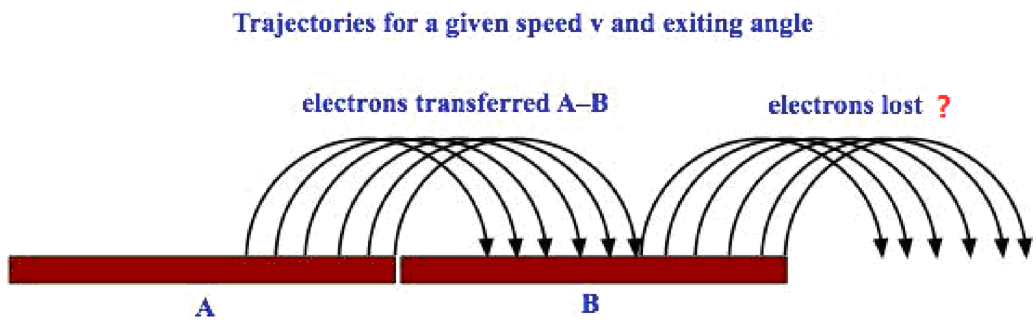
Realization of Maxwell's Hypothesis（Ⅱ）

Preface

In July 2017, Professor Harvey Leff of Reed College (also Professor Emeritus of Physics, California State Polytechnic University) asked us a question about our experiment[1] with his following figure:



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For the past 40 years, we called this question the right most problem.

Professor Zhao Kaihua of Beijing University (plasma expert, Beijing, 1979), Dr. Jack Denu of the United States (Salt Lake City, PQE 2007), and Professor Wulf Wulfhekel of Karlsruhe Institute of Science and Technology (Karlsruhe, 2013) et al, had discussed this question face to face with one of the authors, Xinyong Fu. So far, we, Xinyong Fu and Zitao Fu, have studied and explored the problem many times. We were well prepared in mind to face it again. Now Professor Leff asked us this question once again with a vivid picture, he incented us to investigate into it more carefully and more profoundly.

Thanks to all these friendly discussions, we progressed step by step.

The Right Most Problem

A Graphical Survey on the Various Trajectories of Thermal Electrons in Fu & Fu's Experiment of a Magnetic Demon

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Abstract

This is a graphical survey on the various thermal electron trajectories and their contribution to the output current in Fu & Fu's heat-electric conversion experiment [(1)]. The electrons are emitted at room temperature from two Ag-O-Cs surfaces, A and B, with various exiting angles, speeds and exiting spots. A static uniform magnetic field was applied to the tube, transferring the thermal electrons from A to B (a net electron migration from A to B), and resulting in an electric potential, an output current and power. The experiment was in contradiction to the Kelvin statement of the second law.

Introduction

In our experiment of electron tube FX12-51, two parallel Ag-O-Cs surfaces (work function 0.8eV) ceaselessly emit thermal electrons at room temperature. The speeds of the thermal electrons, as discovered by Richardson with his blocking-potential experiments in 1907~1909, are governed by Maxwell's gas molecule speed distribution law

$$f(v)dv = 4\pi n\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m}{2kT}v^2} v^2 dv \qquad (1)$$

Equation (1) gives the numbers of molecules or thermal electrons in unit space volume and in the speed interval v to v + dv. (So, it is a volume distribution, which is related but a little different from the wall emission distribution, see the appendix of this paper.) At 20°C, the mean velocity of the thermal electrons is 106 km/s, compared to the common gas molecules (400~500 m/s), **they are extremely fast!**

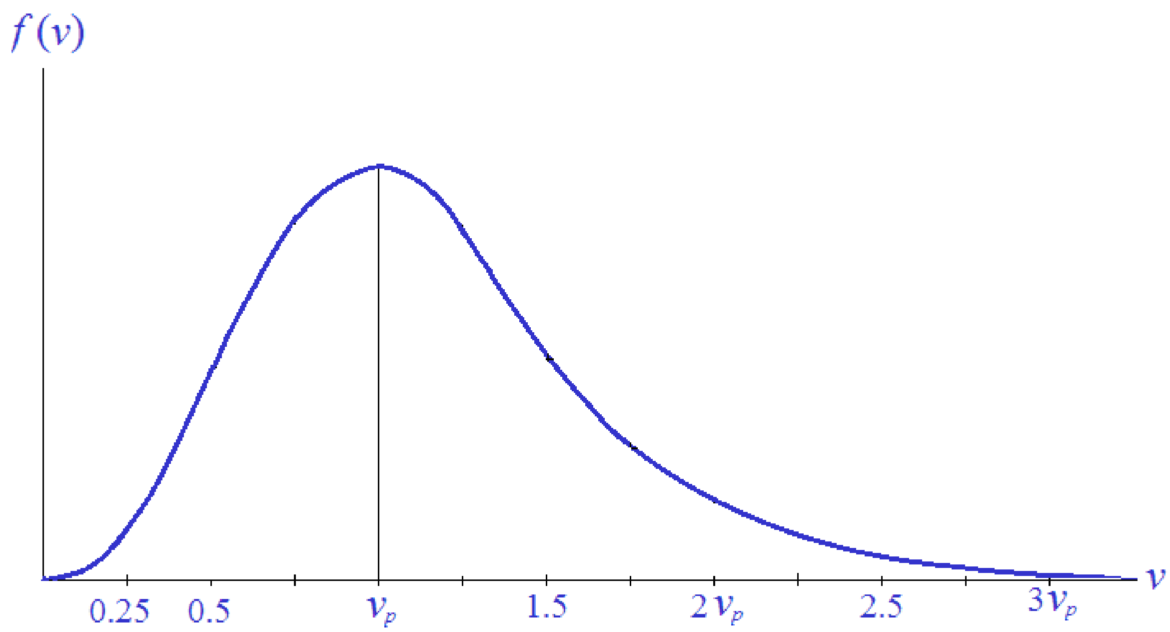


Fig 1 Maxwell's speed distribution law (volume distribution)

Maxwell's speed distribution concentrates in the speed range of 0.25 v_p ~ 2.5v_p, here v_p is **the most probable speed** of the (gas molecule or) thermal electrons at the temperature. Divide the area under Maxwell's distribution graph into nine parts A_0, A_1, A_2, A_3, A_4, A_5 A_6, A_7, A_∞, as shown in Fig 2: for A_0 , $\Delta v = 0.25v_p$, for each part of A_1 to A_7, $\Delta v = 0.5v_p$, and for A_∞, $\Delta v = \infty$ (from $3.75v_p \sim \infty$) .

Calculate the relative numbers of thermal electrons in each part with the help of error functions, the results are list in table 1.

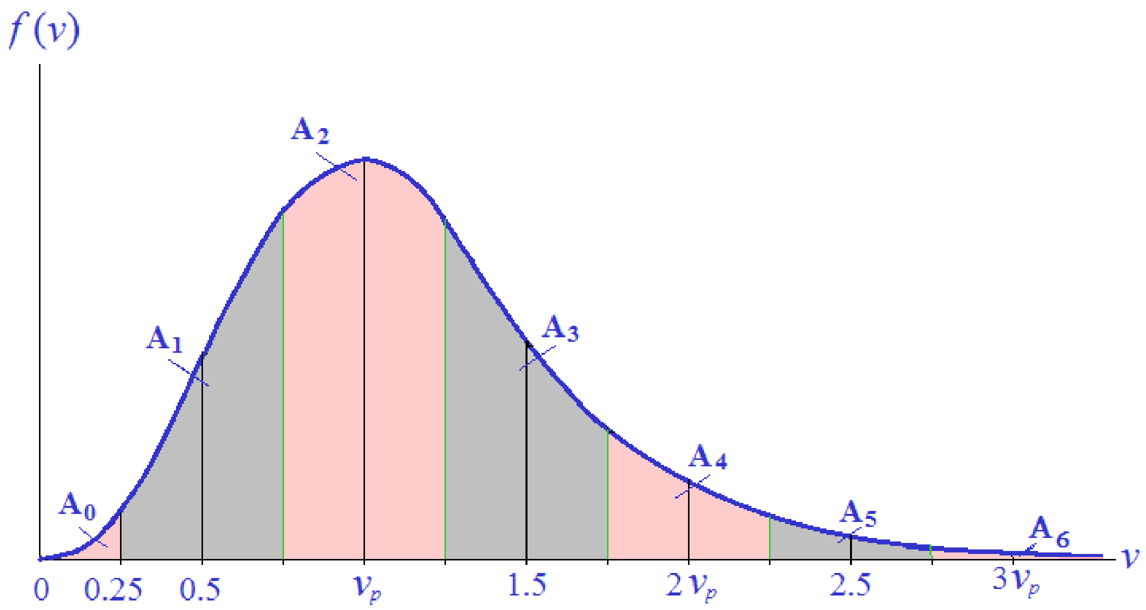


Fig 2 The nine area under Maxwell's speed distribution graph.

The error functions are

$$N_{0\sim v} = \int_0^v N\, 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m}{2kT}v^2} v^2 dv$$

$$= \int_0^v N\, 4\pi \left(\frac{1}{\pi v_p^2}\right)^{\frac{3}{2}} e^{-\frac{v^2}{v_p^2}} v^2 dv = \int_0^v N \frac{4}{\sqrt{\pi}} e^{-x^2} x^2 dx$$

$$\frac{N_{0\sim v}}{N} = \frac{4}{\sqrt{\pi}} \int_0^x e^{-x^2} x^2 dx = \frac{4}{\sqrt{\pi}} \left[\frac{1}{2} \int_0^x e^{-x^2} dx - \frac{1}{2} \int_0^x d(xe^{-x^2}) \right]$$

$$= \left[erf(x) - \frac{2}{\sqrt{\pi}} xe^{-x^2} \right]$$

Range of speed ($x = v/v_p$)	$A_i = \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} = \frac{\Delta N_{0\sim v_{i+1}} - \Delta N_{0\sim v_i}}{N}$
0.0 ~ 0.25 v_p	A_0 = （0.276326 - 0.265004） = 0.011358 = 1.13%
0.0 ~ 0.125 v_p	A_{01} = （0.140222-138860） = 0.001362 = 0.14 %
0.125 ~ 0.25 v	A_{02} = （0.011358-0.001362） = 0.009996 = 0.99%
0.25 ~ 0.75v_p	A_1 = 0.228958 - 0.011358 = 0.217599 = 21.76%
0.75 ~ 1.25v_p	A_2 = 0.627286 - 0.228958 = 0.398328 = 39.83%
1.25 ~ 1.75v_p	A_3 = 0.894316 - 0.627249 = 0.267067 = 26.71%
1.75 ~ 2.25v_p	A_4 = 0.982467 - 0.894316 = 0.088151 = 8.82%
2.25 ~ 2.75v_p	A_5 = 0.998287 - 0.982467 = 0.015820 = 1.58%
2.75 ~ 3.25v_p	A_6 = 0.999900 - 0.998287 = 0.001613 = 0.16%
3.25 ~ 3.75v_p	A_7 = 0.999996 - 0.999901 = 0.000095≈0.01%
3.75v_p ~ ∞	A_∞= 1.000000 - 0.999996 = 0.000004 = 0.0004%

Table 1 The nine areas under Maxwell's distribution graph, $\sum A_i$=100%

In our experiment, electrons of different speed ranges contribute to the output current differently. There are two causes.

The first cause, of course, is Maxwell's speed distribution itself. The numbers of electrons in the different speed ranges are obviously different, hence their contributions to the output current are certainly different.

The second cause is the trajectories of the thermal electrons of different speeds (also related to the exiting angles and exiting spots) are tremendously different. Tremendously different trajectories result in tremendously different contributions to the output current. That is just **the right-most problem**.

For simplicity of discussion, we analyze here the right-most problem with an ideal simple and symmetric tube as shown in Fig 3, which is similar to our actual experimental tube FX12-51, meanwhile has some difference from it, see Fig 4 and Fig 5.

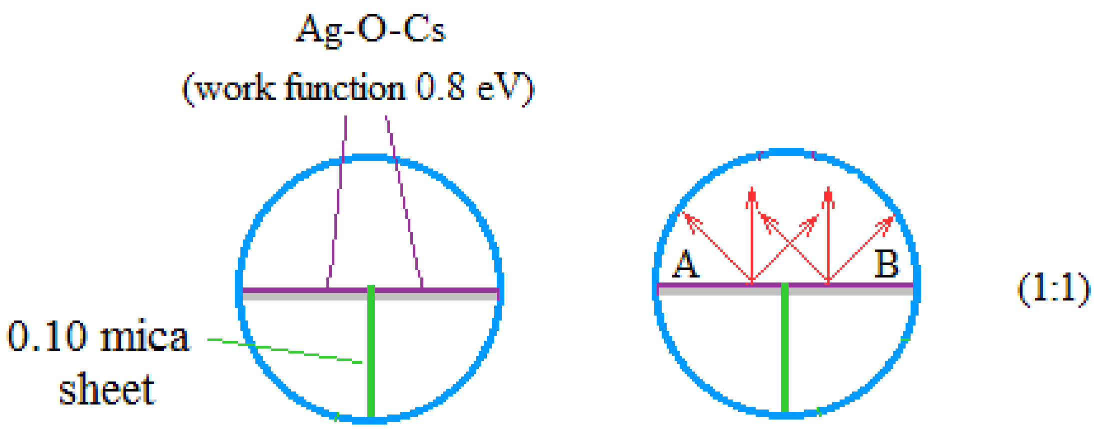


Fig3 The cross section of an ideal symmetric electron tube.

The mean free path of thermal electrons in a vacuum of 10^{-6} mmHg is determined by their collisions with the remnant gas molecules, about 50 meters, which is much greater than the dimensions of our tube, ϕ28mm $\times$ L60 mm. Hence, these collisions may be neglected in our discussion, and the actual free paths of the thermal electrons in the tube may be regarded to be just determined by their collisions with the tube's glass wall and the emitters themselves.

Numerous thermal electrons frequently collide with the glass wall. These collisions are due to the kinetic energy of the quickly flying electrons. At T = 293K (t = 20°C), the average kinetic energy of the electrons is

$$\bar{\varepsilon} = \frac{3}{2}kT = 0.038eV = 6.07\times10^{-21}J\ .$$

So, **the collisions are extremely weak, extremely soft and gentle**. The glass wall is sufficiently hard and smooth for such collisions, and may be regarded here as a perfect elastic solid.

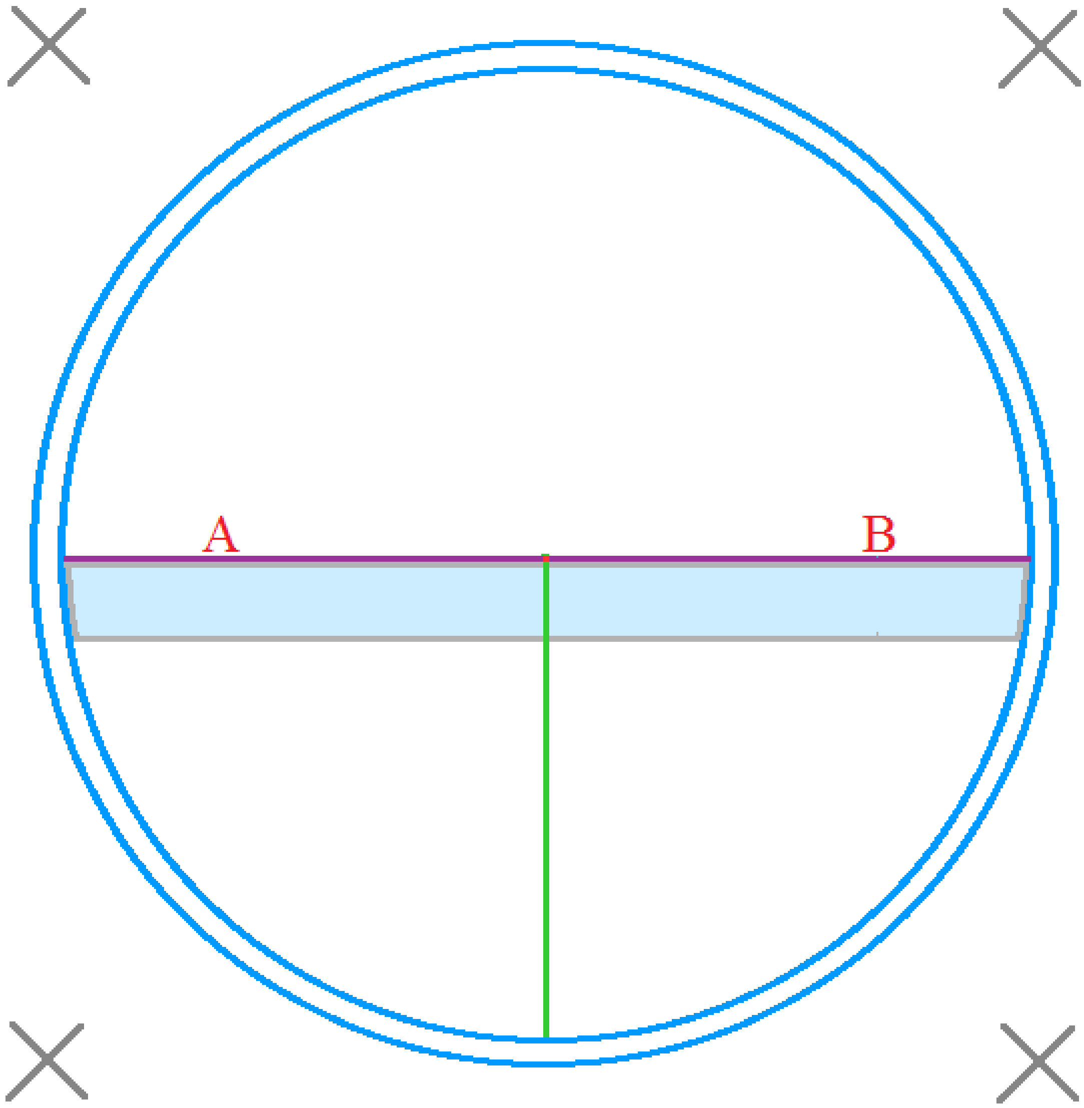


Fig 4 The cross section of the ideal simple and symmetric tube (5:1)

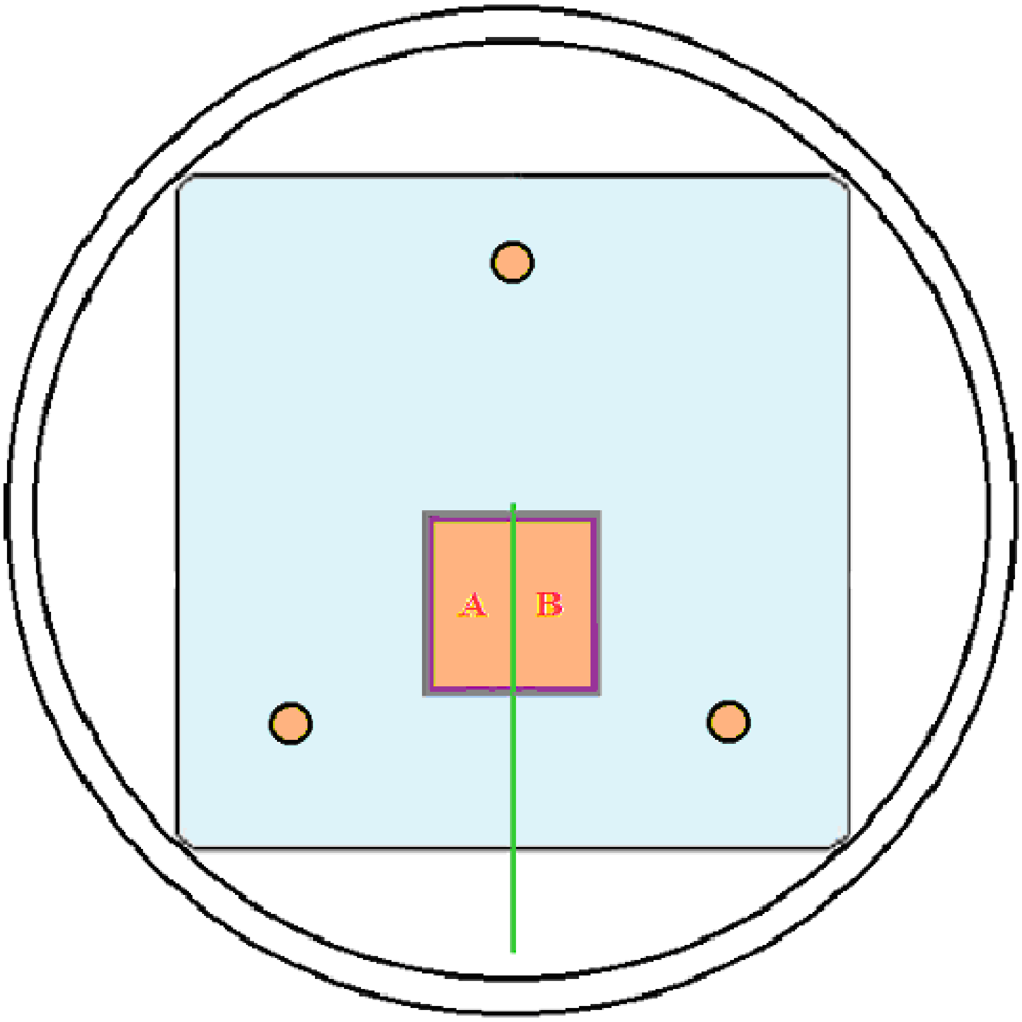


Fig 5 The cross section of our actual experimental electron tube FX12-51 (5:1)

When no magnetic field is applied to the tube as shown in Fig 6(a), the thermal electrons emitted from A or B fly straight forward until they hit the glass wall and bounce back, and finally fall back to the emitters. Statistically, the trajectories of electrons that eventually move from A to B and those that eventually move from B to A are in numerous symmetric pairs, hence, all the electron migrations between A and B cancel each other (A and B like two islands), no net contribution to the output current.

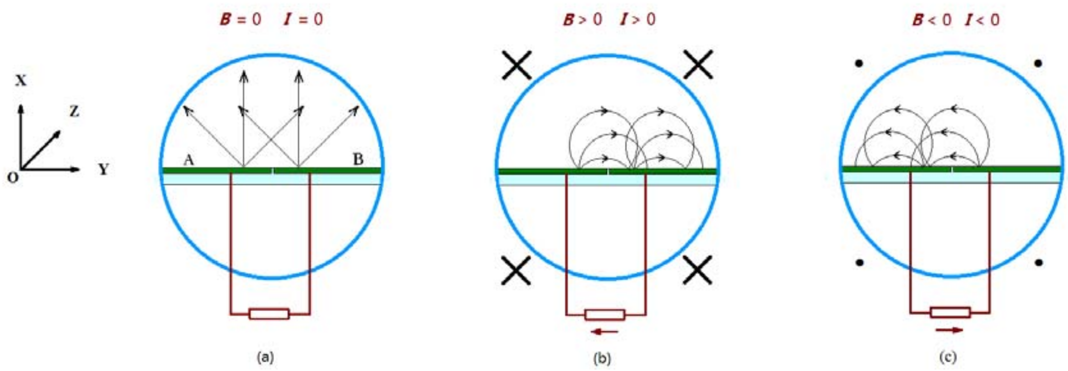


Fig 6 Thermal electrons eject from A or B, with or without a magnetic field.

Now, if a static uniform magnetic field is applied to the electron tube in the direction parallel to the tube axis (the Z-axis), and firstly, directed into the plane of the figure, as shown in Fig 6(b), all the thermal electrons will fly clockwise along circles of different radii according to their speeds

$$R = \frac{m}{eB} v$$

where v is the speed component perpendicular to the tube axis.

We omit here the discussion about the electron's Z-component motion.

In this paper, we are going to use a graphic method to survey the numerous various trajectories of the thermal electrons emitted from A or B with different exiting angles, speeds and exiting spots, and discuss their contributions to the output current.

In this and the next two pages, we demonstrate in four figures the trajectories of thermal electrons emitted vertically ($\theta = 0^{\circ}$) from all the different points of A and B with a speed of $v = v_p$. Besides, we also demonstrate two another figures with exiting angles of $\theta = -30^{\circ}$ and $\theta = 30^{\circ}$, respectively, with some different speeds. These trajectories, **as a**

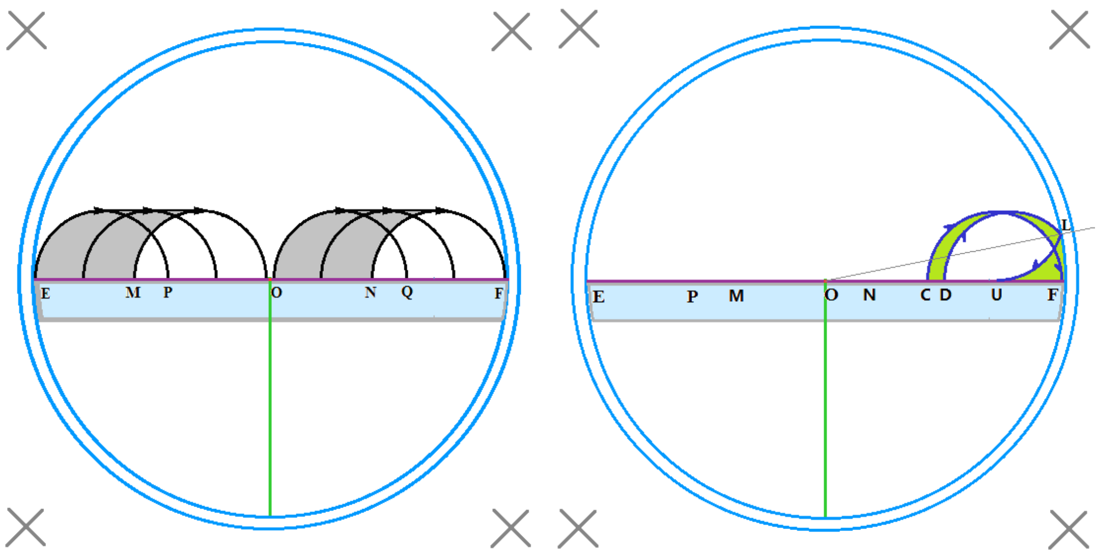
sample (representing all the electron trajectories in the tube), may be classified into four groups.

(1) The first group: A-A = A-directly-A + A-glass-A

A-directly-A Part of the electrons emitted from A directly fall back to A. Examples, Fig 1-1(a), from E to P, M to O, etc., grey.

A-glass-A Part of the electrons emitted from A hit the glass wall and bounce back to A. Examples, Fig 4-3 (a), from P to Q, etc., green.

The trajectories of the first group result in no electron migration between A and B. They are A-A, contribute nothing to the output current.



(a) A-directly-A (grey) & B-directly-B (grey) (b) B-glass-B (green)

No electron migration between A and B due to all the above trajectories.

Fig 1-1 (a) (b) $\theta = 0^{\text{o}}$ $v = v_p$ $R = 4\text{mm}$ $(B = 1.34 \text{ gauss})$

(2) The second group: B-B = B-directly-B + B-glass-B

B-directly-B Part of the electrons emitted from B directly fall back to B. Examples, Fig 1-1(a), from O to Q, N to F, etc., grey.

B-glass-B Part of the electrons emitted from B hit the glass wall and bounce back to B. Examples, Fig 1-1(b), from C to F, D to U, etc., green.

The trajectories of the second group also result in no electron migration between A and B. They are B-B, contributing nothing to the output current, too.

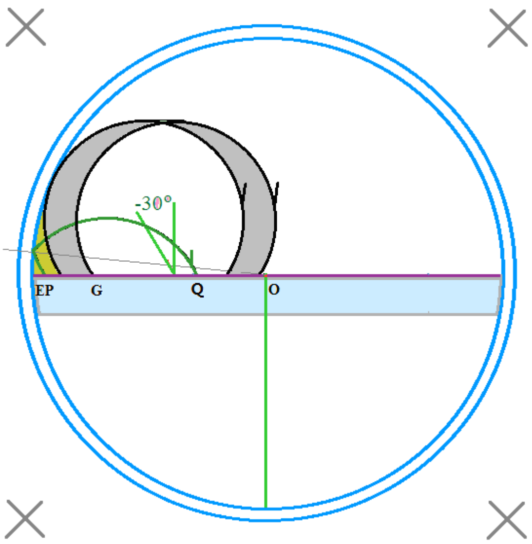


Fig 4-3 (a) A-glass-A (green)
θ = -30° $v = 1.5v_p$ R = 6mm
No electron migration between A and B

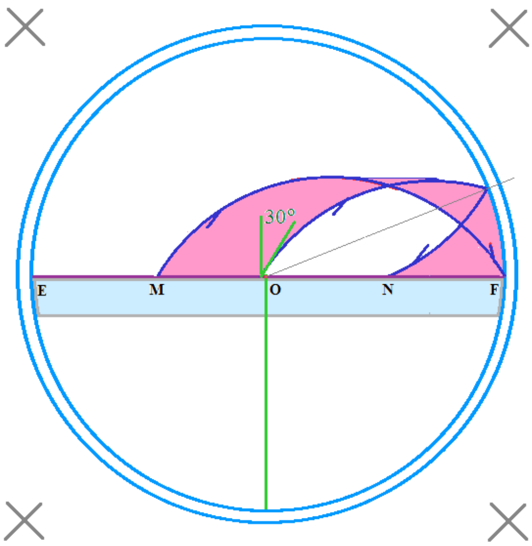
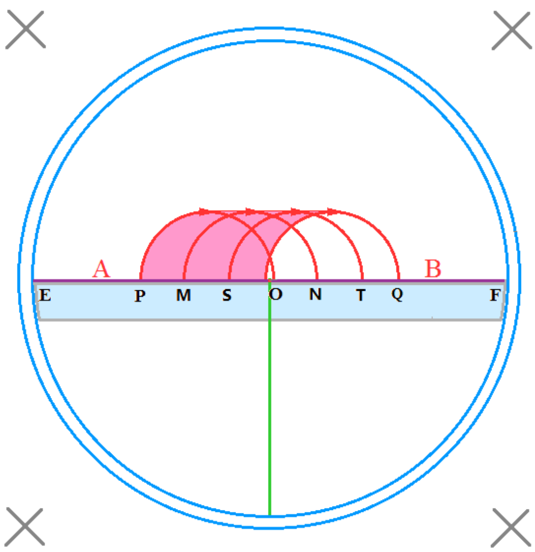
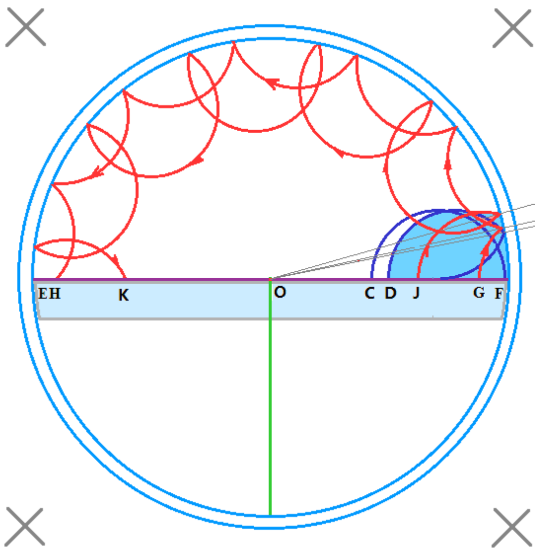


Fig 5-6 (b) A-glass-B (pink)
θ = 30° $v = 3v_p$ R = 12mm
Electrons migrate from MO to FN.



(c) A-directly-B (from PO to OQ, pink)
PO/EO = 40/70 = 0.57



(d) B-glass-A (from J to K, etc., blue)
DF/OF = 35/70 = 0.50

A-B exceeds B-A, the net contribution is 57% - 50% = 7%

Fig 1-1 (c) (d) θ = 0° $v = v_p$ R = 4mm (B = 1.34 gauss)

(3) The third group: A-B = A-directly-B + A-glass-B

A-directly-B Part of the electrons emitted from A directly fall into B. Examples, Fig 1-1(c), from P to O, M to N, S to T, O to Q, etc., pink.

A-glass-B Part of the electrons emitted from A hit the glass wall and bounce back into B. Examples, Fig5-6(b), from M to F, O to N, etc., pink.

The electrons of the third group migrate from A to B. They are A-B, contributing positively to the output current.

(4) The fourth group: B-A = B-glass-A

B-glass-A Part of the electrons emitted from B hit the glass wall and bounce back once or several times, and finally fall into A. Examples, Fig 1-1(d), from G to H, J to K, etc., blue.

The electrons of the fourth part migrate from B to A. They are B-A, contributing negatively to the output current.

A strange thing happens here ---- A certain part of the electrons emitted from A can directly fall into B, but none of the electrons emitted from B can directly fall into A: A-directly-B is fairly possible, but B-directly-A is absolutely impossible. This conclusion is also valid for the electrons of all other exiting angles and speeds. So, such an electron tube in a static magnetic field is an unconventional thermodynamic system: it is sharply in contradiction to Boltzmann's principle of detailed balance.

The ways of electron migration from A to B are of totally different geometrical styles from the ones of electron migration from B to A. So, for the whole tube, the general number of electron migrations from A to B is almost impossible to be exactly equal to the one from B to A. That is, the general contribution of the thermal electrons to the tube's output current is usually, even mostly, not zero.

If the intensity of the magnetic field is different, the output current will follow to be different. By adjusting the intensity of the magnetic induction we may find a maximum output current from the tube (for a given temperature).

If the direction of the magnetic field is opposite, **as shown in Fig 6(c), all the electron trajectories will be changed symmetrically (mirror reflect symmetry, left-right symmetry), and the direction of the output current will be reversed. This is an easy operation, but very significant in our experiment.**

The whole graphical survey consists of two parts, about 140 pages, 80 trajectory figures. The following exiting angles and speeds are selected for drawing the trajectories:

$$\theta = 0^{o}, -15^{o}, 15^{o}, -30^{o}, 30^{o}, -45^{o}, 45^{o}, -60^{o}, 60^{o}, -75^{o}, 75^{o}$$

$$v = 0.5v_{p,} \quad v_{p,} \quad 1.5v_{p,} \quad 2v_{p,} \quad 2.5v_{p,} \quad 3v_{p,} \quad 4.5v_{p.}$$

If the readers have enough time, we suggest them read the whole survey. However, the survey is rather long, some readers may have not so much time to read it completely. If so, we suggest them just read the first part of the survey, which deals with only the thermal electrons emitted vertically from A or B ($\theta = 0^{\circ}$) with different speeds. The first part has only 16 pages, 11 trajectory figures, providing a primary understanding of the whole survey.

The First Part of the survey ($\theta = 0^{\circ}$)

In this first part, we survey all the trajectories of electrons that emitted vertically from the two Ag-O-Cs surfaces with these different speeds,

$v = 0.125v_p$, $0.25v_p$, $0.5v_p$, v_p, $1.5v_p$, $2v_p$, $2.5v_p$, $3v_p$, $4.5v_p$.

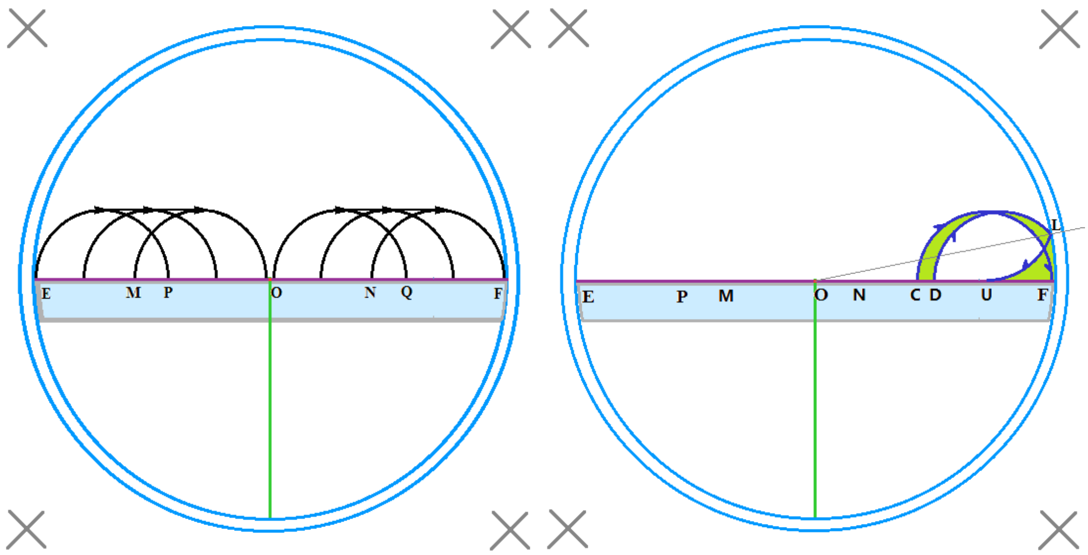
According to Lambert's law, normal is the strongest direction for such emissions ($j \propto \cos\theta$).

For convenience of discussion, we choose a uniform magnetic field of a specific magnetic induction intensity, $B = 1.34 \times 10^{-4}$tesla = 1.34 gausses, to be applied to the ideal electron tube in the direction parallel to the tube axis. In such a magnetic field, the electrons of speed (Z-component) v = 94.3km/s (= v_p, for $t = 20^{\circ}$C) cycle with a radius of R = 4mm (more precisely, 4.002mm). Thus, for the electrons of speed $v = 0.5v_p$, the corresponding radius is R =2mm; for the electrons of speed $v = 2v_p$, R = 8mm, and so on.

1. Trajectories of electrons of $\theta = 0^{\circ}$ and different speeds

First, inspect the trajectories of electrons of $\theta = 0^{\circ}$ and $v = v_p$

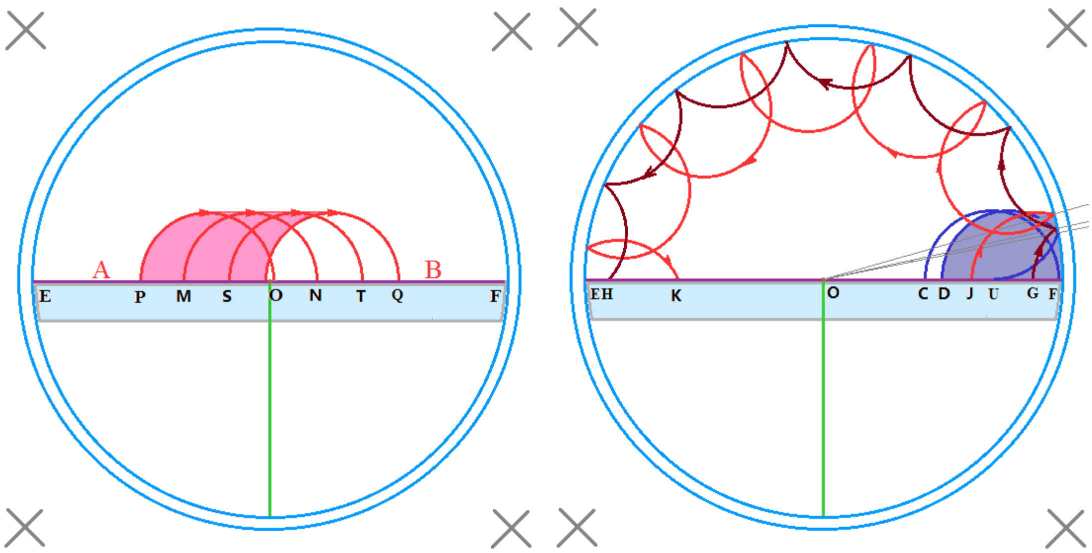
(1) Fig 1-1 $\theta = 0^{\circ}$ $v = v_p$ R = 4mm (B =1.34 gauss)



(a) A-directly-A & B-directly-B (from E to P, M to O)

(b) B-glass-B (from C to F, D to U)

No electron migration between A and B due to these trajectories.



(c) A-directly-B (from PO to OQ)
PO/EO = 40/70 = 0.57 (pink)
(PO=40, EO=70)

(d) B-glass-A (from J to K, etc.)
DF/OF = 35/70 = 0.50 (blue)
(DF=35, OF=70)

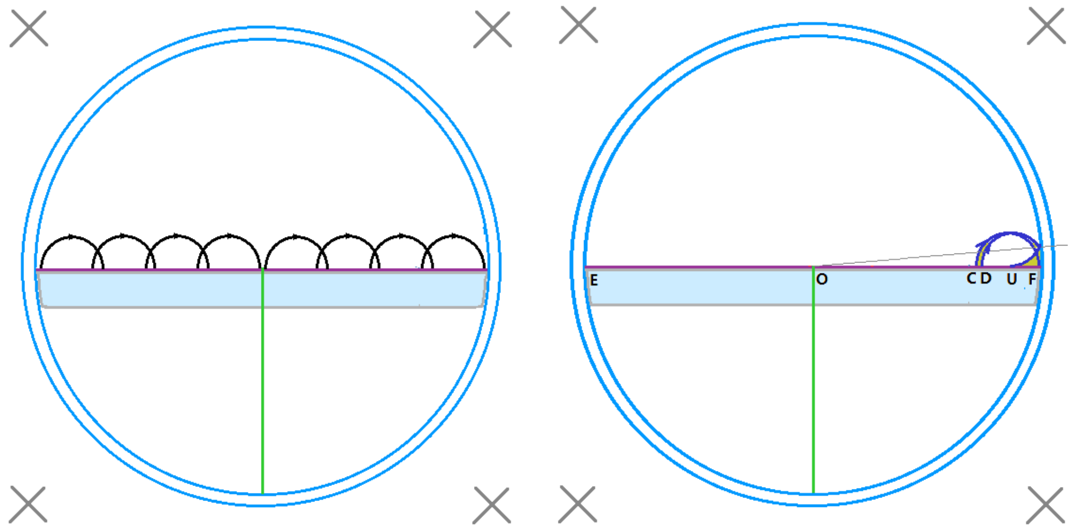
A-B 57% of the electrons of (0°, v_p) emitted from A migrate to B,

B-A 50% of the electrons of (0°, v_p) emitted from B migrate to A.

For all the electrons of (0°, v_p), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current

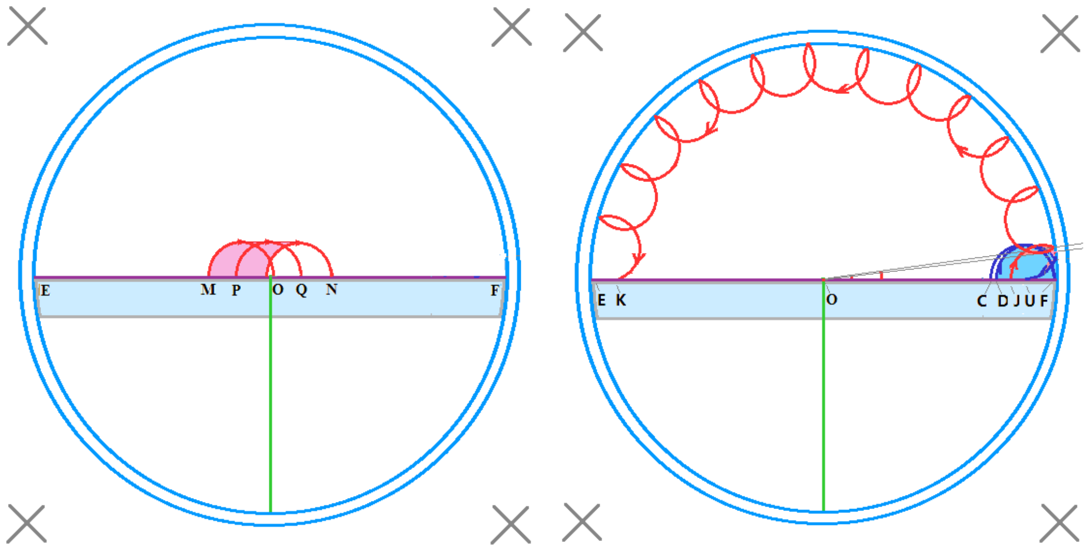
$$D_0^{\circ}(v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{0^{\circ} v_p} = 0.57 - 0.50 = 0.07.$$

(2) Fig 1-2 $\theta = 0^{o}$ $v = 0.5v_p$ $R = 2\text{mm}$



(a) A-directly-A & B-directly-B (b) B-glass-B

No electron migration between A and B due to these trajectories.



(c) A-directly-B (from MO to ON)

MO/EO=20/70=0.29

(MO=29, EO=70)

(d) B-glass-A (from J to K, etc.)

DF/OF=17.5/70=0.25

(DF =17.5 OF = 70)

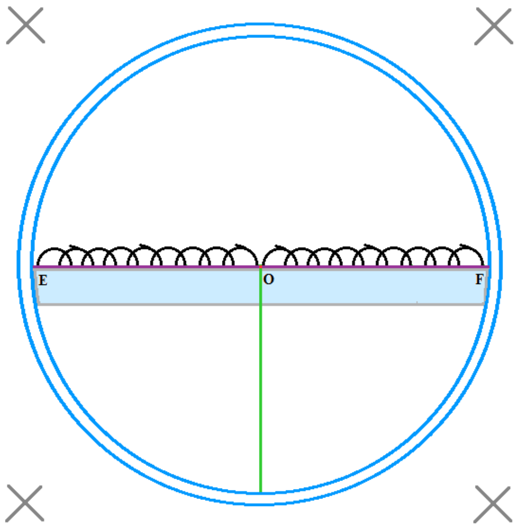
A-B 29% of the electrons of (0^{o}, $0.5v_p$) emitted from A migrate to B.

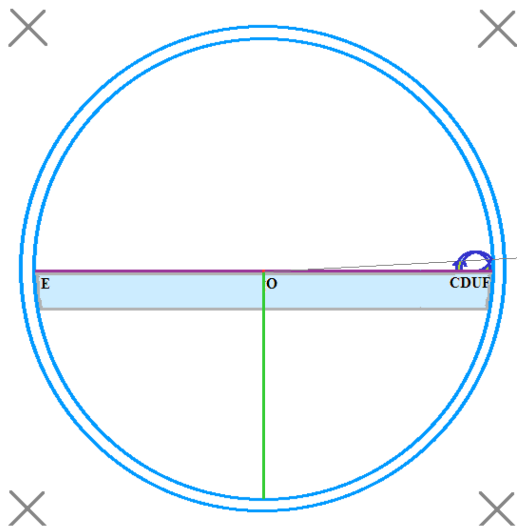
B-A 25% of the electrons of (0^{o}, $0.5v_p$) emitted from B migrate to A.

For all the electrons of (0^{o}, $0.5v_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current

$$D_{0^{o}}(0.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{0^{o},\, 0.5v_p} = 0.29 - 0.25 = 0.04$$

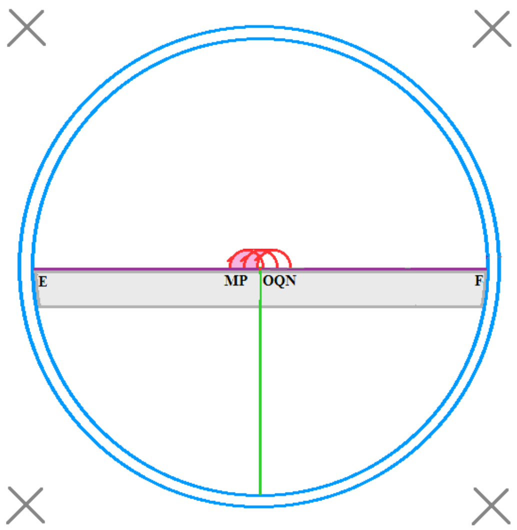
(3) Fig 1-3 $\theta = 0^{o}$ $v = 0.25v_p$ $R = 1\text{mm}$

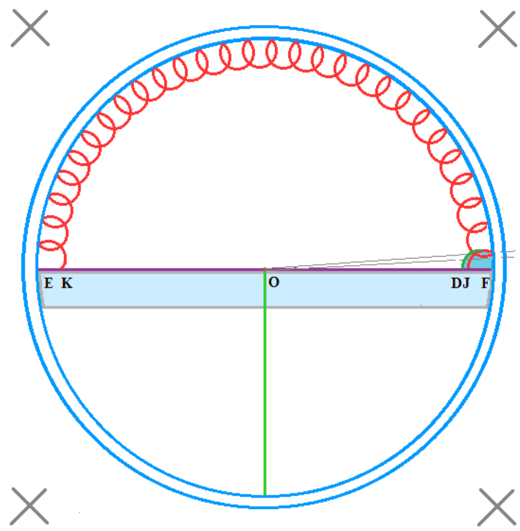




(a) A-directly-A & B-directly-B (b) B-glass-B

No electron migration between A and B due to these trajectories.





(c) A-directly-B (from MO to ON)

MO/EO=10/70=0.14

(MO=10 EO=70)

(d) B-glass-A (from J to K, etc.)

DF/OF=8.5/70=0.12

(DF =8.5 OF = 70)

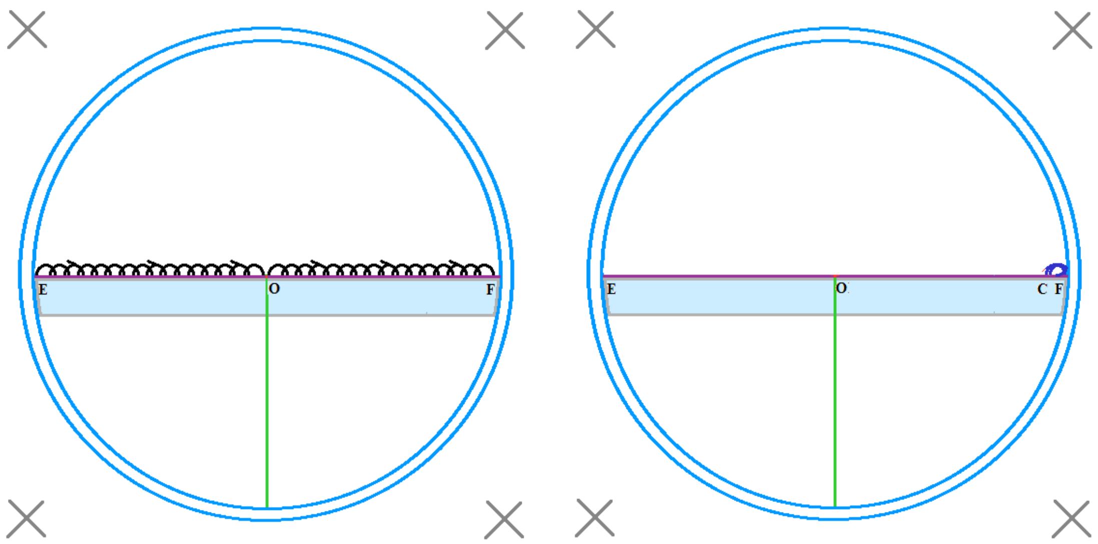
A-B 14% of the electrons of (0^{o}, $0.25v_p$) emitted from A migrate to B.

B-A 12% of the electrons of (0^{o}, $0.25v_p$) emitted from B migrate to A.

For all the electrons of (0^{o}, $0.25v_p$), A-B exceeds B-A, and their difference (equals CD) is the corresponding contribution to the output current.

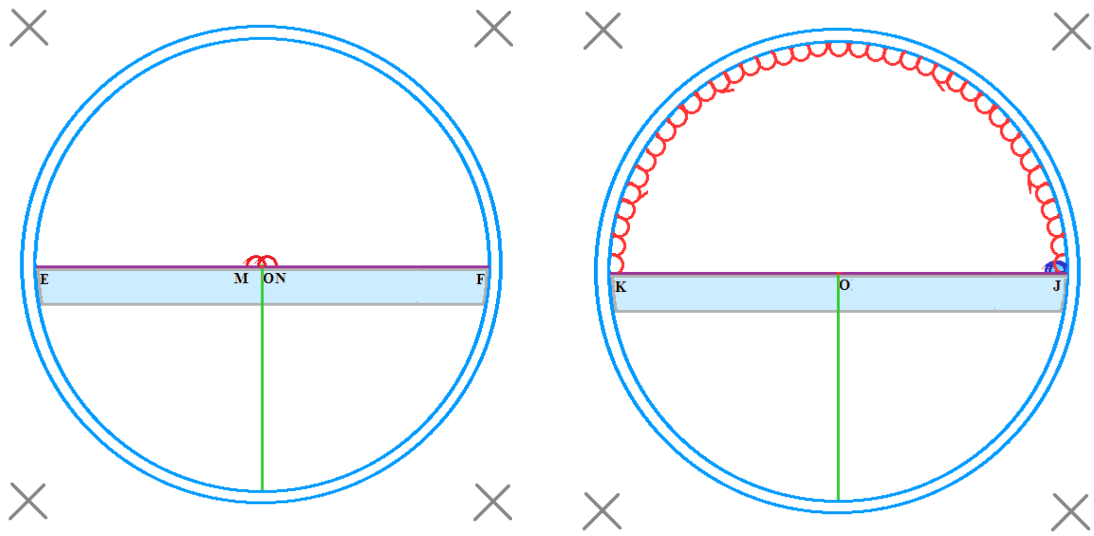
$$D_0{}^{o}\,(0.25v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{0^{o}\ 0.25v_p} = 0.14 - 0.12 = 0.02$$

(4) Fig 1-4 $\theta = 0^{\circ}$ $v = 0.125v_p$ $R = 0.5\mathrm{mm}$



(a) A-directly-A & B-directly-B (b) B-glass-B

No electron migration between A and B due to these trajectories.



(c) A-directly-B (from MO to ON)

MO/EO=5/70=0.071

(MO=5 EO=70)

(d) B-glass-A (from J to K, etc.)

DF/OF=4.5/70=0.064

(DF = 4.5 OF = 70)

A-B 7.1% of the electrons of (0°, $0.125v_p$) emitted from A migrate to B.

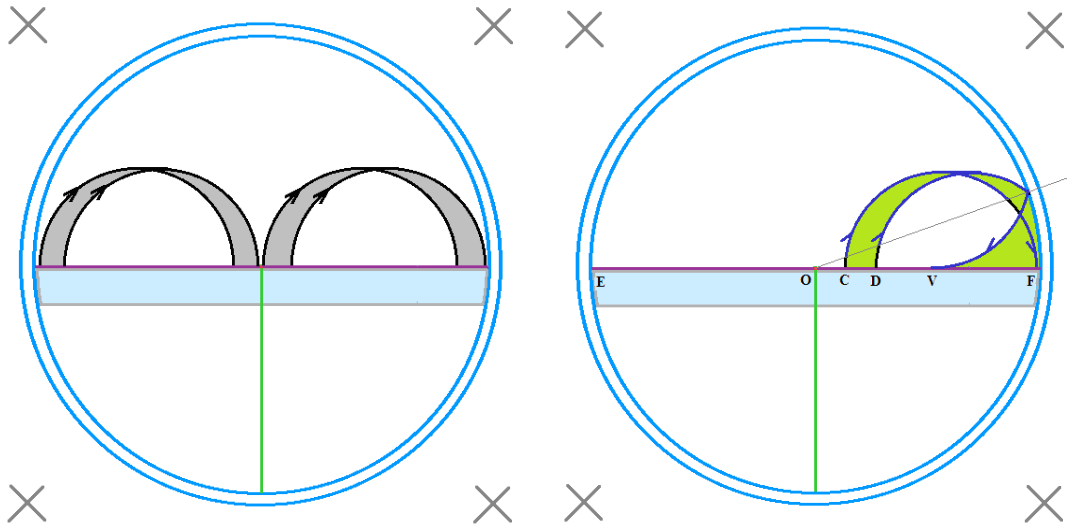
B-A 6.4% of the electrons of (0°, $0.125v_p$) emitted from B migrate to A.

For all the electrons of (0°, $0.125v_p$), migration A-B exceeds B-A, and their difference is the corresponding contribution to the output current

$$D_0^{\circ}(0.125v_p) = \{(\text{A-B}) - (\text{B-A})\}_{0^{\circ}\,0.125v_p} = 0.071 - 0.064 = 0.007$$

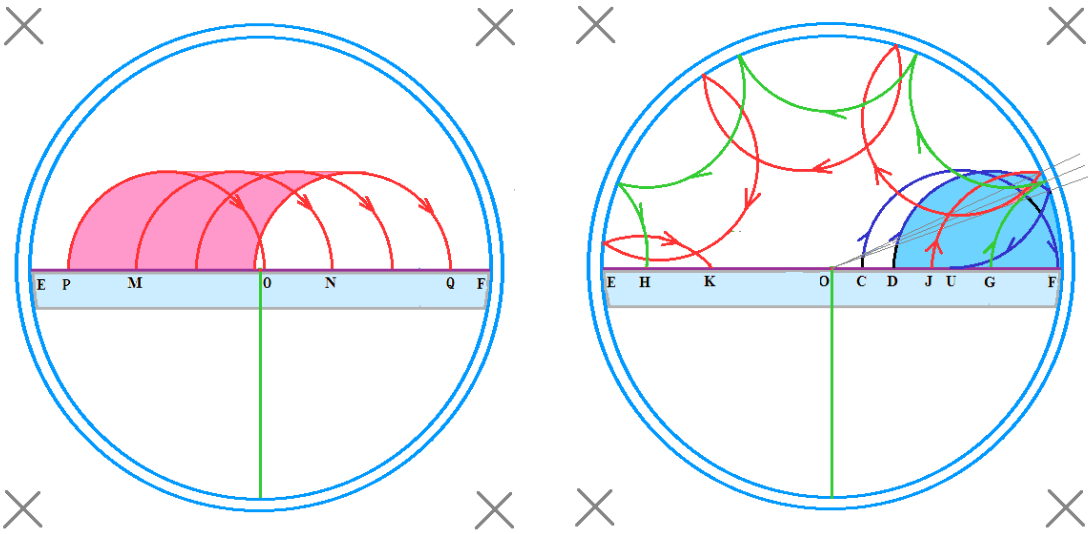
Now inspect the trajectories of the faster electrons.

(5) Fig 1-5 $\theta = 0^{\circ}$ $v = 1.5v_p$ R = 6mm



(a) A-directly-A & B-directly-B (b) B-glass-B

No electron migration between A and B due to these trajectories.



(c) A-directly-B (from PO to OQ)
PO/EO=60/70=0.857
(PO=60 EO=70)

(d) B-glass-A (from J to K)
DF/OF=50/70=0.714
(DF =50 OF = 70)

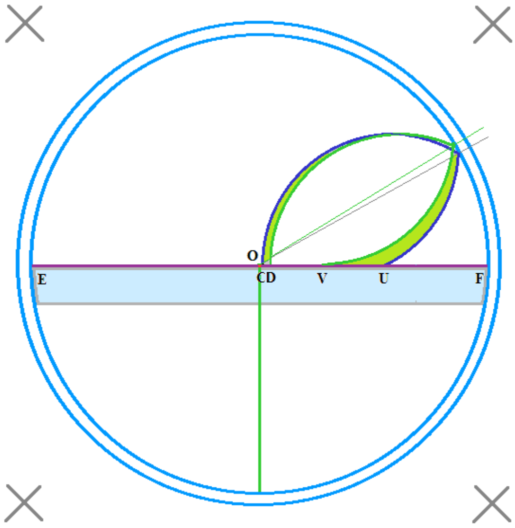
A-B 85.7% of the electrons of (0°, $1.5v_p$) emitted from A migrate to B.

B-A 71.4% of the electrons of (0°, $1.5v_p$)) emitted from B migrate to A.

For all the electrons of (0°, $1.5v_p$), A-B exceeds B-A, and their difference (equals CD) is the corresponding contribution to the output current

$$D_0^{\circ}(1.5v_p) = \{(\text{A-B}) - (\text{B-A})\}_{0^{\circ}\,1..5vp} = 0.857 - 0.714 = 0.14 \,.$$

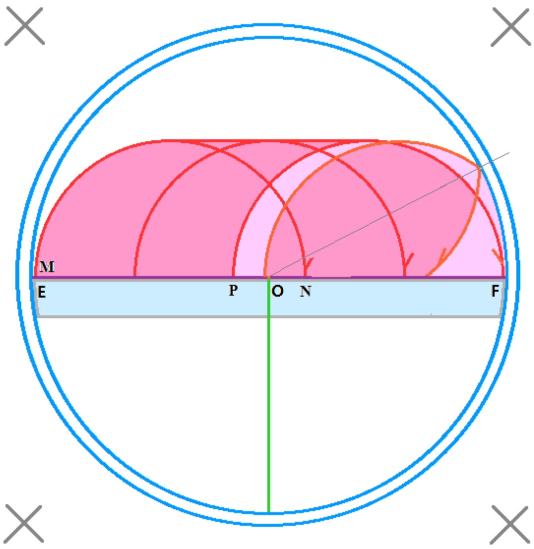
(6) Fig 1-6 $\theta = 0^o$ $v = 2\,v_p$ $R = 8\text{mm}$

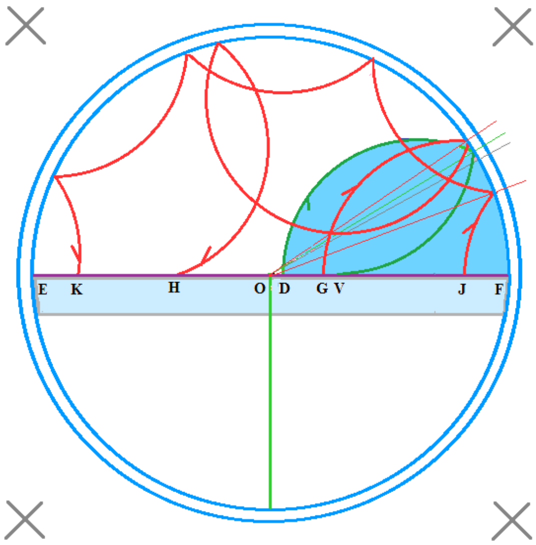


(a) B-glass-B

(CD = 3, green)

No electron migration between A and B due to these trajectories.





(b) A-directly-B & A-glass-B

MO/EO=70/70=1.00

(MP+PO=MO =70, EO=70)

(c) B-glass-A (from J to K, G to H, etc.)

DF/OF=67/70=0.96

(DF =67, OF = 70)

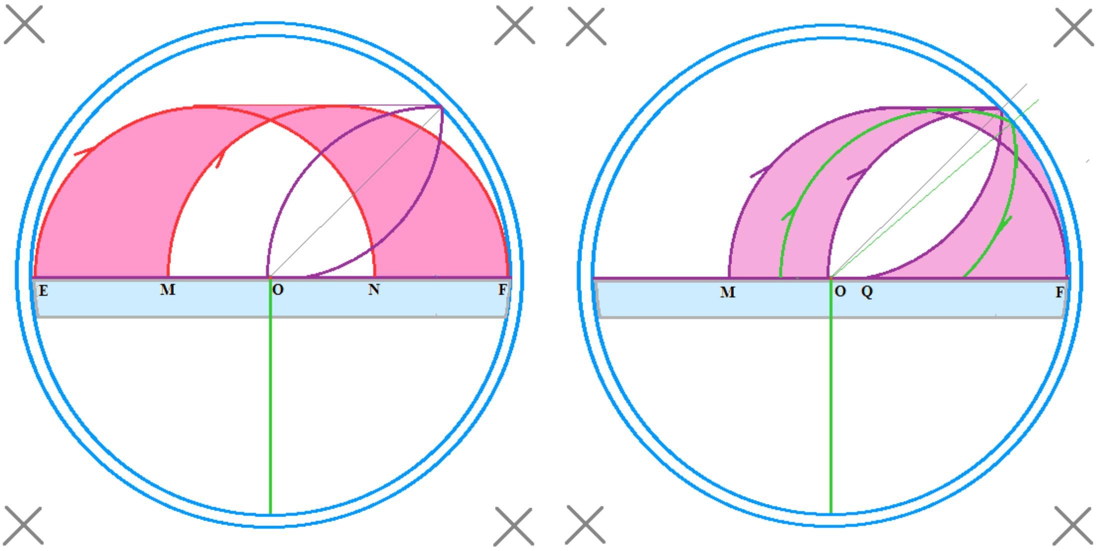
A-B 100% of the electrons of (0^o, $2v_p$) emitted from A migrate to B.

B-A 96% of the electrons of (0^o, $2v_p$)) emitted from B migrate to A.

For all the electrons of (0^o, $2v_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$D_{0^o}(1.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{0^o\ 2v_p} = 1.00 - 0.96 = 0.04.$$

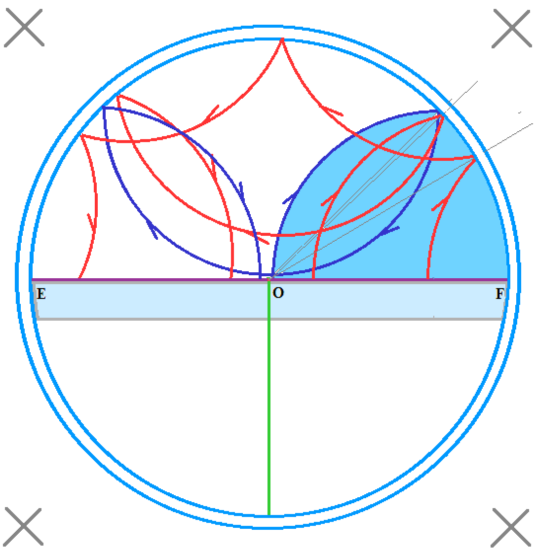
(7) Fig 1-7 $\theta = 0^{o}$ $v = 2.5\ v_p$ $R = 10\text{mm}$



(a) A-directly-B (pink) (b) A-glass-B (pink)

A-B = A-directly-B + A-glass-B \ (EM +MO)/EO =70/70 = 1.00

A-B 100% of the electrons of (0^{o}, $2.5v_p$) emitted from A migrate to B.



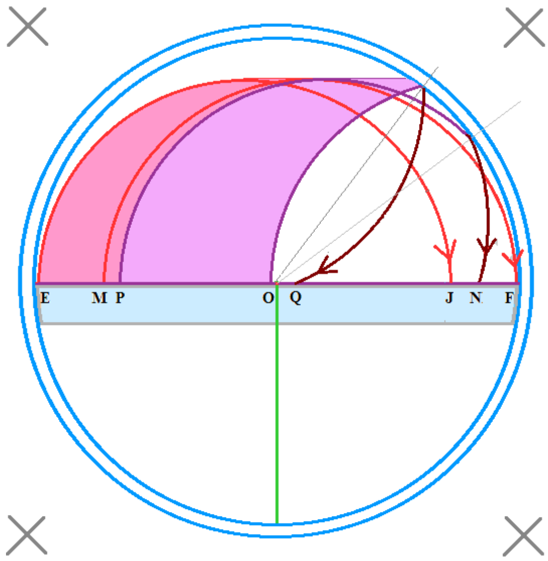
(c) B-glass-A OF/OF = 1.00 (blue)

B-A 100% of the electrons of (0^{o}, $2.5v_p$) emitted from B migrate to A.

For all the electrons of (0^{o}, $2.5v_p$), A-B and B-A cancel each other, there is no net contribution to the output current.

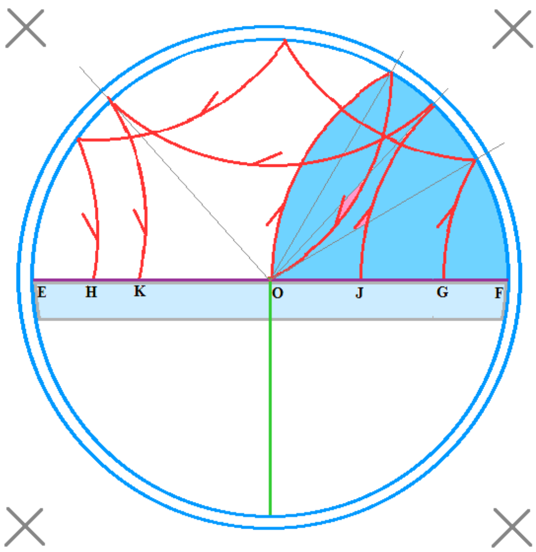
$$D_{0^{o}}(2.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{0^{o}\ 2..5vp} = 1.00 - 1.00 = 0.$$

(8) Fig 1-8 $\theta = 0^{o}$ $v = 3v_p$ $R = 12\text{mm}$



(a) A-B = A-directly-B (pink) + A-glass-B (violet) 70/70=1.00

A-B 100% of the electrons of (0^{o}, $3v_p$) emitted from A migrate to B.



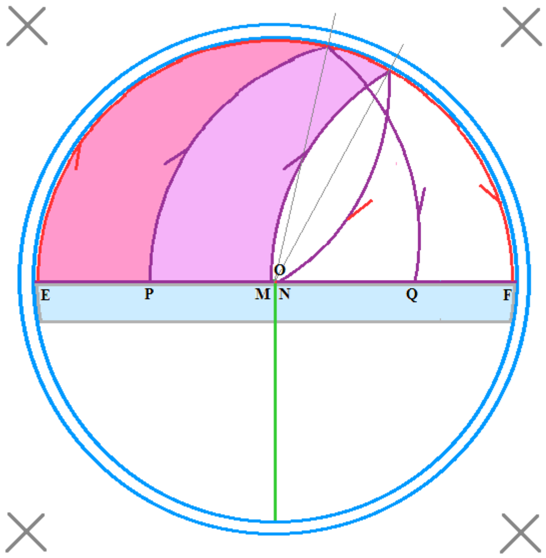
(b) B-glass-A 70/70 = 1.00 (from G to H, J to K, etc. blue)

B-A 100% of the electrons of (0^{o}, $3vp$) emitted from B migrate to A.

For all the electrons of (0^{o}, $3v_p$), A-B and B-A cancel each other, there is no net contribution to the output current..

$$D_{0^{o}}(3v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{0^{o},\,3v_p} = 0.$$

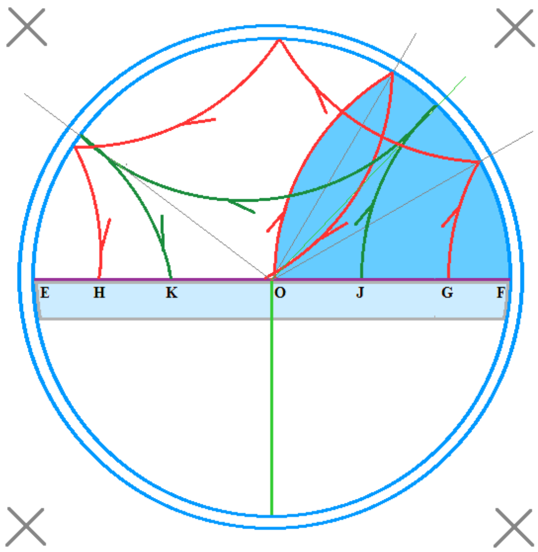
(9) Fig 1-9 $\theta = 0^{o}$ $v = 3.5v_p$ $R = 14\text{mm}$



(a) A-directly-B (pink), A-glass-B (violet)

A-B = A-directly-B +A-glass-B (pink + violet) 70/70 = 1.00

A-B 100% of the electrons of (0^{o}, $3.5v_p$) emitted from A migrate to B.



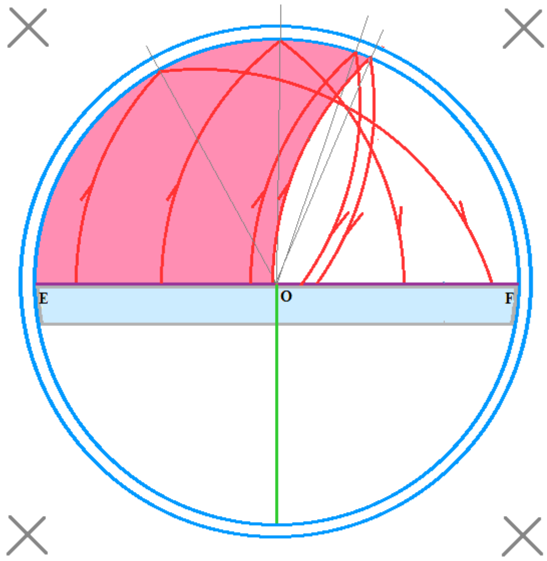
(b) B-glass-A 70/70 = 1.00 (from G to H, etc.) (OF = 70, blue)

B-A 100% of the electrons of (0^{o}, $3.5v_p$) emitted from B migrate to A.

For all the electrons of (0^{o}, $3.5v_p$), migrations A-B and B-A cancel each other, there is no net contribution to the output current.

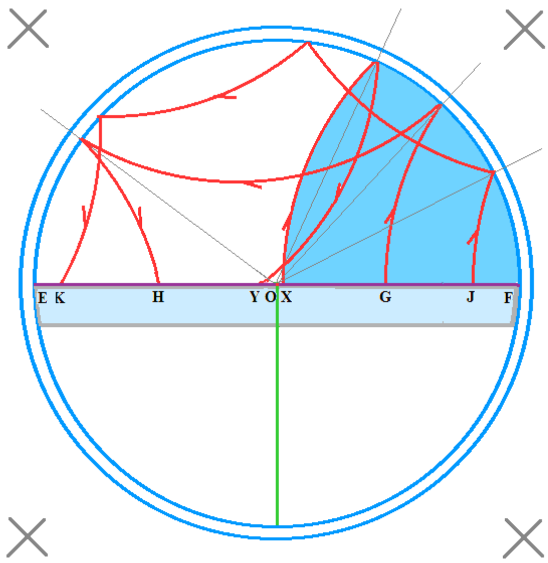
$$D_{0^{o}}(3.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{0^{o}\,3.5v_p} = 1.00 - 1.00 = 0.$$

(10) Fig 1-10 $\theta = 0^{o}$ $v = 4.5\ v_p$ $R = 18\text{mm}$



(a) A-glass-B EO/EO = 70/70 = 1.00 (pink)

A-B 100% of the electrons of (0^{o}, $4.5v_p$) emitted from A migrate to B.



(b) B-glass-A OF/OF = 70/70 =1.00 (blue)

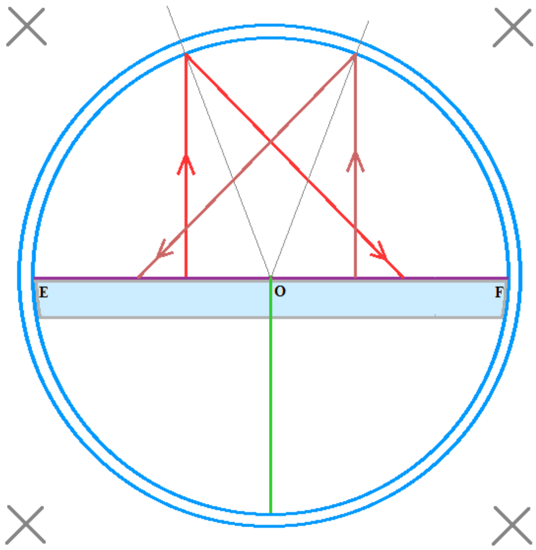
A-B 100% of the electrons of (0^{o}, $4.5v_p$) emitted from B migrate to A.

For all the electrons of (0^{o}, $4.5v_p$), migrations A-B and B-A cancel each other, there is no net contribution to the output current.

$$D_0^{\,o}(4.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{\,0^{o},\,4.5v_p} = 1.00 - 1.00 = 0.$$

(11) Fig 1-11 $\theta = 0^{o}$ $v >> 4.5\ v_p$ $R >> 18\text{mm}$

(i.e., $\theta = 0^{o}$ $v = \infty$ $R = \infty$)



(a) A-glass-B & B-glass-A

For thermal electrons of extremely high speed, their trajectories are approximately straight lines although a magnetic field is applied.

Statistically, the electrons are now in symmetric pairs (mirror reflect symmetry, left-right symmetry), one electron exits from A and the other from B, with the same speeds, symmetric exiting angles and exiting spots. Their contributions to the output current cancel each other.

$D_{0^o}(\infty) = \{(A\text{-}B) - (B\text{-}A)\}_{0^o\ \infty} = 1.00 - 1.00 = 0.$

List all the above contributions $D_{0^o}(v) = \{(A\text{-}B)-(B\text{-}A)\}_{0^o}(v)$ into Tab 1 (1).

Fig 1 (1) is the corresponding graph.

$\theta = 0^o$ $cos\theta = 1$	$\{(A\text{-}B) - (B\text{-}A)\}_{0^o}$	$D_{0^o}(v) =$ $\{(A\text{-}B)-(B\text{-}A)\}_{0^o}(v)\, cos\theta$
Fig 1-4 $v = 0.125v_p$	0.07 - 0.06 = 0.01	0.01
Fig 1-3 $v = 0.25v_p$	0.14 - 0.12 = 0.02	0.02
Fig 1-2 $v = 0.5v_p$	0.29 - 0.25 = 0.04	0.04
Fig 1-1 $v = v_p$	0.57 - 0.50 = 0.07	0.07
Fig 1-5 $v = 1.5v_p$	0.857 - 0.714 = 0.14	0.14
Fig 1-6 $v = 2v_p$	1.00 - 0.96 = 0.04	0.04
Fig 1-7 $v = 2.5v_p$	1.00 - 1.00 = 0	0
Fig 1-8 $v = 3v_p$	1.00 - 1.00 = 0	0
Fig 1-9 $v = 3.5v_p$	1.00 - 1.00 = 0	0
Fig 1-10 $v = 4.5v_p$	1.00 - 1.00 = 0	0
Fig 1-11 $v = \infty$	1.00 - 1.00 = 0	0

Tab. 1 (1) Contributions of electrons of $\theta = 0^o$ with different speeds v

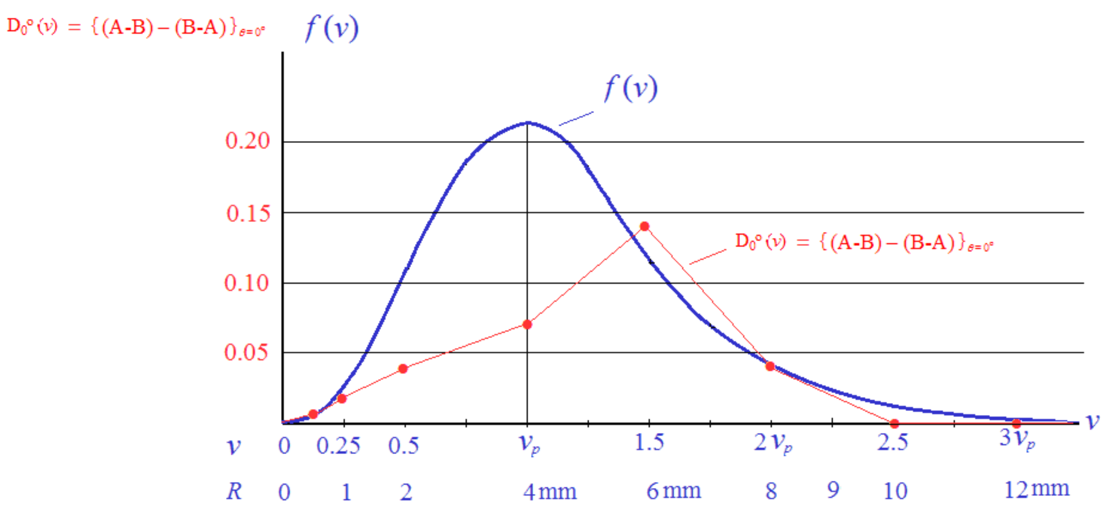


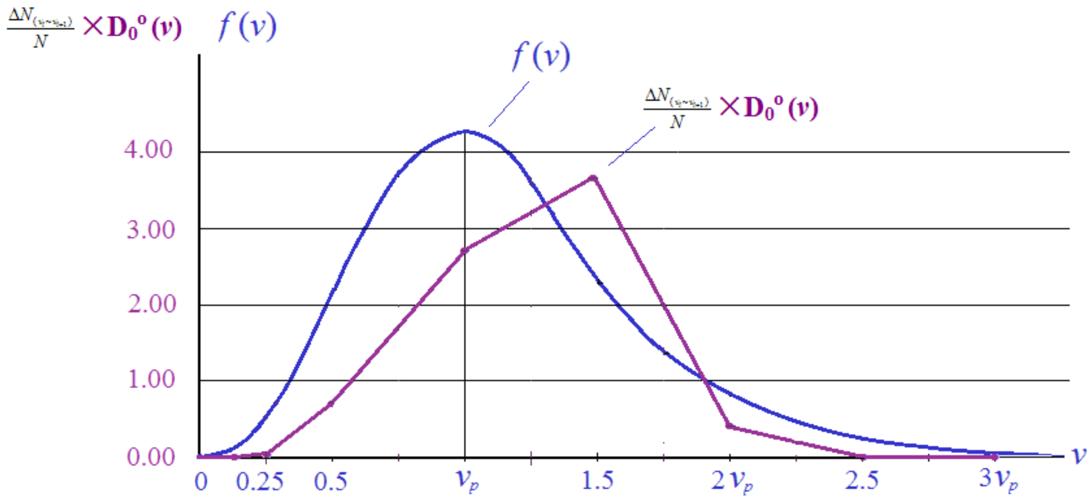
Fig 1(1) Graph of the contributions of electrons of $\theta = 0^{\circ}$ with different speeds.

Take Maxwell's speed distribution into account to derive the actual contributions of the thermal electrons of $\theta = 0^{\circ}$ and different speed ranges, i.e., $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{0^{\circ}}(v) \sim v$.

Speed range $v_i \sim v_{i+1}$	$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N}$	$D_0^{\circ}(v)$	$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_0^{\circ}(v)$
$0.000 \sim 0.125 v_p$	$A_{01} =$ 0. 15%	0. 007	0. 15×0.007×0.5=0.00053=0 (Δ)
$0.125 \sim 0.25 v_p$	$A_{02} =$ 4. 00%	0.02	4×0. 02×0. 5 = 0. 04 (Δ)
$0.25 \sim 0.75 v_p$	$A_1 =$ 18. 74%	0.04	0.7496
$0.75 \sim 1.25 v_p$	$A_2 =$ 39. 83%	0.07	2.7881
$1.25 \sim 1.75 v_p$	$A_3 =$ 26. 71%	0.14	3.7394
$1.75 \sim 2.25 v_p$	$A_4 =$ 8. 82%	0.04	0.3528
$2.25 \sim 2.75 v_p$	$A_5 =$ 1. 58%	0	0
$2.75 \sim 3.25 v_p$	$A_6 =$ 0. 16%	0	0
$3.25 \sim 3.75 v_p$	$A_7 =$ 0.0096%	0	0
$3.75 v_p \sim \infty$	$A_8 =$ 0.0003%	0	0
			$\sum_v \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{0^{\circ}}(v)$ **= 7.6699**

Tab. 1 (2) The actual contributions of electrons $\theta = 0^{\circ}$ with different speeds v.

Fig. 1 (2) is the graph of $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{0^o}(v) \sim v$.



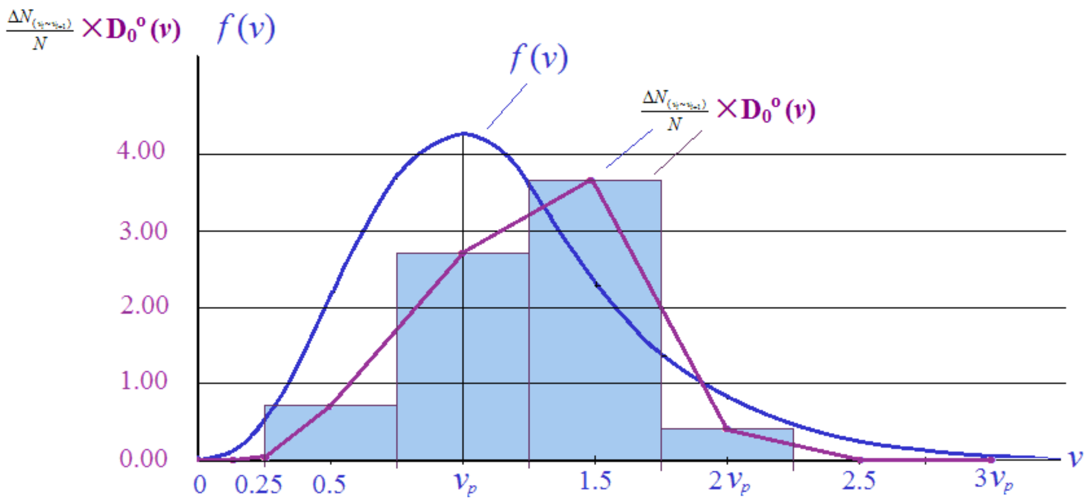


Fig. 1(2) Graph of actual contributions of electrons of $\theta = 0^o$ with different speeds, $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_0^o(v) \sim v$.

From the above discussion, we see, **the general contribution to the output current of all the vertically emitted electrons from A and B is not zero. It is positive**. **This result is sharply in contradiction to Boltzmann's principle of detailed balance.**

The Second Part of the survey

Trajectories of thermal electrons of other exiting angles ($\theta \neq 0^{o}$) with different speeds

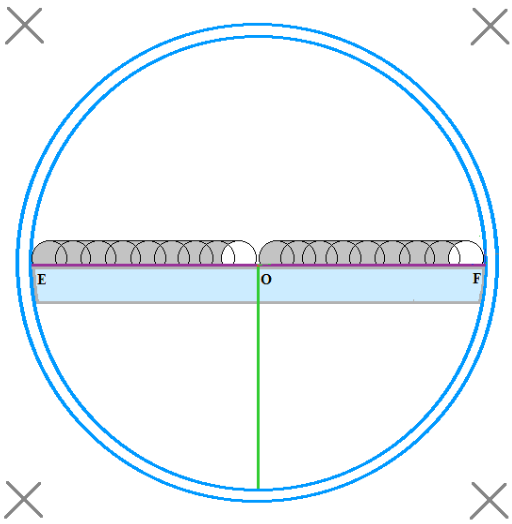
The following exiting angles and speeds are adopted

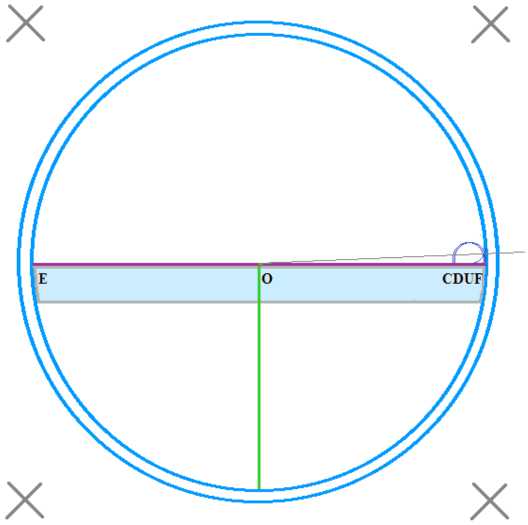
θ **= -15^{o}, 15^{o}, -30^{o}, 30^{o}, -45^{o}, 45^{o}, -60^{o}, 60^{o}, -75^{o}, 75^{o}**

$v = 0.5v_p,\quad v_p,\quad 1.5v_p,\quad 2v_p,\quad 2.5v_p,\quad 3v_p,\quad 4.5v_p.$

2. Trajectories of electrons of θ = -15° and different speeds

(1) Fig 2-1 $\theta = -15^{\circ}$ $v = 0.25v_p$ $R = 1\text{mm}$ ($cos\theta = 0.9659$)

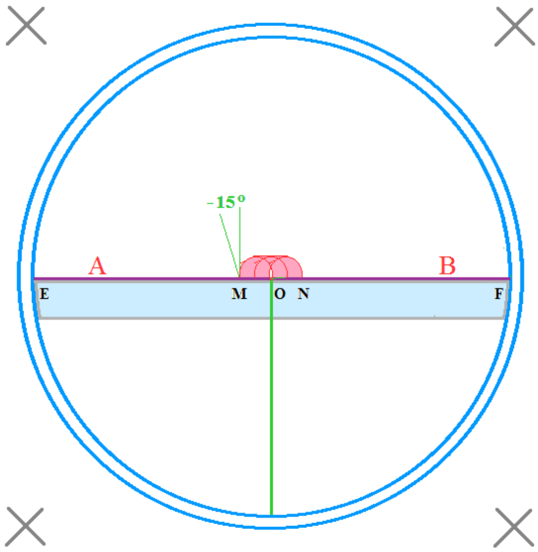


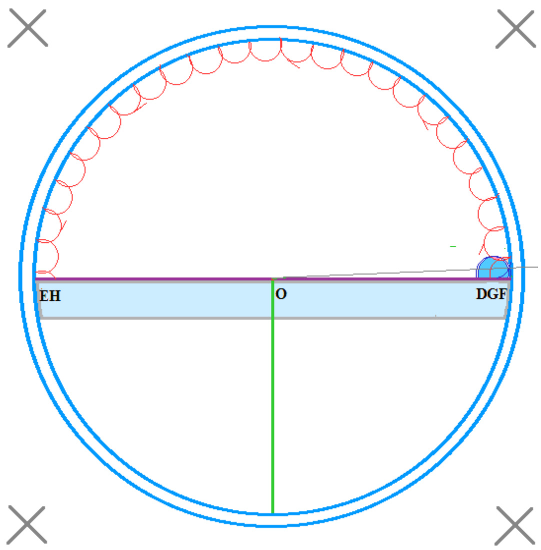


A-directly-A B-directly-B
(grey)

B-glass-B
CD/OF= 0.5/70 = 0.007≈0. 01

No electron migration between A and B due to these trajectories.





A-directly-B
MO/EO = 9.5/70 = 0.136
(from MO to ON, MO = 9.5, pink)

B-Glass-A
DF/OF= 9/70 = 0.129
(from G to H, etc., DF = 9, blue)

A-B 13.6% of the electrons of (-15°, $0.25v_p$) emitted from A migrate to B.

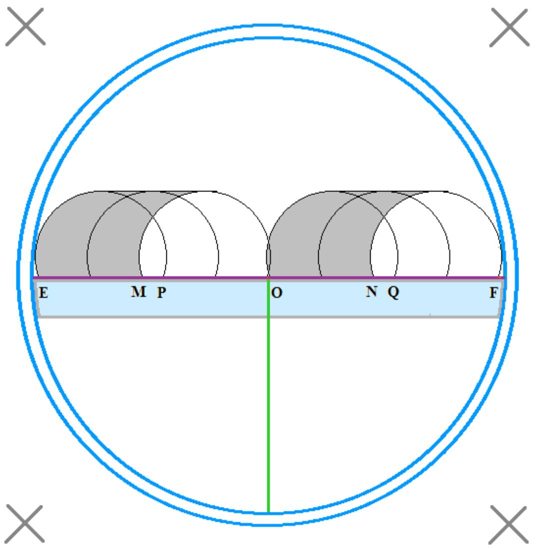
B-A 12.9% of the electrons of (-15°, $0.5v_p$) emitted from B migrate to A.

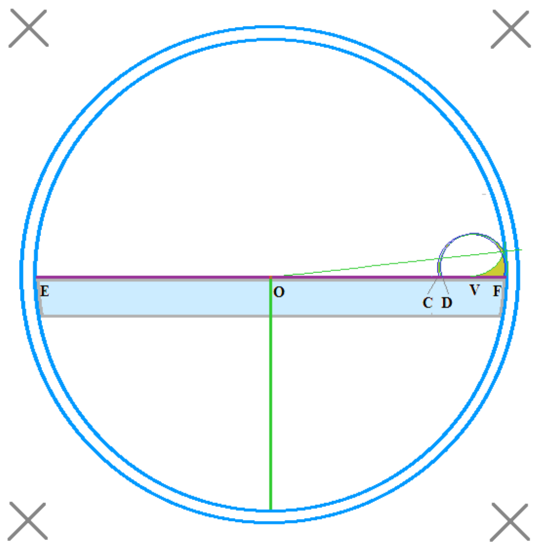
For all the electrons of (-15°, $0.25v_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current

$$\{(A\text{-}B) - (B\text{-}A)\}_{-15^{\circ}\ 0.25v_p} = 0.136 - 0.129 = 0.007$$

$$D_0^{\circ}(0.25v_p) = \quad 0.007 \times cos15^{\circ} = \quad 0.007 \times 0.9659 \approx 0.007$$

(2) Fig 2-2 θ = -15° $v = 0.5v_p$ R = 2mm (B =1.34 gauss)

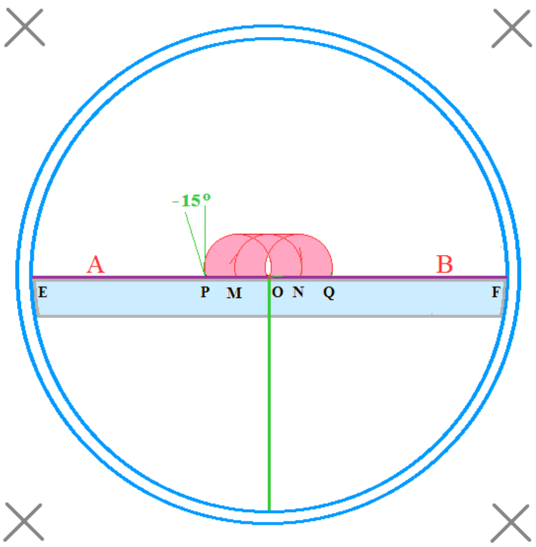


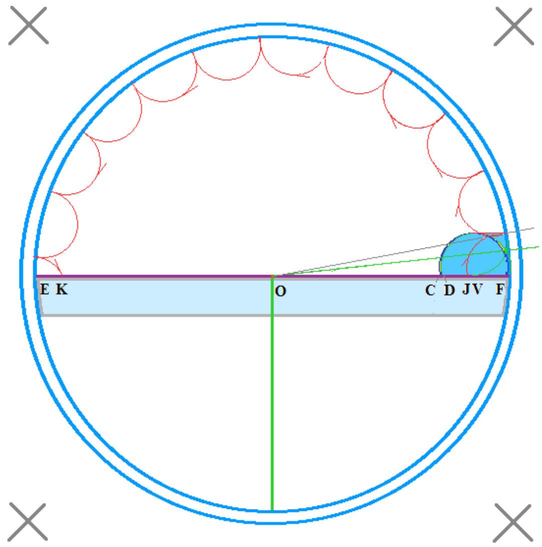


A-directly-A B-directly-B
(from EM to PO, ON to QF, etc,. grey)

B-Glass-B
CD/OF=1/70 = 0.014 (CD = 1, OF =70)

No electron migration between A and B due to these trajectories.





A-directly-B
PO/EO = 19/70 = 0.27
(PO = 19, EO = 70)
(from PO to OQ, pink)

B-Glass-A
DF/OF=18/70 = 0.26
(DF = 18, OF = 70)
(from J to K, etc., pink)

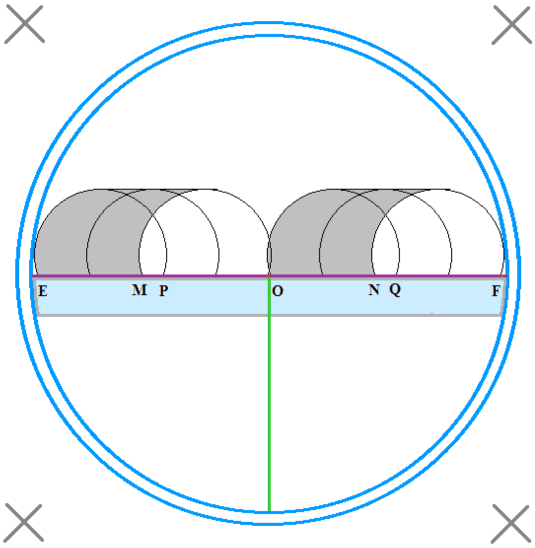
A-B 27% of the electrons of (-15°, $0.5v_p$) emitted from A migrate to B.
B-A 26% of the electrons of (-15°, $0.5v_p$) emitted from B migrate to A.

For all the electrons of (- 15°, $0.5v_p$), A-B exceeds B- A, and their difference is the corresponding contribution to the output current

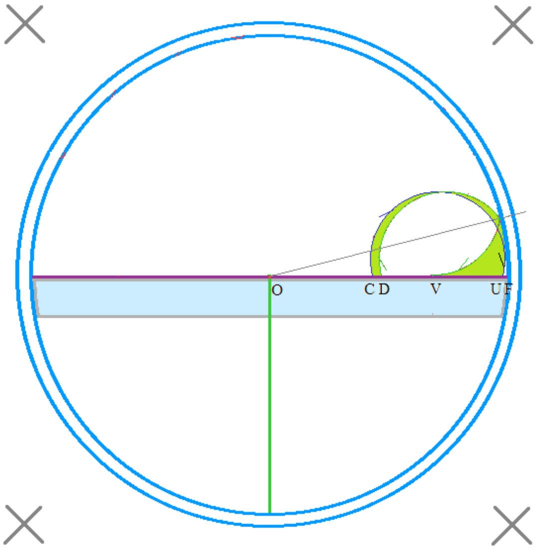
$$\{(A\text{-}B) - (B\text{-}A)\}_{-15^{\circ}\,0.5v_p} = 0.27 - 0.26 = 0.01$$

$$D_{-15^{\circ}}(0.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-15^{\circ}\,0.5v_p} \times \cos 15^{\circ} = 0.01 \times 0.9659 \approx 0.01$$

(3) Fig 2-3 θ = -15° $v = v_p$ R = 4mm

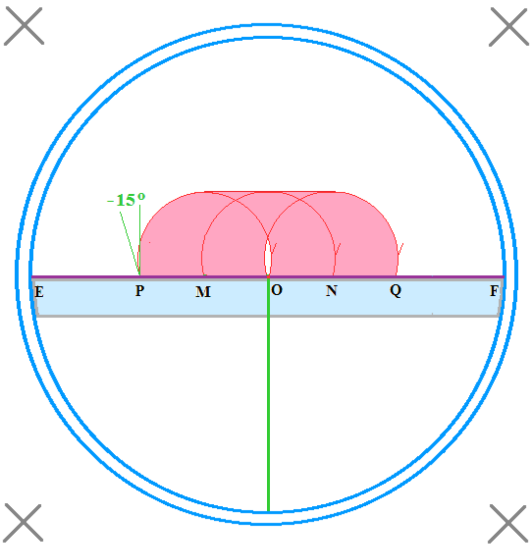


A-directly-A B-directly-B
(from EM to PO, ON to QF, grey)

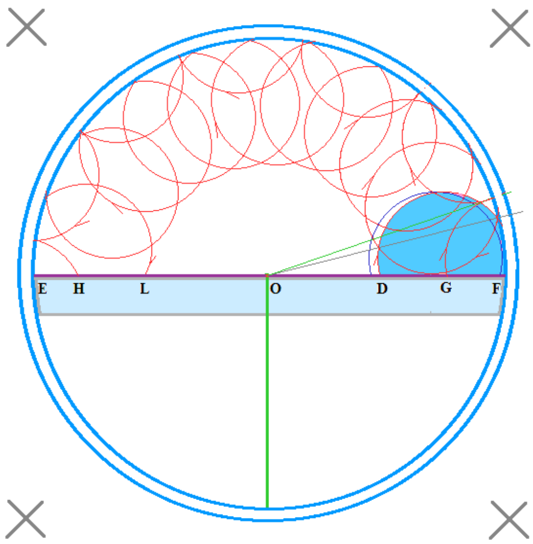


B-Glass-B
CD/OF =2.5/70≈0.04
(CD=2.5, OF =70) (from CD to UV, green.)

No electron migration between A and B due to these trajectories.



A-directly-B
PO/EO=40/70 = 0.57
(PO = 40, EO = 70)
(from P to O, M to N, O to Q, etc., pink)



B-Glass-A
DF/OF=37.5/70 = 0.54
(DF = 37.5, OF = 70)
(from G to H, D to L, etc., blue)

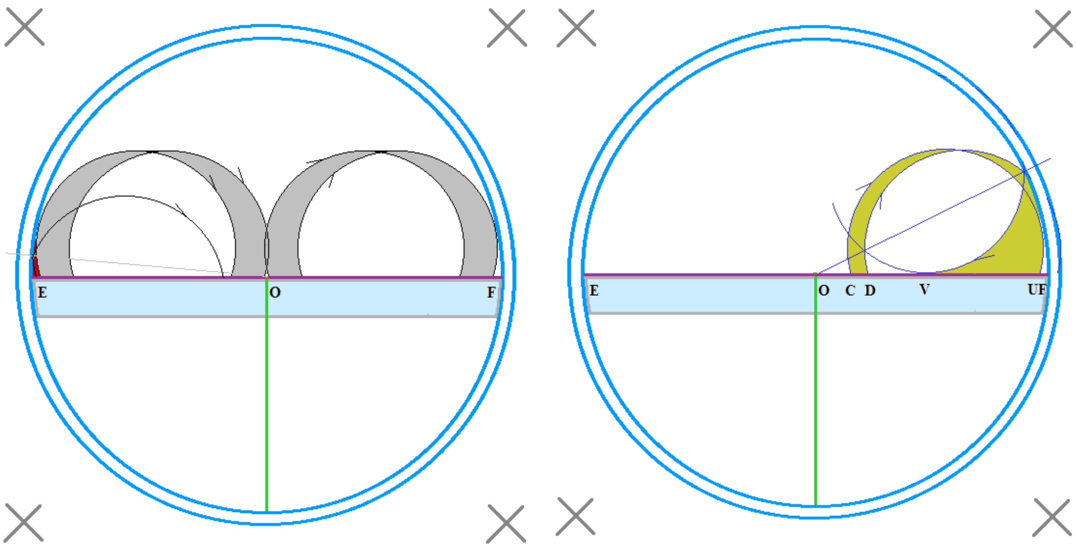
A-B 57% of the electrons of (-15°, v_p) emitted from A migrate to B.
B-A 54% of the electrons of (-15°, v_p) emitted from B migrate to A.

For all the electrons of (- 15°, v_p), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{-15^{\circ}} v_p = 0.57 - 0.54 = 0.03$$

$$D_{-15^{\circ}}(v_p) = 0.03 \times cos15^{\circ} = 0.03 \times 0.9659 \approx 0.03$$

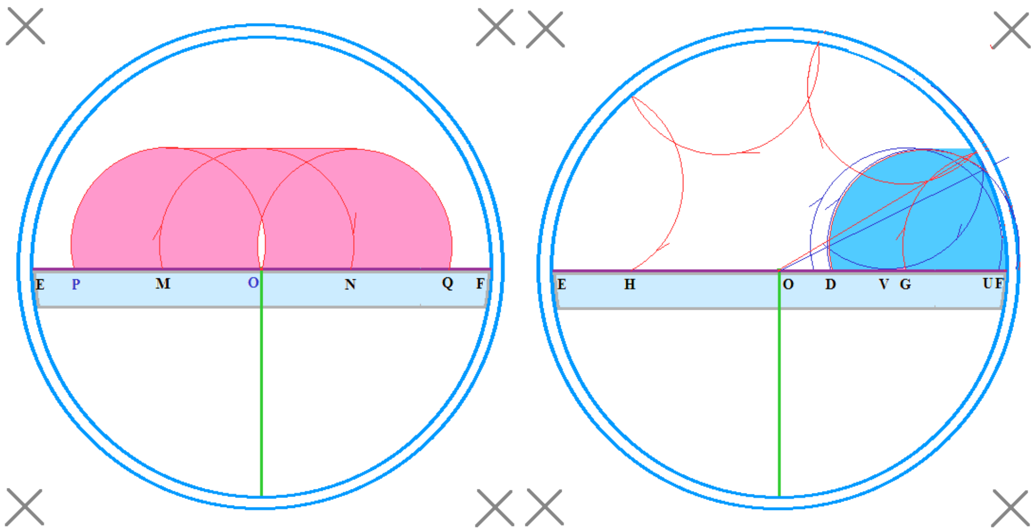
(3) Fig 2-4 $\theta = -15^{o}$ $v = 1.5v_p$ $R = 6$mm



A-glass-A (brown) A-directly-A (grey) B-directly-B (grey) B-Glass-B

CD/OF = 5/70 =0.07

No electron migration between A and B due to these trajectories.



A-directly-B

PO/EO=58/70 = 0.83

(from P to O, M to N, O to Q, etc., PO = 58, pink)

B-glass-A

DF/OF=54/70 = 0.77

(from G to H, etc., blue)

A-B 83% of the electrons of (-15^{o}, $1.5v_p$) emitted from A migrate to B.

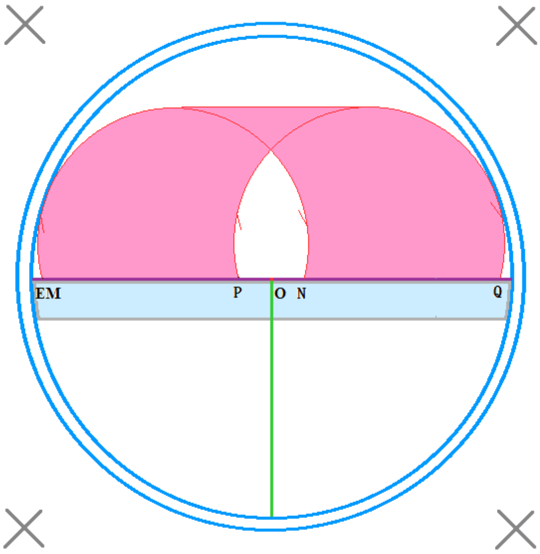
B-A 77% of the electrons of (-15^{o}, $1.5v_p$) emitted from B migrate to A.

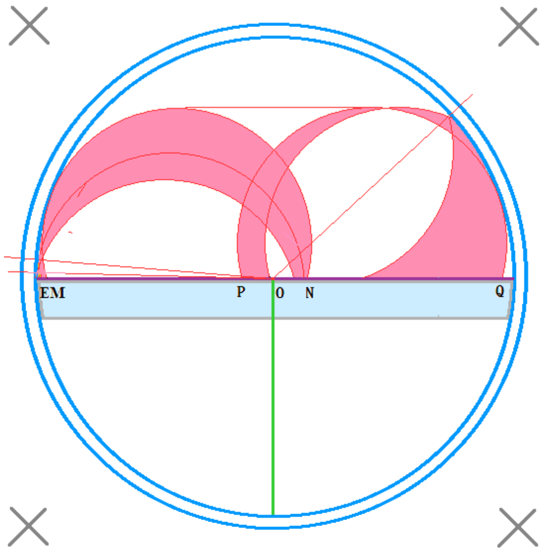
For all the electrons of (-15^{o}, $1.5v_p$), A-B exceeds B-A, and their difference is the contribution to the output current,

$$\{(A\text{-}B) - (B\text{-}A)\}_{-15^{o}\ 1.5v_p} = 0.83 - 0.77 = 0.06$$

$$D_{-15^{o}}(1.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-15^{o}\ 1.5v_p} \times \cos 15^{\circ} = 0.06 \times 0.9659 = 0.058 \approx 0.06$$

(4) Fig 2-5 $\theta = -15^{\circ}$ $v = 2v_p$ $R = 8\text{mm}$



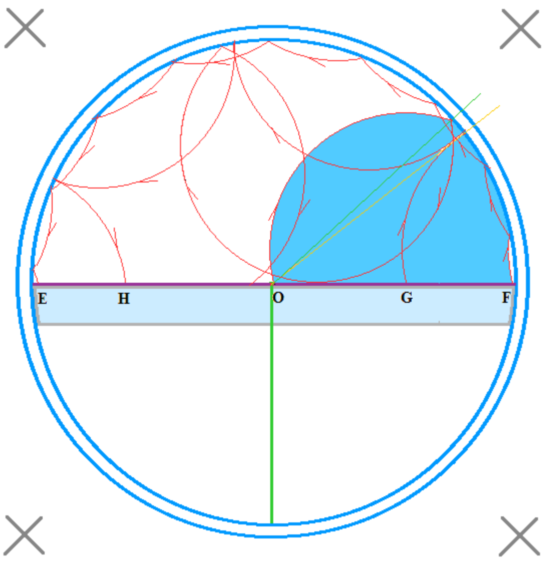


A-directly-B 58/70 = 0.83 (MP=58, EO=70) (from MP to NQ, pink)

A-glass-B 12/70=0.17 (2.5+9.5=12) (EM=2.5, PO=9.5, pink)

EM + MP + PO = 70 70/70 =1.00

A-B 100% of the electrons of (- 15°, $2v_p$) emitted from A migrate to B.



B-glass-A 70/70 = 1.00 (OE = 70)
(from G to H, etc., blue)

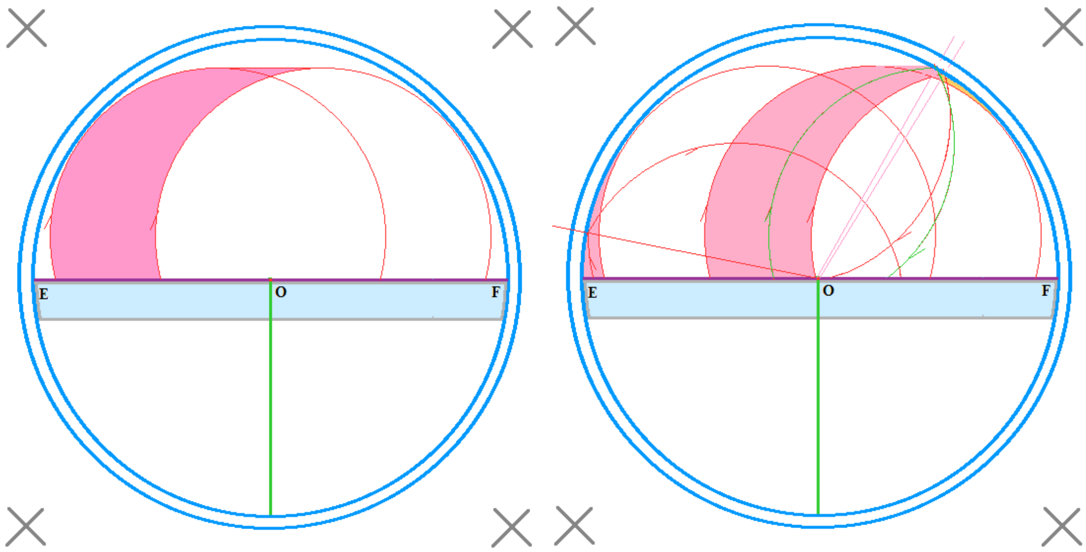
B-A 100% of the electrons of (- 15°, $2v_p$) emitted from B migrate to A.

For all the electrons of (- 15°, $2v_p$), A-B equals B-A, no net contribution to the output current,

$$\{(\text{A-B}) - (\text{B-A})\}_{\theta=-15^{\circ}\ v=2vp} = 1.00 - 1.00 = 0$$

$$D_{-15^{\circ}}(2v_p) = \{(\text{A-B}) - (\text{B-A})\}_{\theta=-15^{\circ}\ v=2vp} \times \cos 15^{\circ} = 0$$

(6) Fig 2-6 $\theta = -15^{o}$ $v = 2.5v_p$ $R = 10\text{mm}$

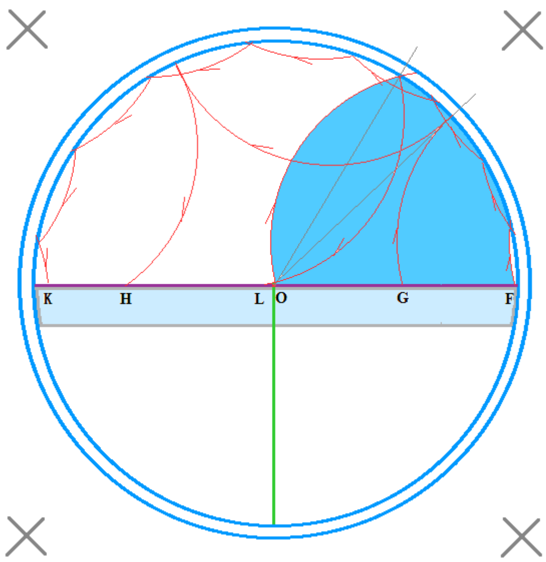


A-directly-B (pink) A-glass-B (pink + pink)

A-B = A-glass-B + A-directly-B + A-glass-B 70/70 = 1.00

(pink + pink + pink = 70)

A-B 100% of the electrons of (-15^{o}, $2.5v_p$) emitted from A migrate to B.



B-Glass-A 70/70 = 1.00

(from G to H, F to K, O to L, etc., blue)

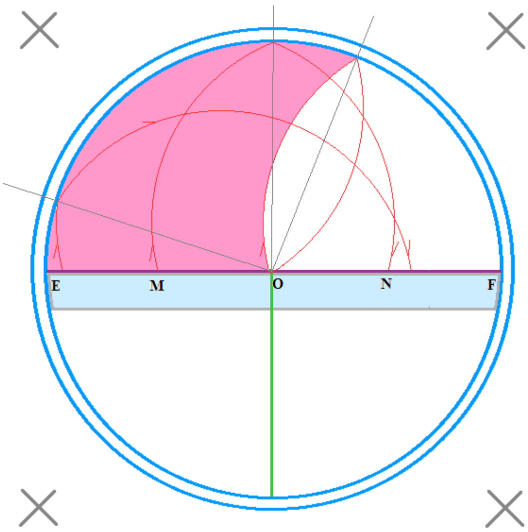
B-A 100% of the electrons of (-15^{o}, $2.5v_p$) emitted from B migrate to A.

For all the electrons of (-15^{o}, $2.5v_p$), A-B equals B-A, and their contribution to the output current cancel each other.

$$\{(\text{A-B}) - (\text{B-A})\}_{\theta=-15^{o}\ v=2.5v_p} = 1.00 - 1.00 = 0$$

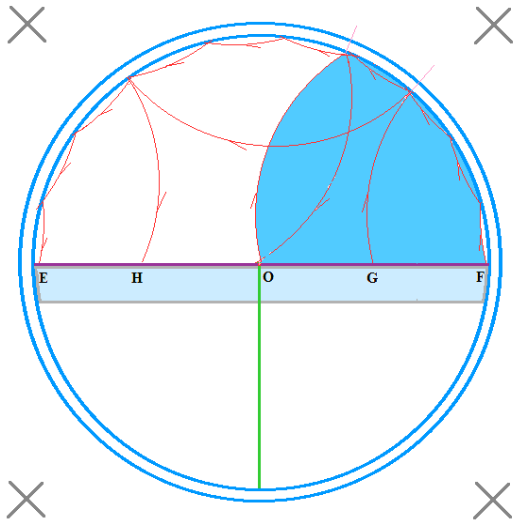
$$D_{-15^{o}}(2.5v_p) = \{(\text{A-B}) - (\text{B-A})\}_{\theta=-15^{o}\ v=2.5v_p} \times cos15^{o} = 0$$

(7) $\theta = -15^{\circ}$ $v = 3v_p$ $R = 10\text{mm}$



A-glass-B 70/70 = 1.00
(from M to N, etc., pink)

A-B 100% of the electrons of (-15°, $3v_p$) emitted from A migrate to B.



B-Glass-A 70/70 = 1.00
(from G to H, etc., blue)

B-A 100% of the electrons of (-15°, $3v_p$) emitted from B migrate to A.

For all the electrons of (-15°, $3v_p$), electron migrations A-B and B-A cancel each other, no net contribution to the output current,

$$\{(\text{A-B}) - (\text{B-A})\}_{\theta=-15^{\circ}\ v=3v_p} = 1.00 - 1.00 = 0$$

$$D_{-15^{\circ}}(3v_p) = \{(\text{A-B}) - (\text{B-A})\}_{\theta=-15^{\circ}\ v=3v_p} \times \cos 15^{\circ} = 0$$

List the Contributions of electrons of θ = -15° with different speeds in Tab 2(1). Fig 2 (1) is the corresponding graph.

$\theta = -15^{o}$	$\{(A\text{-}B) - (B\text{-}A)\}_{-15^{o}}$	$D_{-15^{o}}(v)$
Fig 2-1 $v = 0.25v_p$	0.136 – 0.129 = 0.007	0.007
Fig 2-2 $v = 0.5v_p$	0.27 – 0.26 = 0.01	0.01
Fig 2-3 $v = v_p$	0.57 – 0.54 = 0.03	0.03
Fig 2-4 $v = 1.5v_p$	0.83 – 0.77 = 0.06	0.06
Fig 2-5 $v = 2v_p$	1.00 – 1.00 = 0	0
Fig 2-6 $v = 2.5v_p$	1.00 – 1.00 = 0	0
Fig 2-7 $v = 3v_p$	1.00 – 1.00 = 0	0
Fig 2-8 $v = 4.5v_p$	1.00 - 1.00 = 0	0

Tab 2 (1) Contributions of electrons of θ = -15° with different speeds.

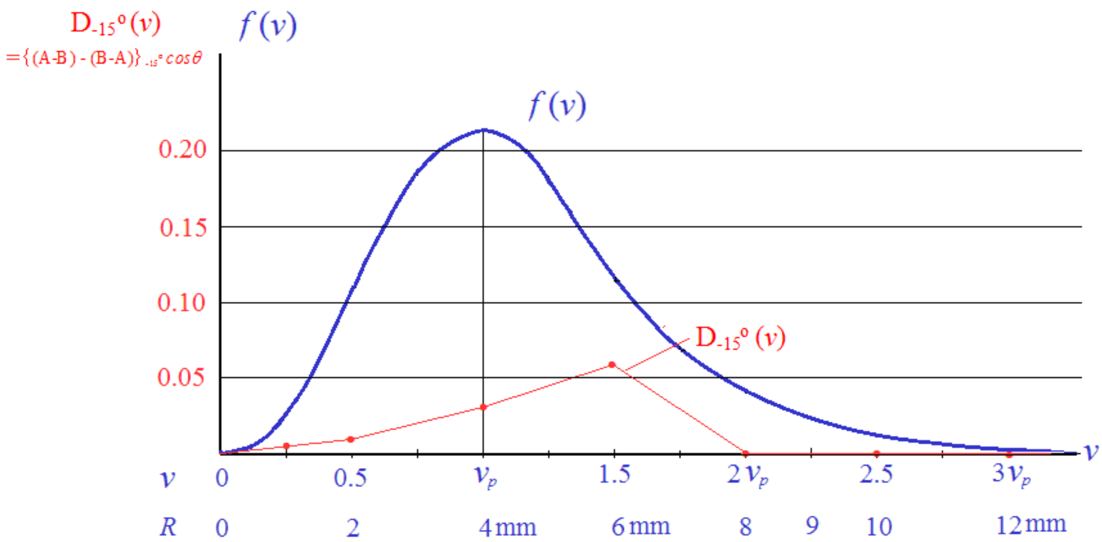


Fig 2 (1) Graph of contributions of electrons of θ = -15° with different speeds

Take Maxwell's speed distribution into account to derive the actual contributions of the thermal electrons of θ = -15° with respect to speeds, i.e., $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-15}^{o}(v) \sim v$. And Fig 2 (2) is the corresponding graph.

Speed range	$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N}$	$D_{15}^{o}(v)$	$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{15}^{o}(v)$
$0.00 \sim 0.25v_p$	$A_0 \approx 0.004$	< 0.007	< 0.000028
$0.25 \sim 0.75v_p$	$A_1 = 18.74\%$	0.01	0.1874
$0.75 \sim 1.25v_p$	$A_2 = 39.83\%$	0.03	1.1949
$1.25 \sim 1.75v_p$	$A_3 = 26.71\%$	0.06	1.6026
$1.75 \sim 2.25v_p$	$A_4 = 8.82\%$	0	0
$2.25 \sim 2.75v_p$	$A_5 = 1.58\%$	0	0
$2.75 \sim 3.25v_p$	$A_6 = 0.16\%$	0	0
$3.25 \sim 3.75v_p$	$A_7 = 0.0096\%$	0	0
$3.75v_p \sim \infty$	$A_8 = 0.0003\%$	0	0
			$\sum_v \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-15}^{o}(v) = 2.9849$

Tab 2 (2) Real contributions of electrons of $\theta = 0^o$ with different speeds.

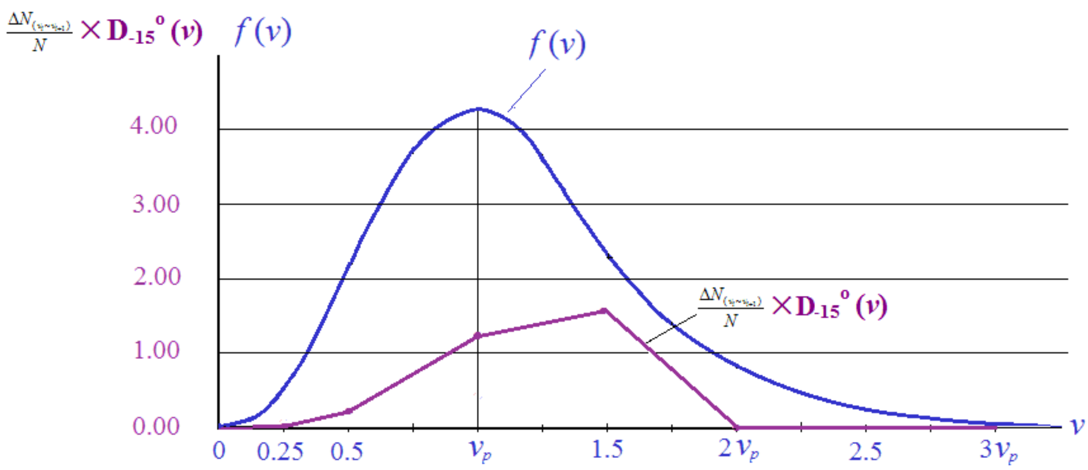
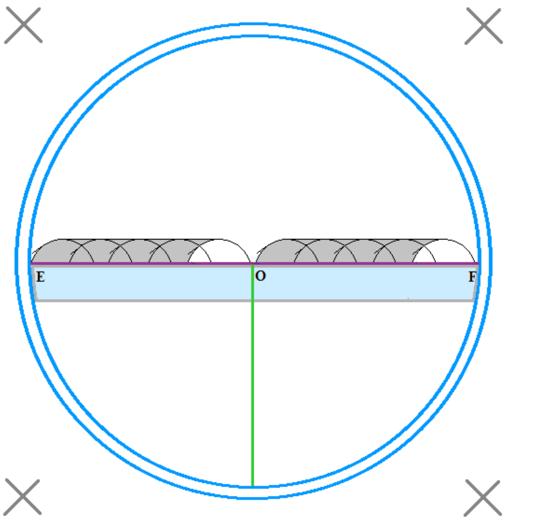


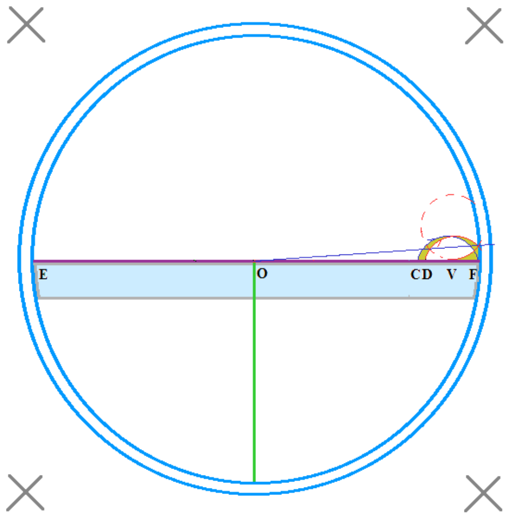
Fig 2 (2) Graph of $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N}$ D_{-15}^{o} $(v) \sim v$.

3. Trajectories of electrons of θ = 15° and different speeds

(1) Fig 3-1 $\theta = 15^{o}$ $v = 0.5v_p$ $R = 2\text{mm}$ ($cos\theta$ = 0.9659)

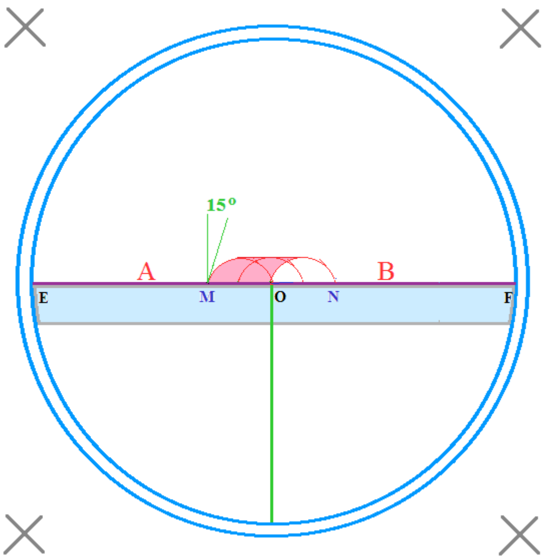


A-directly-A & B-directly-B
(grey)

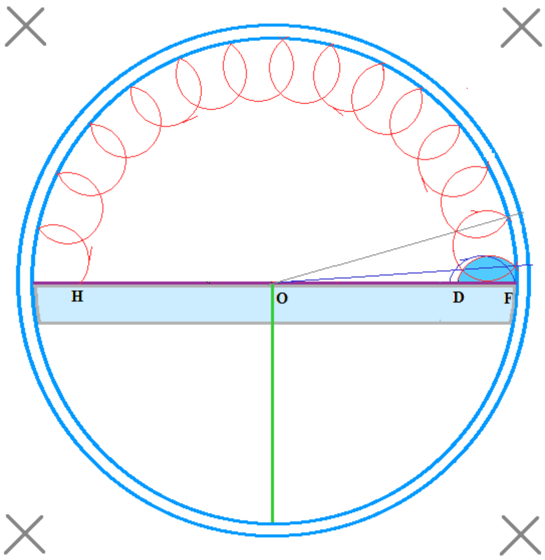


B-glass-B 2.5/70=0.0357
(from CD to FV, CD=2.5, OF=70)

No electron migration between A and B due to these trajectories.



A-directly-B 19/70 = 0.271
(from MO to ON, MO=19, EO=70)



B-glass-A 16.5/70 = 0.235
(from D to H, etc.) (DF=16.5, OF=70)

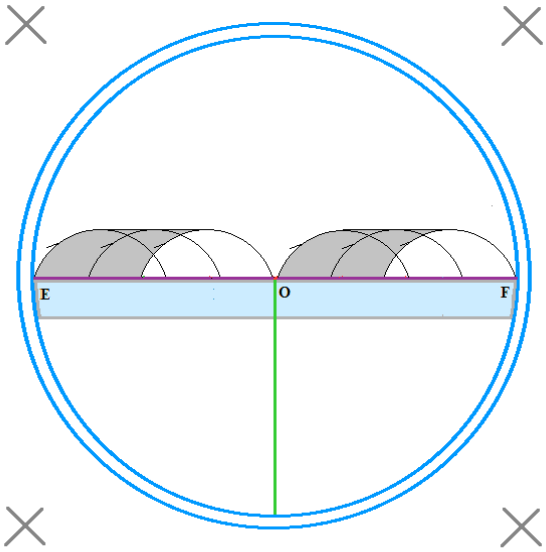
A-B 27% of the electrons of (15°, 0.5v_p) emitted from A migrate to B.
B-A 23.5% of the electrons of (15°, 0.5v_p) emitted from B migrate to A.

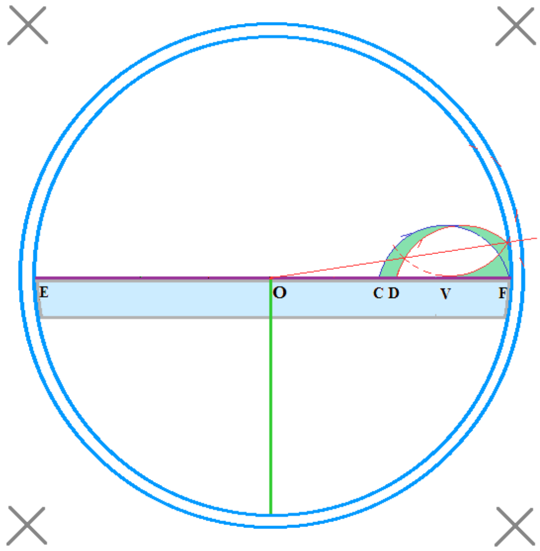
For all the electrons of (15°, 0.5v_p), A-B exceeds B-A, and their difference is the corresponding contribution to the output current

$$\{ (A\text{-}B) - (B\text{-}A) \}_{15^{o}\ 0.5v_p} = 0.271 - 0.235 = 0.036 \approx 0.04$$

$$D_{15^{o}}(0.5v_p) = \{ (A\text{-}B) - (B\text{-}A) \}_{15^{o}\ 0.5v_p} \cos 15^{o} = 0.04 \times 0.9659 \approx 0.04$$

(2) Fig 3-2 $\theta = 15^{\rm o}$ $v = v_p$ $R = 4\text{mm}$ (B =1.34 gauss)

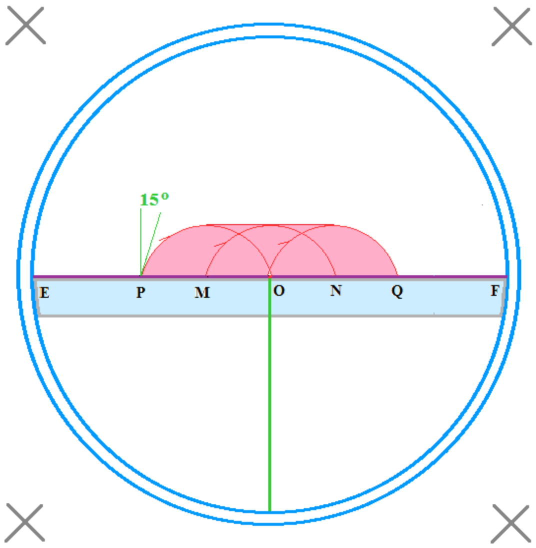


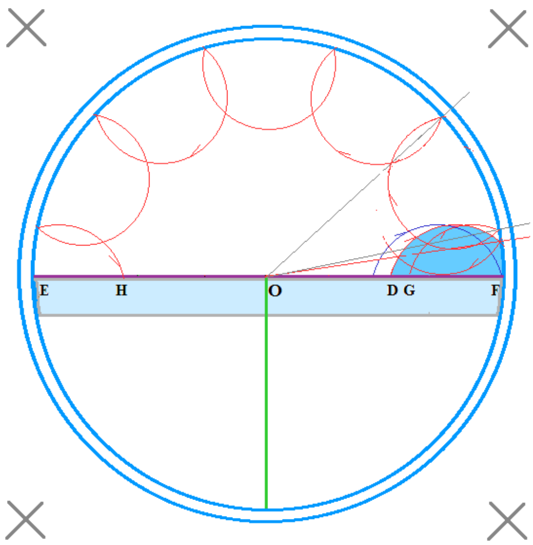


A-directly-A & B-directly-B

B-glass-B

No electron migration between A and B due to these trajectories.





A-directly-B

PO/EO = 39/70 = 0.56 (PO=39)

(from PO to OQ, etc., pink)

B-glass-A

DF/OF = 33.7/70 = 0.48 (DF=33.7)

(from G to H, etc., blue)

A-B 56% of the electrons of ($15^{\rm o}$, v_p) emitted from A migrate to B.

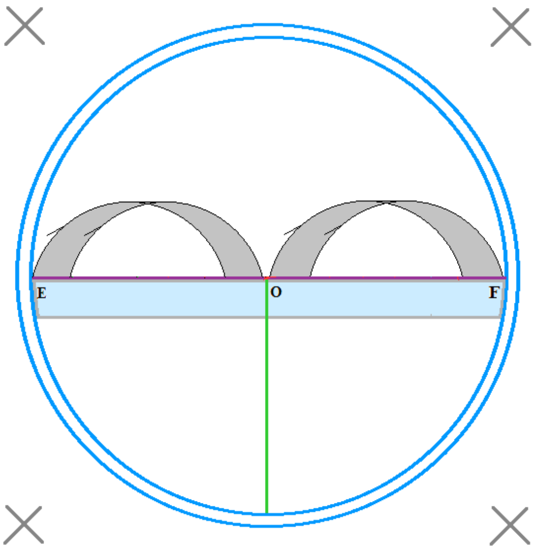
B-A 48% of the electrons of ($15^{\rm o}$, v_p) emitted from B migrate to A.

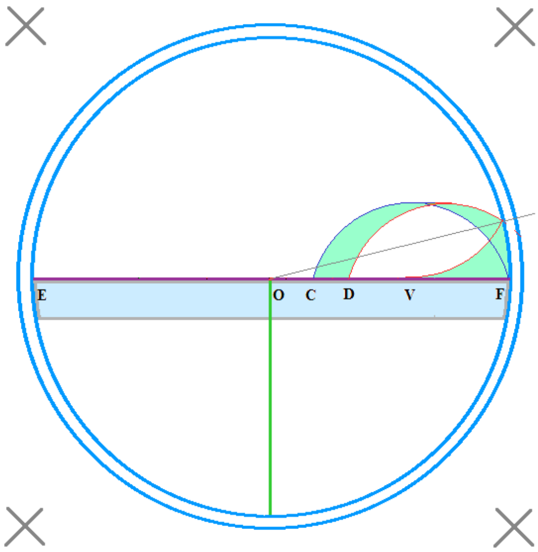
For all the electrons of ($15^{\rm o}$, v_p), A-B exceeds B-A, and their difference is the corresponding contribution to the output current

$$\{\text{(A-B) - (B-A)}\}_{15^{\rm o}v_p} = 0.56 - 0.48 = 0.08$$

$$D_{15^{\rm o}}(v_p) = \{\text{(A-B) - (B-A)}\}_{15^{\rm o}v_p}\cos 15^{\rm o} = 0.08 \times 0.9659 \approx 0.08$$

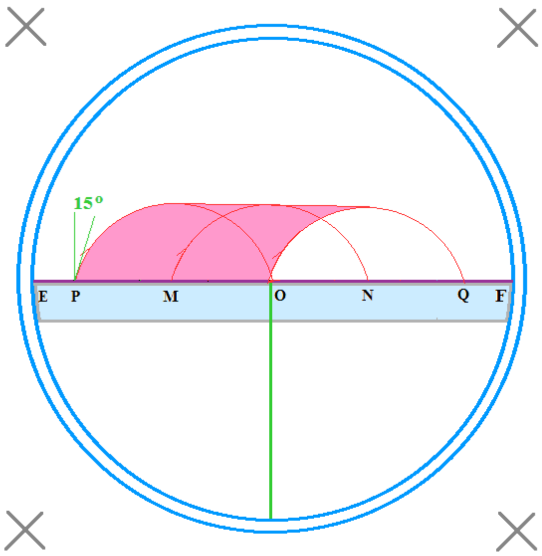
(3) Fig 3-3 θ = 15° $v = 1.5v_p$ R = 6mm

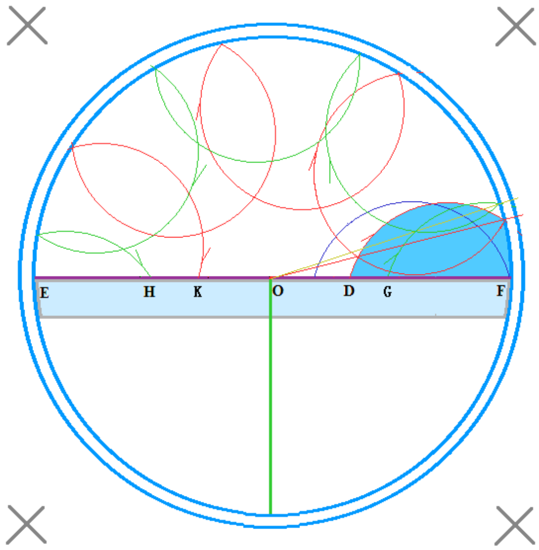




A-directly-A & B-directly-B B-Glass-B

No electron migration between A and B due to the above trajectories.





A-directly-B
PO/EO = 58/70 = 0.83 (PO=58)
(from PO to OQ, pink)

B-glass-A
DF/OF = 47/70 = 0.67 (DF = 47)
(from G to H, D to K, etc., blue)

A-B 83% of the electrons of (15°, $1.5v_p$) emitted from A migrate to B.

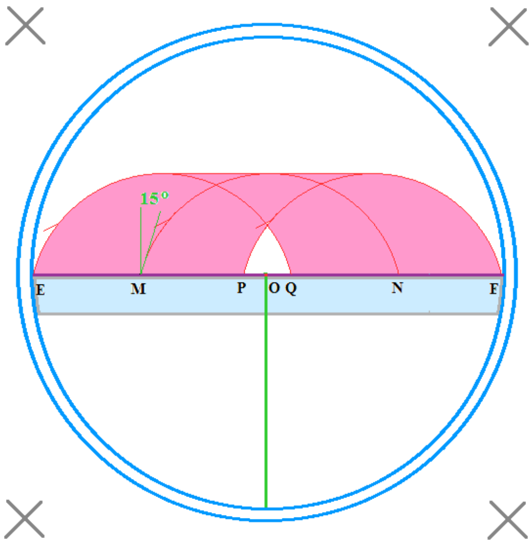
B-A 67% of the electrons of (15°, $1.5v_p$) emitted from B migrate to A.

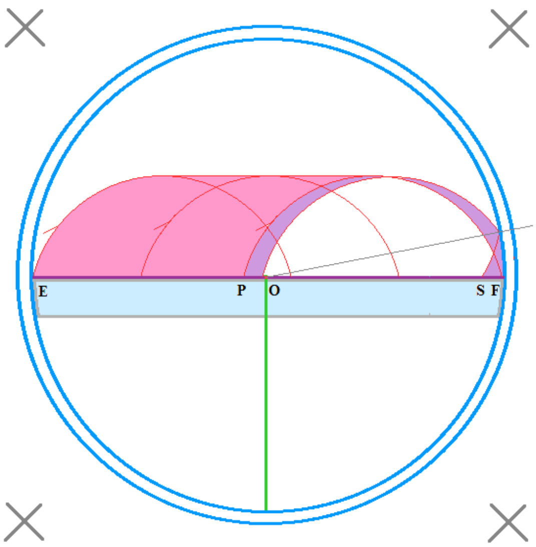
For all the electrons of (15°, v_p), A-B exceeds B-A, and their difference is the corresponding contribution to the output current

$$\{ (A\text{-}B) - (B\text{-}A) \}_{15^{\circ}\,1.5v_p} = 0.83 - 0.67 = 0.16$$

$$D_{15^{\circ}}(1.5v_p) = \{ (A\text{-}B) - (B\text{-}A) \}_{15^{\circ}\,1.5v_p} \cos 15^{\circ} = 0.16 \times 0.9659 \approx 0.16$$

(4) Fig 3-4 $\theta = 15^{o}$ $v = 2v_p$ $R = 8\text{mm}$



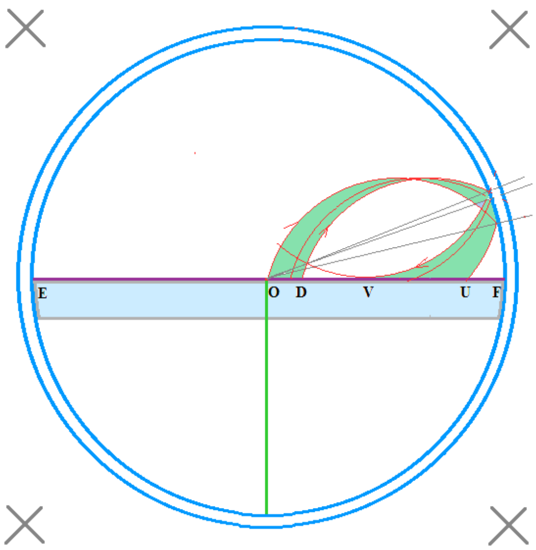


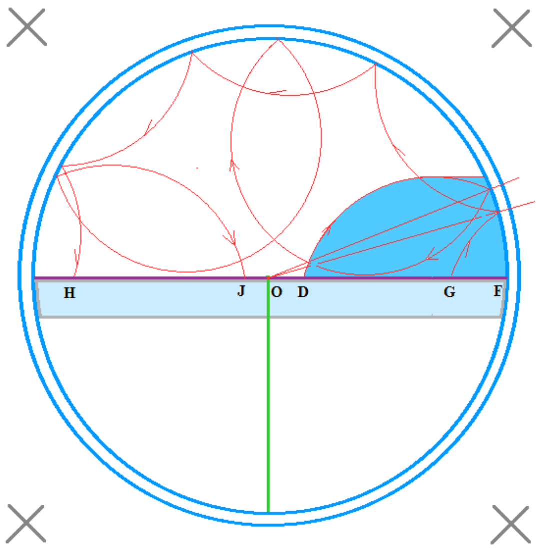
A-directly-B 64/70 = 0.91
(from E to Q, M to N, P to F, pink)

A-glass-B 6/70 = 0.09
(from PO to FS, violet)

A-B = A-directly-B + A-glass-B (64 + 6 =70, pink + violet) 70/70 = 1.00

A-B 100% of the electrons of (15^{o}, $2v_p$) emitted from A migrate to B.





B-glass-B (from O to U, D to F, etc., green)
No electron migration between A and B due to these trajectories.

B-glass-A DF/OF=59.5/70 = 0.85
(from D to J, G to H, etc., blue)

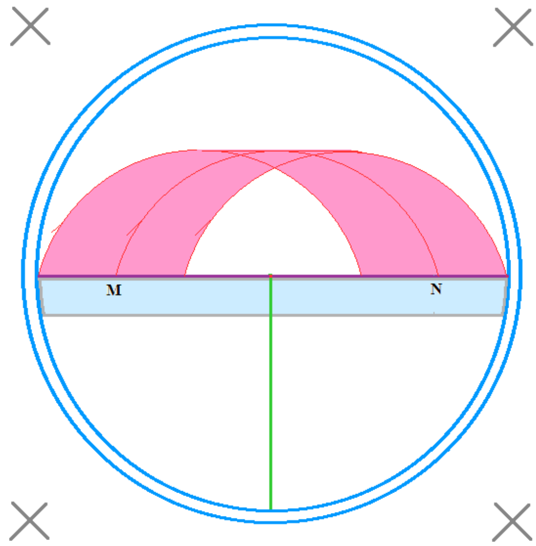
B-A 85% of the electrons of (15^{o}, $2v_p$) emitted from B migrate to A.

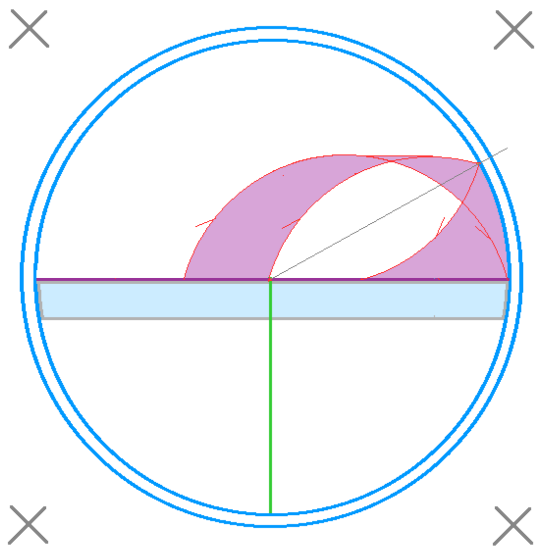
For all the electrons of (15^{o}, $2v_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current

$$\{ \text{(A-B)} - \text{(B-A)} \}_{15^{o}\,2v_p} = 1.00 - 0.85 = 0.15$$

$$D_{15^{o}}(2v_p) = \{ \text{(A-B)} - \text{(B-A)} \}_{15^{o}\,2v_p} \cos 15^{o} = 0.15 \times 0.9659 \approx 0.15$$

(5) Fig 3-5 $\theta = 15^{\circ}$ $v = 2.5v_p$ $R = 10\text{mm}$

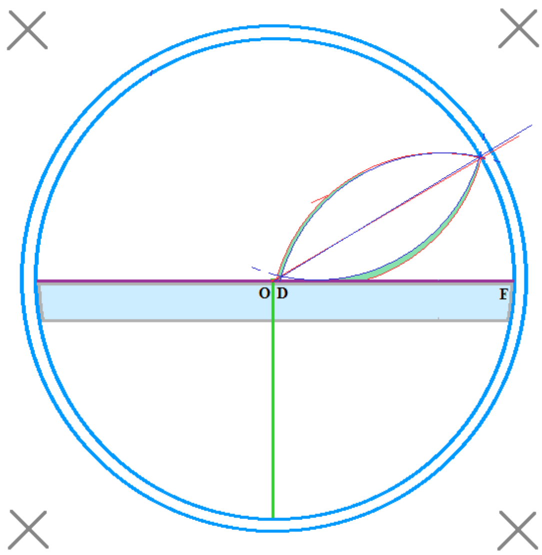


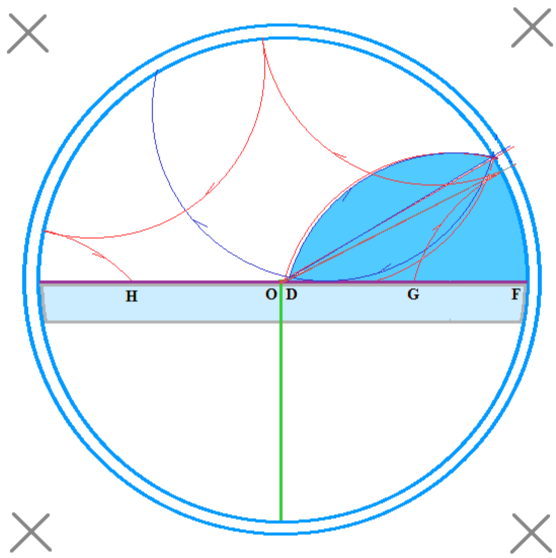


A-directly-B (45/70) (pink) A-glass-B (25/70) (violet)

A-B (total) 70/70 = 1.00 (pink +violet, 45 + 25 = 70)

A-B 100% of the electrons of (15°, $2.5v_p$) emitted from A migrate to B.





B-glass-B 1/70 = 0.014
(OD = 1, OF = 70)
No electron migration between A and B due to these trajectories.

B-glass-A 69/70 = 0.986
(DF = 69, OF =70)
(from G to H, etc., blue)

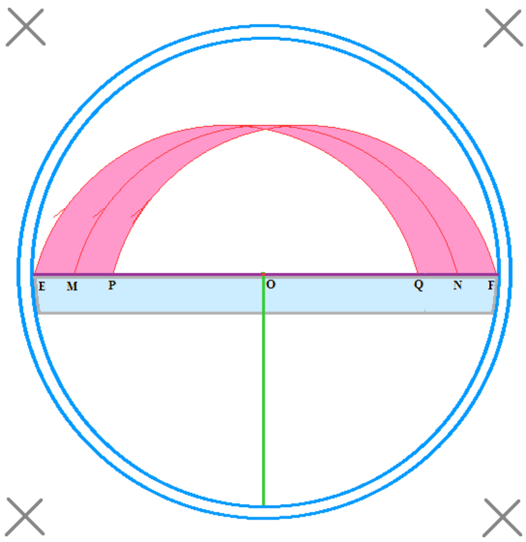
B-A 98.6% of the electrons of (15°, $2.5v_p$) emitted from B migrate to A.

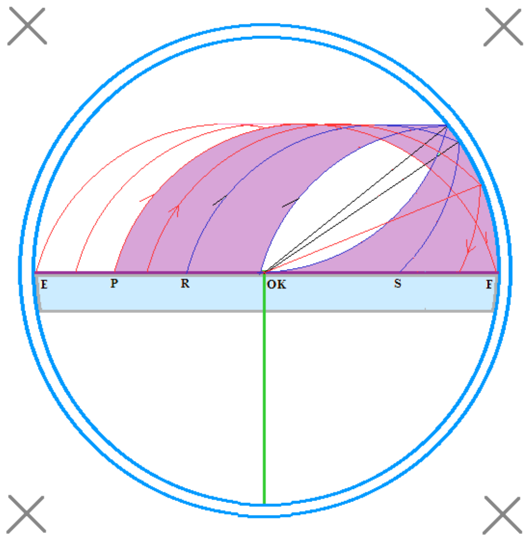
For all the electrons of (15°, $2.5v_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current

$$\{\,(\text{A-B}) - (\text{B-A})\,\}_{15^{\circ}\,2.5v_p} = 1.00 - 0.986 = 0.014$$

$$D_{15^{\circ}}(2.5v_p) = \{\,(\text{A-B}) - (\text{B-A})\,\}_{15^{\circ}\,2.5v_p} \cos 15^{\circ} \approx 0.01$$

(6) Fig 3-6 $\theta = 15^{o}$ $v = 3v_p$ $R = 12$mm





A-directly-B

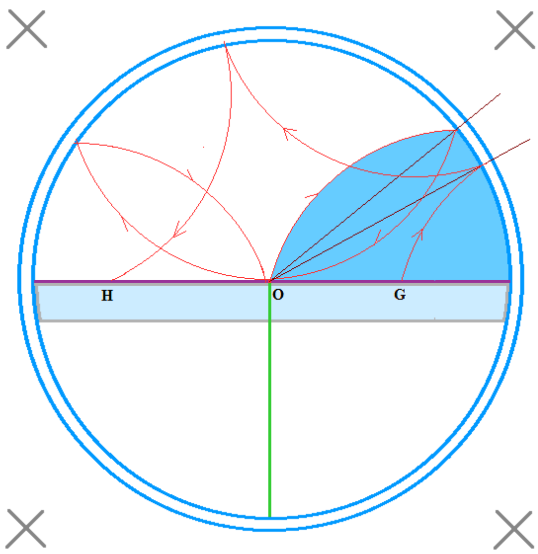
(from E to Q, M to N, P to F, etc., pink)

A-glass-B

(from P to F, O to K, etc., violet)

A-B (total) 70/70 = 1.00 (EP + PO = 20 + 50 = 70)

A-B 100% of the electrons of (15^{o}, $3v_p$) emitted from A migrate to B.



B-glass-A 70/70 = 1.00

(from G to H, O to O, etc., blue)

B-A 100% of the electrons of (15^{o}, $3v_p$) emitted from B migrate to A.

For all the electrons of (15^{o}, $3v_p$), A-B equals B-A, and their contribution to the output current cancel each other.

$$\{ (\text{A-B}) - (\text{B-A}) \}_{15^{o}\,3v_p} = 1.00 - 1.00 = 0$$

$$D_{15^{o}}(3v_p) = \{ (\text{A-B}) - (\text{B-A}) \}_{15^{o}\,3v_p} \times \cos 15^{\circ} = 0$$

Contributions of electrons of exiting angle $\theta = 15^{o}$ and different speeds, Tab 3 (1) and Fig 3 (1).

$\theta = 15^{o}$	{ (A-B) - (B-A) } $_{15^{o}}$	$D_{15^{o}}(v)$ = { (A-B) - (B-A) } $_{15^{o}} \cos\theta$
Fig 3-1 $v = 0.5v_p$	0.271 – 0.235 =0. 04	0.04
Fig 3-2 $v = v_p$	0.56 – 0.48 = 0. 08	0.08
Fig 3-3 $v = 1.5v_p$	0.83 – 0.69 = 0.16	0.16
Fig 3-4 $v = 2v_p$	1.00 – 0.85 = 0.15	0.15
Fig 3-5 $v = 2.5v_p$	1.00 – 0.986=0.014	0.01
Fig 3-6 $v = 3v_p$	1.00 – 1.00 = 0	0
Fig 3-7 $v = 4.5v_p$	1.00 - 1.00 = 0	0

Tab 3 (1) Contributions of electrons of $\theta = 15^{o}$ and different speeds

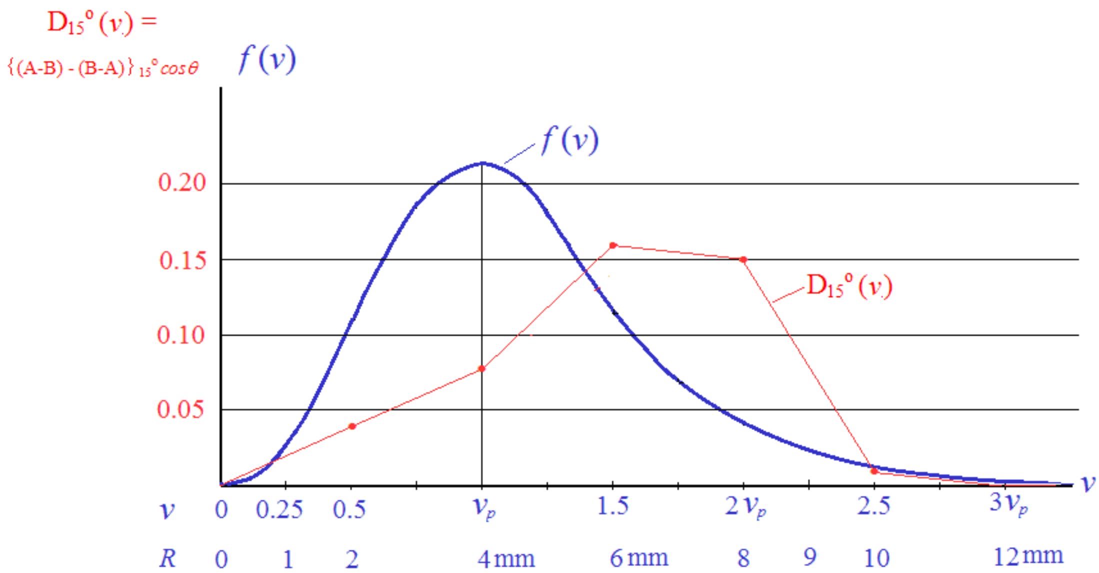


Fig 3 (1) Graph of the contributions of $\theta = 15^{o}$ with different speeds

Take Maxwell's speed distribution into account to derive the actual contributions of the thermal electrons of $\theta = 15^{o}$ with respect to different speeds, i.e., $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{15^{o}}(v) \sim v$. And, Fig 1-12 is the graph.

Speed range	$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N}$	$D_{15^{o}}(v)$	$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{15^{o}}(v)$
$0.00 \sim 0.25v_p$	$A_0 \approx$ 0. 004	≈ 0	$\approx$ 0
$0.25 \sim 0.75v_p$	$A_1 =$ 18. 74%	0.04	0.7496
$0.75 \sim 1.25v_p$	$A_2 =$ 39. 83%	0.08	3.1864
$1.25 \sim 1.75v_p$	$A_3 =$ 26. 71%	0.16	4.2736
$1.75 \sim 2.25v_p$	$A_4 =$ 8. 82%	0.15	1.323
$2.25 \sim 2.75v_p$	$A_5 =$ 1. 58%	0.01	0.016
$2.75 \sim 3.25v_p$	$A_6 =$ 0. 16%	0	0
$3.25v_p \sim \infty$	$A_7A_8 \approx 0.0099\%$	0	0
			$\sum_v \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{15^{o}}(v) = 9.5486$

Tab 3 (2) Actual contributions of electrons of $\theta = 15^{o}$ and different speeds

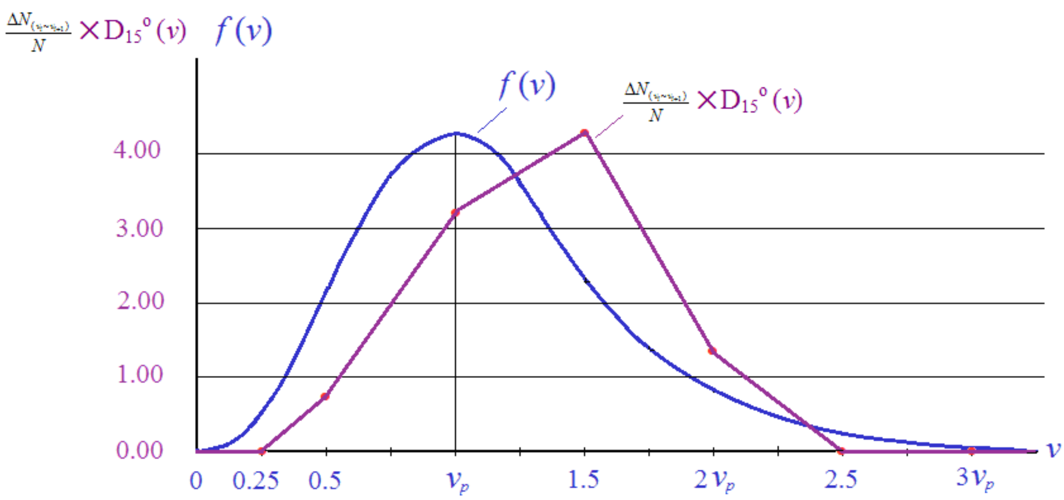
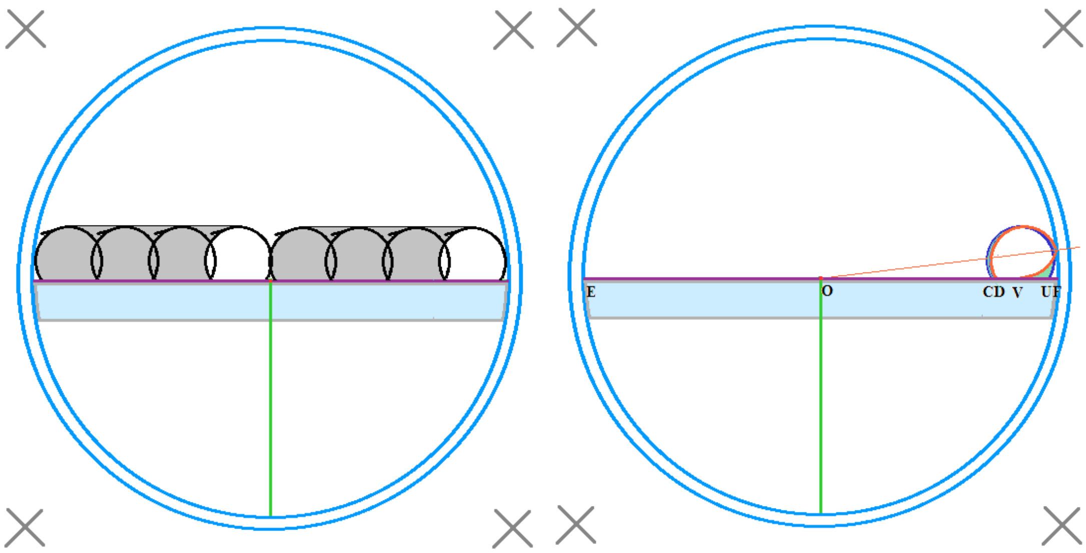


Fig 3 (2) Graph of $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{15^{o}} \sim v$

4. Trajectories of electrons of θ = - 30^{o} and different speeds

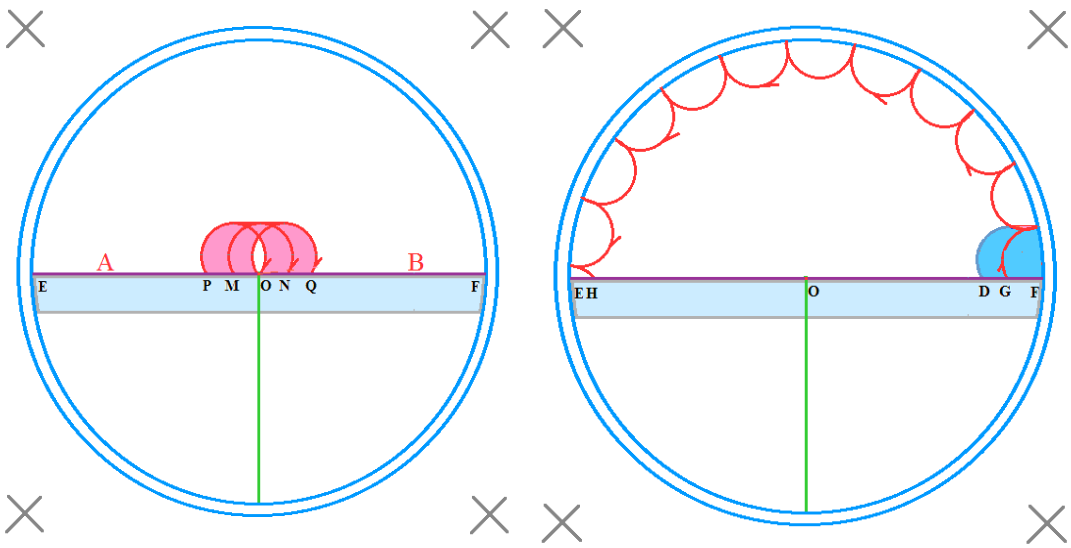
(1) Fig 4-1 θ = - 30^{o} $v = 0.5v_p$ R = 2mm (B = 1.34 gauss)



A-directly-A B-directly-B B-glass-B 1/70 = 0.014

(grey) (CD = 1, OF =70) (from CD to UV)

No electron migration between A and B due to these trajectories.



A-directly-B (from PO to OQ, pink) B-Glass-A (from G to H, etc., blue)

16.5/70 = 0.236 (PO = 16.5, EO = 70) 18/70 = 0.257 (DF =18, OF = 70)

A-B 24% of the electrons of (- 30^{o}, 0.5v_p) emitted from A migrate to B.

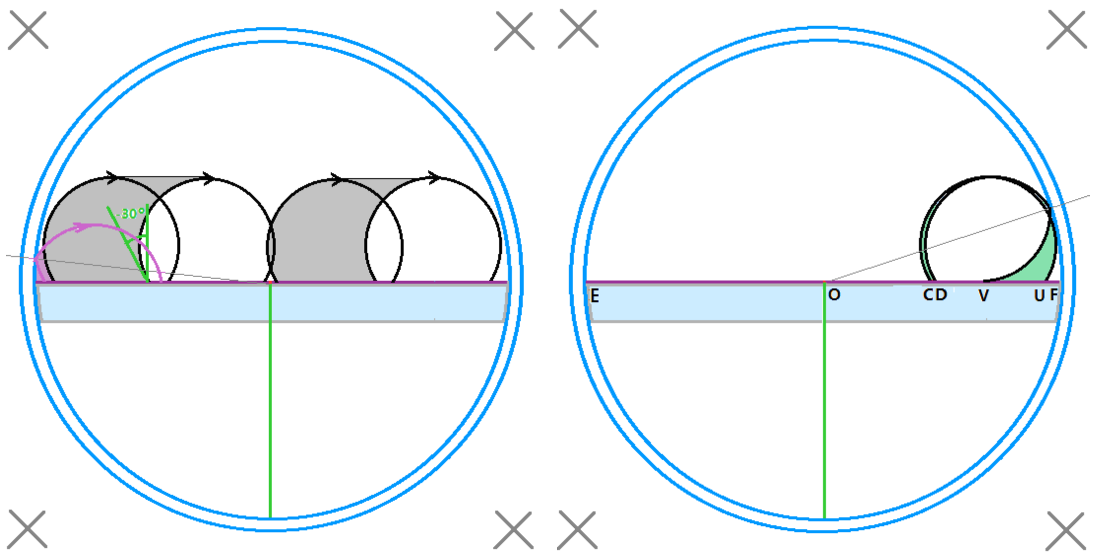
B-A 26% of the electrons of (- 30^{o}, 0.5v_p) emitted from B migrate to A.

For all the electrons of (-30^{o}, 0.5v_p), A-B is less than B-A, and their difference is the corresponding contribution (negative) to the output current.

$$\{ (A\text{-}B) - (B\text{-}A) \}_{-30^{o}\ 0.5v_p} = 0.236 - 0.257 = -0.021$$

$$D_{-30^{o}}(0.5v_p) = \{ (A\text{-}B) - (B\text{-}A) \}_{-30^{o}\ 0.5v_p} \cos 30^{o} = -0.021 \times 0.8660 \approx -0.02$$

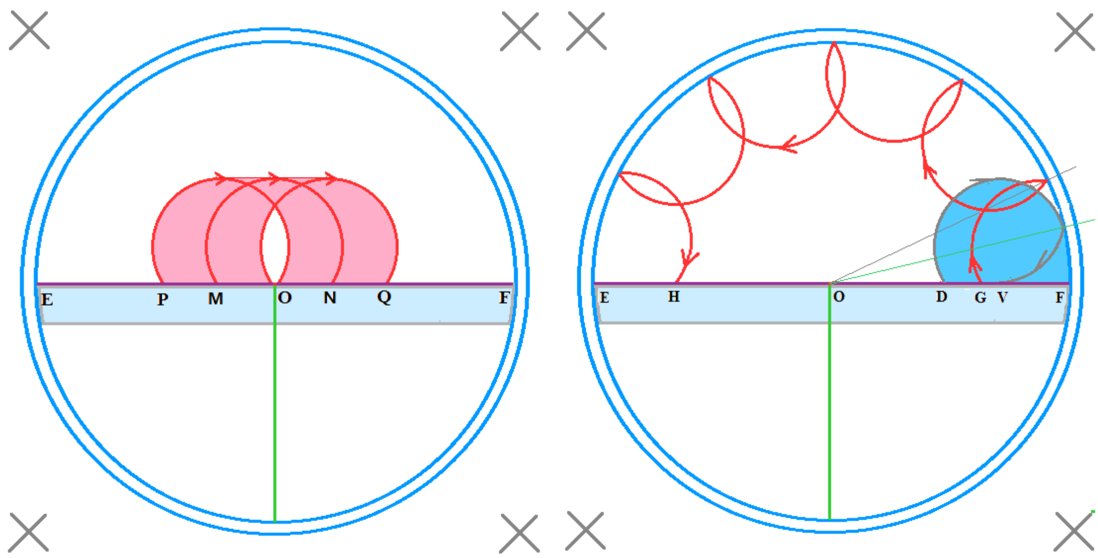
(2) Fig 4-2 $\theta = -30^{o}$ $v = v_p$ $R = 4\text{mm}$ ($\cos -30^{o} = 0.8660$)



A-glass-A (violet) A-directly-A (grey) B-directly-B (grey) B-glass-B

CD/OF = 1.5/70 = 0.021 (green)

No electron migration between A and B due to these trajectories.



A-directly-B PO/EO = 34/70 = 0.49 (PO =34) (from PO to OQ)

B-Glass-A DF/OF = 37/70 = 0.53 (DF = 37) (from G to H, etc., blue)

A-B 49% of the electrons of (-30^{o}, v_p) emitted from A migrate to B.

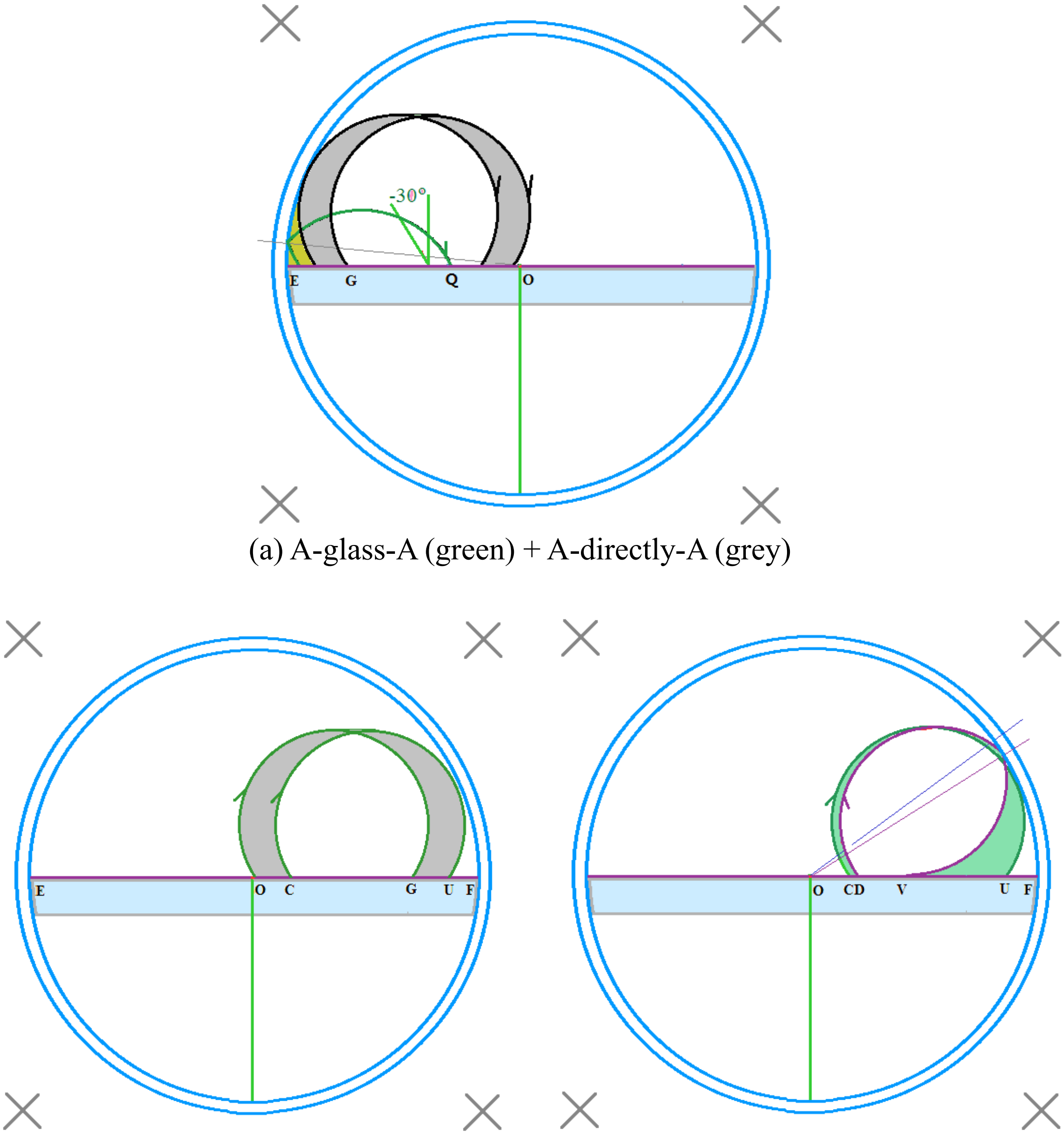
B-A 53% of the electrons of (-30^{o}, v_p) emitted from B migrate to A.

For all the electrons of (-30^{o}, v_p), A-B is less than B-A, and their difference is the corresponding contribution (negative) to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{-30^{o}\,v_p} = 0.49 - 0.53 = -0.04$$

$$D_{-30^{o}}(v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-30^{o}\,v_p} \cos 30^{o} = -0.04 \times 0.8660 \approx -0.03$$

(3) Fig 4-3 $\theta = -30^{\circ}$ $v = 1.5v_p$ $R = 6\text{mm}$

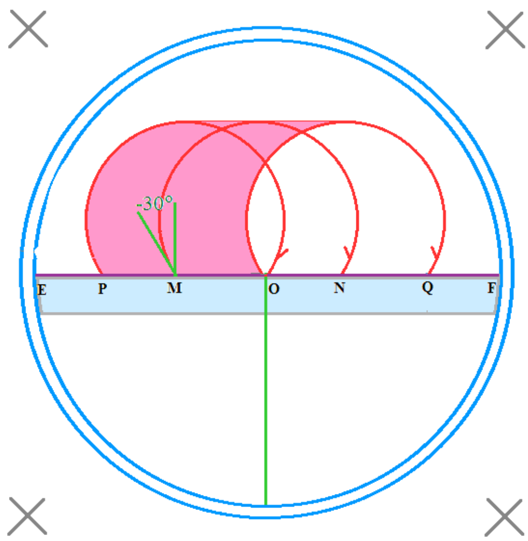


(a) A-glass-A (green) + A-directly-A (grey)

(b) B-directly-B (grey)

(c) B-glass-B (green)

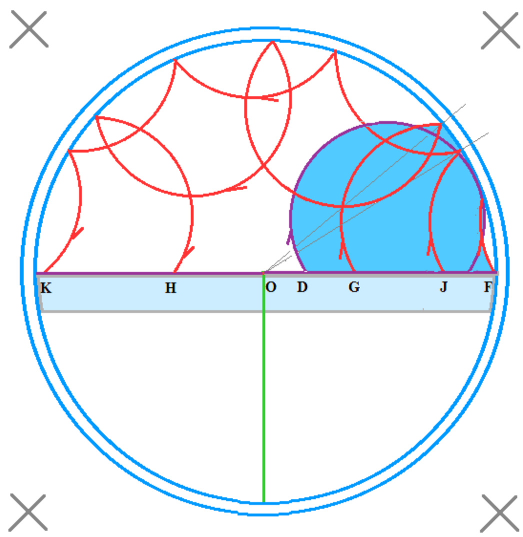
No electron migration between A and B due to these trajectories.



(d) A-directly-B 50/70 = 0.71

(PO = 50, EO =70) (from PO to OQ, pink)

A-B 71% of the electrons of (- 30°, 1.5v_p) emitted from A migrate to B.



(e) B-glass-A 55/70 = 0.79 (DF = 55, OF = 70)

(from G to H, J to K, etc., blue)

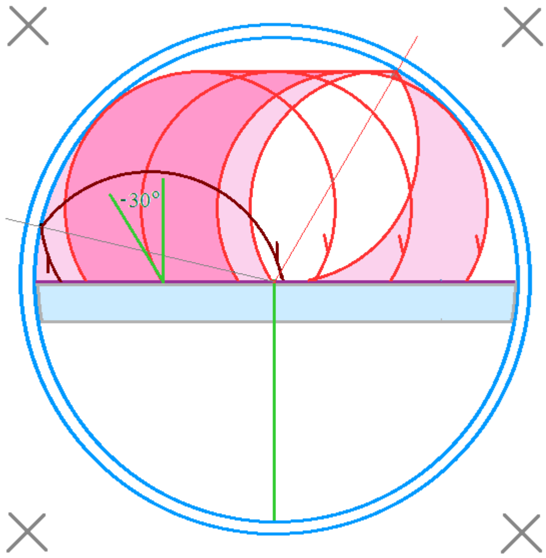
B-A 79% of the electrons of (- 30°, 1.5v_p) emitted from B migrate to A.

For all the electrons of (- 30°, 1.5v_p), A-B is less than B-A, their difference is the corresponding contribution (negative) to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{-30^{o}\ 1.5v_p} = 0.71 - 0.79 = -0.08$$

$$D_{-30^{o}}(1.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-30^{o}\ 1.5v_p} \times \cos 30^{o} = -0.08 \times 0.866 = -0.07$$

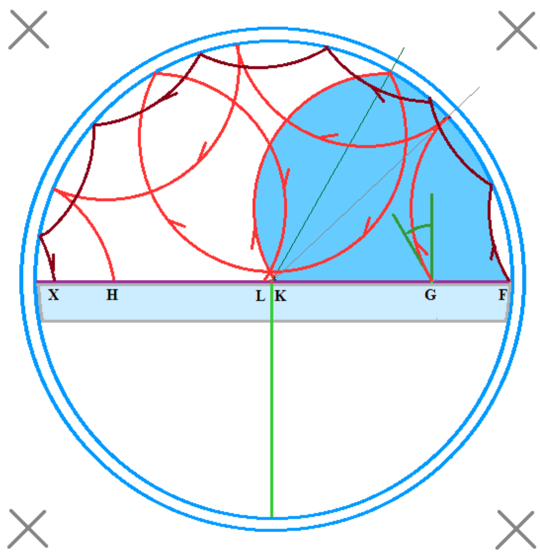
(4) Fig 4-4 θ = - 30° $v = 2\ v_p$ R = 8mm



A-glass-B + A-directly-B + A-glass-B 70/70 =1.00

violet pink violet

A-B 100% of the electrons of (-30°, $2v_p$) emitted from A migrate to B.



B-glass-A 70/70 = 1.00

(from G to H, K to L, F to X, etc. blue)

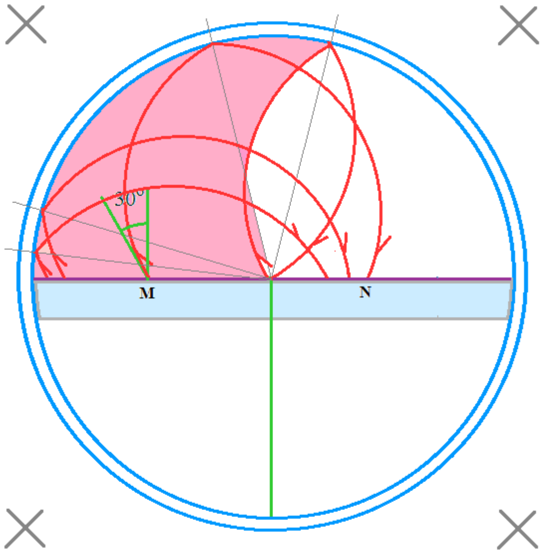
B-A 100% of the electrons (- 30°, $2v_p$) emitted from B migrate to A.

For all the electrons of (- 30°, $2v_p$), A-B and B-A cancel each other, no contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{-30^{\circ}\ 2v_p} = 1.00 - 1.00 = 0$$

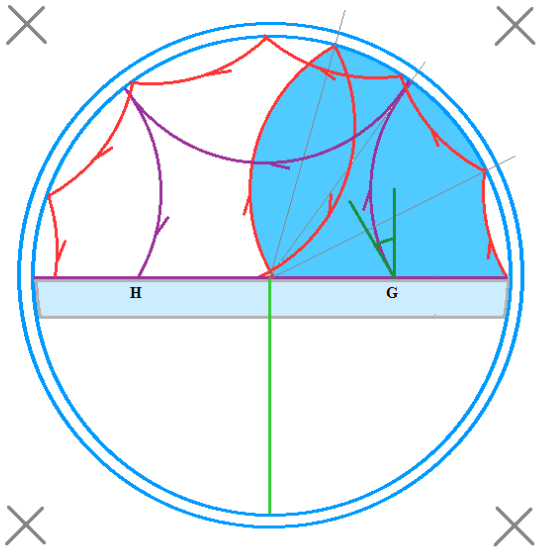
$$D_{-30^{\circ}}(2v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-30^{\circ}\ 2v_p} \times \cos 30^{\circ} = 0$$

(5) Fig 4-5 $\theta = -30^{o}$ $v = 2.5v_p$ $R = 10mm$



A-glass-B 70/70 = 1.00 (pink)

A-B 100% of the electrons of (- 30°, $2.5v_p$) emitted from A migrate to B.



B-glass-A 70/70 = 1.00 (blue)

B-A 100% of the electrons of (- 30°, $2.5v_p$) emitted from B migrate to A.

For all the electrons of (- 30°, $2.5v_p$), A-B and B-A cancel each other, no contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{\theta=-30^{o}\ v=2.5v_p} = 1.00 - 1.00 = 0$$

$$D_{-30^{o}}(2.5v_p) = 0 \times \cos 30^{o} = 0$$

Contributions of electrons of θ = - 30° with different speeds

θ = - 30°	{(A-B) – (B-A)} $_{-30°}$	$D_{-30°}(v)$
Fig 4-1 $v = 0.5v_p$	0.235 - 0.257 = - 0.022	- 0.02
Fig 4-2 $v = v_p$	0.49 – 0.54 = - 0.04	- 0.03
Fig 4-3 $v = 1.5v_p$	0.71 - 0.79 = - 0.08	- 0.07
Fig 4-4 $v = 2v_p$	1.00 – 1.00 = 0	0
Fig 4-5 $v = 2.5v_p$	1.00 – 1.00 = 0	0
Fig 4-6 $v = 3v$	1.00 – 1.00 = 0	0

Tab 4 (1) $D_{-30°}(v)$ = {(A-B) – (B-A)} $_{\theta=-30°}\cos\theta$ ~ v

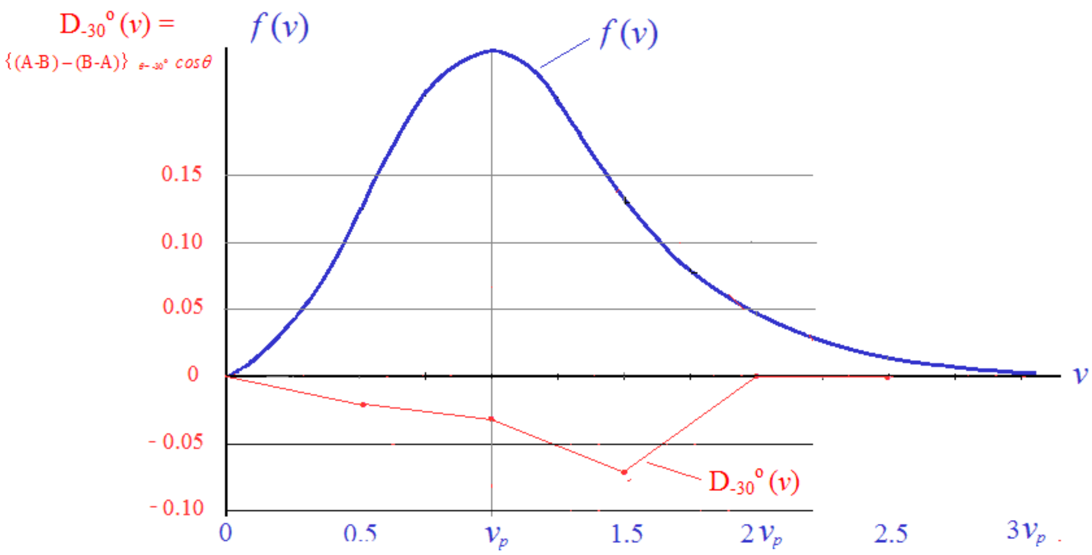


Fig 4 (1) Graph of contributions of electrons of (-30° , v)

Take Maxwell's speed distribution into account to derive the actual contributions of the thermal electrons of θ = -30° with different speeds, i.e., $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-30°}(v) \sim v$, see Tab 4(2).

Fig 4(2) is the corresponding graph.

Speed range	$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N}$	$D_{-30^o}(v)$	$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-30^o}(v)$
$0.00\sim0.25v_p$	$A_0 \approx 0.004$	≈ 0	≈ 0
$0.25\sim0.75v_p$	$A_1 =$ 18.74%	-0.02	- 0.3748
$0.75\sim1.25v_p$	$A_2 =$ 39.83%	-0.03	- 1.1949
$1.25\sim1.75v_p$	$A_3 =$ 26.71%	-0.07	- 1.8697
$1.75\sim2.25v_p$	$A_4 =$ 8.82%	0	0
$2.25\sim2.75v_p$	$A_5 =$ 1.58%	0	0
$2.75\sim3.25v_p$	$A_6 =$ 0.16%	0	0
$3.25v_p\sim\infty$	$A_7A_8 \approx 0.0099\%$	0	0
			$\sum_v \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-30^o}(v) = -3.4394$

Tab 4 (2) Actual contributions of electrons of $\theta = -30^o$ with different speeds v

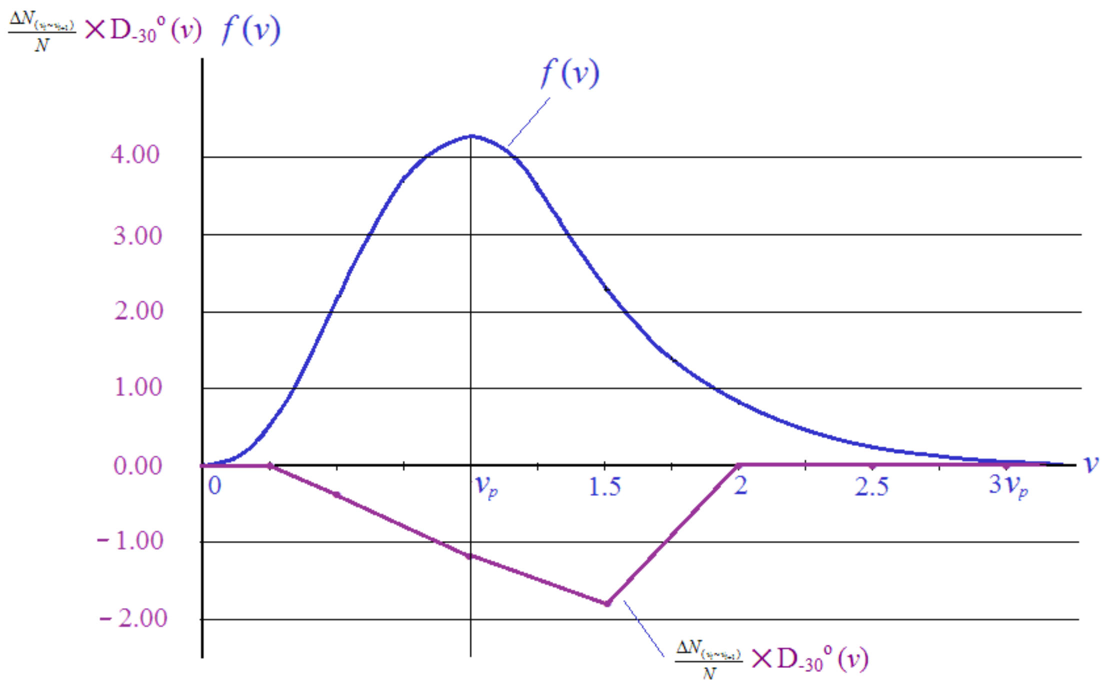
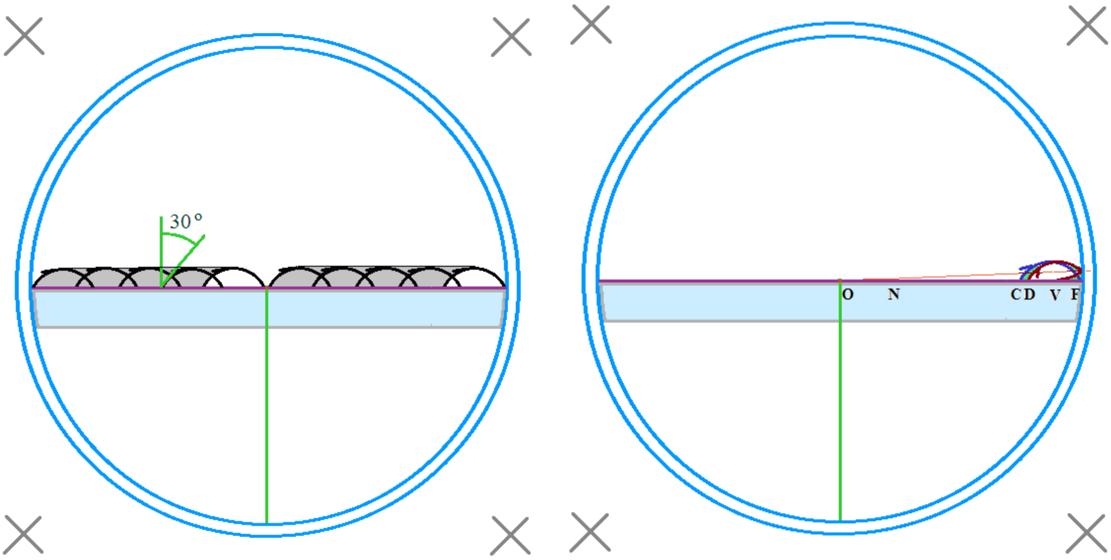


Fig 4 (2) Graph of $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-30^o} \sim v$.

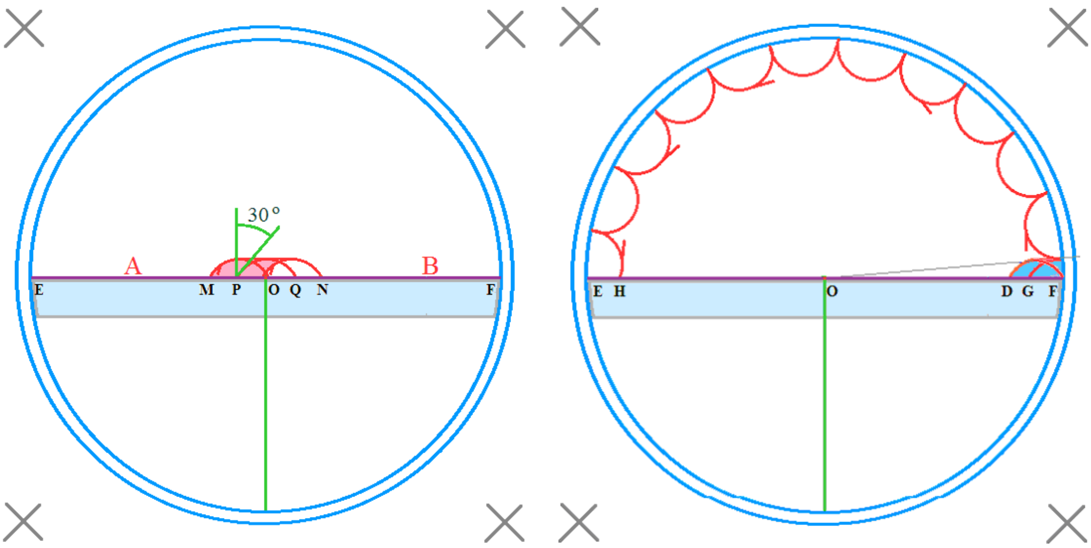
5. Trajectories of electrons of θ = 30° and different speeds

(1) Fig 5-1 $\theta = 30^{o}$ $v = 0.5v_p$ R = 2mm (B = 1.34 gauss)



A-directly-A & B-directly-B (grey) B-glass-B 2/70 =0.029 (CD = 2) (green)

No electron migration between A and B due to these trajectories.



A-directly-B (from MO to ON, pink) B-glass-A (from G to H, etc., blue)

17.5/70 = 0.25 (MO = 17.5, EO = 70) 15.5/70 =0.22 (DF=15.5, OF=70)

A-B 25% of the electrons of (30°, $0.5v_p$) emitted from A migrate to B.

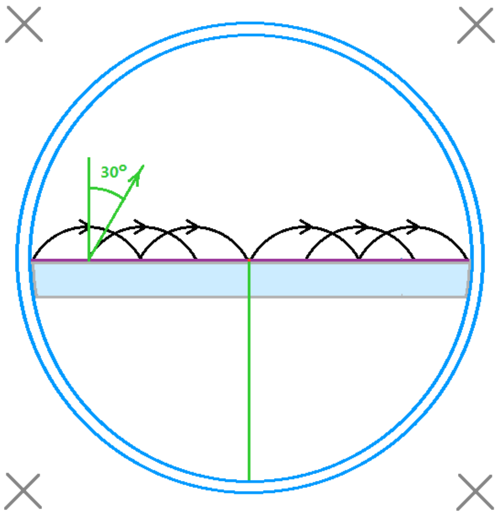
B-A 22% of the electrons of (30°, $0.5v_p$) emitted from B migrate to A.

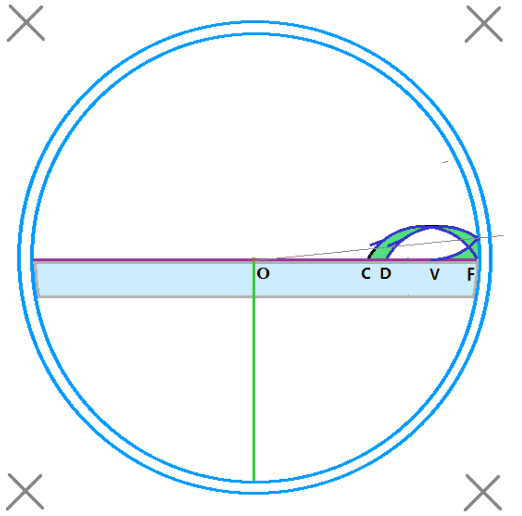
For all the electrons of (30°, $0.5v_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{30^{o}\,0.5v_p} = 0.25 - 0.22 = 0.03$$

$$D_{30^{o}}(0.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{30^{o}\,0.5v_p} \times \cos 30^{o} = 0.03 \times 0.8660 \approx 0.03$$

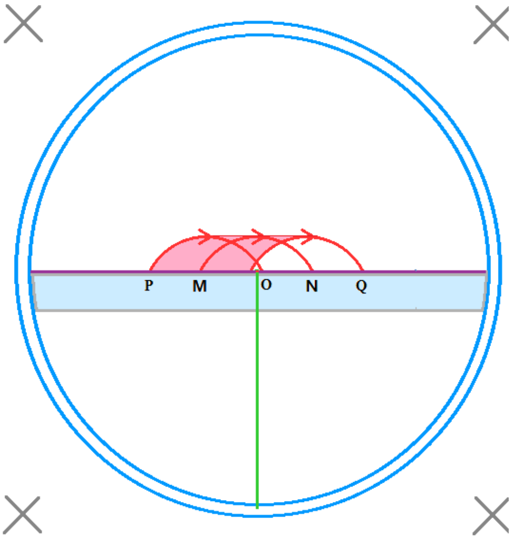
(2) Fig 5-2 $\theta = 30^{\circ}$ $v = v_p$ $R = 4\text{mm}$ ($\cos 30^{\circ} = 0.8660$)

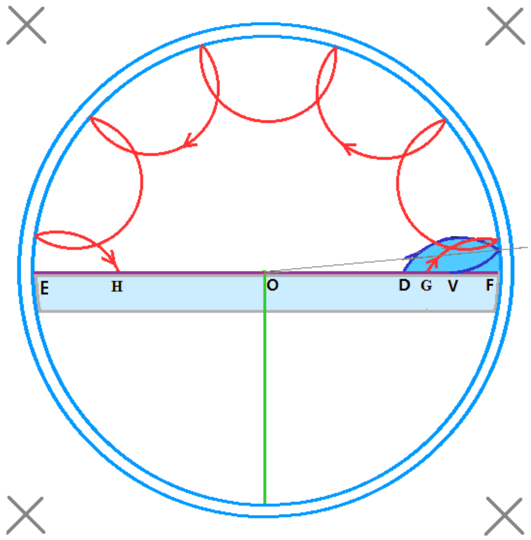




A-directly-A & B- directly-B B-glass-B 5.5/70 = 0.08 (CD = 5.5)

No electron migration between A and B due to these trajectories.





A-directly-B 34.5/70 = 0.49
(PO = 34.5, OE =70) (from PO to OQ)

B-glass-A 29/70 = 0.41
(DF = 29, OF = 70) (from G to H, etc.)

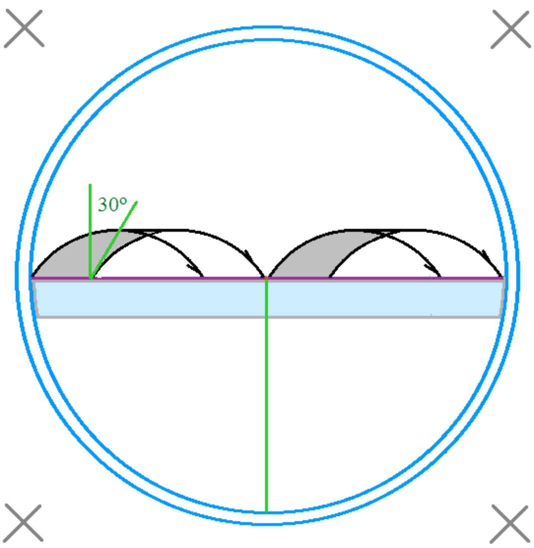
A-B 49% of the electrons of (30°, v_p) emitted from A migrate to B.
B-A 41% of the electrons of (30°, v_p) emitted from B migrate to A.

For all the electrons of (30°, v_p), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

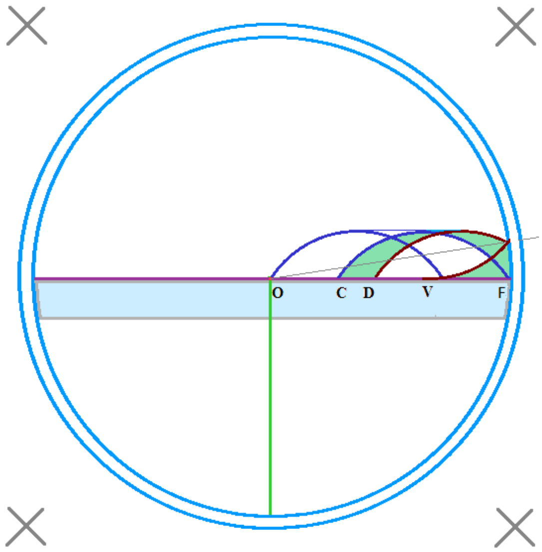
$$\{(\text{A-B}) - (\text{B-A})\}_{\theta = 30^{\circ}\ v = 1v_p} = 0.49 - 0.41 = 0.08$$

$$D_{30^{\circ}}(v_p) = 0.08 \times \cos\theta = 0.08 \times 0.866 = 0.069 = 0.07$$

(3) Fig 5-3 $\theta = 30^{\circ}$ $v = 1.5v_p$ $R = 6\text{mm}$

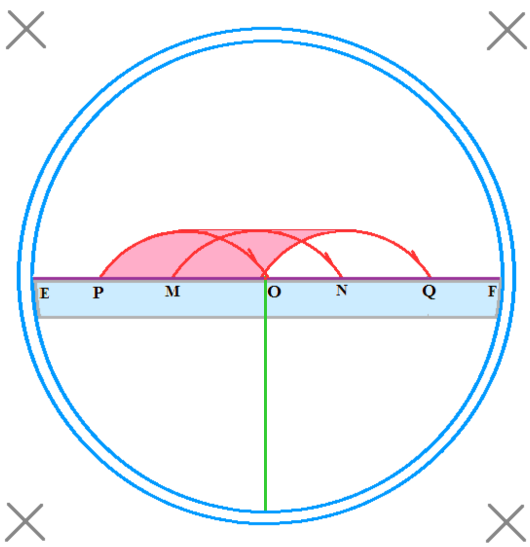


A-directly-A & B-directly-B



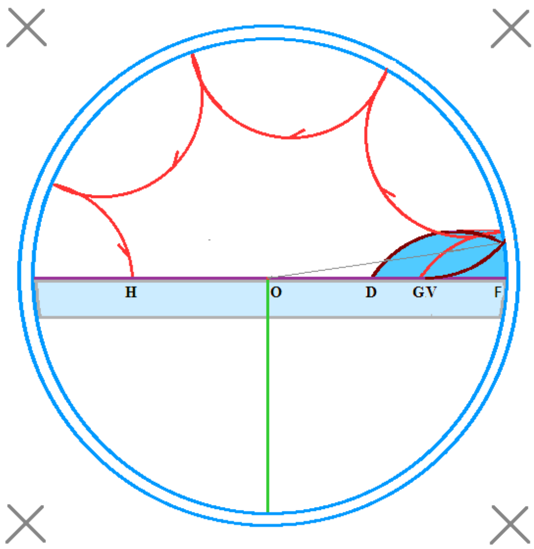
B-glass-B (from CD to FV, CD = 11)

No electron migration between A and B due to these trajectories.



A-directly-B 51/70 = 0.729

(PO=51, EO=70) (from PO to OQ, pink)



B-glass-A 40/70 = 0.57

(DE=40, OF=70) (from G to H, etc., blue)

A-B 73% of the electrons of (30°, $1.5v_p$) emitted from A migrate to B.

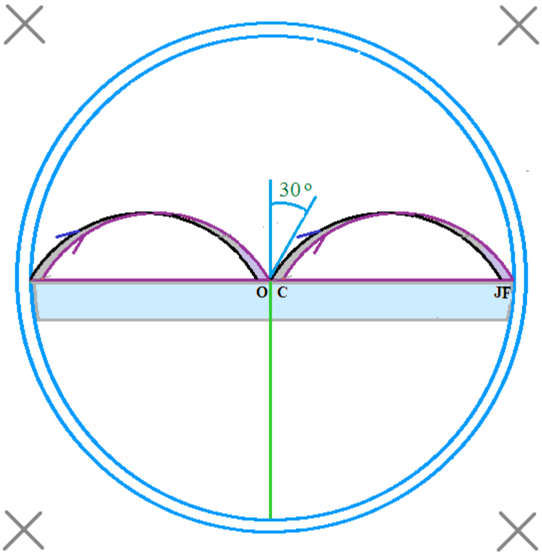
B-A 57% of the electrons of (30°, $1.5v_p$) emitted from B migrate to A.

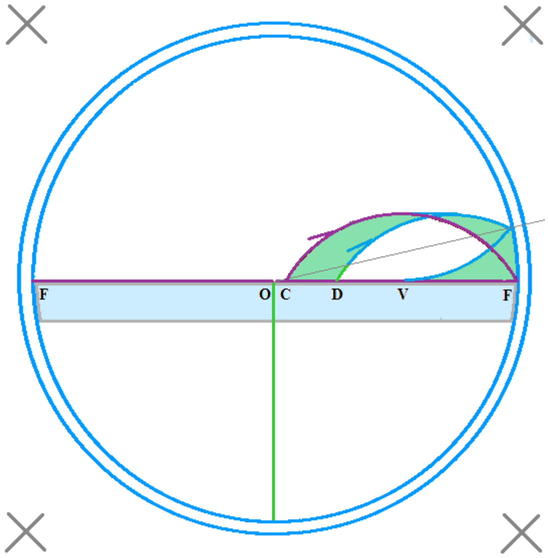
For all the electrons of (30°, $1.5v_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{30^{\circ}\,1.5v_p} = 0.73 - 0.57 = 0.16$$

$$D_{30^{\circ}}(1.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{30^{\circ}\,1.5v_p} \times \cos\theta = 0.16 \times 0.8660 = 0.14$$

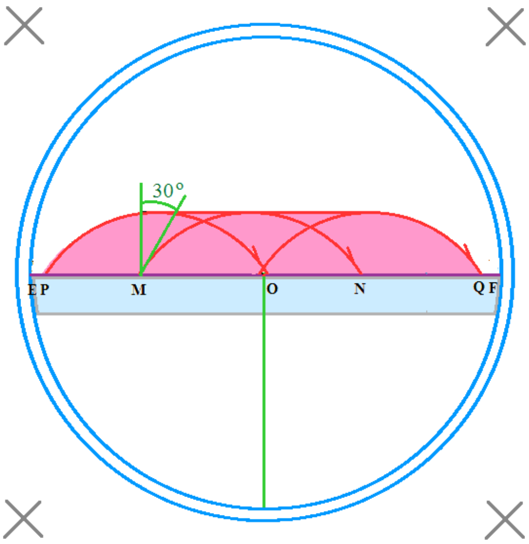
(4) Fig 5-4 $\theta = 30^o$ $v = 2v_p$ $R = 8\text{mm}$

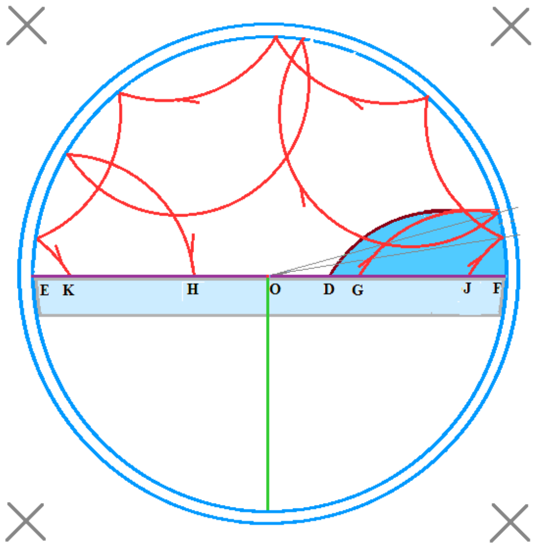




A-directly-A & B-directly-B B-glass-B (CD = 15)

No electron migration between A and B due to these trajectories.





A-directly-B PO/EO = 67.5/70 = 0.96
(PO = 67.5, EO = 70)
(from PO to OQ, pink)

B-glass-A DF/OF = 52.5/70 =0.75
(DF = 52.5, OF = 70)
(from G to H, J to K, etc., blue)

A-B 96% of the electrons of (30^o, $2v_p$) emitted from A migrate to B.

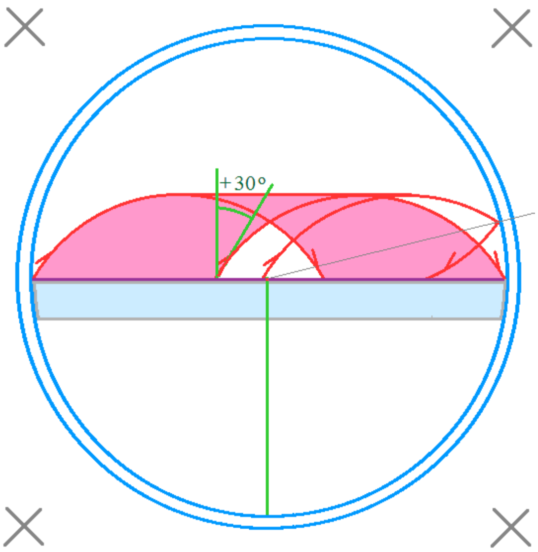
B-A 75% of the electrons of (30^o, $2v_p$) emitted from B migrate to A.

For all the electrons of (30^o, $2v_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

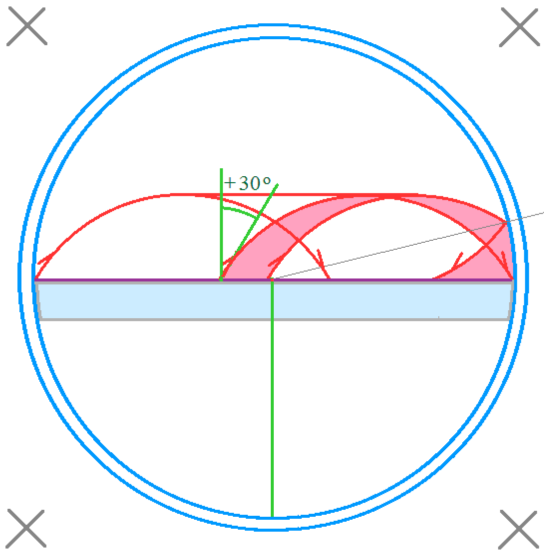
$$\{(A\text{-}B) - (B\text{-}A)\}_{30^o\ 2v_p} = 0.96 - 0.75 = 0.21$$

$$D_{30^o}(2v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{30^o\ 2v_p} \times \cos\theta = 0.21 \times 0.8660 = 0.18$$

(5) Fig 5-5 ***θ* = 30°** ***v* = 2.5*v*$_p$** ***R* = 10mm**



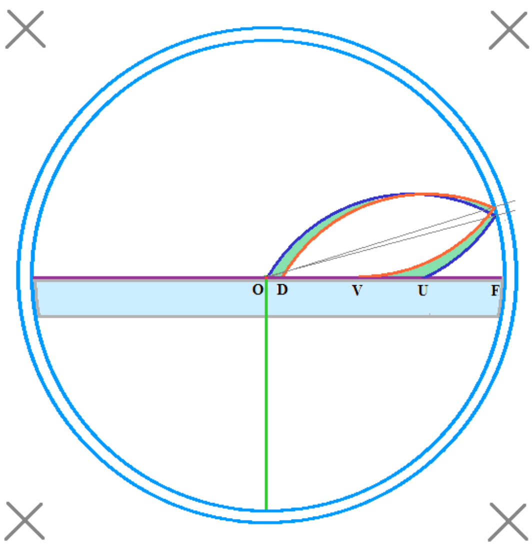
A-directly-B (pink)



A-glass-B (pink)

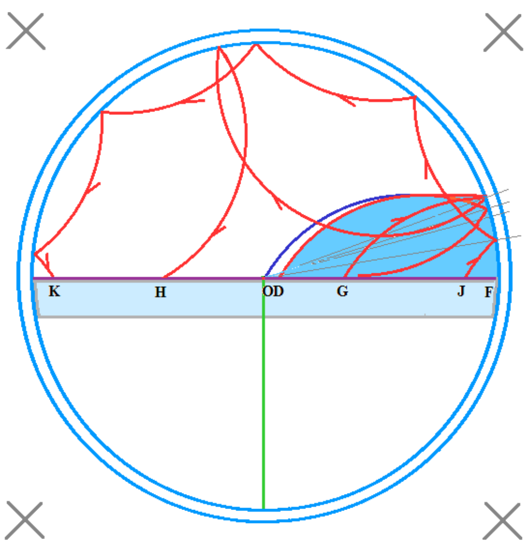
A-B = A-directly-B + A-glass-B (pink + pink) 70/70 = 1.00

A-B 100% of the electrons of (30°, 2.5v_p) emitted from A migrate to B.



B-glass-B OD/OF = 4.5/70 = 0.06

No electron migration between A and B due to these trajectories.



B-glass-A (from G to H. etc., blue.)

DF/OF = 65.5/70 = 0.94

(DF=65.5, OF =70)

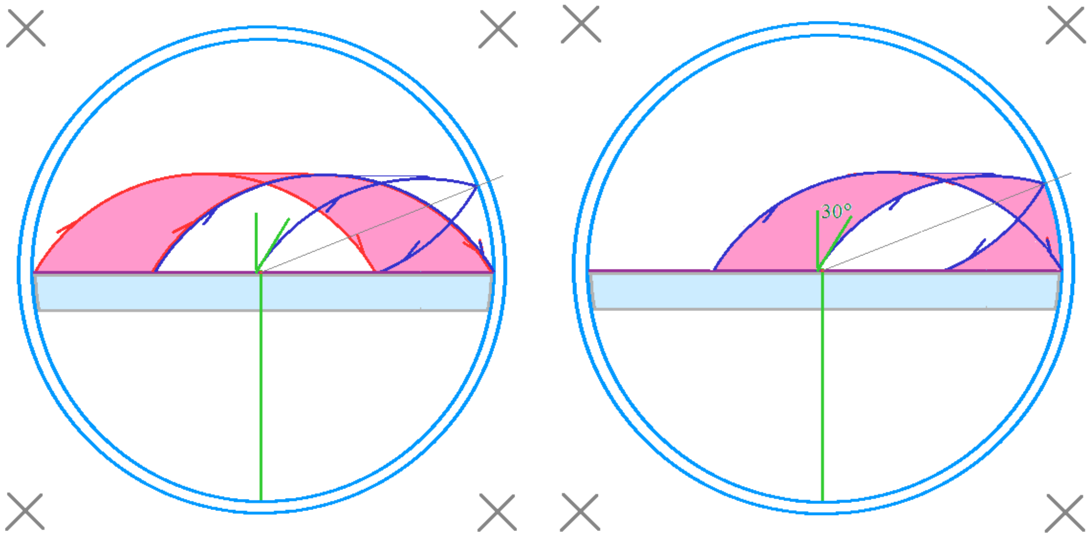
B-A 94% of the electrons of (30°, 2.5vp) emitted from B migrate to A.

For all the electrons of (30°, 2.5v_p), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{30^\circ\, 2.5v_p} = 1.00 - 0.94 = 0.06$$

$$D_{30^\circ}(2.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{30^\circ\, 2.5v_p} \times \cos\theta = 0.06 \times 0.866 = 0.05$$

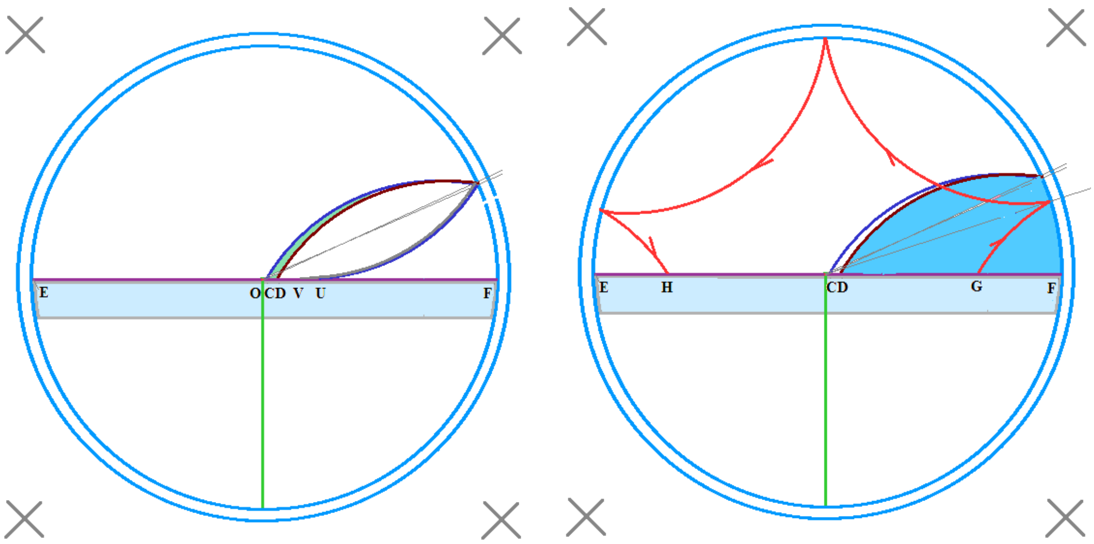
(6) Fig 5-6 $\theta = 30^{o}$ $v = 3v_p$ $R = 12\text{mm}$



(a) A-directly-B (pink) (b) A-glass-B (pink)

A-B = A-directly-B + A-glass-B (pink + pink) 70/70 = 1.00

A-B 100% of the electrons of (30^{o}, $3v_p$) emitted from A migrate to B.



(c) B-glass-B 3/70 = 0.04 (CD = 3)

No electron migration between A and B due to these trajectories.

(d) B-glass-A 67/70 = 0.96 (DF = 67)

(from G to H, etc., blue)

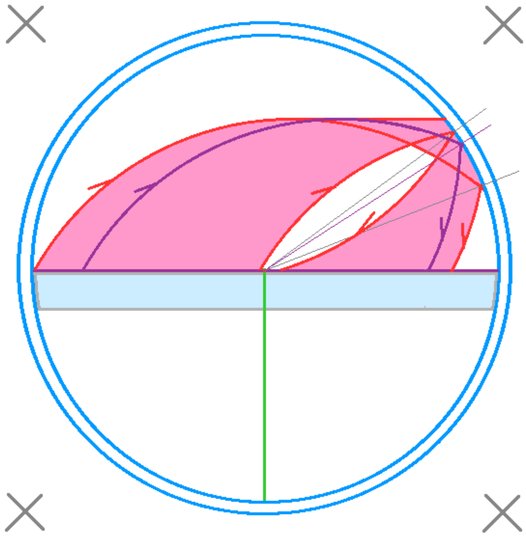
B-A 96% of the electrons of (30^{o}, $3v_p$) emitted from B migrate to A.

For all the electrons of (30^{o}, $3v_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{30^{o}\,3v_p} = 1.00 - 0.96 = 0.04$$

$$D_{30^{o}}(3v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{30^{o}\,3v_p} \times \cos\theta = 0.04 \times 0.8660 = 0.03$$

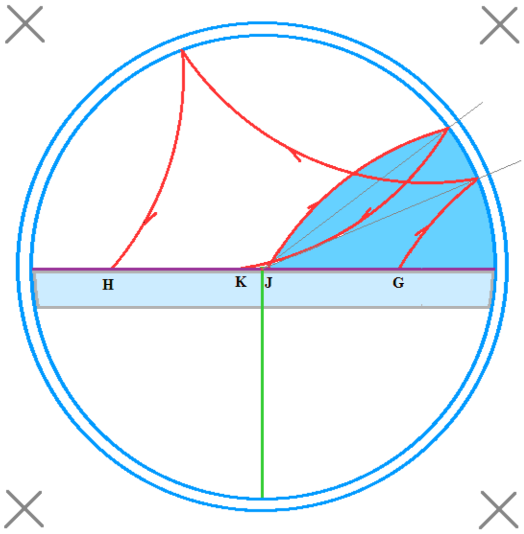
(7) Fig 5-7 $\theta = 30^{o}$ $v = 4.5v_p$ $R = 18\text{mm}$



A-glass-B

70/70 = 1.00 (pink)

A-B 100% of the electrons of (30^{o}, $4.5v_p$) emitted from A migrate to B.



B-glass-A

70/70 = 1.00

(from G to H, J to K, etc., blue.)

B-A 100% of the electrons of (30^{o}, $4.5v_p$) emitted from B migrate to A.

For all the electrons of (30^{o}, $4.5v_p$), A-B and B-A cancel each other, no contribution to the output current.

$$\{(\text{A-B}) - (\text{B-A})_{30^{o}\,4.5v_p} = 1.00 - 1.00 = 0$$
$$D_{30^{o}}(4.5v_p) = 0$$

Contributions of electrons of θ = 30° with different speeds, $D_{30^\circ}(v)$ = $\{(A\text{-}B) - (B\text{-}A)\}_{30^\circ}\cos\theta \sim v$, table 5 (1).

Fig 5 (1) is the corresponding graph.

θ = 30°	$\{(A\text{-}B) - (B\text{-}A)\}_{30^\circ}$	$D_{30^\circ}(v)$
Fig 5-1 $v = 0.5v_p$	0.25 – 0.22 = 0.03	0.03
Fig 5-2 $v = v_p$	0.49 - 0.41 = 0.08	0.07
Fig 5-3 $v = 1.5v_p$	0.73 - 0.57 = 0.16	0.14
Fig 5-4 $v = 2v_p$	0.96 - 0.75 = 0.21	0.18
Fig 5-5 $v = 2.5v_p$	1.00 – 0.94 = 0.06	0.05
Fig 5-6 $v = 3v_p$	1.00 – 0.96 = 0.04	0.03
Fig 5-7 $v = 4.5v_p$	1.00 - 1.00 = 0	0

Tab 5 (1) Contributions of electrons of θ = 30° with different speeds.

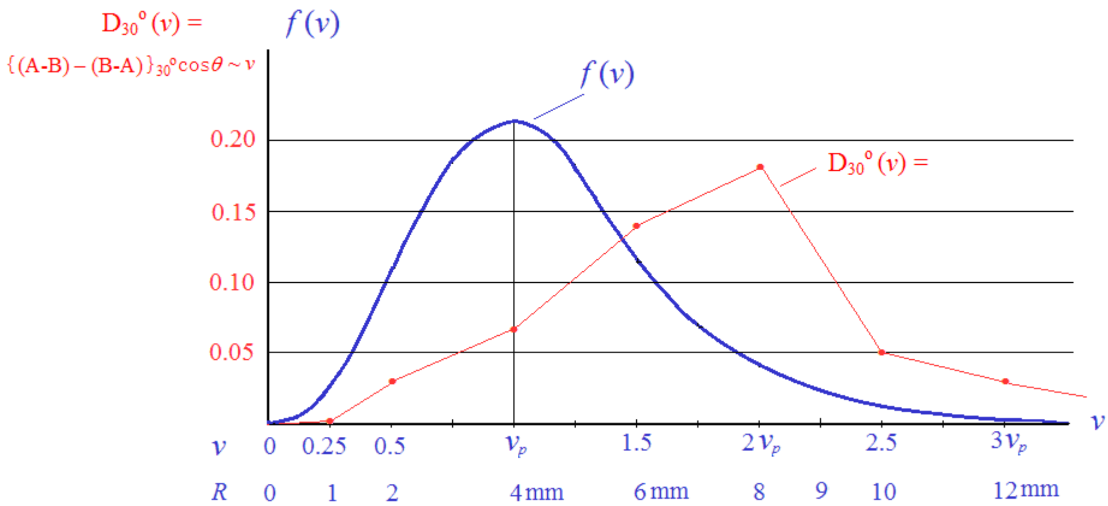


Fig. 5 (1) Graph of $D_{30^\circ}(v) = \{(A\text{-}B) - (B\text{-}A)\}_{30^\circ}\cos\theta \sim v$

Take Maxwell's speed distribution into account to derive the actual contributions of the thermal electrons of θ = -30° with respect to speeds, i.e., $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{30^\circ}(v)$, see Tab 5 (2).

Fig 5 (2) is the corresponding graph.

Speed range	$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N}$	$D_{-45}{}^{o}(v)$	$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-45}{}^{o}(v)$
$0.00 \sim 0.25 v_p$	$A_0 \approx 0.004$	≈ 0	≈ 0
$0.25 \sim 0.75 v_p$	$A_1 = 18.74\%$	0.03	0.4722
$0.75 \sim 1.25 v_p$	$A_2 = 39.83\%$	0.07	2.7881
$1.25 \sim 1.75 v_p$	$A_3 = 26.71\%$	0.14	3.7394
$1.75 \sim 2.25 v_p$	$A_4 = 8.82\%$	0.18	1.5876
$2.25 \sim 2.75 v_p$	$A_5 = 1.58\%$	0.05	0.079
$2.75 \sim 3.25 v_p$	$A_6 = 0.16\%$	0.03	0.0048
$3.25 v_p \sim \infty$	$A_7 A_8 \approx 0.0099\%$	≈ 0	0
			$\sum_v \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{30}{}^{o}(v) = 8.6711$

Tab 5 (2) $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{30}{}^{o} \sim v$.

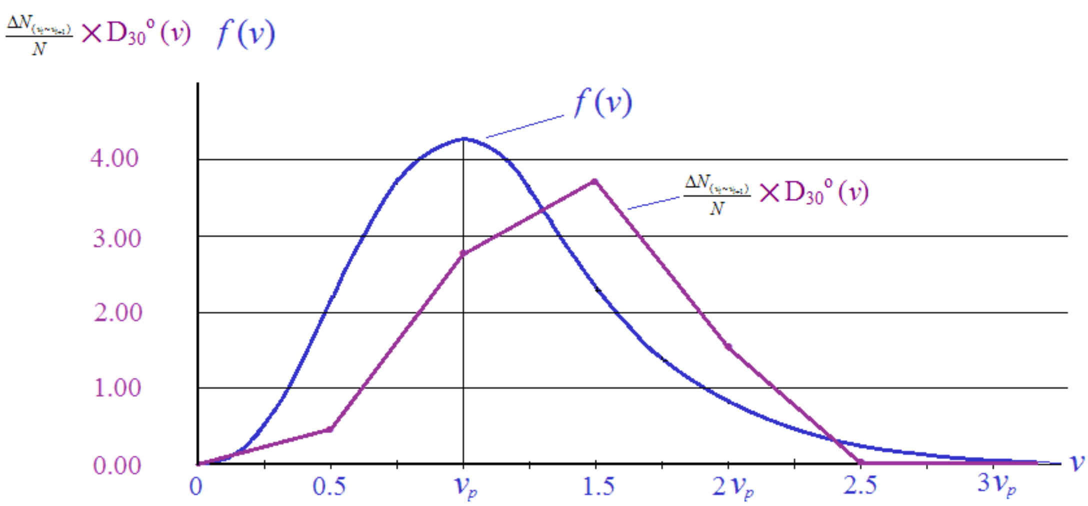
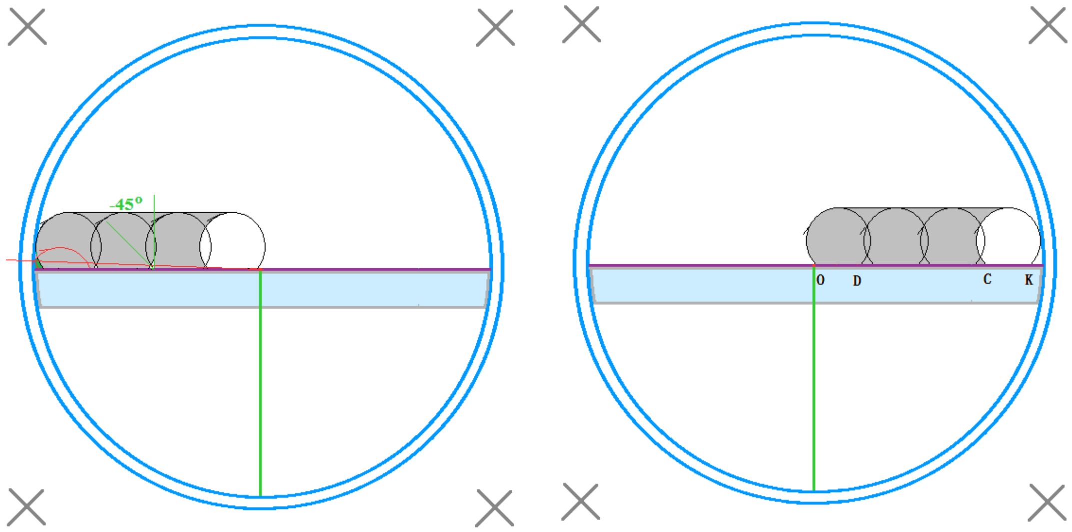


Fig 5 (2) Graph of $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{30}{}^{o} \sim v$.

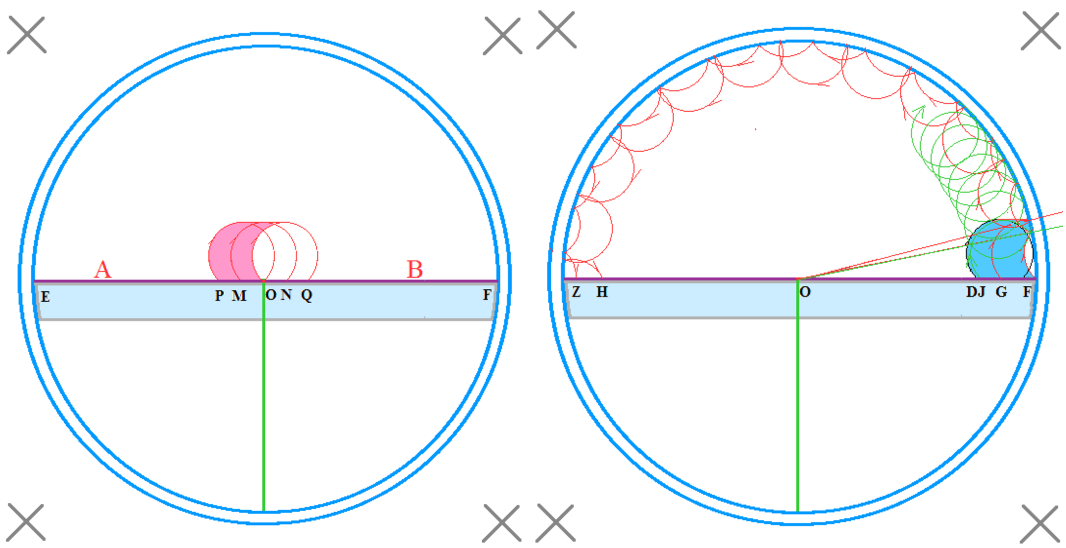
6. Trajectories of electrons of θ = - 45° and different speeds

(1) Fig 6-1 θ = - 45° $v = 0.5v_p$ R = 2mm (B = 1.34 gauss)



A-glass-A (green) & A-directly-A (grey) B-directly-B (grey) (53/70 = 0.96)

No electron migration between A and B due to these trajectories.



A-directly-B 15/70 = 0.21
(PO = 15, EO =70)
(from PO to OQ, pink)

B-glass-A 18/70 = 0.26
(DF=18, OF=70)
(from G to H, F to L, etc., blue.)

A-B 21% of the electrons of (- 45°, $0.5v_p$) emitted from A migrate to B.

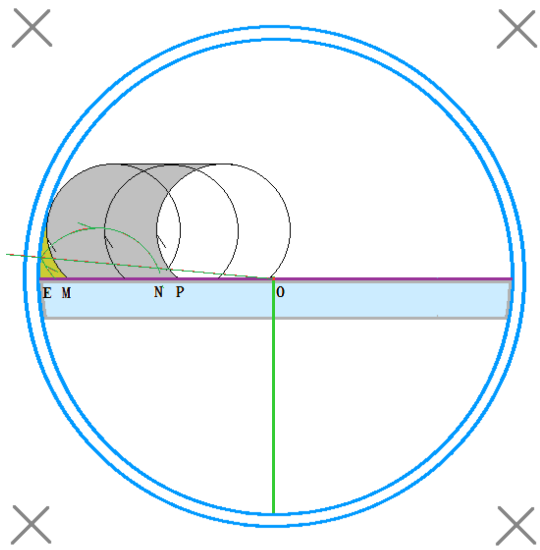
B-A 26% of the electrons of (- 45°, $0.5v_p$) emitted from B migrate to A.

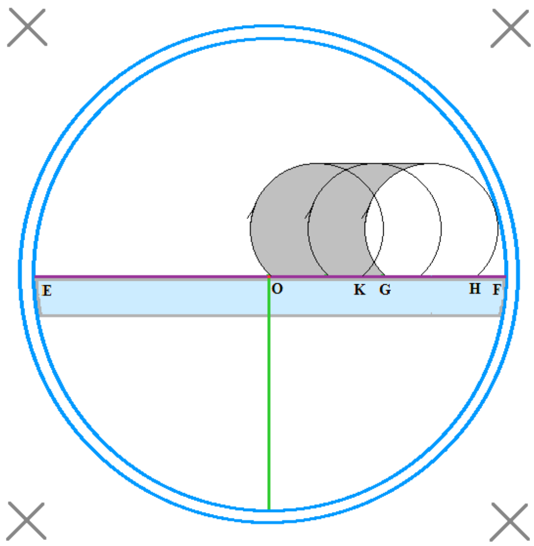
For all the electrons of (θ =-45°, $v = 0.5v_p$), A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

$$\{(A\text{-}B) - (B\text{-}A)\}_{-45^{\circ}\ 0.5v_p} = 0.21 - 0.26 = -0.05$$

$$D_{-45^{\circ}}(0.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-45^{\circ}\ 0.5v_p} \times \cos 45^{\circ} = -0.05 \times 0.7071 = -0.04$$

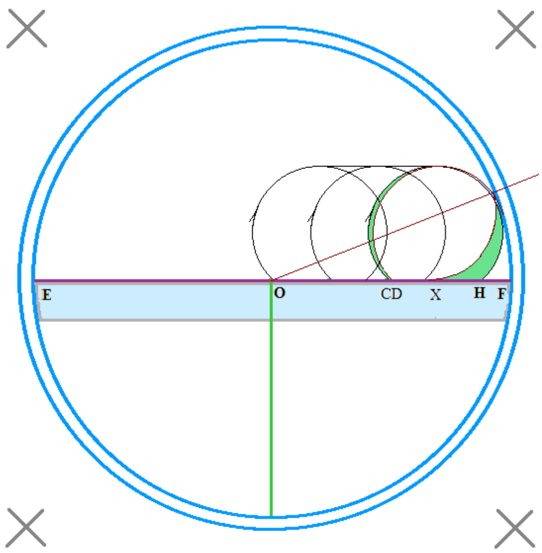
(2) Fig 6-2 θ = - 45° $v = v_p$ R = 4mm (cos -45° = 0.7071)





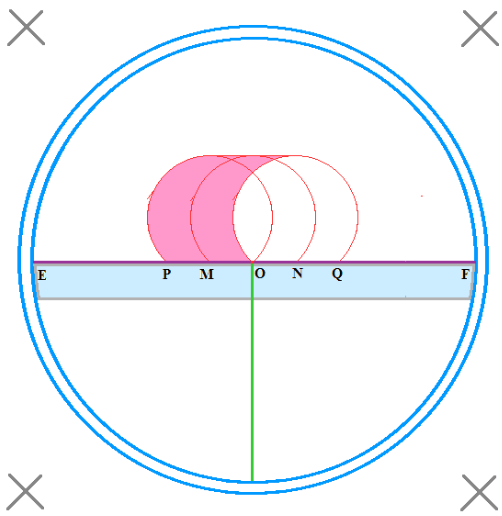
A-glass-A & A-directly-A B-directly-B (from O to K, G to H, etc.)
(green) (grey) (grey)

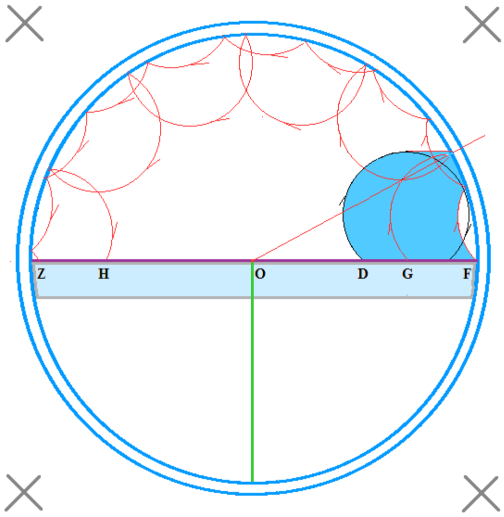
No electron migration between A and B due to these trajectories.



B-glass-B (from CD to HX)
CD = 0.8 (green) 0.8/70 = 0.01

No electron migration between A and B due to these trajectories.





A-directly-B 28/70 = 0.40
(PO = 28, EO =70) (from PO to OQ)
(pink)

B-glass-A 35/70 = 0.50
(DF = 36, OF = 70) (from G to H, etc.)
(blue)

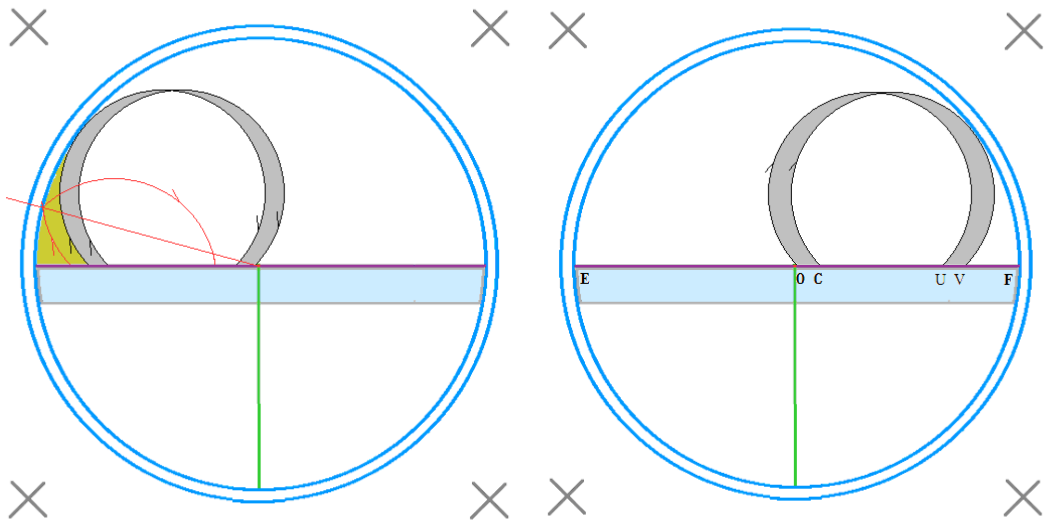
A-B 40% of the electrons of (- 45°, v_p) emitted from A migrate to B.
B-A 51% of the electrons of (- 45°, v_p) emitted from B migrate to A.

For all the electrons of (- 45°, v_p), A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

$$\{(A\text{-}B) - (B\text{-}A)\}_{\theta = -45^{o} \quad v = 1v_p} = 0.40 - 0.50 = -0.10$$

$$D_{-45}{}^{o}(v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-45^{o} \; 1v_p} \cos 45^{o} = -0.10 \times 0.7071 = -0.07$$

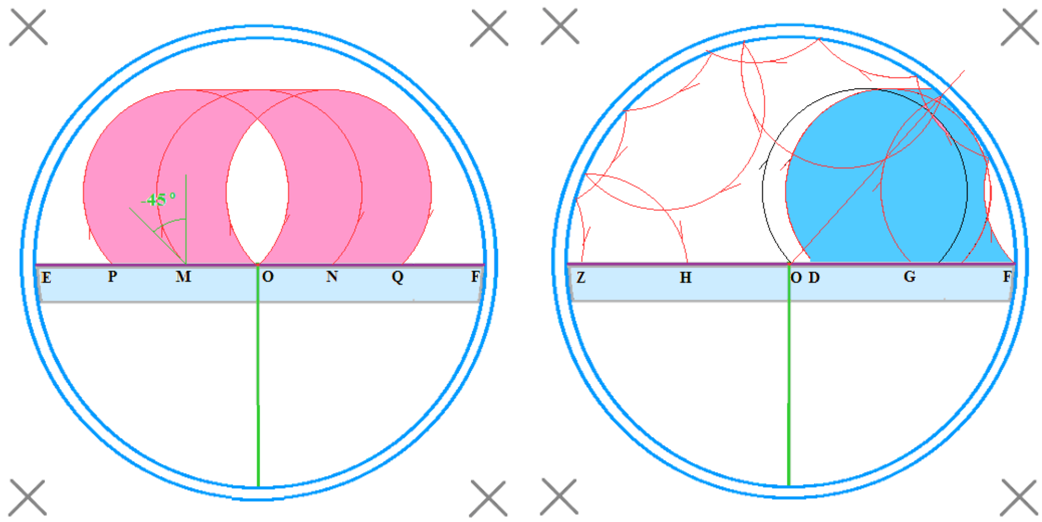
(3) Fig 6-3 $\theta = -45^{o}$ $v = 1.5v_p$ $R = 6\text{mm}$



A-glass-A (17/70) (green)
A-directly-A (7/70) (grey)

B-directly-B
(8/70 = 0.11) (OC = 8, grey)

No electron migration between A and B due to these trajectories.



A-directly-B 46/70 = 0.66
(PO = 46, EO =70)
(from P to O, M to N, O to Q, etc., pink)

B-glass-A 62/70 = 0.86
(DF = 62, OF =70)
(from G to H, F to Z, etc., blue)

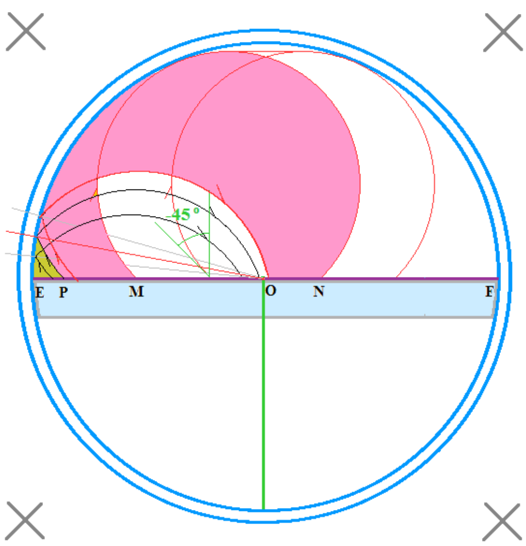
A-B 66% of the electrons of (-45^{o}, $1.5v_p$) emitted from A migrate to B.
B-A 86% of the electrons of (-45^{o}, $1.5v_p$) emitted from B migrate to A.

For all the electrons of (-45^{o}, $1.5v_p$), A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

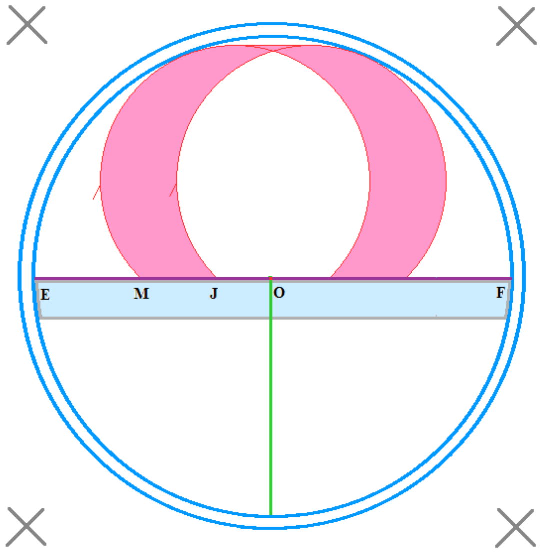
$$\{(A\text{-}B) - (B\text{-}A)\}_{-45^{o}\ 1.5v_p} = 0.66 - 0.86 = -0.20$$

$$D_{-45^{o}}(1.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-45^{o}\ 1.5v_p} \cos 45^{o} = -0.20 \times 0.7071 = -0.14$$

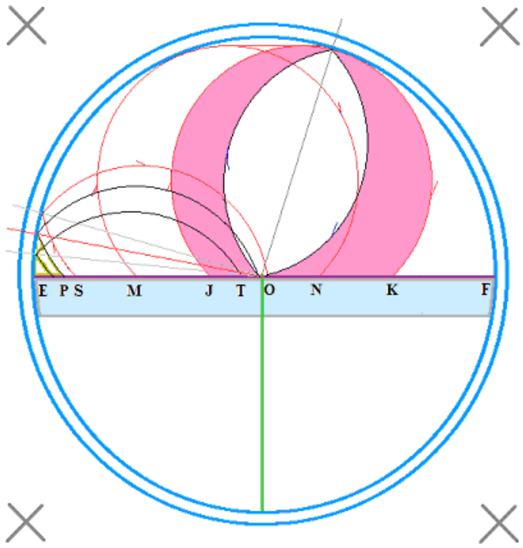
(4) Fig 6-4 θ = - 45° $v = 2v_p$ R = 8mm



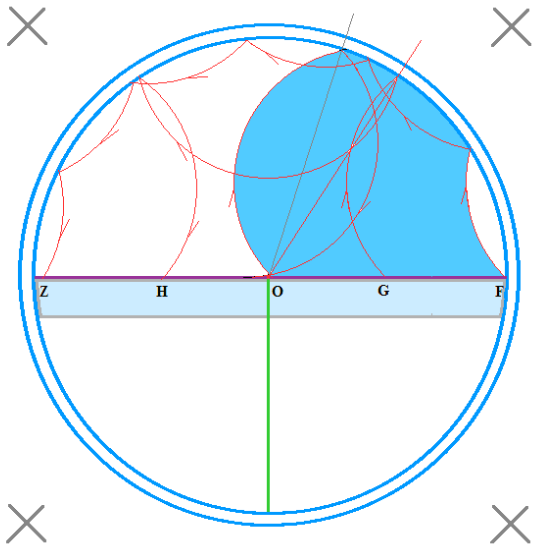
A-glass-A 9/70 = 0.13 (green, EP = 9)
A-glass-B 22/70 = 0.31 (pink, PM =22)



A-directly-B
23/70 = 0.32 (pink, MJ = 23, OE =70)



A-glass-B 16/70 = 0.23 (JO =16, pink)
(from JO to KO)



B-glass-A 70/70 = 1.00
(from G to H, etc., blue, OF=70)

A-B=A-directly-B+A-glass-B 61/70=0.87
(PO = PM + MJ + JO = 22 + 23 + 16 = 61)

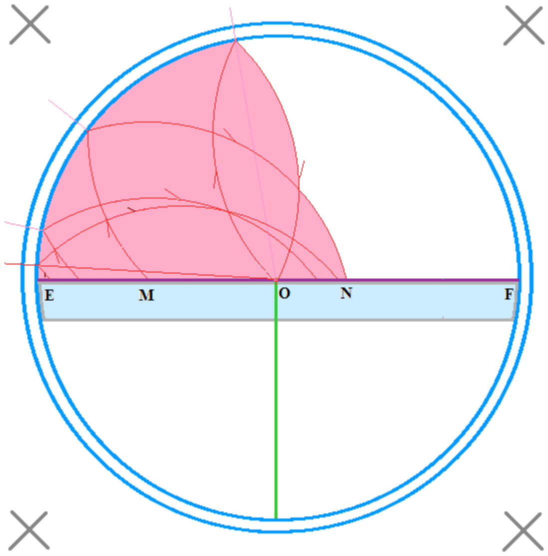
A-B 87% of the electrons of (- 45°, $2v_p$) emitted from A migrate to B.
B-A 100% of the electrons of (- 45°, $2v_p$) emitted from B migrate to A.

For all the electrons of (- 45°, $2v_p$), A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

$$\{(A\text{-}B) - (B\text{-}A)\}_{\theta = -45^{o}\ \ v = 2v_p} = 0.87 - 1.00 = -0.13$$

$$D_{-45^{o}}(2v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-45^{o}\,2v_p}\ cos\ 45^{o} = -0.13 \times 0.7071 = -0.09$$

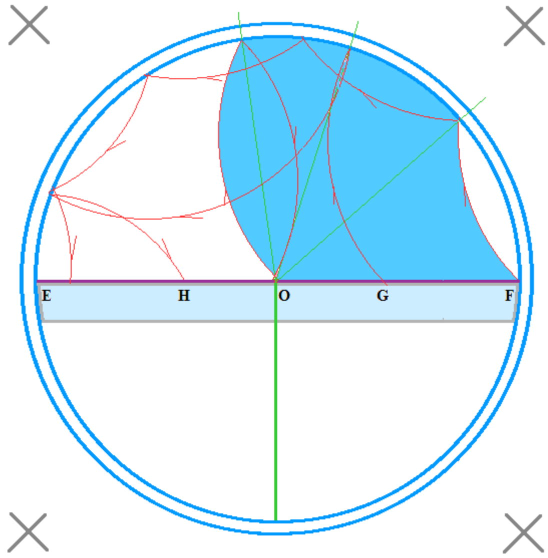
(5) Fig 6-5 θ = -45° $v = 2.5v_p$ R = 10mm



A-glass-B 70/70 = 1.00

(from M to N, etc., pink)

A-B 100% of the electrons of (- 45°, $2.5v_p$) emitted from A migrate to B.



B-glass-A 70/70 = 1.00

(from G to H, etc., blue)

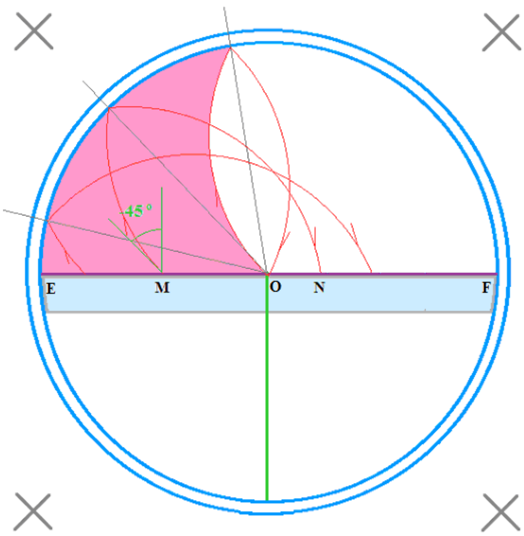
B-A All the electrons of (- 45°, $2.5v_p$) emitted from B migrate to A.

For all the electrons of (- 45°, $2v_p$), A-B and B-A cancel each other, no contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{\theta = -45^{\circ}\ \ v = 2.5v_p} = 1.00 - 1.00 = 0$$

$$D_{-45^{\circ}}(2.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-45^{\circ}\ 2.5v_p} \times \cos 45^{\circ} = 0$$

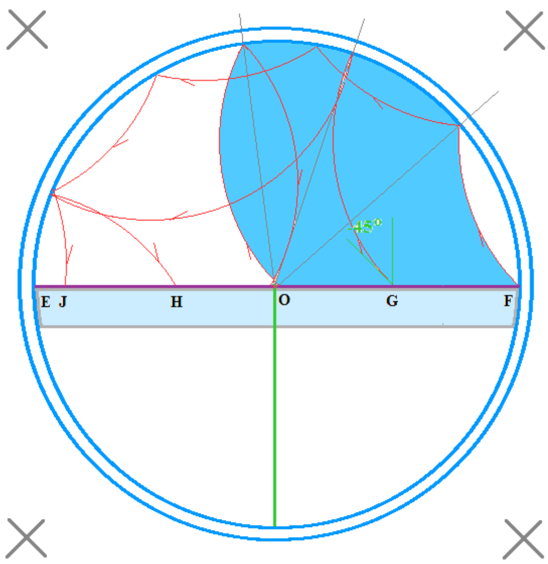
(6) Fig 6-6 θ = -45^{o} $v = 3v_p$ R = 12mm



A-glass-B 70/70 = 1.00

(from M to N, etc., pink)

A-B All of the electrons of (- 45^{o}, $3v_p$) emitted from A migrate to B.



B-glass-A 70/70 = 1.00

(from G to H, F to J, O to O, etc., blue)

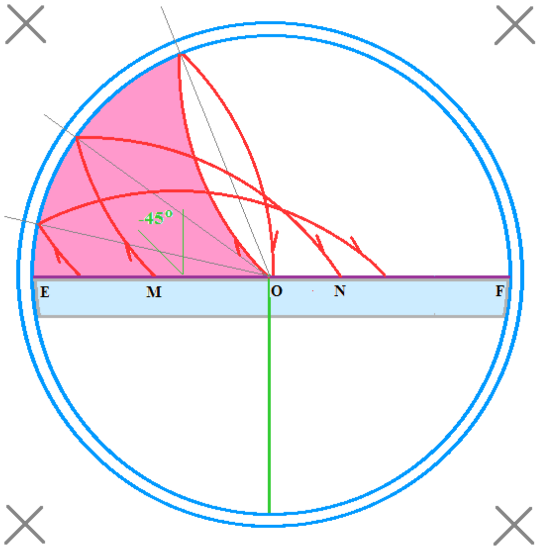
B-A All of the electrons of (- 45^{o}, $3v_p$) emitted from B migrate to A.

For all the electrons of (- 45^{o}, $3v_p$), A-B and B-A cancel each other, no contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{\ \theta = -45^{o}\ v = 3v_p} = 1.00 - 1.00 = 0$$

$$D_{-45^{o}}(3v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{\ -45^{o}\ 3v_p}\ cos\ 45^{o} = 0$$

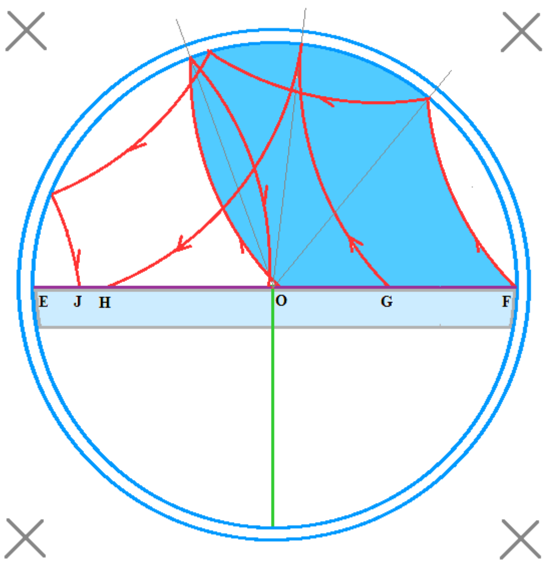
(7) Fig 6-7 θ = -45° $v = 4.5v_p$ R = 18mm



A-glass-B 70/70 = 1.00

(from M to N, etc., pink)

A-B All the electrons of (- 45°, $4.5v_p$) emitted from A migrate to B.



B-glass-A 70/70 = 1.00

(from G to H, F to J, etc., blue)

B-A All the electrons of (- 45°, $4.5v_p$) emitted from B migrate to A.

For all the electrons of (- 45°, $4.5v_p$), A-B and B-A cancel each other, no contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{\theta = -45^o \quad v = 4.5v_p} = 1.00 - 1.00 = 0$$

$$D_{-45^o}(4.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-45^o\ 4.5v_p} \cos 45^o = 0$$

List the contributions of electrons of $\theta = -45^{o}$ with different speeds in table 6(1). Fig 6 (1) is the corresponding graph, $D_{-45^{o}}(v) \sim v$.

$\theta = -45^{o}$	$\{(A\text{-}B) - (B\text{-}A)\}_{-45^{o}}$	$D_{-45^{o}}(v)$
Fig 6-1 $v = 0.5v_p$	0.21 – 0.26 = - 0.05	- 0.04
Fig 6-2 $v = v_p$	0.40 - 0.51 = - 0.11	- 0. 078
Fig 6-3 $v = 1.5v_p$	0.66 – 0.89 = - 0.23	- 0.16
Fig 6-4 $v = 2v_p$	0.87 – 1.00 = - 0.13	- 0.09
Fig 6-5 $v = 2.5v_p$	1.00 – 1.00 = 0	0
Fig 6-6 $v = 3v_p$	1.00 – 1.00 = 0	0
Fig 6-7 $v = 4.5v_p$	1.00 - 1.00 = 0	0

Tab 6(1) $D_{-45^{o}}(v) = (A\text{-}B) - (B\text{-}A)\}_{\theta=-45^{o}} \cos\theta \sim v$.

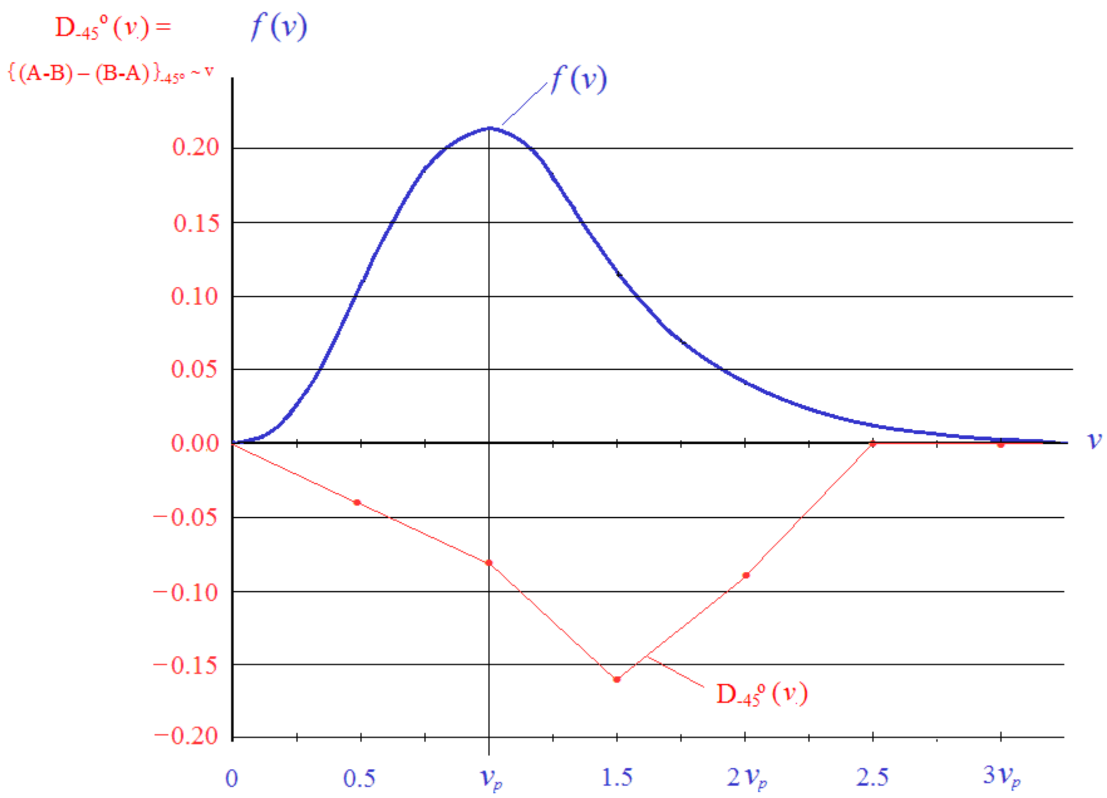


Fig 6 (1) The graph of $D_{-45^{o}}(v) = \{(A\text{-}B) - (B\text{-}A)\}_{\theta=-45^{o}} \cos\theta \sim v$.

Take Maxwell's speed distribution into account to derive the actual contributions of the thermal electrons of $\theta = -45^{o}$ with different speeds, i.e., $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-45^{o}}(v) \sim v$.

Fig 6 (2) is the corresponding graph.

Speed range	$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N}$	$D_{-45^{o}}(v)$	$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-45^{o}}(v)$
$0.00 \sim 0.25v_p$	$A_0 \approx 0.004$	≈ 0	≈ 0
$0.25 \sim 0.75v_p$	$A_1 = 18.74\%$	-0.04	- 0.7496
$0.75 \sim 1.25v_p$	$A_2 = 39.83\%$	-0.08	- 3.1864
$1.25 \sim 1.75v_p$	$A_3 = 26.71\%$	-0.16	- 4.2736
$1.75 \sim 2.25v_p$	$A_4 = 8.82\%$	-0.09	- 0.7938
$2.25 \sim 2.75v_p$	$A_5 = 1.58\%$	0	0
$2.75 \sim 3.25v_p$	$A_6 = 0.16\%$	0	0
$3.25v_p \sim \infty$	$A_7A_8 \approx 0.0099\%$	0	0
			$\Sigma_v \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-45^{o}}(v) = -9.0034$

Tab. 6 (2) $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-45^{o}} \sim v$

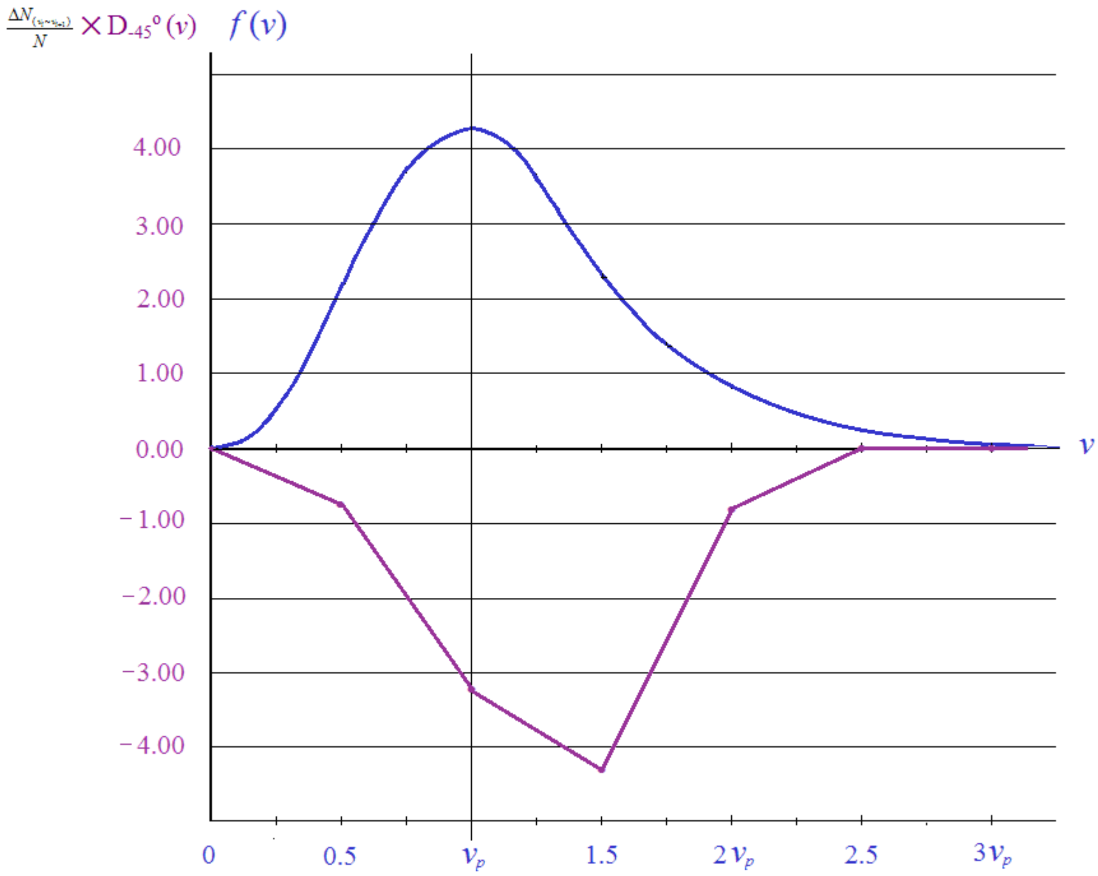
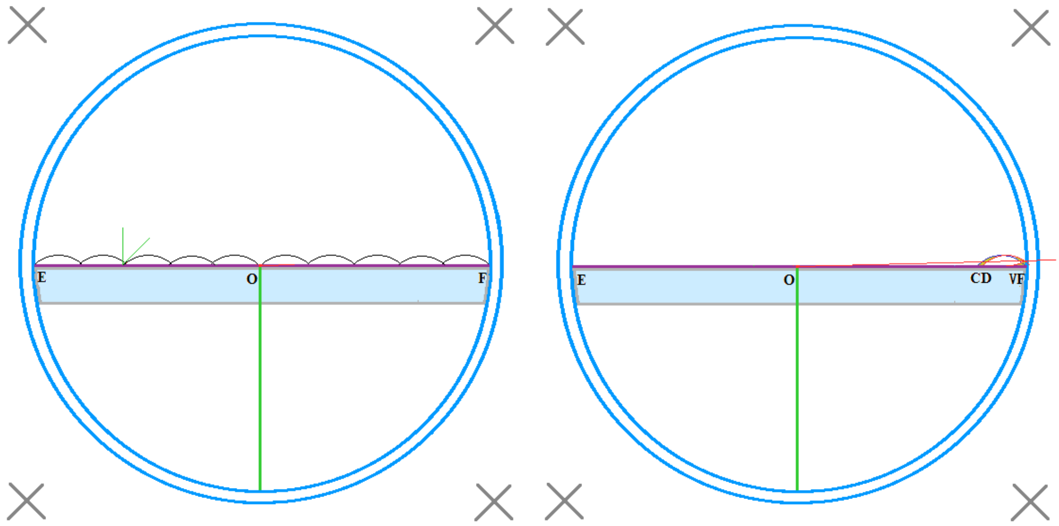


Fig 6 (2) Graph of $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-45}{}^{o} \sim v$

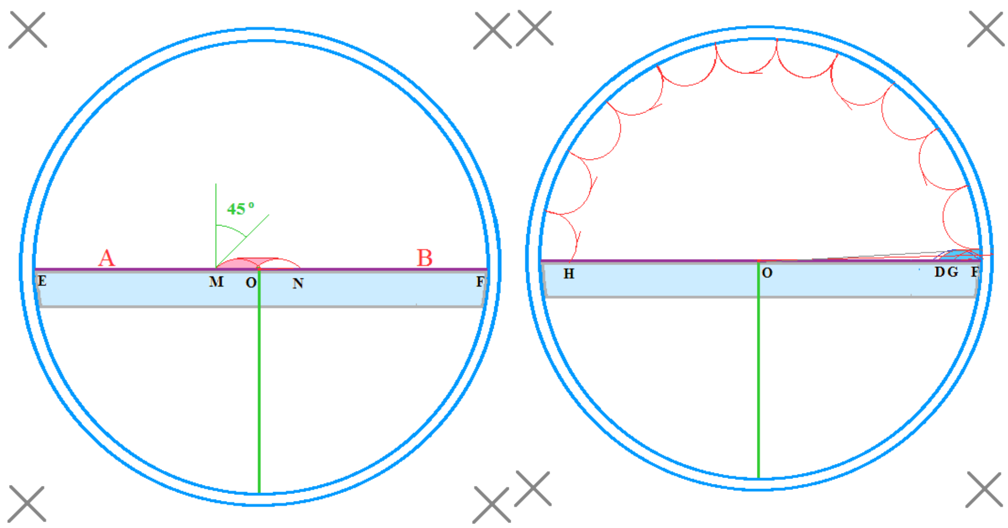
7. Trajectories of electrons of θ = 45° and different speeds

(1) Fig 7-1 $\theta = 45^{o}$ $v = 0.5v_p$ R = 2mm (B = 1.34 gauss)



A-directly-A B-directly-B B-glass-B 1.5/70 = 0.02 (CD = 1.5)

No electron migration between A and B due to these trajectories.



A-directly-B MO/EO = 14/70 = 0.20 B-glass-A DF/OF=12.5/70=0.18

(MO = 12.5 EO=70) (DF=12.5 OF = 70)

A-B 20% of the electrons of (45°, $0.5v_p$) emitted from A migrate to B.

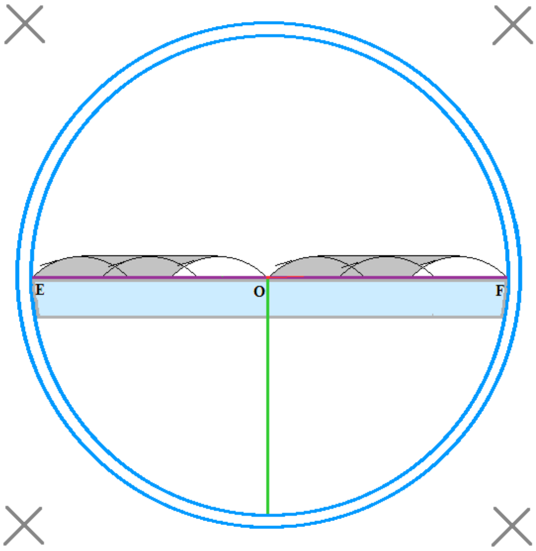
B-A 18% of the electrons of (45°, $0.5v_p$) emitted from B migrate to A.

For all the electrons of (45°, $0.5v_p$), A-B exceeds B-A, and their difference is the contribution to the output current.

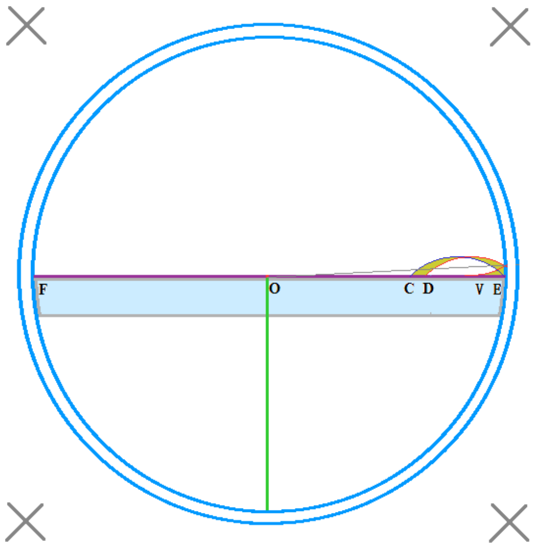
$$\{(A\text{-}B) - (B\text{-}A)\}_{45^{o}\,0.5v_p} = 0.20 - 0.18 = 0.02$$

$$D_{45^{o}}(0.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-45^{o}\,0.5v_p} \cos 45^{o} = 0.02 \times \cos 45^{o} = 0.014$$

(2) Fig 7-2 $\theta = 45^{o}$ $v = v_p$ $R = 4\text{mm}$ ($cos45^{o} = 0.7071$)

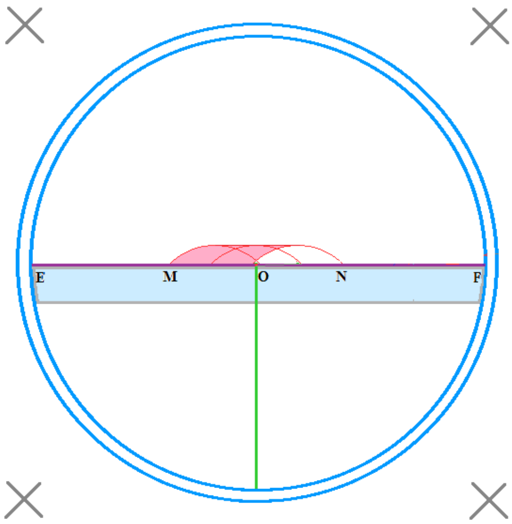


A-directly-A B-directly-B

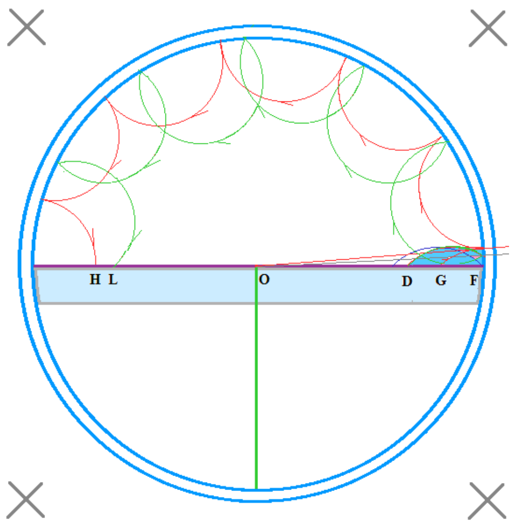


B-glass-B

No electron migration between A and B due to these trajectories.



A-directly-B
(MO = 28, pink)
MO/EO=28/70=0.40



B-glass-A
(DF = 23.5, blue)
DF/OF=23.5/70 = 0.34

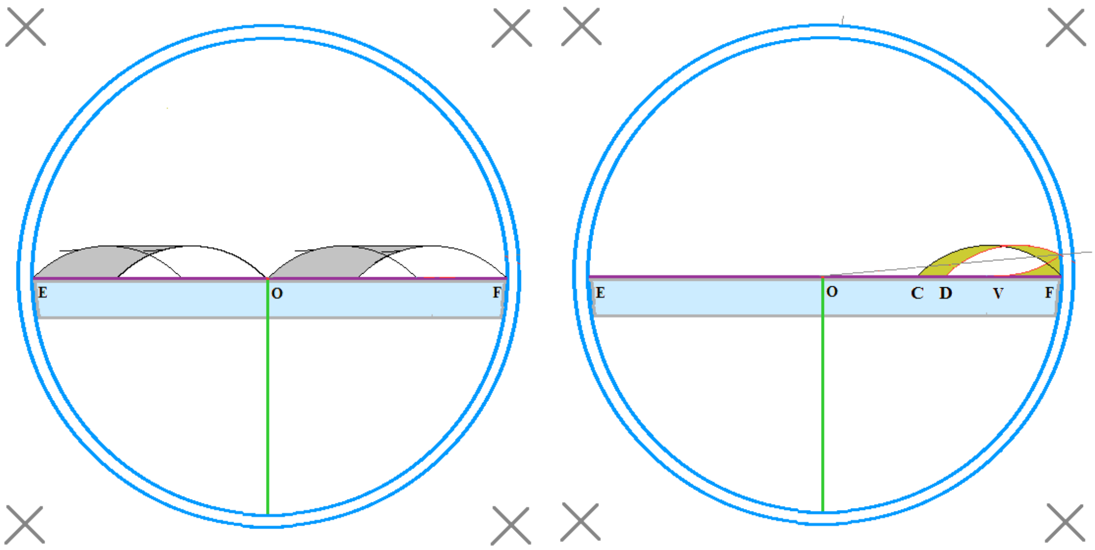
A-B 40% of the electrons of (45^{o}, v_p) emitted from A migrate to B.
B-A 34% of the electrons of (45^{o}, v_p) emitted from B migrate to A.

For all the electrons of (45^{o}, v_p), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{45^{o}\,1v_p} = 0.40 - 0.34 = 0.06$$

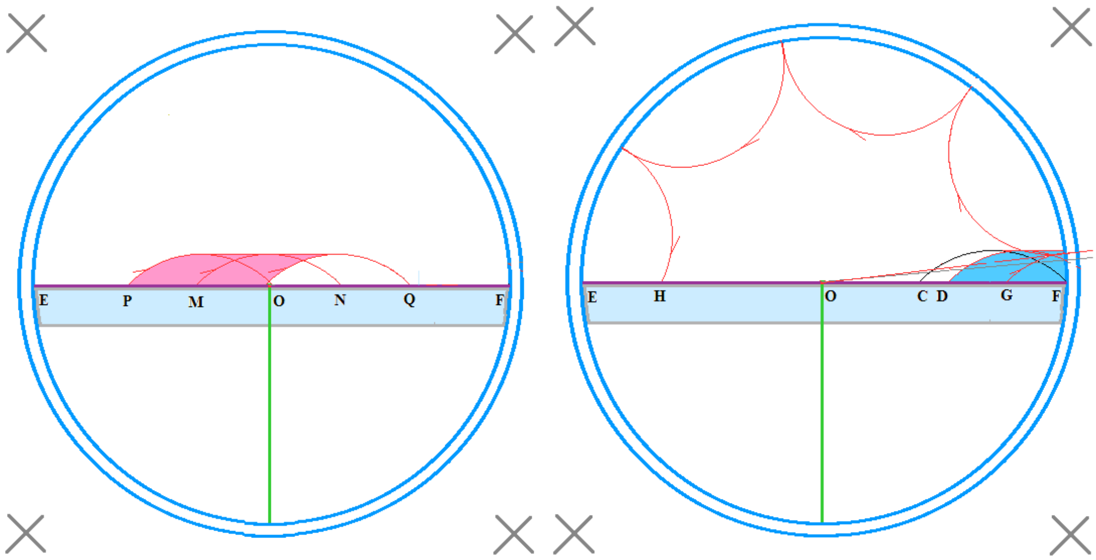
$$D_{45^{o}}(0.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{45^{o}\,1v_p}\ cos\,45^{o} = 0.06 \times 0.7071 = 0.04$$

(3) Fig 7-3 $\theta = 45^{o}$ $v = 1.5v_p$ $R = 6$mm



A-directly-A B-directly-B \ B-glass-B

No electron migration between A and B due to these trajectories.



A-directly-B PO/EO=42.5/70 = 0.61 B-glass-A DF/OF=34/70 = 0.49

(PO = 42.5, pink) (DF = 34, blue)

A-B 61% of the electrons of (45^{o}, $1.5v_p$) emitted from A migrate to B.

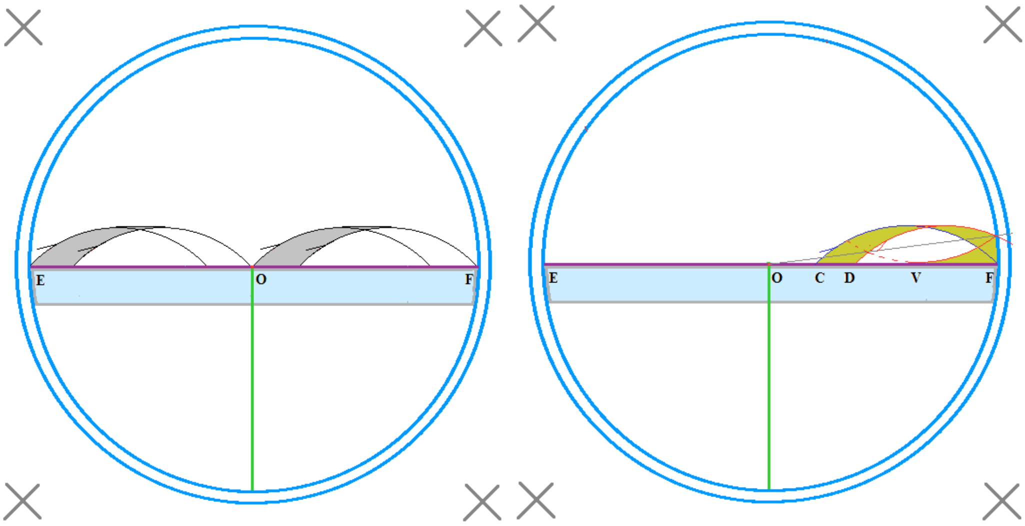
B-A 49% of the electrons of (45^{o}, $1.5v_p$) emitted from B migrate to A.

For all the electrons of (45^{o}, $1.5v_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(\text{A-B}) - (\text{B-A})\}_{45^{o}\,1.5v_p} = 0.61 - 0.49 = 0.12$$

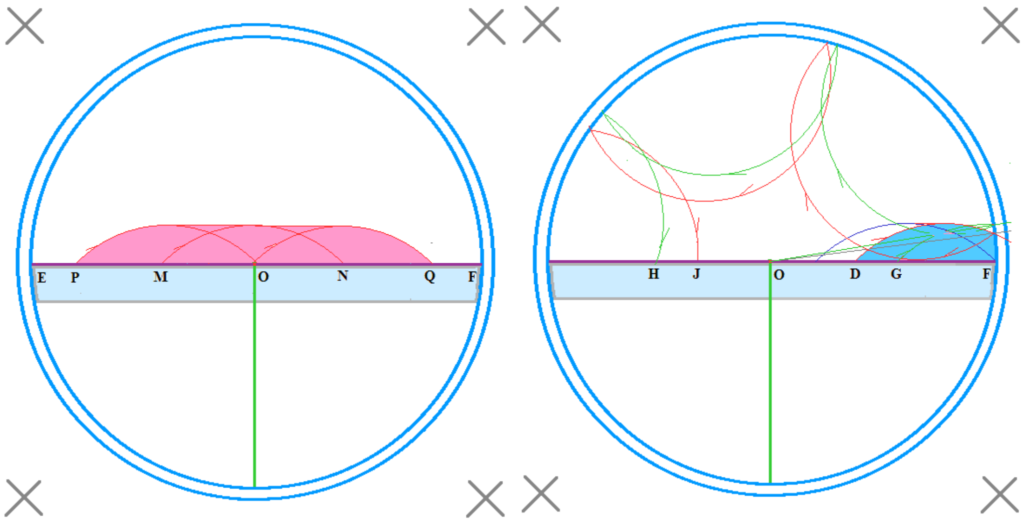
$$D_{45^{o}}(1.5v_p) = \{(\text{A-B}) - (\text{B-A})\}_{45^{o}\,1.5v_p} \cos 45^{o} = 0.12\times\cos45^{o} = 0.0849$$

(4) Fig 7-4 $\theta = 45^{\circ}$ $v = 2v_p$ $R = 8\text{mm}$



A-directly-A B-directly-B B-glass-B

No electron migration between A and B due to these trajectories.



A-directly-B PO/EO = 56.5/70 = 0.81 B-glass-A DF/OF=44/70 = 0.63

(from PO to OQ, pink) (from G to H, D to J, etc., blue)

A-B 81% of the electrons of (45°, $2v_p$) emitted from A migrate to B.

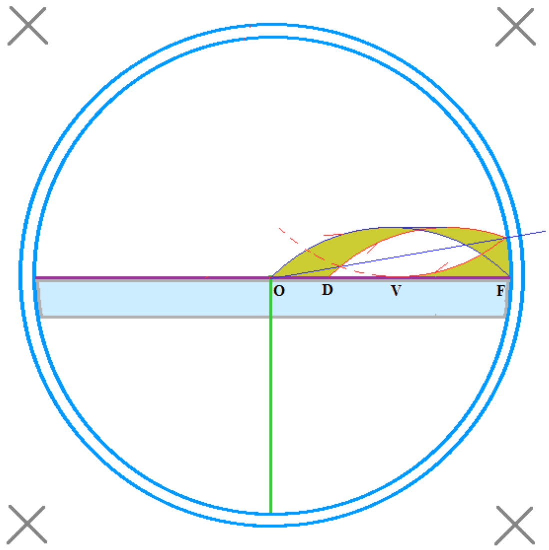
B-A 63% of the electrons of (45°, $2v_p$) emitted from B migrate to A.

For all the electrons of (45°, $2v_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(\text{A-B}) - (\text{B-A})\}_{45^{\circ}\, 2v_p} = 0.81 - 0.63 = 0.18$$

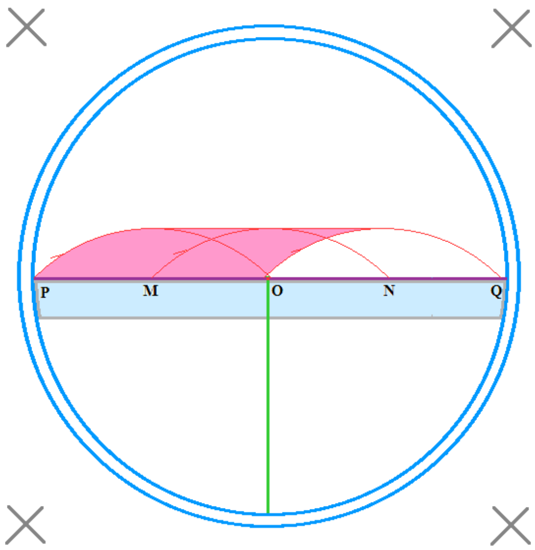
$$D_{45^{\circ}}(2v_p) = \{(\text{A-B}) - (\text{B-A})\}_{45^{\circ}\, 2v_p} \times \cos 45^{\circ} = 0.18 \times 0.7071 = 0.13$$

(5) Fig 7-5 $\theta = 45^{\circ}$ $v = 2.5v_p$ $R = 10\text{mm}$



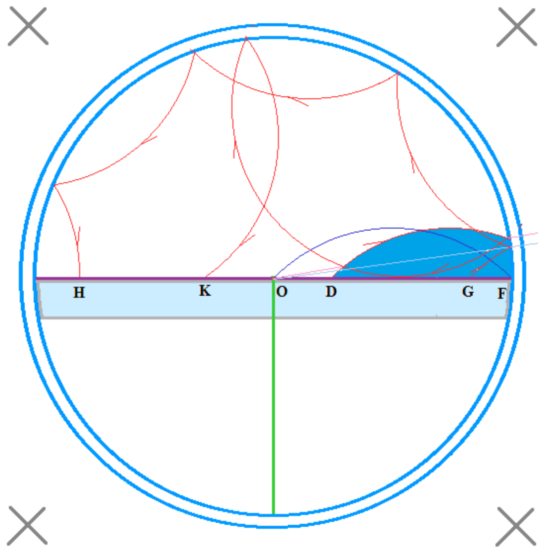
B-glass-B OD/OF= 16/70 = 0.23

No electron migration between A and B due to these trajectories.



A-directly-B PO/OQ=70/70 = 1.00

(from PO to OQ, pink)



B-glass-A DF/OF=54/70 = 0.77

(from G to H, D to K, etc., blue)

A-B 100% of the electrons of (45°, $2.5v_p$) emitted from A migrate to B.

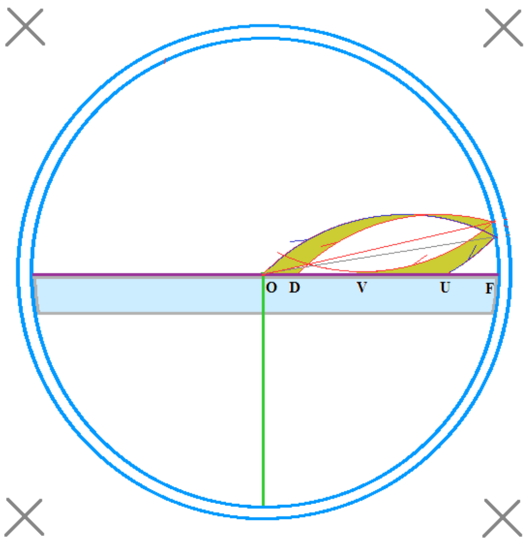
B-A 77% of the electrons of (45°, $2.5v_p$) emitted from B migrate to A.

For all the electrons of (45°, $2.5v_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(\text{A-B}) - (\text{B-A})\}_{45^{\circ}\ 2.5v_p} = 1.00 - 0.77 = 0.23$$

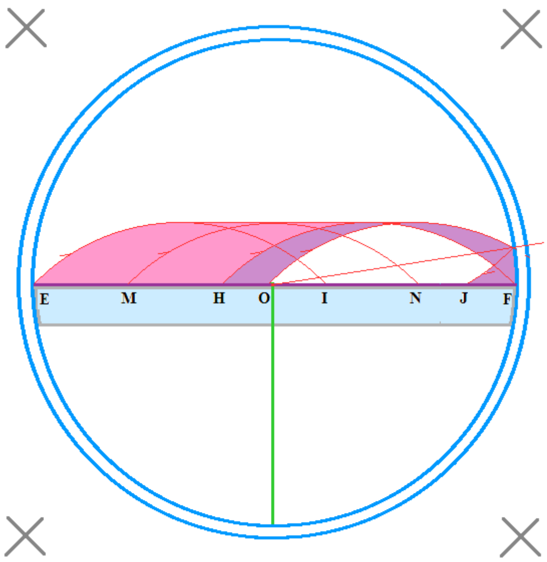
$$D_{45^{\circ}}(2.5v_p) = \{(\text{A-B}) - (\text{B-A})\}_{45^{\circ}\ 2.5v_p} \cos 45^{\circ} = 0.23 \times 0.7071 = 0.16$$

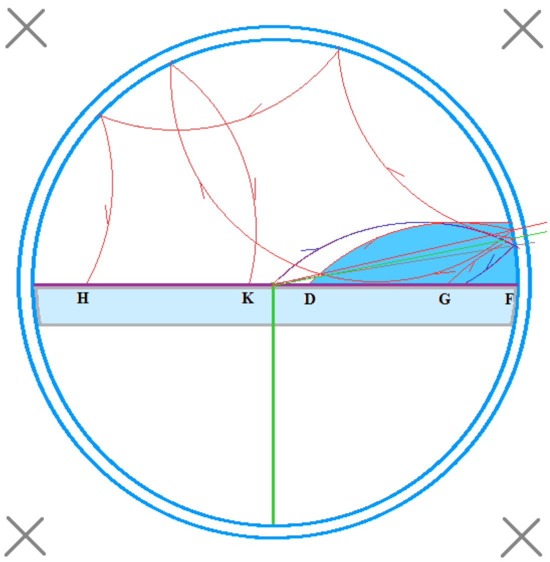
(6) Fig 7-6 $\theta = 45^{\circ}$ $v = 3v_p$ $R = 12\text{mm}$



B-glass-B 10/70 = 0.14 (OD = 10, OF =70)

No electron migration between A and B due to these trajectories.





A-directly-B (from EMH to INF, pink)

A-glass-B (from HO to FJ, violet)

A-B = A-directly-B + A-glass-B 70/70 = 1.00

(FH + HO = 70 pink + violet)

B-glass-A

60/70 = 0.86 (DF = 60, OF =70)

A-B 100% of the electrons of (45°, $3v_p$) emitted from A migrate to B.

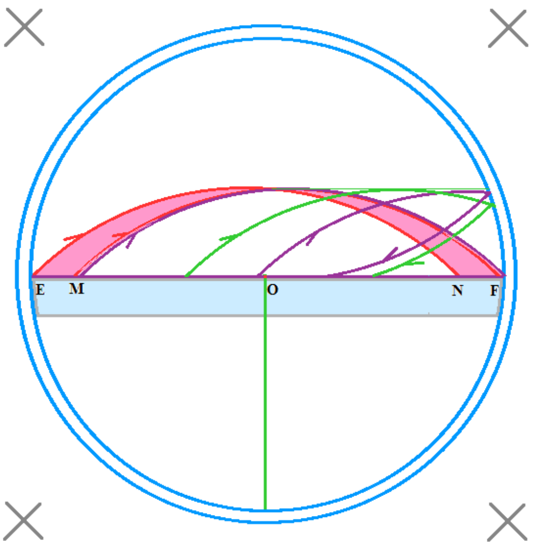
B-A 86% of the electrons of (45°, $3v_p$) emitted from B migrate to A.

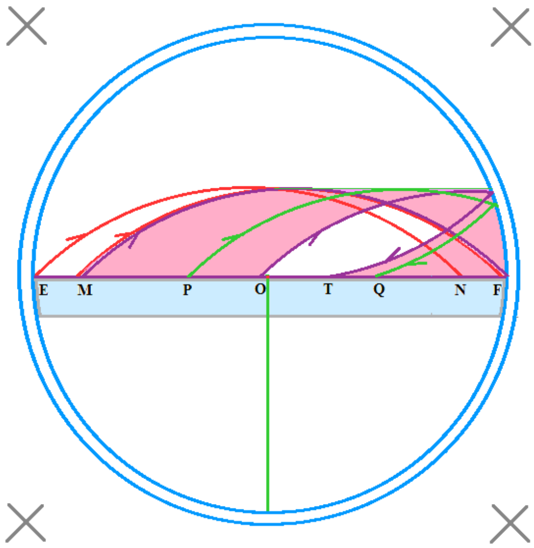
For all the electrons of (45°, $3v_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{45^{\circ}3v_p} = 1.00 - 0.86 = 0.14$$

$$D_{45^{\circ}}(3v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{45^{\circ}3v_p} \times \cos 45^{\circ} = 0.14 \times 0.7071 = 0.10$$

(7) Fig 7-7 θ = 45° $v = 4.5v_p$ R = 18mm

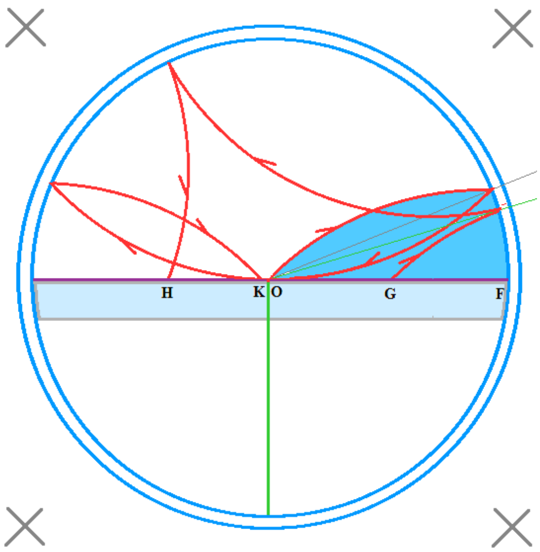




A-directly-B (pink) A-glass-B (pink)

A-B = A-directly-B + A-glass-B (pink +pink) 70/70 = 1.00

A-B 100% of the electrons of (45°, $4.5v_p$) emitted from A migrate to B.



B-glass-A 70/70 = 1.00 (from G to H, O to K, etc., blue)

B-A 100% of the electrons of (45°, $4.5v_p$) emitted from B migrate to A.

For all the electrons of (45°, $4.5v_p$), A-B equals B-A, and their contributions to the output current cancel each other.

$$(\text{A-B}) - (\text{B-A})\}_{45^{\circ}\,4.5v_p} = 1.00 - 1.00 = 0$$

$$D_{45^{\circ}}(4.5v_p) = \{(\text{A-B}) - (\text{B-A})\}_{45^{\circ}\,4.5v_p} \cos 45^{\circ} = 0$$

List the contributions of electrons of $\theta = 45^{\circ}$ with different speeds in Tab 7 (1) , $D_{45^{\circ}}(v) \sim v$. Fig 7 (1) is the corresponding graph.

$\theta = 45^{\circ}$	$\{(A\text{-}B) - (B\text{-}A)\}_{45^{\circ}}$	$D_{45^{\circ}}(v)$
Fig 7-1 $v = 0.5v_p$	0.20 – 0.18 = 0.02	0.014
Fig 7-2 $v = v_p$	0.40 – 0.34 = 0.06	0.04
Fig 7-3 $v = 1.5v_p$	0.61 – 0.49 = 0.12	0.085
Fig 7-4 $v = 2v_p$	0.81 – 0.63 = 0.18	0.13
Fig 7-5 $v = 2.5v_p$	1.00 – 0.77 = 0.23	0.16
Fig 7-6 $v = 3v_p$	1.00 – 0.86 = 0.14	0. 10
Fig 7-7 $v = 4.5v_p$	1.00 - 1.00 = 0	0

Tab 7 (1) Contributions of trajectories of electron of $\theta = 45^{\circ}$ with different speeds, $D_{45^{\circ}}(v) = \{ (A\text{-}B) - (B\text{-}A) \}_{\theta=45^{\circ}} \cos\theta \sim v$.

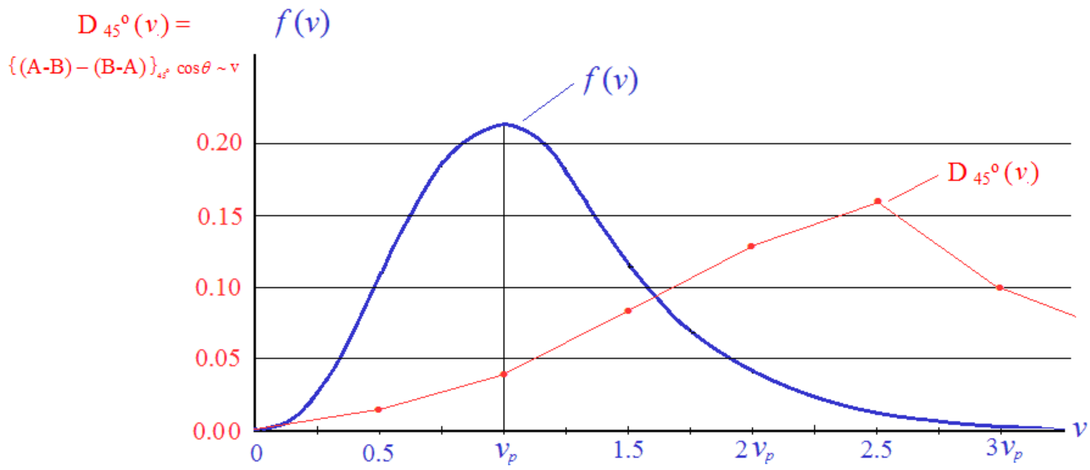


Fig 7 (1) Graph of $D_{45^{\circ}}(v) = \{ (A\text{-}B) - (B\text{-}A) \}_{\theta=45^{\circ}} \cos\theta \sim v$.

Take Maxwell's speed distribution into account to derive the actual contributions of the thermal electrons of $\theta = 45^{\circ}$ with different speeds, i.e., $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{45^{\circ}}(v) \sim v$, as shown in Tab 7 (2).

And Fig 7 (2) is the corresponding graph.

Speed range	$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N}$	$D_{45^o}(v)$	$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{45^o}(v)$
$0.00 \sim 0.25v_p$	$A_0 \approx$ 0. 004	≈ 0	$\approx$ 0
$0.25 \sim 0.75v_p$	$A_1 =$ 18. 74%	0.014	0.2624
$0.75 \sim 1.25v_p$	$A_2 =$ 39. 83%	0.04	1.5932
$1.25 \sim 1.75v_p$	$A_3 =$ 26. 71%	0.085	2.2704
$1.75 \sim 2.25v_p$	$A_4 =$ 8. 82%	0.13	1.1466
$2.25 \sim 2.75v_p$	$A_5 =$ 1. 58%	0.16	0.2528
$2.75 \sim 3.25v_p$	$A_6 =$ 0. 16%	0.10	0.016
$3.25v_p \sim \infty$	$A_7A_8 \approx 0.0099\%$	0	0
			$\sum_v \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{45^o}(v) = 5.5414$

Tab 7 (2) $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{45^o} \sim v$

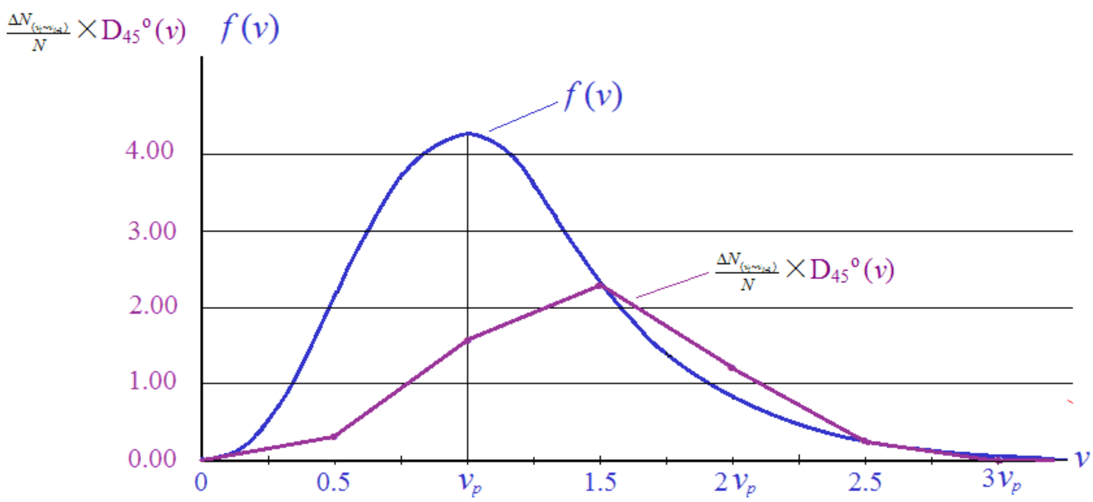
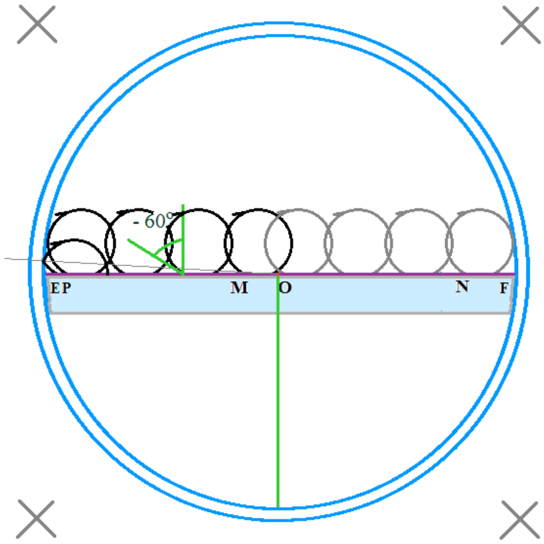


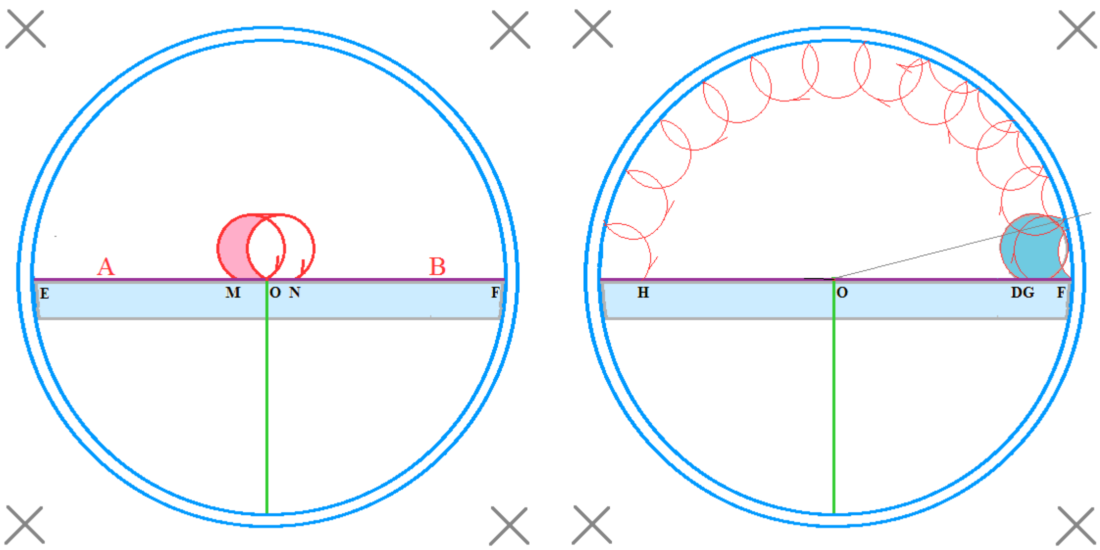
Fig 7 (2) Graph of $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{45^o} \sim v$.

8. Trajectories of electrons of θ = - 60° and different speeds

(1) Fig 8-1 θ = - 60° $v = 0.5v_p$ R = 2mm (B = 1.34 gauss)



A-glass-A & A-directly-A (60/70 = 0.86) (EP+ EM = 60, EO = 70) (black)
B-B (57/70 = 0.81) (ON = 57, OF = 70) (grey)
No electron migration between A and B due to these trajectories.



A-directly-B (from MO to ON, pink) B-glass-A (from G to H, etc., blue)

10/70 = 0.14 (MO = 10, EO = 70) 16/70 = 0.23 (DF =16, OF =70)

A-B 14% of the electrons of (- 60°, $0.5v_p$) emitted from A migrate to B.

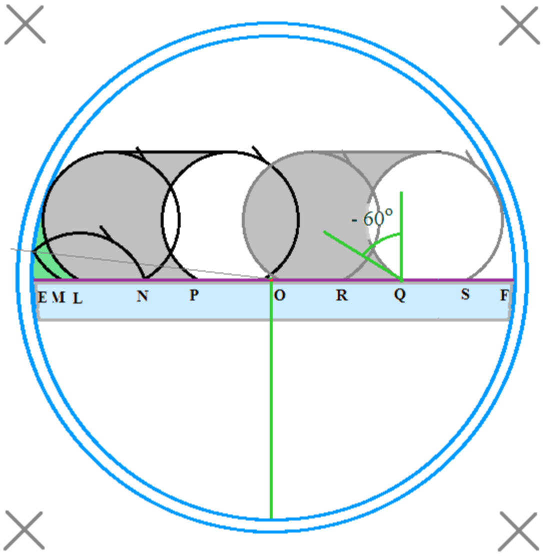
B-A 23% of the electrons of (- 60°, $0.5v_p$) emitted from B migrate to A.

For all the electrons of (-60°, $0.5v_p$), A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

$$\{(A\text{-}B) - (B\text{-}A)\}_{-60^{o}\,0.5v_p} = 0.14 - 0.23 = -0.09$$

$$D_{-60^{o}}(0.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-60^{o}\,0.5v_p} \cos 60^{o} = -0.09 \times 0.50 \approx -0.05$$

(2) Fig 8-2 $\theta = -60^{\circ}$ $v = v_p$ $R = 4\text{mm}$ ($cos\text{-}60^{\circ} = 0.5000$)

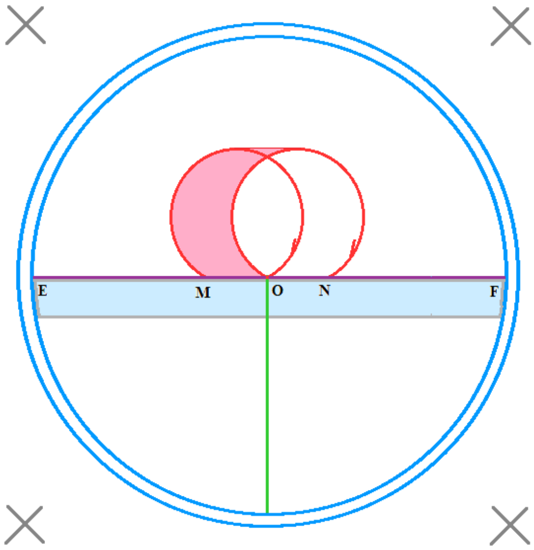


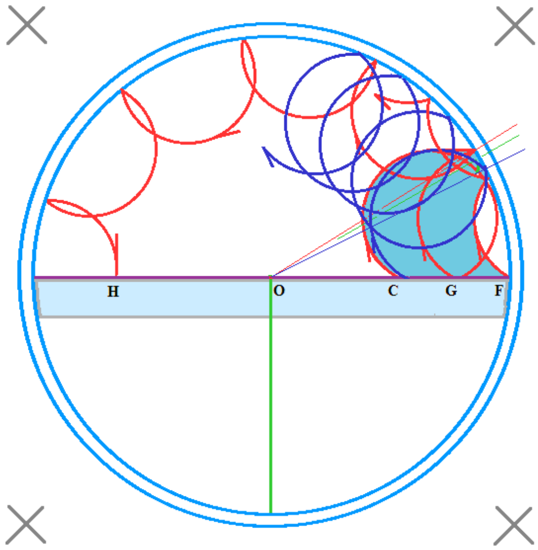
A-glass-A (from M to N, EL, green) A-directly-A (from L to N, P to O,etc., LP, grey)

A-A (total) 49/70 = 0.70 (EP = 49, EO = 70) (EP, green + grey)

B-directly-B 38/70 = 0.54 (OQ = 38) (from O to R, Q to S, etc., grey)

No electron migration between A and B due to these trajectories.





A-directly-B (from MO to ON, pink)

20/70 = 0.29 (MO = 20, EO = 70)

B-glass-A (from G to H, etc., blue)

32/70 = 0.46 (CF = 32, OF = 70)

A-B 29% of the electrons of (- 60°, v_p) emitted from A migrate to B.

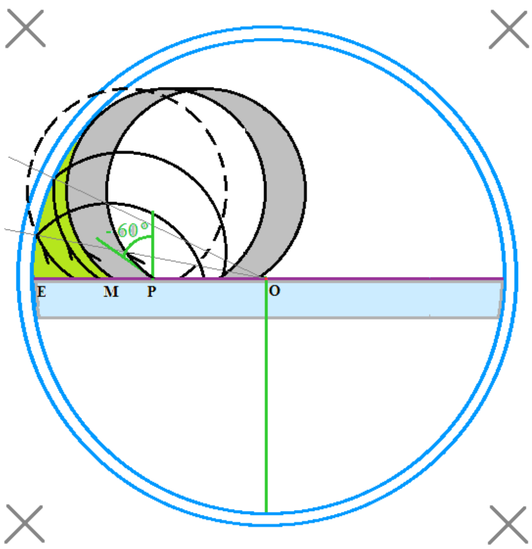
B-A 46% of the electrons of (- 60°, v_p) emitted from B migrate to A.

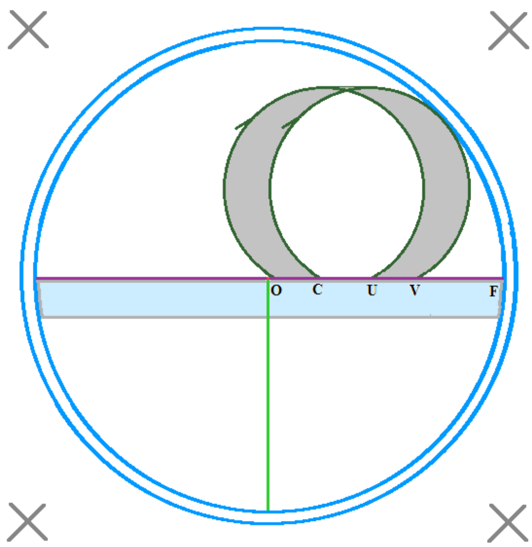
For all the electrons of (-60°, v_p), A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

$$\{(A\text{-}B) - (B\text{-}A)\}_{-60^{\circ}\ 1v_p} = 0.29 - 0.46 = -0.17$$

$$D_{-60^{\circ}}(v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-60^{\circ}\ 1v_p} \times cos\ 60^{\circ} = -0.17 \times 0.50 = -0.09$$

(3) Fig 8-3 θ = - 60° v = 1.5 v_p R = 6mm

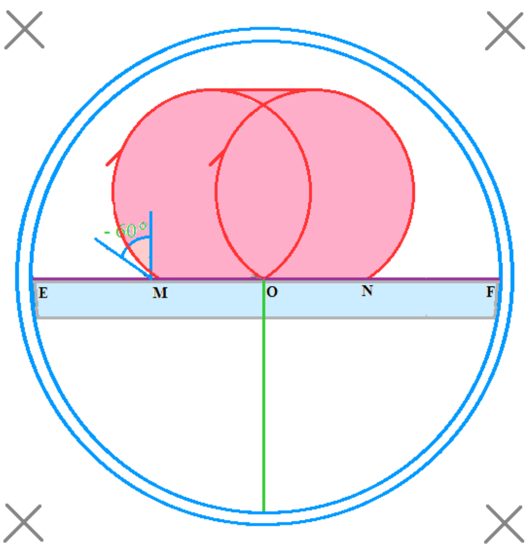


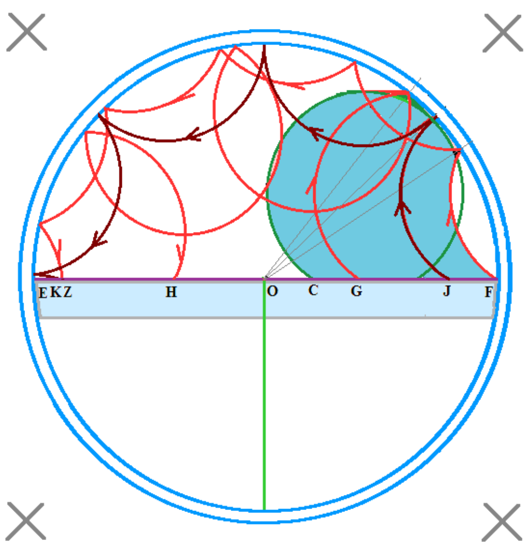


A-glass-A (EM, green)
A-directly-A (MP, grey)

B-directly-B
(from OC to UV, grey)

No electron migration between A and B due to these trajectories.





A-directly-B MO/EO = 33/70 = 0.47
(from MO to ON, pink)

B-glass-A CF/OF = 56/70 = 0.80
(from G to H, J to K, F to Z, etc., blue)

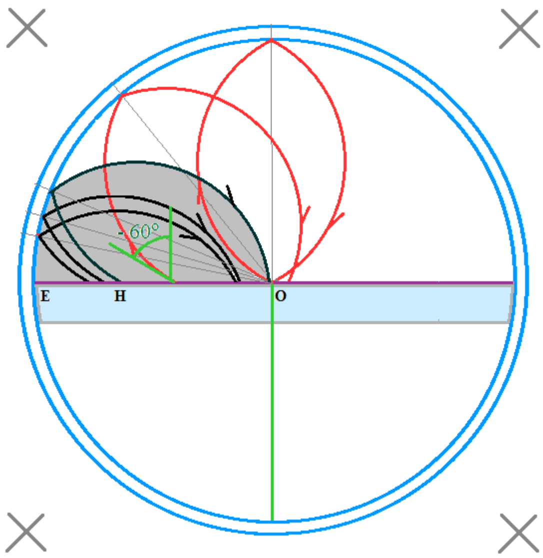
A-B 47% of the electrons of (- 60°, 1.5v_p) emitted from A migrate to B.
B-A 80% of the electrons of (- 60°, 1.5v_p) emitted from B migrate to A.

For all the electrons of (-60°, 1.5v_p), A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

$$\{(A\text{-}B) - (B\text{-}A)\}_{-60^\circ\ 1.5v_p} = 0.47 - 0.80 = -0.33$$

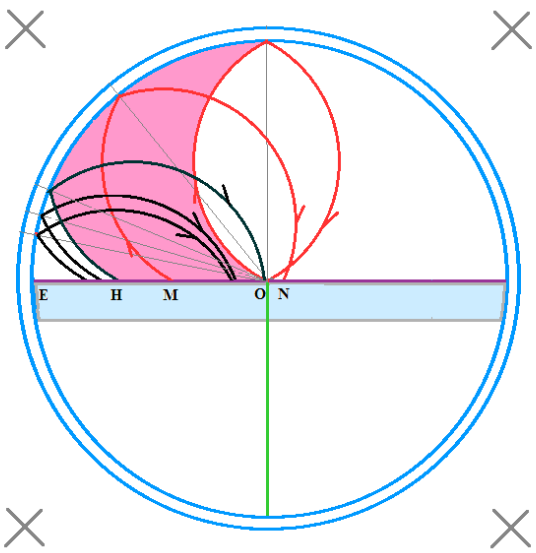
$$D_{-60^\circ}(1.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-60^\circ\ 1.5v_p}\cos 60^\circ = -0.33 \times 0.50 = -0.17$$

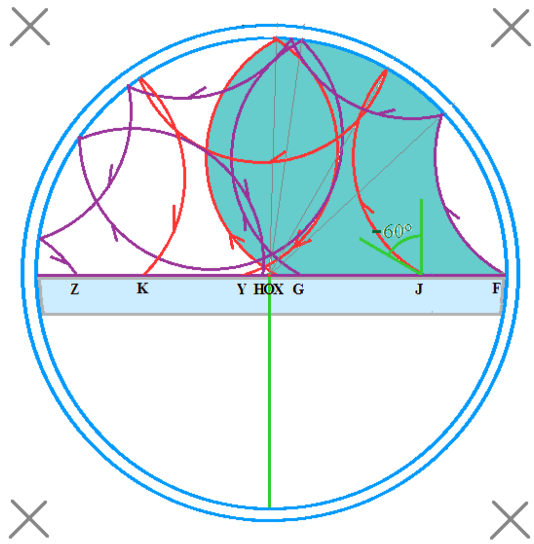
(4) Fig 8-4 θ = - 60° $v = 2\,v_p$ R = 8mm



A-glass-A 26/70 = 0.37 (EH = 26, EO = 70) (grey)

No electron migration between A and B due to these trajectories.





A-glass-B HO/EO =44/70 = 0.63
(from H to O, M to N, O to O, etc., pink)

B-glass-A 70/70 = 1.00
(from G to H, J to K, X to Y, F to Z, blue)

A-B 63% of the electrons of (- 60°, $2v_p$) emitted from A migrate to B.

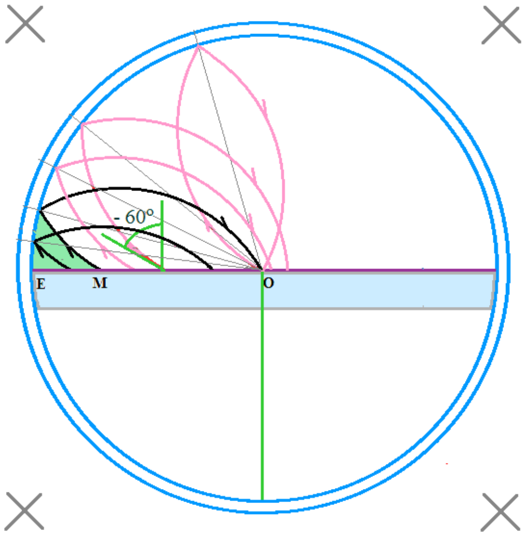
B-A 100% of the electrons of (- 60°, $2v_p$) emitted from B migrate to A.

For all the electrons of (-60°, $2v_p$), A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

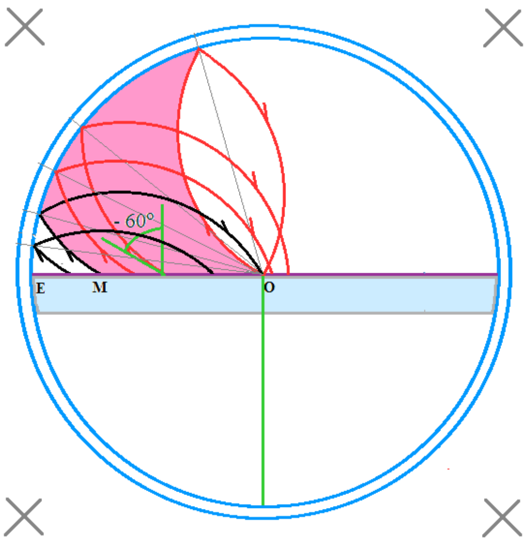
$$\{(A\text{-}B) - (B\text{-}A)\}_{-60^{\circ}\,2v_p} = 0.63 - 1.00 = -0.37$$

$$D_{-60^{\circ}}(2v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-60^{\circ}\,2v_p} \cos 60^{\circ} = -0.37 \times 0.50 = -0.19$$

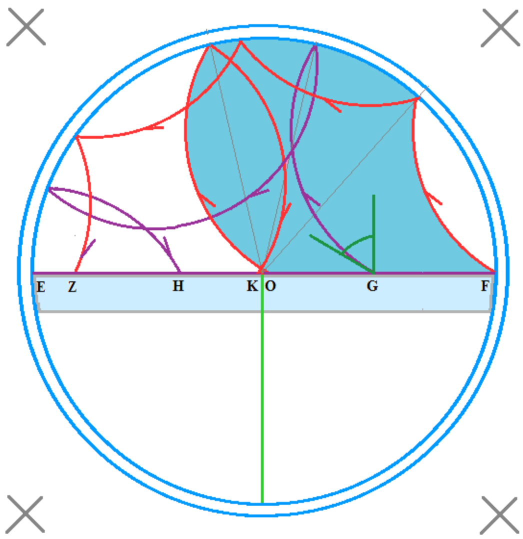
(5) Fig 8-5 θ = - 60° $v = 2.5\ v_p$ R = 10mm



A-glass-A (20/70 = 0.286)
(EM = 20, FO =70) (green)
No electron migration between A and B due to these trajectories.



A-glass-B 50/70 = 0.714
(MO = 50, EO = 70) (pink)



B-glass-A 70/70 = 1.00
(from G to H, O to K, F to Z, blue)

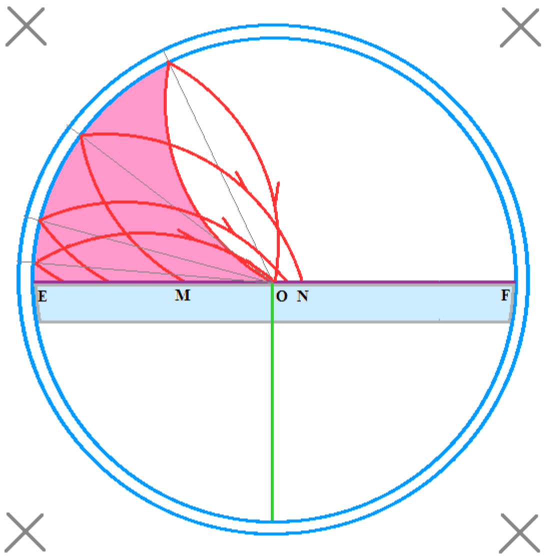
A-B 71% of the electrons of (- 60°, $2.5v_p$) emitted from A migrate to B.
B-A All the electrons of (- 60°, $2.5v_p$) emitted from B migrate to A.

For all the electrons of (-60°, $2.5v_p$), A-B is less than B-A, and their difference is the corresonding contribution to the output current (negative).

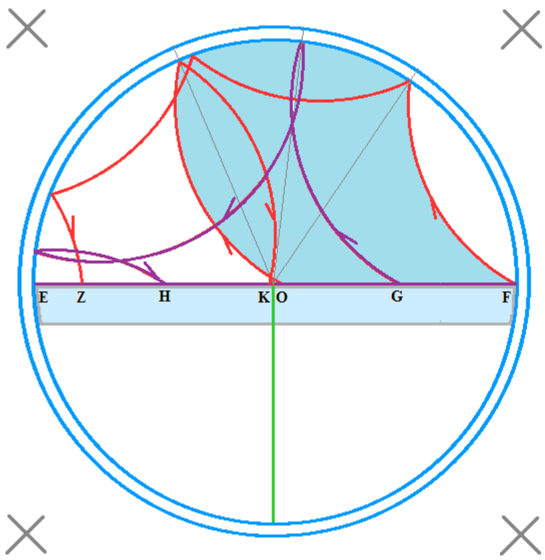
$$\{(A\text{-}B) - (B\text{-}A)\}_{-60^{\circ}\ 2.5vp} = 0.71 - 1.00 = -0.29$$

$$D_{-60^{\circ}}(2.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-60^{\circ}\ 2.5v_p}\ cos\ 60^{\circ} = -\ 0.29 \times 0.50\ =\ -0.15$$

(6) Fig 8-6 θ = - 60° $v = 3\ v_p$ R = 12mm



A-glass-B 70/70 = 1.00 (EO = 70)
(from M to N, etc., pink)



A-glass-A 70/70 = 1.00 (OF = 70)
(from G to H, O to K, F to Z, etc., blue)

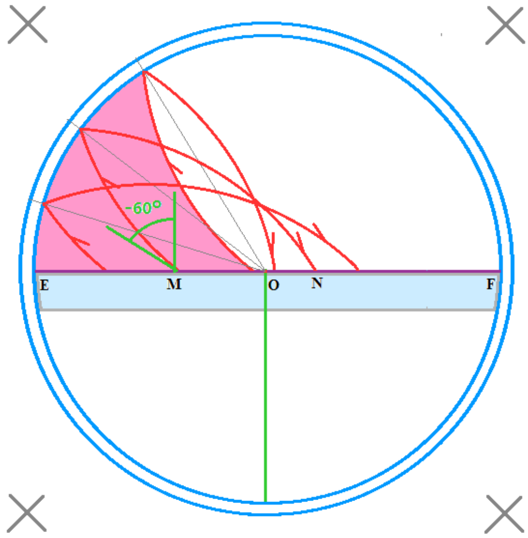
A-B 100% of the electrons of (- 60°, $3v_p$) emitted from A migrate to B.
B-A 100% of the electrons of (- 60°, $3v_p$) emitted from B migrate to A.

For all the electrons of (-60°, $3v_p$), A-B equals B-A and their contributions to the output current cancel each other.

$$\{(A\text{-}B) - (B\text{-}A)\}_{-60^{\circ}\ 3v_p} = 1.00 - 1.00 = 0$$

$$D_{-60^{\circ}}(3v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-60^{\circ}\ 3v_p}\ cos\ 60^{\circ} = 0$$

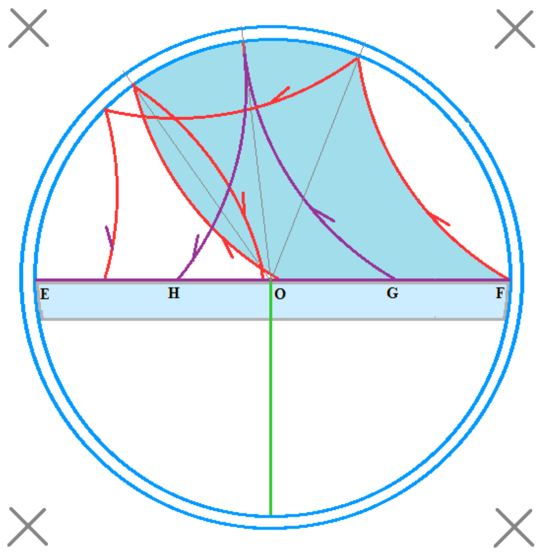
(7) Fig 8-7 θ = - 60° $v = 4.5\ v_p$ R = 18mm



A-glass-B EO/EO = 70/70 = 1.00

(from M to N, etc., pink)

A-B 100% of the electrons of (θ= - 60°, v =4.5v_p) emitted from A migrate to B.



B-glass-A OF/OF = 70/70 = 1.00

(from G to H, etc., blue)

B-A 100% of the electrons of (θ= - 60°, v =4.5v_p) emitted from B migrate to A.

For all the electrons of (-60°, 4.5v_p), A-B equals B-A, and their contributions to the output current cancel each other.

$$\{(A\text{-}B) - (B\text{-}A)\}_{-60^{o}\ 4.5v_p} = 1.00 - 1.00 = 0$$

$$D_{-60^{o}}(4.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-60^{o}\ 4.5v_p}\ \cos 60^{o} = 0$$

Contributions of electrons of exiting angle θ = - 60° with different speeds

θ = - 60°	{(A-B) – (B-A)} $_{60°}$	$D_{60}°(v)$
Fig 8-1 $v = 0.5v_p$	0.14 - 0.23 = - 0.09	- 0.05
Fig 8-2 $v = v_p$	0.29 - 0.46 = - 0.17	- 0.09
Fig 8-3 $v = 1.5v_p$	0.47 - 0.80 = - 0.33	- 0.17
Fig 8-4 $v = 2v_p$	0.63 - 1.00 = - 0.37	- 0.19
Fig 8-5 $v = 2.5v_p$	0.71 - 1.00 = - 0.29	- 0.15
Fig 8-6 $v = 3v_p$	1.00 - 1.00 = 0	0
Fig 8-7 $v = 4.5v_p$	1.00 - 1.00 = 0	0

Tab. 8 (1) $D_{60}°(v)$ = {(A-B) – (B-A)} $_{60}°\cos\theta \sim v$

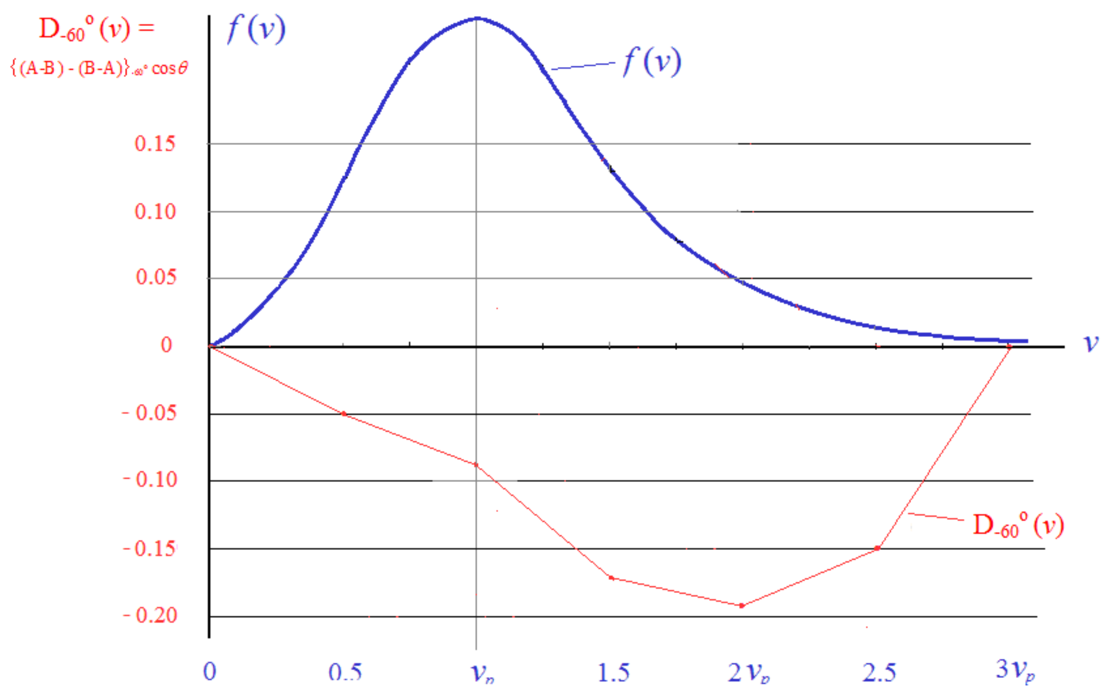


Fig 8 (1) Graph of $D_{60}°(v)$ = (A-B) - (B-A) } $_{-60}°\cos\theta \sim v$

Take Maxwell's speed distribution into account to derive the actual contributions of the thermal electrons of θ = -60° with different speeds, i.e., $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-60^o}(v) \sim v$, see Tab 8 (2).

Fig 8 (2) is the corresponding graph.

Speed range $x = v/v_p$	$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N}$	$D_{-60^o}(v)$	$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-60^o}(v)$
$0.00 \sim 0.25v_p$	$A_0 \approx 0.004$	≈ 0	≈ 0
$0.25 \sim 0.75v_p$	$A_1 = 18.74\%$	- 0.05	- 0.937
$0.75 \sim 1.25v_p$	$A_2 = 39.83\%$	- 0.09	- 3.5847
$1.25 \sim 1.75v_p$	$A_3 = 26.71\%$	- 0.17	- 4.5407
$1.75 \sim 2.25v_p$	$A_4 = 8.82\%$	- 0.19	- 1.6758
$2.25 \sim 2.75v_p$	$A_5 = 1.58\%$	- 0.15	- 0.237
$2.75 \sim 3.25v_p$	$A_6 = 0.16\%$	0	0
$3.25v_p \sim \infty$	$A_7A_8 \approx 0.0099\%$	0	0
			$\sum_v \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-60^o}(v) = -10.9752$

Tab 8 (2) The actual contributions of electrons of θ = - 60° with different speeds, $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-60^o} \sim v$.

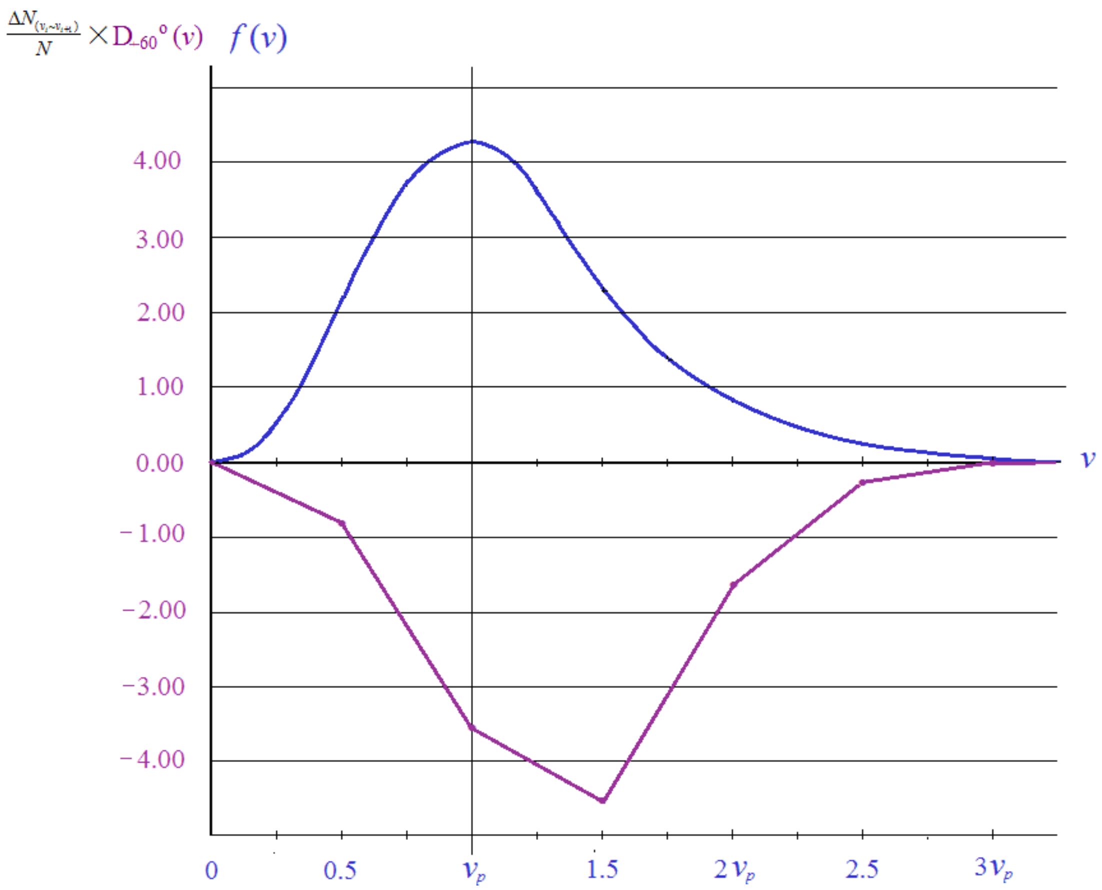
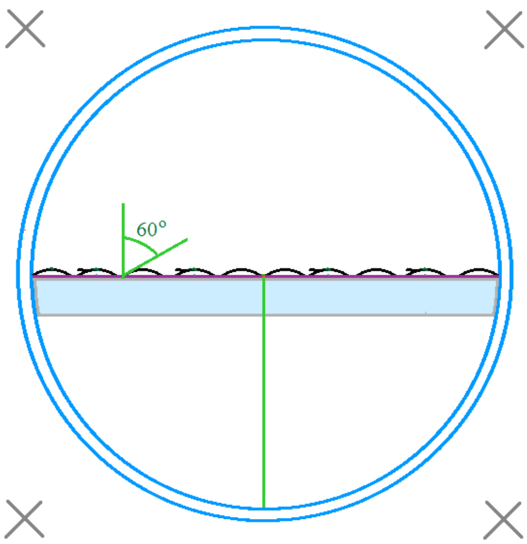


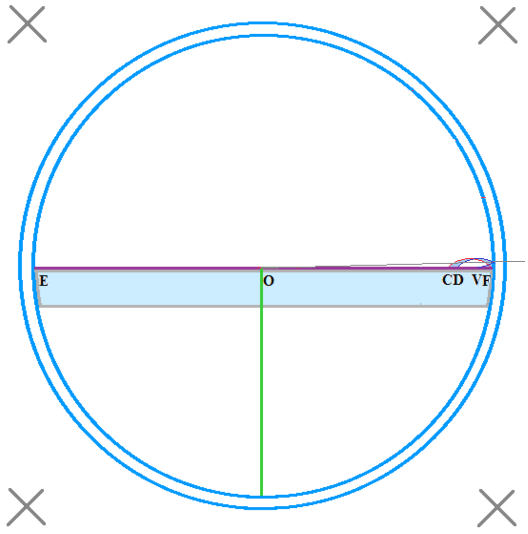
Fig 8 (2) Graph of the actual contributions of electrons of θ = - 60° with different speeds, $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-60^o} \sim v$.

9. Trajectories of electrons of θ = 60° and different speeds

(1) Fig 9-1 $\theta = 60^{\circ}$ $v = 0.5v_p$ $R = 2\mathrm{mm}$ (B = 1.34 gauss)

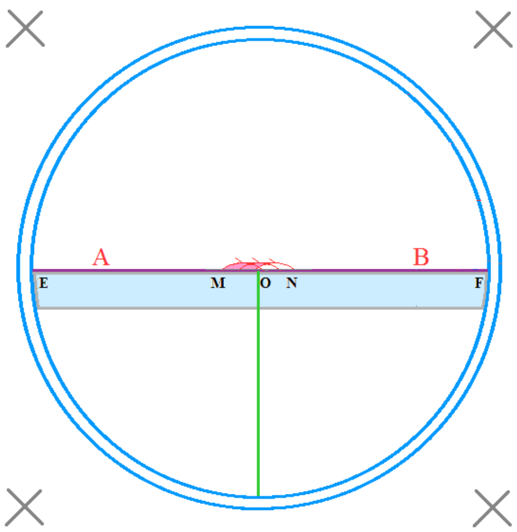


A-directly-A B-directly-B

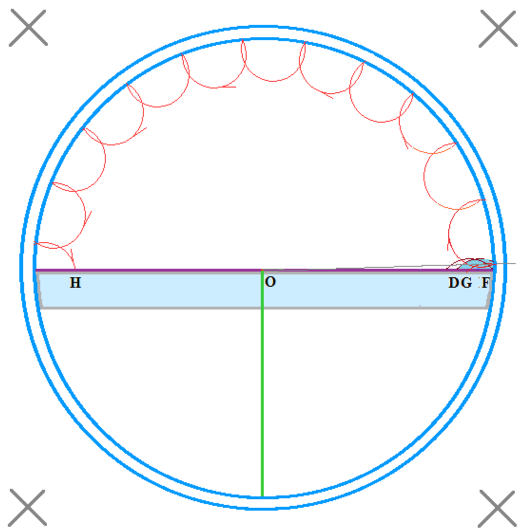


B-glass-B CD /OF = 3/70 = 0.04

No electron migration between A and B due to these trajectories.



A-directly-B MO/EO =12.5/70 = 0.18
(from MO to ON, pink.)



B-glass-A DF/OF =9.5/70 = 0.14
(from G to H, etc., blue)

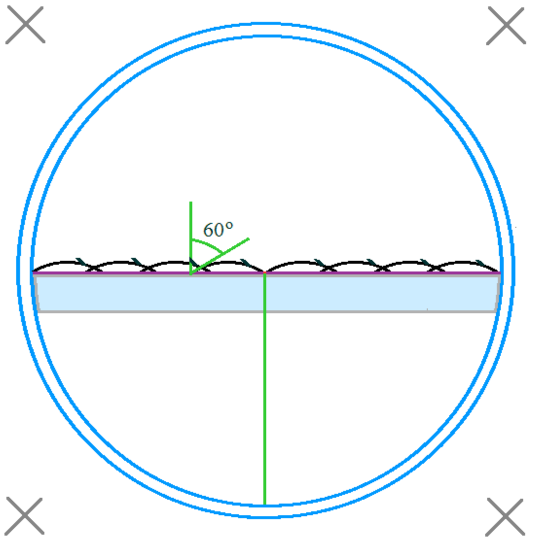
A-B 18% of the electrons of (60°, $0.5v_p$) emitted from A migrate to B.
B-A 14% of the electrons of (60°, $0.5v_p$) emitted from B migrate to A.

For all the electrons of (60°, $0.5v_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

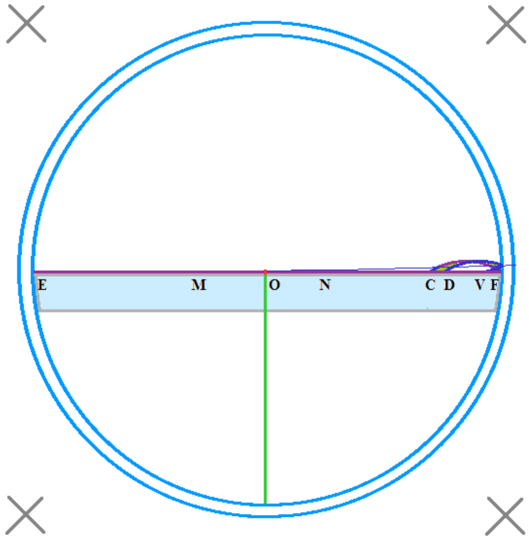
$$\{(\text{A-B}) - (\text{B-A})\}_{60^{\circ} 0.5v_p} = 0.18 - 0.14 = 0.04$$

$$D_{60^{\circ}}(0.5v_p) = \{(\text{A-B}) - (\text{B-A})\}_{60^{\circ} 0.5v_p} \times \cos 60^{\circ} = 0.04 \times 0.50 = 0.02$$

(2) Fig 9-2 $\theta = 60^{\circ}$ $v = v_p$ $R = 4\text{mm}$ $(cos60^{\circ} = 0.5000)$

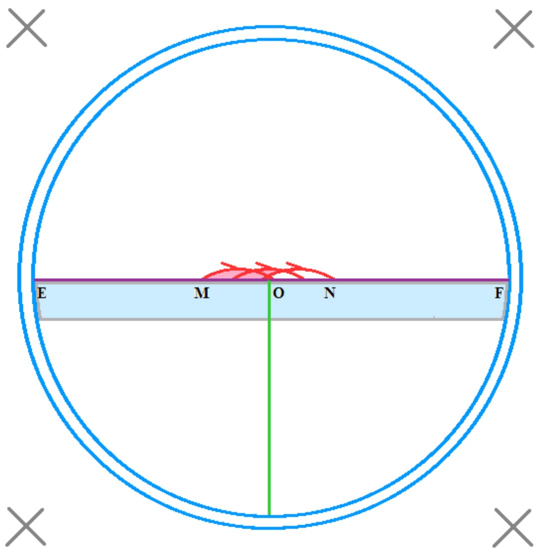


A-directly-A B-directly-B

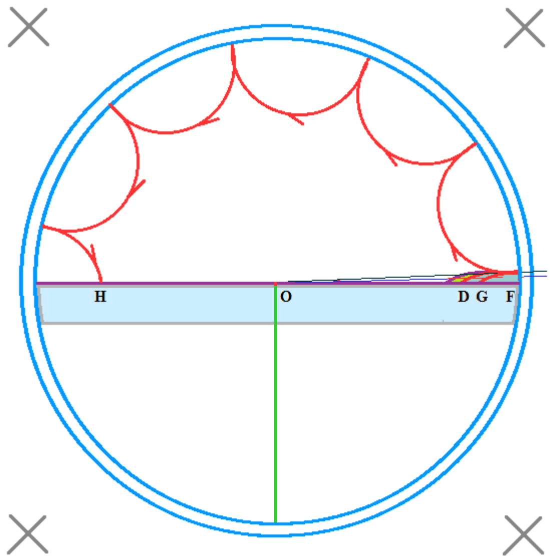


B-glass-B

No electron migration between A and B due to these trajectories.



A-directly-B MO/EO = 20/70 = 0.29
(from MO to ON, pink)



B-glass-A DF/OF=15.5/70 = 0.221
(from G to H, etc., blue)

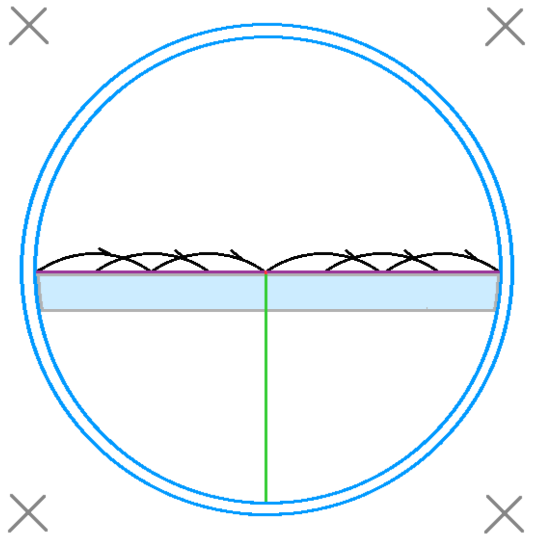
A-B 29% of the electrons of $(60^{\circ}, v_p)$ emitted from A migrate to B.
A-B 22% of the electrons of $(60^{\circ}, v_p)$ emitted from B migrate to A.

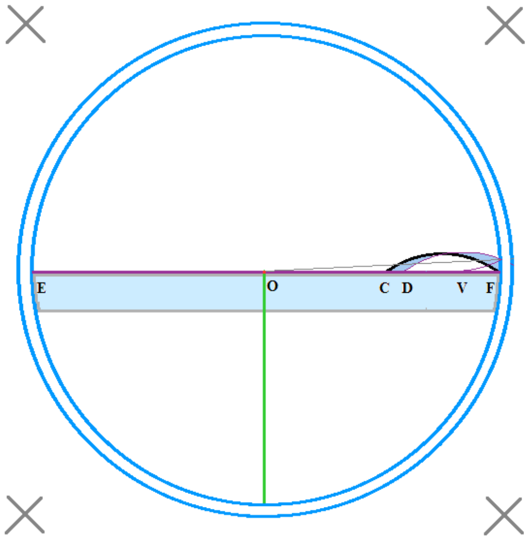
For all the electrons of $(60^{\circ}, v_p)$, A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(\text{A-B}) - (\text{B-A})\}_{\theta=60^{\circ}\ v=1v_p} = 0.29 - 0.22 = 0.07$$

$$D_{60^{\circ}}(v_p) = \{(\text{A-B}) - (\text{B-A})\}_{60^{\circ}\ 1v_p}\, cos\, 60^{\circ} = 0.07 \times 0.50 \approx 0.04$$

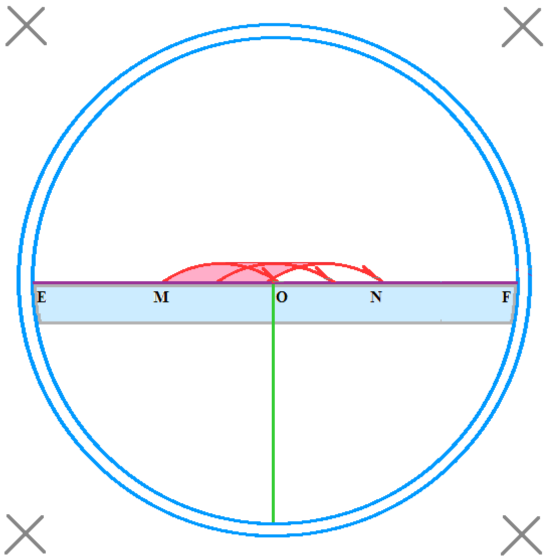
(3) Fig 9-3 $\theta = 60^{o}$ $v = 1.5v_p$ $R = 6\text{mm}$

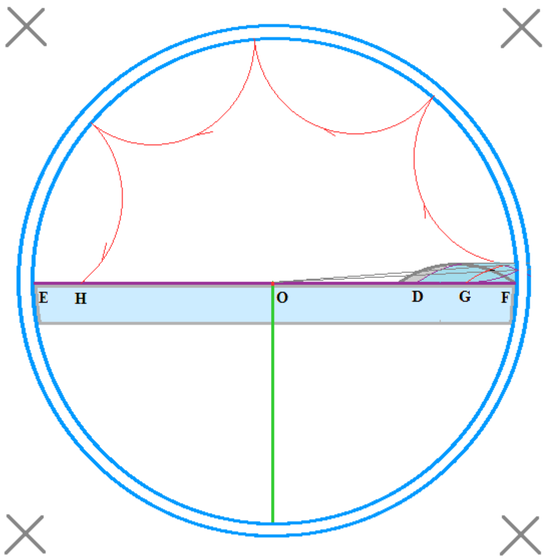




A-directly-A B-directly-B B-glass-B 5/70 = 0.07 (CD = 5)

No electron migration between A and B due to these trajectories.





A-directly-B MO/EO = 33/70 = 0.47 (from MO to ON, pink) B-glass-A DF/OF = 28/70 = 0.40 (from G to H, etc., blue)

A-B 47% of the electrons of (60^{o}, $1.5v_p$) emitted from A migrate to B.

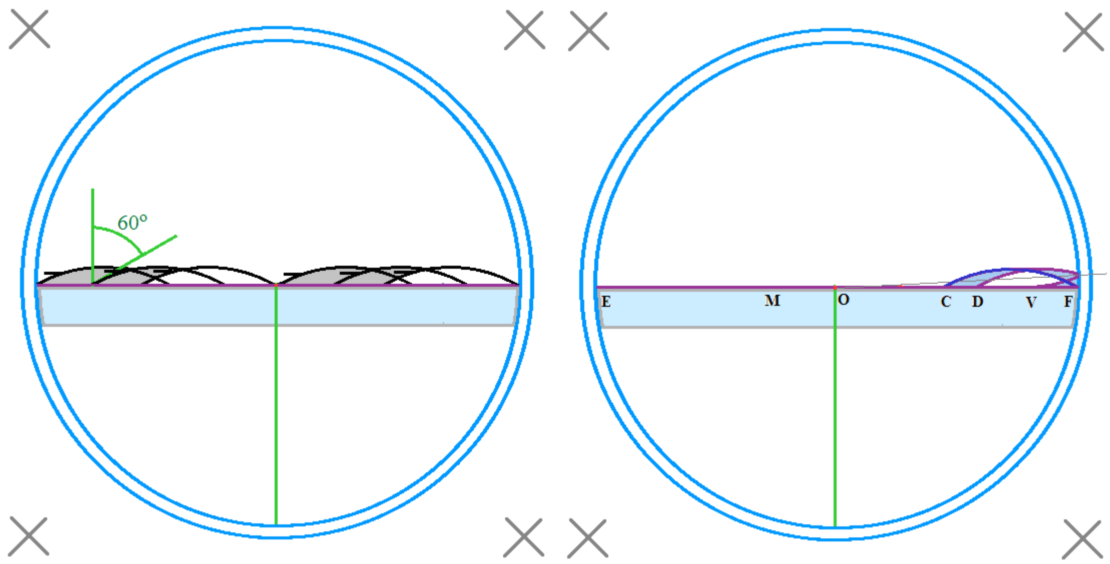
B-A 40% of the electrons of (60^{o}, $1.5v_p$) emitted from B migrate to A.

For all the electrons of ($\theta = 60^{o}$, $v = 1.5v_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(\text{A-B}) - (\text{B-A})\}_{60^{o}\ 1.5v_p} = 0.47 - 0.40 = 0.07$$

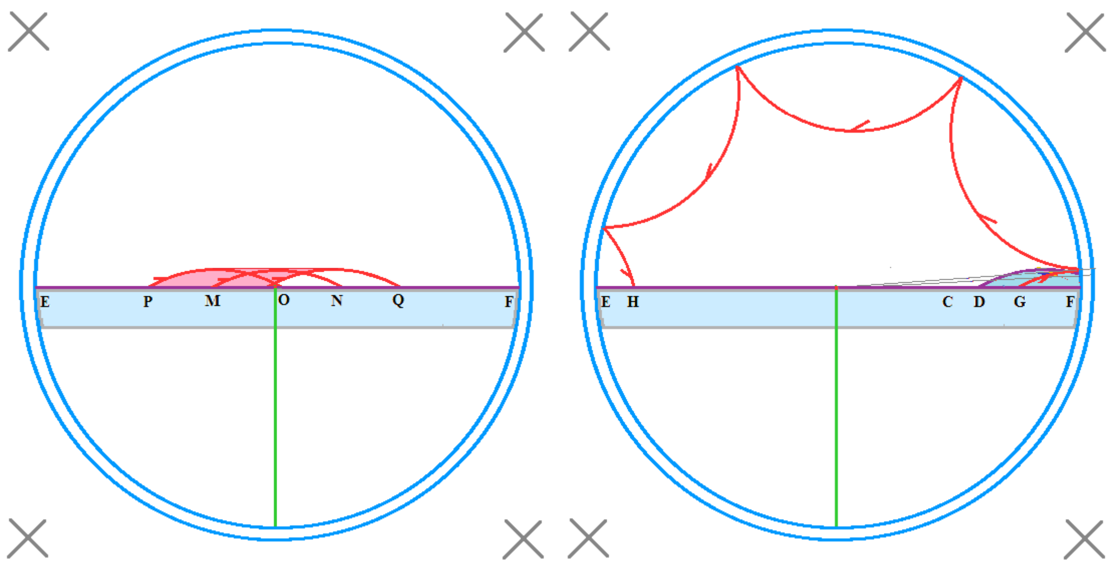
$$D_{60^{o}}(1.5v_p) = \{(\text{A-B}) - (\text{B-A})\}_{60^{o}\ 1.5v_p} \times \cos 60^{o} = 0.07 \times 0.50 = 0.04$$

(4) Fig 9-4 $\theta = 60^{o}$ $v = 2\ v_p$ $R = 8\text{mm}$



A-directly-A B-directly-B B-glass-B CD/OF = 10/70 = 0.14

No electron migration between A and B due to these trajectories.



A-directly-B PO/EO = 39/70 = 0.56 B-glass-A DF.OF = 29/70 = 0.414

(from P to O, M to N, O to Q, etc., pink) (from G to H, etc., blue)

A-B 56% of the electrons of (60^{o}, $2v_p$) emitted from A migrate to B.

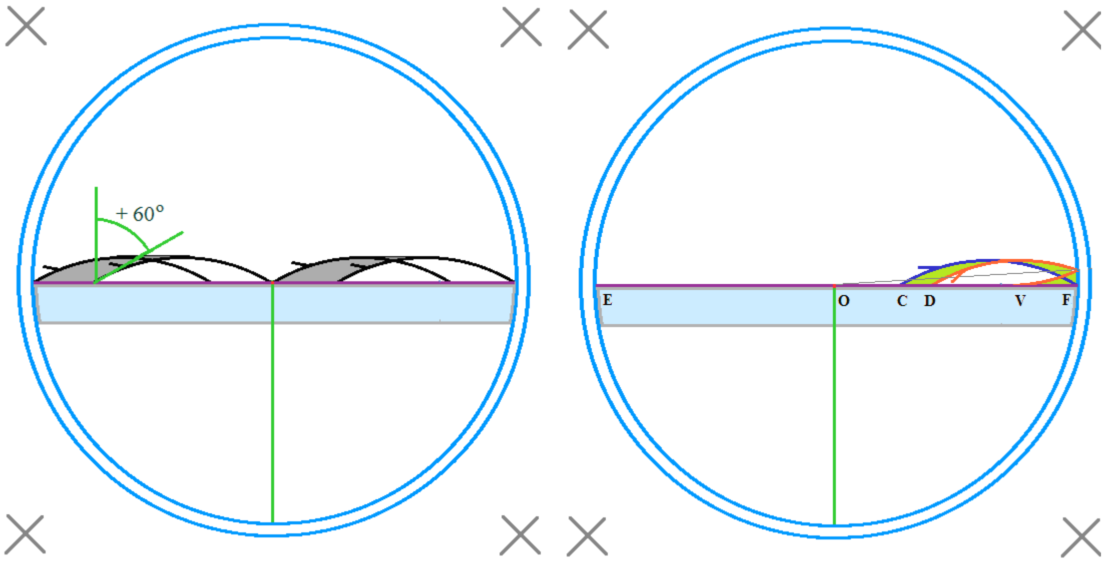
B-A 41% of the electrons of (60^{o}, $2v_p$) emitted from B migrate to A.

For all the electrons of (60^{o}, $2v_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{60^{o}\ 2v_p} = 0.56 - 0.41 = 0.14$$

$$D_{60^{o}}(2v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{60^{o}\ 2v_p} \times \cos 60^{o} = 0.14 \times 0.50 = 0.07$$

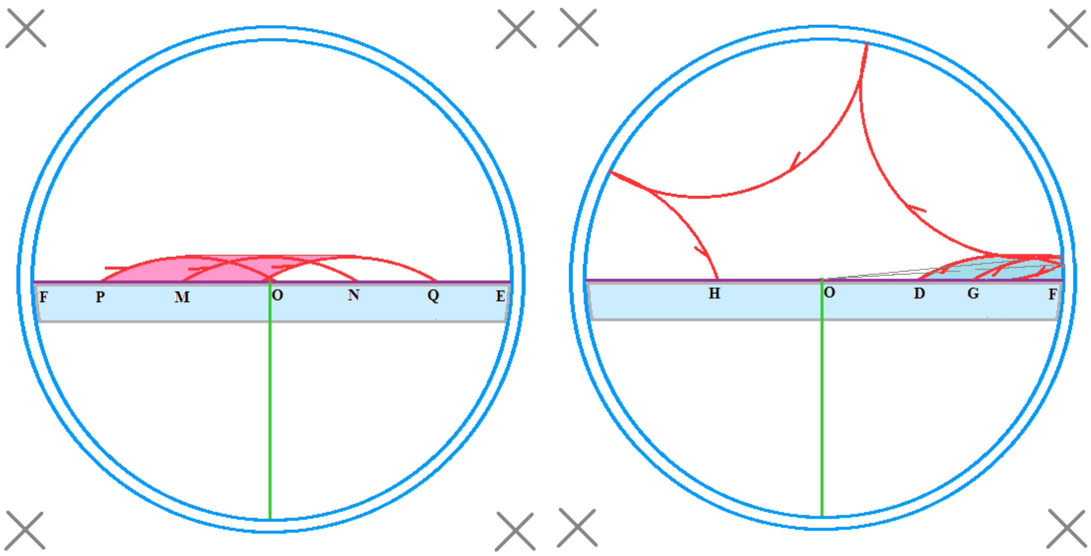
(5) Fig 9-5 $\theta = 60^{\mathrm{o}}$ $v = 2.5v_p$ $R = 10\mathrm{mm}$



A-directly-A B-directly-B B-glass-B CD/OF = 9.5/70 = 0.14

(from CD to FV)

No electron migration between A and B due to these trajectories.



A-directly-B PO/OE=51.5/70=0.74

(from P to O, M to N, O to Q, etc., pink)

A-glass-A DF/OF=42/70=0.60

(from G to H, etc., blue)

A-B 74% of the electrons of (60^{o}, $2.5v_p$) emitted from A migrate to B.

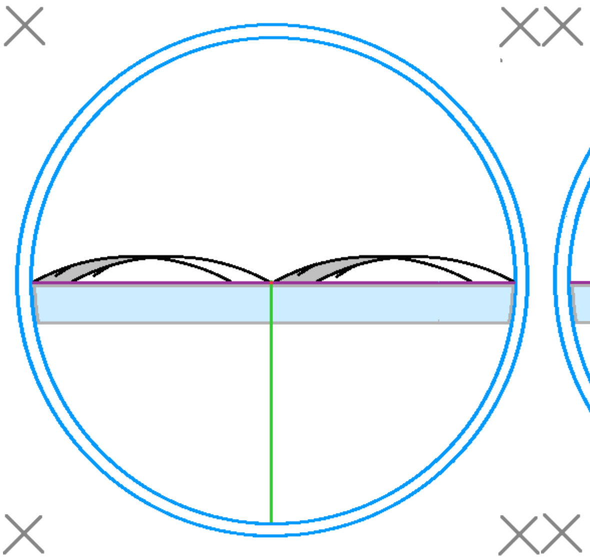
B-A 60% of the electrons of (60^{o}, $2.5v_p$) emitted from B migrate to A.

For all the electrons of (60^{o}, $2.5v_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(\text{A-B}) - (\text{B-A})\}_{60^{\mathrm{o}}\, 2.5v_p} = 0.74 - 0.60 = 0.14$$

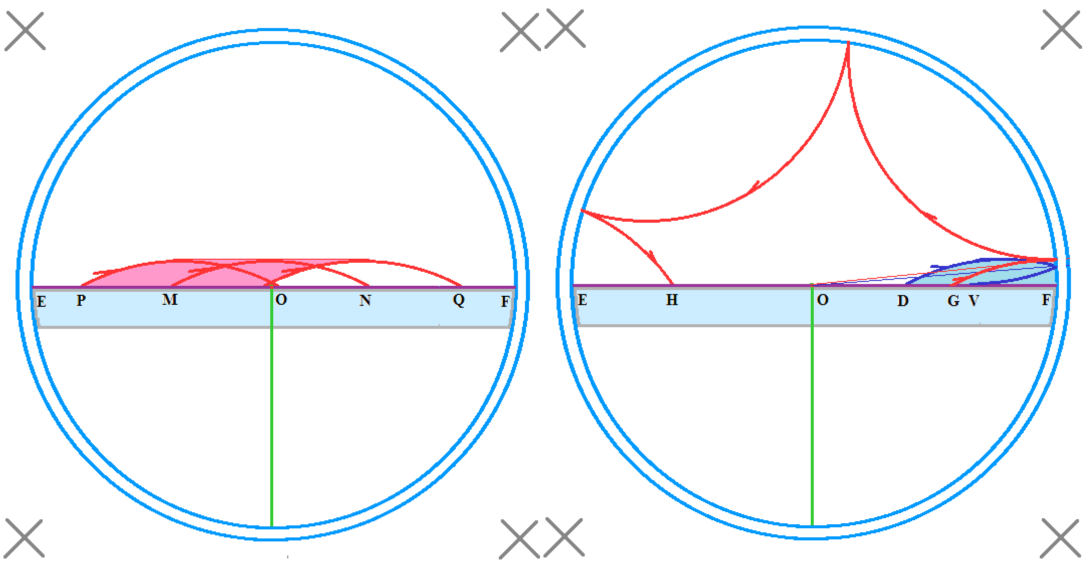
$$D_{60^{\mathrm{o}}}(2.5v_p) = \{(\text{A-B}) - (\text{B-A})\}_{60^{\mathrm{o}}\, 2.5v_p} \times \cos 60^{\mathrm{o}} = 0.14 \times 0.50 = 0.07$$

(6) Fig 9-6 $\theta = 60^{o}$ $v = 3v_p$ $R = 10mm$



A-directly-A B-directly-B B-glass-B

No electron migration between A and B due to these trajectories.



A-directly-B PO/EO = 57/70 = 0.81 B-glass-A DF/OF = 43/70 = 0.61

(from P to O, M to N, O to Q, etc., pink) (from G to H, etc., blue)

A-B 81% of the electrons of (60^{o}, $3v_p$) emitted from A migrate to B.

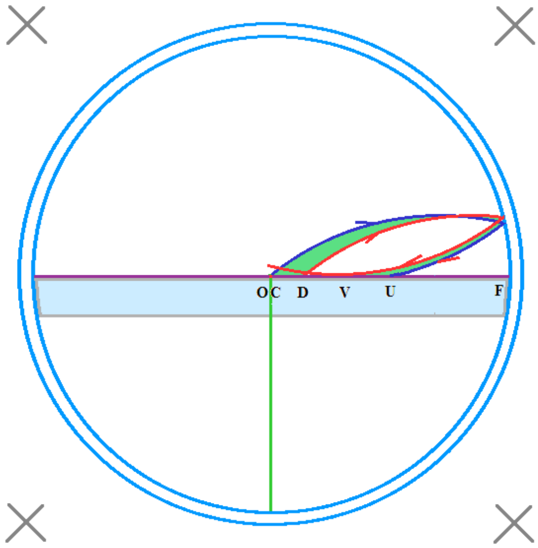
B-A 61% of the electrons of (60^{o}, $3v_p$) emitted from B migrate to A.

For all the electrons of (60^{o}, $3v_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

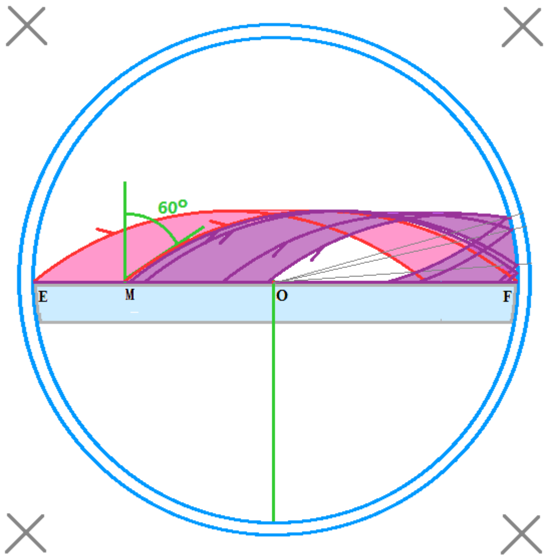
$$\{(A\text{-}B) - (B\text{-}A)\}_{60^{o}\,3v_p} = 0.81 - 0.61 = 0.20$$

$$D_{60^{o}}(1.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{60^{o}\,1.5v_p} \times \cos 60^{o} = 0.20 \times 0.50 = 0.10$$

Fig 9-7 $\theta = 60^{\circ}$ $v = 4.5v_p$ $R = 18$mm



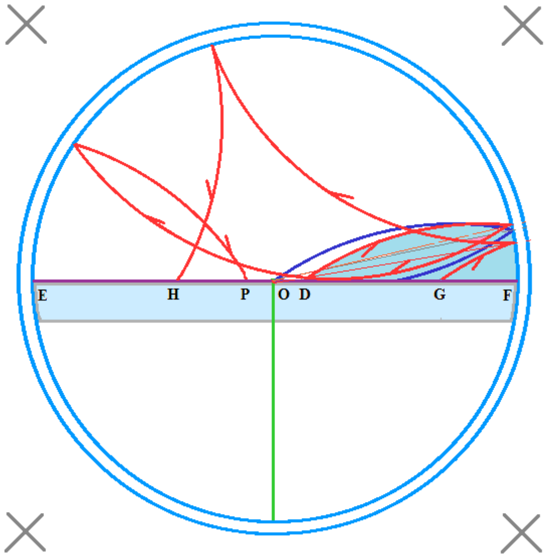
B-glass-B 8.5/70 = 0.12 (OD = CD = 8.5, OF = 70)
(from C to U, D to V, etc., green)
No electron migration between A and B due to these trajectories.



A-directly-B (EM, pink) A-glass-B (MO, violet)

A-B (total) 70/70 = 1.00 (pink + violet, EM+MO=EO=70)

A-B 100% of the electrons of (60°, $4.5v_p$) emitted from A migrate to B.



B-glass-A   DF/OF = 61.5/70 = 0.879
(from G to H, D to P, etc., blue)

B-A   87.9% electrons of (60^o, $4.5v_p$) emitted from B migrate to A.

For all the electrons of (60^o, $4.5v_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{60^o\ 4.5v_p} = 1.00 - 0.879 = 0.121 \approx 0.12$$
$$D_{60^o}(4.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{60^o\ 4.5v_p} \times \cos 60^o = 0.12 \times 0.50 = 0.06$$

Tab 9 (1) is the contributions of electrons of $\theta = 60^{\circ}$ with different speeds. Fig 9 (1) is the corresponding graph.

$\theta = 60^{\circ}$	$\{(\text{A-B})-(\text{B-A})\}_{60^{\circ}} \cos\theta \sim v$	$D_{60}{}^{\circ}(v)$
Fig 10-1 $v = 0.5v_p$	$0.18 - 0.14 = 0.04$	0. 02
Fig 10-2 $v = v_p$	$0.29 - 0.22 = 0.07$	0. 04
Fig 10-3 $v = 1.5v_p$	$0.47 - 0.40 = 0.07$	0. 04
Fig 10-4 $v = 2v_p$	$0.56 - 0.41 = 0.14$	0. 07
Fig 10-5 $v = 2.5v_p$	$0.74 - 0.60 = 0.14$	0. 07
Fig 10-6 $v = 3v_p$	$0.81 - 0.61 = 0.20$	0. 10
Fig 10-7 $v = 4.5v_p$	$1.00 - 0.879 = 0.12$	0. 06

Tab 9 (1) $D_{60}{}^{\circ}(v) = \{(\text{A-B}) - (\text{B-A})\}_{60^{\circ}} \cos\theta \sim v$

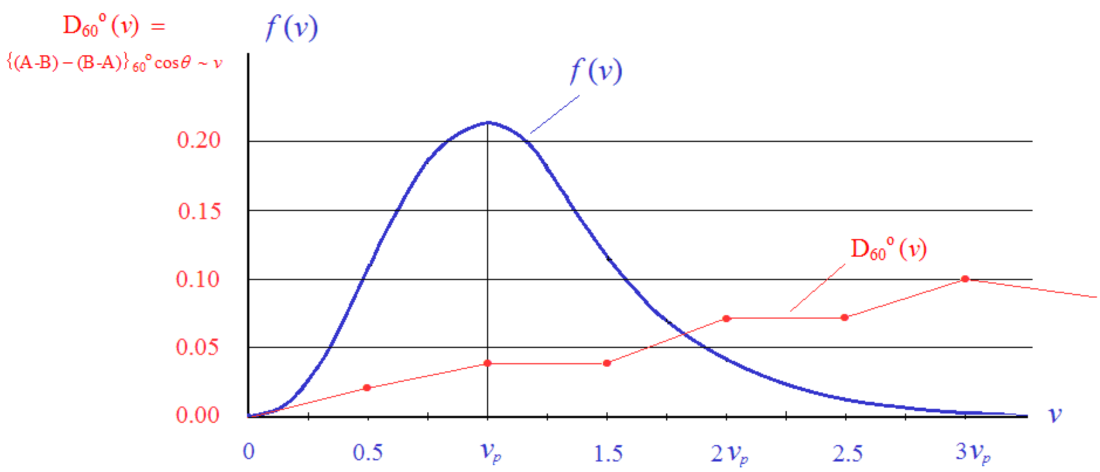


Fig 9 (1) Graph of $D_{60}{}^{\circ}(4.5v_p) = \{(\text{A-B}) - (\text{B-A})\}_{60^{\circ}} \cos\theta \sim v$

Take Maxwell's speed distribution into account to derive the actual contributions of the thermal electrons of $\theta = 60^{\circ}$ with different speeds, $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{60}{}^{\circ}(v) \sim v$, as shown in Tab 9 - 2.

Fig 9 (2) is the corresponding graph.

Speed range	$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N}$	$D_{60}^{o}(v)$	$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{60}^{o}(v)$
$0.00 \sim 0.25v_p$	$A_0 \approx 0.004$	≈ 0	≈ 0
$0.25 \sim 0.75v_p$	$A_1 = 18.74\%$	0.02	0.3748
$0.75 \sim 1.25v_p$	$A_2 = 39.83\%$	0.04	1.5932
$1.25 \sim 1.75v_p$	$A_3 = 26.71\%$	0.04	1.0684
$1.75 \sim 2.25v_p$	$A_4 = 8.82\%$	0.07	0.6174
$2.25 \sim 2.75v_p$	$A_5 = 1.58\%$	0.07	0.1106
$2.75 \sim 3.25v_p$	$A_6 = 0.16\%$	0.10	0.016
$3.25v_p \sim \infty$	$A_7A_8 \approx 0.0099\%$	0.06	≈ 0
			$\sum_v \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{60}^{o}(v) = 3.7804$

Tab. 9 (2) $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{60}^{o} \sim v$

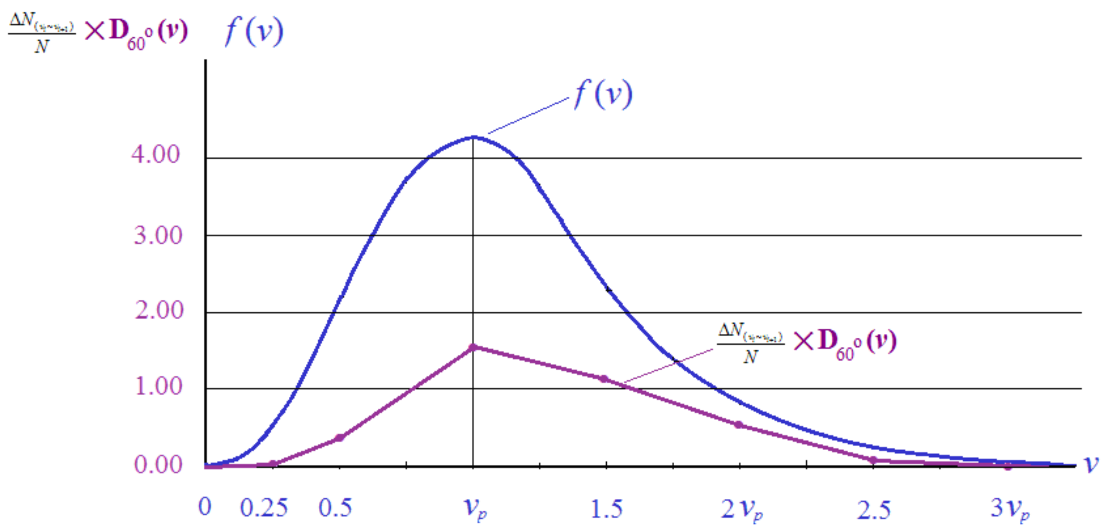
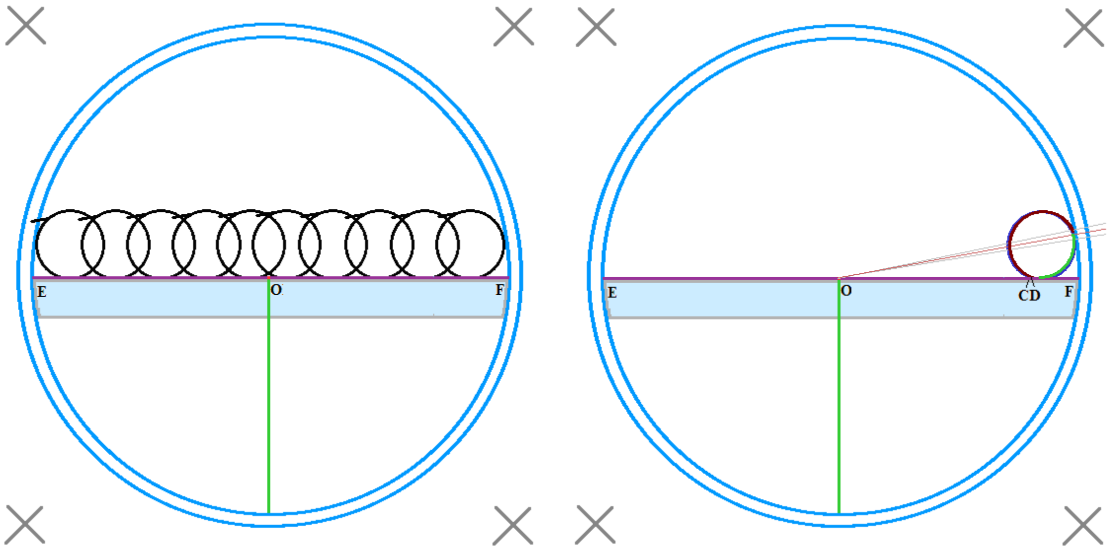


Fig. 9 (2) Graph of $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{60}^{o} \sim v$

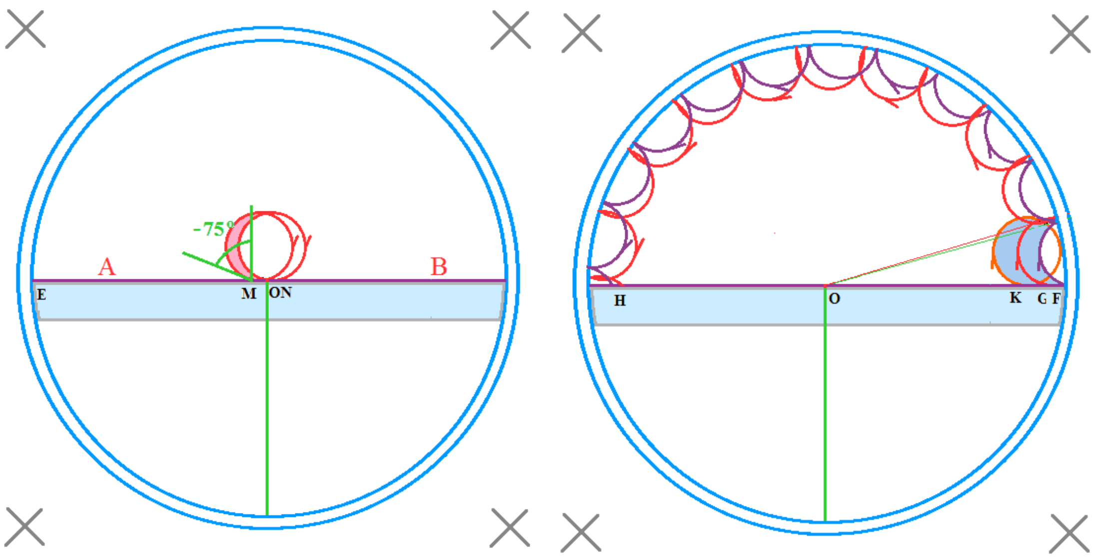
10. Trajectories of electrons of θ = - 75° and different speeds

(1) Fig 10-1 θ = - 75° $v = 0.5v_p$ R = 2mm (B = 1.34 gauss)



A-directly-A B-directly-B B-glass-B (CD≈0)

No electron migration between A and B due to these trajectories.



A-directly-B MO/EO = 6/70 = 0.09
(from MO to ON, MO = 6, pink)

B-glass-A KF/OF = 15/70 = 0.21
(from G to H, etc., KF = 15, blue)

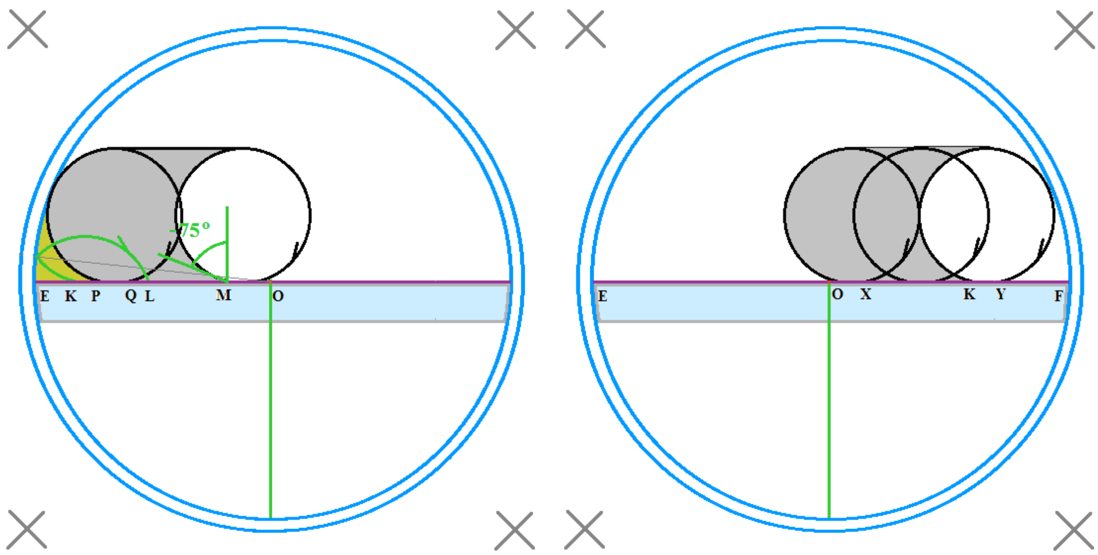
A-B 9% of the electrons of (-75°, $0.5v_p$) emitted from A migrate to B.
B-A 21% of the electrons of (-75°, $0.5v_p$) emitted from B migrate to A.

For all the electrons of (-75°, $0.5v_p$), A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

$$\{(A\text{-}B) - (B\text{-}A)\}_{-75^\circ\ 0.5vp} = 0.09 - 0.21 = -0.12$$

$$D_{-75^\circ}(0.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-75^\circ\ 0.5vp}\ cos75^\circ = -0.12 \times 0.26 = -0.028 \approx -0.03$$

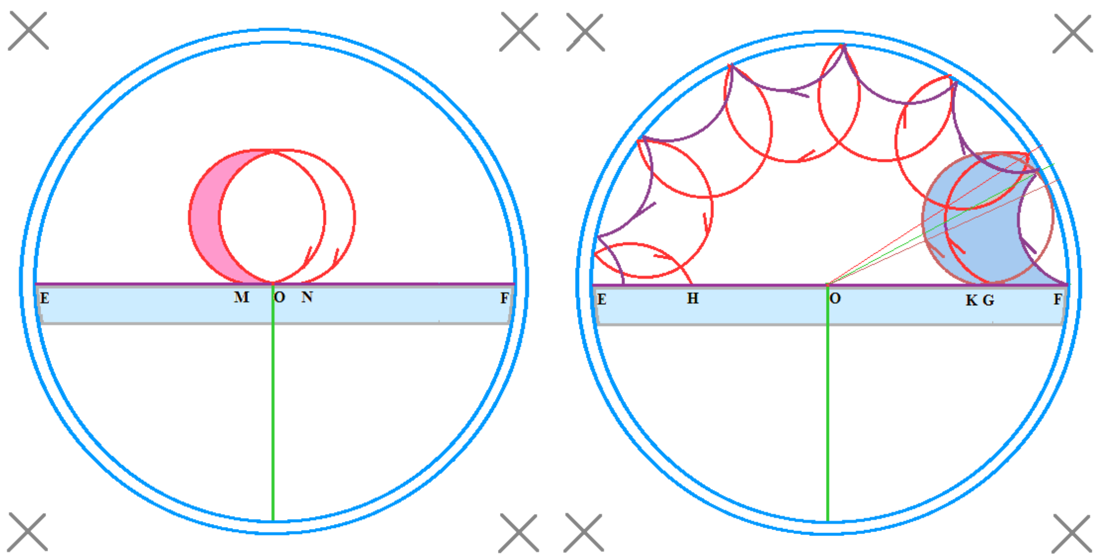
(2) Fig 10-2 θ = - 75° $v = v_p$ R = 4mm (cos-75° = 0.26)



A-glass-A (from K to L, green, EP =20)
A-directly-A (from PM to QO, grey, PM =39)

B-directly-B (from OK to XY, grey)
(B-glass-B CD≈0)

No electron migration between A and B due to these trajectories.



A-directly-B MO/EO = 11/70 =0.16
(from MO to ON, pink, MO = 11)

B-glass-A KF/OF =29/70 = 0.41
(from G to H, etc., blue, KF = 29)

A-B 16% of the electrons of (- 75°, v_p) emitted from A migrate to B.

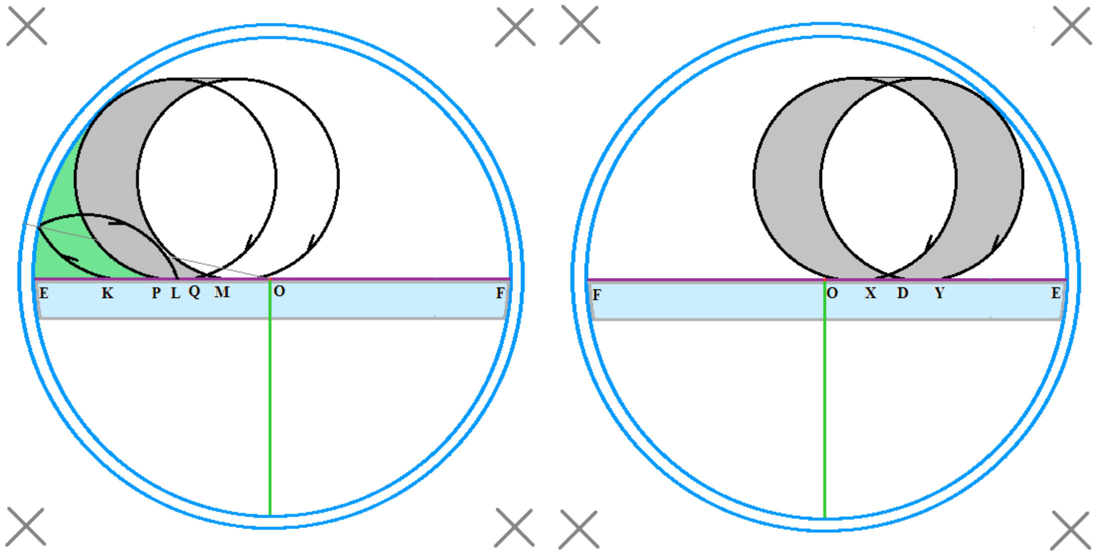
B-A 41% of the electrons of (- 75°, v_p) emitted from B migrate to A.

For all the electrons of (-75°, v_p), A-B is less than B-A, and their difference is the corresponding contribution (negative) to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{-75^{\circ}\ 1v_p} = 0.16 - 0.41 = -0.25$$

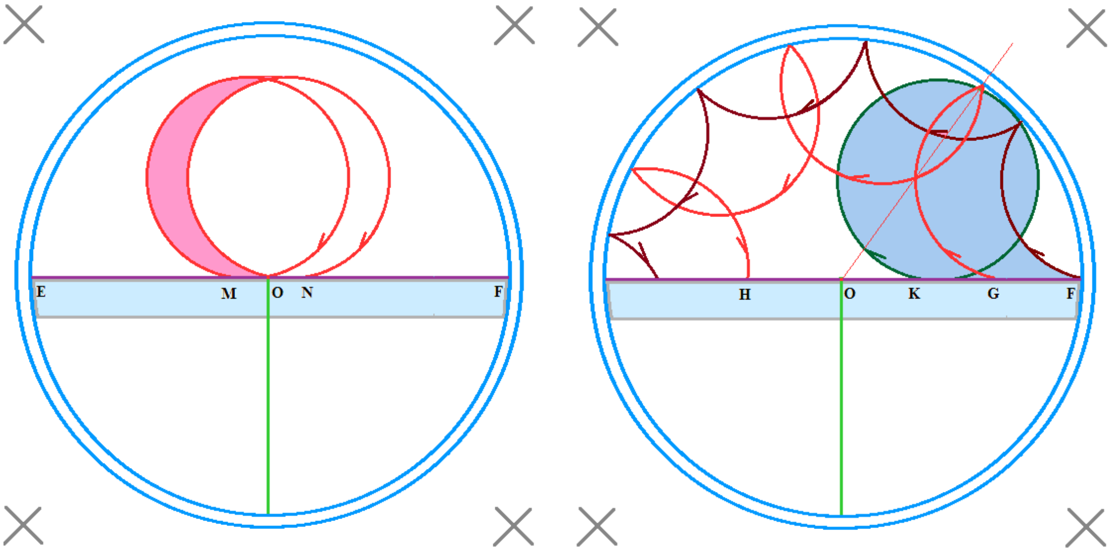
$$D_{-75^{\circ}}(v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-75^{\circ}\ 1v_p} \cos 75^{\circ} = -0.25 \times 0.26 = -0.0647 = -0.06$$

(3) Fig 10-3 θ = - 75° $v = 1.5v_p$ R = 6mm



A-glass-A (from K to L, etc., green) B-directly-B
A-directly-A (from P to Q, M to O, grey) (from O to X, D to Y, grey)

No electron migration between A and B due to these trajectories.



A-directly-B MO/EO = 14/70 = 0.20
(from MO to ON, etc. , pink, MO = 14)

B-glass-A KF/OF = 50/70 = 0.71
(from G to H, etc., blue, KF = 50)

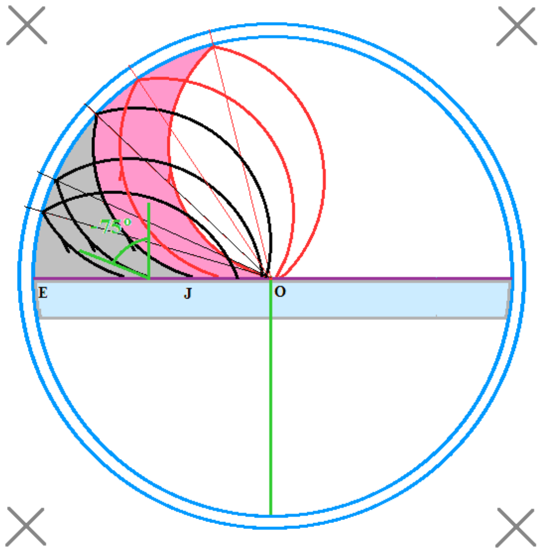
A-B 20% of the electrons of (- 75°, $1.5v_p$) emitted from A migrate to B.
B-A 71% of the electrons of (- 75°, $1.5v_p$) emitted from B migrate to A.

For all the electrons of (-75°, $1.5v_p$), A-B is less than B-A, and their difference is the corresponding contribution (negative) to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{-75^\circ\ 1.5v_p} = 0.20 - 0.71 = -0.51$$

$$D_{-75^\circ}(1.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-75^\circ\ 1.5v_p} \cos 75^\circ = -0.51 \times 0.26 = -0.13$$

(4) Fig 10-4 θ = - 75^{o} $v = 2v_p$ R = 8mm

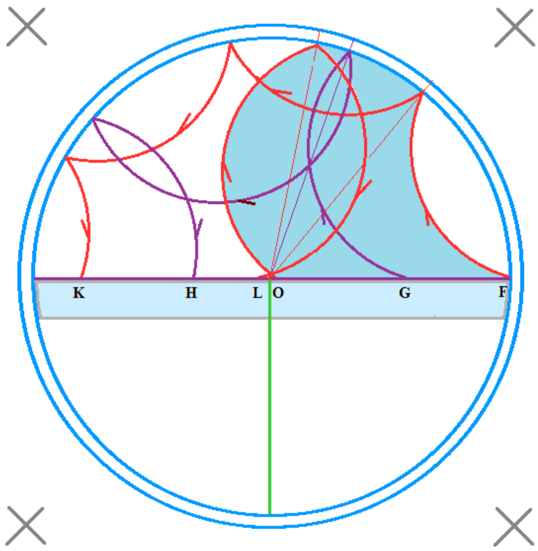


A-glass-A EJ/EO = 47/70 = 0.67 (EJ = 47, EO = 70) (grey)

No electron migration between A and B due to these trajectories.

A-glass-B 23/70 = 0.33 (JO =23, EO =70) (pink)

A-B 33% of the electrons of (-75^{o}, $2v_p$) emitted from A hit the glass wall and bounce back, and migrate to B.



B-glass-A 70/70 = 1.00

(from G to H, F to K, O to L, etc, blue)

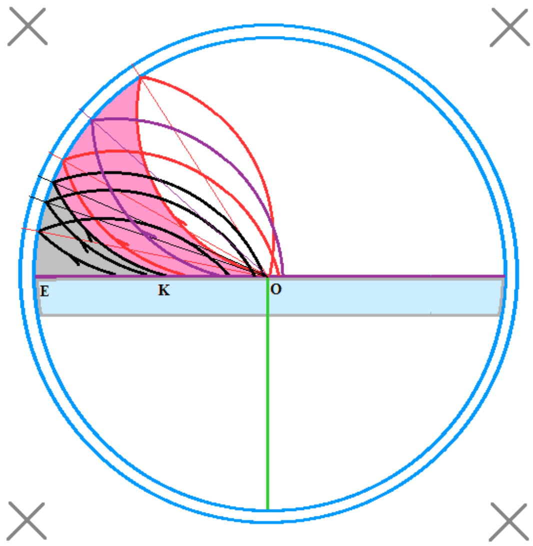
B-A 100% of the electrons of (-75^{o}, $2v_p$) emitted from B migrate to A.

For all the electrons of (-75^{o}, $2v_p$), A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

$$\{(A\text{-}B) - (B\text{-}A)\}_{-75^{o}\ 2v_p} = 0.33 - 1.00 = -0.67$$

$$D_{-75^{o}}(2v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-75^{o}\ 2v_p} \cos 75^{o} = -0.67 \times 0.26 = -0.17$$

(5) Fig 10-5 θ = - 75° $v = 2.5v_p$ R = 10mm

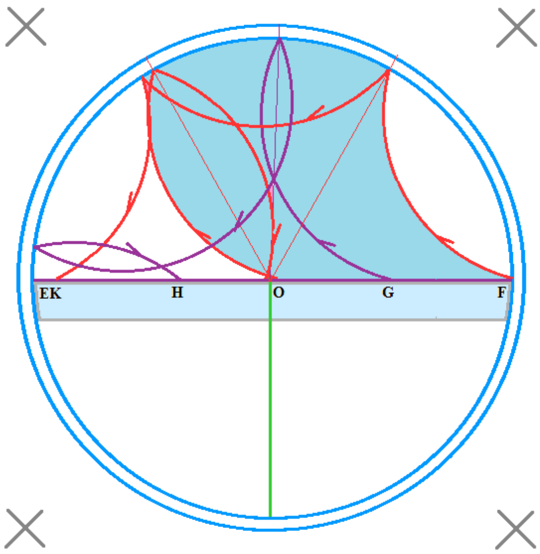


A-glass-A EK/EO = 40/70 = 0.57 (EK = 40, grey)

No electron migration between A and B due to these trajectories.

A-glass-B 30/70 = 0.43 (KO = 30, EO =70, pink)

A-B 43% of the electrons (-75°, $2.5v_p$) emitted from A hit the glass wall and bounce back, and migrate to B.



B-glass-A 70/70=1.00 (from G to H, F to K, O to O, etc., blue)

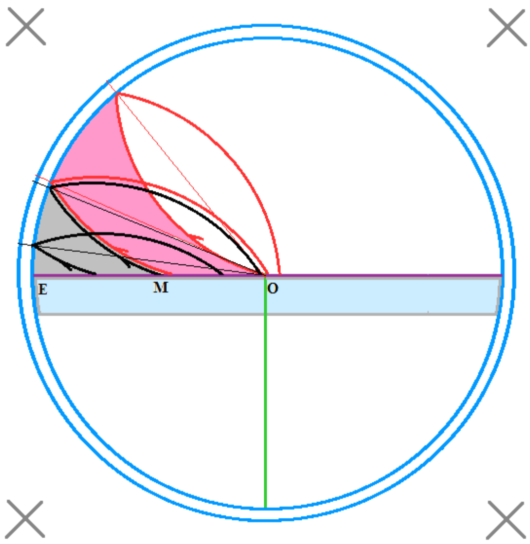
B-A All the electrons of (-75°, $2.5v_p$) emitted from B migrate to A.

For all the electrons of (-75°, $2.5v_p$), A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

$$\{(A\text{-}B)-(B\text{-}A)\}_{-75^{o}\ 2.5v_p} = 0.43 - 1.00 = -0.57$$

$$D_{-75^{o}}(2.5v_p) = \{(A\text{-}B)-(B\text{-}A)\}_{-75^{o}\ 2.5v_p}\ cos75^{o} = -0.57\times 0.26 = -0.15$$

(6) Fig 10-6 $\theta = -75^{o}$ $v = 3v_p$ $R = 10\text{mm}$

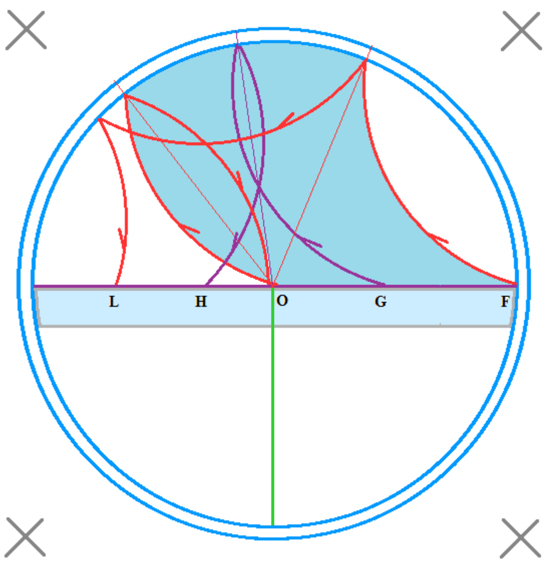


A-glass-A EM/EO = 40/70 = 0.57 (EM = 40, grey)

No electron migration between A and B due to these trajectories.

A-glass-B MO/EO = 30/70 = 0.43 (MO = 30, pink)

A-B 43% of the electrons of (-75^{o}, $3v_p$) emitted from A migrate to B.



B-glass-A 70/70 = 1.00 (from G to H, F to L, O to O, etc., blue)

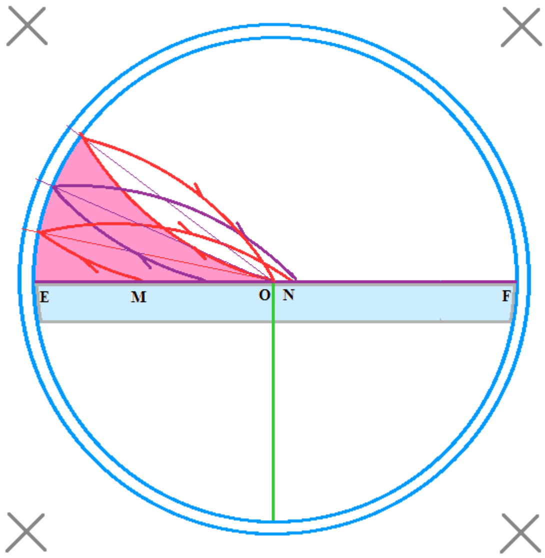
B-A 100% of the electrons of ($\theta = -75^{o}$, $v = 3v_p$) emitted from B migrate to A.

For all the electrons of (-75^{o}, $3v_p$), A-B is less than B-A, and their difference is the corresponding contribution to the output current (negative).

$$\{(A\text{-}B) - (B\text{-}A)\}_{-75^{o}\,3v_p} = 0.43 - 1.00 = -0.57$$

$$D_{-75^{o}}(3v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{-75^{o}\,3v_p} \cos 75^{o} = -0.57 \times 0.26 = -0.15$$

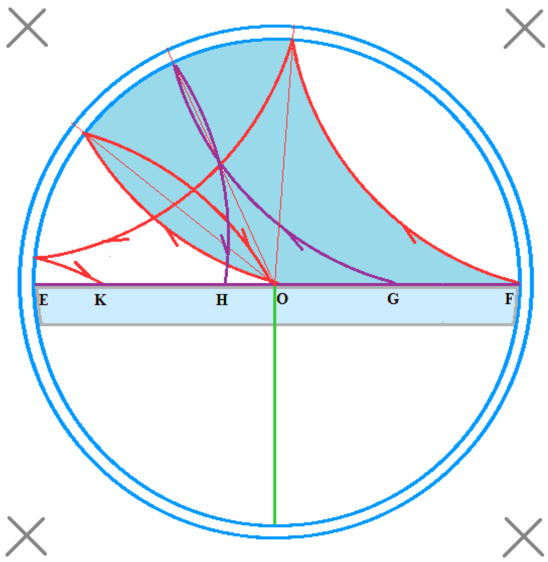
(7) Fig 10-7 θ = - 75° $v = 4.5v_p$ R = 18mm



A-glass-B 70/70 = 1.00

(from M to N, etc., pink)

A-B 100% of the electrons (- 75°, $4.5v_p$) emitted from A migrate to B.



B-glass-A 70/70 = 1.00 (from G to H, F to K, O to O, etc., blue)

B-A 100% of the electrons of (-75°, $4.5v_p$) emitted from B migrate to A.

For all the electrons of (-75°, $4.5v_p$), A-B and B-A cancel each other, no contribution to the output current.

$$\{(\text{A-B}) - (\text{B-A})\}_{-75^{\circ}\ 4.5v_p} = 1.00 - 1.00 = 0$$

$$D_{-75^{\circ}}(4.5v_p) = \{(\text{A-B}) - (\text{B-A})\}_{-75^{\circ}\,4.5v_p} \cos 75^{\circ} = 0$$

List the contributions of electrons of $\theta = -75^{o}$ and different speeds in Tab 10(1). And Fig 10 (1) is the corresponding graph, $D_{-75^{o}}(v)$.

$\theta = -75^{o}$	$\{(A\text{-}B) - (B\text{-}A)\}_{-75^{o}}(v)$	$D_{-75^{o}}(v)$
Fig 10-1 $v = 0.5v_p$	0.10 – 0.21 = - 0.11	- 0. 03
Fig 10-2 $v = v_p$	0.16 – 0.41 = - 0.25	-0. 06
Fig 10-3 $v = 1.5v_p$	0.21 – 0.71 = - 0.50	-0. 13
Fig 10-4 $v = 2v_p$	0.33 – 1.00 = - 0.67	-0. 17
Fig 10-5 $v = 2.5v_p$	0.43 – 1.00 = - 0.57	-0. 15
Fig 10-6 $v = 3v_p$	0.43 – 1.00 = - 0.57	-0. 15
Fig 10-7 $v = 4.5v_p$	1.00 – 1.00 = 0	0

Tab 10 (1) $D_{-75^{o}}(v) = \{(A\text{-}B) - (B\text{-}A)\}_{-75^{o}} \cos\theta \sim v$

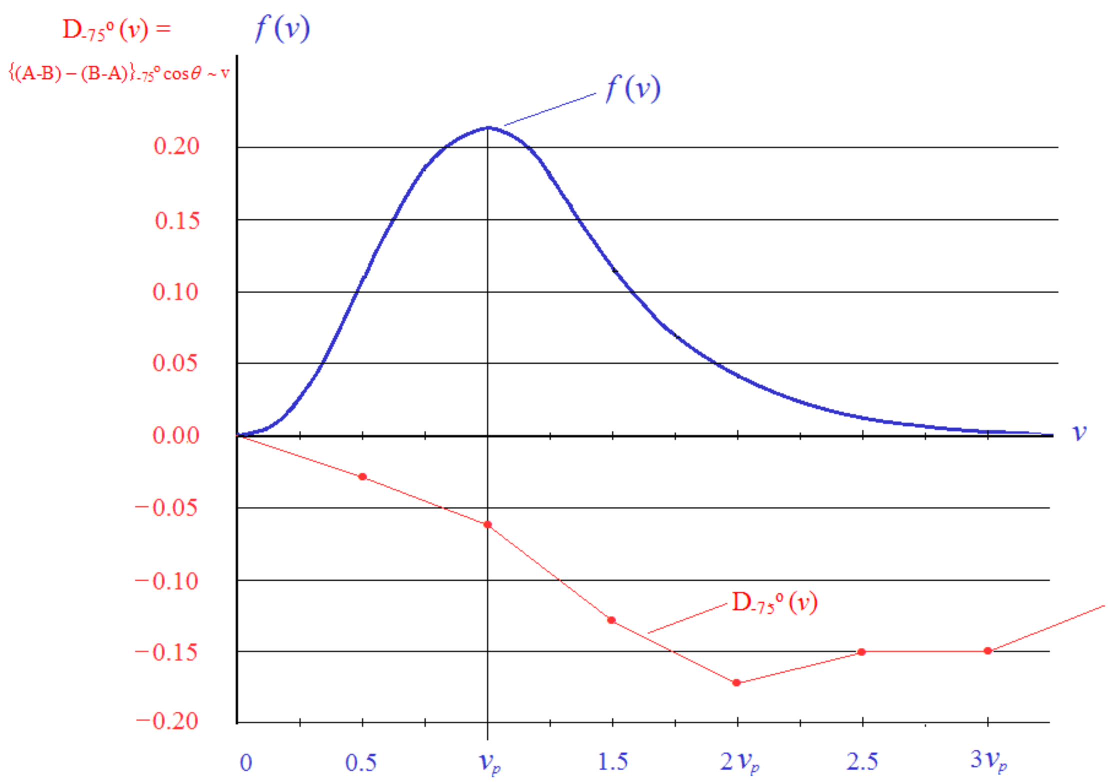


Fig 10 (1) $D_{-75^{o}}(v) = \{(A\text{-}B) - (B\text{-}A)\}_{\theta=-75^{o}} \cos\theta \sim v$

Take Maxwell's speed distribution into account to derive the actual contributions of the thermal electrons of $\theta = -75^{o}$ with different speeds, i.e., $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-75^{o}} \sim v$, as shown in Tab 10 (2).

Fig 10 (2) is the corresponding graph.

Speed range	$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N}$	$D_{-75^{o}}(v)$	$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{75^{o}} \sim v$
$0.00 \sim 0.25 v_p$	$A_0 \approx 0.004$	≈ 0	≈ 0
$0.25 \sim 0.75 v_p$	$A_1 = 18.74\%$	- 0.03	- 0.5622
$0.75 \sim 1.25 v_p$	$A_2 = 39.83\%$	- 0.06	- 2.3898
$1.25 \sim 1.75 v_p$	$A_3 = 26.71\%$	- 0.13	- 3.4723
$1.75 \sim 2.25 v_p$	$A_4 = 8.82\%$	- 0.17	- 1.4994
$2.25 \sim 2.75 v_p$	$A_5 = 1.58\%$	- 0.15	- 0.237
$2.75 \sim 3.25 v_p$	$A_6 = 0.16\%$	- 0.15	- 0.024
$3.25 v_p \sim \infty$	$A_7 A_8 \approx 0.0099\%$	0	≈ 0
			$\sum_v \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-75^{o}}(v) = -8.1847$

Tab. 10 (2) The actual contributions of electrons of $\theta = -75^{o}$ with different speeds, $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-75^{o}} \sim v$

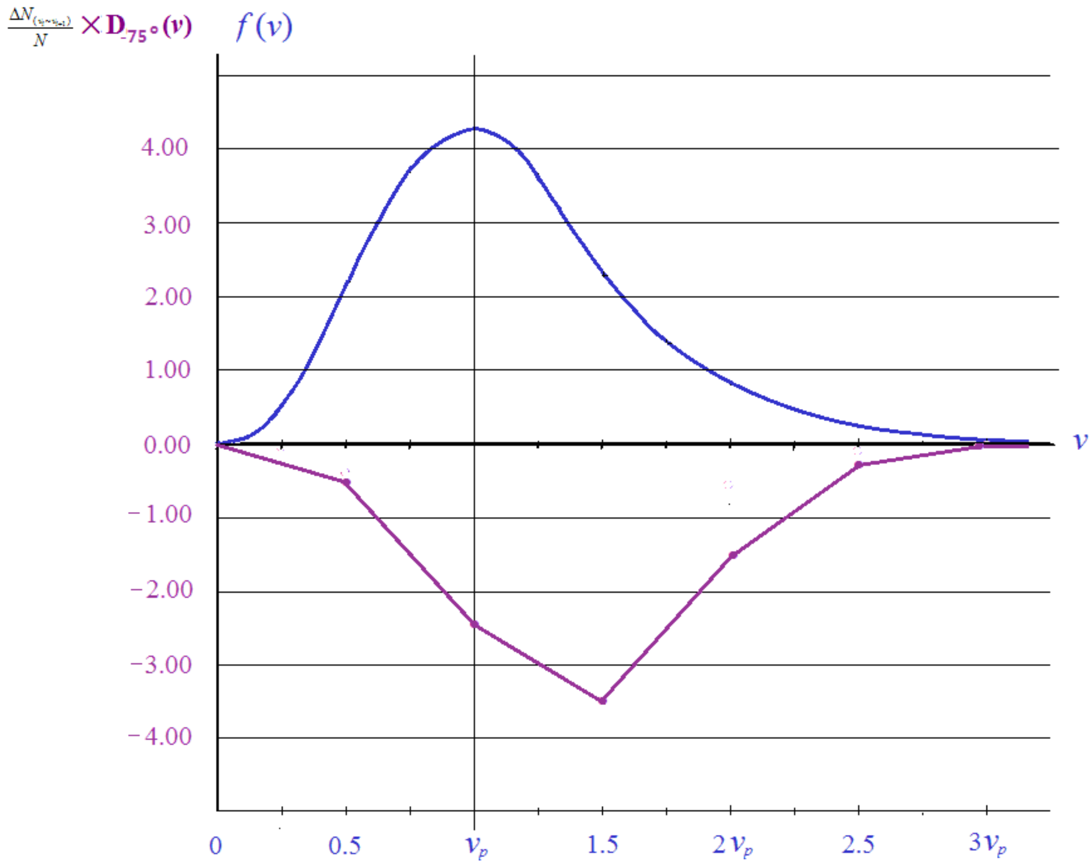
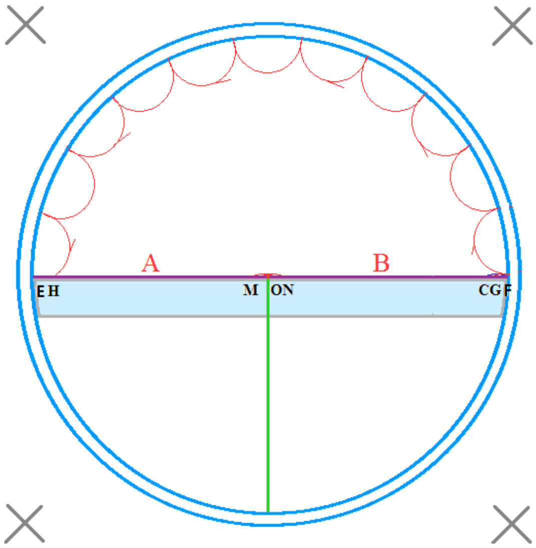


Fig 10 (2) Graph of the actual contributions of electrons of θ = - 75^o

with different speeds, $\dfrac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-75^o} \sim v$.

11. Trajectories of electrons of θ = 75° and different speeds

(1) Fig 11-1 $\theta = 75^o$ $v = 0.5v_p$ R = 2mm (B = 1.34 gauss)



(2.5 : 1, see next page 5 : 1) (refer to Fig.11-3, alike)

A-directly-B MO/EO = 5/70 = 0.07 (MO = 5, from MO to ON)

A-B 7% of the electrons of (75°, $0.5v_p$) emitted from A migrate to B.

B-glass-B 0.5/70 = 0.007

(CD = 0.5, OE = 70) (from CD to FV)

B-glass-A 4.5/70 = 0.06 (from G to H, etc., DF = 4)

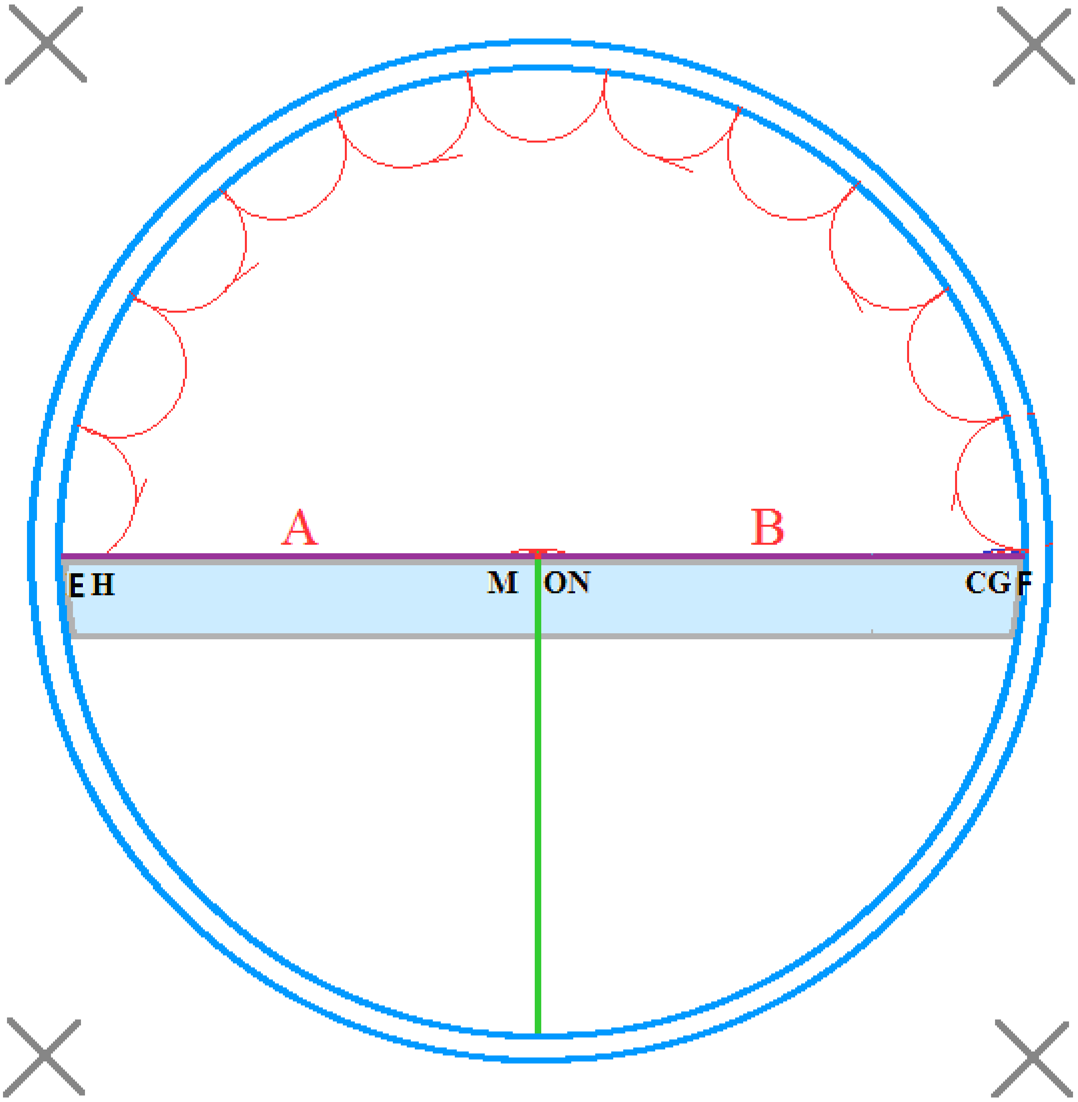
B-A 6% of the electrons of (75°, $0.5v_p$) emitted from B migrate to A.

(Here accurate drawing is difficult, refer to the 5 : 1 figure on next page.)

For all the electrons of (75°, $0.5v_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{75^o\,0.5v_p} = 0.07 - 0.06 = 0.01$$

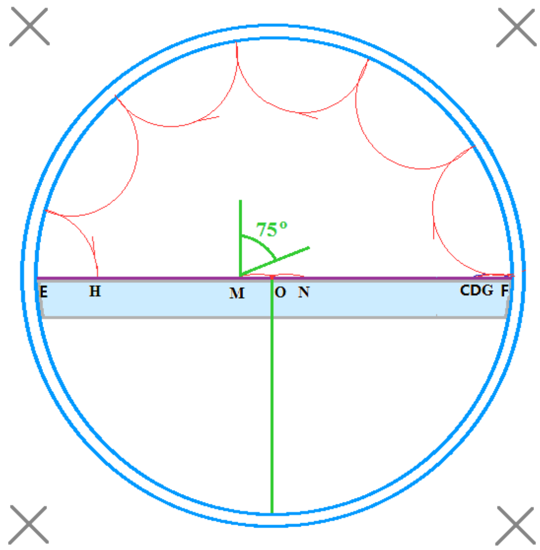
$$D_{75^o}(0.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{75^o\,0.5v_p} \cos 75^o = 0.01 \times 0.2588 = 0.0023 \approx 0.00$$



(5 : 1)

Fig 11-1 $\theta = 75^{\circ}$ $v = 0.5v_p$ $R = 2\text{mm}$

(2) Fig 11-2 θ $= 75^{\mathrm{o}}$ $v = v_p$ $R = 4\mathrm{mm}$ ($cos75^{\mathrm{o}} = 0.2588$)



(2.5 : 1, see next page 5 : 1) (refer to Fig.11-3, alike)

A-directly-B 10/70 = 0.14 (MO = 10, EO = 70)

A-B 14% of the electrons of (75^{o}, v_p) emitted from A migrate to B.

B-glass-B 1/70 = 0.014 (CD =1, OF = 70)

B-glass-A 9/70 = 0.13 (DF = 9, OF = 70) (from G to H, etc.)

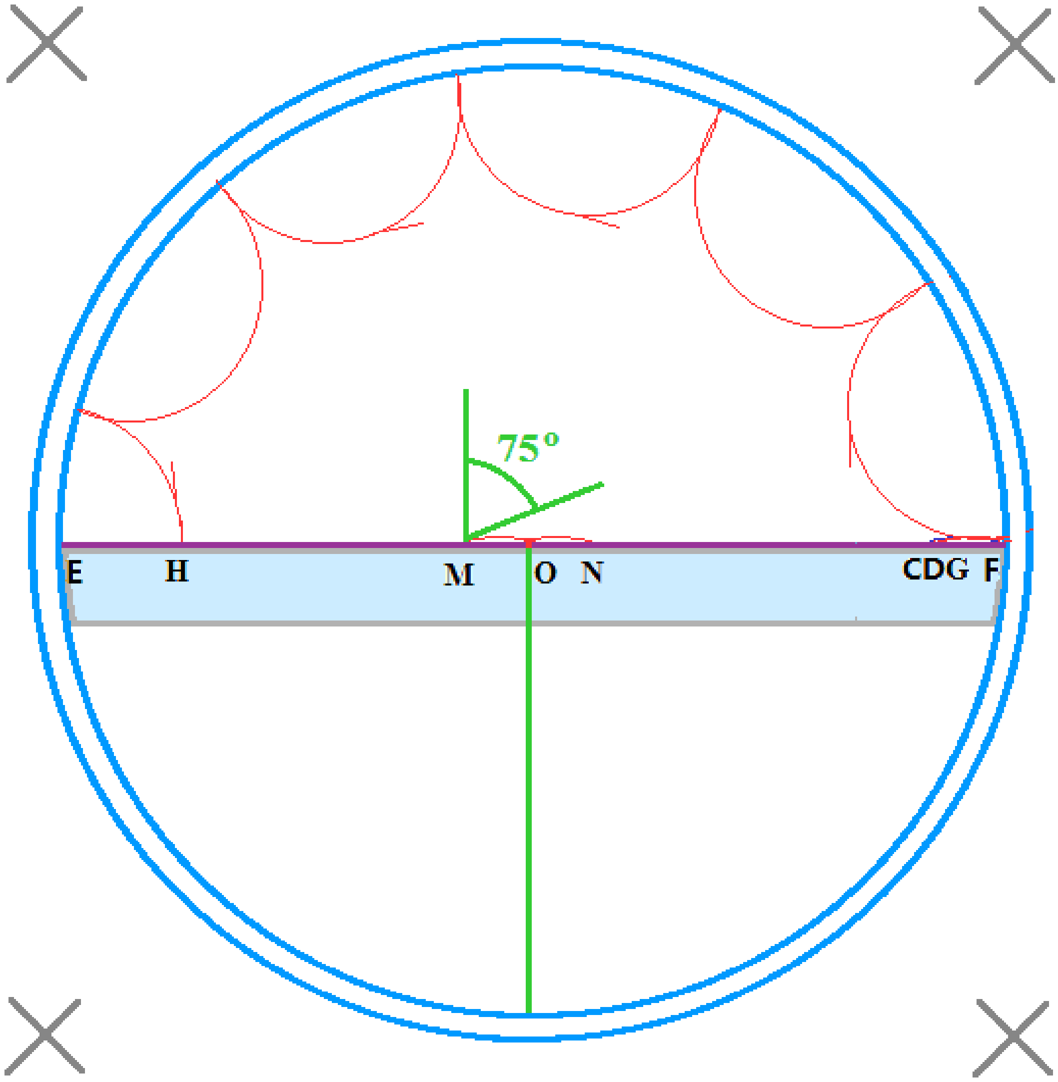
B-A 13% of the electrons of (75^{o}, v_p) emitted from B migrate to A.

(Here accurate drawing is difficult, refer to the 5 : 1 figure on next page.)

For all the electrons of (75^{o}, v_p), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{75^{\mathrm{o}}\ 1v_p} = 0.14 - 0.13 = 0.01$$

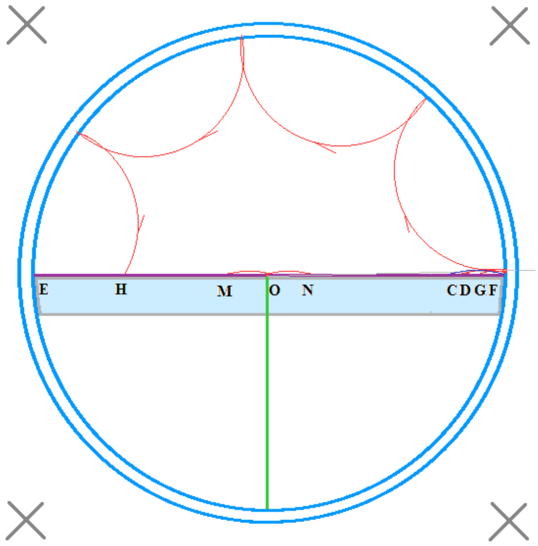
$$D_{75^{\mathrm{o}}}(v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{75^{\mathrm{o}}\ 1v_p} cos75^{\mathrm{o}} = 0.01 \times cos75^{\circ} = 0.002588 \approx 0.00$$



(5 : 1)

Fig 11-2 θ = 75° $v = v_p$ R = 4mm

(3) Fig 11-3 $\theta = 75^{\circ}$ $v = 1.5v_p$ $R = 6\text{mm}$



(2.5 : 1)

A-directly-B MO/EO = 13/70= 0.186 (MO = 13, from MO to ON)

A-B 18.6% of the electrons of (75°, $1.5v_p$) emitted from A migrate to B.

B-glass-B 2/70 = 0.028 (CD =2, OF = 70)

B-glass-A DF/OF = 11/70 = 0.157 (DF = 11, from G to H, etc.)

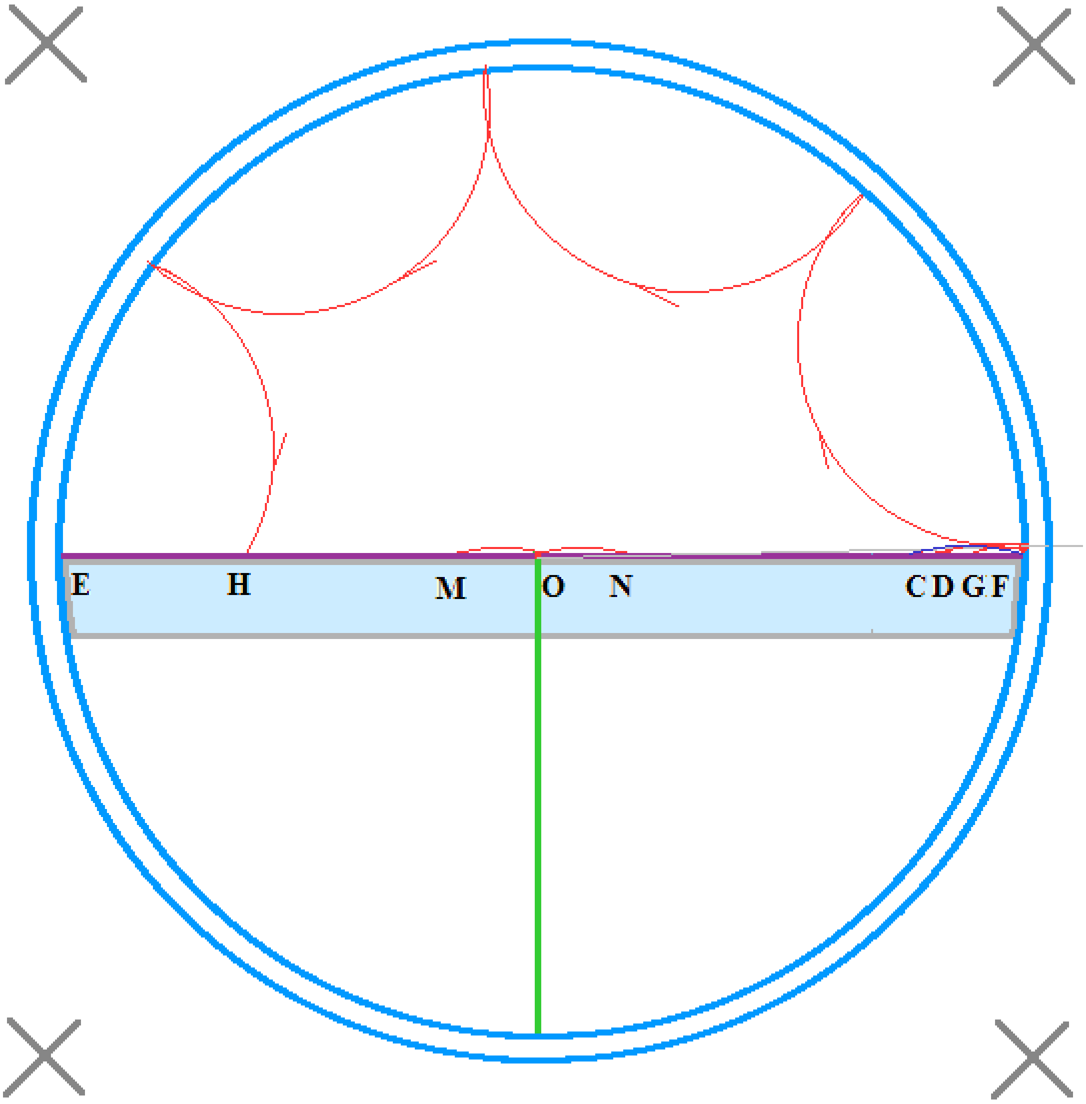
B-A 15.7% of the electrons of (75°, $1.5v_p$) emitted from B migrate to A.

(Here accurate drawing is difficult, refer to the 5 : 1 figure on next page.)

For all the electrons of (75°, $1.5v_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{75^{\circ}\,1.5v_p} = 0.186 - 0.157 = 0.029$$

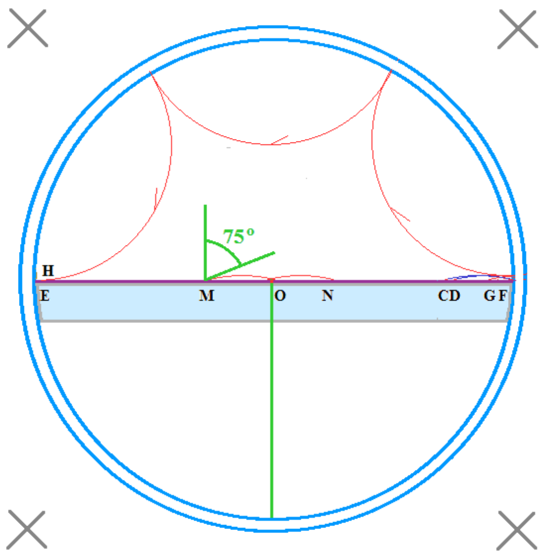
$$D_{75^{\circ}}(1.5v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{75^{\circ}\,1.5v_p} \cos 75^{\circ} = 0.029 \times 0.2588 = 0.0075 \approx 0.01$$



(5 : 1)

Fig 11-4 $\theta = 75^{\circ}$ $v = 1.5v_p$ $R = 6\text{mm}$

(4) Fig 11-4 $\theta = 75^{\circ}$ $v = 2v_p$ $R = 8\text{mm}$



(2.5 : 1, See next page)

A-directly-B 20/70 = 0.2857 (MO = 20, EO =70)

A-B 28.6% of the electrons of (75°, $2v_p$) emitted from A migrate to B.

B-glass-B 3/70 = 0.04 (CD = 3, OF =70)

B-glass-A 17/70 = 0.2428 (DF =17, OF =70) (from G to H, etc.)

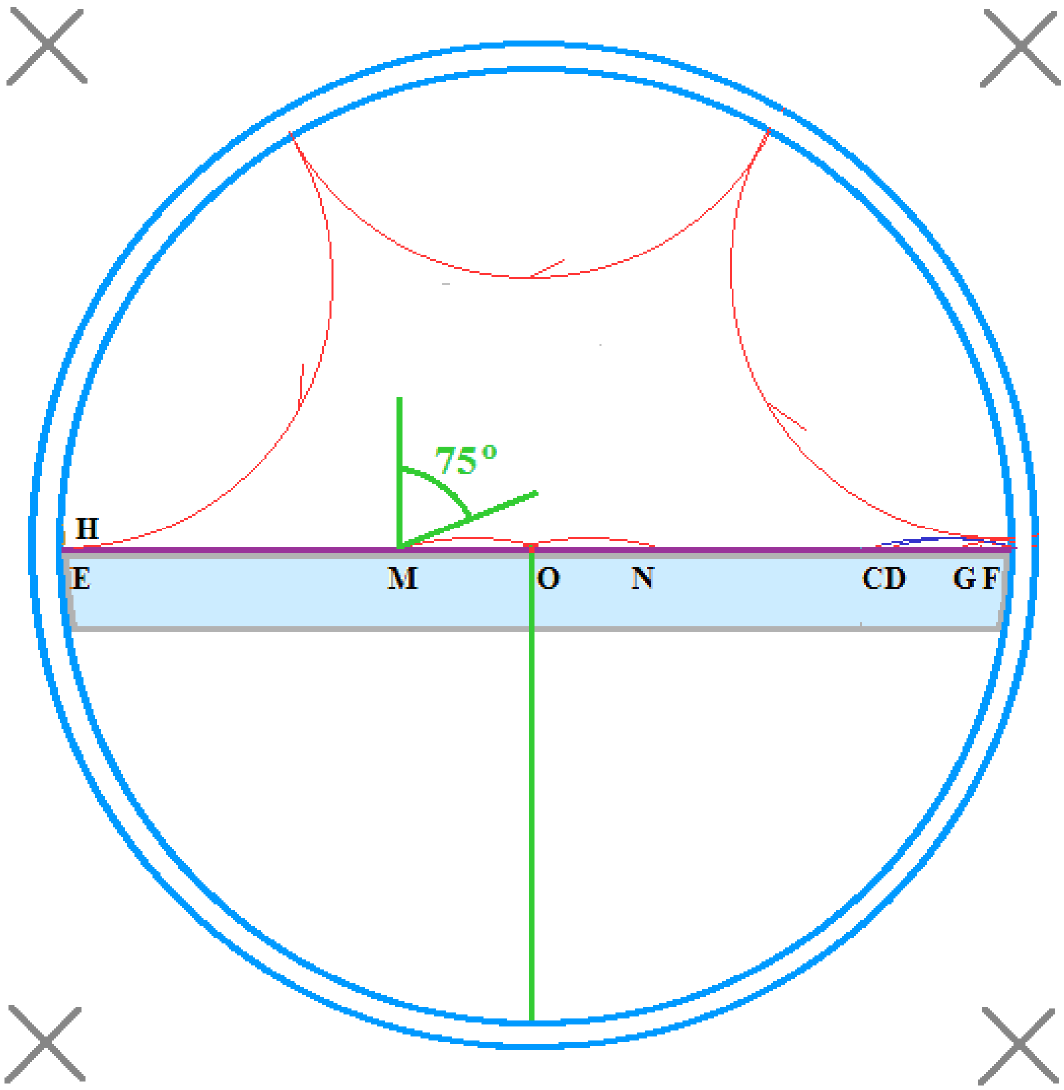
B-A 24.3% of the electrons of (75°, $2v_p$) emitted from B migrate to A.

(Here accurate drawing is difficult, refer to the 5 : 1 figure on next page.)

For all the electrons of (75°, $2v_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{75^{\circ}\ 1v_p} = 0.286 - 0.243 = 0.04$$

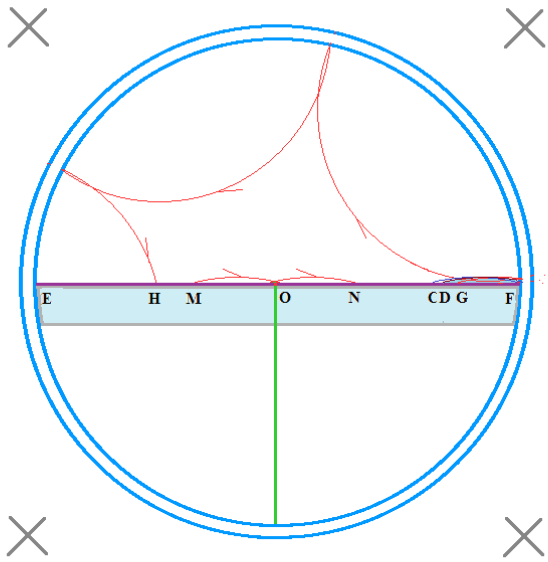
$$D_{75^{\circ}}(2v_p) = \{(A\text{-}B) - (B\text{-}A)\}_{75^{\circ}\ v_p} cos75^{\circ} = 0.04 \times 0.2588 = 0.0103 \approx 0.01$$



(5 : 1)

Fig 11-5 θ = 75° $v = 2v_p$ R = 8mm

(4) Fig 11-5 $\theta = 75^{\circ}$ $v = 2.5v_p$ $R = 10\text{mm}$



(2.5 : 1, see next page)

A-directly-B MO/EO =24/70 = 0.343 (from MO to ON)

A-B 34% of the electrons of (75°, $2.5v_p$) emitted from A migrate to B.

B-glass-B 4/70 = 0.057 (CD = 4, OF = 70)

B-glass-A DF/OF = 20/70 = 0.286 (from G to H, etc.)

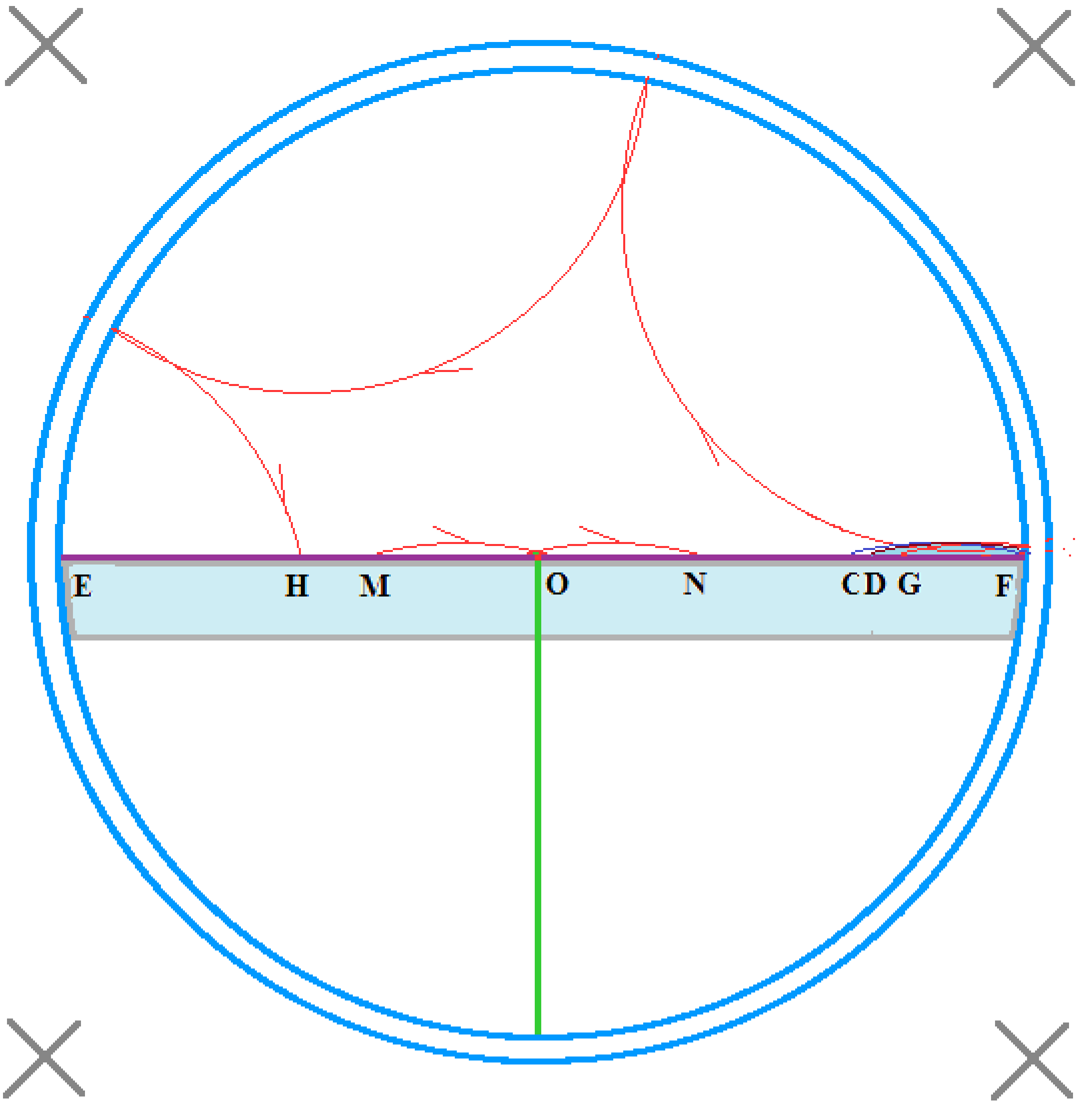
B-A 29% of the electrons (75°, $2.5v_p$) emitted from B migrate to A.

(Here accurate drawing is difficult, refer to the 5 : 1 figure on next page.)

For all the electrons of (75°, $2.5v_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(A\text{-}B) - (B\text{-}A)\}_{75^{\circ}\ 2.5v_p} = 0.343 - 0.286 = 0.057$$

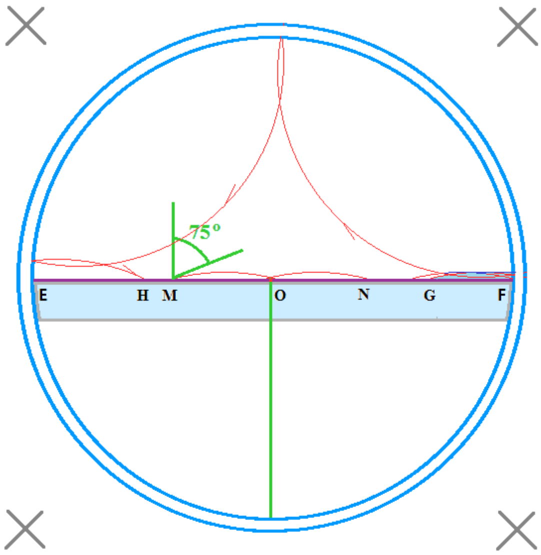
$$D_{75^{\circ}}(2.5v_p) = \{(A\text{-}B)-(B\text{-}A)\}_{75^{\circ}\ 2.5v_p} \cos 75^{\circ} = 0.057 \times 0.2588 = 0.0148 \approx 0.015$$



(5 : 1)

Fig 11-5-1 $\theta = 75^{o}$ $v = 2.5v_p$ $R = 10\text{mm}$

(6) Fig 11-6 $\theta = 75^{o}$ $v = 3v_p$ $R = 12\text{mm}$



(2.5 : 1, see next page)

A-directly-B 29/70 = 0.414 (MO = 29, EO = 70)

A-B 41% of the electrons of (75^{o}, $3v_p$) emitted from A migrate to B.

B-glass-B (5/70 = 0.071) (CD = 5, OF = 70)

B-glass-A 24/70 = 0.342 (DF = 24, OF = 70) (from G to H, etc., blue)

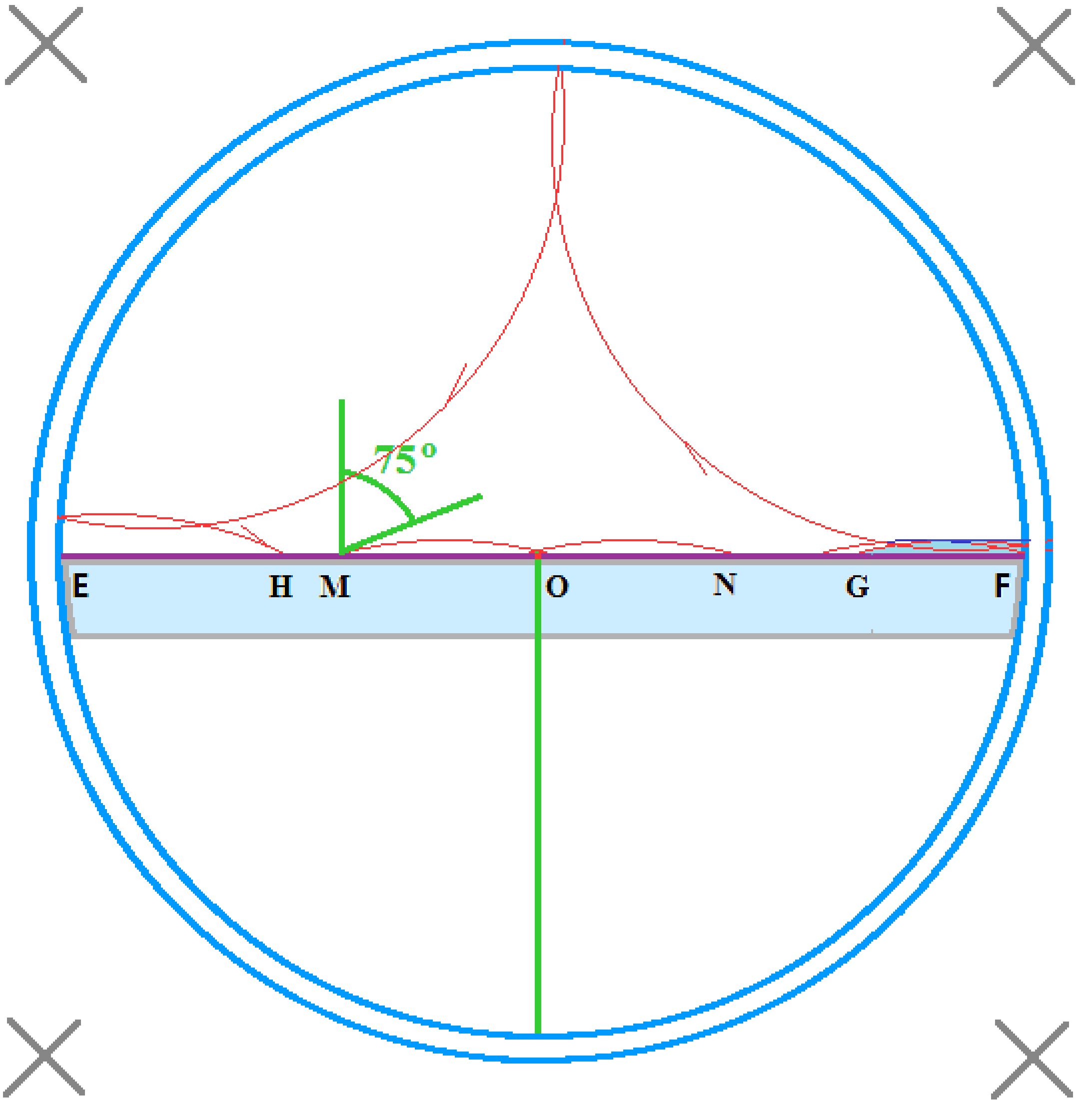
B-A 34% of the electrons (75^{o}, $3v_p$) emitted from B migrate to A.

(Here accurate drawing is difficult. refer to the 5 : 1 figure on next page.)

For all the electrons of (75^{o}, $3v_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(\text{A-B}) - (\text{B-A})\}_{75^{o}\ 3v_p} = 0.414 - 0.342 = 0.072$$

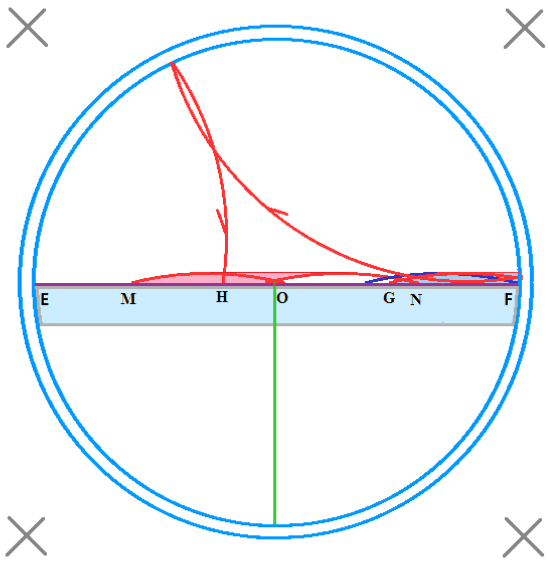
$$D_{75^{o}}(3v_p) = \{(\text{A-B}) - (\text{B-A})\}_{75^{o}\ 3v_p} \cos 75^{o} = 0.072 \times 0.2588 = 0.0186 = 0.02$$



(5 : 1)

Fig 11-6 θ = 75° $v = 3v_p$ R = 12mm

7) Fig 11-7 $\theta = 75^{o}$ $v = 4.5v_p$ $R = 18\text{mm}$



(2.5 : 1, see next page)

A-directly-B 45/70 = 0.642 (MO = 45, EO =70) (from Mo to ON, pink)

A-B 64% of the electrons of (75^{o}, $4.5v_p$) emitted from A migrate to B.

B-glass-B 6/70 = 0.086 (CD = 6, OF =70)

B-glass-A 39/70 = 0.557 (DF = 39, OF =70) (from G to H, etc. blue)

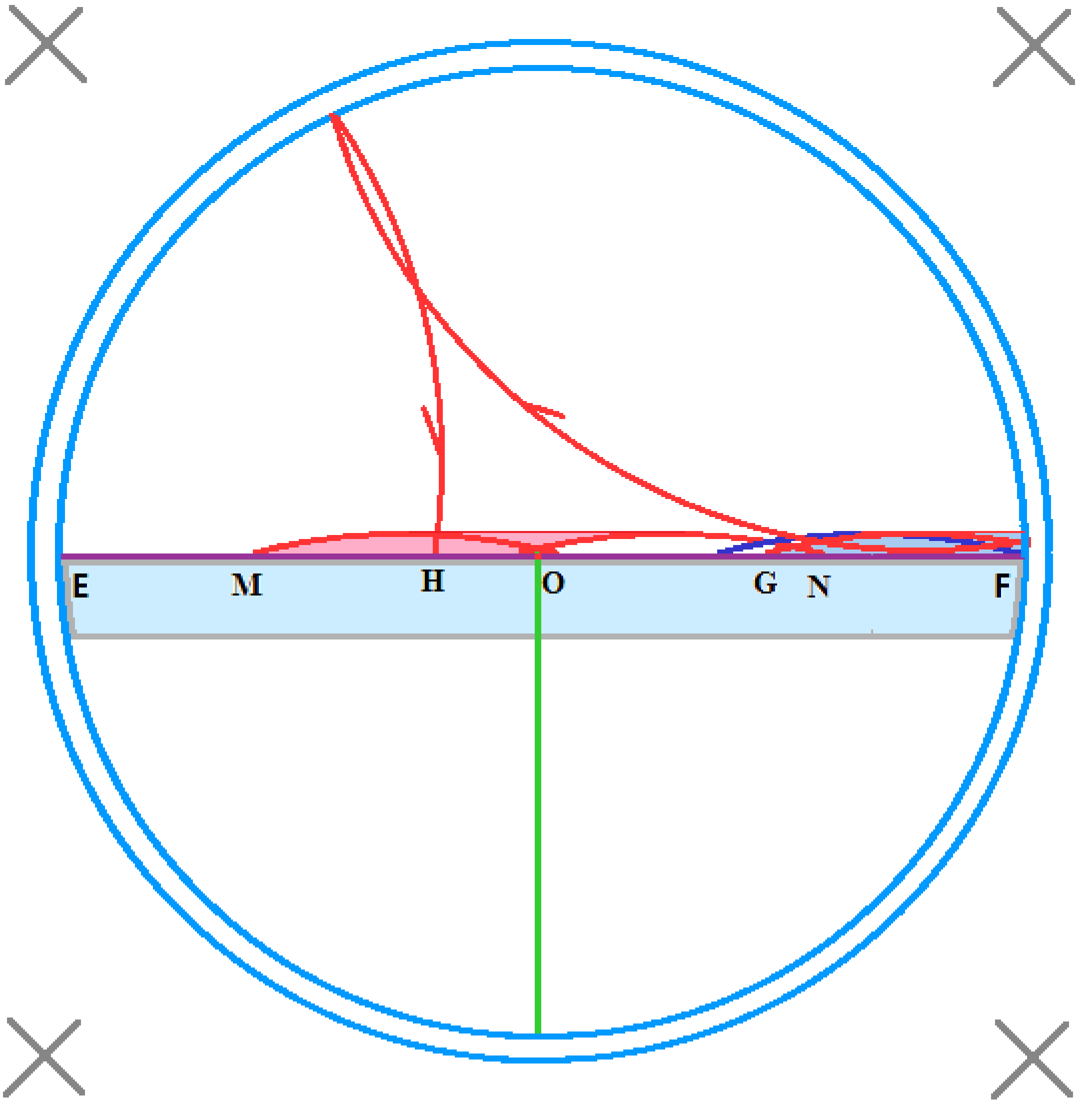
B-A 55.7% of the electrons of (75^{o}, $4.5v_p$) emitted from B migrate to A.

(Here accurate drawing is difficult. Refer to the 5 : 1 figure on next page.)

For all the electrons of (75^{o}, $4.5v_p$), A-B exceeds B-A, and their difference is the corresponding contribution to the output current.

$$\{(\text{A-B}) - (\text{B-A})\}_{75^{o}\,4.5\,v.p} = 0.642 - 0.557 = 0.085$$

$$D_{75^{o}}(4.5v_p) = \{(\text{A-B})-(\text{B-A})\}_{75^{o}\,.4.5v_p} \cos 75^{o} = 0.085 \times 0.2588 = 0.0219 \approx 0.02$$



(5 : 1)

Fig 11-7-1 θ = 75° $v = 4.5v_p$ R = 18mm

Contributions of electrons of θ = 75^{o} with different speeds, $D_{75^{o}}(v)$. And Fig 11 (1) is the corresponding graph.

$\theta = 75^{o}$ $\cos\theta = 0.2588$	$\{(A\text{-}B) - (B\text{-}A)\}_{\theta=75^{o}}$	$D_{75^{o}}(v) =$ $\{(A\text{-}B) - (B\text{-}A)\}_{75^{o}} \cos\theta$
Fig 7-1 $v = 0.5v_p$	0.07 – 0.06 = 0.01	0. 00
Fig 7-2 $v = v_p$	0.14 – 0.13 = 0.01	0. 00
Fig 7-3 $v = 1.5v_p$	0.186 – 0.157 = 0.029	0. 01
Fig 7-4 $v = 2v_p$	0.286 – 0.243 = 0.04	0. 01
Fig 7-5 $v = 2.5v_p$	0.34 – 0.29 = 0.05	0. 015
Fig 7-6 $v = 3v_p$	0.41 – 0.34 = 0.07	0. 02
Fig 7-7 $v = 4.5v_p$	0.64 – 0.56 = 0.08	0. 02

Tab 11 (1) $D_{75^{o}}(v) = \{(A\text{-}B) - (B\text{-}A)\}_{75^{o}4.5v_p} \cos 75^{o}$

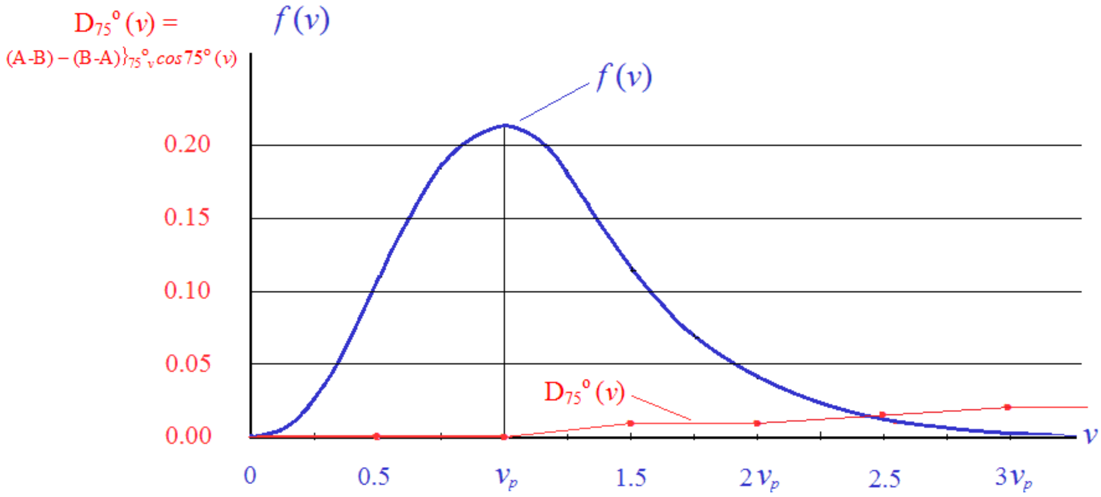


Fig 11 (1) Graph of Contributions of electrons of θ = 75^{o} with different speeds

Take Maxwell's speed distribution into account to derive the actual contributions of the thermal electrons of $\theta = 75^{o}$ with different speeds, i.e., $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{75^{o}} \sim v$. And Fig 11 (2) is the corresponding graph.

Speed range $x = v/v_p$	$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N}$	$D_{75}^{o}(v)$	$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{75}^{o}(v)$
$0.00 \sim 0.25v_p$	$A_0 \approx 0.004$	≈ 0	≈ 0
$0.25 \sim 0.75v_p$	$A_1 = 18.74\%$	0.00	0.0000
$0.75 \sim 1.25v_p$	$A_2 = 39.83\%$	0.00	0.0000
$1.25 \sim 1.75v_p$	$A_3 = 26.71\%$	0.01	0.2671
$1.75 \sim 2.25v_p$	$A_4 = 8.82\%$	0.01	0.0882
$2.25 \sim 2.75v_p$	$A_5 = 1.58\%$	0.015	0.0237
$2.75 \sim 3.25v_p$	$A_6 = 0.16\%$	0.02	0.0032
$3.25v_p \sim \infty$	$A_7 A_8 \approx 0.0099\%$	0.02	≈ 0
			$\sum_v \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{75}^{o}(v) = 0.3822$

Tab 11 (2) $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{75}^{o} \sim v$

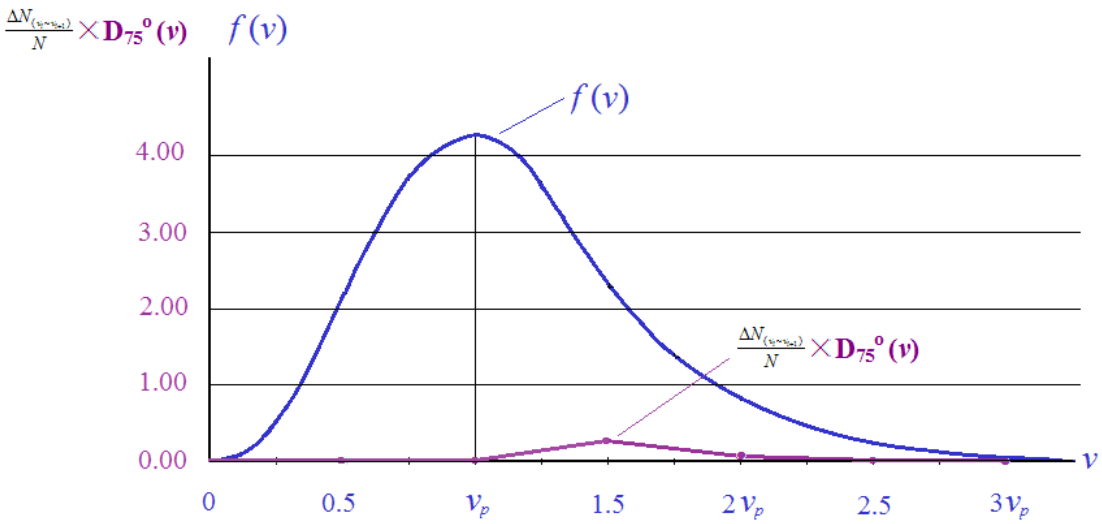


Fig 11 (2) Graph of $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{75}^{o} \sim v$

Discussion

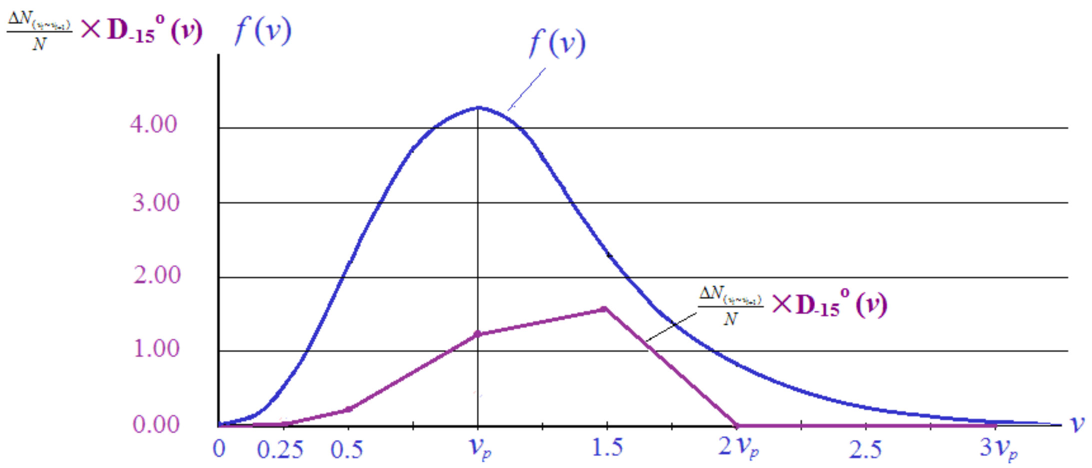
Altogether, there are eleven graphs of $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times \mathrm{D}_\theta \sim v$ (corresponding to eleven different exiting angles θ): seven are positive graphs and four negative graphs.

(1) The seven positive graphs

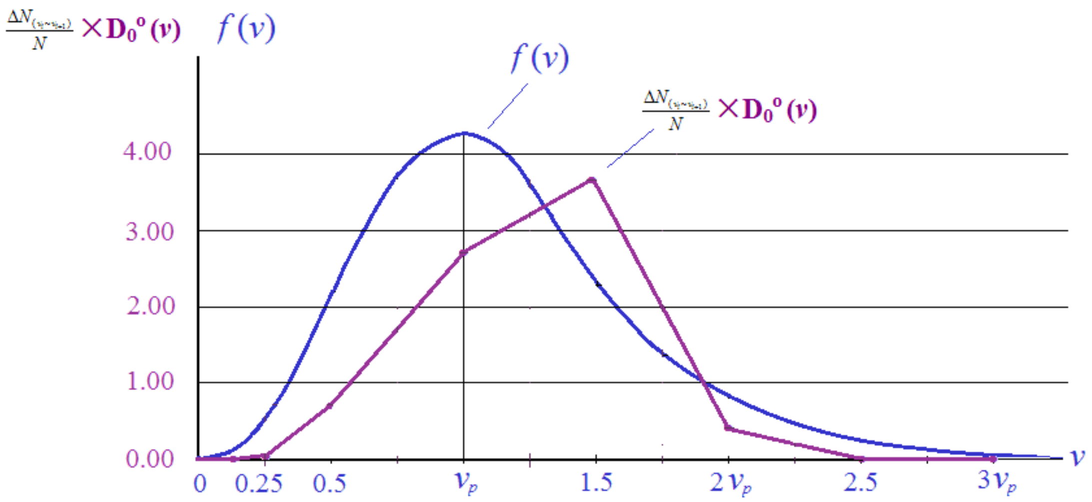
For θ= -15 °, 0° , 15°, 30°, 45°, 60°, 75 °, $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times \mathrm{D}_\theta$ are positive.

And in any of these graphs, for any speed, the electron migration of A-B exceeds the one of B-A.

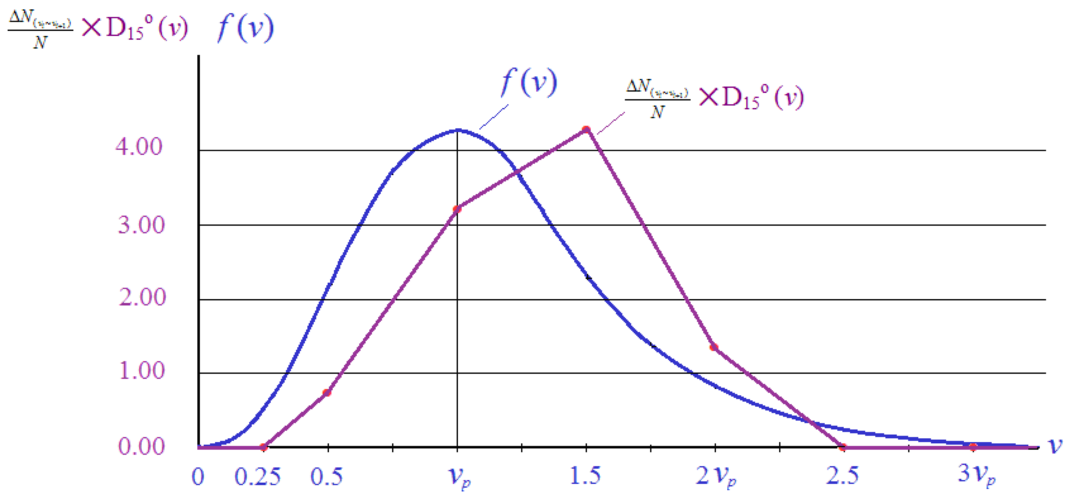
θ= -15 °



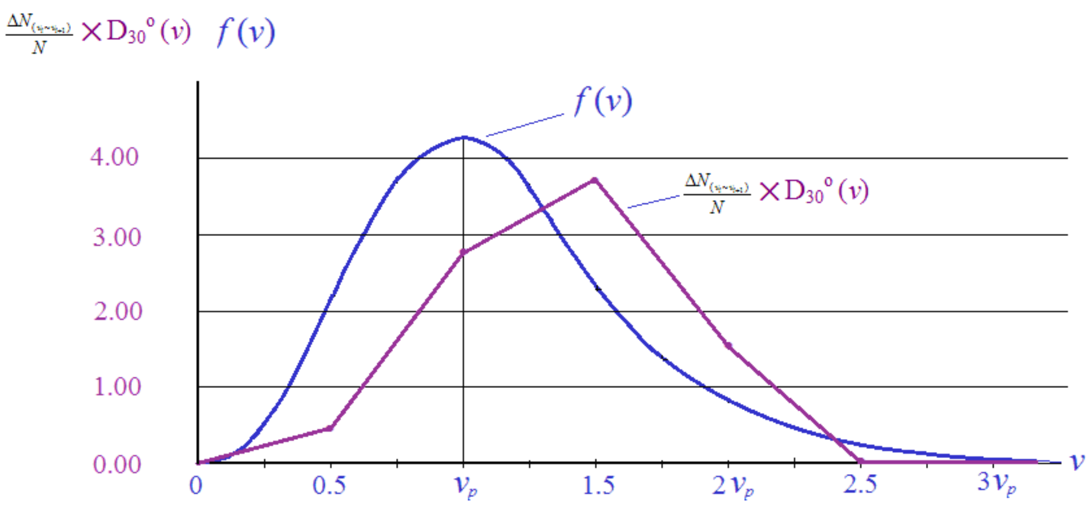
$\theta = 0°$



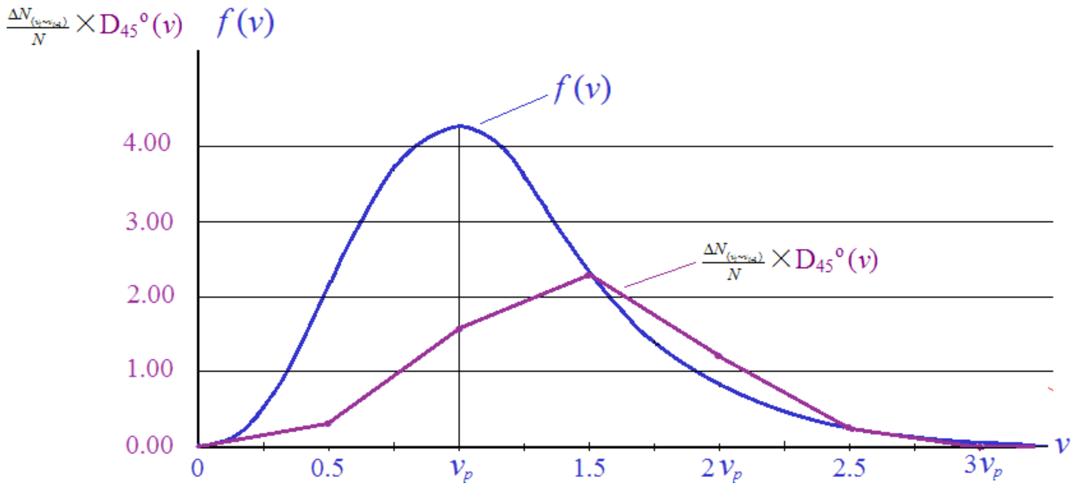
$\theta = 15°$



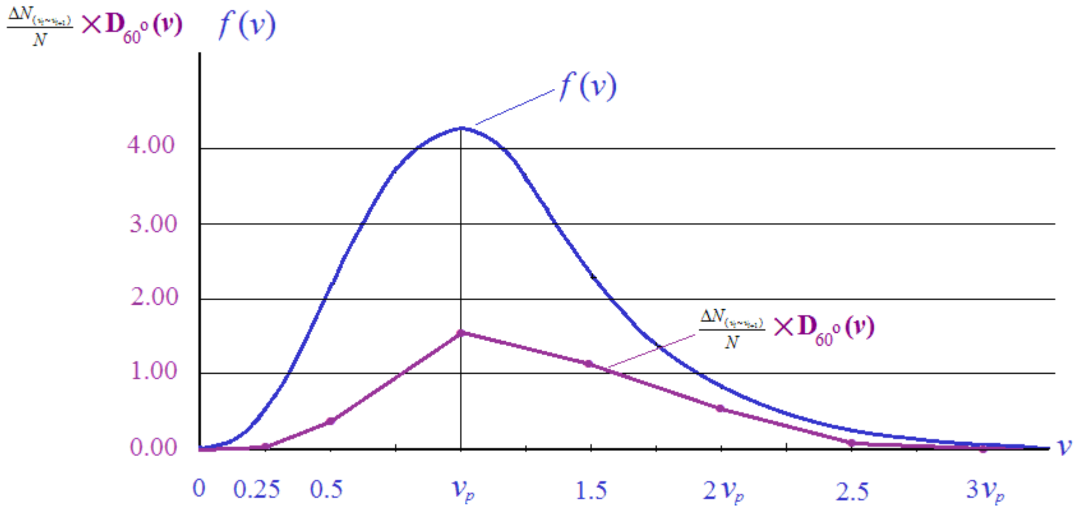
$\theta = 30°$



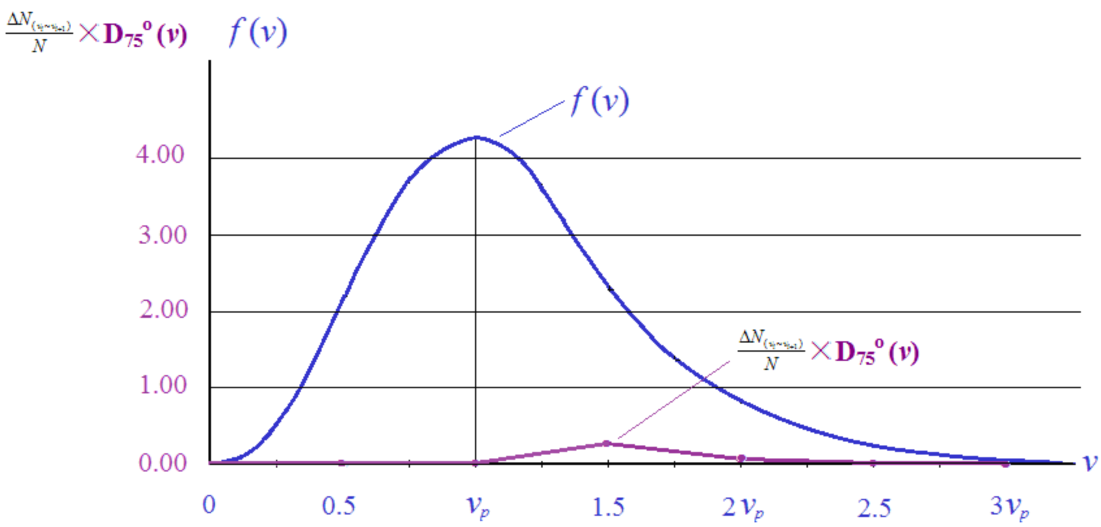
$\theta = 45^{\circ}$



$\theta = 60^{\circ}$



$\theta = 75^{\circ}$

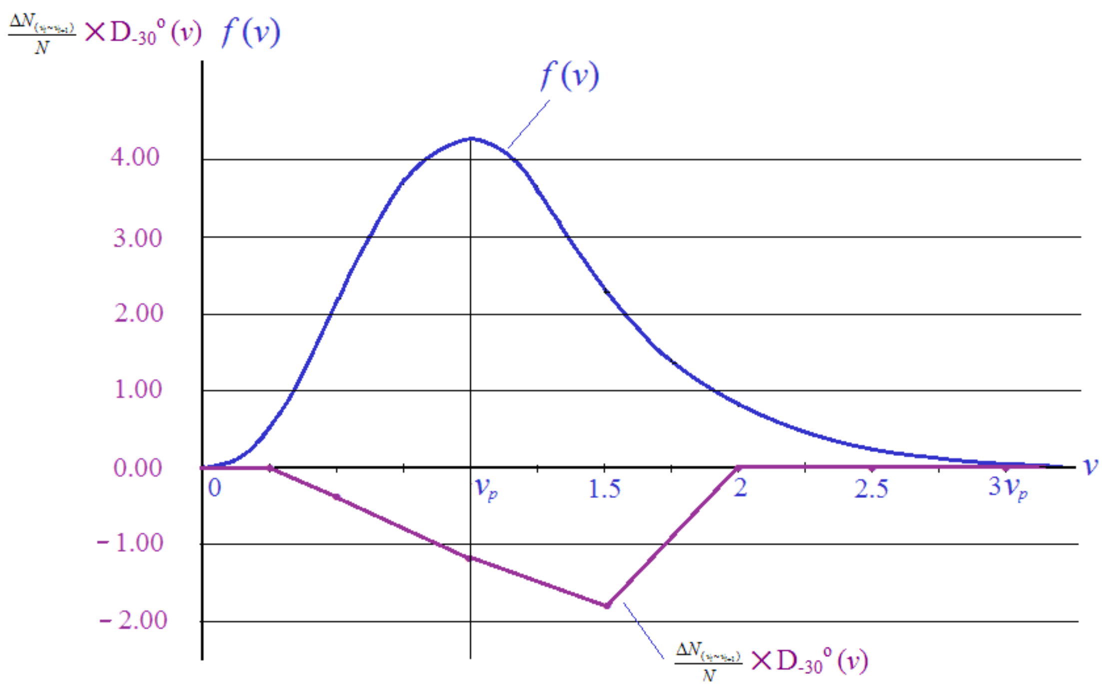


(2) The four negative graphs

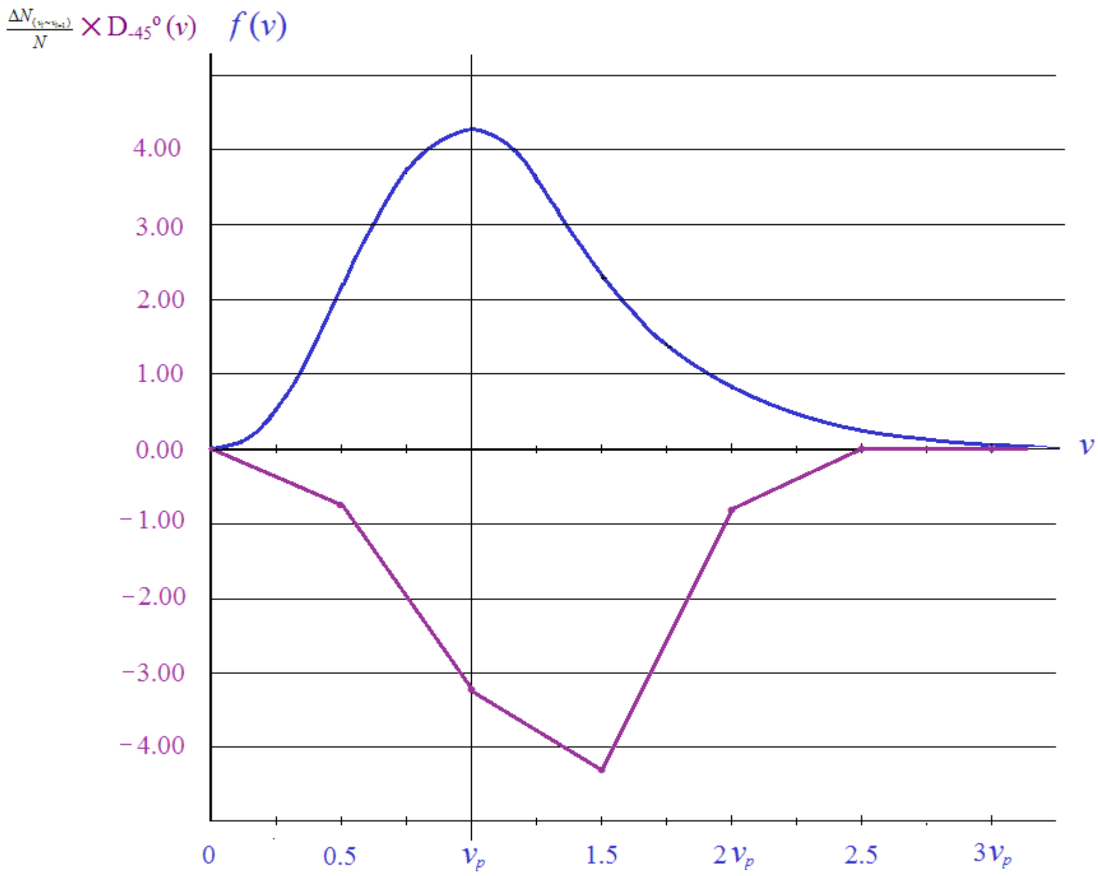
For θ= -30 º, -45º, -60º, -75 º, $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{\theta}$ are all negative.

In these graphs, for any speed, the electron migration of A-B is less than the one of B-A.

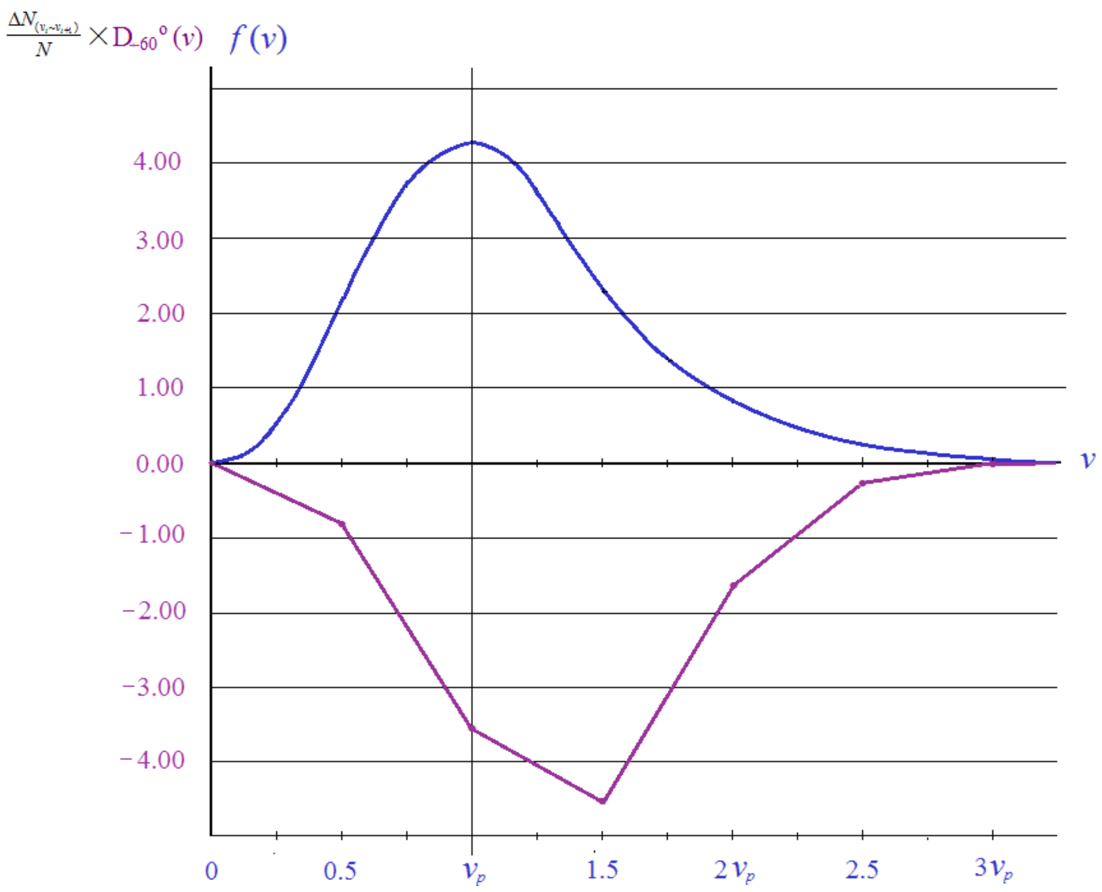
θ= -30º



$\theta = -45^{\circ}$

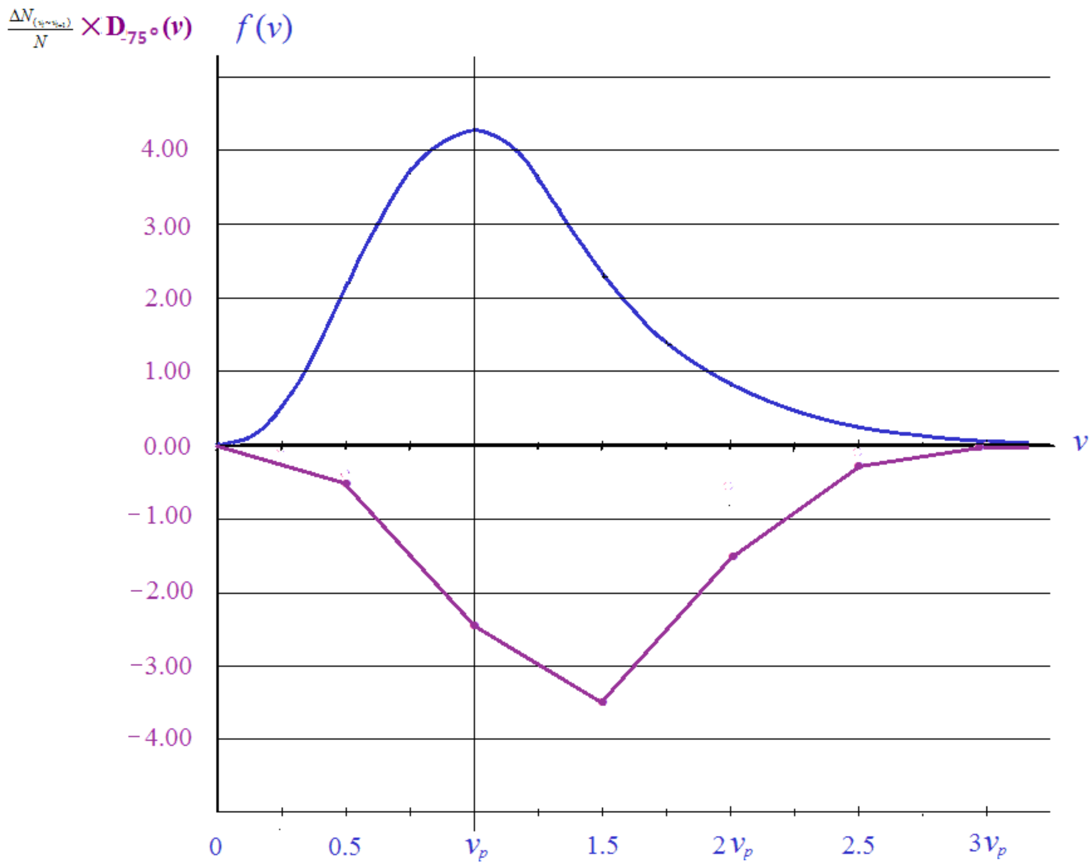


θ = - 60 °



$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-60°}(v)$ $f(v)$
4.00
3.00
2.00
1.00
0.00
-1.00
-2.00
-3.00
-4.00
v
0
0.5
v_p
1.5
$2v_p$
2.5
$3v_p$

θ = - 75°



$\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times \mathbf{D}_{-75°}(v)$
$f(v)$
4.00
3.00
2.00
1.00
0.00
−1.00
−2.00
−3.00
−4.00
v
0
0.5
v_p
1.5
$2v_p$
2.5
$3v_p$

The areas below the eleven graphs $\frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_\theta \sim v$ are the relative contributions to the output current of the electrons of the corresponding exiting angles, also, seven are positive, and four are negative.

The areas of the seven positive ones are:

$\theta = -15^{\circ}$ $\quad \Sigma_v \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-15^{\circ}}(v) = 2.9849$

$\theta = 0^{\circ}$ $\quad \Sigma_v \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{0^{\circ}}(v) = 7.6349.$

$\theta = 15^{\circ}$ $\quad \Sigma_v \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{15^{\circ}}(v) = 9.5486$

$\theta = 30^{\circ}$ $\quad \Sigma_v \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{30^{\circ}}(v) = 8.6395$

$\theta = 45^{\circ}$ $\quad \Sigma_v \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{45^{\circ}}(v) = 5.5414$

$\theta = 60^{\circ}$ $\quad \Sigma_v \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{60^{\circ}}(v) = 3.7804$

$\theta = 75^{\circ}$ $\quad \Sigma_v \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{75^{\circ}}(v) = 0.3822$

Their sum is

$$\Sigma_{\theta(-15^{\circ} \sim +75^{\circ})} \Sigma_i \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{\theta}(v) = 38.5119$$

The areas of the four negative ones are:

$\theta = -30^{\circ}$ $\quad \Sigma_v \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-30^{\circ}}(v) = - 3.4394$

$\theta = -45^{\circ}$ $\quad \Sigma_v \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-45^{\circ}}(v) = - 9.0034$

$\theta = -60^{o}$ $\quad \sum_{v} \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-60^{o}}(v) = -10.9752$

$\theta = -75^{o}$ $\quad \sum_{v} \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{-75^{o}}(v) = -8.1847$

Their sum is

$$\sum_{\theta(-30^{o} \sim -75^{o})} \sum_{i} \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{\theta}(v) = -31.6027$$

The algebraic sum of the positive and negative sums is proportional to the general output current,

$$\sum_{\theta(-75^{o} \sim +75^{o})} \sum_{i} \frac{\Delta N_{(v_i \sim v_{i+1})}}{N} \times D_{\theta}(v) = 38.5119 - 31.6027 = 6.9092.$$

The final result is positive. That means, when the direction of the magnetic field is positive (directed into the plane of the figure, as shown in Fig 6(b)), the **general net electron migration between A and B** is from A to B, and, the output current of the tube is positive.

If the direction of the magnetic field is opposite (directed out off the plane of the figure, as shown in Fig 6(c)), all the trajectories change symmetrically (mirror reflect symmetrically, left-right symmetrically), and **the general net electron migration between A and B is from B to A**, **the general output current of the tube is negative**.

These conclusions coincide with the results of the experiments of most of our electron tubes.

The experiments enlightened us more.

In the above graphical survey of the electron trajectories of various angles and speeds and exiting spots, we have actually assumed that the work function of the various parts of the two Ag-O-Cs surfaces in a tube is uniform. After 20 year's electron tube manufacture and numerous measurements of the output currents in experiments, we realized, that was often true, but sometimes, that might be not true, i.e., the work function of the various parts of the two Ag-O-Cs surfaces in a tube might not be uniform.

The oxidizing of the silver film might not be uniform. The anode for oxygen discharge was high at the top part of the tube, and the distances from the anode to the different parts of the surfaces of A and B were different. Hence, the extent of oxidization (by thin oxygen discharge) of these different parts might not be uniform.

The shapes and sizes of the anode and the two cathodes, and their relative positions, might not be standard as designed. They might be different from tube to tube.

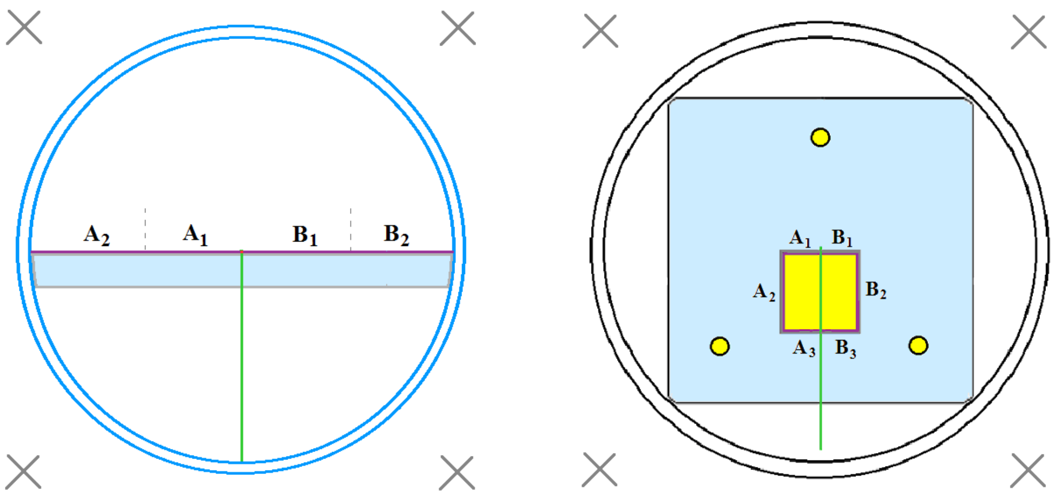
The cesium taking in during manufacture was mainly a rush. Hence, the distribution of cesium steam to the different parts of the two surfaces A and B might be not uniform. For different tubes, the distribution might be different.

And so on.

What kinds of different results may be derived in our experiments due to the non-uniform of the work function on the different parts of the two emitters?

Let us see some imaginary but reasonable situations.

If the work functions of the two central parts A_1 and B_1 of the two emitters as shown in Fig 8(a) are both 0.81eV, they will emit less thermal electrons. And if in the same time, the work functions of the two side parts A_2 and B_2 are both 0.79eV, they will emit more thermal electrons. Thus, according to the method of our graphical survey, when the magnetic field is applied to the tube, the number of electron trajectories



(a) (b)

Fig. 8 The work functions of the various parts of A and B may be different.

of A-B will decrease, and the number of electron trajectories of B-A will increase, and the general net electron migration will change, even to be a negative one, i.e., from B to A. Thus, the output current will be negative.

And, in such a case, if the direction of the magnetic field is reversed, the direction of the net electron migration will also be reversed, i.e., from A to B, and the output current will be positive.

The experiment with several our electron tubes behaved this way. We call these tubes **the negative-current tubes**.

If the work functions of A_1 and B_1 are both 0.79eV, and the work functions of A_2 and B_2 are both 0.81eV, what will happen?

If the work functions of A_1 and A_2 are both 0.79eV, and the work functions of B_1 and B_2 are both 0.81eV, what will happen?

If the work functions of A_1 and A_2 are both 0.81eV, and the work functions of B_1 and B_2 are both 0.79eV, what will happen?

And so on.

It is not difficult for the interested readers to guess the answers of these questions.

And the answers coincide with the experiments of some of our tubes.

Nevertheless, the most probable situation is still, when a positive magnetic field is applied to the tube, a positive output current, as illustrated by Fig 2 (b). That is, the net thermal electron migration is from A to B, and we call these tubes **the positive-current tubes**. We think, mostly, in these tubes, the work functions of the different parts of A and B are fundamentally uniform.

Anyway, when a magnetic field is applied to the tube, the net electron migrations between A and B is usually not exactly zero. There is an output current, maybe positive, may be negative. All these currents are stable macroscopic ones (a DC!) produced from the chaos of the electron tube and the ambient air.

So, we realized: **the magnetic field is a successful demon, which accomplishes its job quietly and easily, without any expenditure of work**.

In physics, experiment is always the final criterion for major arguments. We once again suggest the readers have a careful watch at our experiment video, about 30 minutes:

https://www.youtube.com/watch?v=PyrtC2nQ_UU.

Reference

〔1〕 Xinyong Fu and Zitao Fu: *Realization of Maxwell's Hypothesis, a Heat-Electric Conversion in Contradiction to the Kelvin Statement* of the Second Law of Thermodynamics, Preprints.org2016070028.v6 (2016 ~ 2018)

〔2〕Xinyong Fu and Zitao Fu: *Realization of Maxwell's Hypothesis,* arxiv.org/physics/0311104v3 (2003 ~ 2012)

〔3〕 Xinyong Fu, *An approach to Realize Maxwell's Hypothesis*, Energy Conversion and Management, Vol.22 pp1-3 (1982)

Appendix: About Maxwell's gas molecule (and thermal electron) speed distribution law

(1) Speed distribution of three dimensions or two dimensions

In our design and experiment, we only consider the thermal electron's motion in the XOY plane. Their motion along the Z axis (the direction of the magnetic field) is not important and may be neglected.

The original Maxwell's speed distribution is of three dimensions

$$f(v_x, v_y, v_z)dv_x dv_y dv_z = n(\frac{m}{2\pi kT})^{3/2} e^{-\frac{m}{2kT}(v_x{}^2+v_y{}^2+v_z{}^2)} dv_x dv_y dv_z \,. \quad (1)$$

It is the number of gas molecules **in unit volume** and in the infinitesimal speed intervals dv_x dv_y dv_z. So, it is a **volume distribution**, and the speed is of three dimensions.

By integration over v_z, equation (1) can be reduced to a two-dimensional one

$$f(v_x, v_y)dv_x dv_y = n(\frac{m}{2\pi kT})e^{-\frac{m}{2kT}(v_x{}^2+v_y{}^2)} dv_x dv_y \,.$$

Let $u^2 = v_x^2 + v_y^2$, we derive

$$f(u)du = n(\frac{m}{2\pi kT})e^{-\frac{m}{2kT}u^2} 2\pi u du$$

$$f(u)du = n\frac{m}{kT} e^{-\frac{m}{2\pi kT}u^2} u du \quad (2)$$

It is the number of gas molecules **in unit volume** and in the infinitesimal speed intervals du (from u to $u+du$). This is also a **volume distribution**, and speed u is of two dimensions, using cylindrical coordinates.

(2) Volume distribution and **beam distribution**

There are two kinds of speed distributions of gas molecules (or thermal electrons), namely, volume distribution and beam distribution.

(A) **Volume distribution**

Usually, Maxwell gas molecule speed distribution is a **volume distribution**, the speed v is of three dimensions, it often takes the form

$$f(v)dv = 4\pi n\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m}{2kT}v^2} v^2 dv \qquad (3)$$

where $f(v)dv$ is the number of molecules of the infinitesimal speed interval dv (from v to $v + dv$) **in unit volume**, and v is of three dimensions.

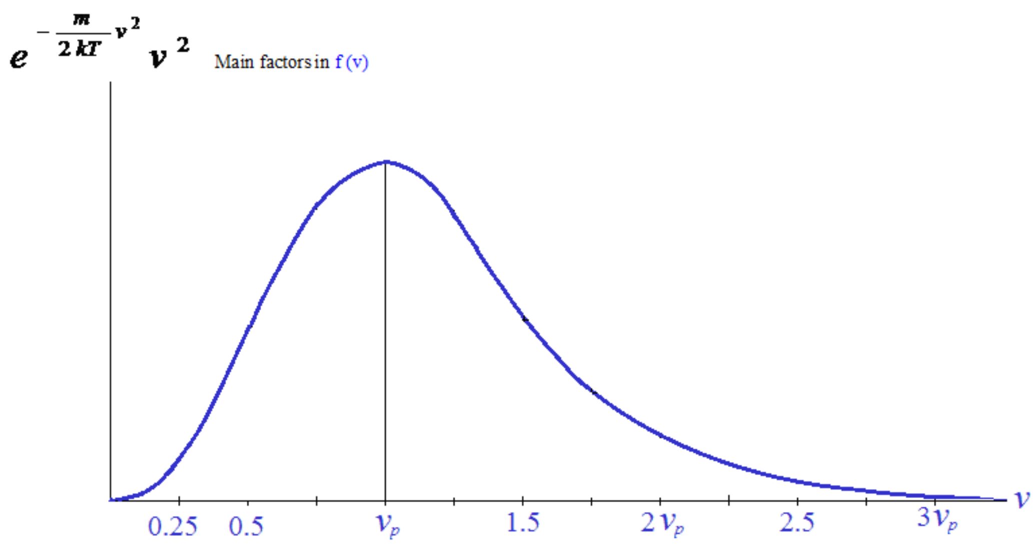


Fig 1 Volume distribution, three-dimension speed

As is well known, we may easily derive the three characteristic speeds and the mean kinetic energy of the molecules of this distribution:

$$v_p = \sqrt{\frac{2kT}{m}} \quad \bar{v} = \sqrt{\frac{8kT}{\pi m}} \quad \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}} \quad \bar{\varepsilon} = \frac{3}{2}kT$$

(b) **Beam distribution**

It is a gas molecule hole-passing (or wall colliding) speed distribution, or, a thermal electron wall (surface) emission speed distribution. In a word, we just call them the **beam distribution.**

The speed in a beam distribution may also be of three dimensions or of two dimensions. What we interested mainly in this paper is **the beam distribution of two dimensions**, discussed as follows.

Define $d\Gamma(u, \theta)$ as the number of gas molecules of an infinitesimal speed interval du (from u to $u + du$) that pass through a hole of unit area ($dA = 1$) in unit time ($dt = 1$) in the direction of $d\theta$ (from θ to $\theta + d\theta$).

$d\Gamma(u, \theta)$ may also be the number of thermal electrons of infinitesimal speed interval du that ejected from unit area in unit time in the direction of $d\theta$.

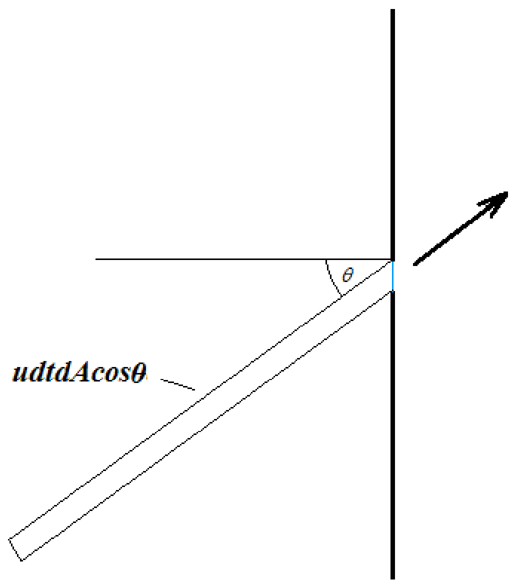


Fig 2 the number of molecules passing dA in dt in the direction of $d\theta$.

As shown in Fig 2, at time *t*, all the molecules in the volume of a cylinder $(udtdA \cos\theta)$ with a speed of $u \sim u + du$ in the direction of $\theta \sim \theta + d\theta$ will pass through *dA* in *dt*, and their number is

$$d\Gamma(u,\theta) = f(u)du \times (udtdA \cos\theta)\frac{d\theta}{2\pi}$$

$$= n\frac{m}{2\pi kT}e^{-\frac{m}{2kT}u^2}udu \times udtdA \cos\theta d\theta$$

$$= n\frac{m}{2\pi kT}e^{-\frac{m}{2kT}u^2}u^2du\ \mathrm{dtdA} \cos\theta d\theta$$

It may also be the number of the thermal electrons ejected in the direction of $d\theta$ in the duration of time *dt* from unit area *dA* of a wall.

$$d\Gamma(u,\ \theta) = n\frac{m}{2\pi kT}e^{-\frac{m}{2kT}u^2}u^2du \cos\theta d\theta \tag{4}$$

The factor $cos\theta$ represents Lambert's law.

The left factor about *u* represents the speed distribution of the molecules (electrons) in the beam, and it need to be normalized

$$b(u)du = Ae^{-\frac{m}{2kT}u^2}u^2du\ ,$$

$$\int b(u)du = \int Ae^{-\frac{m}{2kT}u^2}u^2du = A\frac{1}{2}\left[\frac{2kT}{m}\right] = 1$$

$$A = \frac{m}{kT}$$

$$b(u)du = \frac{m}{kT}e^{-\frac{m}{2kT}u^2}u^2du \tag{5}$$

This is the ratio of the number of molecules of speed interval $u \sim u + du$ in the beam to the number of the molecules of all speeds in the same beam.

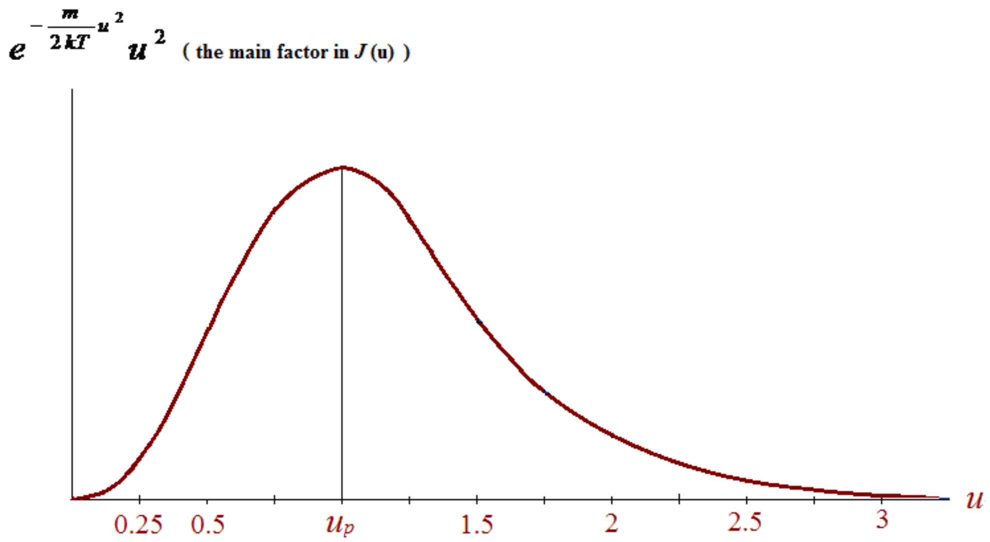


Fig 3 Beam distribution, two-dimensional speed

We may also easily derive the three characteristic speeds and the mean kinetic energy of the molecules (or electrons) of this beam distribution, the speed is of two dimensions,

$$u_p = \sqrt{\frac{2kT}{m}} \quad \overline{u} = \sqrt{\frac{8kT}{\pi m}} \quad \sqrt{\overline{u^2}} = \sqrt{\frac{3kT}{m}} \quad \overline{\varepsilon} = \frac{3}{2}kT \quad .$$

They are exactly alike to the ones of the gas speed volume distribution of three dimensions.

Equation (3) and equation (5) are alike, and consequently their graphs Fig 1 and Fig 3 are alike, too. That means, their relative numbers of molecules in different speed ranges are alike.

From page 2 to page 8, symbol v is the speed of three dimensions, the related speed distribution is the ordinary form of Maxwell's distribution law, equation (2), a volume distribution. And the relative numbers of gas molecules in different speed ranges is shown in Tab 1.

From page 9 to page 138, in our graphical survey of the various trajectories of thermal electrons, we are dealing with the motion of the thermal electrons in the XOY plane (as a magnetic field is applied), and the speeds of these electrons are governed by the beam speed distribution, equation (5). We still use the symbol *v* to represent the electron's speed component in the XOY plane. So, this *v* is actually *u*. Tab 1 is also valid for the electrons in the beam, just replace *v* by *u*.

We do not mention the relation of this *v* and that u in the discussion of our survey, just for avoiding too much explanation at that time.

(c) **Beam molecule speed distribution of three dimensions**

The original Maxwell's gas molecule speed distribution is

$$dn(dv_x, dv_y, dv_z) = n\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m}{2kT}v_x^2+v_y^2+v_z^2} dv_x dv_y dv_z$$

in spherical coordinates, we have

$$dn(v,\theta,\varphi) = n\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m}{2kT}v^2} v^2 dv \sin\theta d\theta d\varphi \qquad (6)$$

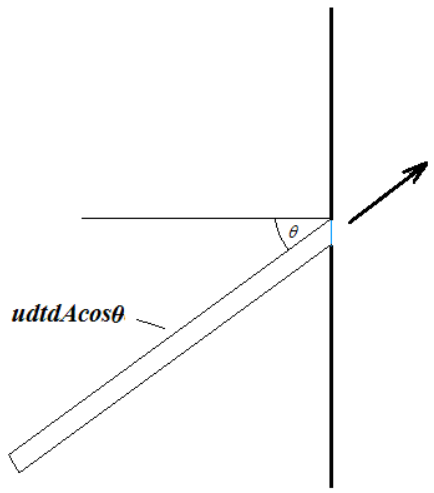
Integrate over θ and ϕ, we derive equation (3),

$$f(v)dv = 4\pi n\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m}{2kT}v^2} v^2 dv$$

This is, as mentioned above, a **volume speed distributio**n, the speed *v* is of three dimensions.

The collision number for the case of three dimensions

For this case, the collision number of molecules on *dA* and in *dt* in the direction of $\theta \sim \theta + d\theta$ and $\phi \sim \phi + d\phi$, as shown in the figure below, is



$$d\Gamma(v,\theta,\varphi) = n\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m}{2kT}v^2} v^2 dv \ \sin\theta d\theta d\varphi \times vdtdA \quad \cos\theta$$

$$= n\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{m}{2kT}v^2} v^3 dv \ \sin\theta \ \cos\theta d\theta d\varphi$$

Integrate over θ(from −π/2 to π/2) and ϕ(from 0 to 2π), we derive

$$d\Gamma(v) = n\pi\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m}{2kT}v^2} v^3 dv$$

This is the number of molecules of speed range $v \sim v + dv$ passing through dA in dt.

The corresponding general number of molecules of all the speed passing through dA in dt is the **wall-collision number**,

$$\Gamma = \int n\pi\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m}{2kT}v^2} v^3 dv$$

$$= n\pi\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} \int e^{-\frac{m}{2kT}v^2} v^3 dv$$

$$= n\pi\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} \times \frac{1}{2}\left(\frac{2kT}{m}\right)^2 = \frac{1}{4}n\left(\frac{8kT}{\pi m}\right)^{\frac{1}{2}}$$

$$= \frac{1}{4}n\bar{v}$$

In these molecules, the probability of a molecule to have a speed within the infinitesimal range ($v \sim v + dv$) is

$$b(v)dv = \frac{d\Gamma(v)dv}{\Gamma}$$

$$= (n\pi(\frac{m}{2\pi kT})^{3/2} e^{-\frac{m}{2kT}v^2} v^3 dv) \div \left[\frac{1}{4}n\left(\frac{8kT}{\pi m}\right)^{\frac{1}{2}}\right]$$

$$= 4\pi\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}}\left(\frac{\pi m}{4\times 2kT}\right)^{\frac{1}{2}} e^{-\frac{m}{2kT}v^2} v^3 dv$$

$$= 2\left(\frac{m}{2kT}\right)^2 e^{-\frac{m}{2kT}v^2} v^3 dv$$

$A = 2\left(\frac{m}{2kT}\right)^2$ is normalization constant

So, the beam speed distribution of three dimensions is

$$b(v)dv = \frac{d\Gamma(v)}{\Gamma} = 2\left(\frac{m}{2kT}\right)^2 e^{-\frac{m}{2kT}v^2} v^3 dv \qquad (7)$$

It is also easy to find the three characteristic speeds and the mean kinetic energy for the molecules or thermal electrons in such a beam

$$v_p = \sqrt{\frac{3kT}{m}} \quad \bar{v} = \sqrt{\frac{9\pi kT}{8m}} \quad \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}} \quad \bar{\varepsilon} = 2kT \quad .$$

Equations (1), (2), (3), (5), (6) and (7) are all Maxwell's speed distributions, different forms for different situations.