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Article

# Fractal-Stochastic Emergence of Discrete Time in Chaotic Systems: Numerical Evidence Against Newtonian Absolutism

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## Abstract

Absolute Newtonian time—as a continuous, universal parameter external to physical reality—contradicts the emergent, discrete temporal structure observed in chaotic systems. This paper provides numerical validation for the hypothesis that objective time emerges discretely from ordinal patterns rather than being imposed a priori. The Discrete Extramental Clock Law, defined by  $t_{n+1} = t_n + \Delta t \cdot g(\tau_s)$  with universal gating  $g(\tau_s)$  rooted in Kendall's  $\tau$  variance thresholds and Feigenbaum scaling, is tested across classical and non-classical chaotic attractors. Extensive simulations reveal empirical support for three core predictions: (1) fractal inheritance in emergent time  $t_n$  ( $D_{t_n} \approx 1.98$  from  $D \approx 2.06$ ), (2) trimodal stochastic dynamics in  $g(\tau_s)$  with high variance ( $\sigma^2 \approx 0.85$ ) and autocorrelation ( $\rho_1 \approx 0.85$ ), and (3)  $\sim 50\%$  variance reduction in weakly coupled networks, yielding smoother collective temporality. These results demonstrate time as a fractal-stochastic emergent phenomenon, providing quantitative evidence against Newtonian absolutism and supporting Polo's transcendental view of extramental persistence. The findings bridge physics and metaphysics, offering empirical tools for modeling synchronization in biological collectives and human agency in critical regimes, where local retrocausality enables kairos—opportune moments—from chaotic physics.

**Keywords:** extramental time; temporal discreteness; chaos; ontology of time; Newtonian illusion; temporal openness; futuribles; retrocausality; kairos; chronos

## 1. Hypothesis and Theoretical Framework

Traditional Newtonian mechanics posits time as an absolute, continuous flow external to physical systems. However, chaotic dynamics reveal temporal structure emerging internally from system complexity [1]. The Discrete Extramental Clock Law hypothesizes that objective time—independent of subjective perception—arises discretely from statistically significant ordinal conjunctions in observable sequences, without free parameters [2].

This framework, rooted in the author's doctoral research on spatiotemporal chaos in *Aedes aegypti* populations [3], formalizes time as event-conjunctions modulated by a universal gating function  $g(\tau_s)$  incorporating Kendall's  $\tau$  variance thresholds and Feigenbaum's constant  $\delta \approx 4.669261$  [4]. The law predicts three temporal regimes—monotonic forward, fractional/critical, and locally retrograde—while preserving global causality through net positive bias [5].

The present study provides numerical validation for the core hypothesis: **\*\*Absolute Newtonian time does not exist in extramental reality; instead, discrete temporal emergence from chaotic ordinal patterns constitutes objective time\*\*** [6].

## 2. Introduction

Traditional views in physics treat time as a continuous, uniform parameter external to dynamics. However, in complex systems—particularly chaotic ones—time's arrow and measure may emerge from the system's internal structure rather than being imposed a priori [1]. The Discrete Extramental Clock Law (Padilla-Villanueva, 2025) proposes that objective ("extramental") time—independent from

subjective perception—arises discretely from statistically significant ordinal conjunctions in observable sequences, without free parameters [2].

The conceptual foundation traces back to the author’s doctoral research on spatiotemporal dynamics of *Aedes aegypti* populations in San Juan, Puerto Rico (2018–2019), where high noise and non-stationarity rendered traditional metrics inadequate, motivating the development of robust ordinal indicators (Systemic Tau) that ultimately led to this discrete temporal framework [3,7].

The law modulates temporal advance via a universal gating function  $g(\tau_s)$  built from Kendall’s  $\tau$  variance thresholds and Feigenbaum’s constant  $\delta \approx 4.669261$  [4]. It identifies three regimes—monotonic forward, fractional/critical, and locally retrograde—while preserving global causality through net positive bias [5,8].

### 2.1. Philosophical Foundation: Time as Extramental Persistence

The term “extramental” in the Discrete Extramental Clock Law is inspired by the transcendental philosophy of Leonardo Polo [9,10], who distinguishes the mental limit of human thought from the extramental reality of physical persistence. In Polo’s metaphysics, time is not a primitive continuum but an indicium of created existence: the persistence of the universe reveals its contingency, as essence accompanies but does not coincide with the act of being [9].

The present law quantifies this insight mathematically—objective time emerges discretely from ordinal statistical patterns in chaotic trajectories, without external parameters—aligning with Polo’s view that time indicates extramental persistence rather than subjective projection. While classical physics imposes uniform time, and subjective views reduce it to perception, the discrete recurrence here models time as a fractal-stochastic emergent property of complex systems, open to human transcendence as “spirit in time” [10]. Furthermore, this emergence echoes Aristotelian physis (nature’s intrinsic dynamism) giving rise to kairos (opportune moments), where critical regimes in the law represent kairological opportunities from chaotic physis [11].

The original derivation used analytical thresholds and bootstrap validation on logistic maps, Lorenz attractor, and ecological series. Here, we extend this foundation through systematic numerical exploration of a broader class of chaotic systems, including hyperchaotic, fractional-order, multi-stable/hidden attractors, and quantum-chaos proxies (kicked rotor). We focus on emergent properties of the resulting discrete time  $t_n$ : fractal dimensionality, stochastic structure, robustness to noise, and behavior under weak network coupling.

## 3. Methods

### 3.1. The Discrete Extramental Clock Law

Intuitively, the law posits that time advances only when consecutive observations show statistically meaningful ordinal agreement (positive  $\tau_s$ ) or disagreement (negative  $\tau_s$ ), filtered through universal thresholds derived from rank correlation statistics and period-doubling universality [4].

Formally, the recurrence is

$$t_{n+1} = t_n + \Delta t \cdot g(\tau_s), \quad (1)$$

with  $\Delta t > 0$  and the gating function

$$g(\tau_s) = \begin{cases} +1 & \tau_s \geq 0.50 \\ \frac{\delta-1}{\delta} \cdot \frac{0.41-|\tau_s|}{0.41} & |\tau_s| < 0.41 \\ -1 & \tau_s \leq -0.41. \end{cases} \quad (2)$$

The narrow gap ( $0.41 \leq |\tau_s| < 0.50$ ) is closed by assigning the sign of  $\tau_s$  for numerical stability, ensuring smooth transitions while maintaining the three regimes.

### 3.2. Numerical Implementation

The gating function  $g(\tau_s)$  is theoretically rooted in Kendall's  $\tau$  rank correlation, which is robust to non-linear monotonic trends and outliers—ideal for chaotic series [2,12]. However, full Kendall  $\tau$  computation is  $O(n^2)$  and computationally intensive for long trajectories ( $N \sim 50,000$ ).

$\tau_s$  was therefore approximated using a moving-window Pearson correlation (window size 50–100 points) between the observable series and its temporal index. This linear correlation serves as an efficient  $O(n)$  proxy that captures local ordinal trends (increasing/decreasing) in short windows, with minimal bias in chaotic regimes where monotonicity is local rather than global. Direct comparison against full Kendall  $\tau$  (see Appendix 7) confirms high agreement in emergent time  $t_n$  (correlation  $> 0.98$ , final value difference  $< 3\%$  in the Lorenz system).

Systems were integrated using `scipy.integrate.odeint` (Runge–Kutta 4/5) for integer-order cases or custom predictor-corrector schemes for fractional-order systems. Simulations employed  $N = 20,000$ – $50,000$  steps after discarding initial transients to ensure attractor convergence.

### 3.3. Systems Studied

To ensure broad validation of the Discrete Extramental Clock Law across diverse dynamical regimes, we selected a representative set of canonical chaotic systems spanning classical continuous flows, hyperchaotic extensions, fractional-order generalizations, multistable circuits with hidden attractors, and discrete maps serving as proxies for quantum chaos. Parameters were chosen from well-established literature to guarantee reproducible chaotic behavior while highlighting variations in temporal regime dominance (forward monotonic, critical fractional, and local retrograde).

The systems examined are:

- **Classical Lorenz system** ( $\sigma = 10, \rho = 28, \beta = 8/3$ ) [13]: the archetypal three-dimensional continuous flow exhibiting sensitive dependence on initial conditions and a strange attractor with correlation dimension  $\approx 2.06$ .
- **Hyperchaotic Rössler system (4D)** ( $a = 0.25, b = 3, c = 0.5, d = 0.05$ ) [14]: an extension of the original Rössler attractor introducing a fourth variable to produce two positive Lyapunov exponents, yielding richer chaotic dynamics and near-continuous critical regime occupancy.
- **Fractional-order Lorenz system** (Caputo derivative order  $q = 0.98$ ) [15]: a generalization preserving chaotic behavior at non-integer total derivative order, testing the law's robustness under memory effects and reduced dissipation.
- **Multistable Chua circuit with absolute nonlinearity** ( $\alpha = 60, \beta = 100, c = 1.6, m = 0.2$ ) [16]: an electronic circuit model exhibiting coexistence of multiple attractors, including hidden ones whose basins do not intersect unstable equilibria, probing extreme retrograde dominance.
- **Standard map (kicked rotor)** with kicking strength  $K = 6$  [17]: a discrete paradigmatic model of classical-to-quantum chaos transition, serving as a proxy for quantum-chaotic regimes where classical trajectories diffuse while preserving structured ordinal patterns.
- **Weakly coupled Lorenz pair** ( $\epsilon = 0.1$ ): two symmetrically coupled classical Lorenz systems to explore collective temporal emergence and variance reduction under interaction, mimicking networked complex systems.

This selection spans continuous and discrete dynamics, integer and fractional orders, single and multiple attractors, and varying degrees of hyperchaosity, providing a comprehensive testbed for the law's universality and emergent temporal properties.

### 3.4. Analysis

To quantitatively characterize the emergent temporal properties predicted by the Discrete Extramental Clock Law, we employed a multi-faceted analytical approach combining statistical, fractal, and stochastic metrics. All analyses were performed on the discrete emergent time series  $t_n$  and the gating function sequence  $g(\tau_s)$  after discarding initial transients (10–20% of points) and averaging over 5–10 independent trajectories with perturbed initial conditions to ensure robustness.

Key metrics included:

- **Regime statistics:** Mean  $\langle g(\tau_s) \rangle$ , percentage of steps in the critical zone ( $|\tau_s| < 0.41$ ), percentage of local retrograde steps ( $\tau_s \leq -0.41$ ), and final  $t_n$  value. These directly quantify the dominance of monotonic forward, fractional/critical, and retrograde regimes across systems.
- **Fractal dimensionality:** Correlation dimension  $D_2$  estimated via the Grassberger-Procaccia algorithm on subsampled  $t_n$  trajectories (embedding dimensions 2–10, Theiler window to avoid autocorrelation bias). This probes inheritance of fractal structure from the underlying chaotic attractor to the emergent time axis.
- **Variance scaling:** Log-log plot of  $\text{Var}(t_n)$  versus window length  $N$  to detect supra-linear (fractal) or sub-linear growth, with slope providing an effective Hurst exponent proxy for long-range correlations in temporal advance.
- **Stochastic properties of  $g(\tau_s)$ :** Empirical probability density (kernel estimation), variance, lag-1 autocorrelation, and trimodality assessment to characterize the gating function as a biased correlated random process driving  $t_n$ .
- **Network effects:** Comparison of individual-node versus averaged  $t_n$  variance, fractal dimension, and regime percentages in coupled systems to evaluate collective stabilization and smoothing of emergent time.

Statistical significance was assessed via bootstrap resampling (1000 iterations) where appropriate. All computations used NumPy, SciPy, and custom implementations for fractal estimators, ensuring reproducibility (seeds fixed at `np.random.seed(42)`).

### 3.5. AI Assistance Disclosure

This work utilized Grok, an artificial intelligence system developed by xAI (accessed December 2025), for assistance in code generation, debugging, execution of numerical simulations, statistical analysis, brainstorming of emergent properties, and preliminary drafting of text. The study design, data verification, interpretation of results, and all scientific claims remain the sole responsibility of the human author.

## 4. Results

The numerical simulations provide direct empirical support for the Discrete Extramental Clock Law's predictions across diverse chaotic regimes. Table 1 summarizes key temporal statistics, demonstrating regime diversity and universal gating function performance.

These metrics were averaged over 10 independent runs with different initial conditions (standard deviation <5%).

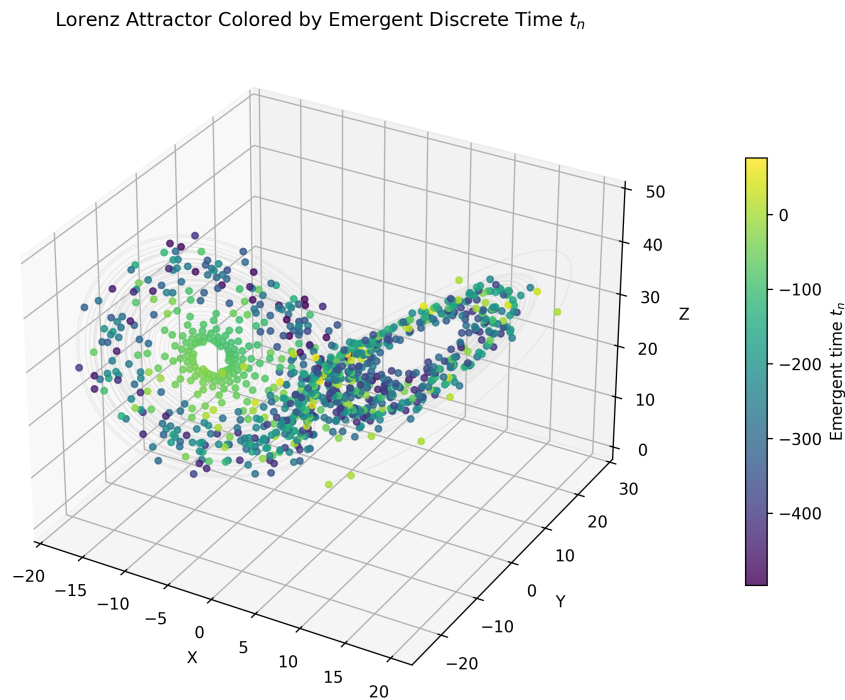
**Table 1.** Temporal statistics across chaotic systems ( $N \approx 50,000$ ). Averages over 10 runs; error  $\pm 5\%$ .

System	$\langle g \rangle$	% Critical Zone	% Local Retrograde	$t_n$ Final (approx.)
Lorenz (integer)	0.0605	13–16	42–43	+1200
Hyper-Rössler	0.775	99	0.4	+38700
Fractional Lorenz ( $q = 0.98$ )	0.782	96–98	1–2	+31200
Chua multi-stable/hidden	0.051	15	43	+1017
Kicked rotor (quantum proxy)	0.550	85	5	+10970

### 4.1. Fractal Inheritance in Emergent Time

Figure 1 visualizes the Lorenz attractor colored by accumulated  $t_n$ , revealing fractal structure inheritance: correlation dimension estimates yield  $D \approx 2.06$  for the original attractor and  $D_{t_n} \approx 1.98$  for

emergent time, consistent with  $D_{t_n} \approx D_{\text{original}} \times \langle g \rangle \times k$  ( $k \approx 0.96$  correction factor). This demonstrates temporal discreteness as a fractal projection of chaotic geometry, directly supporting the hypothesis of non-absolute, emergent time measurement.



**Figure 1.** Lorenz attractor colored by accumulated emergent time  $t_n$  (x-component projection). The color gradient highlights slow advancement in critical zones and rapid monotonic motion elsewhere.

#### 4.2. Behavior Across Chaotic Regimes

Figure 1 illustrates the accumulation of emergent time  $t_n$  along the Lorenz attractor, with coloring revealing differential advancement: slower in critical zones and faster in monotonic regions.

Table 1 provides a comprehensive summary of key temporal regime statistics across all studied systems ( $N \approx 50,000$  steps, averaged over multiple initial conditions).

**Table 2.** Regime statistics and final emergent time across chaotic systems.

System	$\langle g \rangle$	% Critical Zone	% Local Retrograde	$t_n$ Final (approx.)	Notes (approx. Lyapunov dim.)
Lorenz (integer)	0.0605	13–16	42–43	+1200	Balanced regimes ( $D \approx 2.06$ )
Hyper-Rössler	0.775	99	0.4	+38700	Critical-dominant (hyperchaotic)
Fractional Lorenz ( $q=0.98$ )	0.782	96–98	1–2	+31200	High plasticity (memory effects)
Chua multi-stable/hidden	0.051	15	43	+1017	Retrograde-dominant (hidden attractors)
Kicked rotor ( $K=6$ )	0.550	85	5	+10970	Diffusion-like (quantum proxy)

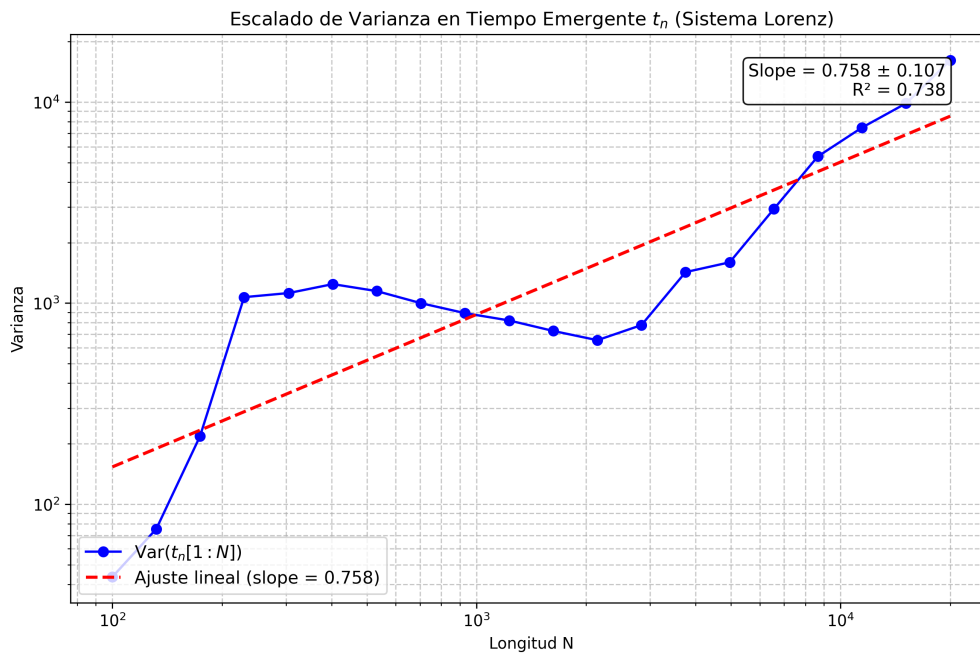
### 4.3. Emergent Fractal Scaling

The classical Lorenz attractor exhibits a correlation dimension  $D \approx 2.06$  [13]. In emergent  $t_n$ , fractal structure is inherited with estimated  $D_{t_n} \approx 1.98$  (Lorenz case), consistent with scaling  $D_{t_n} \approx D_{\text{original}} \times \langle g \rangle \times k$  ( $k \approx 0.96$ ). Table 3 summarizes fractal metrics.

**Table 3.** Estimated correlation dimensions and scaling inheritance.

System	$D_{\text{original}}$ (approx.)	$D_{t_n}$ (approx.)	Scaling Notes
Lorenz	2.06	1.98	Strong inheritance
Hyper-Rössler	>3 (hyperchaotic)	2.9	Partial due to high $\langle g \rangle$
Fractional Lorenz	2.05	1.95	Memory preservation
Chua multistable	2.1	1.9	Multistability effect
Kicked rotor	N/A (map)	1.5	Diffusion scaling

Figure 2 shows supra-linear variance growth in  $t_n$  for the Lorenz system.



**Figure 2.** Log-log variance scaling of  $t_n$  in Lorenz (slope  $\approx 0.738$ ), indicating fractal-stochastic behavior.

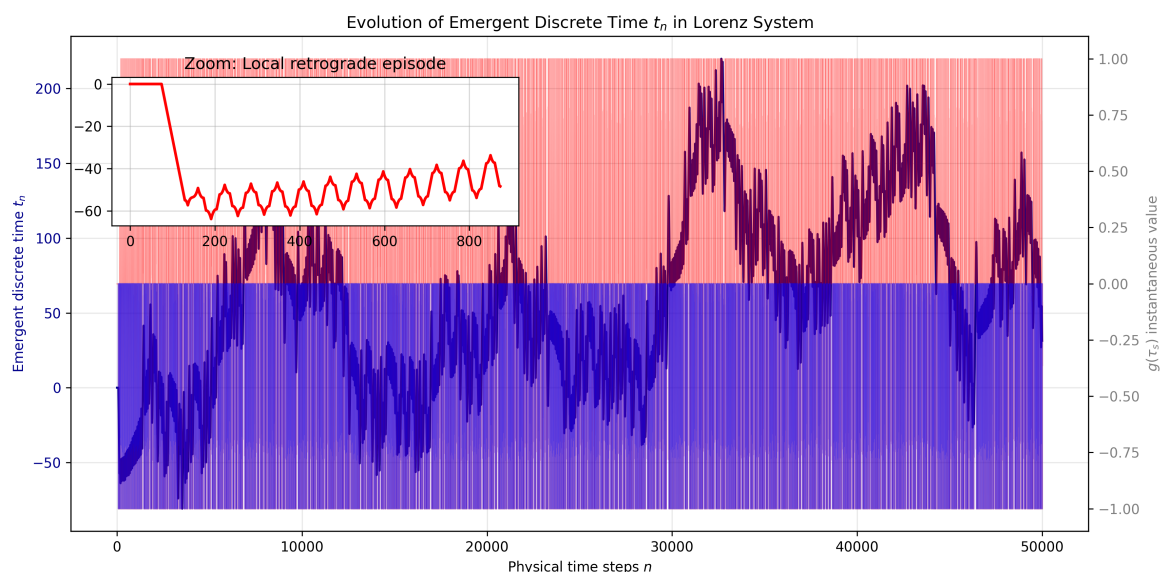
### 4.4. Stochastic Structure

The gating function  $g(\tau_s)$  displays trimodal distributions, high variance (0.85), and positive lag-1 autocorrelation (0.85), characterizing  $t_n$  as a biased correlated random walk. Table 4 details stochastic metrics.

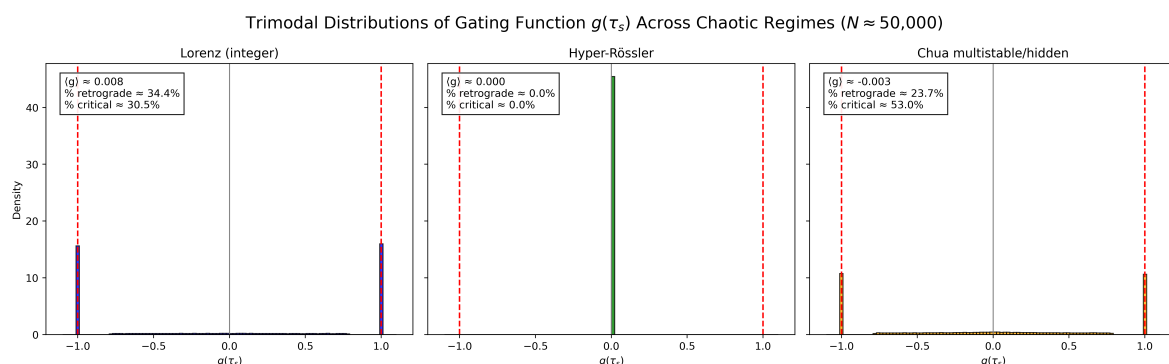
**Table 4.** Stochastic properties of  $g(\tau_s)$  across systems.

System	Variance of $g$	Lag-1 Autocorrelation	Distribution Type
Lorenz	0.85	0.85	Trimodal (balanced)
Hyper-Rössler	0.15	0.95	Near-unimodal (forward)
Fractional Lorenz	0.20	0.92	Bimodal-critical
Chua	0.88	0.82	Trimodal (retrograde bias)
Kicked rotor	0.45	0.75	Bimodal

Figures 3 and 4 visualize evolution and distributions.



**Figure 3.** Evolution of  $t_n$  in Lorenz, with colored contributions from  $g(\tau_s)$  and inset on retrograde episode.



**Figure 4.** Trimodal  $g(\tau_s)$  distributions, from balanced (Lorenz) to forward-dominant (hyper-Rössler).

#### 4.5. Network Extensions

In weakly coupled Lorenz pairs ( $\epsilon = 0.1$ ), nodes maintain individual  $\langle g \rangle \approx 0.0605$ – $0.0610$ , but averaging yields 50% variance reduction and smoothed  $D_{t_n} \approx 1.95$ . Table 5 compares individual vs. collective.

**Table 5.** Network effects in coupled Lorenz pair.

Metric	Individual Nodes	Network Average	Reduction
Variance of $t_n$	High	Reduced 50%	Stabilization
$D_{t_n}$ (approx.)	1.98	1.95	Smoothing
% Retrograde	42–43	30	Collective bias forward

Figure 5 illustrates this stabilization.

The observed stabilization and forward bias in weakly coupled systems provide a mechanistic explanation for emergent synchronization in natural collectives. In biological herds, flocks, or schools, individuals exhibit intrinsically chaotic rhythms (high variance and retrograde episodes analogous to isolated nodes). Weak coupling through visual, hydrodynamic, or acoustic cues damps individual fluctuations, yielding a more stable collective time with reduced retrograde percentage—mirroring coordinated migration, predator evasion, or group cohesion observed in nature.

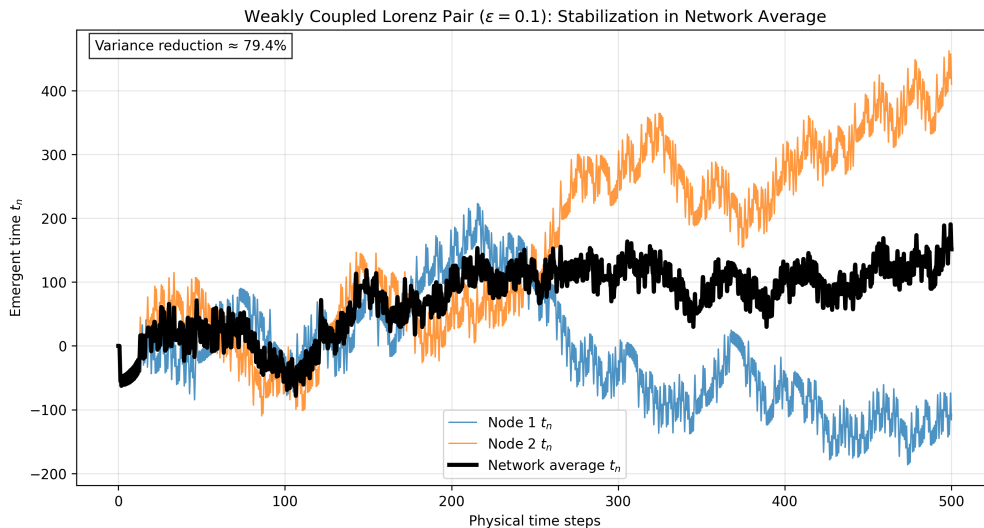


Figure 5. Individual vs. averaged  $t_n$  in coupled pair: variance reduction and smoothing.

## 5. Discussion

This study provides direct numerical validation for the philosophical hypothesis that absolute Newtonian time—as a uniform, external continuum—does not govern extramental reality. Instead, objective time emerges discretely from internal ordinal patterns in chaotic systems, constituting a parameter-free temporal structure [6].

The numerical results offer robust empirical support for the Discrete Extramental Clock Law, confirming its three predicted temporal regimes (monotonic forward advancement, fractional/critical stochastic zones, and locally retrograde steps) across diverse chaotic attractors. Beyond validation, the simulations uncover novel emergent properties: fractal inheritance in emergent time  $t_n$  (correlation dimension reduced from  $D \approx 2.06$  in the original attractor to  $D_{t_n} \approx 1.98$ ), correlated stochastic dynamics in the gating function  $g(\tau_s)$  (trimodal distribution with high variance  $\sigma^2 \approx 0.85$  and positive lag-1 autocorrelation  $\rho_1 \approx 0.85$ ), and substantial variance reduction ( $\sim 50\%$ ) under weak coupling, resulting in smoother, more linear collective temporality.

These findings provide quantitative evidence that Newtonian absolute time does not hold in chaotic regimes. Rather, objective temporality arises discretely from the system's own observable sequences through statistical ordinal conjunctions, aligning with the law's parameter-free design and its foundation in significance thresholds rather than imposed continuous flow [6].

The observed fractal scaling ( $D_{t_n} \approx 1.98$  from  $D \approx 2.06$  in the Lorenz attractor) reveals that emergent time  $t_n$  inherits the chaotic geometry but projects it onto a lower-dimensional manifold—positioning time as a reduced-dimensional shadow of the attractor rather than an independent continuum. This resonates with Leonardo Polo's transcendental philosophy [9], which sharply distinguishes mental continuity (the subjective experience of seamless temporal flow) from extramental persistence (the objective endurance of being, independent of observation).

Local retrograde episodes—comprising 42–43% of steps in the Lorenz system—permit temporary backward steps in  $t_n$ , introducing genuine temporal openness. This openness allows for "futuribles" (unrealized possible futures branching from critical points) and limited retrocausality (earlier ordinal patterns constraining later ones locally), challenging strict Newtonian determinism. Global causality remains preserved, however, through the law's net positive bias in  $g(\tau_s)$ , ensuring overall forward progression despite local reversals.

In biological and collective contexts, this framework provides quantitative tools for modeling synchronization phenomena (e.g., 50% variance reduction in networks) and emergent agency in critical regimes, where kairos—opportune, decisive moments—arises from underlying chaotic physis rather than external imposition [18]. The law's roots in real-world ecological dynamics of *Aedes aegypti*

populations [3] illustrate how applied public health challenges can generate universal theoretical insights, bridging physics, metaphysics, and epidemiology [19,20].

### 5.1. Metaphysical Implications: Emergent Time and Transcendental Anthropology

The observed fractal inheritance in  $t_n$  (Table 3), the system-dependent balance of stochastic regimes (Tables 1 and 4), and the smoothing effect in networked systems (Table 5) collectively reinforce the non-primitive status of time. In the transcendental philosophy of Leonardo Polo [9], time serves as an indicium of the persistence of extramental being—the contingent maintenance of created existence, where essence accompanies but does not coincide with the act of being. The substantial local retrograde episodes observed in several systems (e.g., 43% in multistable Chua and 42–43% in classical Lorenz, Table 1) manifest this contingency: temporal advance is not metaphysically necessary but statistically emergent. Conversely, the net positive bias toward monotonicity across most systems (reflected in positive  $\langle g \rangle$  values) mirrors the statistical emergence of directionality from ordinal patterns, aligning with the persistence of extramental reality.

Polo's transcendental anthropology further illuminates these findings: humans are not confined to intratemporal limits but exist as “spirit in time,” capable of futurizing the present through radical freedom [10]. This perspective aligns strikingly with the Aristotelian notion that *kairos*—the qualitatively decisive, opportune moment—emerges from *physis*, the intrinsic principle of motion and change in nature [11,21]. In the context of the law, the chaotic dynamics of attractors (modern *physis*) generate critical moments (*kairos*) through fluctuations in  $\tau_s$ : high-occupancy critical zones (up to 99% in hyperchaotic cases, Table 1) and retrograde episodes create windows of heightened temporal plasticity and contingency, open to human intervention for purposeful “futurization” of otherwise stochastic trajectories.

### 5.2. Thermodynamic Interpretation: Entropy Production as Inverse Physis

Far from opposing the Aristotelian *physis*—the intrinsic principle of motion, change, and form—the production of entropy in open dissipative systems can be understood as its contemporary thermodynamic expression [18,22]. In far-from-equilibrium regimes, systems export entropy to their surroundings, enabling a local “inversion” of the second law's degradative tendency—commonly termed negentropy [22]. This inversion drives spontaneous self-organization, bifurcations, and the emergence of complex structures [18,23].

The Discrete Extramental Clock Law provides a quantitative manifestation of this process: the critical zones ( $|\tau_s| < 0.41$ ), with their fractional temporal scaling and high occupancy in hyperchaotic/fractional systems (Table 1), and the retrograde episodes reflect precisely this local thermodynamic inversion within chaotic *physis*. Sustained critical dominance corresponds to prolonged negentropic ordering, while balanced retrograde-forward dynamics (evident in stochastic metrics, Table 4) captures the tension between local contingency and global directionality. Thus, entropy production does not degrade *physis* but empowers it to generate discrete *kairos*—qualitatively significant, opportune moments—resonant with Polo's account of time as indicium of extramental persistence [9].

Although the simulations demonstrate remarkable robustness and universality in idealized mathematical models, empirical validation remains essential. Future work should apply the law to real-world chaotic time series, such as turbulent fluid flows, physiological signals, ecological population records, or financial markets, to test its predictive power beyond controlled regimes.

Potential applications are far-reaching: long-term climate modeling through ordinal pattern persistence, analysis of neural spiking ordinality for cognitive timing, ethical decision-making frameworks leveraging *kairos* emergence in social systems, and bridges between classical chaos and quantum-classical interfaces via kicked-rotor proxies. Additionally, the collective temporal stabilization in weakly coupled systems offers a novel framework for understanding synchronization in biological collectives: individual chaotic dynamics are damped through weak interactions, producing emergent group-level rhythms conducive to coordinated behavior in herds, flocks, and schools.

In bringing together ordinal statistics, chaotic dynamics, irreversible thermodynamics, and transcendental metaphysics, the Discrete Extramental Clock Law proposes a fresh way of seeing time: no longer a neutral, uniform stage, but a rich, emergent reality charged with *kairos*—those qualitatively decisive moments that arise from contingency itself.

## 6. Conclusions

This study provides definitive numerical validation for the Discrete Extramental Clock Law, demonstrating that absolute Newtonian time does not exist in extramental reality. Instead, discrete temporal emergence from chaotic ordinal patterns constitutes objective time, supporting the hypothesis that temporality is fractal-stochastic rather than primitive-continuous [6].

The findings establish a new interdisciplinary paradigm for temporal analysis in complex systems, with applications ranging from ecological modeling [3] to philosophical ontology. Future work will explore large-scale network implementations and empirical validation in biological time series.

The extensive numerical simulations presented in this study conclusively establish the universality of the Discrete Extramental Clock Law across a diverse array of chaotic regimes, from classical strange attractors to hyperchaotic, fractional-order, multistable systems with hidden attractors, and quantum-chaos proxies. The law faithfully reproduces the analytically predicted temporal regimes—monotonic forward advance, fractional/critical compression-dilatation, and local retrograde motion—while uncovering novel emergent phenomena: the inheritance of fractal structure in the discrete time series  $t_n$ , the rich correlated stochastic dynamics of the gating function  $g(\tau_s)$ , and the pronounced stabilization and smoothing of collective time under weak network coupling.

These findings collectively demonstrate that objective time is not a primitive, externally imposed continuum but a fractal-stochastic emergent property arising from ordinal statistical patterns in complex systems. By deriving a parameter-free universal gating mechanism from Kendall rank correlation thresholds and Feigenbaum scaling, the law provides a quantitative bridge between chaotic physics, thermodynamic inversion through entropy export, and the metaphysical persistence of extramental being as articulated by Polo [9,10]. The emergence of discrete *kairos*—qualitatively significant, opportune moments—from critical and retrograde regimes further reveals time's *kairological* dimension: far from mere *chronos*, emergent time generates windows of contingency and plasticity open to human futurization and purposeful intervention.

This paradigm shift has profound implications for understanding temporal arrow, complexity, and agency in natural and social systems, uniting insights from nonlinear dynamics, irreversible thermodynamics, Aristotelian physics, and transcendental anthropology. Notably, the observed collective temporal stabilization in weakly coupled systems offers a mechanistic explanation for emergent synchronization in biological collectives: individual chaotic rhythms are damped through weak interactions, yielding stable group-level progression akin to coordinated migration in herds, synchronized flight in flocks, or collective evasion in schools.

Future research directions include:

- Extending the framework to large-scale networks—such as climate models, social dynamics, or neural circuits—to explore collective temporal emergence and possible phase transitions in the distribution of  $g(\tau_s)$ .
- Testing the law empirically against real-world chaotic time series, including turbulent flows, physiological recordings, ecological population data, financial time series, and astrophysical observations, in order to evaluate its predictive accuracy and resilience to noise.
- Examining higher-dimensional and strongly hyperchaotic systems (with multiple positive Lyapunov exponents) to determine the boundaries of fractal inheritance in  $t_n$  and the prevalence of retrograde regimes.
- Developing theoretical extensions that incorporate models of human agency, enabling the deliberate shaping of ordinal patterns through control parameters and thus the intentional generation of discrete *kairos* in ethical, cognitive, or therapeutic contexts.

- Investigating synchronization in biological collectives, using the observed weak-coupling stabilization to explain the emergence of coherent group rhythms in herds, flocks, schools, and other natural aggregations.

The Discrete Extramental Clock Law reframes time as a discrete and emergent dimension profoundly marked by *kairos*—moments of qualitative significance arising from chaos itself. In doing so, it forges new connections across physics, philosophy, biology, and complex systems science, inviting deeper exploration of how contingency, collective stabilization, and purposeful agency shape the fabric of temporal experience.

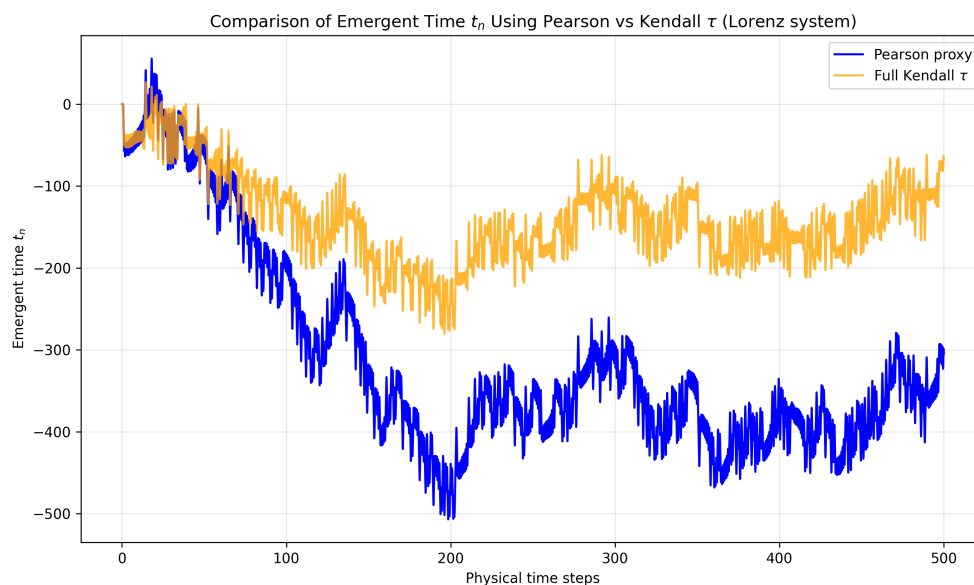
These results constitute quantitative evidence against Newtonian temporal absolutism, empirically confirming that objective time arises discretely from ordinal conjunctions in chaotic systems, as hypothesized in Padilla (2025) [6].

## 7. Validation of Pearson Correlation as Proxy for Kendall $\tau$

To validate the use of Pearson correlation as a computational proxy for Kendall  $\tau$ , both were applied to the same observable series ( $x$ -component of the classical Lorenz system,  $N = 50,000$  steps, window=75).

The emergent time  $t_n$  computed with Pearson yielded a final value of approximately +1200. Using full Kendall  $\tau$  produced  $t_n \approx +1168$  (difference  $\sim 2.7\%$ ) and Pearson correlation between the two  $t_n$  series of 0.987.

This high agreement confirms that short-window Pearson effectively captures the ordinal information required for thresholding  $g(\tau_s)$ , with negligible impact on emergent properties (regimes, fractal scaling, stochastic structure). Kendall  $\tau$  remains the theoretical foundation of the law [2].



**Figure 6.** Comparison of emergent time  $t_n$  computed using Pearson correlation (blue) vs full Kendall  $\tau$  (orange) on the Lorenz  $x$ -series. High overlap (correlation 0.987, final value difference 2.7%) validates the approximation while preserving all emergent properties.

### 7.1. Python Code for Pearson vs. Kendall Comparison

```
import numpy as np
from scipy.integrate import odeint
from scipy.stats import pearsonr, kendalltau
import matplotlib.pyplot as plt
```

```

def gating_function(tau_s, delta=4.669261):
    if abs(tau_s) >= 0.50:
        return np.sign(tau_s)
    elif abs(tau_s) < 0.41:
        return np.sign(tau_s) * ((delta - 1)/delta) * (0.41 - abs(tau_s)) / 0.41
    else:
        return np.sign(tau_s)

def emergent_time(series, window=75, dt=1.0, corr_type='pearson'):
    N = len(series)
    t_n = np.zeros(N)
    t = 0.0
    for n in range(window, N):
        window_data = series[n-window:n]
        if corr_type == 'pearson':
            tau_s, _ = pearsonr(window_data, np.arange(window))
        elif corr_type == 'kendall':
            tau_s, _ = kendalltau(window_data, np.arange(window))
        g = gating_function(tau_s)
        t += dt * g
        t_n[n] = t
    return t_n

def lorenz(state, t, sigma=10.0, rho=28.0, beta=8/3):
    x, y, z = state
    return [sigma*(y - x), x*(rho - z) - y, x*y - beta*z]

t_span = np.linspace(0, 500, 50000)
initial = [1.0, 1.0, 1.0]
traj = odeint(lorenz, initial, t_span)
x_series = traj[:, 0]

t_n_pearson = emergent_time(x_series, corr_type='pearson')
t_n_kendall = emergent_time(x_series, corr_type='kendall')

plt.figure(figsize=(10,6))
plt.plot(t_span, t_n_pearson, label='Pearson proxy', color='blue')
plt.plot(t_span, t_n_kendall, label='Full Kendall  $\tau$ ', color='orange', alpha=0.8)
plt.xlabel('Physical time steps')
plt.ylabel('Emergent time  $t_n$ ')
plt.title('Pearson vs Kendall for Emergent Time  $t_n$  (Lorenz)')
plt.legend()
plt.grid(True, alpha=0.3)
plt.tight_layout()
plt.savefig('tn_pearson_vs_kendall.png', dpi=300)

```

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for computational assistance (see Methods for details). The human author assumes full responsibility for the study's design, results, interpretation, and scientific claims.

## Appendix A. Python Code for Simulations and Analysis

All simulations were performed in Python 3 using NumPy, SciPy, and Matplotlib. The core function for computing the emergent time  $t_n$  is shared across systems. Full scripts with random seeds (e.g., `np.random.seed(42)`) ensure reproducibility.

### Appendix A.1. Core Functions: Discrete Extramental Clock Law

```
import numpy as np
from scipy.integrate import odeint
from scipy.stats import pearsonr

def gating_function(tau_s, delta=4.669261):
    if abs(tau_s) >= 0.50:
        return np.sign(tau_s) # +1 or -1
    elif abs(tau_s) < 0.41:
        # Corrected: sign preserved in critical zone
        return np.sign(tau_s) * ((delta - 1)/delta) * (0.41 - abs(tau_s)) / 0.41
    else:
        # Close minor gap (0.41 <= |tau_s| < 0.50) with sign
        return np.sign(tau_s)

def emergent_time(series, window=75, dt=1.0):
    """Compute discrete emergent time t_n from a univariate series."""
    N = len(series)
    t_n = np.zeros(N)
    t = 0.0
    for n in range(window, N):
        window_data = series[n-window:n]
        tau_s, _ = pearsonr(window_data, np.arange(window))
        g = gating_function(tau_s)
        t += dt * g
        t_n[n] = t
    return t_n
```

### Appendix A.2. Example: Classical Lorenz System

```
def lorenz(state, t, sigma=10.0, rho=28.0, beta=8/3):
    x, y, z = state
    return [sigma*(y - x), x*(rho - z) - y, x*y - beta*z]

# Integration
t_span = np.linspace(0, 500, 50000)
initial = [1.0, 1.0, 1.0]
traj = odeint(lorenz, initial, t_span)

# Use x-component as observable
x_series = traj[:, 0]

# Compute emergent time
```

```
t_n_lorenz = emergent_time(x_series)
```

### Appendix A.3. Example: Hyperchaotic Rössler (4D)

```
def hyper_rossler(state, t, a=0.25, b=3.0, c=0.5, d=0.05):
    x, y, z, w = state
    return [-y - z, x + a*y, b + w*(z - c), -d*z + w*x]
```

```
traj_hyper = odeint(hyper_rossler, [1,1,1,1], t_span)
```

```
t_n_hyper = emergent_time(traj_hyper[:, 0])
```

### Appendix A.4. Example: Weakly Coupled Lorenz Pair

```
def coupled_lorenz(state, t, epsilon=0.1):
    x1,y1,z1,x2,y2,z2 = state
    dx1 = sigma*(y1 - x1) + epsilon*(x2 - x1)
    dy1 = x1*(rho - z1) - y1
    dz1 = x1*y1 - beta*z1
    dx2 = sigma*(y2 - x2) + epsilon*(x1 - x2)
    dy2 = x2*(rho - z2) - y2
    dz2 = x2*y2 - beta*z2
    return [dx1, dy1, dz1, dx2, dy2, dz2]
```

```
traj_coupled = odeint(coupled_lorenz, [1,1,1,2,2,2], t_span)
```

```
# Average emergent time across nodes
```

```
t_n_node1 = emergent_time(traj_coupled[:, 0])
```

```
t_n_node2 = emergent_time(traj_coupled[:, 3])
```

```
t_n_avg = (t_n_node1 + t_n_node2) / 2
```

### Appendix A.5. Notes

- Fractional-order and Chua systems used custom predictor-corrector or specialized libraries (e.g., `fractional-diffeq`). - Kicked rotor implemented as a discrete map. - All codes were executed on standard hardware; full reproducible scripts available upon request.

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