

Co-degree and Degree of classes of Neutrosophic Hypergraphs

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Abstract

New setting is introduced to study types of coloring numbers, degree of vertices, degree of hyperedges, co-degree of vertices, co-degree of hyperedges, neutrosophic degree of vertices, neutrosophic degree of hyperedges, neutrosophic co-degree of vertices, neutrosophic co-degree of hyperedges, neutrosophic number of vertices, neutrosophic number of hyperedges in neutrosophic hypergraphs. Different types of procedures including neutrosophic (r, n) -regular hypergraphs and neutrosophic complete r -partite hypergraphs are proposed in this way, some results are obtained. General classes of neutrosophic hypergraphs are used to obtain chromatic number, the representatives of the colors, degree of vertices, degree of hyperedges, co-degree of vertices, co-degree of hyperedges, neutrosophic degree of vertices, neutrosophic degree of hyperedges, neutrosophic co-degree of vertices, neutrosophic co-degree of hyperedges, neutrosophic number of vertices, neutrosophic number of hyperedges in neutrosophic hypergraphs. Using colors to assign to the vertices of neutrosophic hypergraphs and characterizing representatives of the colors are applied in neutrosophic (r, n) -regular hypergraphs and neutrosophic complete r -partite hypergraphs. Some questions and problems are posed concerning ways to do further studies on this topic. Using different ways of study on neutrosophic hypergraphs to get new results about number, degree and co-degree in the way that some number, degree and co-degree get understandable perspective. Neutrosophic (r, n) -regular hypergraphs and neutrosophic complete r -partite hypergraphs are studied to investigate about the notions, coloring, the representatives of the colors, degree of vertices, degree of hyperedges, co-degree of vertices, co-degree of hyperedges, neutrosophic degree of vertices, neutrosophic degree of hyperedges, neutrosophic co-degree of vertices, neutrosophic co-degree of hyperedges, neutrosophic number of vertices, neutrosophic number of hyperedges in neutrosophic (r, n) -regular hypergraphs and neutrosophic complete r -partite hypergraphs. In this way, sets of representatives of colors, degree of vertices, degree of hyperedges, co-degree of vertices, co-degree of hyperedges, neutrosophic degree of vertices, neutrosophic degree of hyperedges, neutrosophic co-degree of vertices, neutrosophic co-degree of hyperedges, neutrosophic number of vertices, neutrosophic number of hyperedges have key points to get new results but in some cases, there are usages of sets and numbers instead of optimal ones. Simultaneously, notions chromatic number, the representatives of the colors, degree of vertices, degree of hyperedges, co-degree of vertices, co-degree of hyperedges, neutrosophic degree of vertices, neutrosophic degree of hyperedges, neutrosophic co-degree of vertices, neutrosophic co-degree of hyperedges, neutrosophic

number of vertices, neutrosophic number of hyperedges are applied into neutrosophic hypergraphs, especially, neutrosophic (r, n) -regular hypergraphs and neutrosophic complete r -partite hypergraphs to get sensible results about their structures. Basic familiarities with neutrosophic hypergraphs theory and hypergraph theory are proposed for this article.

Keywords: Degree, Coloring, Co-degree
AMS Subject Classification: 05C17, 05C22, 05E45

1 Background

Fuzzy set in Ref. [15], neutrosophic set in Ref. [2], related definitions of other sets in Refs. [2, 13, 14], graphs and new notions on them in Refs. [5–11], neutrosophic graphs in Ref. [3], studies on neutrosophic graphs in Ref. [1], relevant definitions of other graphs based on fuzzy graphs in Ref. [12], related definitions of other graphs based on neutrosophic graphs in Ref. [4], are proposed.

In this section, I use two subsections to illustrate a perspective about the background of this study.

1.1 Motivation and Contributions

In this study, there’s an idea which could be considered as a motivation.

Question 1.1. *Is it possible to use mixed versions of ideas concerning “neutrosophic degree”, “neutrosophic co-degree” and “neutrosophic coloring” to define some notions which are applied to neutrosophic hypergraphs?*

It’s motivation to find notions to use in any classes of neutrosophic hypergraphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Connections amid two items have key roles to assign colors and introducing different types of degree of vertices, degree of hyperedges, co-degree of vertices, co-degree of hyperedges, neutrosophic degree of vertices, neutrosophic degree of hyperedges, neutrosophic co-degree of vertices, neutrosophic co-degree of hyperedges, neutrosophic number of vertices, neutrosophic number of hyperedges in neutrosophic hypergraphs. Thus they’re used to define new ideas which conclude to the structure of coloring, degree and co-degree. The concept of having general neutrosophic hyperedge inspires me to study the behavior of general neutrosophic hyperedge in the way that, types of coloring numbers, degree of vertices, degree of hyperedges, co-degree of vertices, co-degree of hyperedges, neutrosophic degree of vertices, neutrosophic degree of hyperedges, neutrosophic co-degree of vertices, neutrosophic co-degree of hyperedges, neutrosophic number of vertices, neutrosophic number of hyperedges in neutrosophic hypergraphs are introduced.

The framework of this study is as follows. In the beginning, I introduced basic definitions to clarify about preliminaries. In section “New Ideas For Neutrosophic Hypergraphs”, new notions of coloring, degree of vertices, degree of hyperedges, co-degree of vertices, co-degree of hyperedges, neutrosophic degree of vertices, neutrosophic degree of hyperedges, neutrosophic co-degree of vertices, neutrosophic co-degree of hyperedges, neutrosophic number of vertices, neutrosophic number of hyperedges in neutrosophic hypergraphs are introduced. In section “Applications in Time Table and Scheduling”, one application is posed for neutrosophic hypergraphs concerning time table and scheduling when the suspicions are about choosing some subjects. In section “Open Problems”, some problems and questions for further studies are proposed. In section “Conclusion and Closing Remarks”, gentle discussion about

results and applications are featured. In section “Conclusion and Closing Remarks”, a brief overview concerning advantages and limitations of this study alongside conclusions are formed.

1.2 Preliminaries

Definition 1.2. (Graph).

$G = (V, E)$ is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Definition 1.3. (Hypergraph).

$H = (V, E)$ is called a **hypergraph** if V is a set of objects and for every nonnegative integer $t \leq n$, E is a set of t -subsets of V where V is called **vertex set** and E is called **hyperedge set**.

Definition 1.4. (Neutrosophic Hypergraph).

$NHG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic hypergraph** if it's hypergraph, $\sigma_i : V \rightarrow [0, 1]$, $\mu_i : E \rightarrow [0, 1]$, and for every $v_1 v_2 \cdots v_t \in E$,

$$\mu(v_1 v_2 \cdots v_t) \leq \sigma(v_1) \wedge \sigma(v_2) \wedge \cdots \wedge \sigma(v_t).$$

(i) : σ is called **neutrosophic vertex set**.

(ii) : μ is called **neutrosophic hyperedge set**.

(iii) : $|V|$ is called **order** of NHG and it's denoted by $\mathcal{O}(NHG)$.

(iv) : $\sum_{v \in V} \sigma(v)$ is called **neutrosophic order** of NHG and it's denoted by $\mathcal{O}_n(NHG)$.

(v) : $|E|$ is called **size** of NHG and it's denoted by $\mathcal{S}(NHG)$.

(vi) : $\sum_{e \in E} \mu(e)$ is called **neutrosophic size** of NHG and it's denoted by $\mathcal{S}_n(NHG)$.

Example 1.5. Assume Figure (1).

(i) : Neutrosophic hyperedge $n_1 n_2 n_3$ has three neutrosophic vertices.

(ii) : Neutrosophic hyperedge $n_3 n_4 n_5 n_6$ has four neutrosophic vertices.

(iii) : Neutrosophic hyperedge $n_1 n_7 n_8 n_9 n_5 n_6$ has six neutrosophic vertices.

(iv) : $\sigma = \{(n_1, (0.99, 0.98, 0.55)), (n_2, (0.74, 0.64, 0.46)), (n_3, (0.99, 0.98, 0.55)), (n_4, (0.54, 0.24, 0.16)), (n_5, (0.99, 0.98, 0.55)), (n_6, (0.99, 0.98, 0.55)), (n_7, (0.99, 0.98, 0.55)), (n_8, (0.99, 0.98, 0.55)), (n_9, (0.99, 0.98, 0.55))\}$ is neutrosophic vertex set.

(v) : $\mu = \{(e_1, (0.01, 0.01, 0.01)), (e_2, (0.01, 0.01, 0.01)), (e_3, (0.01, 0.01, 0.01))\}$ is neutrosophic hyperedge set.

(vi) : $\mathcal{O}(NHG) = 9$.

(vii) : $\mathcal{O}_n(NHG) = (8.21, 7.74, 4.47)$.

(viii) : $\mathcal{S}(NHG) = 3$.

(ix) : $\mathcal{S}_n(NHG) = (0.03, 0.03, 0.03)$.

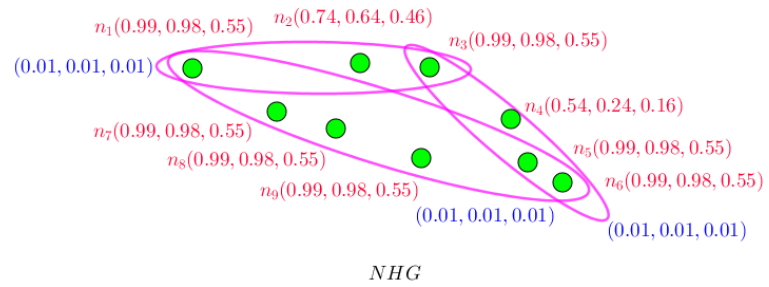


Figure 1. There are three neutrosophic hyperedges and two neutrosophic vertices.

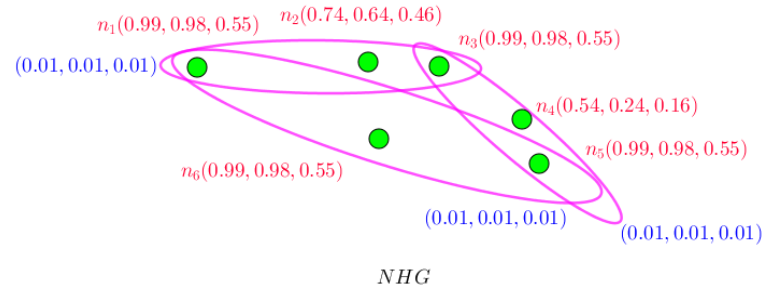


Figure 2. $NHG = (V, E, \sigma, \mu)$ is neutrosophic edge 3-regular hypergraph

Definition 1.6. (Neutrosophic Edge t -Regular Hypergraph).

A neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ is called a **neutrosophic edge t -regular hypergraph** if every neutrosophic hyperedge has only t neutrosophic vertices.

Question 1.7. What-if all neutrosophic hypergraphs are either edge t -regular or not?

In the following, there are some directions which clarify the existence of some neutrosophic hypergraphs which are either edge t -regular or not.

Example 1.8. Two neutrosophic hypergraphs are presented such that one of them is edge t -regular and another isn't.

(i) : Assume Figure (1). It isn't neutrosophic edge t -regular hypergraph.

(ii) : Suppose Figure (2). It's neutrosophic edge 3-regular hypergraph.

Definition 1.9. (Neutrosophic vertex t -Regular Hypergraph).

A neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ is called a **neutrosophic vertex t -regular hypergraph** if every neutrosophic vertex is incident to only t neutrosophic hyperedges.

Example 1.10. Three neutrosophic hypergraphs are presented such that one of them is vertex t -regular and others aren't.

(i) : Consider Figure (1). It isn't neutrosophic vertex t -regular hypergraph.

(ii) : Suppose Figure (2). It's neutrosophic edge 3-regular hypergraph but It isn't neutrosophic vertex 3-regular hypergraph.

(iii) : Assume Figure (3). It's neutrosophic vertex 2-regular hypergraph but It isn't neutrosophic edge t -regular hypergraph.

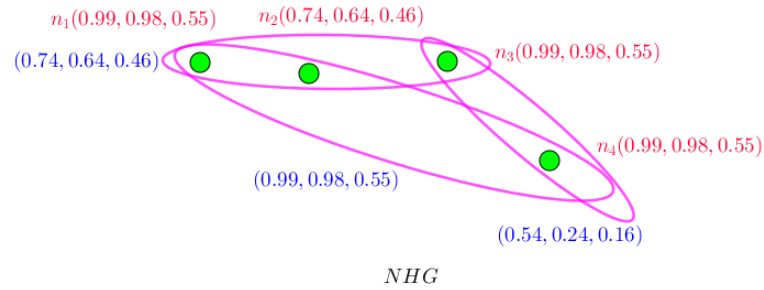


Figure 3. $NHG = (V, E, \sigma, \mu)$ is neutrosophic strong hypergraph.

Definition 1.11. (Neutrosophic Strong Hypergraph).

A neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ is called a **neutrosophic strong hypergraph** if it's hypergraph and for every $v_1 v_2 \cdots v_t \in E$,

$$\mu(v_1 v_2 \cdots v_t) = \sigma(v_1) \wedge \sigma(v_2) \wedge \cdots \wedge \sigma(v_t).$$

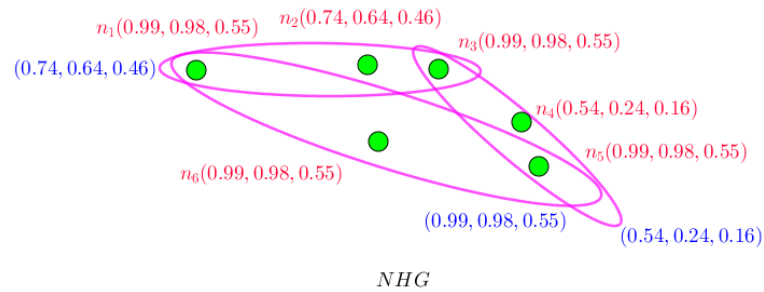


Figure 4. $NHG = (V, E, \sigma, \mu)$ is neutrosophic strong hypergraph.

Example 1.12. Three neutrosophic hypergraphs are presented such that one of them is neutrosophic strong hypergraph and others aren't.

- (i) : Consider Figure (1). It isn't neutrosophic strong hypergraph.
- (ii) : Assume Figure (2). It isn't neutrosophic strong hypergraph.
- (iii) : Suppose Figure (3). It isn't neutrosophic strong hypergraph.
- (iv) : Assume Figure (4). It's neutrosophic strong hypergraph. It's also neutrosophic edge 3-regular hypergraph but it isn't neutrosophic vertex t -regular hypergraph.

Definition 1.13. (Neutrosophic Strong Hypergraph).

Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. A neutrosophic hyperedge $v_1 v_2 \cdots v_t \in E$ is called a **neutrosophic strong hyperedge** if

$$\mu(v_1 v_2 \cdots v_t) = \sigma(v_1) \wedge \sigma(v_2) \wedge \cdots \wedge \sigma(v_t).$$

Proposition 1.14. Assume neutrosophic strong hypergraph $NHG = (V, E, \sigma, \mu)$. Then all neutrosophic hyperedges are neutrosophic strong.

Definition 1.15. (Neutrosophic Hyperpath).

A path $v_0, E_0, v_1, v_1, E_1, v_2, \cdots, v_{t-1}, E_{t-1}, v_t$, is called **neutrosophic hyperpath** such that v_{i-1} and v_i have incident to E_{i-1} for all nonnegative integers $0 \leq i \leq t$. In this case, $t - 1$ is called **length** of neutrosophic hyperpath. Also, if x and y are two

neutrosophic vertices, then maximum length of neutrosophic hyperpaths from x to y , is called **neutrosophic hyperdistance** and it's denoted by $d(x, y)$. If $v_0 = v_t$, then it's called **neutrosophic hypercycle**.

Example 1.16. Assume Figure (1).

- (i) : $n_1, E_1, n_3, E_2, n_6, E_3, n_1$ is a neutrosophic hypercycle.
- (ii) : $n_1, E_1, n_n, E_2, n_6, E_3, n_1$ isn't neither neutrosophic hypercycle nor neutrosophic hyperpath.
- (iii) : $n_1 E_1 n_3 E_2 n_6 E_3 n_1$ isn't neither neutrosophic hypercycle nor neutrosophic hyperpath.
- (iv) : n_1, n_3, n_6, n_1 isn't neither neutrosophic hypercycle nor neutrosophic hyperpath.
- (v) : $n_1 E_1, n_3, E_2, n_6, E_3, n_1$ isn't neither neutrosophic hypercycle nor neutrosophic hyperpath.
- (vi) : $n_1, E_1, n_3, E_2, n_6, E_3, n_7$ is a neutrosophic hyperpath.
- (vii) : Neutrosophic hyperdistance amid n_1 and n_4 is two.
- (viii) : Neutrosophic hyperdistance amid n_1 and n_7 is one.
- (ix) : Neutrosophic hyperdistance amid n_1 and n_2 is one.
- (x) : Neutrosophic hyperdistance amid two given neutrosophic vertices is either one or two.

2 New Ideas For Neutrosophic Hypergraphs

Question 2.1. What-if the notion of complete proposes some classes of neutrosophic hypergraphs?

In the setting of neutrosophic hypergraphs, the notion of complete have introduced some classes. Since the vertex could have any number of arbitrary hyperedges. This notion is too close to the notion of regularity. Thus the idea of complete has an obvious structure in that, every hyperedge has n vertices so there's only one hyperedge.

Definition 2.2. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. It's denoted by NHG_n^r and it's (r, n) - **regular** if every hyperedge has exactly r vertices in the way that, all r -subsets of the vertices have a unique hyperedge where $r \leq n$ and $|V| = n$.

Example 2.3. In Figure (5), NHG_4^3 is shown.

Definition 2.4. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$.

- (i) : **Maximum number** is maximum number of hyperedges which are incident to a vertex and it's denoted by $\Delta(NHG)$;
- (ii) : **Minimum number** is minimum number of hyperedges which are incident to a vertex and it's denoted by $\delta(NHG)$;
- (iii) : **Maximum value** is maximum value of vertices and it's denoted by $\Delta_n(NHG)$;
- (iv) : **Minimum value** is minimum value of vertices and it's denoted by $\delta_n(NHG)$.

Example 2.5. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ as Figure (5).

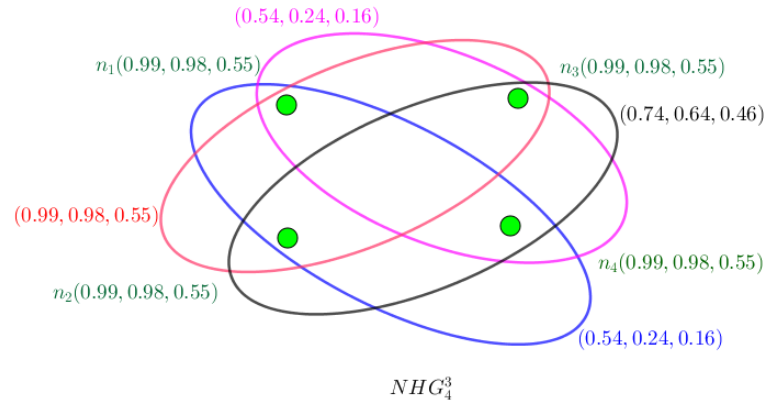


Figure 5. $NHG_4^3 = (V, E, \sigma, \mu)$ is neutrosophic $(3, 4)$ - regular hypergraph.

(i) : $\Delta(NHG) = 3$;

(ii) : $\delta(NHG) = 3$;

(iii) : $\Delta_n(NHG) = (0.99, 0.98, 0.55)$;

(iv) : $\delta_n(NHG) = (0.99, 0.98, 0.55)$.

Proposition 2.6. Assume neutrosophic hypergraph $NHG_n^r = (V, E, \sigma, \mu)$ which is (r, n) - regular. Then $\Delta(NHG) = \delta(NHG)$.

Proof. Consider neutrosophic hypergraph $NHG_n^r = (V, E, \sigma, \mu)$ which is (r, n) - regular. Every hyperedge has same number of vertices. Hyperedges are distinct. It implies the number of hyperedges which are incident to every vertex is the same. \square

Proposition 2.7. Assume neutrosophic hypergraph $NHG_n^r = (V, E, \sigma, \mu)$ which is (r, n) - regular. Then the number of hyperedges equals to n choose r .

Proof. Suppose neutrosophic hypergraph $NHG_n^r = (V, E, \sigma, \mu)$ which is (r, n) - regular. Every hyperedge has r vertices. Thus r -subsets of n form hyperedges. It induces n choose r . \square

Proposition 2.8. Assume neutrosophic hypergraph $NHG_n^r = (V, E, \sigma, \mu)$ which is (r, n) - regular. Then

(i) : Chromatic number is at least r ;

(ii) : Chromatic number is at most Δr ;

(iii) : Neutrosophic chromatic number is at most $\Delta_n r$.

Proof. (i). Suppose $NHG_n^r = (V, E, \sigma, \mu)$. Every hyperedge has r vertices. It implies the set of representatives has at least r members. Hence chromatic number is at least r .

(ii). Suppose $NHG_n^r = (V, E, \sigma, \mu)$. Every hyperedge has r vertices. It implies the set of representatives has at least r members. If all vertices have at least one common hyperedge, then chromatic number is at most Δr . Thus chromatic number is at most Δr .

(iii). Consider $NHG_n^r = (V, E, \sigma, \mu)$. Every hyperedge has r vertices. It implies the set of representatives has at least r members. If all vertices have at least one common hyperedge, then neutrosophic chromatic number is at most $\Delta_n r$. Thus neutrosophic chromatic number is at most $\Delta_n r$. \square

Question 2.9. What-if the notion of complete proposes some classes of neutrosophic hypergraphs with some parts?

In the setting of neutrosophic hypergraphs, when every part has specific attribute inside and outside, the notion of complete is applied to parts to form the idea of completeness.

Definition 2.10. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. It's denoted by $NHG_{n_1, n_2, \dots, n_r}^r$ and it's **complete r -partite** if V can be partitioned into r non-empty parts, V_i , and every hyperedge has only one vertex from each part where n_i is the number of vertices in part V_i .

Example 2.11. In Figure (6), $NHG_{3,3,3}^3 = (V, E, \sigma, \mu)$ is shown.

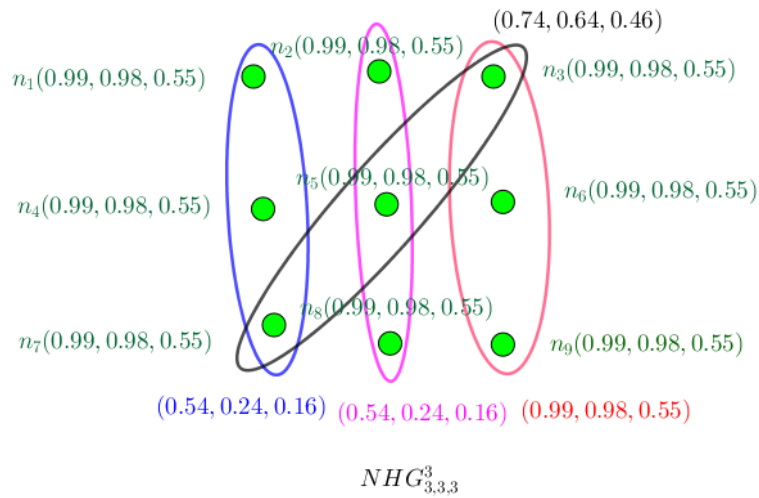


Figure 6. $NHG_{3,3,3}^3 = (V, E, \sigma, \mu)$ is neutrosophic complete 3-partite hypergraph.

Proposition 2.12. For any given r , the number of neutrosophic complete r -partite hypergraph $NHG_{p_1, p_2, \dots, p_r}^r = (V, E, \sigma, \mu)$ is at most

$$p_1 \times p_2 \times \dots \times p_r.$$

Proof. Assume r is given. Consider $NHG_{p_1, p_2, \dots, p_r}^r = (V, E, \sigma, \mu)$ is neutrosophic complete r -partite hypergraph. Any possible hyperedge has to choose exactly one vertex from every part. First part has p_1 vertices. Thus there are p_1 choices. Second part has p_2 vertices and et cetera. Thus for any given r , the number of neutrosophic complete r -partite hypergraph $NHG_{p_1, p_2, \dots, p_r}^r = (V, E, \sigma, \mu)$ is at most

$$p_1 \times p_2 \times \dots \times p_r.$$

□

Proposition 2.13. Assume neutrosophic complete r -partite hypergraph $NHG_{n_1, n_2, \dots, n_r}^r = (V, E, \sigma, \mu)$. Then

(i) : Chromatic number is at least r ;

(ii) : Neutrosophic chromatic number is at least

$$\min_{X \subseteq V \text{ and } X \text{ is } r\text{-subset}} \sum_{x \in X} \sigma(x).$$

Proof. (i). Suppose neutrosophic complete r -partite hypergraph $NHG_{n_1, n_2, \dots, n_r}^r$. Every hyperedge has r vertices. It implies the set of representatives has r members. Hence chromatic number is least r .

(ii). Consider neutrosophic complete r -partite hypergraph $NHG_{n_1, n_2, \dots, n_r}^r$. Every hyperedge has r vertices. It implies the set of representatives has r members. If all vertices have at least one common hyperedge, then neutrosophic chromatic number is at least $\min_{X \subseteq V \text{ and } X \text{ is } r\text{-subset}} \sum_{x \in X} \sigma(x)$. \square

Definition 2.14. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$.

(i) : A **neutrosophic number** of vertices x_1, x_2, \dots, x_n is

$$\sum_{i=1}^n \sigma(x_i).$$

(ii) : A **neutrosophic number** of hyperedges e_1, e_2, \dots, e_n is

$$\sum_{i=1}^n \mu(e_i).$$

Example 2.15. I get some clarifications about new definitions.

(i) : In Figure (5), NHG_4^3 is shown.

(a) : A neutrosophic number of vertices n_1, n_2, n_3 is

$$\sum_{i=1}^3 \sigma(n_i) = (2.97, 2.94, 1.65).$$

(b) : A neutrosophic number of hyperedges e_1, e_2, e_3 is

$$\sum_{i=1}^3 \sigma(e_i) = (1.82, 1.12, 0.78).$$

where $e_1 = (0.54, 0.24, 0.16), e_2 = (0.74, 0.64, 0.46), e_3 = (0.54, 0.24, 0.16)$.

(ii) : In Figure (6), $NHG_{3,3,3}^3 = (V, E, \sigma, \mu)$ is shown.

(a) : A neutrosophic number of vertices n_1, n_2, n_3 is

$$\sum_{i=1}^3 \sigma(n_i) = (2.97, 2.94, 1.65).$$

(b) : A neutrosophic number of hyperedges e_1, e_2, e_3 is

$$\sum_{i=1}^3 \sigma(e_i) = (1.82, 1.12, 0.78).$$

where $e_1 = (0.54, 0.24, 0.16), e_2 = (0.74, 0.64, 0.46), e_3 = (0.54, 0.24, 0.16)$.

Proposition 2.16. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. A neutrosophic number of vertices is at least δ_n and at most \mathcal{O}_n .

Proof. Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Let v be a given vertex. Then $\sigma(v) \geq \min_{v \in V} \sigma(v)$. Thus $\sigma(v) \geq \delta_n$. So a neutrosophic number of vertices is at least δ_n . $\sigma(v) \leq \sum_{v \in V} \sigma(v)$. Thus $\sigma(v) \leq \mathcal{O}_n$. So a neutrosophic number of vertices is at most \mathcal{O}_n . Hence a neutrosophic number of vertices is at least δ_n and at most \mathcal{O}_n . \square

Proposition 2.17. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. A neutrosophic number of hyperedges is at least δ_n^e and at most \mathcal{S}_n where $\delta_n^e = \min_{e \in E} \mu(e)$.

Proof. Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Let e be a given hyperedge. Then $\mu(e) \geq \min_{e \in E} \mu(e)$. Thus $\mu(v) \geq \delta_n^e$. So a neutrosophic number of hyperedges is at least δ_n^e . $\mu(e) \leq \Sigma_{e \in E} \mu(e)$. Thus $\mu(e) \leq \mathcal{S}_n$. So a neutrosophic number of hyperedges is at most \mathcal{S}_n . Hence a neutrosophic number of hyperedges is at least δ_n^e and at most \mathcal{S}_n . \square

Definition 2.18. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$.

- (i) : A **degree** of vertex x is the number of hyperedges which are incident to x .
- (ii) : A **neutrosophic degree** of vertex x is the neutrosophic number of hyperedges which are incident to x .
- (iii) : A **degree** of hyperedge e is the number of vertices which e is incident to them.
- (iv) : A **neutrosophic degree** of hyperedge e is the neutrosophic number of vertices which e is incident to them.
- (v) : A **co-degree** of vertices x_1, x_2, \dots, x_n is the number of hyperedges which are incident to x_1, x_2, \dots, x_n .
- (vi) : A **neutrosophic co-degree** of vertices x_1, x_2, \dots, x_n is the neutrosophic number of hyperedges which are incident to x_1, x_2, \dots, x_n .
- (vii) : A **co-degree** of hyperedges e_1, e_2, \dots, e_n is the number of vertices which e_1, e_2, \dots, e_n are incident to them.
- (viii) : A **neutrosophic co-degree** of hyperedges e_1, e_2, \dots, e_n is the neutrosophic number of vertices which e_1, e_2, \dots, e_n are incident to them.

Example 2.19. I get some clarifications about new definitions.

- (i) : In Figure (5), NHG_4^3 is shown.
 - (a) : A degree of any vertex is 3.
 - (b) : A neutrosophic degree of vertex n_1 is (2.07, 1.46, 0.87).
 - (c) : A degree of hyperedge e where $\mu(e) = (0.99, 0.98, 0.55)$ is 3.
 - (d) : A neutrosophic degree of hyperedge e where $\mu(e) = (0.99, 0.98, 0.55)$ is (2.97, 2.94, 1.65).
 - (e) : A co-degree of vertices n_1, n_3 is 2.
 - (f) : A neutrosophic co-degree of vertices n_1, n_3 is (1.53, 1.22, 0.71).
 - (g) : A co-degree of hyperedges e_1, e_2 where $\mu(e_1) = (0.99, 0.98, 0.55)$ and $\mu(e_2) = (0.54, 0.24, 0.16)$ is 2.
 - (h) : A neutrosophic co-degree of hyperedges e_1, e_2 where $\mu(e_1) = (0.99, 0.98, 0.55)$ and $\mu(e_2) = (0.54, 0.24, 0.16)$ is (1.98, 1.96, 1.1).
- (ii) : In Figure (6), $NHG_{3,3,3}^3 = (V, E, \sigma, \mu)$ is shown.
 - (a) : A degree of any vertex $n_1, n_2, n_4, n_6, n_8, n_9$ is 1 and degree of any vertex n_3, n_5, n_7 is 2.
 - (b) : A neutrosophic degree of vertex $n_1, n_2, n_4, n_6, n_8, n_9$ is (0.99, 0.98, 0.55) and degree of any vertex n_3, n_5, n_7 is (1.98, 1.96, 1.1).
 - (c) : A degree of any hyperedge is 3.
 - (d) : A neutrosophic degree of hyperedge is (2.97, 2.94, 1.65).

(e) : A co-degree of vertices n_1, n_4 is 1. 252

(f) : A neutrosophic co-degree of vertices n_1, n_4 is $(0.54, 0.24, 0.16)$. 253

(g) : A co-degree of hyperedges e_1, e_2 where $\mu(e_1) = (0.99, 0.98, 0.55)$ and $\mu(e_2) = (0.54, 0.24, 0.16)$ is 1. 254
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(h) : A neutrosophic co-degree of hyperedges e_1, e_2 where $\mu(e_1) = (0.99, 0.98, 0.55)$ and $\mu(e_2) = (0.54, 0.24, 0.16)$ is $(0.99, 0.98, 0.55)$. 256
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Proposition 2.20. Assume neutrosophic complete r -partite hypergraph $NHG_{p_1, p_2, \dots, p_r}^r = (V, E, \sigma, \mu)$. 258
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(i) : A degree of vertex x is at most

$$p_2 \times \dots \times p_r.$$

(ii) : A degree of hyperedge e is r . 260

(iii) : A co-degree of vertices x_1, x_2, \dots, x_t is at most

$$p_{t+1} \times \dots \times p_r.$$

(iv) : A co-degree of hyperedges e_1, e_2, \dots, e_t is $r - t$. 261

Proof. (i). Suppose neutrosophic complete r -partite hypergraph $NHG_{p_1, p_2, \dots, p_r}^r = (V, E, \sigma, \mu)$. Vertex x belongs to part first part. x is chosen so for second part, there are p_2 choices and et cetera. By it's neutrosophic complete r -partite hypergraph $NHG_{p_1, p_2, \dots, p_r}^r = (V, E, \sigma, \mu)$, possible choice from every part is exactly one vertex. It induces for second part, one vertex has to be chosen and et cetera. Therefore the number of neutrosophic complete r -partite hypergraph $NHG_{p_1, p_2, \dots, p_r}^r = (V, E, \sigma, \mu)$, when x is chosen, introduces biggest possible number of degree of x which is $p_2 \times \dots \times p_r$. Hence a degree of vertex x is at most

$$p_2 \times \dots \times p_r.$$

(ii). Consider neutrosophic complete r -partite hypergraph $NHG_{p_1, p_2, \dots, p_r}^r = (V, E, \sigma, \mu)$. Vertex x belongs to part first part. x is chosen so for second part, there is one choice and et cetera. By it's neutrosophic complete r -partite hypergraph $NHG_{p_1, p_2, \dots, p_r}^r = (V, E, \sigma, \mu)$, possible choice from every part is exactly one vertex. It induces for second part, one vertex has to be chosen and et cetera. Therefore neutrosophic complete r -partite hypergraph $NHG_{p_1, p_2, \dots, p_r}^r = (V, E, \sigma, \mu)$ introduces exact number of degree of e which is r . Hence a degree of hyperedge e is

$$r.$$

(iii). Suppose neutrosophic complete r -partite hypergraph $NHG_{p_1, p_2, \dots, p_r}^r = (V, E, \sigma, \mu)$. Vertices x_1, x_2, \dots, x_t belong to part first part, second part, ..., and part t . x_1, x_2, \dots, x_t are chosen so for part $t + 1$, there are p_{t+1} choices and et cetera. By it's neutrosophic complete r -partite hypergraph $NHG_{p_1, p_2, \dots, p_r}^r = (V, E, \sigma, \mu)$, possible choice from every part is exactly one vertex. It induces for part $t + 1$, one vertex has to be chosen and et cetera. Therefore the number of neutrosophic complete r -partite hypergraph $NHG_{p_1, p_2, \dots, p_r}^r = (V, E, \sigma, \mu)$, when x_1, x_2, \dots, x_t are chosen, introduces biggest possible number of co-degree of x_1, x_2, \dots, x_t which is $p_{t+1} \times \dots \times p_r$. Hence a co-degree of vertices x_1, x_2, \dots, x_t is at most

$$p_{t+1} \times \dots \times p_r.$$

(iv). Consider neutrosophic complete r -partite hypergraph $NHG_{p_1, p_2, \dots, p_r}^r = (V, E, \sigma, \mu)$. Vertex x belongs to part first part. x is chosen so for second part, there is one choice and et cetera. By it's neutrosophic complete r -partite hypergraph $NHG_{p_1, p_2, \dots, p_r}^r = (V, E, \sigma, \mu)$, possible choice from every part is exactly one vertex. It induces for second part, one vertex has to be chosen and et cetera. Therefore neutrosophic complete r -partite hypergraph $NHG_{p_1, p_2, \dots, p_r}^r = (V, E, \sigma, \mu)$ introduces exact number of co-degree of e_1, e_2, \dots, e_t which is $r - t$. Hence a co-degree of hyperedges e_1, e_2, \dots, e_t is

$$r - t.$$

□

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Proposition 2.21. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ where E is power set of V . Then the number of hyperedges is

$$2^n.$$

Proof. Consider neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ where E is power set of V . The cardinality of E is 2^n . The number of hyperedges is

$$2^n.$$

□

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Proposition 2.22. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ where E is power set of V . Then

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(i) : A degree of vertex x is

$$2^{n-1}.$$

(ii) : A degree of hyperedge e is at most

$$\mathcal{O}$$

and at least

$$0.$$

(iii) : A co-degree of vertices x_1, x_2, \dots, x_t is at most

$$2^{n-t}.$$

(iv) : A co-degree of hyperedges e_1, e_2, \dots, e_t is at most

$$\mathcal{O} - t$$

and at least

$$0.$$

Proof. (i). Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ where E is power set of V . Vertex x is chosen. Thus all hyperedges have to have x . It induces E' is power set of $V \setminus \{x\}$. The cardinality of E' is 2^{n-1} . So the number of hyperedges which are incident to x , is 2^{n-1} . It implies a degree of vertex x is

$$2^{n-1}.$$

(ii). Consider neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ where E is power set of V . Hyperedge e is chosen. Thus a hyperedge has either all vertices or no vertex. It

induces for hyperedge e , the number of vertices is either \mathcal{O} or 0. Then a degree of hyperedge e is at most

$$\mathcal{O}$$

and at least

$$0.$$

(iii). Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ where E is power set of V . Vertices x_1, x_2, \dots, x_t are chosen. Thus all hyperedges have to have x_1, x_2, \dots, x_t . It induces E' is power set of $V \setminus \{x_1, x_2, \dots, x_t\}$. The cardinality of E' is 2^{n-t} . So the number of hyperedges which are incident to x_1, x_2, \dots, x_t , is 2^{n-t} . It implies a co-degree of vertices x_1, x_2, \dots, x_t is

$$2^{n-t}.$$

(iv). Consider neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ where E is power set of V . Hyperedges e_1, e_2, \dots, e_t are chosen. Thus hyperedges e_1, e_2, \dots, e_t don't have all vertices. Since one edge is incident to all vertices and there's no second edge to be incident to all vertices. It implies hyperedges e_1, e_2, \dots, e_t have all vertices excluding only t vertices or no vertex. It induces for hyperedges e_1, e_2, \dots, e_t , the number of vertices is either $\mathcal{O} - t$ or 0. Hence a co-degree of hyperedges e_1, e_2, \dots, e_t is at most

$$\mathcal{O} - t$$

and at least

$$0.$$

□

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Proposition 2.23. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ where E is power set of V . Then

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(i) : Chromatic number is

$$\mathcal{O};$$

(ii) : Neutrosophic chromatic number is

$$\mathcal{O}_n.$$

Proof. (i). Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ where E is power set of V . Every hyperedge has either of $0, 1, 2, \dots, \mathcal{O}$ vertices but for any of two vertices, there's at least one hyperedge which is incident to them. Furthermore, all vertices have at least one common hyperedge which is V . Since $V \in E$ and V is also a hyperedge. It implies the set of representatives has \mathcal{O} members. Hence chromatic number is

$$\mathcal{O}.$$

(ii). Consider neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ where E is power set of V . Every hyperedge has either of $0, 1, 2, \dots, \mathcal{O}$ vertices but for any of two vertices, there's at least one hyperedge which is incident to them. Furthermore, all vertices have at least one common hyperedge which is V . Since $V \in E$ and V is also a hyperedge. It implies the set of representatives has \mathcal{O} members. Hence neutrosophic chromatic number is

$$\mathcal{O}_n.$$

□

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3 Applications in Time Table and Scheduling

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has important to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive section. Beyond that, sometimes sections are not the same.

Step 3. (Model) As Figure (7), the situation is designed as a model. The model uses data to assign every section and to assign to relation amid section, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relation amid them. Table (1), clarifies about the assigned numbers to these situation.

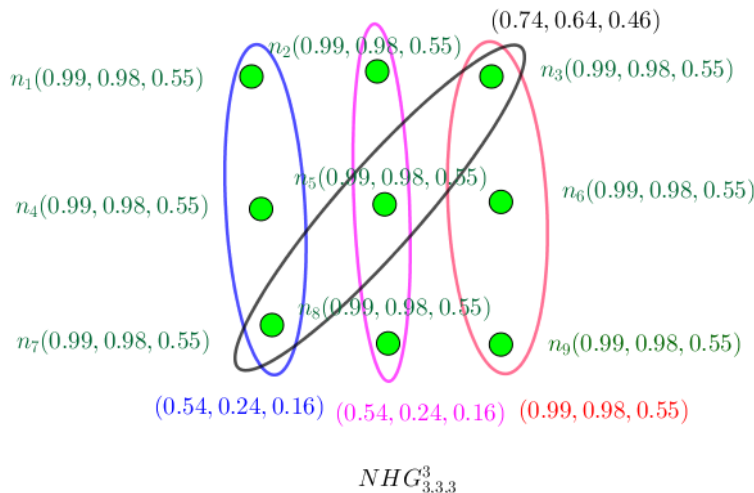


Figure 7. Vertices are suspicions about choosing them.

Table 1. Scheduling concerns its Subjects and its Connections as a Neutrosophic Hypergraph in a Model.

Sections of NHG	n_1	$n_2 \dots$	n_9
Values	$(0.99, 0.98, 0.55)$	$(0.74, 0.64, 0.46) \dots$	$(0.99, 0.98, 0.55)$
Connections of NHG	E_1, E_2	E_3	E_4
Values	$(0.54, 0.24, 0.16)$	$(0.99, 0.98, 0.55)$	$(0.74, 0.64, 0.46)$

Step 4. (Solution) As Figure (7) shows, $NHG^3_{3,3,3} = (V, E, \sigma, \mu)$ is neutrosophic complete 3-partite hypergraph as model, proposes to use different types of degree of vertices, degree of hyperedges, co-degree of vertices, co-degree of hyperedges, neutrosophic number of vertices, neutrosophic number of hyperedges and et cetera.

(i) : The notions of neutrosophic number are applied on vertices and hyperedges.

(a) : A neutrosophic number of vertices n_1, n_2, n_3 is

$$\Sigma_{i=1}^3 \sigma(n_i) = (2.97, 2.94, 1.65).$$

(b) : A neutrosophic number of hyperedges e_1, e_2, e_3 is

$$\Sigma_{i=1}^3 \sigma(e_i) = (1.82, 1.12, 0.78).$$

where $e_1 = (0.54, 0.24, 0.16), e_2 = (0.74, 0.64, 0.46), e_3 = (0.54, 0.24, 0.16)$.

(ii) : The notions of degree, co-degree, neutrosophic degree and neutrosophic co-degree are applied on vertices and hyperedges.

(a) : A degree of any vertex $n_1, n_2, n_4, n_6, n_8, n_9$ is 1 and degree of any vertex n_3, n_5, n_7 is 2.

(b) : A neutrosophic degree of vertex $n_1, n_2, n_4, n_6, n_8, n_9$ is $(0.99, 0.98, 0.55)$ and degree of any vertex n_3, n_5, n_7 is $(1.98, 1.96, 1.1)$.

(c) : A degree of any hyperedge is 3.

(d) : A neutrosophic degree of hyperedge is $(2.97, 2.94, 1.65)$.

(e) : A co-degree of vertices n_1, n_4 is 1.

(f) : A neutrosophic co-degree of vertices n_1, n_4 is $(0.54, 0.24, 0.16)$.

(g) : A co-degree of hyperedges e_1, e_2 where $\mu(e_1) = (0.99, 0.98, 0.55)$ and $\mu(e_2) = (0.54, 0.24, 0.16)$ is 1.

(h) : A neutrosophic co-degree of hyperedges e_1, e_2 where $\mu(e_1) = (0.99, 0.98, 0.55)$ and $\mu(e_2) = (0.54, 0.24, 0.16)$ is $(0.99, 0.98, 0.55)$.

4 Open Problems

The different types of degree of vertices, degree of hyperedges, co-degree of vertices, co-degree of hyperedges, neutrosophic number of vertices, neutrosophic number of hyperedges are introduced on neutrosophic hypergraphs. Thus,

Question 4.1. *Is it possible to use other types neutrosophic hyperedges to define different types of degree and co-degree in neutrosophic hypergraphs?*

Question 4.2. *Are existed some connections amid degree and co-degree inside this concept and external connections with other types of neutrosophic degree and neutrosophic co-degree in neutrosophic hypergraphs?*

Question 4.3. *Is it possible to construct some classes on neutrosophic hypergraphs which have “nice” behavior?*

Question 4.4. *Which applications do make an independent study to apply these types degree, co-degree, neutrosophic degree and neutrosophic co-degree in neutrosophic hypergraphs?*

Problem 4.5. *Which parameters are related to this parameter?*

Problem 4.6. *Which approaches do work to construct applications to create independent study?*

Problem 4.7. *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

5 Conclusion and Closing Remarks

This study introduces different types of degree of vertices, degree of hyperedges, co-degree of vertices, co-degree of hyperedges, neutrosophic degree of vertices, neutrosophic degree of hyperedges, neutrosophic co-degree of vertices, neutrosophic co-degree of hyperedges, neutrosophic number of vertices, neutrosophic number of hyperedges in neutrosophic hypergraphs. The connections of neutrosophic vertices which are clarified by general hyperedges differ them from each other and and put them in different categories to represent one representative for each color. Further studies could be about changes in the settings to compare this notion amid different settings of neutrosophic hypergraphs theory. One way is finding some relations amid these definitions of notions to make sensible definitions. In Table (2), some limitations and some advantages of this study are pointed out.

Table 2. A Brief Overview about Advantages and Limitations of this study

Advantages	Limitations
1. Defining degree	1. General Results
2. Defining co-degree	
3. Defining neutrosophic degree	2. Connections With Parameters
4. Applying colortring	
5. Defining neutrosophic co-degree	3. Connections of Results

References

1. M. Akram, and G. Shahzadi, *Operations on Single-Valued Neutrosophic Graphs*, Journal of uncertain systems 11 (1) (2017) 1-26.
2. K. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets Syst. 20 (1986) 87-96.
3. S. Broumi, M. Talea, A. Bakali and F. Smarandache, *Single-valued neutrosophic graphs*, Journal of New Theory 10 (2016) 86-101.
4. N. Shah, and A. Hussain, *Neutrosophic soft graphs*, Neutrosophic Set and Systems 11 (2016) 31-44.
5. Henry Garrett, *Big Sets Of Vertices*, Preprints 2021, 2021060189 (doi: 10.20944/preprints202106.0189.v1).
6. Henry Garrett, *Locating And Location Number*, Preprints 2021, 2021060206 (doi: 10.20944/preprints202106.0206.v1).
7. Henry Garrett, *Metric Dimensions Of Graphs*, Preprints 2021, 2021060392 (doi: 10.20944/preprints202106.0392.v1).
8. Henry Garrett, *New Graph Of Graph*, Preprints 2021, 2021060323 (doi: 10.20944/preprints202106.0323.v1).
9. Henry Garrett, *Numbers Based On Edges*, Preprints 2021, 2021060315 (doi: 10.20944/preprints202106.0315.v1).

10. Henry Garrett, *Matroid And Its Outlines*, Preprints 2021, 2021060146 (doi: 10.20944/preprints202106.0146.v1).

357
358

11. Henry Garrett, *Matroid And Its Relations*, Preprints 2021, 2021060080 (doi: 10.20944/preprints202106.0080.v1).

359
360

12. A. Shannon and K.T. Atanassov, *A first step to a theory of the intuitionistic fuzzy graphs*, Proceeding of FUBEST (Lakov, D., Ed.) Sofia (1994) 59-61.

361
362

13. F. Smarandache, *A Unifying field in logics neutrosophy: Neutrosophic probability, set and logic, Rehoboth: American Research Press (1998).*

363
364

14. H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, *Single-valued neutrosophic sets*, Multispace and Multistructure 4 (2010) 410-413.

365
366

15. L. A. Zadeh, *Fuzzy sets*, Information and Control 8 (1965) 338-353.

367