Spin g factors and neutrino magnetic moment of elementary fermions Jae-Kwang Hwang

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Abstract;

The spin magnetic moments and spin g factors (g_s = -2) of electron, muon and tau are explained based on the electric charges (EC) and lepton charges (LC) in terms of the three-dimensional quantized space model. The spin g factors of electron, muon and tau are g_s = -2 which is the sum of the EC g factor (g_{EC} = -1) and the LC g factor (g_{LC} = -1). The spin g factor (g_s = -2) of the electron is predicted by the Dirac's equation. The orbit g factors of electron, muon and tau are g_L = g_{EC} = -1 from the EC g factor (g_{EC} = -1) without the contribution of the LC g factor (g_{LC} = -1). The spin g factors of the elementary fermions are calculated from the equation of g_s = g_{EC} + g_{LC} + g_{CC} where g_{EC} = EC/|EC|, g_{LC} = LC/|LC| and g_{CC} = CC/|CC|. For example, the spin g factors of the neutrinos and dark matters are g_s = -1. The spin g factors of the u and d quarks are g_s = 0 and g_s = -2, respectively. The g factor problem of neutrinos with the non-zero LC charges are solved by the LC Coulomb force of $F_c(LC) \approx 0$. It is, for the first time, proposed that the binary motion (fluctuations) of the m_{EC} and m_{LC} masses for the electron, muon and tau leptons make the anomalous g factor. This binary motion could be originated from the virtual particle processes including the photons. Also, the weak force (beta) decay is closely related to the binary motion of the m_{EC} and m_{LC} for the electron, muon and tau leptons.

Key words; Anomalous spin g factor; Lepton charge (LC); Neutrino magnetic moment; Weak force decay; EC and LC partial masses; Three-dimensional quantized space model (TQSM).

1. Introduction

The spin magnetic moments of the electron, muon and tau leptons have been actively studied [1-30]. The experimental spin magnetic moments of these leptons have been successfully reproduced by the quantum electrodynamics (QED) calculations [1-30] but, for further studies, could need the new ideas beyond the standard model. The electrically charged leptons of e, μ and τ have the spin magnetic moments with the spin g factor of g_s =-2. The spin g factor (g_s = -2) of the electron is predicted by the Dirac's equation. The observed non-zero values of $a = (g_{eff} - g_s)/g_s = (g_{eff} + 2)/(-g_{eff} + g_{eff} + g_{ef$ 2) indicate that there are the anomalous magnetic moments for the electrically charged leptons. The anomalous magnetic moment (a) values have been calculated successfully through the virtual particle processes of photons, leptons, weak force bosons and hadrons including the QED calculations in terms of the standard model (SM). As well known in the classical electrodynamics, the orbit magnetic moment has the g factor of $g_L = -1$ for the rotational motion of an electron. The spin angular momentum (s) can be replaced with the orbital angular momentum (L) in the magnetic moment equation. Then the g factor value is changed from $g_s = -2$ to $g_L = -1$. It is thought that the spin angular momentum of $s = \frac{1}{2}$ makes the twice more contribution than the orbital angular momentum of L to the magnetic moment of an electron. The reason why the g factors are different for the spin angular momentum and orbital angular momentum is not clearly explained in the standard model (SM). I think that this is the g factor problem.

All charged elementary fermions have the (rest) masses. In other words, the electric charges (EC) of the elementary fermions are created based on the masses. And the neutrinos have the zero masses in terms of the standard model (SM). The neutrinos have the zero EC charges and zero spin magnetic moment because of the zero masses in SM. But the neutrino spin of ½ should be based on the non-zero neutrino masses. The elementary fermions with the zero masses cannot have the

Same origins of $g_{\mbox{\tiny S}}$ and $g_{\mbox{\tiny L}}$ for the electron in terms of TQSM

The electron with (EC,LC) = (-1,-2/3) has $g_s = -2$ and $g_L = -1$.

$$\overrightarrow{\mu_S} = \frac{e}{2m} g_S \vec{s} = (-2) \frac{e}{2m} \vec{s} \qquad \mu_{EC} \neq 0, \mu_{LC} \neq 0 \text{ for the spin motion}$$

$$\overrightarrow{\mu_S} = \frac{e}{2m} g_S \vec{s} = (g_{EC} + g_{LC}) \frac{e}{2m} \vec{s} = \overrightarrow{\mu_{EC}} + \overrightarrow{\mu_{LC}} \qquad \frac{s(m_{EC})}{m_{EC}} = \frac{s(m_{LC})}{m_{LC}} = \frac{s}{m}$$

$$\overrightarrow{\mu_S} = \frac{e}{2m} g_S (1 + a) \vec{s} \qquad \text{g}_{EC} = \text{EC/|EC|=-1}, \ \text{g}_{LC} = \text{LC/|LC|=-1} \qquad \text{m} = \text{m}_{EC} + \text{m}_{LC}$$

The $a\vec{s}$ term comes from the binary motion (fluctuations) of m_{EC} and m_{LC} through the Z boson force. The m_{EC} and m_{LC} masses are not merged but overlapped.

$$\begin{split} F_c(EC) &\neq 0, F_c(LC) = 0, \quad \mu_{EC} \neq 0, \mu_{LC} = 0 \text{ for the orbital motion} \\ \overrightarrow{\mu_L} &= \frac{e}{2m} g_L \overrightarrow{L} = (g_{EC} + g_{LC}) \frac{e}{2m} \overrightarrow{L} = \overrightarrow{\mu_{EC}} + \overrightarrow{\mu_{LC}} = g_{EC} \frac{e}{2m} \overrightarrow{L} \\ \overrightarrow{\mu_L} &= \frac{e}{2m} g_L \overrightarrow{L} = (-1) \frac{e}{2m} \overrightarrow{L} \\ \end{split}$$

$$g_L = g_{EC} = EC/|EC| = -1$$

Fig. 1. The spin and orbit g factors are compared in terms of TQSM model [31]. See Ref. [32] for the lepton Coulomb force, Fc(LC) and Partial masses. See Figs. 2-4, too.

spin of ½. The neutrinos cannot have both the zero mass and the spin of ½ in terms of the SM model. And it is known that the neutrinos have the very small masses. This means that the massive neutrinos with the spin of ½ can have the very small negative EC charges and spin magnetic moment in the SM model. This does not follow the systematics of the EC charges for the leptons and quarks in the SM model. I think that this is the neutrino mass problem.

The g factor problem and neutrino mass problem cannot be solved in terms of the present SM model. The radical new ideas for the extension of the standard model are required to solve these two problems as shown in Figs. 1 and 2. The present three-dimensional quantized space model (TQSM) [31] (see Figs. 3 - 6) is considered as the extended standard model in the present work. The electric charges (EC), lepton charges (LC) and color charges (CC) of all elementary fermions [31] are tabulated as shown in Fig. 3. The leptons have only the electric charges (EC) in terms of SM. However, the leptons have the electric charges (EC) and lepton charges (LC) in terms of the three-dimensional quantized space model (TQSM) [31] as shown in Fig. 3. If the spin magnetic moments for the leptons are made by the EC and LC charges, the spin magnetic moment term of the Dirac's equation is the total spin magnetic moment including the EC and LC contributions in Fig. 1. If the electron rotates around the positively EC charged proton with LC = 0, the orbit magnetic moment of the electron includes only the EC contribution but not the LC contribution as shown in Fig. 1. The anomalous magnetic moment is explained by using the binary motion (fluctuations) of m_{EC} and m_{LC} with the distance (d) by the Z boson force. The binary motion in terms of the TQSM model in Fig. 1 could be originated from the virtual particle processes of

photons, leptons, weak force bosons and hadrons including the QED calculations in terms of the standard model (SM). The spin magnetic moment of the electron neutrino is shown by using the LC magnetic moment in Fig. 2. Therefore, the spin magnetic moments of the leptons including the

Spin magnetic moment of the electron neutrino in TQSM $v_{e}\text{-}e \text{ elastic scattering} \quad v_{e}\text{: (EC,LC)} = (0,-2/3), \quad e\text{: (-1,-2/3)}, \quad W^{+}\text{: (1,0)}$ $m(v_{e}) = m_{EC} + m_{LC} = m_{LC} \quad \mu_{S}(v_{e}) = \mu_{EC} + \mu_{LC} = \mu_{LC}$ $m_{EC} = 0, m_{LC} \neq 0 \quad \text{Ve} \quad \text{Ve$

Fig. 2. Spin magnetic moment of the electron neutrino. See Ref. [32] for the lepton Coulomb force, Fc(LC). See Figs. 1 and 3-4, too.

$$E = c\Delta t\Delta V = |q|\Delta V = mc^2$$

For the elementary fermion with the charge (q) configuration of (EC,LC,CC),

$$E = mc^2 = (|EC| + |LC| + |CC|))\Delta V = (m_{EC} + m_{LC} + m_{CC}) c^2$$

$$s = \frac{1}{2} = s(m_{EC}) + s(m_{IC}) + s(m_{CC}) \propto m\omega$$

$$|EC|: |LC|: |CC| = m_{EC}: m_{LC}: m_{CC} = s(m_{EC}): s(m_{LC}): s(m_{CC})$$

$$\frac{s(m_{EC})}{m_{EC}} = \frac{s(m_{LC})}{m_{LC}} = \frac{s(m_{CC})}{m_{CC}} = \frac{s}{m}$$

If the rest mass (m) of the elementary fermion is given, partial masses and partial spins can be calculated by using the charge(q) configuration of (EC,LC,CC). The calculated tables are shown for all elementary fermions of the 3 dark matters (3 bastons), 9 leptons and 27 quarks in the present work. Note that the volume (Δ V) and angular velocity (ω) are the same for m, m_{EC}, m_{LC} and m_{CC}. Then the partial spin is proportional to the partial mass.

Fig. 3. The masses, spins and charges are closely related to each other [32]. By using these relations, the partial masses used in the g factors can be calculated.

neutrinos are newly discussed by including the LC charge effects in the present work. The effective charge is defined as $Q_{\rm eff} = (k_1/k_2)^{0.5}Q \approx 0$ for $k_1 \approx 0$. Q is EC for the dark matters, LC for the

leptons and CC for the quarks in Fig. 2. The EC and LC partial masses of the leptons given in Figs. 3 and 4 [32] are used to explain the g factors of the leptons in the present work.

Leptons (I)		mass: MeV/c²		Leptons	(I)	mass: MeV/c²		
	ν_{e}	ν_{μ}	$\nu_{ au}$		е	μ	τ	
EC	0	0	0	EC	-1	-1	-1	
LC	-2/3	-5/3	-8/3	LC	-2/3	-5/3	-8/3	
m _{EC}	0	0	0	m _{EC}	0.307	39.62	484.6	
m _{LC}	m(v _e)	$m(v_{\mu})$	$m(v_{\tau})$	m _{LC}	0.204	66.04	1292.2	
m(l)	$m(v_e)$	$m(v_{\mu})$	$m(v_{\tau})$	m(l)	0.511	105.66	1776.8	
s(m _{EC})	0	0	0	s(m _{EC})	3/10	3/16	3/22	
s(m _{LC})	1/2	1/2	1/2	s(m _{LC})	2/10	5/16	8/22	
s(I)	1/2	1/2	1/2	s(I)	1/2	1/2	1/2	

From the lepton (e(-1,-2/3)/ μ (-1,-5/3)) - p(1,0) deep inelastic scattering, spins (s(m_{EC})) of the u and d quarks are determined. Total spins (s = ½) of the u and d quarks should include the CC charge effect of A(CC=-5)₃. Note that p(1,0) is the lepton-like particle. The neutrino masses (m_{LC}) are not zero because the lepton charges (LC) are not zero.

Fig. 4. Lepton spins are calculated for the EC and LC charge masses [31,32] which are used to explain the g factors.

For the electron with s= ½ and (EC,LC) = (-1,-2/3),

Spin magnetic moment $\overrightarrow{\mu_S} = \frac{e}{2m} g_S \vec{s} = \frac{e}{2m} (g_{EC} + g_{LC}) \vec{s} = \overrightarrow{\mu_{EC}} + \overrightarrow{\mu_{LC}}$ $g_{EC} = -1$ EC = -1 $g_{LC} = -2/3$ $g_{EC} = \frac{s(m_{LC})}{m_{LC}} = \frac{s}{m}$ $g_{EC} = \frac{s(m_{EC})}{m_{EC}} =$

The electron has the internal structure of m_{EC} and m_{LC} . The spin g factor (g_s =-2) of the electron is the sum of the EC g factor (g_{EC} = -1) and LC g factor (g_{LC} = -1). The additional g factor (ag_s) comes from the binary motion (fluctuations) of m_{EC} and m_{LC} through the Z boson force. The ag_s and g_s and g_s are not merged but overlapped.

Fig. 5. The spin and orbit g factors are compared in terms of TQSM model [31,32].

2. Magnetic moments and spin g factors of the elementary fermions

Dark matters (bastons), leptons and quarks [31] have the charge configurations of (EC), (EC,LC) and (EC,LC,CC), respectively. The charges of these elementary fermions can make the spin magnetic moments and spin g factors. In Fig. 1 and 2, two cases of the electron and electron neutrino are taken into consideration. In the standard model (SM), the spin magnetic moments of the leptons are derived only by the electric charges (EC). But in the present TQSM model, the spin magnetic moments of the leptons are derived by both the electric charges (EC) and lepton charges (LC) as shown in Figs. 1 and 2. The electron has the charge configuration of (EC,LC) = (-1,-2/3). Then the spin g factor (g_s) of the electron is the sum of the EC g factor (g_{EC}) and the LC g factor (g_{LC}) in Figs. 1 and 14. The observed spin g factors (g_s) of the electron with q = (-1,-2/3) and the muon with q = (-1,-5/3) in Fig. 3 is -2. The spin g factor of -2 is reproduced by the Dirac's equation for the electron. g_{EC} is -1 and g_{LC} is -1. The g_{LC} value is -1 for the electron with LC = -2/3 and the muon with LC = -5/3. From these observations, the g_{EC} and g_{LC} are defined as $g_{EC} = EC/|EC| = -1$

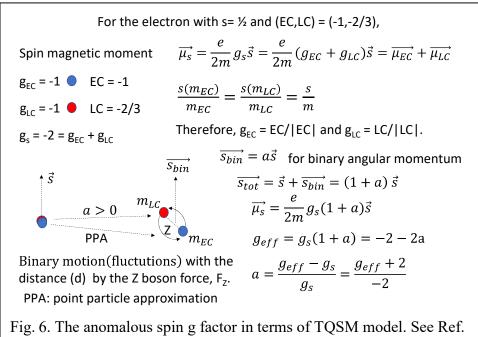


Fig. 6. The anomalous spin g factor in terms of TQSM model. See Ref. [32] for the Z boson force and partial masses.

and $g_{LC} = LC/|LC| = -1$. Also, the orbit magnetic g factor (g_L) is -1 for the electron in Figs. 1 and 5 - 7. The orbit magnetic moment of the electron is made only by the EC Coulomb $(F_c(EC))$ force but not by the LC Coulomb force $(F_c(LC))$ because the LC Coulomb force is zero. The proton has the zero LC charge (LC=0) in Figs. 1 and 5 and the LC Coulomb constant (k_1) is nearly zero [32]. This indicates that the orbit magnetic moment is the electric magnetic moment with $g_L = g_{EC} = -1$. Therefore, the reason why the spin g factor is -2 and orbit g factor is -1 for the electron is explained by introducing the lepton charge (LC) for the electron and muon.

In Fig. 5 - 9, the anomalous g factors of the electron, muon and tau are explained by using the binary motion of the m_{Ec} and m_{LC} calculated in Figs. 3 and 4 in terms of TQSM. The experimental spin g factors (g_{eff}) of the EC charged leptons are different from the predicted s[in g factor ($g_s = -$

2). The difference is defined as the anomalous g factor (a = $(g_{eff}-g_s)/g_s$). The electron, muon and tau have two masses of m_{EC} and m_{LC} . If these two masses are fluctuated in the binary motion, we can calculate the binary angular momentum (s_{bin} = as) which is added to the spin (s) of ½ in Figs. 6 - 9. Then the anomalous g factor can be written as g_{eff} = $g_s(1+a)$. This binary motion with the distance of d is made by the Z(0,0) boson force. The Z boson has the experimental mass of 91 10^{-11} eV/c². The graviton mass is taken as 3.2 10^{-31} eV/c² in Fig. 7. For the binary motion,

$$s_{bin} = m_{EC} m_{LC} (\frac{G_Z d}{m_{EC} + m_{LC}})^{0.5} = as$$

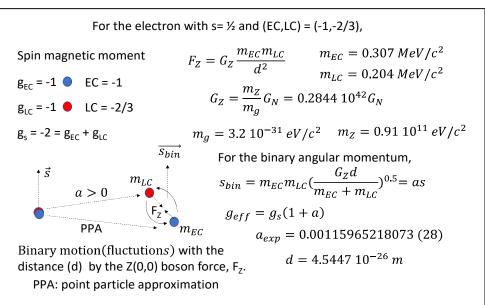


Fig. 7. The anomalous spin g factor of the electron. See Ref. [32] for the Z boson force and partial masses.

The experimental value of $a_{exp} = 0.00115965218073(28)$ [23] gives the distance of $d = 4.5447 \cdot 10^{-1}$ ²⁶ m for the electron. This means that the binary motion with the distance of d can predict the anomalous spin g factor when the Z boson force and graviton mass are properly given. The same calculations are tried for the muon and tau. In Fig. 8, the experimental value of $a_{exp} = 0.0011659209$ (6) [1-4] gives the distance of $d = 5.3580 \ 10^{-33} \ m$ for the muon. The anomalous magnetic moment of the tau lepton seems to be negative in Fig. 9. Therefore, the anomalous g factor is $a_{exp} = -0.018$ (17) which gives the distance of $d = 2.6972 \cdot 10^{-34}$ m. The better and more experimental data are needed for the further research because the experimental a_{exp} values are relatively large compared with those of the electron and muon. The binary motion in terms of the present TQSM model could be originated from the virtual particle processes of photons, leptons, weak force bosons and hadrons including the QED calculations in terms of the standard model (SM). In Fig. 9, the g factors of the tau leptons are shown. The experimental anomalous g factors of the tau lepton [14,16,17] have the negative values. But the standard model predicts the positive anomalous g factor for the tau lepton. The negative anomalous g factor of the tau lepton means in terms of TQSM that the binary angular moment has the negative value with the opposite direction to the spin direction as shown in Fig. 9.

The magnetic moment measurement is very difficult because the neutrinos have the zero electric charge (EC). The upper limit of the spin magnetic moment of the electron neutrino from the ν -e

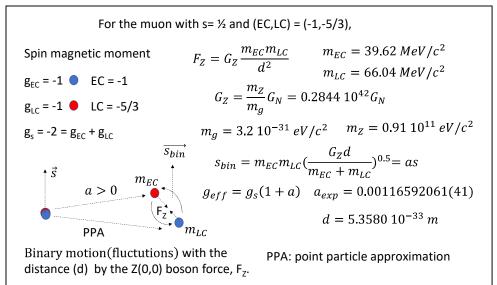


Fig. 8. The anomalous spin g factor of the muon. See Ref. [32] for the Z boson force and partial masses.

elastic scattering has been given in the SM model. The spin magnetic moment is discussed in terms of the TQSM model in Fig. 2. In the TQSM model, the neutrinos have the LC charges in Fig. 4.

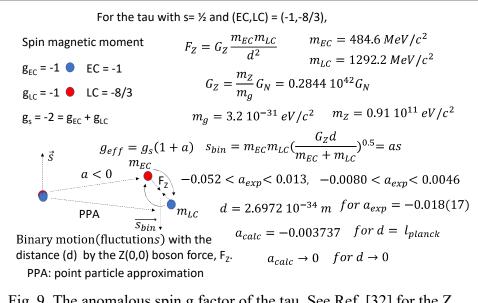
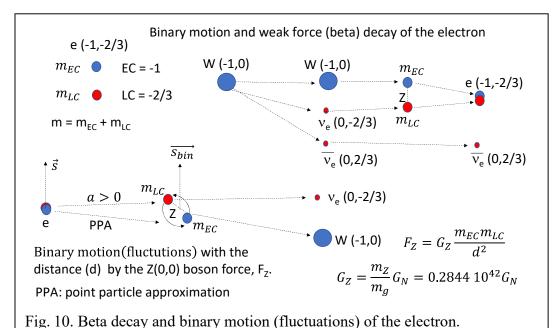


Fig. 9. The anomalous spin g factor of the tau. See Ref. [32] for the Z boson force and partial masses.

This means that the neutrinos have the LC magnetic moments. But the LC coulomb force between the electron and electron neutrino [32] is nearly zero in Figs. 2 and 5. The effective charge is defined as $Q_{\rm eff} = (k_1/k_2)^{0.5}Q \approx 0$ for $k_1 \approx 0$. Q is EC for the dark matters, LC for the leptons and CC for the quarks. Therefore, in the v_e and electron elastic scattering experiment, it will be very difficult to measure the LC spin magnetic moment because of $F_c(LC) \approx 0$ in Fig. 2. To measure

the LC spin magnetic moments of the neutrinos the new experiments are needed. It is thought that the observed magnetic moment of the electron neutrino comes from the loop virtual process of W^+ and e in Fig. 2. The experimental loop magnetic moment has the upper limit of 4.5 10^{-12} μ_B [7] for the electron neutrino.



Spi	Spin g factors of elementary fermions ($g_{\rm S}=g_{\rm EC}+g_{\rm LC}+g_{\rm CC}$)											
	Bastons (EC), γ(0)			Leptons(EC,LC), γ(0,0)				Quarks(EC,LC,CC), γ(0,0,0)				
	EC				EC				EC			
X1	-2/3	-1			0	0	0	0	2/3	1	1	1
X2	-5/3	-1			-1	-1	-1	-1	-1/3	-1	-1	-1
Х3	-8/3	-1			-2	-1	-1	-1	-4/3	-1	-1	-1
Total	-5				-3				-1			
	$g_{FC} = EC/ EC ,$ LC							LC				
X4	g _{LC} = LC/ LC , g _{CC} = CC/ CC				-2/3	-1	-1	-1	0	0	0	0
X5					-5/3	-1	-1	-1	-1	-1	-1	-1
Х6				-8/3	-1	-1	-1	-2	-1	-1	-1	
Total	$- \overrightarrow{\mu_s} = \frac{e}{2m} g_s \vec{s} -$				-5				-3			
	$g_s = -1$, $a = 0$ for dark matters								CC			
X7	$g_s = -1$, $a = 0$ for neutrinos,							-2/3(r)	-1	-1	-1	
Х8	$g_s = -2$ for e, μ , τ , d, Q1							-5/3(g)	-1	-1	-1	
Х9	$g_s=0$ for u, $g_s=-1$ for c, t,							-8/3(b)	-1	-1	-1	
Total								-5				

Fig. 11. The spin g factors of the elementary fermions.

The beta (weak force) decay of the leptons can be explained by using the binary motion of the m_{EC} and m_{LC} in Fig. 10. The beta decay of the electron is shown as one example in Fig. 10. The neutrinos with the charge configuration of (0,LC) have the anti-neutrinos with the charge configurations of (0,-LC) in Fig. 4. The electron neutrino, muon neutrino and tau neutrino have the charge configurations of (0,-2/3), (0,-5/3) and (0,-8/3), respectively. Finally, in Fig. 10, the proposed spin g factors are shown for the dark matters (bastons), leptons and quarks. By using these spin g factors, the spin magnetic moments can be given as the equation of $\mu_s = (g_s e s)/(2m) = (g_s e)/(4m)$. Note that the dark matters and neutrinos do not have the anomalous g factor because the dark matters and neutrinos have one mass of m_{EC} and m_{LC} , respectively. In other words, the dark matters and neutrinos do not have the binary motion (fluctuations). And the u quark has the spin g factor of $g_s = 0$ and the d quark has the spin g factor of $g_s = -2$.

3. Summary and conclusions

TIn the present work, the electric charge, lepton charge and color charge are expressed as EC, LC and CC. The elementary fermions are made of the three bastons (dark matters) with the charge configuration of (EC), 9 leptons with the charge configuration of (EC,LC) and 27 quarks with the charge configuration of (EC,LC,CC). The spin magnetic moments and spin g factors ($g_s = -2$) of electron, muon and tau are explained based on the electric charges (EC) and lepton charges (LC). The spin g factors of electron, muon and tau are $g_s = -2$ which is the sum of the EC g factor (g_{EC} = -1) and the LC g factor (g_{LC} = -1). The spin g factor (g_s = -2) of the electron is predicted by the Dirac's equation. The orbit g factors of electron, muon and tau are $g_L = g_{EC} = -1$ from the EC g factor ($g_{EC} = -1$) without the contribution of the LC g factor ($g_{LC} = -1$). The spin g factors of the elementary fermions are calculated from the equation of $g_s = g_{EC} + g_{LC} + g_{CC}$ where $g_{EC} = EC/|EC|$, $g_{LC} = LC/|LC|$ and $g_{CC} = CC/|CC|$. For example, the spin g factors of the neutrinos and dark matters are $g_s = -1$. The spin g factors of the u and d quarks are $g_s = 0$ and $g_s = -2$, respectively. The g factor problem of neutrinos with the non-zero LC charges are solved by the LC Coulomb force of F_c(LC) ≈ 0 . To measure the LC spin magnetic moments of the neutrinos the new experiments are needed. It is thought that the observed magnetic moment of the electron neutrino comes from the loop virtual process of W⁺ and e. It is, for the first time, proposed that the binary motion (fluctuations) of the m_{EC} and m_{LC} masses for the electron, muon and tau leptons make the anomalous g factor. This binary motion could be originated from the virtual particle processes including the photons. Also, the weak force (beta) decay is closely related to the binary motion of the m_{EC} and m_{LC} for the electron, muon and tau leptons. The experimental anomalous g factors of the tau lepton have the negative values. But the standard model predicts the positive anomalous g factor for the tau lepton. The negative anomalous g factor of the tau lepton means in terms of TQSM that the binary angular moment has the negative value with the opposite direction to the spin direction. The effective charge is defined as $Q_{\rm eff} = (k_1/k_2)^{0.5}Q \approx 0$ for $k_1 \approx 0$. Q is EC for the dark matters, LC for the leptons and CC for the quarks.

References

- [1] T. Albahri et al., Phys. Accel. Beams 24, 044002 (2021).
- [2] T. Albahri et al., Phys. Rev. A103, 042208 (2021).
- [3] T. Albahri et al., Phys. Rev. D103, 072002 (2021).
- [4] B. Abi et al., Phys. Rev. Lett. 126, 141801 (2021).
- [5] E. Balzani, S. Laporta and M. Passera, arXiv:2112.05704v1 (2021).

- [6] F. Jegerlehner and A. Nyffeler, Phys. Rept. 477, 1 (2009).
- [7] N. Viaux et al., Phys. Rev. Lett. 111, 231301 (2013).
- [8] E. Aprile et al., Phys. Rev. D102, 072004 (2020).
- [9] K.S. Babu, S. Jana and M. Lindner, JHEP 10, 040 (2020).
- [10] V. Brdar et al., JCAP 01, 039 (2021).
- [11] G.C. McLaughlin and C. Volpe, Phys. Lett. B591, 229 (2004).
- [12] M. Wen, C.H. Keitel and H. Bauke, Phys. Rev. A95, 042102 (2017).
- [13] M. Frasca, A. Ghoshal and S. Groote, arXiv:2109.05041v2 (2021).
- [14] L. Beresford and J. Liu, Phys. Rev. D102, 113008 (2020).
- [15] T. Aoyama, T. Kinoshita and M. Nio, Atoms 7, 28 (2019).
- [16] J. Abdallah et al., Eur. Phys. J. C35, 159 (2004).
- [17] S. Eidelman and M. Passera, Mod. Phys. Lett. A22, 159 (2007).
- [18] T. Aoyama et al., Phys. Rept. 887, 1 (2020).
- [19] A. Gerardin, Eur. Phys. J. A57, 116 (2021).
- [20] D. Giusti and S. Simula, Phys. Rev. D102, 054503 (2020).
- [21] G. Colangelo et al., arXiv:2110.05493v1 (2021).
- [22] S. Laporta and E. Remiddi, Phys. Lett. B379, 283 (1996).
- [23] D. Hanneke, S.F. Hoogerheide and G. Gabrielse, Phys. Rev. A83, 052122 (2011).
- [24] T. Beier et al., Phys. Rev. Lett. 88, 011603 (2002).
- [25] D. Hanneke, S. Fogwell and G. Gabrielse, Phys. Rev. Lett. 100, 120801 (2008).
- [26] T.V. Holten, Appl. Sci. 9, 1471 (2019).
- [27] S. Borsanyi et al., Nature 593, 51 (2021).
- [28] F. Kohler et al., J. Phys. B: At. Mol. Opt. Phys. 48, 144032 (2015).
- [29] K. Pachucki et al., Phys. Rev. A72, 022108 (2005).
- [30] D. Castelvecchi, Nature 593, 18 (2021).
- [31] Jae-Kwang Hwang, Mod. Phys. Lett. A32, 1730023 (2017).
- [32] J.K. Hwang, Preprints 2021, 2021020395 (2021).