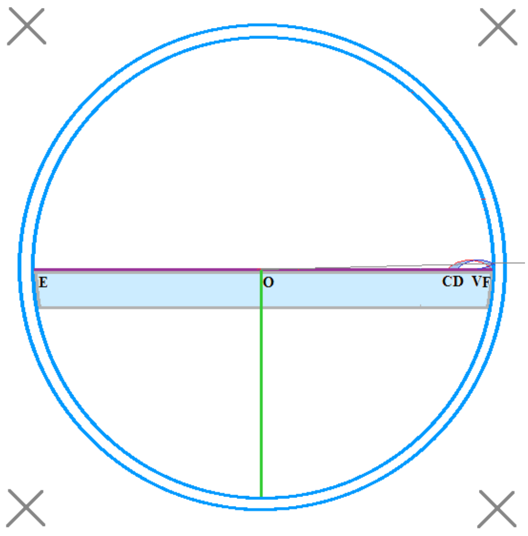


Fig 8 (1) Graph of the actual contributions of electrons of θ = - 60° with different speeds, $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{-60^\circ} \sim u$.

9. Trajectories of electrons of θ = 60° and different speeds

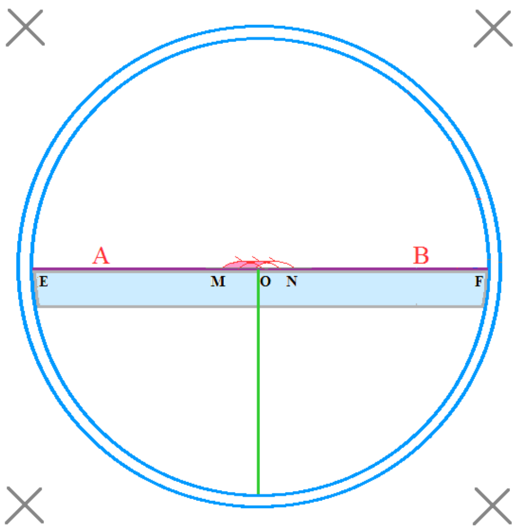
(1) Fig 9-1 θ = 60° $u = 0.5u_p$ R = 2mm (B = 1.34 gauss)

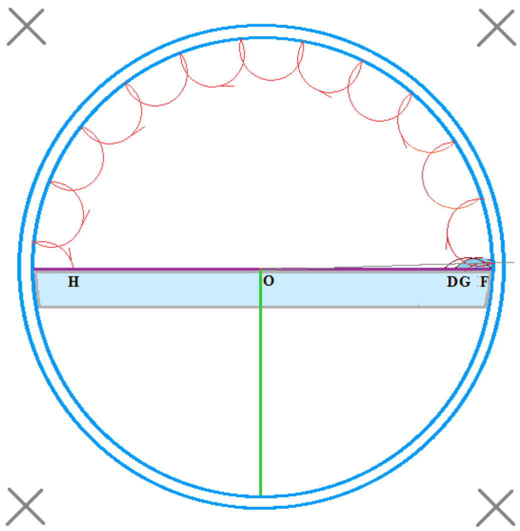




A-directly-A B-directly-B B-glass-B CD /OF = 3/70 = 0.04

No electron migration between A and B due to these trajectories.





A-directly-B
(from MO to ON, red.)
MO/EO =12.5/70 = 0.18

B-glass-A
(from G to H, etc., blue)
DF/OF = 9.5/70 = 0.14

A-B 18% of the electrons of (60°, $0.5u_p$) emitted from A migrate to B.

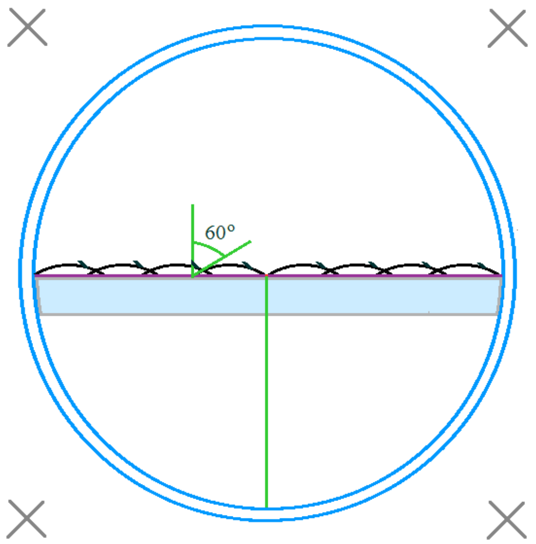
B-A 14% of the electrons of (60°, $0.5u_p$) emitted from B migrate to A.

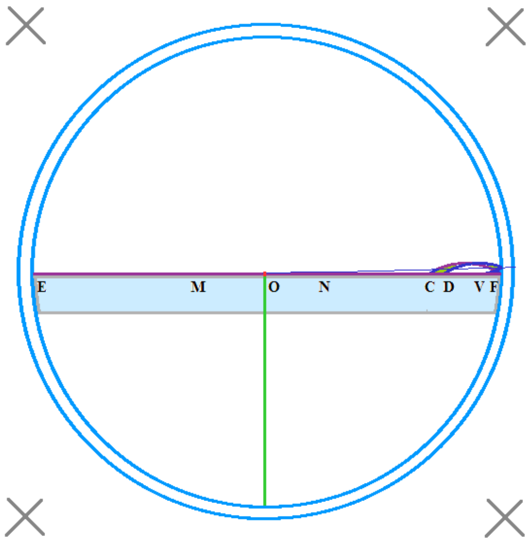
For all the electrons of (60°, $0.5u_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{60^\circ\, 0.5u_p} = 0.18 - 0.14 = 0.04$$

$$D_{60^\circ}(0.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{60^\circ\, 0.5u_p} \times \cos 60^\circ = 0.04 \times 0.50 = 0.02$$

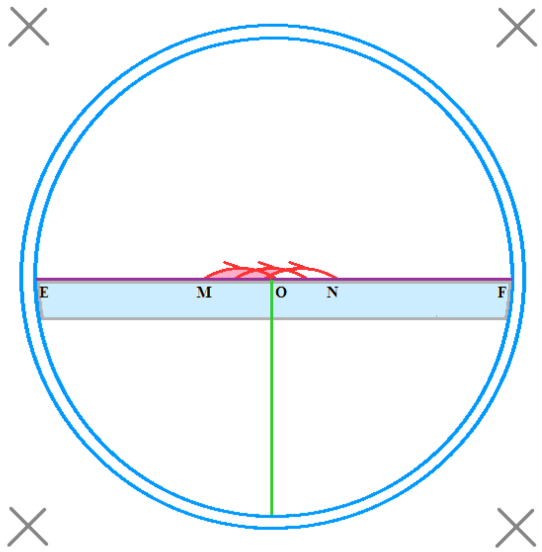
(2) Fig 9-2 $\theta = 60^{\circ}$ $u = u_p$ R = 4mm ($cos 60^{\circ} = 0.5000$)

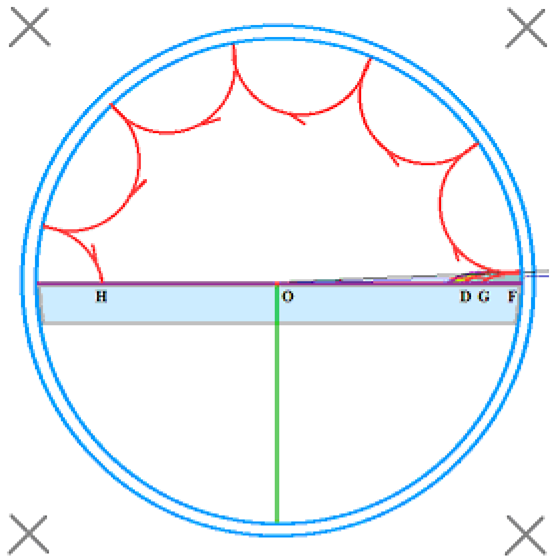




A-directly-A B-directly-B B-glass-B

No electron migration between A and B due to these trajectories.





A-directly-B
(from MO to ON, red)
MO/EO = 20/70 = 0.29

B-glass-A
(from G to H, etc., blue)
DF/OF=15.5/70 = 0.221

A-B 29% of the electrons of (60°, u_p) emitted from A migrate to B.

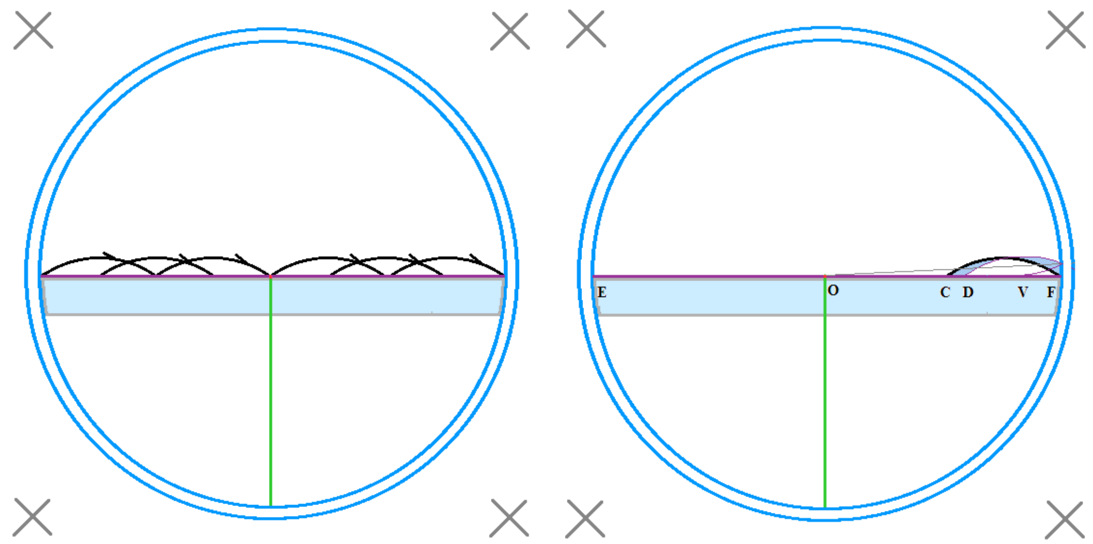
A-B 22% of the electrons of (60°, u_p) emitted from B migrate to A.

For all the electrons of (60°, u_p), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{60^{\circ}\, u_p} = 0.29 - 0.22 = 0.07$$

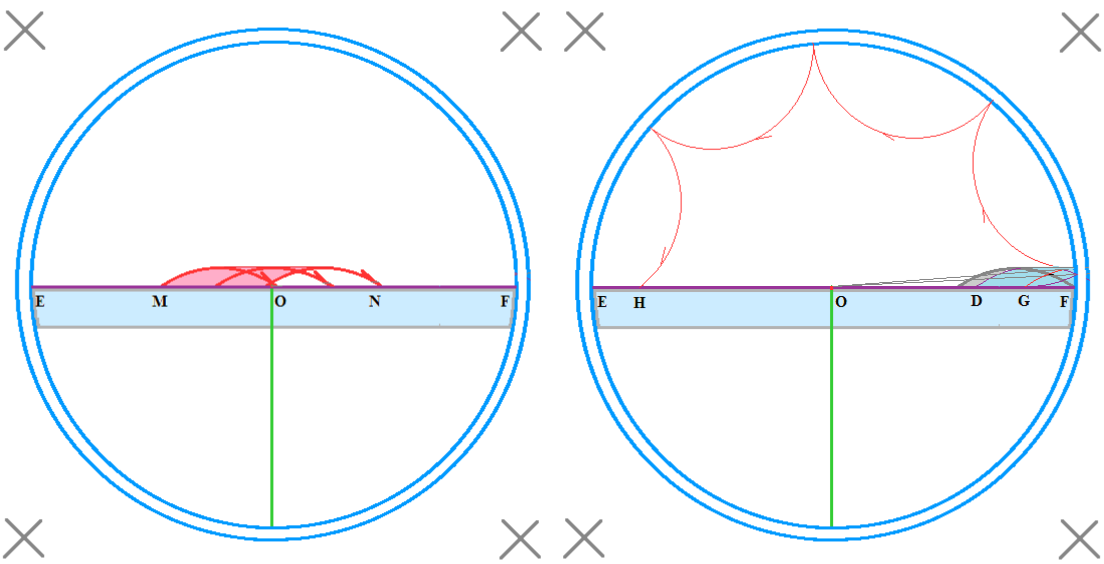
$$D_{60^{\circ}}(u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{60^{\circ}\, u_p} \cos 60^{\circ} = 0.07 \times 0.50 \approx 0.04$$

(3) Fig 9-3 θ = 60° $u = 1.5u_p$ R = 6mm



A-directly-A B-directly-B B-glass-B 5/70 = 0.07 (CD = 5)

No electron migration between A and B due to these trajectories.



A-directly-B MO/EO = 33/70 = 0.47 B-glass-A DF/OF = 28/70 = 0.40

(from MO to ON, red) (from G to H, etc., blue)

MO/EO = 33/70 = 0.47 DF/OF = 28/70 = 0.40

A-B 47% of the electrons of (60°, $1.5u_p$) emitted from A migrate to B.

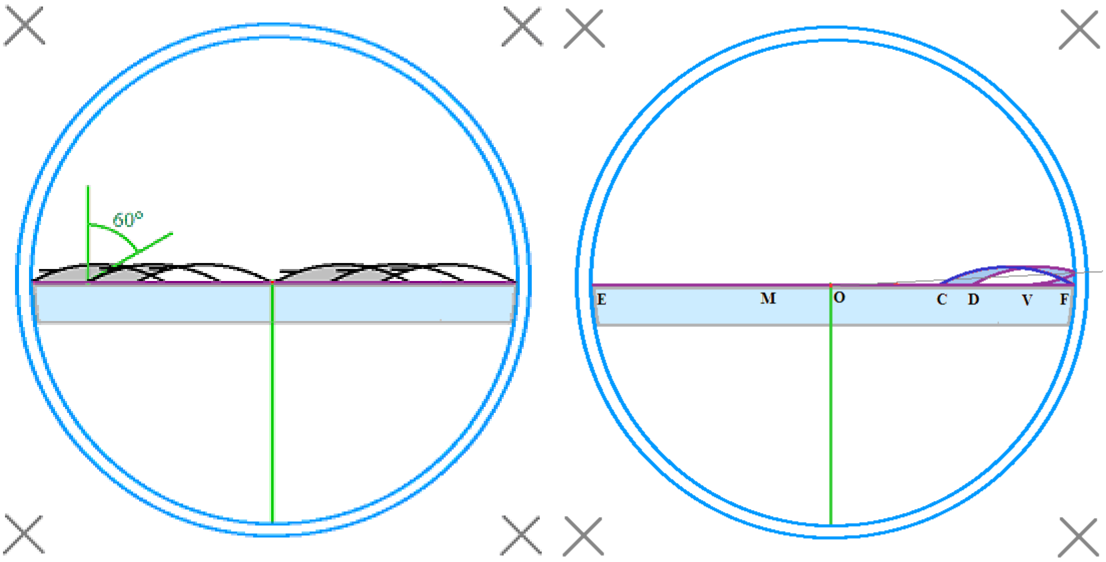
B-A 40% of the electrons of (60°, $1.5u_p$) emitted from B migrate to A.

For all the electrons of (θ = 60°, $u = 1.5u_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{60^\circ\ 1.5u_p} = 0.47 - 0.40 = 0.07$$

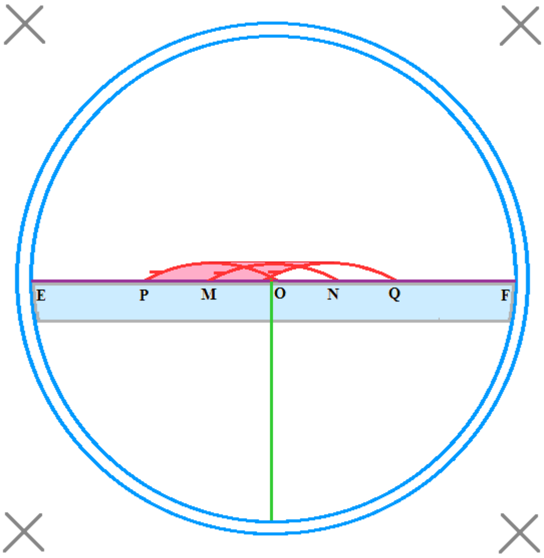
$$D_{60^\circ}(1.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{60^\circ\ 1.5u_p} \times \cos 60^\circ = 0.07 \times 0.50 = 0.04$$

(4) Fig 9-4 θ = 60° $u = 2u_p$ R = 8mm



A-directly-A B-directly-B B-glass-B CD/OF = 10/70 = 0.14

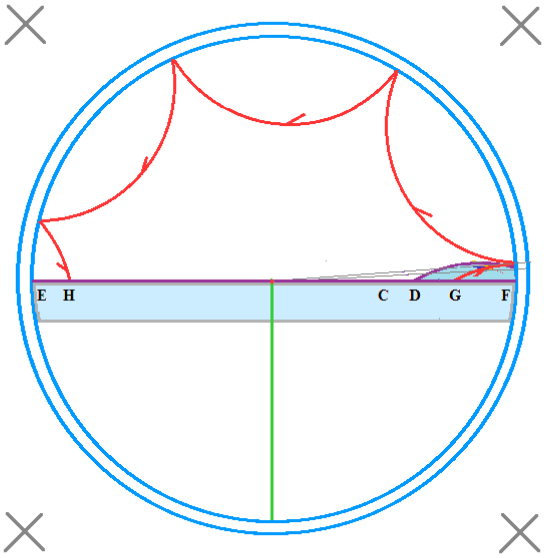
No electron migration between A and B due to these trajectories.



A-directly-B

(from P to O, M to N, O to Q, etc., red)

PO/EO = 39/70 = 0.56 (PO = 39)



B-glass-A

(from G to H, etc., blue)

DF.OF = 29/70 = 0.414 (DF = 29)

A-B 56% of the electrons of (60°, $2u_p$) emitted from A migrate to B.

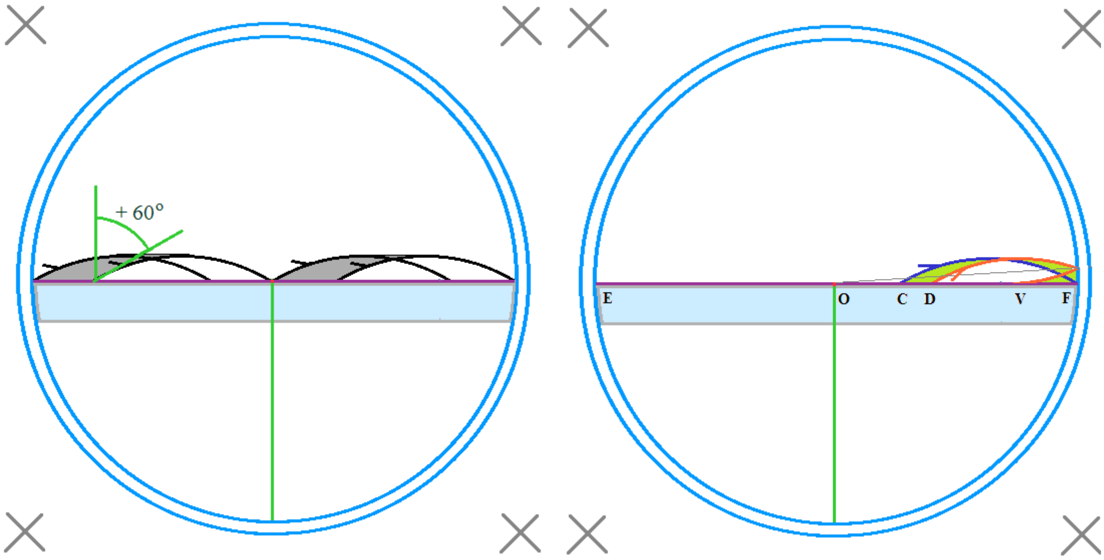
B-A 41% of the electrons of (60°, $2u_p$) emitted from B migrate to A.

For all the electrons of (60°, $2u_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{60^\circ\ 2up} = 0.56 - 0.41 = 0.14$$

$$D_{60^\circ}(2u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{60^\circ\ 2up} \times \cos 60^\circ = 0.14 \times 0.50 = 0.07$$

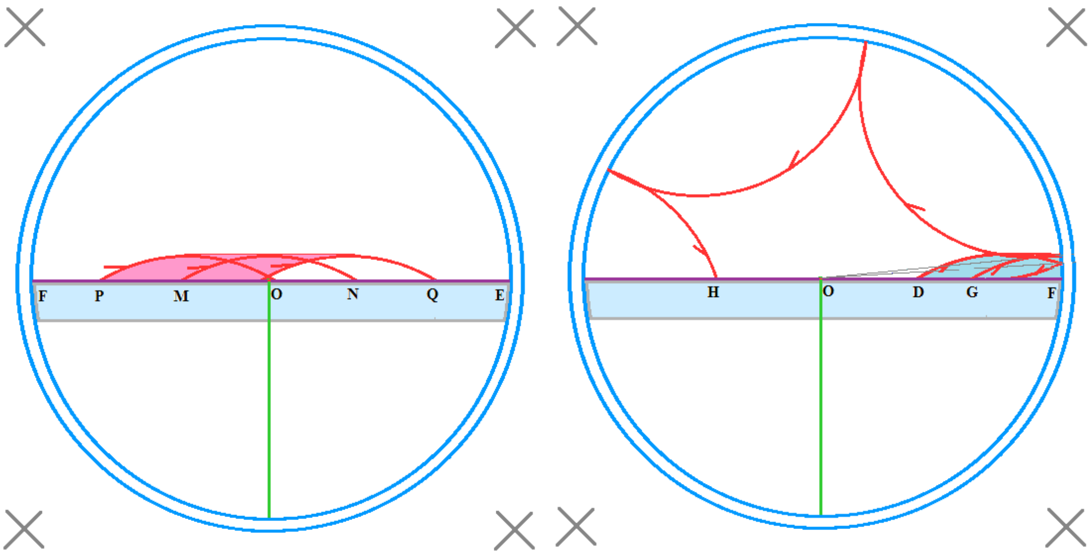
(5) Fig 9-5 θ = 60° $u = 2.5u_p$ R = 10mm



A-directly-A B-directly-B B-glass-B

CD/OF = 9.5/70 = 0.14

No electron migration between A and B due to these trajectories.



A-directly-B
(from P to O, M to N, O to Q, etc., red)
PO/OE=51.5/70=0.74

A-glass-A
(from G to H, etc., blue)
DF/OF=42/70=0.60

A-B 74% of the electrons of (60°, $2.5u_p$) emitted from A migrate to B.

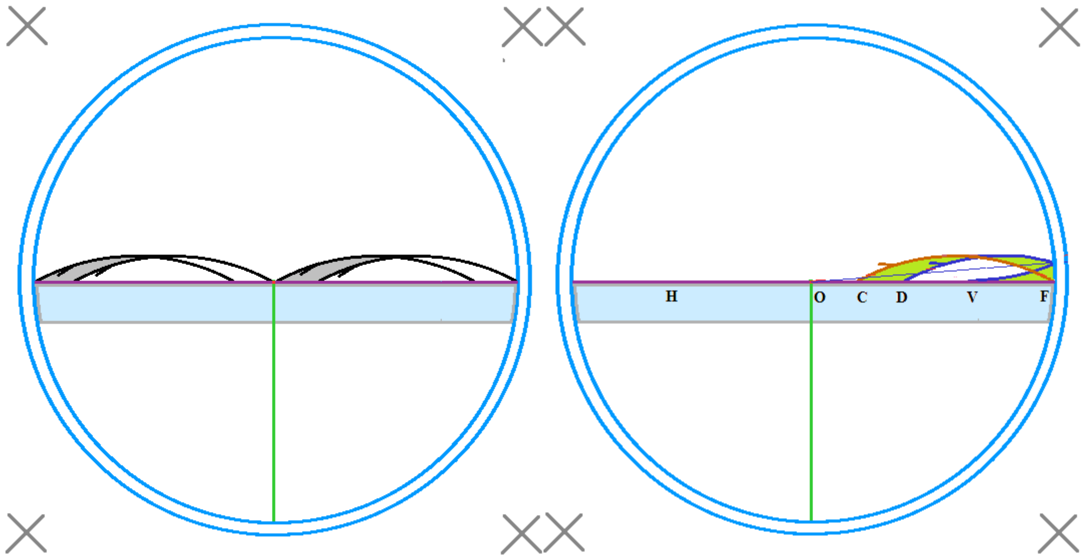
B-A 60% of the electrons of (60°, $2.5u_p$) emitted from B migrate to A.

For all the electrons of (60°, $2.5u_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{60^\circ\, 2.5up} = 0.74 - 0.60 = 0.14$$

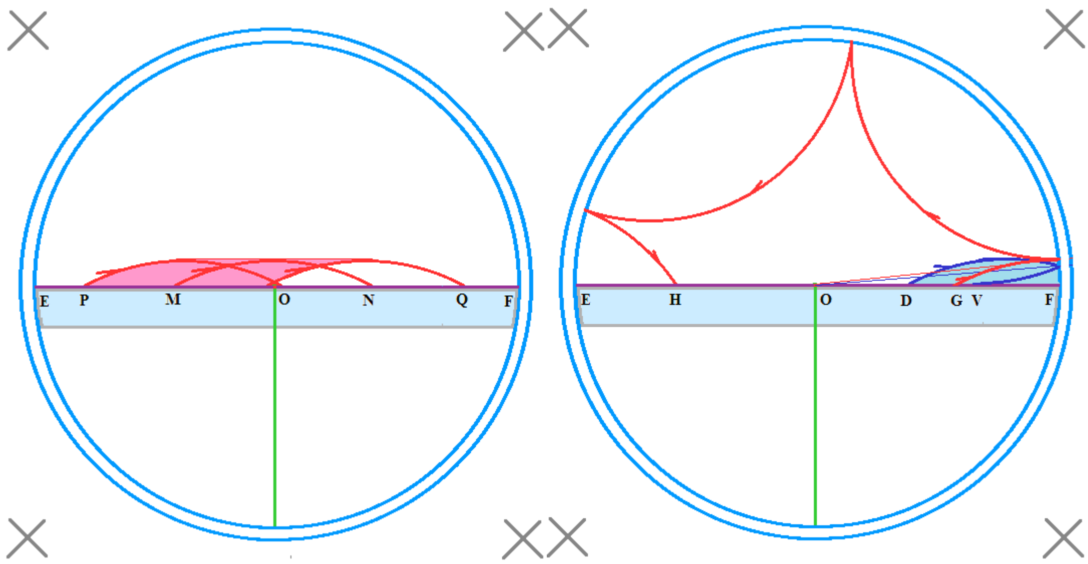
$$D_{60^\circ}(2.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{60^\circ\, 2.5up} \times \cos 60^\circ = 0.14 \times 0.50 = 0.07$$

(6) Fig 9-6 $\theta = 60^{o}$ $u = 3u_p$ $R = 10\text{mm}$



A-directly-A B-directly-B B-glass-B

No electron migration between A and B due to these trajectories.



A-directly-B
(from P to O, M to N, O to Q, etc., red)
PO/EO = 57/70 = 0.81

B-glass-A
(from G to H, etc., blue)
DF/OF = 43/70 = 0.61

A-B 81% of the electrons of (60^{o}, $3u_p$) emitted from A migrate to B.

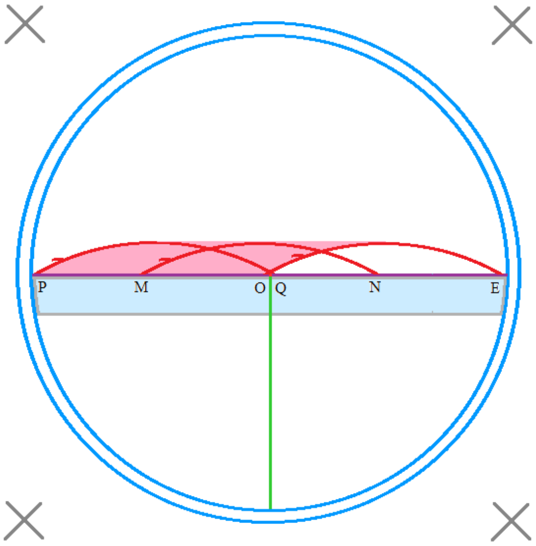
B-A 61% of the electrons of (60^{o}, $3u_p$) emitted from B migrate to A.

For all the electrons of (60^{o}, $3u_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

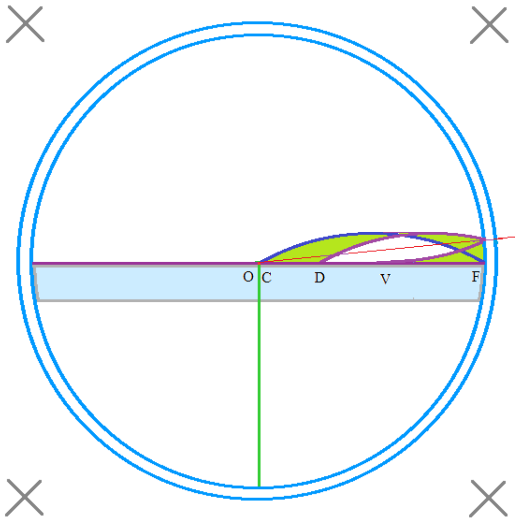
$$\{(A\text{-}B) - (B\text{-}A)\}_{60^{o}\,3up} = 0.81 - 0.61 = 0.20$$

$$D_{60^{o}}(3u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{60^{o}\,3up} \times \cos 60^{o} = 0.20 \times 0.50 = 0.10$$

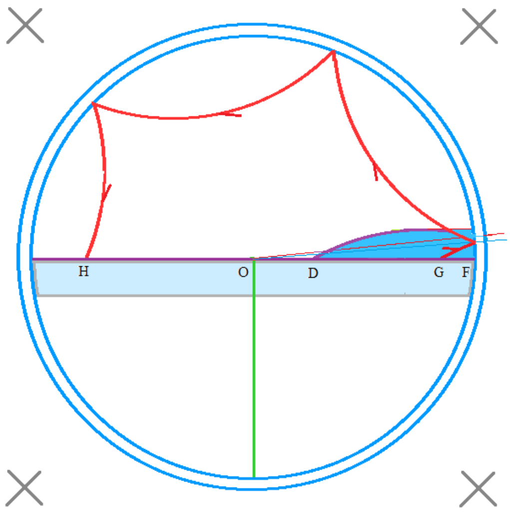
(7) Fig 9-7 θ = 60° $u = 3.5u_p$ R = 14mm



A-directly-B
(from P to O, M to N, O to E, etc., red)
PO/EO = 70/70 = 1.0



B-glass-B
(from CD to FV, etc., green)
CD/OF = 17.5/70 = 0.25



B-glass-A
(from G to H, etc., blue)
DF/OF = 52.7/70 = 0.75

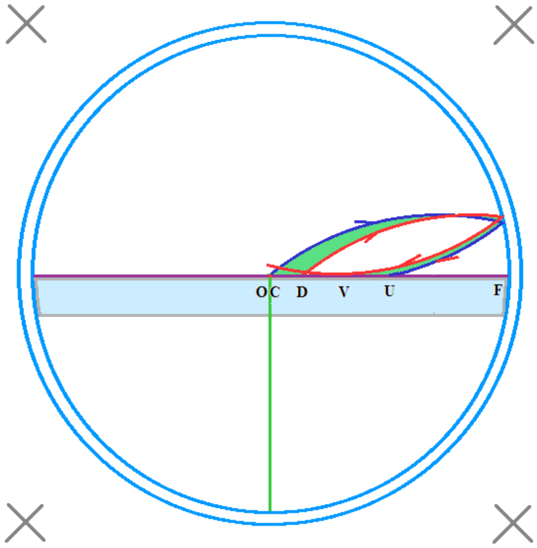
A-B 100% of the electrons of (60°, $3.5u_p$) emitted from A migrate to B.
B-A 75% of the electrons of (60°, $3.5u_p$) emitted from B migrate to A.

For all the electrons of (60°,$3.5u_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

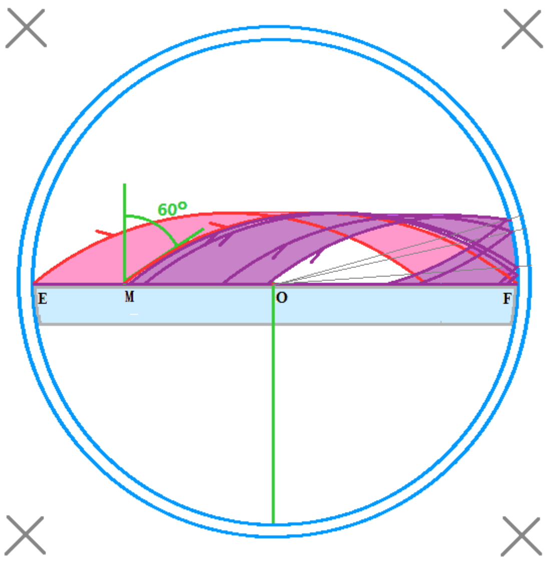
$$\{(A\text{-}B) - (B\text{-}A)\}_{60^\circ\, 3.5u_p} = 1.00 - 0.75 = 0.25$$

$$D_{60^\circ}(3.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{60^\circ\, 3.5u_p} \times \cos 60^\circ = 0.25 \times 0.50 = 0.125$$

(8) Fig 9-8 $\theta = 60^o$ $u = 4.5u_p$ $R = 18mm$



B-glass-B
(from C to U, D to V, etc., green)
8.5/70 = 0.12 (OD = CD = 8.5, OF = 70)
No electron migration between A and B due to these trajectories.

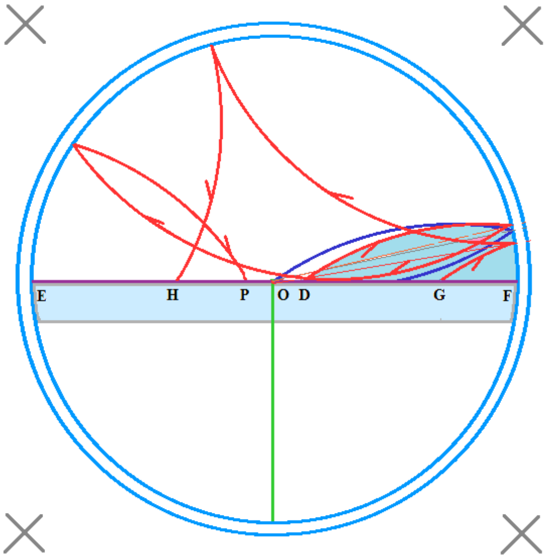


A-directly-B (from EM, red)   A-glass-B (from MO, violet)

A-B (total) (red + violet, EM+MO=EO=70)

70/70 = 1.00

A-B 100% of the electrons of (60^o, $4.5u_p$) emitted from A migrate to B.



B-glass-A

(from G to H, D to P, etc., blue)

DF/OF = 61.5/70 = 0.879

B-A 87.9% electrons of (60°, 4.5u_p) emitted from B migrate to A.

For all the electrons of (60°, 4.5u_p), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{60^o\ 4.5u_p} = 1.00 - 0.879 = 0.121 \approx 0.12$$
$$D_{60^o}(4.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{60^o\ 4.5u_p} \times \cos 60^o = 0.12 \times 0.50 = 0.06$$

Every 100 + 100 electrons (1.00 = 100% from A, 1.00 = 100% from B) of exiting angle $\theta = 60^o$ and of different speeds contribute to the electron migration between A and B differently, as list in Tab 9 (1) ($cos60^o$ = 0.5000),

$$D_{45^o}(u) = \{(A\text{-}B) - (B\text{-}A)\}_{60^o} \cos\theta \sim u ,$$

speed u	$\{(A\text{-}B)–(B\text{-}A)\}_{60^o} \sim u$	$D_{60^o}(u)$ $\{(A\text{-}B)–(B\text{-}A)\}_{60^o} \cos\theta \sim u$
Fig 9-1 $u = 0.5u_p$	0.18 – 0.14 = 0.04	0.09 – 0.07 = 0.02
Fig 9-2 $u = u_p$	0.29 – 0.22 = 0.07	0.145 -0.11 = 0.035
Fig 9-3 $u = 1.5u_p$	0.47 – 0.40 = 0.07	0.235 – 0.20 = 0.035
Fig 9-4 $u = 2u_p$	0.56 – 0.41 = 0.14	0.28 – 0.205 = 0.07
Fig 9-5 $u = 2.5u_p$	0.74 – 0.60 = 0.14	0.37 – 0.30 = 0.07
Fig 9-6 $u = 3u_p$	0.81 – 0.61 = 0.20	0.405 - 0.305 = 0.100
Fig 9-7 $u = 3.5u_p$	1.00 – 0.75 = 0.25	0.5 – 0.375 = 0.125
Fig 9-8 $u = 4.5u_p$	1.00 – 0.879 = 0.12	0.5 – 0.4395 = 0.0605

Tab 9 (1) $D_{60^o}(u) = \{(A\text{-}B) – (B\text{-}A)\}_{60^o} \cos\theta \sim u$

Fig 9(1) is the corresponding graph.

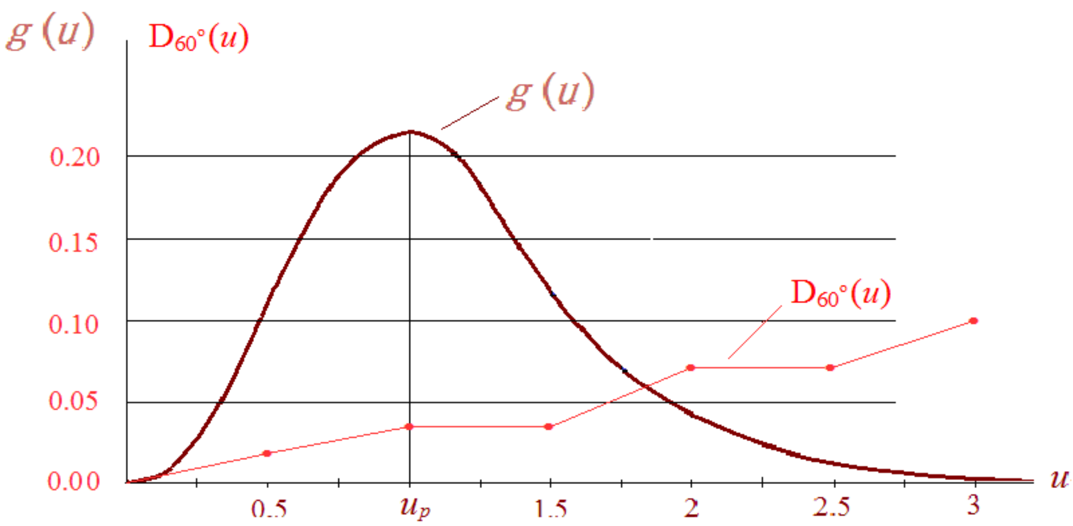


Fig 9(1) $D_{60^o}(u) = \{(A\text{-}B) - (B\text{-}A)\}_{60^o} \cos\theta \sim u$

Take Maxwell's speed distribution $g(u)$ into account to derive the actual contributions of the thermal electrons of θ = 60° with different speed ranges, $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{60^o}(u) \Delta u \sim u$, as shown in Tab 9 - 2.

Speed range Δu	$\frac{\Delta N_{(u_i \sim u_{i+1})}}{N}$	$D_{60^o}(u)$ {(A-B)–(B-A)} $_{60^o}$ $\cos\theta \sim$ \u	$\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{60^o}(u) \Delta u$
$0.00 \sim 0.25u_p$	$A_0 \approx$ 0. 004	≈ 0	≈ 0
$0.25 \sim 0.75u_p$	$A_1 =$ 18. 74%	0.09 – 0.07 = 0.02	1.6866–1.3118=0.3748
$0.75 \sim 1.25u_p$	$A_2 =$ 39. 83%	0.145 -0.11 = 0.035	5.7754-4.3813=1.3941
$1.25 \sim 1.75u_p$	$A_3 =$ 26. 71%	0.235 – 0.20 = 0.035	6.2769-5.3420=0.9349
$1.75 \sim 2.25u_p$	$A_4 =$ 8. 82%	0.28 – 0.21 = 0.07	2.4696-1.8522=0.6174
$2.25 \sim 2.75u_p$	$A_5 =$ 1. 58%	0.37 – 0.30 = 0.07	0.5846-0.4740=0.1106
$2.75 \sim 3.25u_p$	$A_6 =$ 0. 16%	0.405 - 0.305 = 0.100	0.0648-0.0488=0.0160
$3.25 \sim 3.75u_p$	$A_7 =$ 0.0096%	0.5 – 0.375 = 0.125	0.0048-0.0036=0.0012
$3.75u_p \sim \infty$	$A_8 \approx$ 0.0003%	0.5 – 0.4395 = 0.0605	0.00015-0.00013=0.00002
$\sum_u \frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{60^o}(u) \Delta u$ = 16.86285 – 13.4138 = 3.449			

Tab. 9 (2) $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{60^o} \Delta u \sim u$

Fig 9 (2) is the corresponding graph.

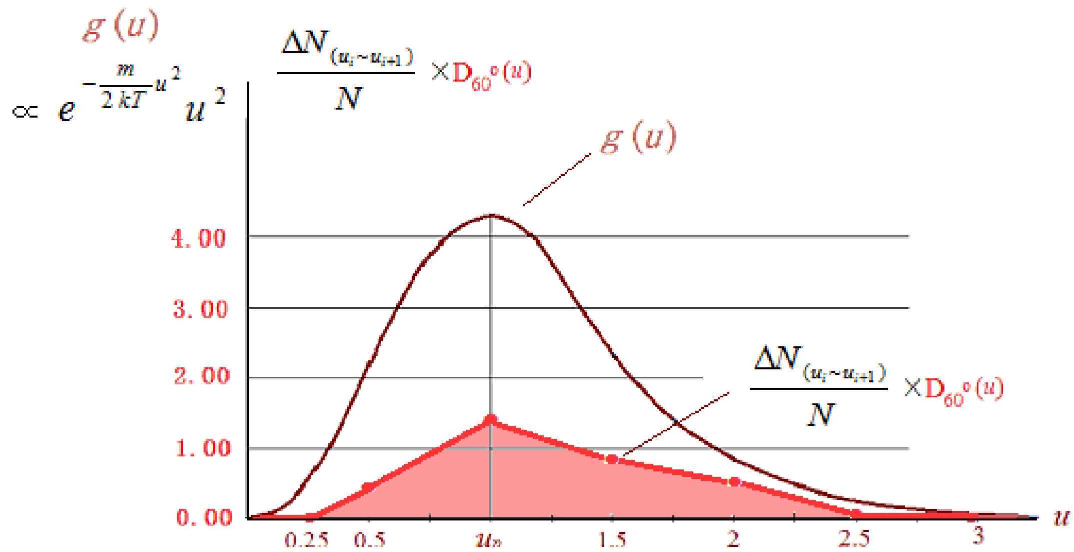
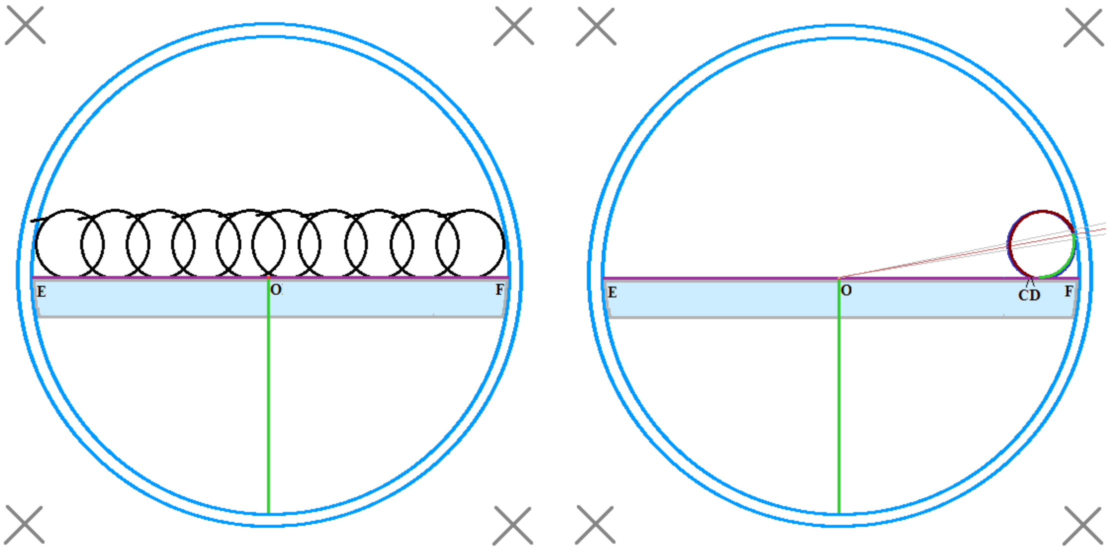


Fig. 9 (2) Graph of $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{60^o} \sim u$

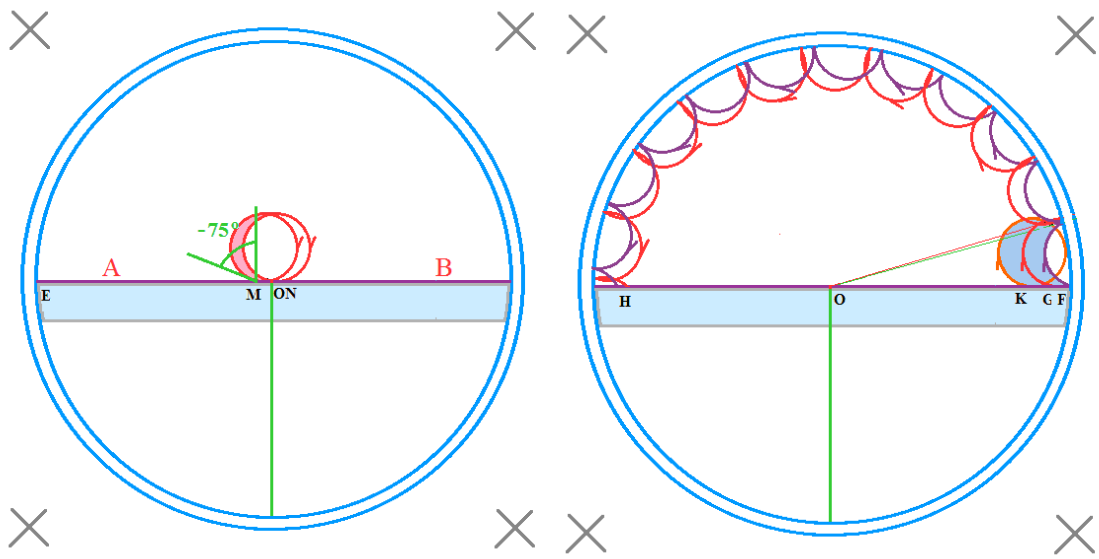
10. Trajectories of electrons of θ = - 75° and different speeds

(1) Fig 10-1 θ = - 75° $u = 0.5u_p$ R = 2mm (B = 1.34 gauss)



A-directly-A B-directly-B B-glass-B (CD≈0)

No electron migration between A and B due to these trajectories.



A-directly-B (from MO to ON, MO = 6, red) MO/EO = 6/70 = 0.09

B-glass-A (from G to H, etc., KF = 15, blue) KF/OF = 15/70 = 0.21

A-B 9% of the electrons of (-75°, $0.5u_p$) emitted from A migrate to B.

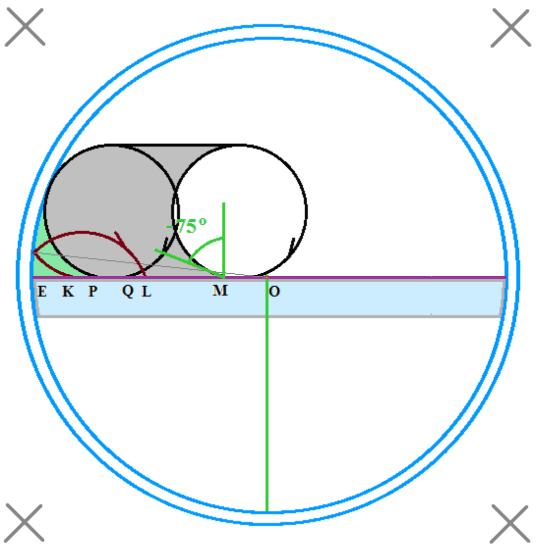
B-A 21% of the electrons of (-75°, $0.5u_p$) emitted from B migrate to A.

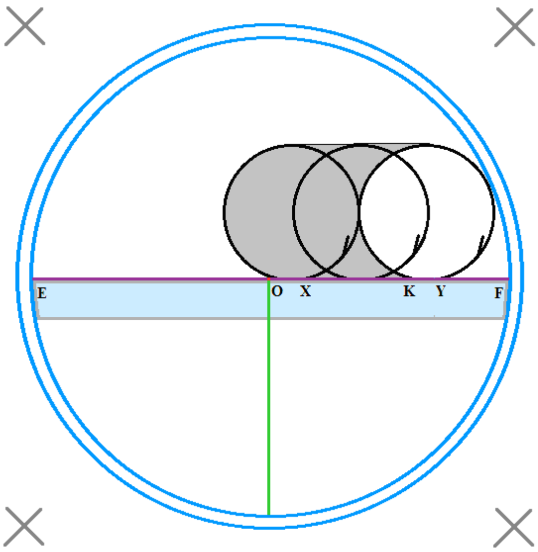
For all the electrons of (-75°, $0.5u_p$), migration A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

$$\{(A\text{-}B) - (B\text{-}A)\}_{-75^\circ\ 0.5u_p} = 0.09 - 0.21 = -0.12$$

$$D_{-75^\circ}(0.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-75^\circ\ 0.5u_p} \cos 75^\circ = -0.12 \times 0.26 = -0.028 \approx -0.03$$

(2) Fig 10-2 θ = - 75° $u = u_p$ R = 4mm (cos-75° = 0.2588)

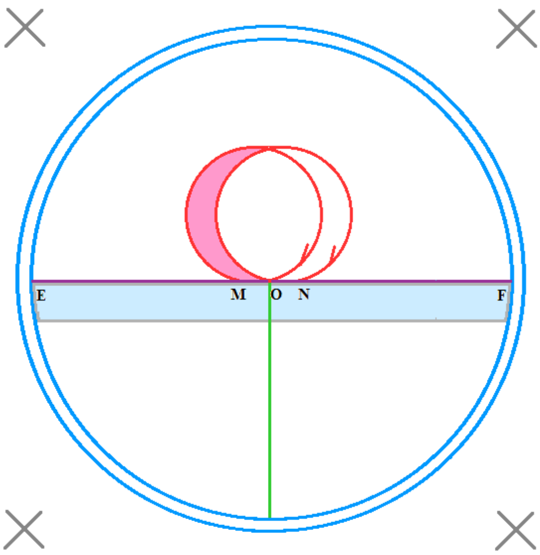


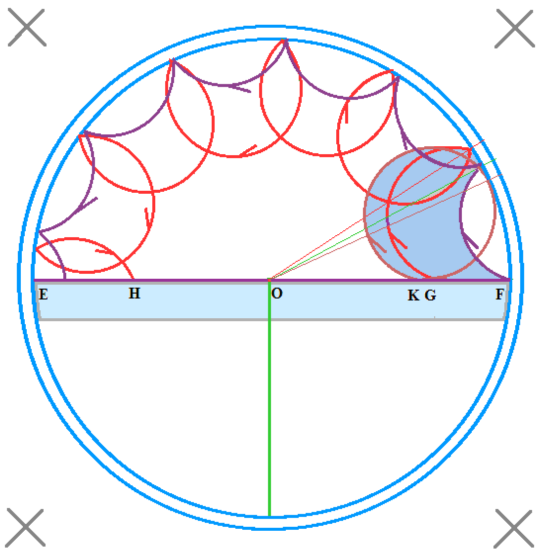


A-glass-A (from K to L, etc, green, EP =20) B-directly-B (from OK to XY, grey)

A-directly-A (from PM to QO, grey, PM =39) (B-glass-B CD≈0)

No electron migration between A and B due to these trajectories.





A-directly-B
(from MO to ON, red , MO = 11)
MO/EO = 11/70 =0.16

B-glass-A
(from G to H, etc., blue, KF = 29)
KF/OF =29/70 = 0.41

A-B 16% of the electrons of (- 75°, u_p) emitted from A migrate to B.

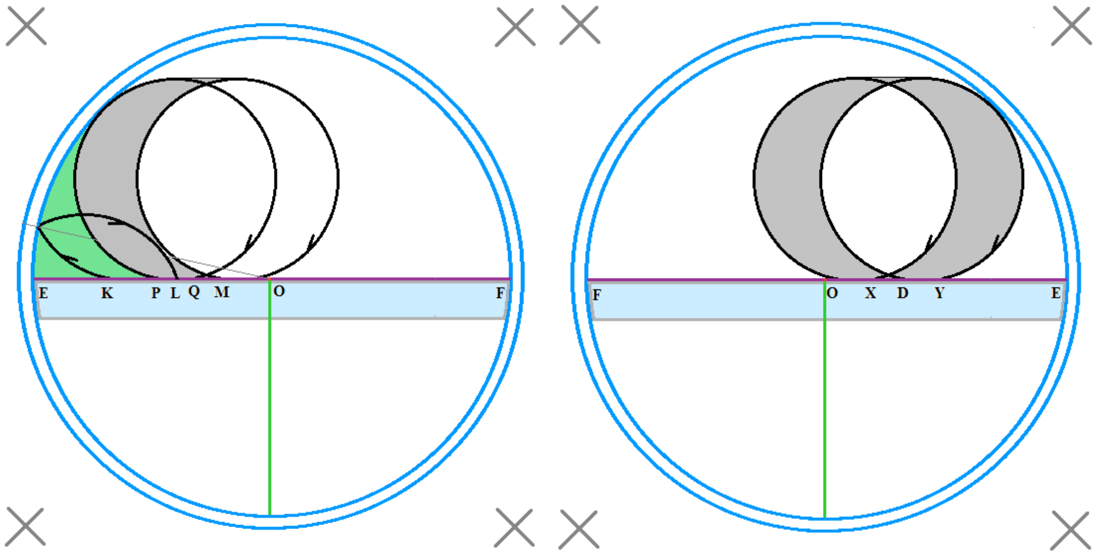
B-A 41% of the electrons of (- 75°, u_p) emitted from B migrate to A.

For all the electrons of (-75°, u_p), migration A-B is less than B-A, and their difference is the corresponding contribution (negative) to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{-75^\circ\, up} = 0.16 - 0.41 = -0.25$$

$$D_{-75^\circ}(u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-75^\circ\, up}\, cos75^\circ = -0.25 \times 0.26 = -0.0647 = -0.06$$

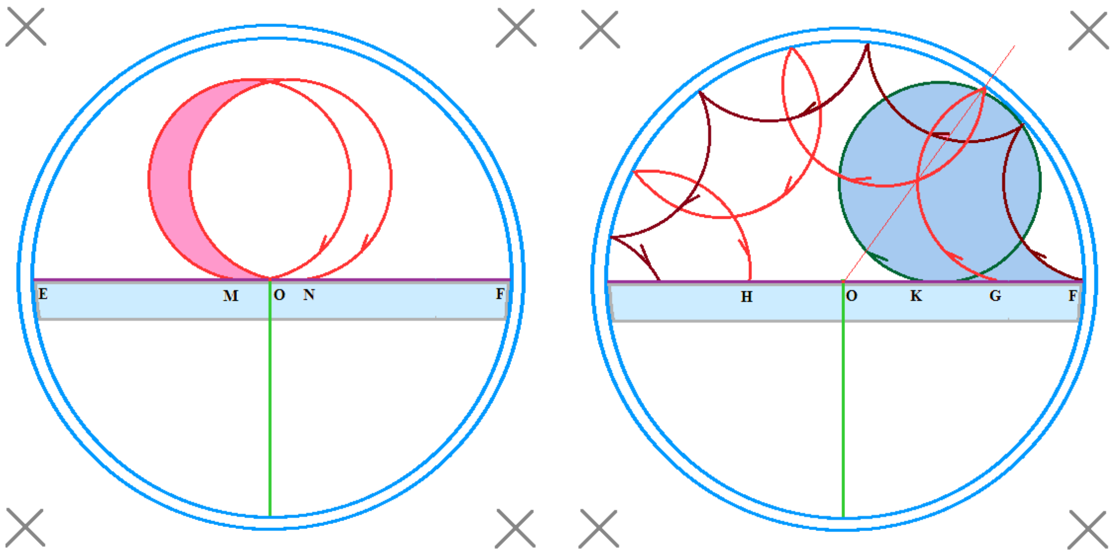
(3) Fig 10-3 θ = - 75° $u = 1.5u_p$ R = 6mm



A-glass-A (from K to L, etc., green) B-directly-B

A-directly-A (from P to Q, M to O, etc., grey) (from O to X, D to Y, etc., grey)

No electron migration between A and B due to these trajectories.



A-directly-B

(from MO to ON, etc., red, MO = 14)

MO/EO = 14/70 = 0.20

B-glass-A

(from G to H, etc., blue, KF = 50)

KF/OF = 50/70 = 0.71

A-B 20% of the electrons of (- 75°, $1.5u_p$) emitted from A migrate to B.

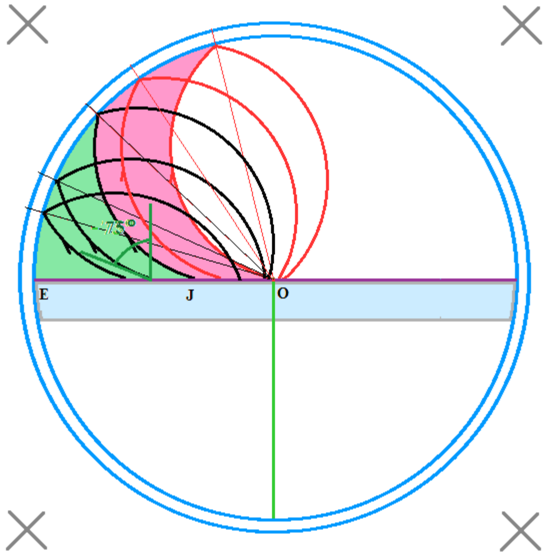
B-A 71% of the electrons of (- 75°, $1.5u_p$) emitted from B migrate to A.

For all the electrons of (-75°, $1.5u_p$), migration A-B is less than B-A, and their difference is the corresponding contribution (negative) to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{-75^\circ\ 1.5u_p} = 0.20 - 0.71 = -0.51$$

$$D_{-75^\circ}(1.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-75^\circ\ 1.5u_p}\ \cos 75^\circ = -0.51 \times 0.26 = -0.13$$

(4) Fig 10-4 θ = - 75° $u = 2u_p$ R = 8mm



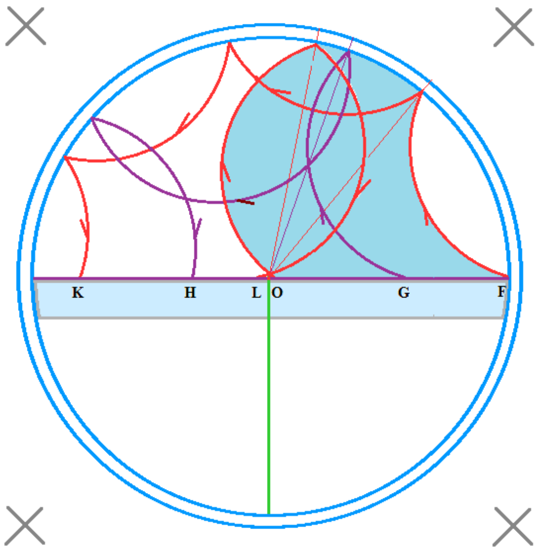
A-glass-A EJ/EO = 47/70 = 0.67 (EJ = 47, EO = 70) (green)

No electron migration between A and B due to these trajectories.

A-glass-B (JO =23, EO =70) (red)

JO/EO = 23/70 = 0.33

A-B 33% of the electrons of (-75°, $2u_p$) emitted from A migrate to B.



B-glass-A

(from G to H, F to K, O to L, etc, blue，OE= 70)

70/70 = 1.00

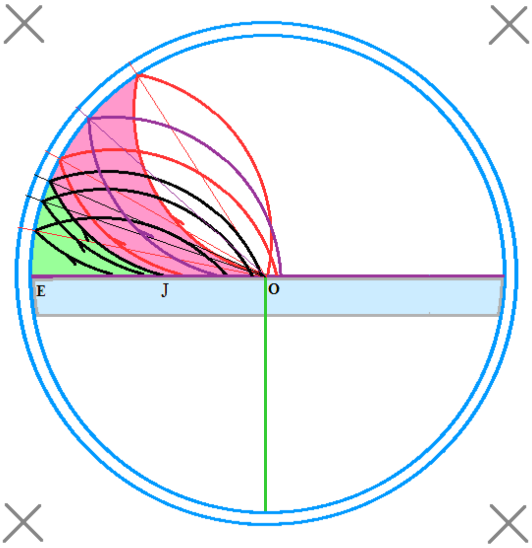
B-A 100% of the electrons of (-75°, $2u_p$) emitted from B migrate to A.

For all the electrons of (-75°, $2u_p$), migrate A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

$\{(A\text{-}B) - (B\text{-}A)\}_{-75^\circ\ 2u_p} = 0.33 - 1.00 = -0.67$

$D_{-75^\circ}(2u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-75^\circ\ 2u_p} \cos 75^\circ = -0.67 \times 0.26 = -0.17$

(5) Fig 10-5 θ = - 75° u =2.5u_p R = 10mm



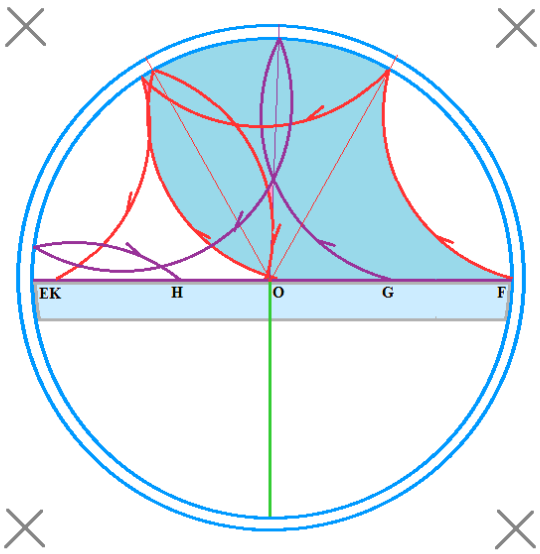
A-glass-A EJ/EO = 40/70 = 0.57 (EJ = 40, green)

No electron migration between A and B due to these trajectories.

A-glass-B (JO = 30, EO =70, red)

JO/EO = 30/70 = 0.43

A-B 43% of the electrons (-75°, 2.5u_p) emitted from A migrate to B.



B-glass-A

(from G to H, F to K, O to O, etc., blue)

70/70=1.00

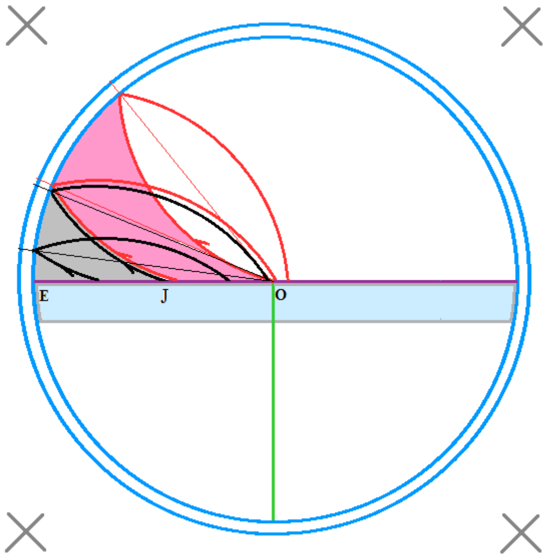
B-A All the electrons of (-75°, 2.5u_p) emitted from B migrate to A.

For all the electrons of (-75°, 2.5u_p), migration A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

$\{(A\text{-}B) - (B\text{-}A)\}_{-75^\circ\ 2.5u_p} = 0.43 - 1.00 = -0.57$

$D_{-75^\circ}(2.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-75^\circ\ 2.5u_p} \cos 75^\circ = -0.57 \times 0.26 = -0.15$

(6) Fig 10-6 θ = - 75° $u = 3u_p$ R = 10mm



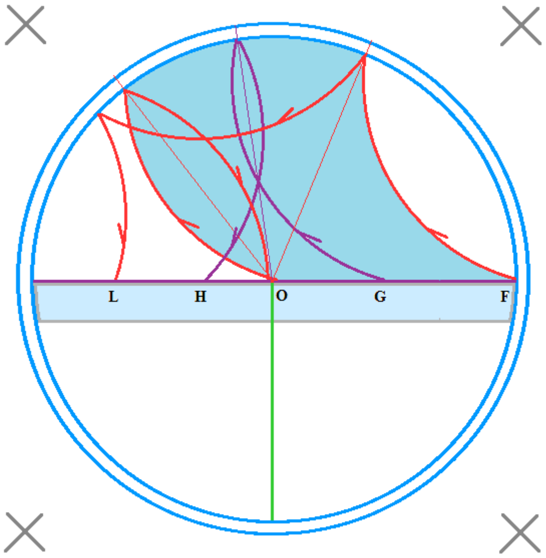
A-glass-A EJ/EO = 38/70 = 0.54 (EJ = 38, grey)

No electron migration between A and B due to these trajectories.

A-glass-B (JO = 32, red)

JO/EO = 32/70 = 0.46

A-B 46% of the electrons of (- 75°, $3u_p$) emitted from A migrate to B.



B-glass-A

(from G to H, F to L, O to O, etc., blue)

70/70 = 1.00

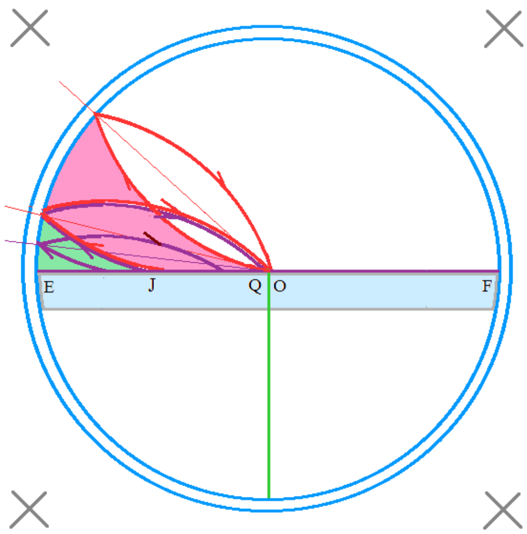
B-A 100% of the electrons of (- 75°, $3u_p$) emitted from B migrate to A.

For all the electrons of (-75°, $3u_p$), migration A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

$\{(A\text{-}B) - (B\text{-}A)\}_{-75^\circ\,3up} = 0.46 - 1.00 = -0.54$

$D_{-75^\circ}(3u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-75^\circ\,3up}\cos 75^\circ = -0.54 \times 0.26 = -0.14$

(7) Fig 10-7 θ = - 75° $u = 3.5u_p$ R = 14mm



A-glass-B

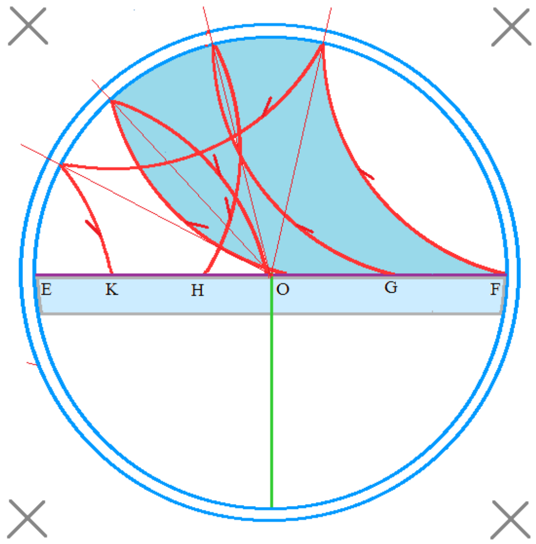
(from M to O, etc., green) (EJ = 34)

A-direct-B

(from JO to B, red, JO = 36)

JO/EO = 36/70 = 0.51

A-B 51% of the electrons (- 75°, 3.5 u_p) emitted from A migrate to B.



B-glass-A

(from G to H, F to K, O to O, etc., blue)

70/70 = 1.00

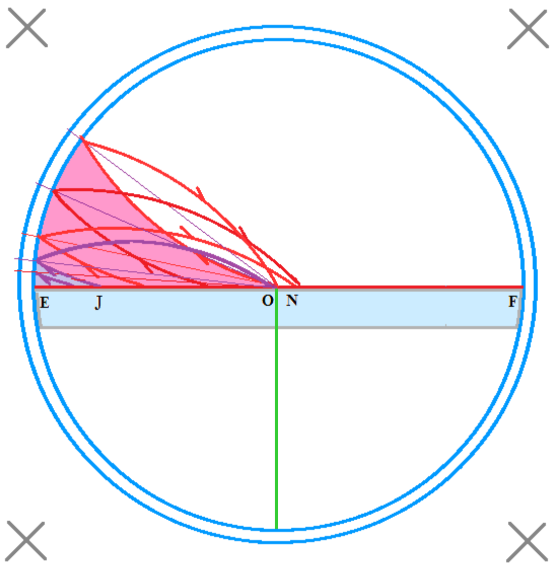
B-A 100% of the electrons of (-75°, 3.5u_p) emitted from B migrate to A.

For all the electrons of (-75°, 3.5u_p), migration A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

$\{(A\text{-}B) - (B\text{-}A)\}_{-75^{\circ}\ 3.5up} = 0.51 - 1.00 = -0.49$

$D_{-75^{\circ}}(3u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-75^{\circ}\ 3up}\ cos75^{\circ} = -0.49 \times 0.26 = -0.13$

(7) Fig 10-8 θ = - 75° u = 4.5u_p R = 18mm

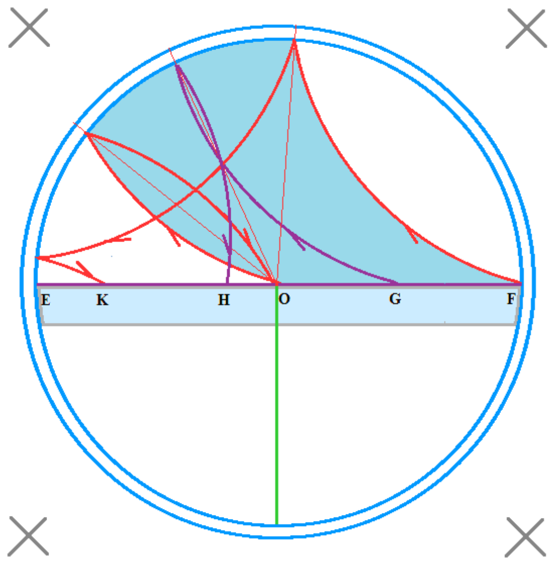


A-glass-B

(from J to O, etc., JO = 50, red. EO = 70.)

50/70 = 0.71

A-B 71% of the electrons (- 75°, 4.5u_p) emitted from A migrate to B.



B-glass-A

(from G to H, F to K, O to O, etc., blue)

70/70 = 1.00

B-A 100% of the electrons of (-75°, 4.5u_p) emitted from B migrate to A.

For all the electrons of (-75°, 4.5u_p), migration A-B and B-A cancel each other, no contribution to the output current.

$\{(A\text{-}B) - (B\text{-}A)\}_{-75^\circ\ 4.5u_p} = 0.71 - 1.00 = 0.29$

$D_{-75^\circ}(4.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-75^\circ\ 4.5u_p}\ cos\text{-}75^\circ = -0.29 \times 0.26 = -0.075$

Every 100 + 100 electrons (1.00 = 100% from A, and, 1.00 = 100% from B) of exiting angle $\theta = -75^{o}$ and of different speeds contribute to the electron migration between A and B differently, as list in Tab 10 (1) (cos-75^{o} = 0.2588),

$$D_{-75^o}(u) = \{(A\text{-}B) - (B\text{-}A)\}_{-75^o} \cos\theta \sim u .$$

speed u	$\{(A\text{-}B)\text{–}(B\text{-}A)\}_{-75^o}(u)$	$(A\text{-}B)\text{–}(B\text{-}A)\}_{-75^o} \cos\theta \sim u$
Fig 10-1 $u = 0.5u_p$	0.10 – 0.21 = - 0.11	0.0260-0.0546 =-0.0286
Fig 10-2 $u = u_p$	0.16 – 0.41 = - 0.25	0.0416-0.1066= -0.065
Fig 10-3 $u = 1.5u_p$	0.21 – 0.71 = - 0.50	0.0546-0.1846=-0.1300
Fig 10-4 $u = 2u_p$	0.33 – 1.00 = - 0.67	0.0858=0.2600=0.1742
Fig 10-5 $u = 2.5u_p$	0.43 – 1.00 = - 0.57	0.1118- 0.2600=-0.1482
Fig 10-6 $u = 3u_p$	0.46 – 1.00 = -0.54	0.1196- 0.2600=-0.1404
Fig 10-7 $u = 3.5u_p$	0.51 – 1.00 = -0.49	0.1326-0.2600=-0.1274
Fig 10-8 $u = 4.5u_p$	0.71 – 1.00 = -0.29	0.1846-0.26 = -0.0754

Tab 10 (1) $D_{-75^o}(u) = \{(A\text{-}B) - (B\text{-}A)\}_{-75^o} \cos\theta \sim u$

And Fig 10(1) is the corresponding graph.

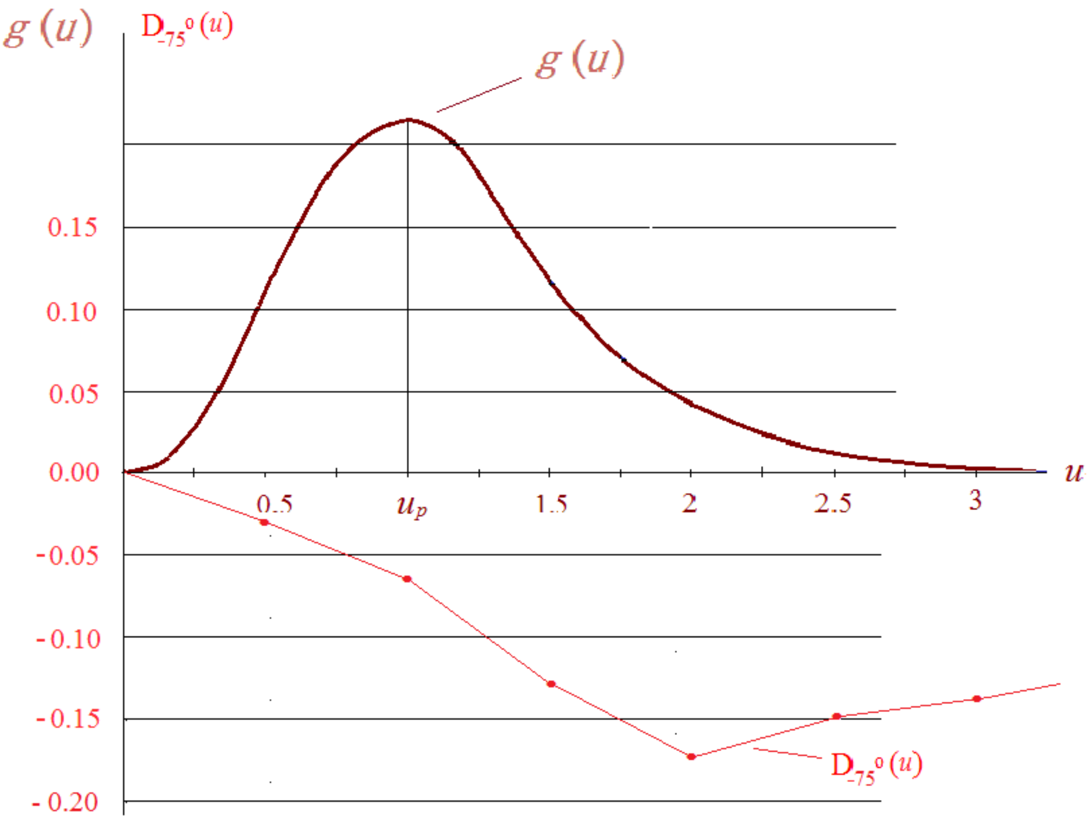


Fig 10 (1) $D_{-75^o}(u) = \{(A\text{-}B) - (B\text{-}A)\}_{-75^o} \cos\theta \sim u$

Take Maxwell's speed distribution $g(u)$ into account to derive the actual contributions of the thermal electrons of θ = -75° with different speed ranges, i.e., $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{-75^o} \varDelta u \sim u$, as shown in Tab 10 (2).

Speed range $\varDelta u$	$\frac{\Delta N_{(u_i \sim u_{i+1})}}{N}$	$D_{-75^o}(u)$ {(A-B)–(B-A)}$_{-75^o}\cos\theta \sim u$	$\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{75^o} \varDelta u$
$0.00 \sim 0.25u_p$	$A_0 \approx$ 0. 004	$\approx$ 0	$\approx$ 0
$0.25 \sim 0.75u_p$	A_1 = 18. 74%	0.0260-0.0546 =-0.0286	0.4872-1.0232=- 0.536
$0.75 \sim 1.25u_p$	A_2 = 39. 83%	0.0416-0.1066= -0. 065	1.6569-4.2459=-2.589
$1.25 \sim 1.75u_p$	A_3 = 26. 71%	0.0546-0.1846=-0.1300	1.4585-4.9307=-3.4722
$1.75 \sim 2.25u_p$	A_4 = 8. 82%	0.0858=0.2600=0.1742	0.7568-2.2932=-1.5364
$2.25 \sim 2.75u_p$	A_5= 1. 58%	0.1118- 0.2600=-0.1482	0.1766-0.4108=-0.2342
$2.75 \sim 3.25u_p$	A_6 = 0. 16%	0.1196- 0.2600=-0.1404	0.0191-0.0416=-0.0225
$3.25 \sim 3.75u_p$	A_7= .0096%	0.1326-0.2600=-0.1274	0.0013-0.0025=0.0012
$3.75u_p \sim \infty$	$A_8 \approx$.0003%	0.1846-0.26 = 0	0.00006-0.00008$\approx$0
$\sum_u \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-75^o}(u) \varDelta u$ = 4.5591 – 12.948 = -8.3889			

Tab. 10 (2) The actual contributions of electrons of θ = - 75° with different speed ranges, $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{-75^o} \varDelta u \sim u$

Fig 10 (2) is the corresponding graph.

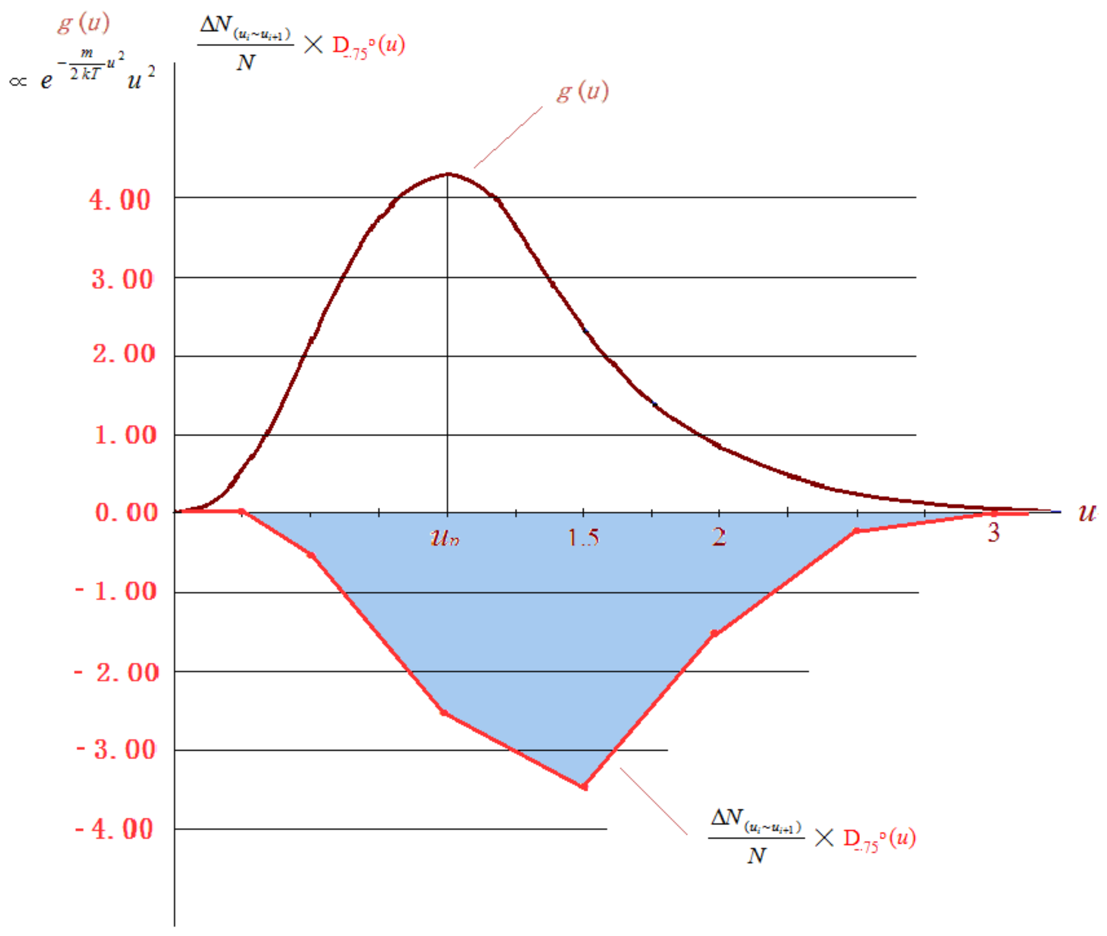
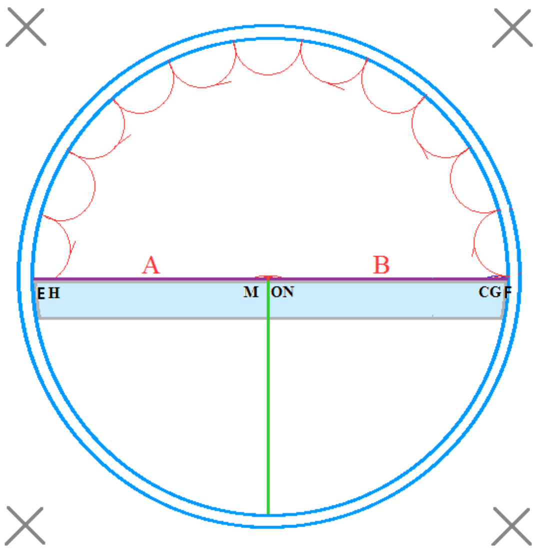


Fig 10 (2) Graph of the actual contributions of electrons of

θ = - 75° with different speeds, $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-75^o} \Delta u \sim u$.

11. Trajectories of electrons of θ = 75º with different speeds

(1) Fig 11-1 $\theta = 75^o$ $u = 0.5u_p$ R = 2mm (B = 1.34 gauss)



$\theta = 75^o$ $u = 0.5u_p$ R = 2mm 2,5 : 1
(see the next page, a 5 : 1 figure)
(also refer to Fig.11-3, an alike figure)

A-directly-B (MO = 5, EO = 70, from MO to ON)
MO/EO = 5/70 = 0.07

A-B 7% of the electrons of (75º, $0.5u_p$) emitted from A migrate to B.

B-glass-B 0.5/70 = 0.007
(CD = 0.5, OF = 70) (from CD to FV)

B-glass-A (from G to H, etc., DF = 4)
4.5/70 = 0.06

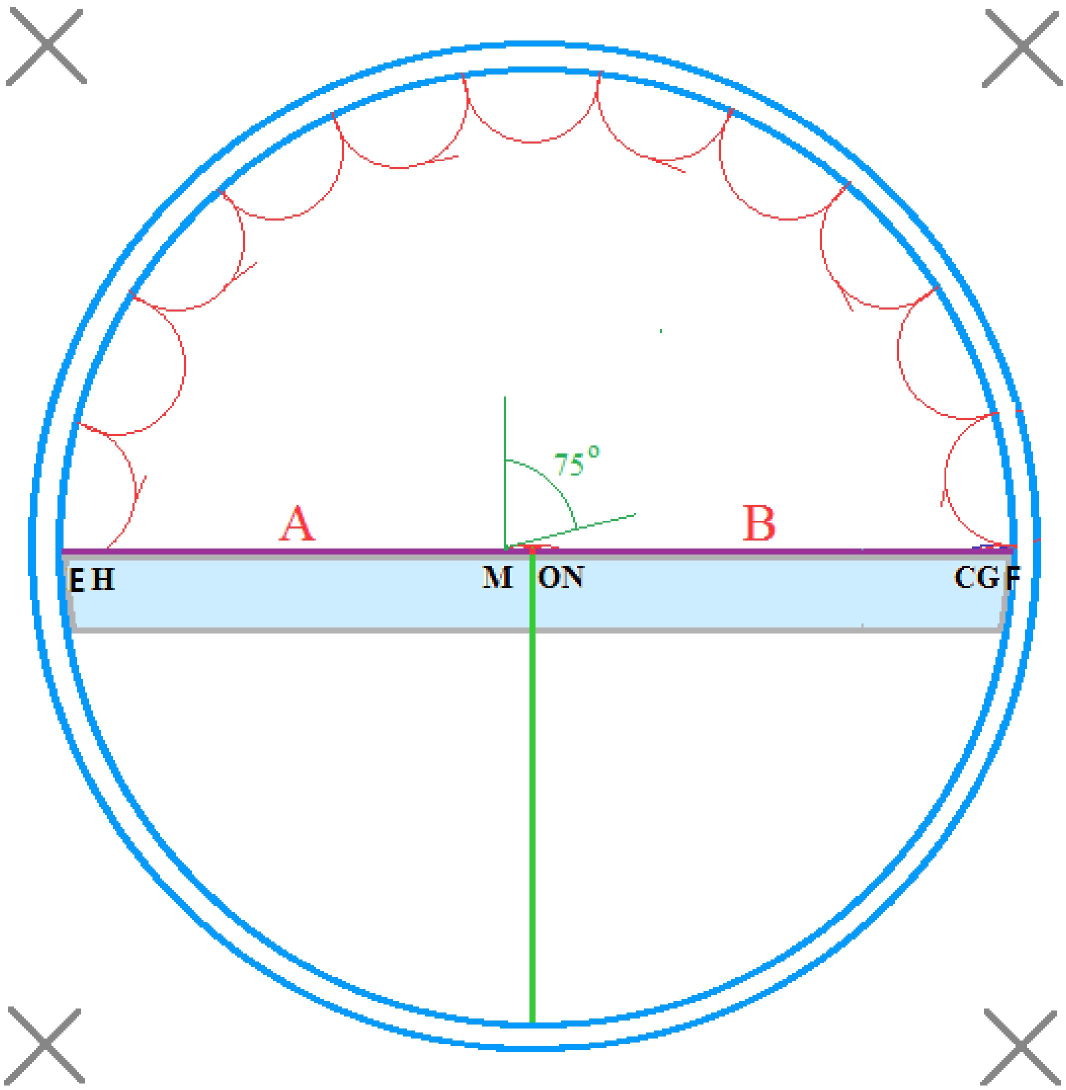
B-A 6% of the electrons of (75º, $0.5u_p$) emitted from B migrate to A.

(Here accurate drawing is difficult, only approximate estimation.)

For all the electrons of (75º, $0.5u_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{75^o\ 0.5u_p} = 0.07 - 0.06 = 0.01$$

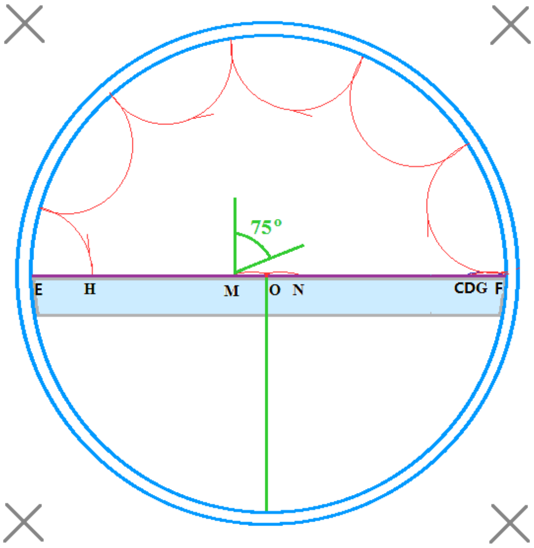
$$D_{75^o}(0.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{75^o\ 0.5u_p} \cos 75^o = 0.01 \times 0.2588 = 0.0023 \approx 0.00$$



(5 : 1)

(Also Fig 11-1) θ = 75° $u = 0.5u_p$ R = 2mm

(2) Fig 11-2 $\theta = 75^o$ $u = u_p$ R = 4mm ($cos75^o = 0.2588$)



$\theta = 75^o$ $u = u_p$ R = 2mm 2,5 : 1

(see the next page, a figure of 5 : 1)

(also refer to Fig.11-3, a better like figure)

A-directly-B (MO = 10, EO = 70)

10/70 = 0.14

A-B 14% of the electrons of (75^o, u_p) emitted from A migrate to B.

B-glass-B 1/70 = 0.014 (CD =1, OF = 70)

B-glass-A (DF = 9, OF = 70) (from G to H, etc.)

9/70 = 0.13

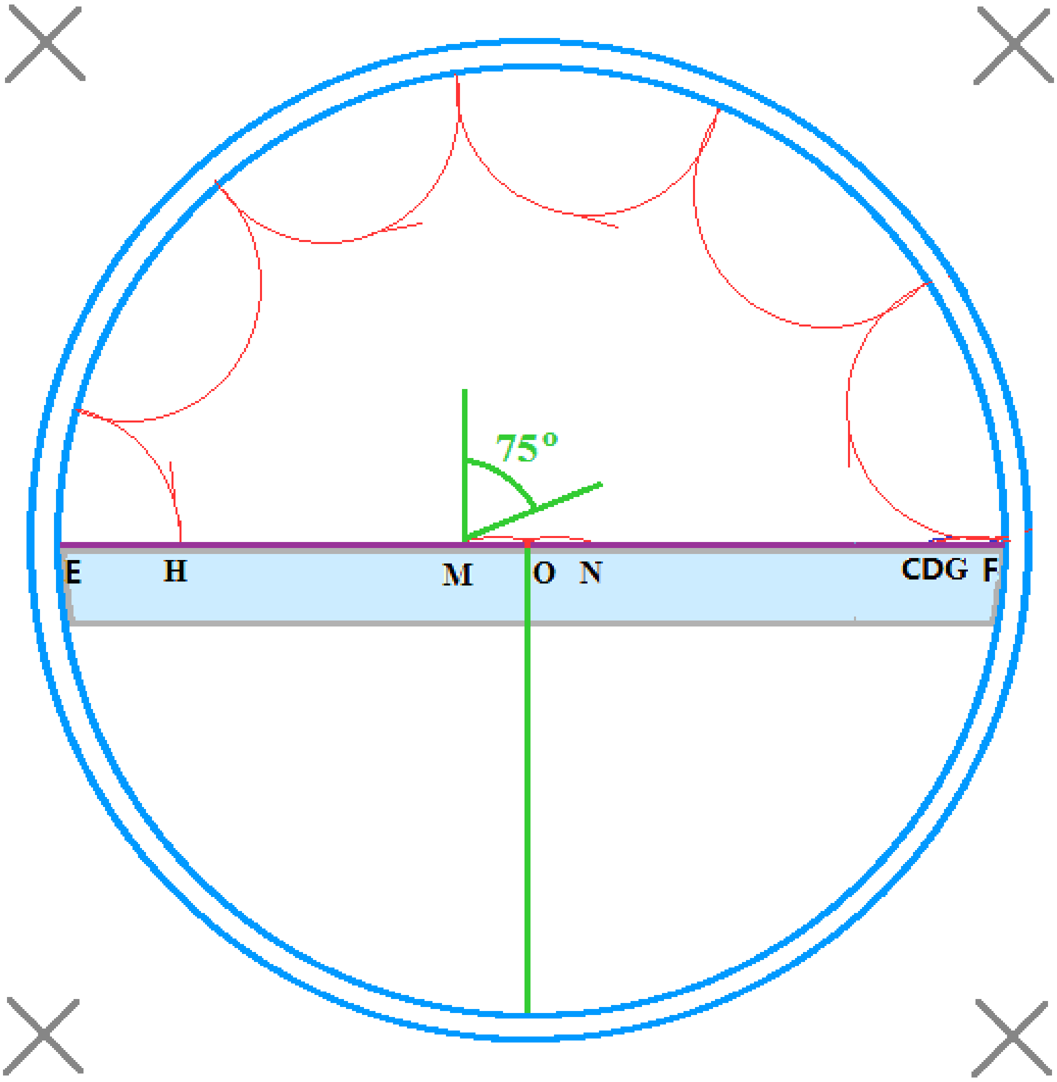
B-A 13% of the electrons of (75^o, u_p) emitted from B migrate to A.

(Here accurate drawing is difficult, only approximate estimation.)

For all the electrons of (75^o, u_p), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$\{(A\text{-}B) – (B\text{-}A)\}_{75^o\ u_p} = 0.14 – 0.13 = 0.01$

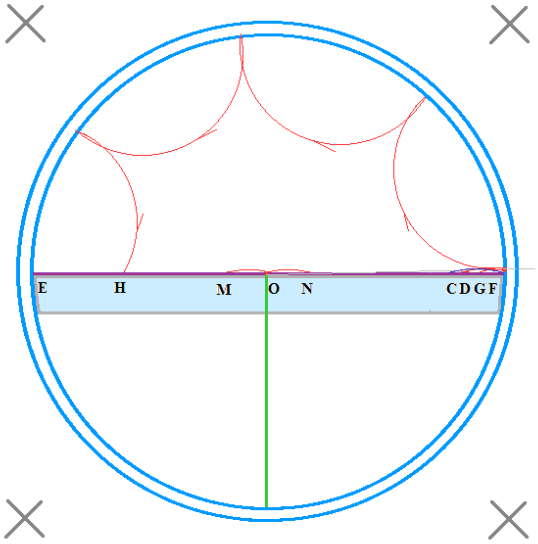
$D_{75^o}(u_p) = \{(A\text{-}B) – (B\text{-}A)\}_{75^o\ u_p} cos75^o = 0.01 \times cos75^o = 0.002588 \approx 0.00$



(5 : 1)

(also Fig 11-2) θ = 75° $u = u_p$ R = 4mm

(3) Fig 11-3 θ = 75° $u = 1.5u_p$ R = 6mm



θ = 75° $u = 1.5u_p$ R = 6mm 2.5 : 1

(see the next page, a figure of 5 : 1)

A-directly-B (MO = 13, from MO to ON)

MO/EO = 13/70= 0.186

A-B 18.6% of the electrons of (75°, $1.5u_p$) emitted from A migrate to B.

B-glass-B 2/70 = 0.028 (CD =2, OF = 70)

B-glass-A (DF = 11, from G to H, etc.)

DF/OF = 11/70 = 0.157

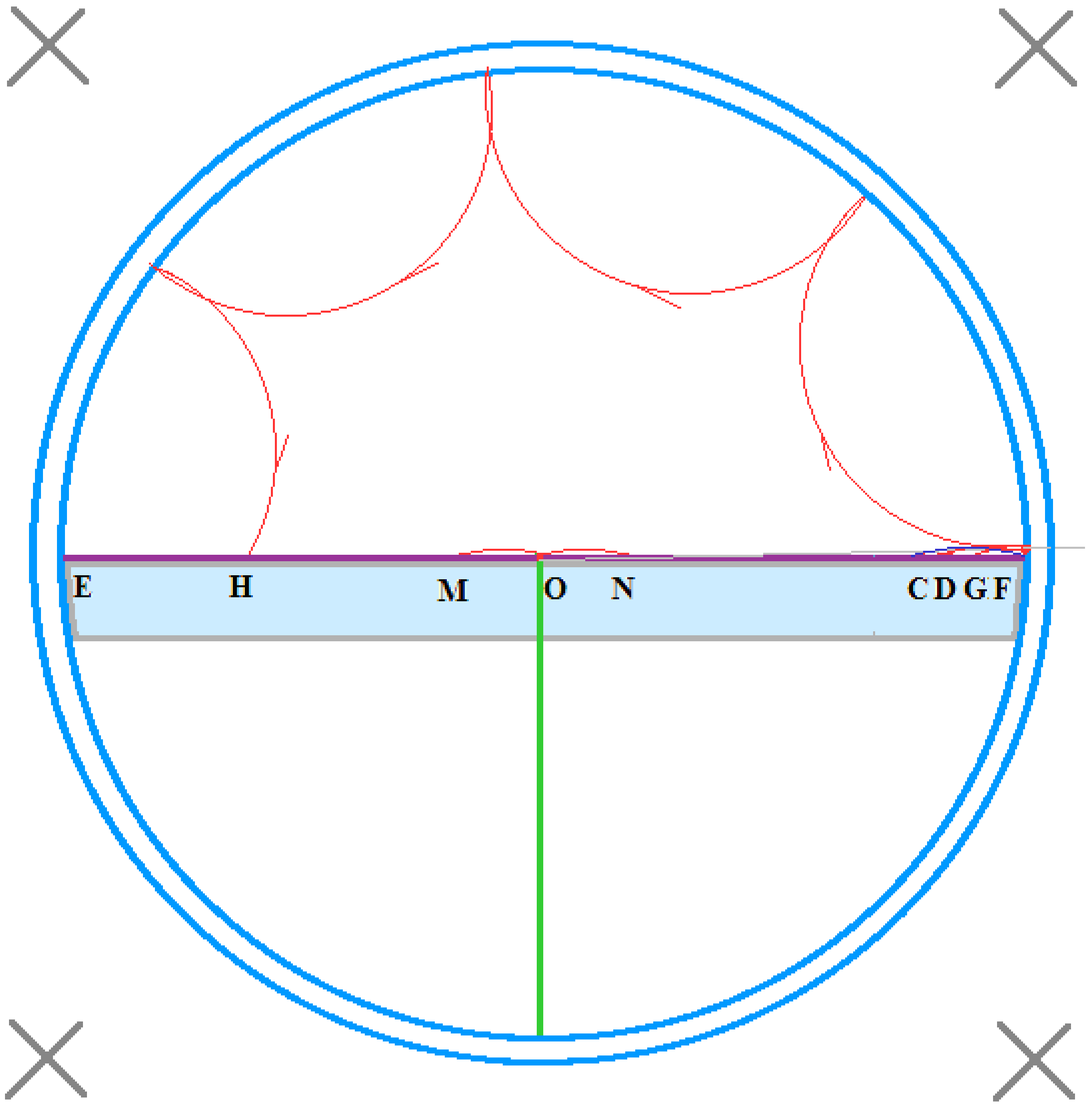
B-A 15.7% of the electrons of (75°, $1.5u_p$) emitted from B migrate to A.

(Here accurate drawing is difficult, only approximate estimation.)

For all the electrons of (75°, $1.5u_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{75^\circ\ 1.5u_p} = 0.186 - 0.157 = 0.029$$

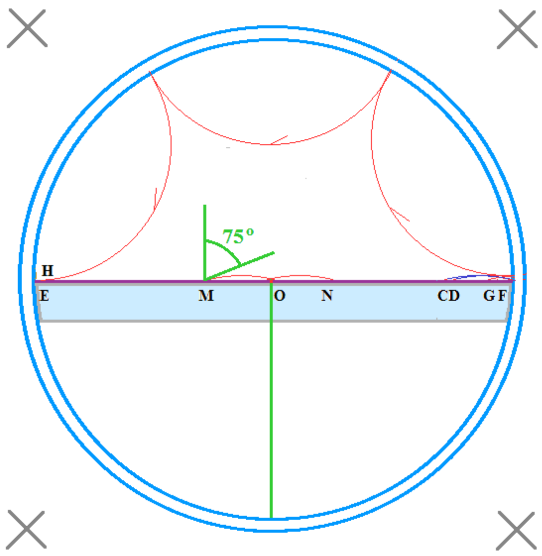
$$D_{75^\circ}(1.5u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{75^\circ\ 1.5u_p} \cos 75^\circ = 0.029 \times 0.2588 = 0.0075 \approx 0.01$$



(5 : 1)

(also Fig 11-4) θ = 75° u = 1.5u_p R = 6mm

(4) Fig 11-4 θ = 75° $u = 2u_p$ R = 8mm



θ = 75° $u = 2u_p$ R = 8mm 2.5 : 1

(see the next page, a figure of 5 : 1)

A-directly-B (MO = 20, EO =70)

20/70 = 0.2857

A-B 28.6% of the electrons of (75°, $2u_p$) emitted from A migrate to B.

B-glass-B 3/70 = 0.04 (CD = 3, OF =70)

B-glass-A (DF =17, OF =70) (from G to H, etc.)

17/70 = 0.2428

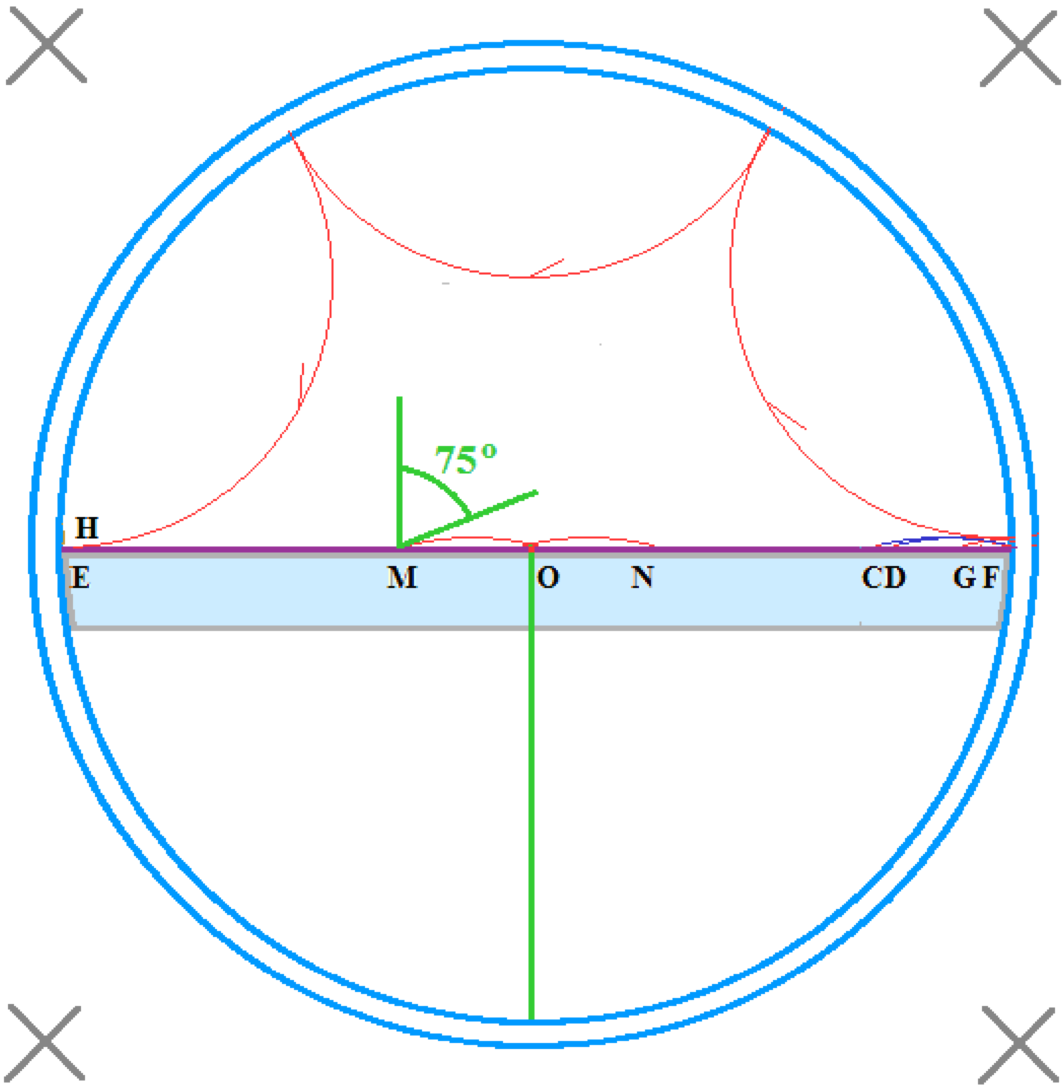
B-A 24.3% of the electrons of (75°, $2u_p$) emitted from B migrate to A.

(Here accurate drawing is difficult, only approximate estimation.)

For all the electrons of (75°, $2u_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{75^\circ\, up} = 0.286 - 0.243 = 0.04$$

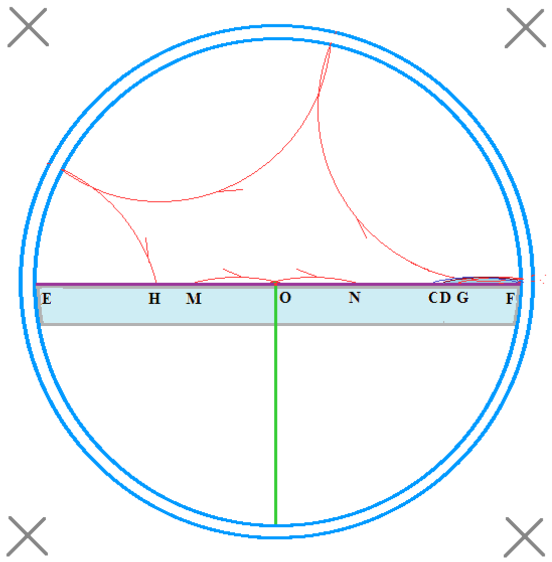
$$D_{75^\circ}(2u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{75^\circ\, up}\ \cos 75^\circ = 0.04 \times 0.2588 = 0.0103 \approx 0.01$$



(5 : 1)

(also Fig 11-5) θ = 75° $u = 2u_p$ R = 8mm

(5) Fig 11-5 $\theta = 75^o$ $u = 2.5u_p$ $R = 10mm$



$\theta = 75^o$ $u = 2.5u_p$ $R = 10mm$ 2.5 : 1
(see the next page, a 5 : 1 figure)

A-directly-B (from MO to ON, MO = 24, EO = 70)

MO/EO =24/70 = 0.343

A-B 34% of the electrons of (75°, $2.5u_p$) emitted from A migrate to B.

B-glass-B 4/70 = 0.057 (CD = 4, OF = 70)

B-glass-A (from G to H, etc., DF = 20)

DF/OF = 20/70 = 0.286

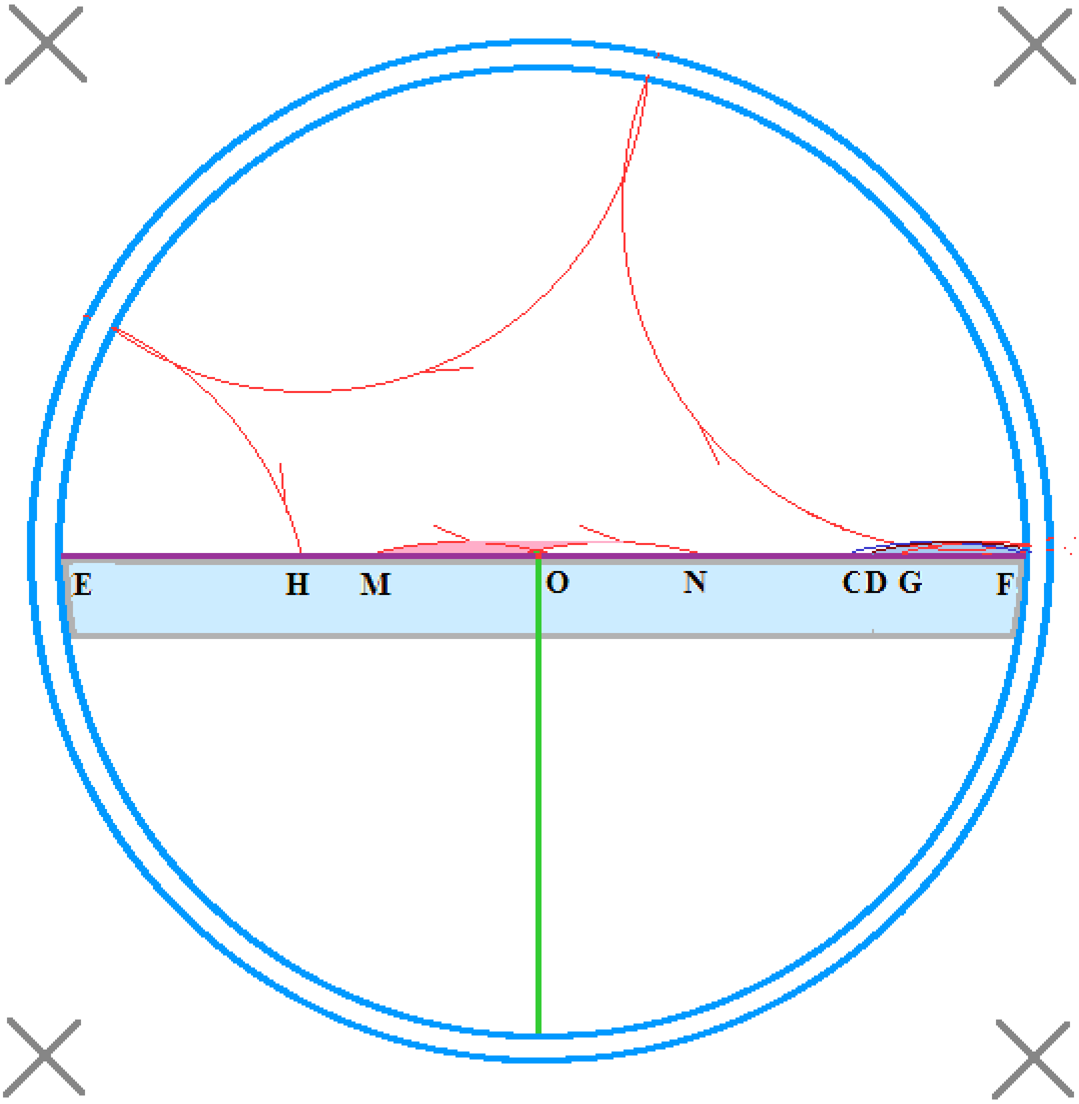
B-A 29% of the electrons (75°, $2.5u_p$) emitted from B migrate to A.

(Here accurate drawing is difficult, only approximate estimation.)

For all the electrons of (75°, $2.5u_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$\{(A\text{-}B) - (B\text{-}A)\}_{75^o\ 2.5up} = 0.343 - 0.286 = 0.057$

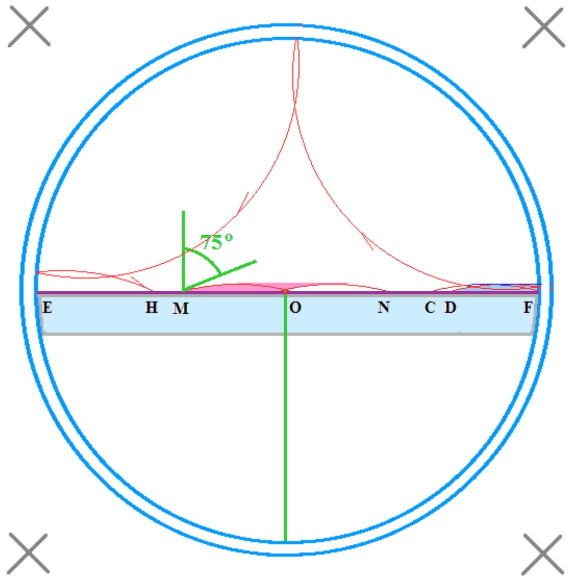
$D_{75^o}(2.5u_p) = \{(A\text{-}B)–(B\text{-}A)\}_{75^o\ 2.5up}\ cos75^o = 0.057 \times 0.2588 = 0.0148 \approx 0.015$



(5 : 1)

(also Fig 11-5) θ = 75° $u = 2.5u_p$ R = 10mm

(6) Fig 11-6 θ = 75° $u = 3u_p$ R = 12mm



θ = 75° $u = 3u_p$ R = 12mm 2.5 : 1

(see the next page, a figure of 5 : 1)

A-directly-B (MO = 29, EO = 70.)

29/70 = 0.414

A-B 41% of the electrons of (75°, $3u_p$) emitted from A migrate to B.

B-glass-B (5/70 = 0.071) (CD = 5, OF = 70)

B-glass-A (DF = 24, OF = 70) (from D to H, etc., blue)

24/70 = 0.342

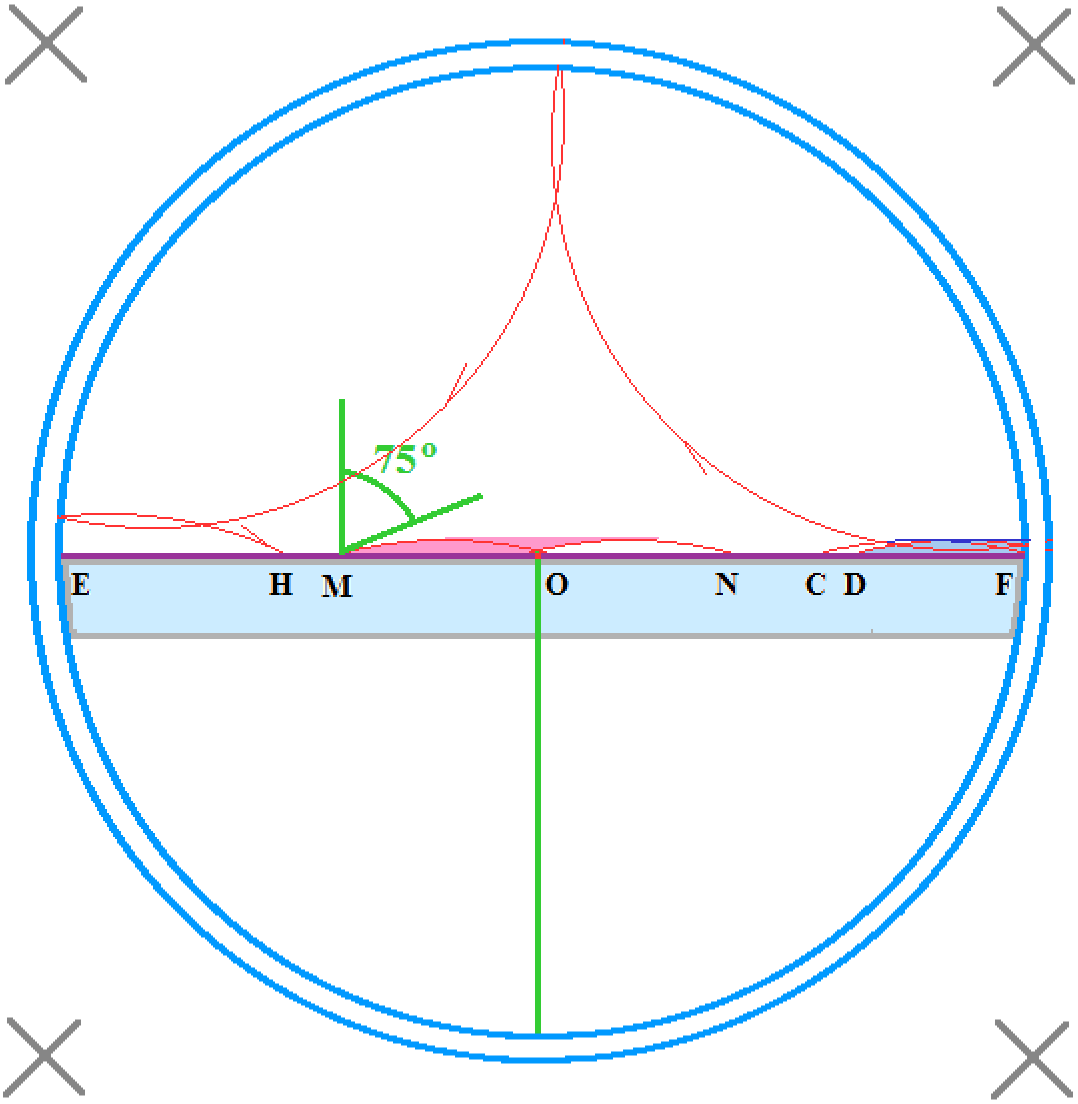
B-A 34% of the electrons (75°, $3u_p$) emitted from B migrate to A.

(Here accurate drawing is difficult, only approximate estimation.)

For all the electrons of (75°, $3u_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

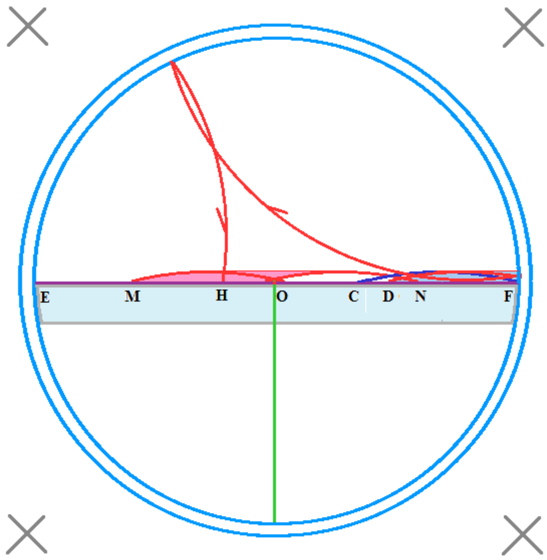
$$\{(A\text{-}B) - (B\text{-}A)\}_{75^\circ\, 3u_p} = 0.414 - 0.342 = 0.072$$

$$D_{75^\circ}(3u_p) = \{(A\text{-}B) - (B\text{-}A)\}_{75^\circ\, 3u_p} \cos 75^\circ = 0.072 \times 0.2588 = 0.0186 = 0.02$$



(also Fig 11-6) θ = 75° $u = 3u_p$ R = 12mm 5 : 1

(7) Fig 11-7 $\theta = 75^{o}$ $u = 4.5u_p$ R = 18mm



$\theta = 75^{o}$ $u = 4.5u_p$ R = 18mm (2.5 : 1)
(see the next page, a figure of 5 : 1)

A-directly-B (MO = 45, EO =70) (from MO to ON, red)

45/70 = 0.642

A-B 64% of the electrons of (75^{o}, $4.5u_p$) emitted from A migrate to B.

B-glass-B 6/70 = 0.086 (CD = 6, OF =70)

B-glass-A (DF = 39, OF =70) (from D to H, etc. blue)

39/70 = 0.557

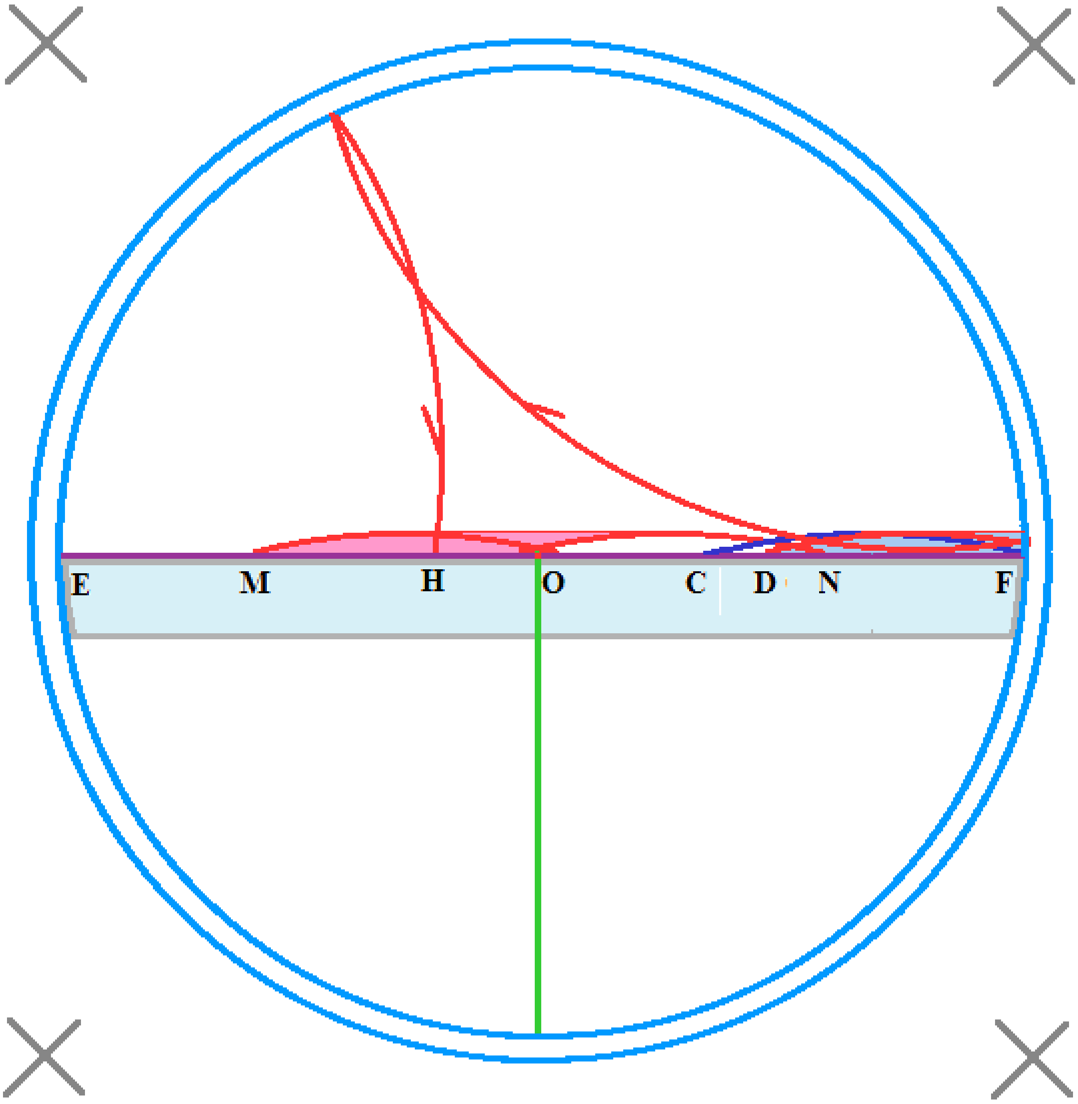
B-A 55.7% of the electrons of (75^{o}, $4.5u_p$) emitted from B migrate to A.

(Here accurate drawing is difficult, only approximate estimation.)

For all the electrons of (75^{o}, $4.5u_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$\{(A\text{-}B) - (B\text{-}A)\}_{75^{o}\ 4.5u_p} = 0.642 - 0.557 = 0.085$

$D_{75^{o}}(4.5u_p) = \{(A\text{-}B)-(B\text{-}A)\}_{75^{o}\ .4.5u_p} \cos 75^{o} = 0.085 \times 0.2588 = 0.0219 \approx 0.02$



(5 : 1)

(also Fig 11-7) θ = 75° u = 4.5u_p R = 18mm

Every 100 + 100 electrons (1.00 = 100% from A, 1.00 = 100% from B) of exiting angle $\theta = 75^o$ and of different speeds contribute to the electron migration between A and B differently, as list in Tab 11 (1), and Fig 11 (1) is the corresponding graph ($cos75^o = 0.2588$),

$$D_{-75^o}(u) = \{(A\text{-}B) - (B\text{-}A)\}_{75^o} \cos\theta \sim u .$$

speed u	$\{(A\text{-}B) - (B\text{-}A)\}_{75^o}$	$D_{75^o}(u) = \{(A\text{-}B) - (B\text{-}A)\}_{75^o} \cos\theta$
Fig 11-1 $u = 0.5u_p$	0.07 – 0.06 = 0.01	0.0181-0.0155=0.0026
Fig 11-2 $u = u_p$	0.14 – 0.13 = 0.01	0.0362-0.0336=0.0026
Fig 11-3 $u = 1.5u_p$	0.186 – 0.157 = 0.029	0.0481-0.0406=0.0075
Fig 11-4 $u = 2u_p$	0.286 – 0.243 = 0.04	0.0740-0.0629=0.0111
Fig 11-5 $u = 2.5u_p$	0.343– 0.286 = 0.057	0.0888-0.0740=0.0148
Fig11-6 $u = 3u_p$	0.414 – 0.342 = 0.072	0.1071-0.0885=0.0186
Fig11-7 $u = 4.5u_p$	0.642– 0.557 = 0.085	0.1661-0.1442=0.0219

Tab 11 (1) $D_{75^o}(u) = \{(A\text{-}B) - (B\text{-}A)\}_{75^o} cos75^o$

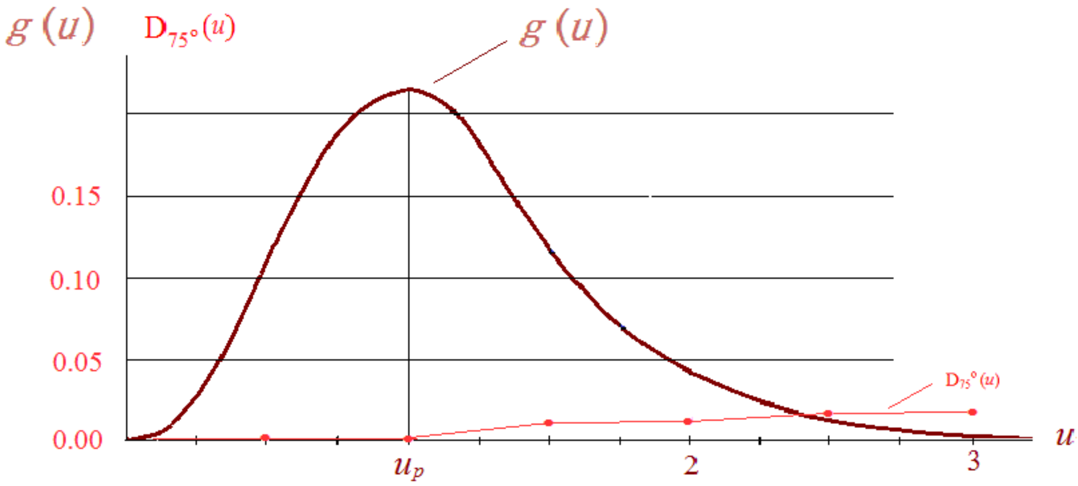


Fig 11 (1) Graph of C**on**tributions of electrons of θ = 75° and different speeds

Take Maxwell's speed distribution $g(u)$ into account to derive the actual contributions of the thermal electrons of θ = 75° with different speed ranges, i.e., $\times \frac{\Delta N_{(u_i \sim u_{i+1})}}{N} D_{75^o} \varDelta u \sim u$.

Speed range $\varDelta u$	$\frac{\Delta N_{(u_i \sim u_{i+1})}}{N}$	$D_{75^o}(u)$ = { (A-B) - (B-A) } $_{75^o} \cos\theta$	$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{75^o}(u) \varDelta u$
$0.25 \sim 0.75u_p$	A_1 = 18. 74%	0.0181-0.0155=0.0026	0.3392-0.2904=0.0488
$0.75 \sim 1.25u_p$	A_2 = 39. 83%	0.0362-0.0336=0.0026	1.4418-1.3383=0.1035
$1.25 \sim 1.75u_p$	A_3 = 26. 71%	0.0481-0.0406=0.0075	1.2848-1.0844=0.2004
$1.75 \sim 2.25u_p$	A_4 = 8. 82%	0.0740-0.0629=0.0111	0.6527-0.5548=0.0979
$2.25 \sim 2.75u_p$	A_5= 1. 58%	0.0888-0.0740=0.0148	0.1403-0.1169=0.0234
$2.75 \sim 3.25u_p$	A_6 = 0. 16%	0.1071-0.0885=0.0186	0.0171-0.0142=0.0029
$3.25 \sim 3.75u_p$	$A_7 \approx$ 0.0096%	0.1661-0.1442=0.0219	0.0016-0.0014=0.0002
$3.75u_p \sim \infty$	$A_8 \approx$ 0.0003%	≈ 0	≈ 0
$\sum_u \frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{75^o}(u) \varDelta u$ = 3.8775 – 3.4004 = 0.4771			

Tab 11 (2) $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{75^o} \varDelta u \sim u$

And Fig 11 (2) is the corresponding graph.

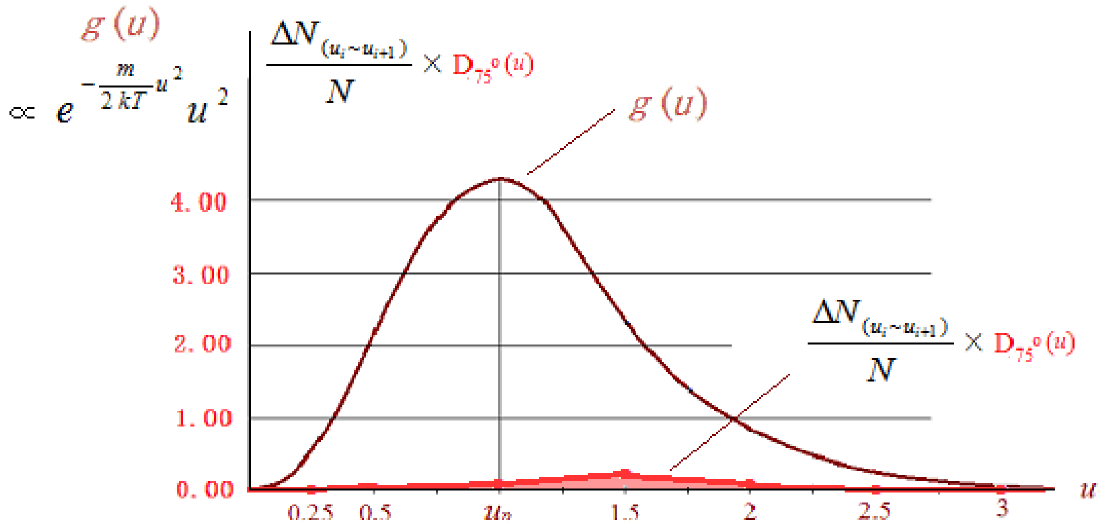


Fig 11 (2) Graph of $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{75^o} \sim u$

Discussion

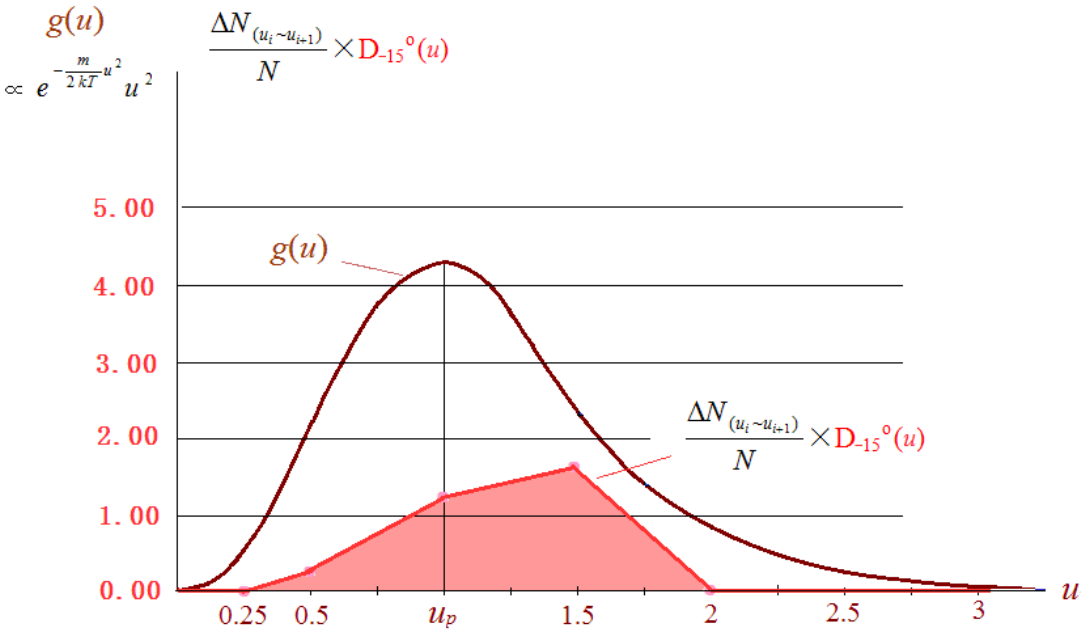
Altogether, we have eleven graphs of $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{\theta} \sim u$, corresponding to eleven different exiting angles θ. Among them, seven are positive graphs and four are negative graphs.

(1) The seven positive graphs of $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{\theta}(u)$ are for

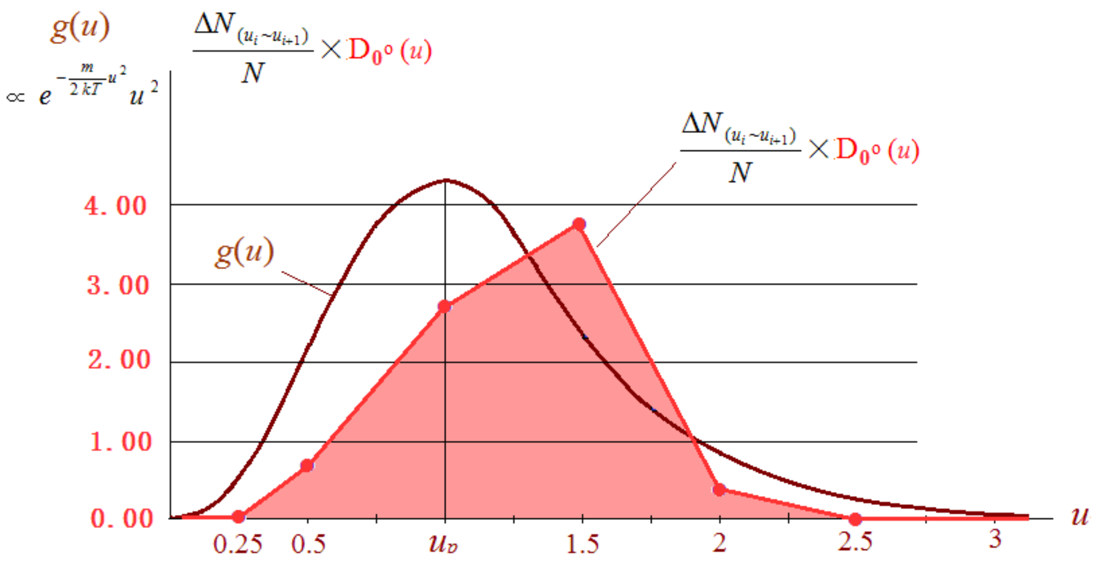
$$\theta = -15^{o}, 0^{o}, 15^{o}, 30^{o}, 45^{o}, 60^{o}, 75^{o}.$$

In any of these seven graphs, for any speed, thermal electron migration of A-B exceeds the one of B-A, and the net migration is positive (red).

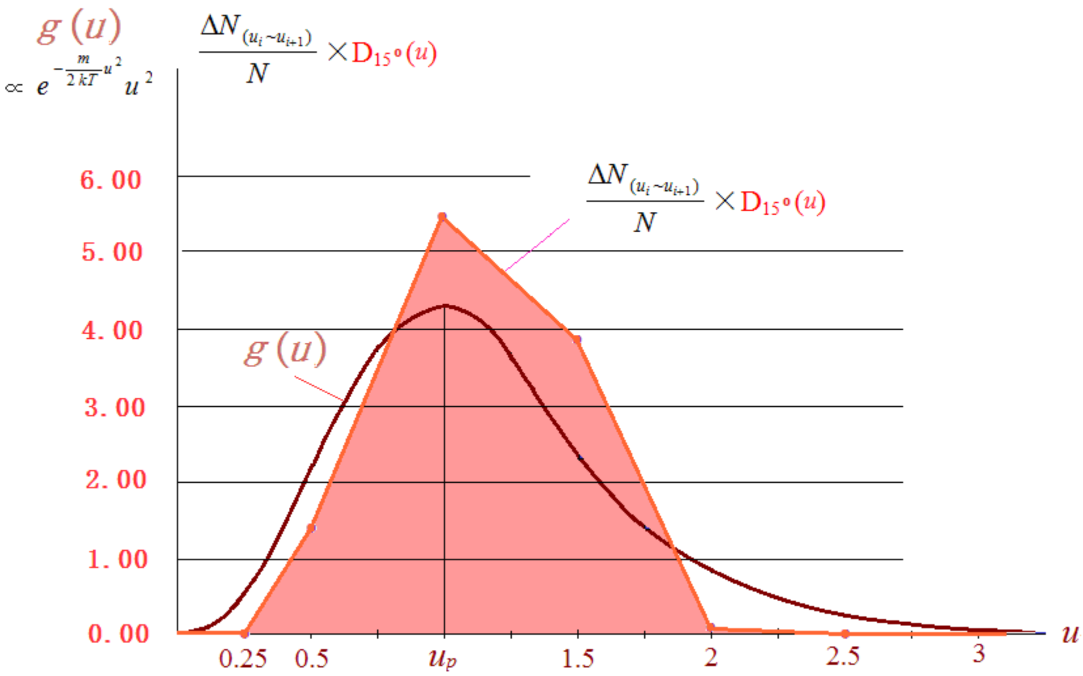
$\theta = -15^{o}$



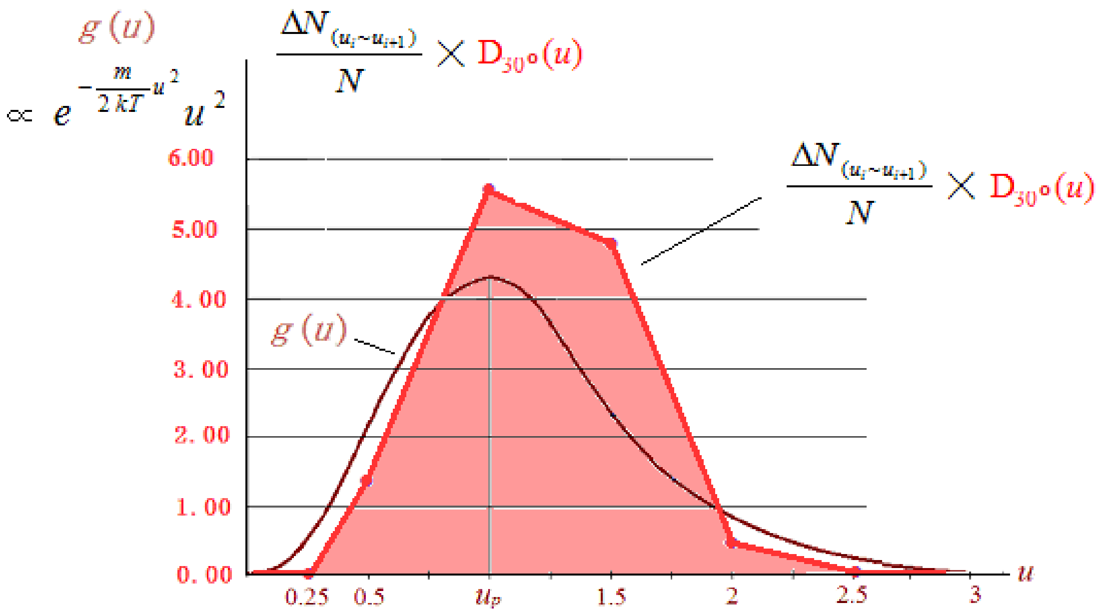
$\theta = 0°$



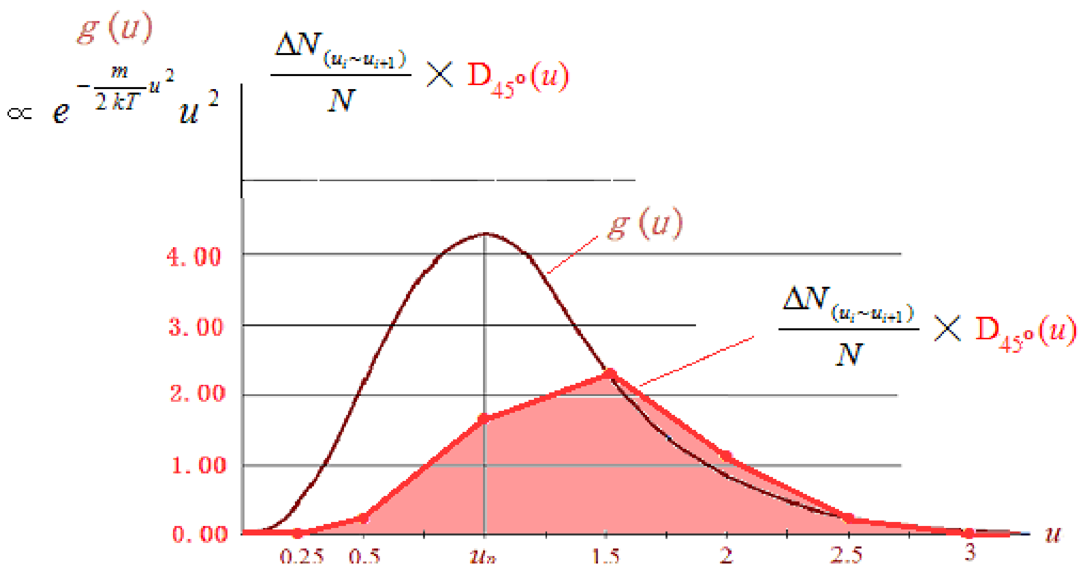
$\theta = 15°$



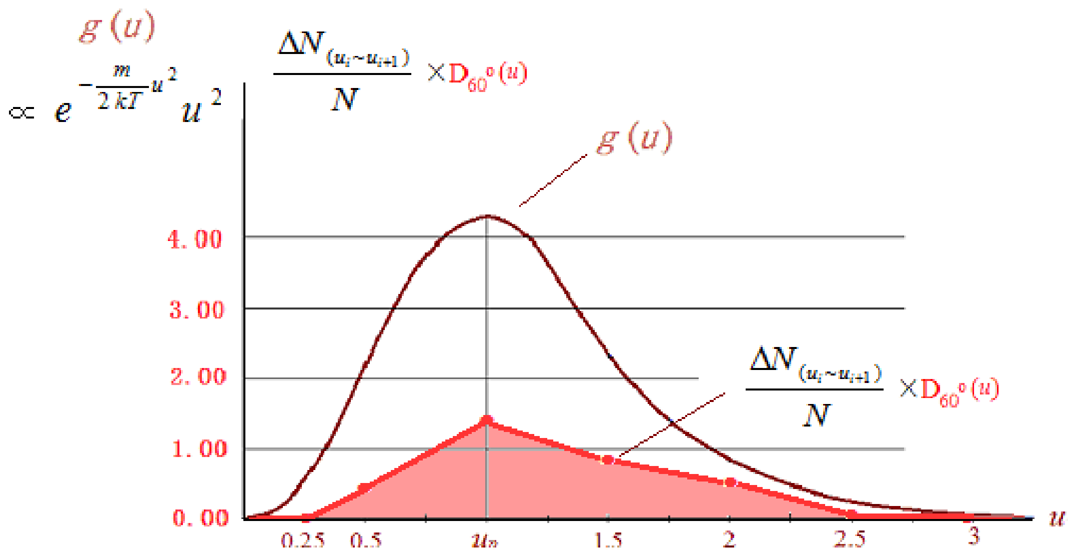
$\theta = 30^{o}$



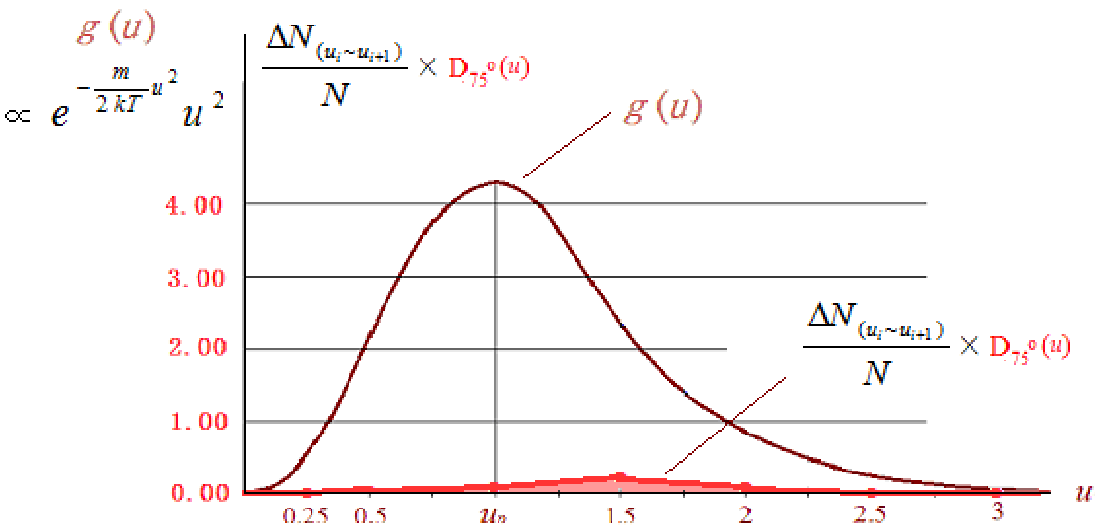
$\theta = 45^{o}$



$\theta = 60°$



$\theta = 75°$

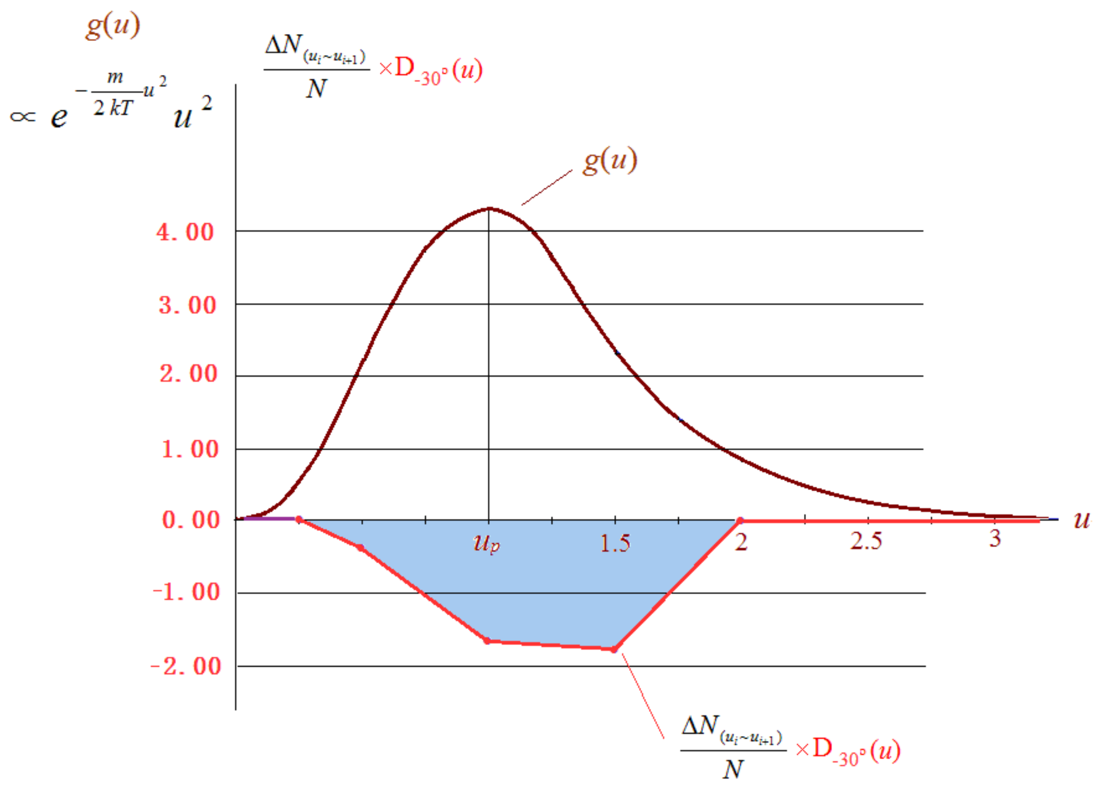


(2) The four negative graphs of $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_\theta$ are for

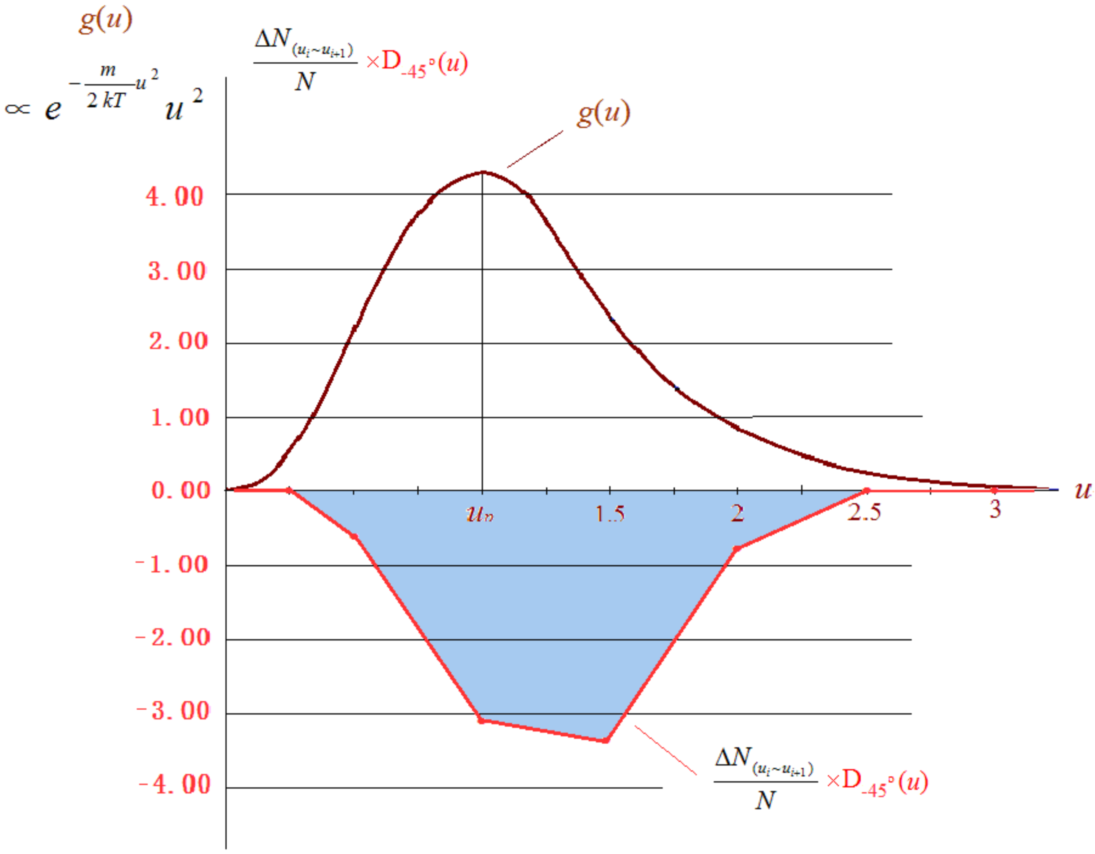
$\theta = -30°, -45°, -60°, -75°,$

In any of these four graphs, for any speed, thermal electron migration of A-B is less than the one of B-A, and the net migration is negative (blue).

$\theta = -30°$

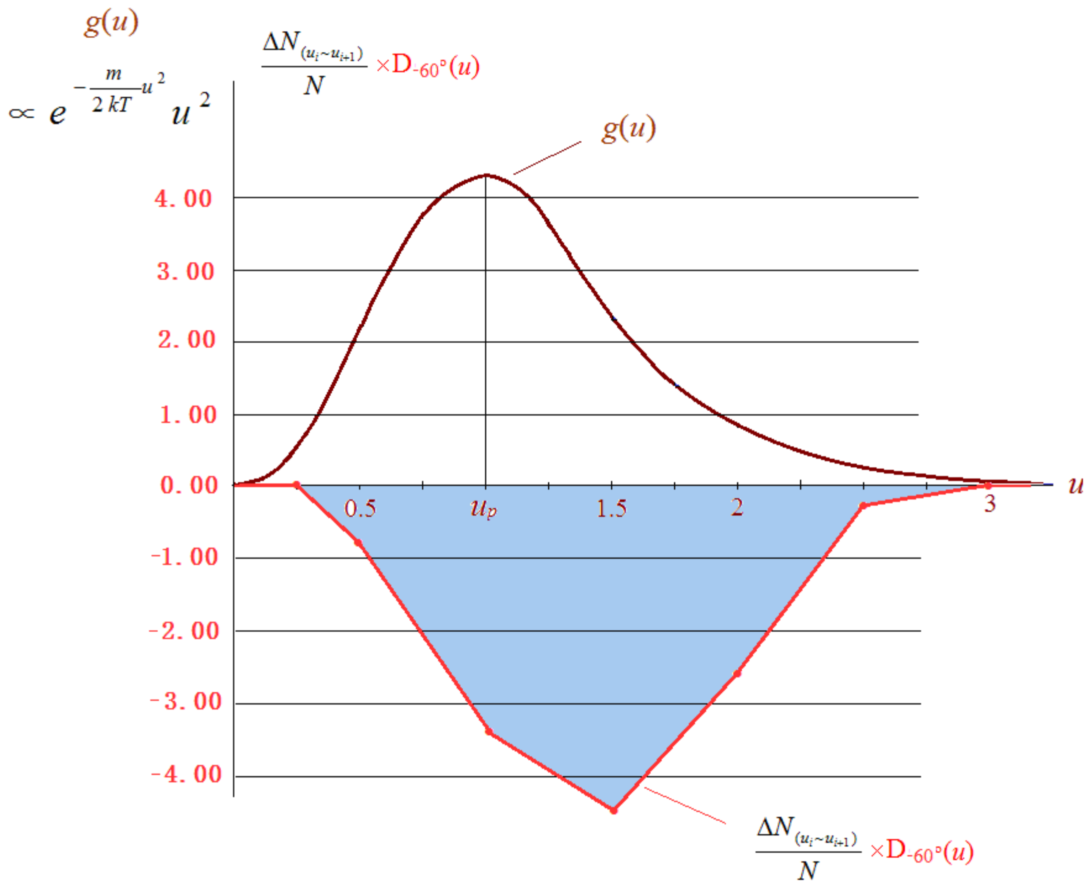


θ = - 45°



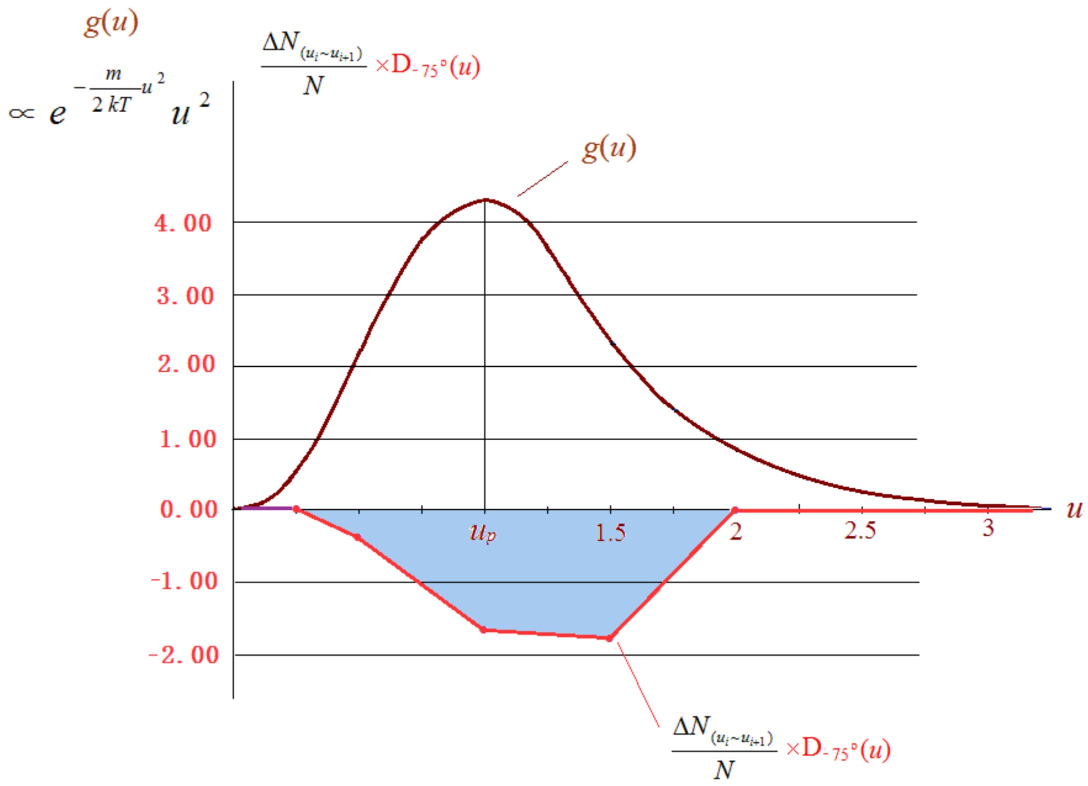
g(u)
∝ e^(−(m/2kT)u²) u²
ΔN(ui~ui+1)/N ×D-45°(u)
g(u)
4.00
3.00
2.00
1.00
0.00
-1.00
-2.00
-3.00
-4.00
u
un
1.5
2
2.5
3
ΔN(ui~ui+1)/N ×D-45°(u)

$\theta = -60^{o}$



$g(u)$
$\propto e^{-\frac{m}{2kT}u^2} u^2$
$\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{-60^o}(u)$
$g(u)$
4.00
3.00
2.00
1.00
0.00
-1.00
-2.00
-3.00
-4.00
0.5
u_p
1.5
2
3
u
$\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{-60^o}(u)$

θ = - 75 °



g(u)
∝ e^{-m/(2kT) u^2} u^2
ΔN(ui~ui+1)/N ×D-75°(u)
g(u)
4.00
3.00
2.00
1.00
0.00
-1.00
-2.00
up
1.5
2
2.5
3
u
ΔN(ui~ui+1)/N ×D-75°(u)

The areas below the eleven graphs $\frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_\theta \sim u$ represent the relative contributions to the output current of the thermal electrons of the corresponding eleven exiting angles.

The areas under the seven positive graphs are:

$\theta = -15°$ $\quad \sum_u \frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{-15°}(u) \Delta u$ = 58.5852–55.9509=2.6343

$\theta = 0°$ $\quad \sum_u \frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{0°}(u) \Delta u$ = 61.5981–53.888 = 7.7101

$\theta = 15°$ $\quad \sum_u \frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{15°}(u) \Delta u$ = 78.0799-67.2541 = 10.8258

$\theta = 30°$ $\quad \sum_u \frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{30°}(u) \Delta u$ = 64.3536–52.1645 = 12.1891

$\theta = 45°$ $\quad \sum_u \frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{45°}(u) \Delta u$ = 31.7231–26.1096 = 5.6135

$\theta = 60°$ $\quad \sum_u \frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{60°}(u) \Delta u$ = 16.8628–13.4139 = 3.4489

$\theta = 75°$ $\quad \sum_u \frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{75°}(u) \Delta u$ = 3.8775 – 3.4004 = 0.4771

Their sum is

$$\sum_{\theta(-15° \sim +75°)} \sum_u \frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_\theta(u) \Delta u$$

= 315.0802 – 272.1814 = 42.8988

The areas under the four negative graphs are:

$\theta = -30°$ $\quad \sum_u \frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{-30°}(u) \Delta u$ = 46.291-50.1498 = -3.8588

$\theta = -45°$ $\quad \sum_u \frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{-45°}(u) \Delta u$ = 33.1758–42.3881= -9.2123

$\theta = -60^{o}$ $\sum_u \frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{-60^o}(u) \Delta u = 16.7883 – 27.285 = -10.4967$

$\theta = -75^{o}$ $\sum_u \frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_{-75^o}(u) \Delta u = 4.5591 – 12.948 = -8.3889$

Their sum is

$$\sum_{\theta(-30^o \sim -75^o)} \sum_u \frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_\theta(u) \Delta u = 100.8142 – 132.7709 = -31.9567$$

The general algebraic sum of the positive and negative ones is proportional to the total output current,

$$\sum_{\theta(\sim -30^o \sim -75^o \text{and} -15^o \sim +75^o)} \sum_u \frac{\Delta N_{(u_i \sim u_{i+1})}}{N} \times D_\theta(u) \Delta u$$

$$= 415.8944 – 404.9523 = 42.8988 – 31.9567 = 10.9421.$$

So, the final result is positive. That means, when the direction of the magnetic field is positive (directed into the plane of the figure, as shown in Fig.6 (b)), **the general net electron migration between A and B** is from A to B, and the output current of the tube is positive.

As described previously, if the direction of the magnetic field is opposite, directed out off the plane of the figure, as shown in Fig.2 (c), all the trajectories will change symmetrically (mirror reflect symmetrically, left-right symmetrically), and **the general net electron migration between A and B is from B to A**, **the total output current of the tube is negative**.

These conclusions coincide with the results of the experiments of many of our electron tubes.

Nevertheless, the experiment enlightened us more.

First of all, in the above graphical survey of the electron trajectories, we have actually assumed that the work function of the various parts of the two Ag-O-Cs surfaces in a tube is uniform.

Actually that might not be true.

The oxidization of the silver film on A and B might not be uniform. In our actual electron tube, FX12-51 for example, as shown in Fig.8 (b), P, used as anode for oxygen discharge, was high at the top part of the tube, and the distances from P to the different parts of the surfaces of A and B (the top surfaces A_1 and B_1, the side surfaces A_2 and B_2 , and the bottom surfaces A_3 and B_3) were different. Hence, the extent of oxidization (by thin-oxygen discharge) of these different parts might not be uniform.

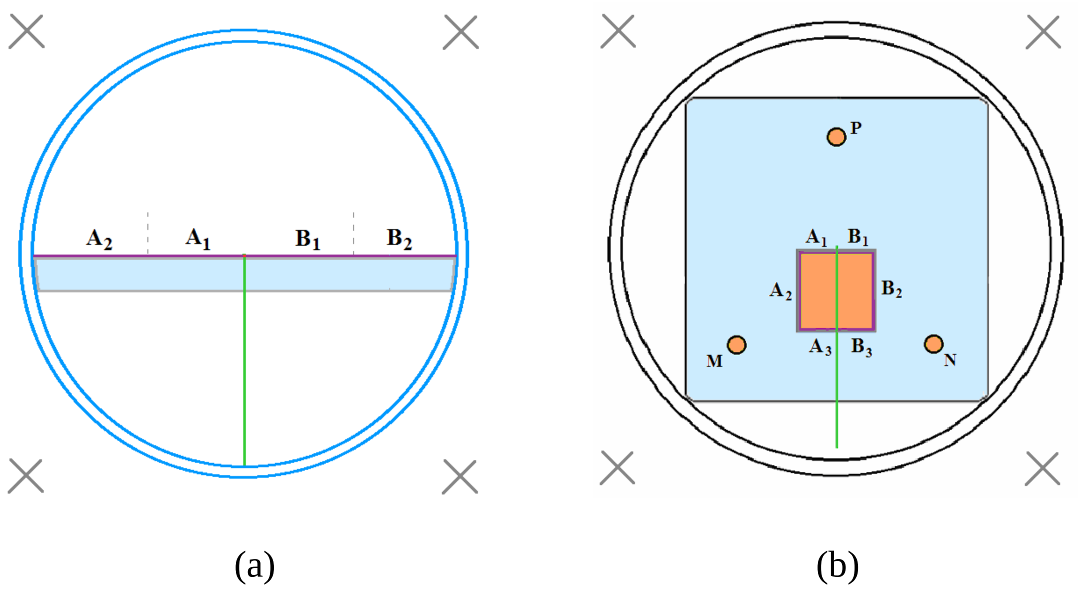


Fig. 8 The work functions of the various parts of A and B may be not uniform.

Manufacture practice told us that oxidization should be moderate. Either too much or not enough is adverse for the emission of thermal electron of the Ag-O-Cs.

Let us see some imaginary but reasonable situations.

If the work functions of the two central parts A_1 and B_1 of the two emitters, as shown in Fig.8 (a) and Fig.8 (b), are both 0.81eV, they will emit less thermal electrons. And if in the same time, the work functions of the two side parts A_2 and B_2 are both optimum, as of 0.79eV, they will emit more thermal electrons. Thus, according to the method of our graphical survey, when a positive magnetic field is applied to the tube, the number of electron trajectories of A-B will decrease, and the one of B-A will increase, and the general net electron migration A-B will decrease, even become a negative one, i.e., the general electron migration from B to A exceeds the one from A to B, so the output current may be a negative one.

And, in such a case, if the direction of the magnetic field is opposite, the direction of the net electron migration will also be reversed, i.e., from A to B, the output current will be positive.

The experiment with a part of our electron tubes behaved this way. We call these tubes **the negative-current tubes**.

Nevertheless, the most probable situation is still, when a positive magnetic field is applied to the tube, a positive output current, as illustrated by Fig 2 (b). That is, the net thermal electron migration is from A to B. And we call these tubes **the positive-current tubes**.

We also realized in our practice that the size and shape of the two emitters, and especially their relative position in the tube, might also change the behavior of the tube in the magnetic field considerably.

Anyway, when a magnetic field is applied to the tube, **the net electron migration between A and B is either positive or negative, mostly not zero**. So there is usually **a stable output current**, maybe positive, may be negative.

In any case, the experiments proved that, **the magnetic field is a successful demon, which accomplished its job quietly and easily, without any expenditure of work**.

Appendix

Maxwell's speed distribution —— the beam distribution of three dimensional speed for gas molecules or thermal electrons

The original Maxwell's gas molecule speed distribution is, as mentioned previously,

$$dn(dv_x, dv_y, dv_z) = n(\frac{m}{2\pi kT})^{3/2} e^{-\frac{m}{2kT}v_x^2+v_y^2+v_z^2} dv_x dv_y dv_z \quad (1)$$

where dn (dv_x, dv_y, dv_z) is the number of gas molecules or thermal electrons **in unit volume** and in the speed range of dv_x dv_y dv_z. It is a volume distribution, and the speed is of three dimensions.

For a spherical coordinate system, we may have

$$dn(v,\theta,\varphi) = n(\frac{m}{2\pi kT})^{3/2} e^{-\frac{m}{2kT}v^2} v^2 dv \sin\theta d\theta d\varphi \qquad (2)$$

Integrate (2) over θ (from 0 to π) and ϕ (from 0 to 2π), we derive equation (3),

$$f(v)dv = 4\pi n(\frac{m}{2\pi kT})^{3/2} e^{-\frac{m}{2kT}v^2} v^2 dv \qquad (3)$$

This is, as have been mentioned, also a **volume speed distributio**n, the speed *v* is of three dimensions.

To find the beam speed distribution (hole passing distribution, wall collision distribution or wall emission distribution) of three dimensions, let us first consider the numbers of gas molecules or thermal electrons at time *t* in a cylinder $vdtdA\cos\theta$ as shown in Fig.1, and find out the gas molecules or thermal electrons with a speed of *v* to *v*+*dv*, in the direction within $\theta \sim \theta + d\theta$ and $\phi \sim \phi + d\phi$. All these gas molecules or thermal electrons will pass area *dA* in the time duration *dt*.

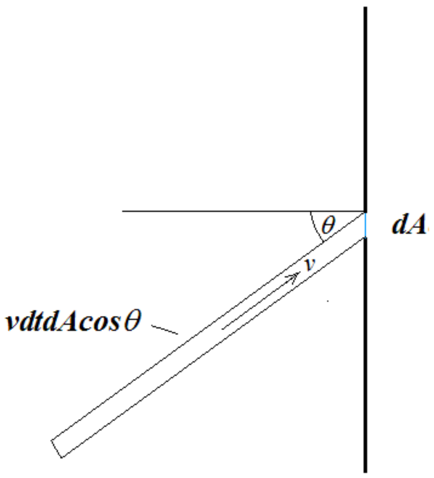


Fig 1 the molecules or thermal electrons in the cylinder $vdtdA\cos\theta$ at time *t* of speed $v \sim v+dv$, of directions $\theta \sim \theta + d\theta$ and $\phi \sim \phi + d\phi$ will pass *dA* in *dt*.

For dA =1 and dt =1 in the direction of $d\theta$ and $d\phi$ the number of passing gas molecules or thermal electrons of v to $v+dv$ is

$$d\Gamma(v,\theta,\varphi) = n\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m}{2kT}v^2} v^2 dv \sin\theta d\theta d\varphi \times (v dt dA \cos\theta)$$

$$= n\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{m}{2kT}v^2} v^3 dv \sin\theta \cos\theta d\theta d\varphi$$

Integrate it over θ (from 0 to π/2) and ϕ (from 0 to 2π), we derive

$$d\Gamma(v) = n\pi\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m}{2kT}v^2} v^3 dv \tag{4}$$

$$\left(\int_0^{\frac{\pi}{2}} \sin\theta \cos\theta d\theta \int_0^{2\pi} d\varphi = \frac{1}{2}\sin^2\frac{\pi}{2} \bullet 2\pi = \pi\right)$$

This is the number of gas molecules or thermal electrons of speed range v ~ $v + dv$ colliding or passing through dA = 1 in dt = 1. It is also the number of thermal electrons of speed range v ~ $v + dv$ emitted from the area dA = 1 on the surface of an emitter in time duration dt = 1.

The general number of molecules or thermal electrons of all the speed colliding on (or passing through, or emitting from) dA = 1 in dt = 1 is the so called **wall-collision number** (or, **wall-emission number)**.

$$\Gamma = \int n\pi\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m}{2kT}v^2} v^3 dv = n\pi\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} \int e^{-\frac{m}{2kT}v^2} v^3 dv$$

$$= n\pi\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} \times \frac{1}{2}\left(\frac{2kT}{m}\right)^2 = \frac{1}{4} n\left(\frac{8kT}{\pi m}\right)^{\frac{1}{2}}$$

Briefly,

$$\Gamma = \frac{1}{4} n\bar{v} \tag{5}$$

The probability of a gas molecule or a thermal electron in the beam and have a speed within the speed range $v \sim v + dv$ is

$$g(v)dv = \frac{d\Gamma(v)}{\Gamma}$$

$$= (n\pi(\frac{m}{2\pi kT})^{3/2} e^{-\frac{m}{2kT}v^2} v^3 dv) \div \left[\frac{1}{4} n \left(\frac{8kT}{\pi m} \right)^{\frac{1}{2}} \right]$$

$$= 4\pi \left(\frac{m}{2\pi kT} \right)^{\frac{3}{2}} \left(\frac{\pi m}{4 \times 2kT} \right)^{\frac{1}{2}} e^{-\frac{m}{2kT}v^2} v^3 dv$$

$$= 2\left(\frac{m}{2kT} \right)^2 e^{-\frac{m}{2kT}v^2} v^3 dv$$

Briefly,

$$g(v)dv = 2\left(\frac{m}{2kT} \right)^2 e^{-\frac{m}{2kT}v^2} v^3 dv \qquad (6)$$

This is **the beam speed distribution** of gas molecules or thermal electrons of three dimensions. Where $A = 2\left(\frac{m}{2kT} \right)^2$ is the normalization constant

$$\int g(v)dv = \int \frac{d\Gamma(v)}{\Gamma} = \int 2\left(\frac{m}{2kT} \right)^2 e^{-\frac{m}{2kT}v^2} v^3 dv = 1$$

It is also easy to find the three characteristic speeds and the mean kinetic energy for the gas molecules or thermal electrons in such a beam

$$v_p = \sqrt{\frac{3kT}{m}} \quad \bar{v} = \sqrt{\frac{9\pi kT}{8m}} \quad \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}} \quad \bar{\varepsilon} = 2kT \quad (7)$$

To sum up, equation (3) is **the volume speed distribution** for the thermal electrons existing in the vacuum, as the so called “space charges”, which is in equilibrium with some thermal electron emitter (a hot

$$e^{-\frac{m}{2kT}u^2}u^2$$

filament, for example), the speed in consideration is of three dimensions.

What Richardson, Germer, et al measured in 1907 ~ 1925 was the wall emission distribution of the speeds of the thermal electrons just emitted from the hot filament. It was a beam distribution, and was governed by equation (6), the speed of the electrons in consideration is of three dimensions.